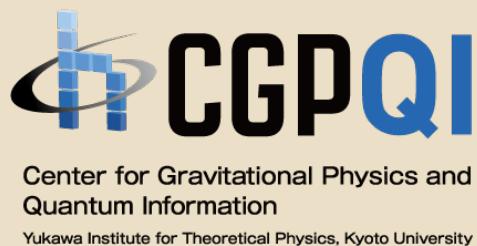


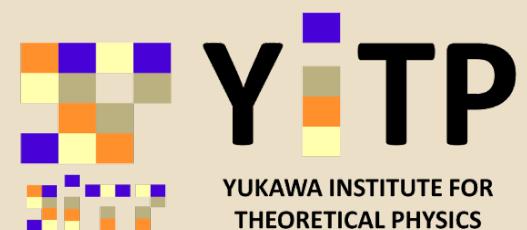


June 28,  
2022

# Hybrid inflation and waterfall field in string theory from D7-branes



Osmin Lacombe  
(YITP, Kyoto University)



Based on 2007.10362, 2109.03243

In collaboration with: Ignatios Antoniadis (LPTHE, Paris)  
George K. Leontaris (University of Ioannina)

# Introduction

- Inflation paradigm and dark energy content are well grounded frameworks explaining observations
- Scalar field inflation UV origin from string theory ?  
*(lot of models, Swampland program constraints)*
- Moduli fields can be good candidates for scalar field inflation:  
natural candidate is internal volume  $\mathcal{V}$
- Motivated by a general framework of **perturbative** moduli stabilization, we construct **hybrid inflation** model
- **hybrid inflation** mechanism can be applied more generally

# Outline of the talk

1. Introduction
2. Inflationary scenario from moduli stabilization
3. Hybrid inflation and waterfall fields
4. A concrete (toy) model
5. Summary & Outlook

# Type IIB moduli stabilization (1)

- Internal Calabi-Yau (CY) geometry characterized by VEVs of metric **moduli**: **complex structure (cs) moduli**  $U_a$ , **Kähler moduli**  $T_i$  (*volume moduli, include  $\mathcal{V}$* )
- massless moduli: unfixed VEV, have to be stabilized (*scalar potential*)
- Light fields (*initially massless*)  $\rightarrow$  SUGRA EFT described by superpotential  $\mathcal{W}$  and Kähler potential  $\mathcal{K}$

Gukov, Vafa, Witten '99

$$\kappa^2 \mathcal{K} = -2 \ln(\mathcal{V}) - \ln(S + \bar{S}) - \ln\left(i \int_{X_6} \Omega \wedge \bar{\Omega}\right),$$


  
 volume dependence      axion-dilaton dependence       $U_a$  cs moduli dependence

$$\mathcal{W} = \int_{X_6} G_3 \wedge \Omega$$


  
 $U_a$  cs moduli dependence

- Massless IIB spectrum  $\exists$  p-forms, e.g.  $C_2, B_2$  with  $F_3 = dC_2, H_3 = dB_2$   
 $\rightarrow$  background of (*quantized*) ISD 3-form fluxes:  $G_3 \equiv F_3 - iSH_3, \star G_3 = iG_3$

# Type IIB moduli stabilization (2)

- SUGRA scalar potential  $V_F = e^{\kappa^2 \mathcal{K}} \left( \sum_{a,\bar{b}} \mathcal{K}^{a\bar{b}} D_a \mathcal{W} \bar{D}_{\bar{b}} \bar{\mathcal{W}} \right)$   $D_i = \partial_i + \mathcal{K}_i$   
*no-scale structure of  $\mathcal{K}$*   
*(Kähler moduli dependent part)* *only complex str.*

- minimization:  $D_a \mathcal{W} = 0$ , leaves constant  $\mathcal{W}_0$  *cs moduli stabilized*
- Kähler moduli stabilization:

- 1) Through **non-perturbative** superpotential for Kähler moduli  
 (KKLT, LVS) *Kachru, Kallosh, Linde, Trivedi '03* *Balasubramanian et al. '05*  
*Colon, Quevedo, Suruliz '05*  
 total superpotential  $\mathcal{W} = \mathcal{W}_0 + \mathcal{W}_{\text{NP}}$  *breaks no-scale structure*  
 $V_F$  minimized for  $D_\tau \mathcal{W} = 0$  *stabilizes  $\tau$  (AdS vacuum, can uplift to dS)*

- 2) Through **perturbative** contributions only.  $\mathcal{W}$  *cannot be corrected*,  $\mathcal{K}$  can  
 $\mathcal{R}^4 \in 10d$  SUGRA action  $\rightarrow$  *compactification*  $\rightarrow$  4d localized EH  $\mathcal{R}_{(4)}$   
 $\rightarrow$  *Planck mass* corrections, namely  $\mathcal{K}$  corrections, in  $\alpha'$  and  $g_s$ :  
 leading ones  $\xi$ , logarithmic  $\gamma$  *localized sources with 2d transverse volume*

*Becker, Becker, Haack, Louis '02*

*Conlon, Pedro '10 Grimm, Savelli, Weissenbacher '13*

*3*

*Antoniadis, Chen, Leontaris '18, '19*

*Burgess, Quevedo '22 Gao, Hebecker, Schreyer, Venken '22*

# New moduli stabilization scenario

- Consider 3 "orthogonal" magnetized D7, worldvolumes (4-cycle) moduli  $\tau_i$

$$\kappa^2 \mathcal{K} \sim -2 \ln \left( \sqrt{\tau_1 \tau_2 \tau_3} + \xi + \sum_k \gamma_k \ln \tau_k \right) = -2 \ln (\mathcal{V} + \xi + \gamma \ln \mathcal{V})$$

$\rightarrow$  include quantum corrections , for simplicity, identical  $\gamma_k = 2\gamma$

- F-part scalar potential:  $V_F = \frac{3\mathcal{W}_0^2}{2\kappa^4 \mathcal{V}^3} (2\gamma(\ln \mathcal{V} - 4) + \xi) + \dots$

- Include D-terms from U(1) fluxes:  $V_D = \frac{d_1}{\kappa^4 \tau_1^3} + \frac{d_2}{\kappa^4 \tau_2^3} + \frac{d_3}{\kappa^4 \tau_3^3} + \dots \equiv d$

minimize  $V_D$  = stabilize ratios:  $\frac{\tau_1}{\tau_2} = \left( \frac{d_1}{d_2} \right)^{1/3}$ ,  $\frac{\tau_1 \tau_2^2}{\tau_3} = \left( \frac{d_1 d_2}{d_3^2} \right)^{1/3} \rightarrow V_D = \frac{3(d_1 d_2 d_3)^{1/3}}{\kappa^4 \mathcal{V}^2}$

