New Methods for Old Problems: Vacua of Supergravity Theories

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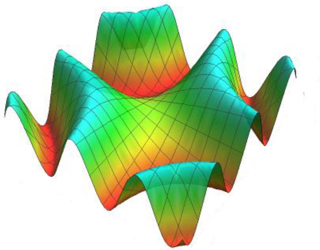
Why looking for vacua?

- Useful for understanding features of QFTs thanks to the AdS/CFT conjecture
  (New CFTs? Do terminal theories exist? Study of theories without Lagrangian in 6D)



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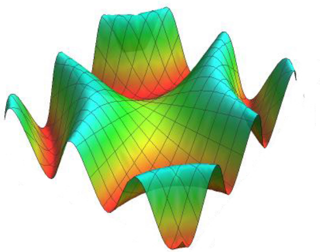
  Do families of vacua depending on continuous parameters exist?



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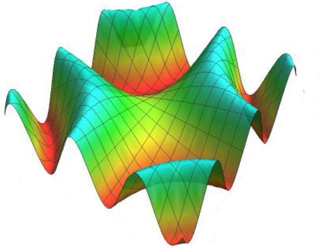

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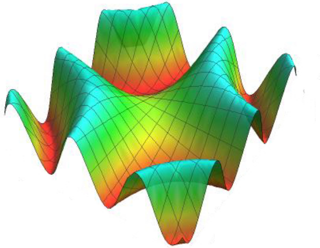
- Better grasp on the theory



Maximal Gauged Supergravities

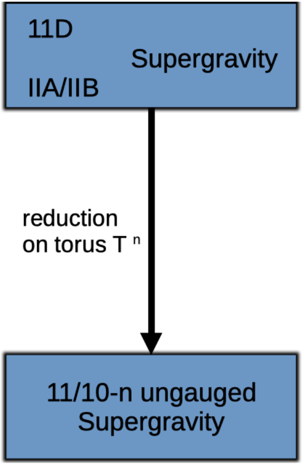
Maximal gauged supergravities in different dimensions have a natural decriptions in terms of the embedding tensor $\Theta_M{}^\alpha$.

[de Wit, Samtleben, Trigiante,2003]

The scalar manifold is the homogeneous coset space $E_{n(n)}/H$. (we will not consider the Trombone symmetry)

Supersymmetry and gauge symmetries impose consistency constraints on the embedding tensor $\Theta_M{}^\alpha$, such as $\Theta \in \mathbf{351}$ of $E_{6(6)}$ in $D = 5$.

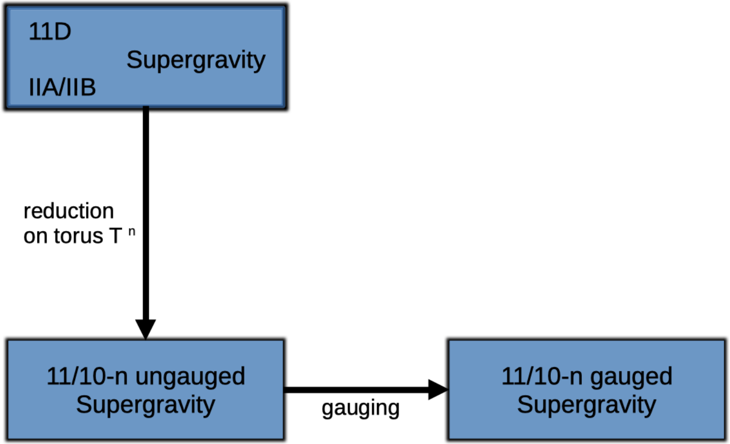
Compactifications



To obtain maximal ungauged supergravities in lower dimensions, it is necessary to reduce higher dimensional supergravities on n-dimensional tori.

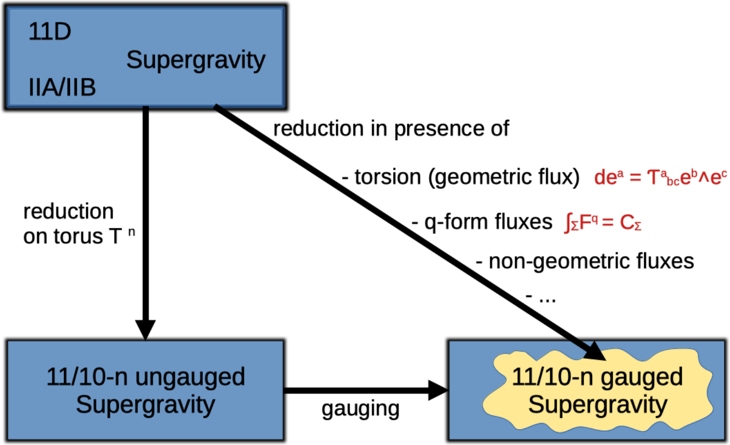
Compactifications

Picking a particular gauge results in a gauged Supergravity.