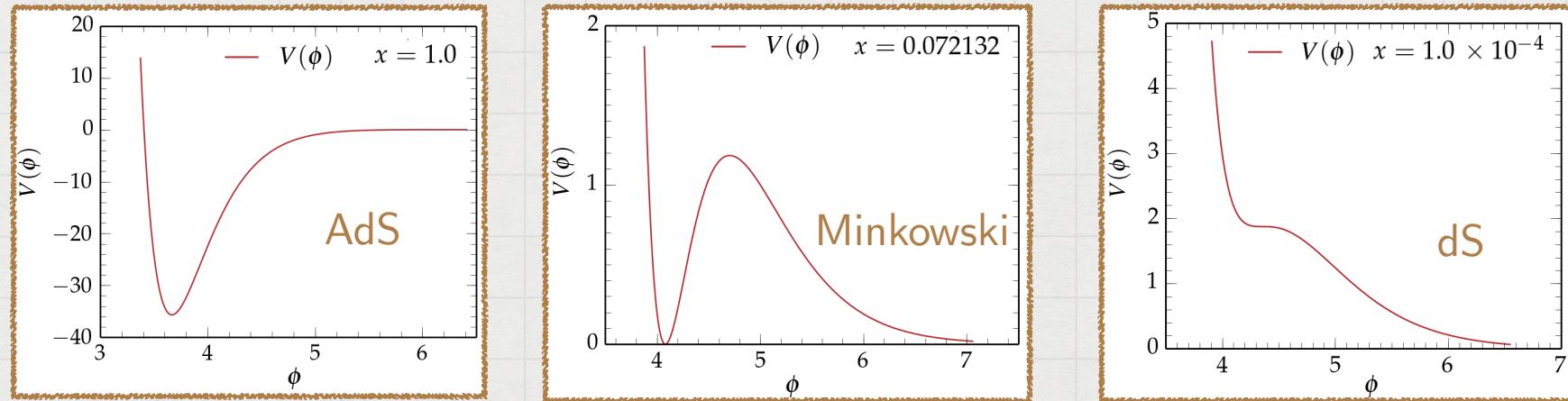
- Natural inflaton candidate  $\mathcal{V}$  canonically normalized:  $\phi = \sqrt{3/2} \ln(\mathcal{V})$ .  
*Check: transverse moduli are heavier than inflaton*

- inflaton scalar potential**  $V(\phi) \simeq -\frac{C}{\kappa^4} e^{-3\sqrt{\frac{3}{2}}\phi} \left( \sqrt{\frac{3}{2}}\phi - 4 + q + \frac{3}{2}\sigma e^{\sqrt{\frac{3}{2}}\phi} \right)$

$$q \equiv \frac{\xi}{2\gamma}, \quad C \equiv -3\mathcal{W}_0^2 \gamma > 0 \quad \sigma \equiv \frac{2(d_1 d_2 d_3)^{1/3}}{9\mathcal{W}_0^2 \gamma} \equiv -\exp\left(q - \frac{16}{3} - x\right)$$

# Inflation in our IIB moduli stab. model

- Shape of the potential only depends on  $x$  parameter ( $\leftrightarrow D\text{-terms}$ )  
 $q$  scales towards large volumes:  $\mathcal{V}_- = \exp(\sqrt{3/2})\phi_- = e^{-q} \exp\left(\frac{13}{3} - W_0(-e^{-x-1})\right)$



- Numerical resolution of:  $H'(\phi) = \mp \frac{1}{\sqrt{2}} \sqrt{3H^2(\phi) - \kappa^2 V(\phi)}$  for  $\neq$  values of  $x$ , compute  $\eta(\phi), \epsilon(\phi), N(\phi)$
- Observational constraints give  $x = 3.3 \times 10^{-4}$ , *horizon exit near inflection point*
- Prediction:  $r = 16\epsilon_* = 4 \times 10^{-4}$ , scale:  $H_* \simeq \kappa \sqrt{V_*/3} \simeq 5.28 \times 10^9$  TeV
- Value of the minimum cannot be tuned ( $x$  fixed by inflationary phase)
- Shape of the minimum (very shallow). How does inflation end ?

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# Hybrid inflation and waterfall fields

- Hybrid inflation: inflaton  $\phi$  and interacting field  $S$  (**waterfall field**)

$$\mathcal{L}(\phi, S) = (\partial_\mu \phi)^2 + (\partial_\mu S)^2 - V(\phi) - V_S(\phi, S)$$

with  $V_S(\phi, S) = 1/2 (-M^2 + f(\phi)) S^2 + \lambda/4 S^4 + \dots$  effective mass  $m_S^2 = -M^2 + f(\phi)$

- $S$  becomes tachyonic  $m_S^2 < 0$  when  $f(\phi) < M^2$  (for  $\phi < \phi_c$  for monotonic  $f$ )

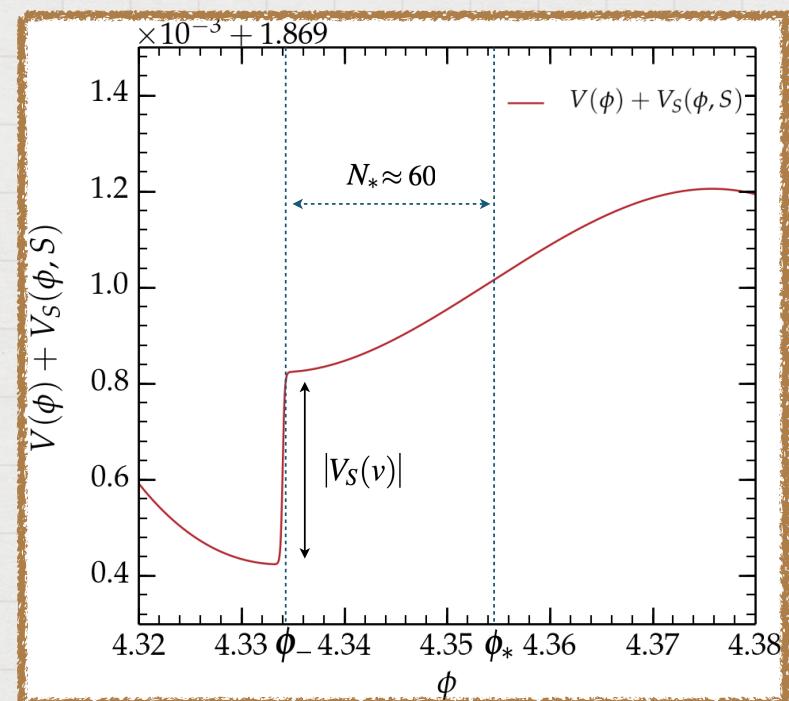
phase transition to  $\langle S \rangle = \pm \frac{|m_S|}{\sqrt{\lambda}} \equiv \pm v$

$$V_S(\phi, v) = -\frac{m_S^4}{4\lambda}(\phi) < 0$$

→ waterfall direction

*steep*: ends inflation,

*large*: lowers the global minimum



# Waterfall field in string theory (1)

- Which field has mass depending on the inflaton  $\phi$ ? (here volume  $\mathcal{V}$ )  
*i.e. related to internal cycles volumes*      e.g. 