Gauging Procedure

The Embedding Tensor parameterise every possible deformations of the theory.



Quadratic Constraints

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Generators of the gauge group are picked by means of the embedding tensor by

$$X_M = \Theta_M{}^\alpha t_\alpha$$

The Embedding tensor must satisfy also some Quadratic Constraints (QC):

$$f_{\beta\gamma}{}^\alpha \Theta_M{}^\beta \Theta_N{}^\gamma - (t_\beta)_N{}^P \Theta_M{}^\beta \Theta_P{}^\alpha = 0$$

or

$$[X_M, X_N] = -X_{MN}{}^P X_P$$

Looking for vacua means solving the QC and the EOM.

Equations of Motion

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Potentials in gauged supergravities is given by the difference of the fermionic shifts A1 and A2 (which are function of the embedding tensor):

$$V = \alpha |A2|^2 - \beta |A1|^2$$

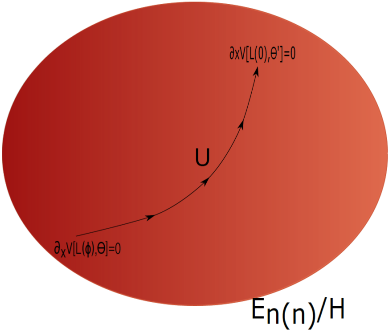
$\alpha = \frac{1}{3}, \frac{1}{8}$ and $\beta = 3, 15$ respectively in $D = 5, 7$.

The EOM are obtained by varying the potential along the directions of the coset manifold $E_{n(n)}/H$, represented by Σ, giving rise to quadratic expressions in the fermionic shift. e.g. in $D = 5$

$$\left(\frac{4}{3} A1^{mq} A2_{m,ijk} \Omega_{lq} + 2 A2^{m,npq} A2_{n,mij} \Omega_{pk} \Omega_{lq} \right) \Sigma^{ijkl} = 0$$

Method and Techniques

We look for solutions to the Quadratic Constraints toghether with the EOM.



Every point of the scalar manifold can be reached by an $E_{n(n)}$ transformation

[G. Dall'Agata , G.Inverso, 2012]

We look at extrema of the scalar potential without fixing the gauging, Θ, a priori!

Analitic Techniques: eXtended Linearization

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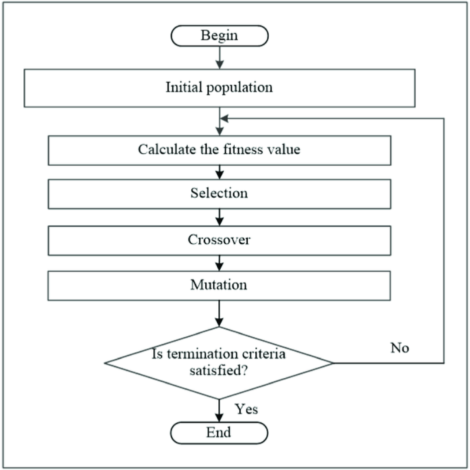
Given a second order system of multivariate equations l_j, denoting with x^k terms of degree k: $\prod_{i=1}^{k} x_{j_i}$, and with $\mathcal{I}_D$ the space generated by $x^k l$ with $0 \leq k \leq D - 2$ [N.T. Courtois, A. Klimov, J. Patarin, A. Shamir, 2000]:

- **Multiply:** Create equations $\prod_{i=1}^{k} x_{j_i} l_j \in \mathcal{I}_D$, with $k \leq D - 2$
- **Linearise:** Linearise the system by introducing variables $y_{j_1 j_2 \ldots j_l} = x_{i_1} x_{i_2} \ldots x_{i_l}$
- **Solve:** When the linearisation technique produces an equation with only one variable, solve it (with Berlekamp's algorithm).
- **Repeat:** Insert the root in the system, simplify, and repeat until every root is found.