$$X_6 = T^2 \times T^2 \times T^2$$

$$\mathcal{V} = \mathcal{A}_{45} \times \mathcal{A}_{67} \times A_{89}$$

- In string theory: **open strings (matter field)** mass contributions from

w.s. electrical coupling  $S_e = \frac{1}{4\pi\alpha'} \left[ q \int d\tau A_\mu \partial_\tau X^\mu \right]_{\sigma=\pi}^{\sigma=0} = \frac{1}{4\pi\alpha'} \int d\tau (A_\mu^a - A_\mu^b) \partial_\tau X^\mu$

→ Wilson lines (or brane separation):  $m^2 > 0$   $\leftrightarrow$  int. cycles volumes

*background  $A_n \rightarrow T$ -dual to brane separation*  $X_a^n = \theta_n^a R$ , with  $A_n^a = \theta_n^a / 2\pi R'$

$$M_{ab}^2 = \sum_{j=p+1}^{D-1} \left( \frac{X_a^j - X_b^j}{2\pi\alpha'} \right)^2 + \text{oscillator contr.}$$

→ Internal magnetic fluxes:  $m^2 < 0$  (*for spin -*)  $\leftrightarrow$  int. cycles volumes

*magnetic field (2-torus):*  $A_n^a = \frac{1}{2} H_{nm}^a X^m$  w.s. term induces modified b.c.

$$(\partial_\sigma X^m + 2\pi\alpha' H_{mn} X^n) \Big|_{\sigma=0}^{\sigma=\pi} = 0 \quad \rightarrow T\text{-dual to brane at angles } \theta^b, \text{ same b.c.}$$

# SUSY breaking with magnetic fluxes

- Modified b.c.  $\implies$  modified oscillator solutions, oscillator shift:

e.g.  $T^2$  on (45)  $\zeta_a = \frac{1}{\pi} \text{Arctan}(2\pi\alpha' q_a H_{(45)}^a) = \frac{1}{\pi} \text{Arctan} \left( \frac{q_a \alpha' k^{(45)}}{\mathcal{A}_{45}} \right) \approx \frac{q_a \alpha' k^{(45)}}{\pi \mathcal{A}_{45}}$

internal area dependence!

- Flux quantized:  $m \int_{T^2} H = 2\pi n \rightarrow 2\pi H_{(45)}^a \mathcal{A}_{45} = k_a^{(45)} \quad k_a^{(45)} = \frac{n_a^{(45)}}{m_a^{(45)}} \in \mathbb{Q},$

- Extract spectrum: partition function or field theory

*Bachas '95  
Angelantonj et al. '00*

$$\text{mass of charged state} \quad 2\alpha' M^2 = \underbrace{(2n_{45} + 1)}_{\text{Landau levels}} \left| \zeta_L^{(i)} + \zeta_R^{(i)} \right| + \underbrace{2\Sigma_{45}}_{\text{internal helicity}} \left( \zeta_L^{(i)} + \zeta_R^{(i)} \right)$$

$\rightarrow$  Can lead to **tachyons** for  $n_{45} = 0$  and  $\Sigma_{45} = -1 \rightarrow \alpha' M^2 < 0$

$\rightarrow$  masses depends on spin: SUSY is broken

# Waterfall field in string theory (2)

- Open strings between two D-branes

*separated in (67)  $\Delta x_{ab}$ , magnetized on (45)  $k_{ab}^{(45)}$*

*lowest-lying state:  $a'm^2 = -\frac{|k_{ab}^{(45)}|}{\pi \mathcal{A}_{45}} + \frac{\Delta x_{ab} \mathcal{A}_{67}}{\alpha'}$   $\approx -\frac{|k_{ab}^{(45)}|}{\pi r_{45} \mathcal{V}^{1/3}} + \Delta x_{ab} r_{67} \mathcal{V}^{1/3}$*

- If  $\mathcal{A}_{45} = r_{45} \mathcal{V}^{1/3}$ ,  $\mathcal{A}_{67} = r_{67} \mathcal{V}^{1/3}$  with fixed  $r_{45}, r_{67}$   
*(as in our moduli stab. scenario, with e.g.  $\tau_3 \sim \mathcal{A}_{45} \mathcal{A}_{67}$ )*
- Mass depending on internal volume  $\mathcal{V}$ , i.e. inflaton  $\phi$ , as required for waterfall field
- Explicit construction with ingredients required for moduli stab. + waterfall field ?

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# Toy model in toroidal orbifold

- Build a model containing moduli stab. ingredients:  
*3 magnetized D7-brane with orthogonal co-volumes*
- Toy model: not CY but toroidal orbifold  $T_1^2 \times T_2^2 \times T_3^2 / \mathbb{Z}_2 \times \mathbb{Z}_2$

- Brane configuration:

	(45)	(67)	(89)
D7 <sub>1</sub>	•	⊗	×
D7 <sub>2</sub>	×	•	⊗
D7 <sub>3</sub>	⊗	×	•

- brane localized in this plane
- × worldvolume spanning this plane
- ⊗ worldvolume magnetization

→ magnetic field on each D7 necessary to have 3  $d_i \neq 0$  coefficients

# Spectrum (1)

- Find spectrum and inspect possible tachyons  
 $\rightarrow$  mass depends on internal spins of each state of our specific model
- Lowest-lying states have the following masses

	$D7_1$	$D7_2$	$D7_3$
$D7_1$	$\alpha' m^2 = -2 \zeta_1^{(2)} $	$\alpha' m^2 =  \zeta_2^{(3)}  -  \zeta_1^{(2)} $	$\alpha' m^2 =  \zeta_1^{(2)}  -  \zeta_3^{(1)} $
$D7_2$		$\alpha' m^2 = -2 \zeta_2^{(3)} $	$\alpha' m^2 =  \zeta_3^{(1)}  -  \zeta_2^{(3)} $
$D7_3$			$\alpha' m^2 = -2 \zeta_3^{(1)} $

recall  $\zeta_a = \frac{1}{\pi} \text{Arctan}(2\pi\alpha' q_a H_{(45)}^a) = \frac{1}{\pi} \text{Arctan} \left( \frac{q_a \alpha' k^{(45)}}{\mathcal{A}_{45}} \right) \approx \frac{q_a \alpha' k^{(45)}}{\pi \mathcal{A}_{45}}$

$$2\pi H_{(45)}^a \mathcal{A}_{45} = k_a^{(45)}$$

$$k_a^{(45)} = \frac{n_a^{(45)}}{m_a^{(45)}} \in \mathbb{Q},$$

- To eliminate mixed state tachyons  $D7_i - D7_j$  :  $|\zeta_1^{(2)}| = |\zeta_2^{(3)}| = |\zeta_3^{(1)}|$
- Left with doubly charged states tachyons  $D7_i - D7_i$  (brane and image)

# Spectrum (2)

- To eliminate doubly charged tachyons: Wilson lines/brane separations

	(45)	(67)	(89)
D7 <sub>1</sub>	•	⊗	×
D7 <sub>2</sub>	×	•	⊗
D7 <sub>3</sub>	⊗	×	•



	(45)	(67)	(89)
D7 <sub>1</sub>	•	⊗	×
D7 <sub>2</sub>	×	•	⊗
D7 <sub>3</sub>	⊗	×	•

→ lowest-lying states

$$y = f(x_2, U_2)$$

$$a_i = g(A_3, U_i)$$

$$\alpha' m_{11}^2 \approx -\frac{2\alpha' |k_1^{(2)}|}{\pi \mathcal{A}_2} + \frac{\alpha' a_1^2}{\mathcal{A}_3} \approx -\frac{2 |k_1^{(2)}|}{\pi r_2 \mathcal{V}^{1/3}} + \frac{a_1^2}{r_3 \mathcal{V}^{1/3}} > 0 \quad \text{freedom in } a_1$$

$$\alpha' m_{22}^2 \approx -\frac{2\alpha' |k_2^{(3)}|}{\pi \mathcal{A}_3} + \frac{y \mathcal{A}_2}{\alpha'} \approx -\frac{2 |k_2^{(3)}|}{\pi r_3 \mathcal{V}^{1/3}} + y r_2 \mathcal{V}^{1/3} \quad \text{waterfall field } \equiv \varphi_-$$

$$\alpha' m_{33}^2 \approx -\frac{2\alpha' |k_3^{(1)}|}{\pi \mathcal{A}_1} + \frac{\alpha' a_3^2}{\mathcal{A}_2} \approx -\frac{2 |k_3^{(1)}|}{\pi r_1 \mathcal{V}^{1/3}} + \frac{a_3^2}{r_2 \mathcal{V}^{1/3}} > 0 \quad \text{freedom in } a_3$$

Moduli ratio stabilization as described before  $\tau_i \leftrightarrow \mathcal{A}_i \rightarrow \mathcal{A}_i = r_i \mathcal{V}^{1/3}$

# Effective theory scalar potential

- Magnetic field contribution: D-term (*SUSY breaking*)

$$\begin{aligned}
 V_D &= \sum_a \frac{g_{U(1)_a}^2}{2} \left( \xi_a + \sum_n q_a^n |\varphi_a^n|^2 \right)^2 + \dots & m_{H_2}^2 \equiv 2g_{U(1)_2}^2 \xi_2 = \frac{2|\zeta_2^{(3)}|}{\alpha'} \approx \frac{2|k_2^{(3)}|}{\pi} \frac{g_s^2}{\kappa^2 \mathcal{V}} \frac{\alpha'}{\mathcal{A}_3} \\
 &= \sum_{a=1,3} \underbrace{\frac{g_{U(1)_a}^2}{2} \xi_a^2}_{d_a} + \underbrace{\frac{g_{U(1)_2}^2}{2} \left( \xi_2 - 2|\varphi_-|^2 + \dots \right)^2}_{\text{red}} + \dots \\
 &+ \frac{1}{g_{U(1)_a}^2} \propto \frac{\mathcal{V}}{g_s} \frac{\alpha'}{\mathcal{A}_a} \quad \rightarrow \quad \xi_a = \dots, \quad d_a \propto \frac{1}{2} g_s^3 \left( \frac{k_a^{(j)}}{\pi} \right)^2
 \end{aligned}$$

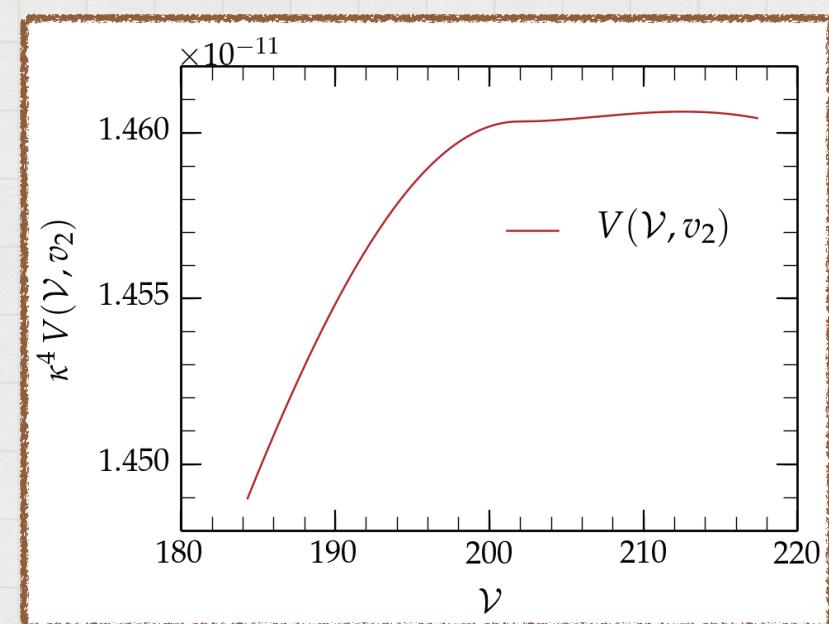
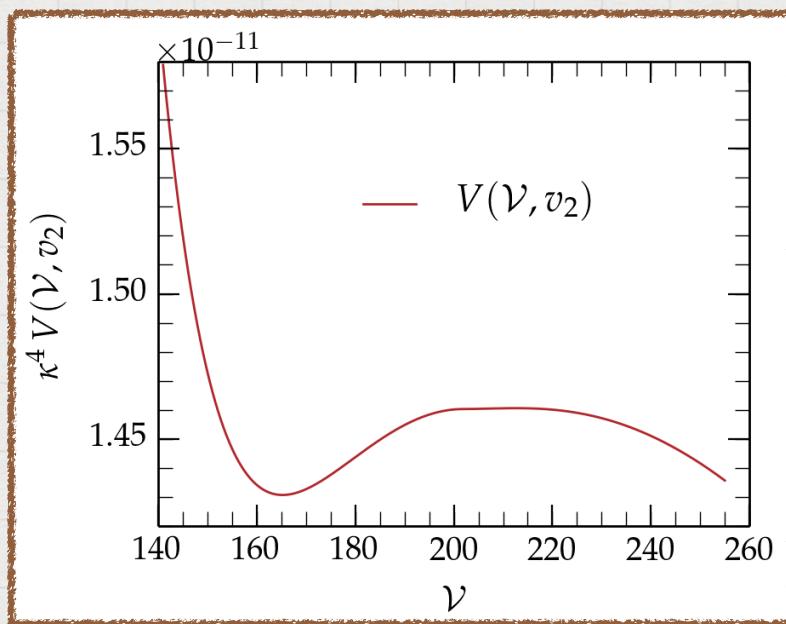
- Wilson line or brane separation: F-term from superpotential (*SUSY*)

$$V_F \ni \kappa^{-4} \sum_i \left| \frac{\partial \mathcal{W}_{tach}}{\kappa \partial \varphi_i} \right|^2 + \dots = m_{x_2}^2 |\varphi_-|^2 + \kappa^2 m_{x_2}^2 |\varphi_-|^4 + \dots$$