Numerical Techniques: Genetic Algorithms



Genetic Algorithms (GA) maximize (minimize) functions by evolving a population, increasing the chances of the fittest individuals to reproduce and carring their phenotypes to next generations.

Genetic Algorithms

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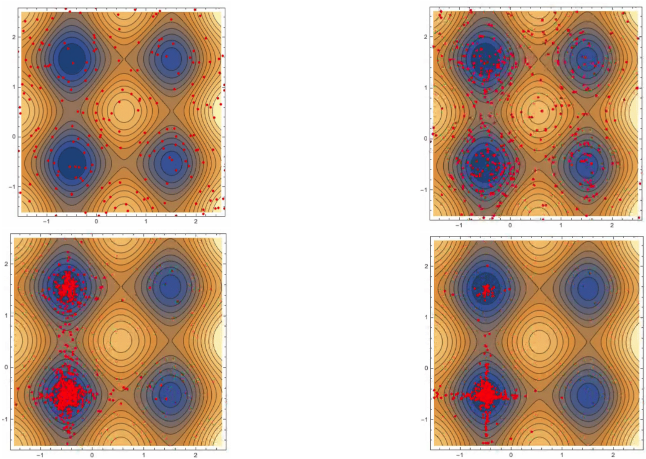
For instance, finding the minimum of $x^2 + 5\sin(3x) + \frac{y^2}{20} + 4\sin(3y)$:



In our case: $f : \mathbb{R}^n -> \mathbb{R}$ with $n > 50$.

CMA-ES

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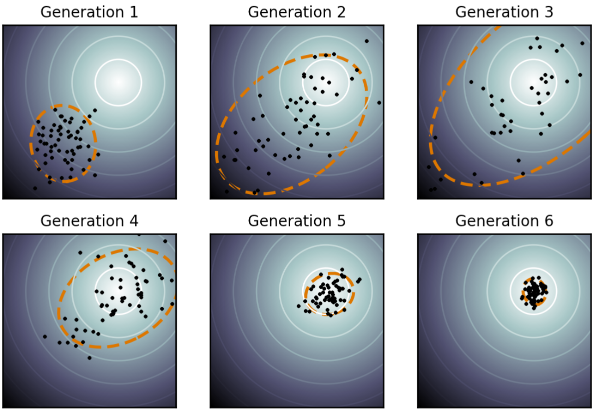
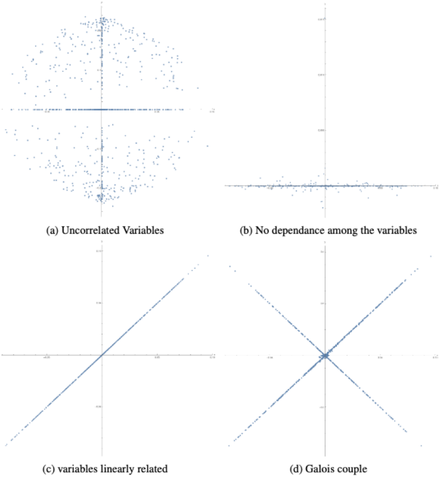
Candidate solutions are sampled according to a multivariate normal distribution in $\mathbb{R}^n$ [Hansen N., Ostermeier A., 2001]



CMA is a method to update the covariance matrix of this distribution

Data Analysis

Residual symmetries of the vacua, vanishing parameters of the embedding tensor and reconstruction of the relations among the variables, allow to obtain analytical results.



Results

The vacua found in 5D are:

vacuum	Susy	G_{gauge}	G_{res}
A1	8	$SO(6)$	$SO(6)$
A2	0	$SO(6)$	$SO(5)$
A3	0	$SO(6)$	$SU(3)$
A4	2	$SO(6)$	$SU(2)\mathrm{X}U(1)$
M0	0,2,4,6	$U(1) \ltimes \mathbb{R}^{16}$	U(1)
M1	4	$U(1) \ltimes \mathbb{R}^{16}$	$U(1)$
M2	2	$SO^*(6) = SU(3,1)$	$SU(3)\mathrm{X}U(1)$
M3	4	$SO^*(4) \ltimes \mathbb{R}^8$	$U(2)$
M4	0	$[SO(3,1) \times SO(2,1)] \ltimes \mathbb{R}^8$	$U(2)$
D1	0	$SO(3,3)$	$SO(3)^2$
D2	0	$SO^*(6) = SU(3,1)$	$SU(2)$