- Total scalar potential:  $V(\mathcal{V}, \varphi_-) = V_F(\mathcal{V}) + V_F(\mathcal{V}, \varphi_-) + V_D(\mathcal{V}, \varphi_-) + \dots$   
*inflationary phase: as described before*

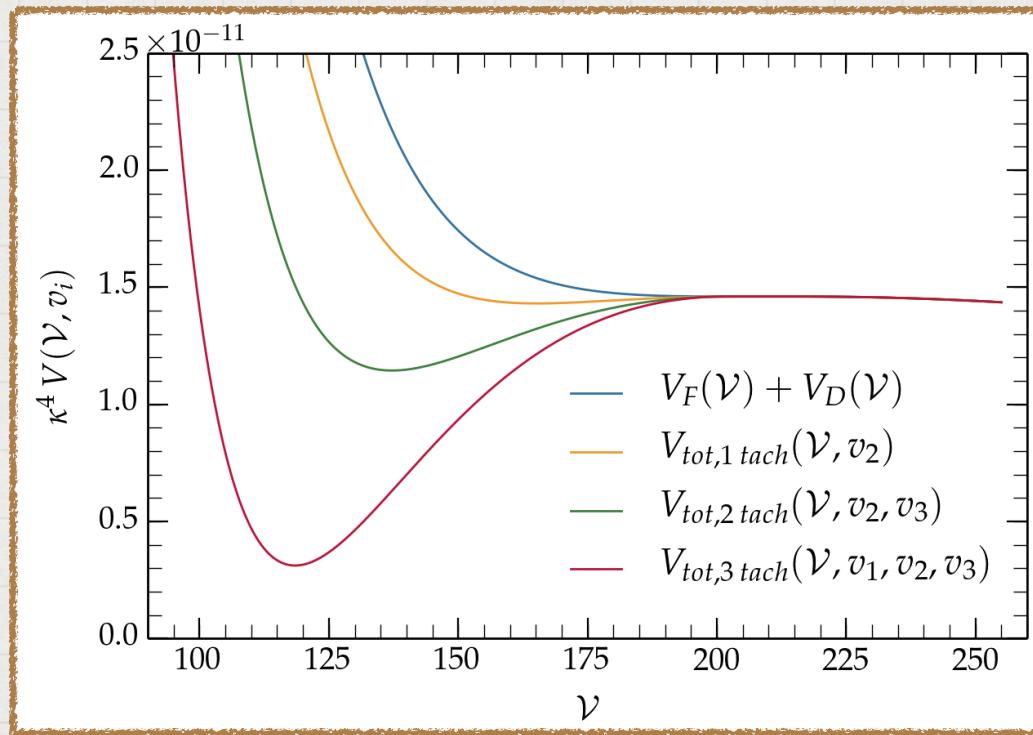
# Global minimum

- Global min.:  $V(\mathcal{V}, \varphi_-) = \frac{C}{\kappa^4} \left( -\frac{\ln \mathcal{V} - 4 + q}{\mathcal{V}^3} - \frac{3\sigma}{2\mathcal{V}^2} \right) + \frac{1}{2} m_S^2(\mathcal{V}) |\varphi_-|^2 + \frac{\lambda(\mathcal{V})}{4} |\varphi_-|^4$
- when  $\mathcal{V} < \mathcal{V}_c$        $m_S^2(\mathcal{V}) < 0, \rightarrow \langle \varphi_- \rangle \equiv v_2(\mathcal{V}) \neq 0 \rightarrow -\frac{m_S^4}{4\lambda}(\mathcal{V})$   
 depends on: fluxes, brane position  
 → choose it near  $\mathcal{V}_-$



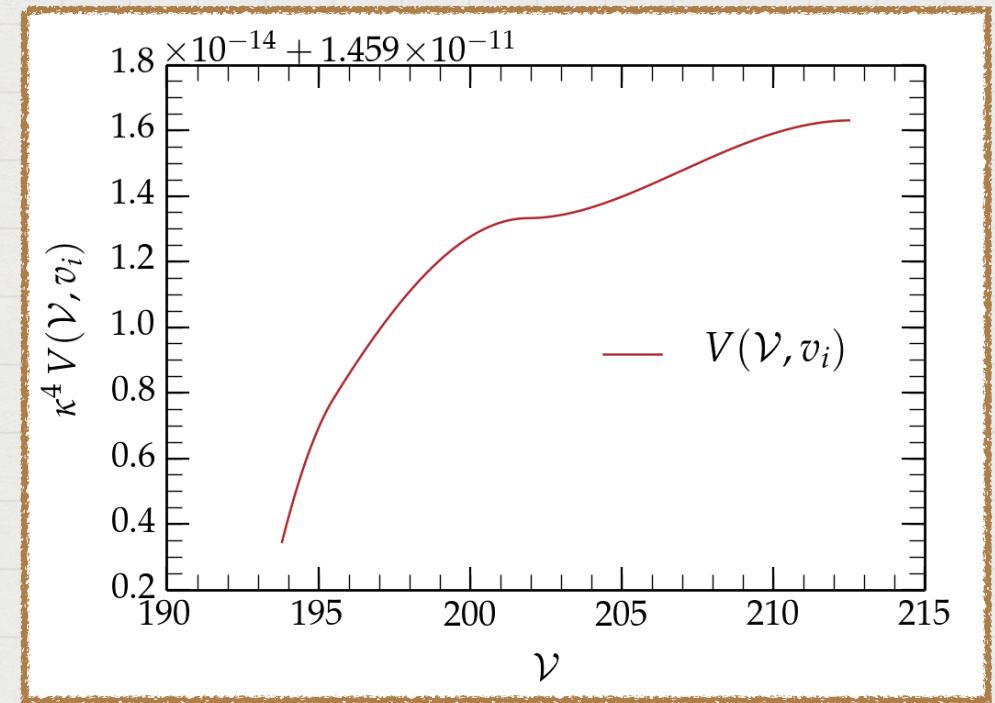
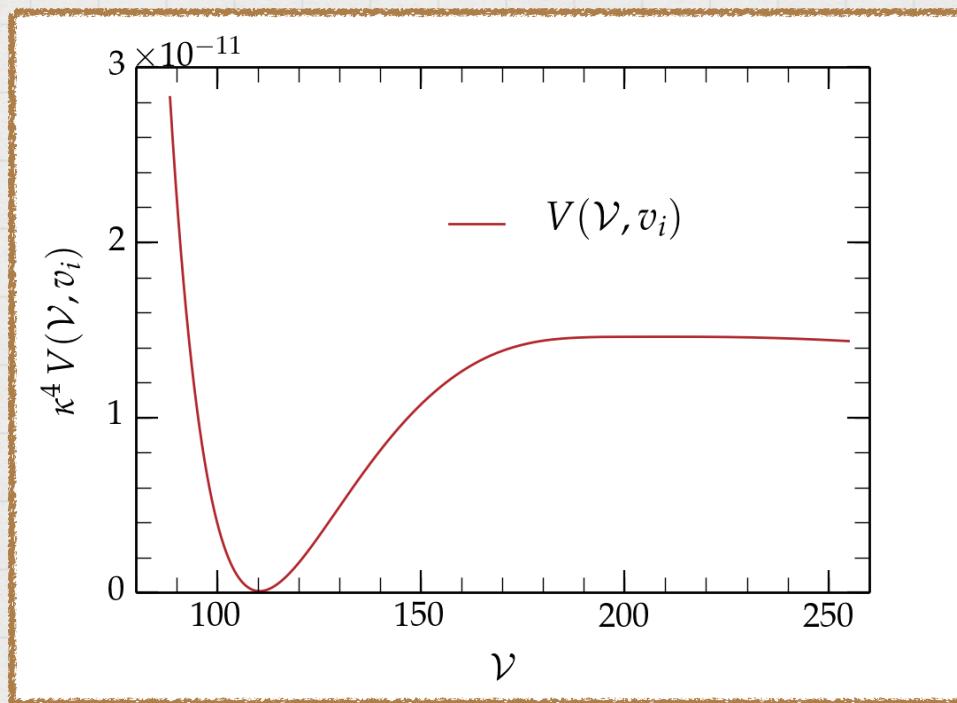
# Tuning the vacuum energy

- Cannot tune value minimum: *all parameters* used to tune  $x, \mathcal{V}_c$
- Invoke new physics near the minimum. With ingredients of our model ?
- Additional tachyons (*other doubly charged states*), not sufficient



# Tuning the vacuum energy

- Cannot tune value minimum: *all parameters* used to tune  $x, \mathcal{V}_c$
- Invoke new physics near the minimum. With ingredients of our model ?
- Add a 4th brane parallel to one of the initial stack !



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# Summary & Outlook

- Toroidal orbifold model with magnetic fluxes:
  - ✓ IIB moduli stabilization through perturbative corrections to  $\mathcal{K}$
  - ✓ effective theory scalar potential accommodate inflationary phase
  - ✓ hybrid inflation with waterfall field (*doubly charged state*)
  - ✓ possibility to tune vacuum energy (*several waterfall fields*)
- *Toy model* but generic ingredients
- Future directions:
  - More generic models, CY configurations
  - Models which also contain SM physics
  - Quintessence modes after false vacuum decay
  - Yoga Dark energy realization ?    *Burgess, Dineen, Quevedo '21*

Thank you

for  
all

# Back-up slides



# Perturbative Kähler potential corrections

- Fully perturbative moduli stab. ?  $\mathcal{W}$  cannot be corrected, only  $\mathcal{K}$   
 $\rightarrow$  Planck mass corrections (bulk corrections, upon compactification) ( $\leftrightarrow \mathcal{R}$ )

- 10d effective action of IIB superstrings:  $\exists \mathcal{R}$  (EH),  $\mathcal{R}^4$  terms  
 $\rightarrow$  upon compactification:  $\mathcal{R}^4$  terms induce 4d localized EH terms

$$\propto \chi(X_6) \times \mathcal{R}_{(4)} \quad \chi(X_6) = \frac{3}{4\pi^3} \int_{X_6} \mathcal{R} \wedge \mathcal{R} \wedge \mathcal{R} \quad \begin{matrix} \text{Grisaru et al. '86} \\ \text{Antoniadis et al. '96, '03} \end{matrix}$$

- Gravitons from localized EH terms towards D7/07 (which effectively travel in the 2d transverse space) induce log. radiative corrections

$$\delta S_{\text{grav}} \sim -\frac{4\zeta(2)}{(2\pi)^3} \chi(X_6) \int_{M_4} \left( \sum_{k=1}^3 e^{2\phi} T_k \ln(R_\perp^k/w) \right) \mathcal{R}_{(4)}$$

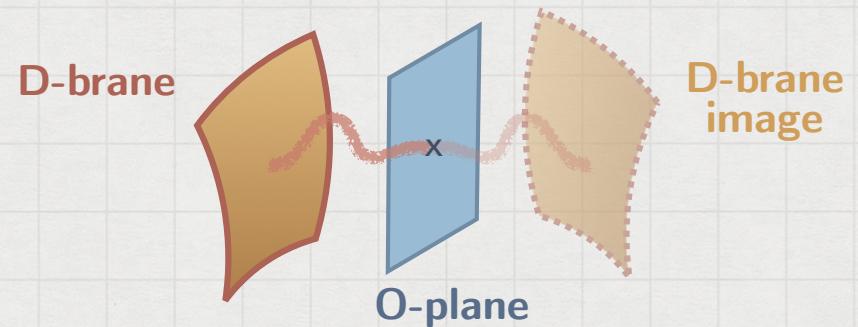
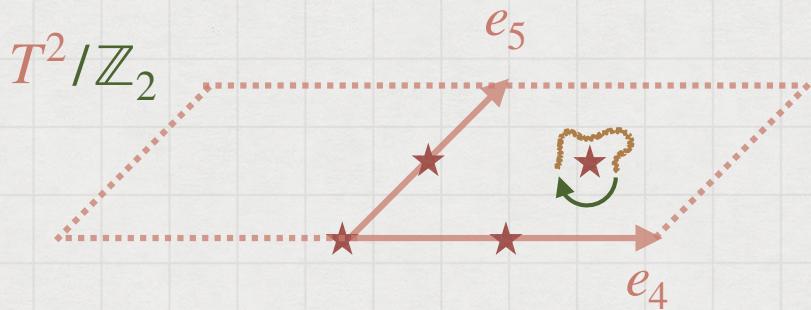
$R_\perp^k$  size of  $\perp$ -space  
 $w$  localization width  
 $T_k$  brane tension

Antoniadis, Chen, Leontaris '18, '19

- $\mathcal{R}_{(4)} \leftrightarrow$  Planck mass (hence  $\mathcal{K}$ ) corrections, break no-scale structure

# Toroidal orbifolds

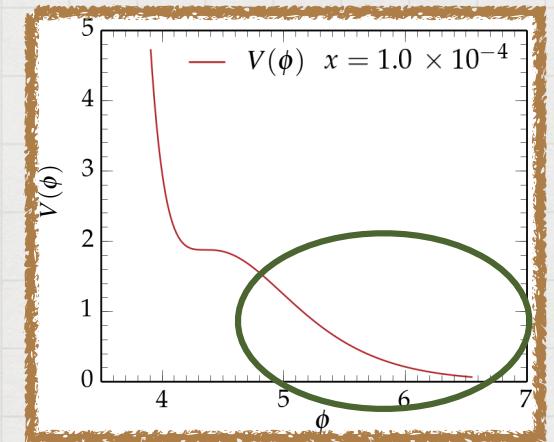
- Toroidal orbifold : torus+discrete symmetry group  
*identification of space-time points:*  $\mathbf{X} \sim g\mathbf{X} + 2\pi\mathbf{L}, \quad g \in G$
- Allows for **twisted** b.c.:
  - for closed strings, close up to action of  $g$
  - for open strings, attached to **O-planes**, new objects located at orientifold fixed points:  $\mathbf{X}_{\text{f.p.}} = g\mathbf{X}_{\text{f.p.}}(\tau, \sigma) + 2\pi\mathbf{L}, \quad g \in G,$