Some of them were already known [Bobev, Fischbacher, Gautason, Pilch,2020] [Gunaydin, Romans, Warner,1985]

Results

The vacua found in 7D are:

vacuum	susy	G_{gauge}	G_{res}
A1	4	$SO(5)$	$SO(5)$
A2	0	$SO(5)$	$SO(4)$
M1	0	$U(1)\ltimes\mathbb{R}^4$	$U(1)$
M2	0	$U(1)\ltimes\mathbb{R}^6$	$U(1)$

Some of them were already known [K. Pilch, P. van Nieuwenhuizen, and P. K. Townsend, 1984] [M. Pernici, K. Pilch, P. van Nieuwenhuizen, and N. P. Warner,1985]

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The spectra have also been computed:

Vacuum	$\frac{4}{9}\Lambda m^2_{3/2}$	$\Lambda m^2_{1/2}$	Λm^2_{scal}	Λm^2_{vec}	Λm^2_{tens}
AdS1	1_8	$0_8\,,\ \left[\frac{1}{4}\right]_{40}\,,\ \left[\frac{9}{4}\right]_8$	$-4_{20}, -3_{20}, 0_2$	0_{15}	1_{12}
AdS2	$\left[\frac{32}{27}\right]_8$	$0_{40},\ \left[\frac{8}{3}\right]_8\,,\ \left[\frac{675}{128}\right]_8$	$-2_{20}, 8_1, 0_7\,,\ \left[-\frac{16}{3}\right]_{14}$	$0_{10}\,,\ \left[\frac{8}{3}\right]_5$	$\left[\frac{2}{3}\right]_{10}\,,\ 6_2$
AdS3	$2_2\,,\ \left[\frac{98}{81}\right]$	$0_8\,,\ \left[\frac{1}{2}\right]_{16}\,,\ \left[\frac{25}{18}\right]_{18},$ $\left[\frac{25}{8}\right]_2\,,\ \left[\frac{2025}{392}\right]_6\,,\ \left[\frac{121}{18}\right]_6$	$\left[-\frac{40}{9}\right]_{12},\ \left[-\frac{16}{9}\right]_{12},$ $8_1, 0_{17}$	$0_8,\ \left[\frac{32}{9}\right]_6\,,\ 8_1$	$\left[\frac{8}{9}\right]_6\,,\ \left[\frac{32}{9}\right]_6$
AdS4	$\left[\frac{49}{36}\right]_4,\ \left[-\frac{16}{9}\right]_2,$ 1_2	$\left[\frac{1}{16}\right]_4,\ \left[\frac{1}{4}\right]_6,\ \left[\frac{9}{16}\right]_4$ $1_2,\ \left[\frac{25}{16}\right]_4,\ \left[\frac{9}{4}\right]_4,\ \left[\frac{49}{16}\right]_8$ $\left[\frac{225}{64}\right]_2,\ 4_2,\ \left[\frac{225}{49}\right]_4$ $0_{12},\ \left[\frac{29}{4} \pm \sqrt{7}\right]_2$	$0_{13},\ [-4]_3,\ \left[-\frac{15}{4}\right]_{12}$ $\left[-\frac{55}{16}\right]_4,\ [-3]_2,\ \left[-\frac{39}{16}\right]_4$ $3_2,\ [4 \pm 2\sqrt{7}]_1$	$0_4,\ \left[\frac{9}{16}\right]_4,\ \left[\frac{5}{4}\right]_2,$ $\left[\frac{65}{16}\right]_4,\ 6_1$	$\left[\frac{9}{4}\right]_2,\ \left[\frac{9}{16}\right]_4,$ $\left[\frac{25}{16}\right]_4,\ \left[\frac{25}{4}\right]_2$

The spectra are analytical!

Future Perspectives

- Numerical Scan in D=5 with 351 parameters with BFGS algorithm (with T. Fischbacher and F.F. Gautason)
- Numerical Scan in D=4 with 912 parameters (with T. Fischbacher and F.F. Gautason and al.)
- Explore some ideas around numerically-assisted fully automatic generation of stringent completeness proofs (with T. Fischbacher and F.F. Gautason and al.)
- Compare optimization analysis with Reinforcement Learning techniques in D=7

Thank you
for the attention !