- Partition functions have to include **twisted** sectors, and project on  $g$  invariant states using projector  $1/2(1 + g)$

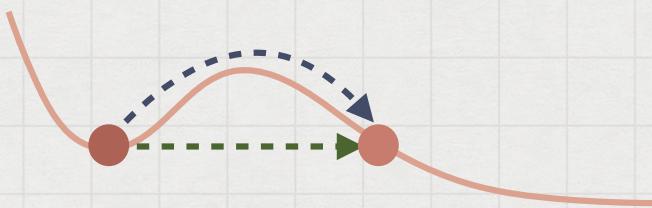
# Stability of the vacuum

- Probability of escaping to the runaway direction ?



# Stability of the vacuum

- Probability of escaping to the runaway direction ?



Tunneling : Coleman-de Luccia instanton  
 Classical « jump » : Hawking-Moss transition

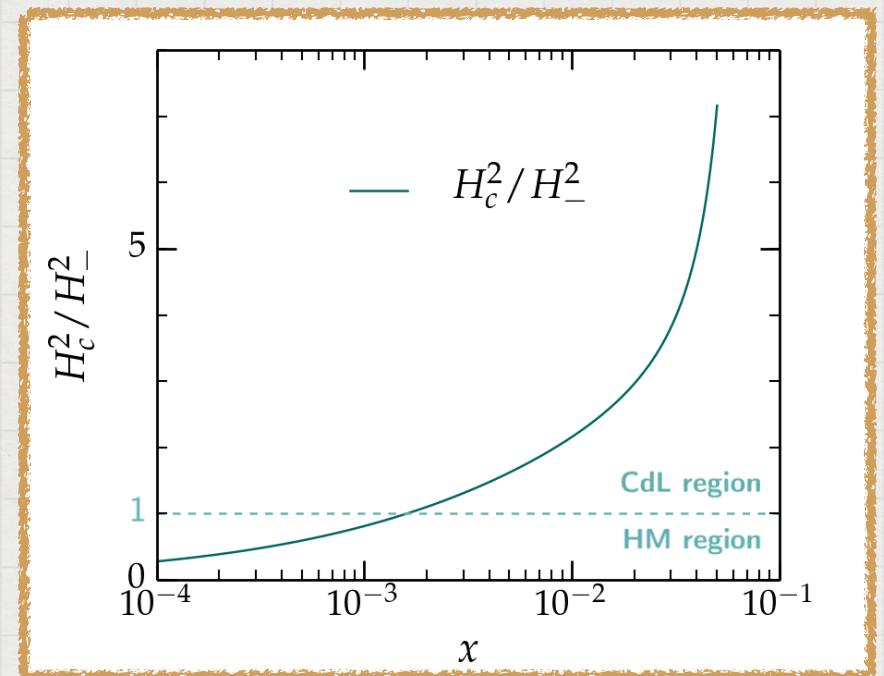
- Compute rate:  $\Gamma = Ae^{-B}$  with  $B = S_E(\varphi) - S_E(\varphi_-)$ ,

$B$  : energy cost of  $\varphi$  **bounce** solutions  
*(Euclidean action analysis)*

- CdL solutions exist for

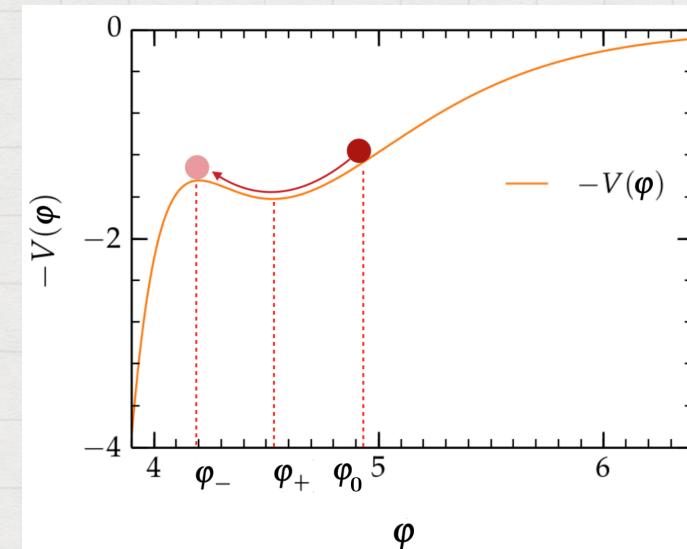
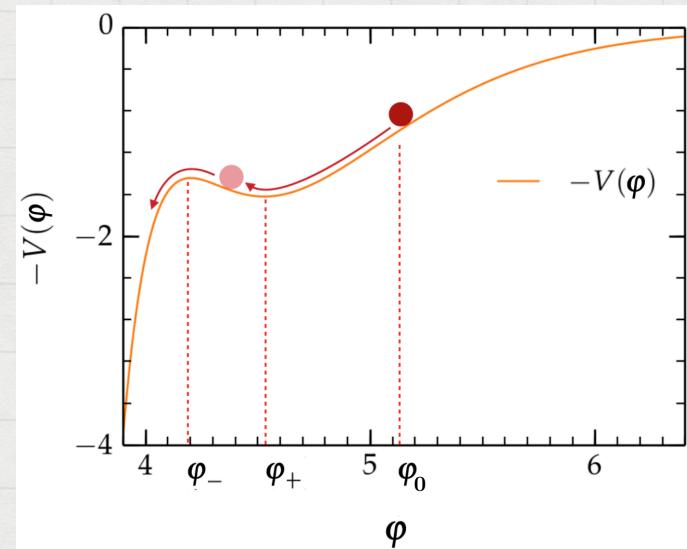
$$H_- = H(\varphi_-) < H_c \quad \begin{matrix} \text{Jensen, Steinhardt '84} \\ \text{Balek, Demetrian '04} \end{matrix}$$

- HM estimate  $B \approx \frac{8\pi^2 \Delta V}{3H_*^4} \approx 3.3 \times 10^9$



# Bounce solutions

- Instanton solutions (*Euclidean action analysis*), between false  $\varphi_-$  and true vacua  $\varphi_+$ , **bounce solutions**
- Compute  $\varphi$  numerically (with  $S_E + \text{b.c.}$ ) with *overshooting/undershooting* method



- HM bounce  $\varphi_+$ :  $B = S_E(\varphi_+) - S_E(\varphi_-) = -\frac{24\pi^2}{\kappa^4 V(\varphi_+)} + \frac{24\pi^2}{\kappa^4 V(\varphi_-)} \simeq \frac{8\pi^2 \Delta V}{3H_*^4} \approx 3.3 \times 10^9$

# Vacuum amplitudes

- Vacuum amplitudes: propagation open string glued back to it self, with(out) orientation reversal, *corresponding to annulus (Möbius strip) topologies*. For 1 unique state  $\phi$ :

$$Z(\phi, \tau_2) = \langle \phi | e^{-(2\pi\alpha' p^+ \tau_2)H} | \phi \rangle = \langle \phi | e^{-\tau_2 \pi \alpha' \sum_i p^i p^i} q^{\frac{1}{2}(N+E_0)} | \phi \rangle \quad q \equiv e^{2\pi i \tau_2}$$

- Sum over states,  $\tau_2$  :  $A = \frac{N^2}{2} \int_{\tau_2 > 0} \frac{d\tau_2}{\tau_2^2} \frac{1}{\tau_2^{(D-2)/2}} \text{tr}_{\mathcal{H}_{\text{osc}}} q^{\frac{1}{2}(N+E_0)}$
- Toroidal orbifold model

$\exists$  bosonic coord. contribution:  
 $\frac{1}{\eta(\tau_2)}$

$$8A_0 = (N_1 \bar{N}_1 W_1 \tilde{P}_2 P_3 + N_2 \bar{N}_2 P_1 W_2 \tilde{P}_3 + N_3 \bar{N}_3 \tilde{P}_1 P_2 W_3) T_{oo}(0,0,0)$$

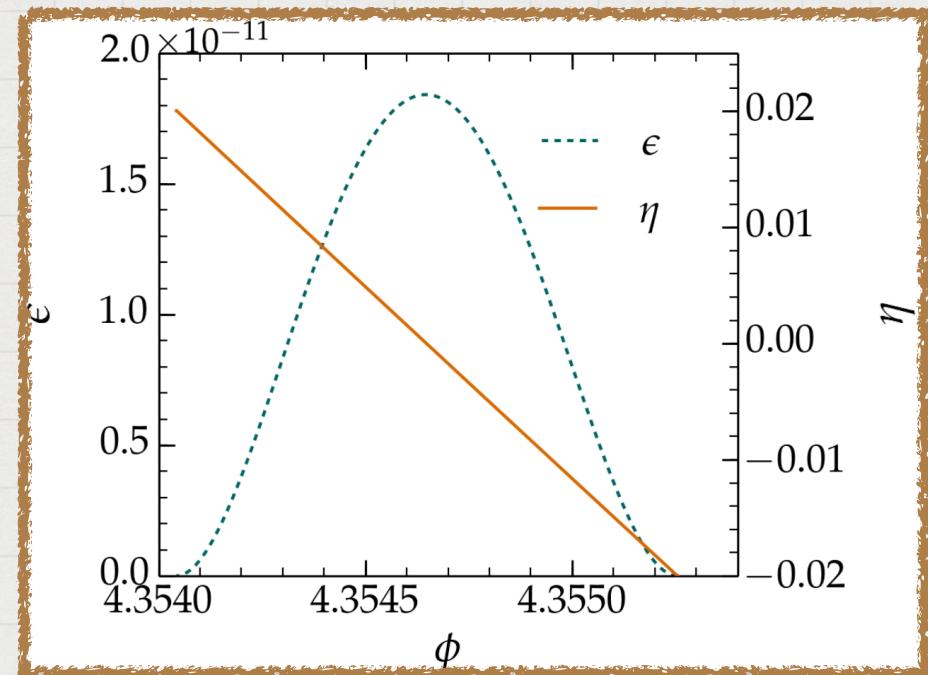
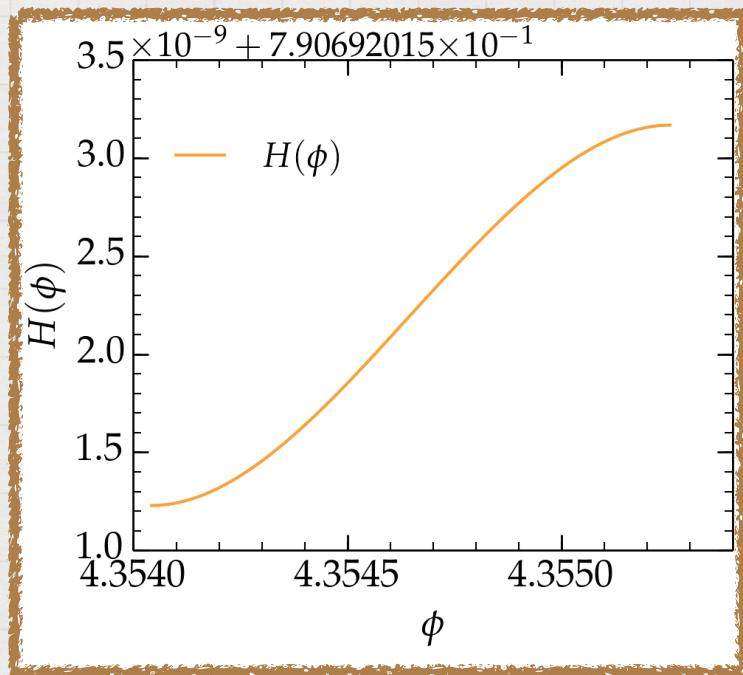
$$4A_1 = -i \left( N_1 N_2 T_{fo}(0, \zeta_1^{(2)}\tau, \zeta_2^{(3)}\tau) + \bar{N}_1 \bar{N}_2 T_{fo}(0, -\zeta_1^{(2)}\tau, \zeta_2^{(3)}\tau) \right) \frac{k_2^{(3)}\eta^3}{\vartheta_4(0)\vartheta_4(\zeta_1^{(2)}\tau)\vartheta_1(\zeta_2^{(3)}\tau)} + \dots$$

$$8A_2 = -i N_1^2 W_1 P_3 T_{oo}(0, 2\zeta_1^{(2)}\tau, 0) \frac{2k_1^{(2)}\eta}{\vartheta_1(2\zeta_1^{(2)}\tau)} + i \bar{N}_1^2 W_1 P_3 T_{oo}(0, -2\zeta_1^{(2)}\tau, 0) \frac{2k_1^{(2)}\eta}{\vartheta_1(-2\zeta_1^{(2)}\tau)} + \dots$$

$\mathbb{Z}_2 \times \mathbb{Z}_2$  characters,  
from  $so(2)$  ones  
function of  $\tau$  through  $q$

# Possibilities for inflation: hilltop

- Possibilities : hilltop with  $\eta = \eta_* = -0.02$  at maximum ( $\epsilon = 0$ )



# Possibilities for inflation: hilltop

- Possibilities : ~~hilltop~~ with  $\eta = \eta_* = -0.02$  at maximum ( $\epsilon = 0$ )

