

Seesaw mechanism in the R-parity violating supersymmetric standard model with the gauged flavor $U(1)_X$ symmetry

SUSY 2022 @ Ioannina

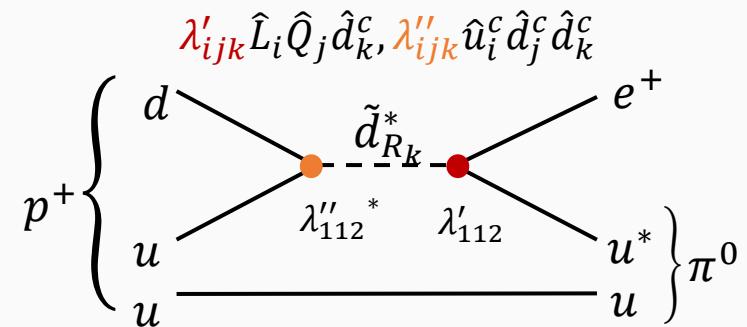
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This talk is based on [AH, arXiv:2112.10337], accepted from PTEP

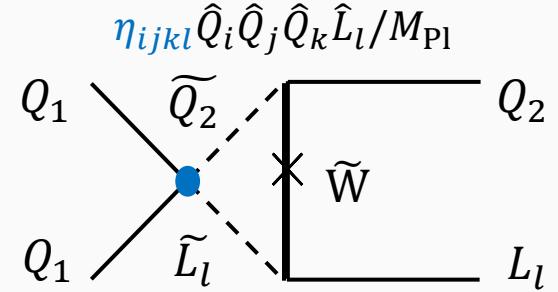
Alternative to R-parity

- Supersymmetric standard models (SSMs)

Baryon(B)/Lepton(L) number violations e.g. $\tau_{p \rightarrow e^+ \pi} > 8.2 \times 10^{33} \text{yr}$ [PDG 2018]



$$\Rightarrow |\lambda'_{l1k} \lambda''_{12k}| < 10^{-27} \times \left(\frac{m_{\tilde{d}_{Rk}}}{100 \text{GeV}} \right) \quad [\text{Barbier et al. 2005}]$$



$$\Rightarrow \eta_{112l} < 10^{-7} \quad [\text{Ellis et al., 1983}]$$

→ Non-renormalizable operators contribute to the proton decay

- Alternative discrete symmetries to R-parity

e.g.) B_3, P_6, M_3 [Ibanez & Ross, 1992] [Dreiner, Luhn, & Thormeier, 2006] [Lee, Luhn, Matchev, 2008]

- Phenomenological applications in RpV scenarios : Yukawa hierarchy, neutrino mass, axion, ...

[Dreiner, Luhn, Murayama & Thormeier, 2007] [Allanach & Kom, 2008] [Colucci, Dreiner and Ubaldi, 2019]

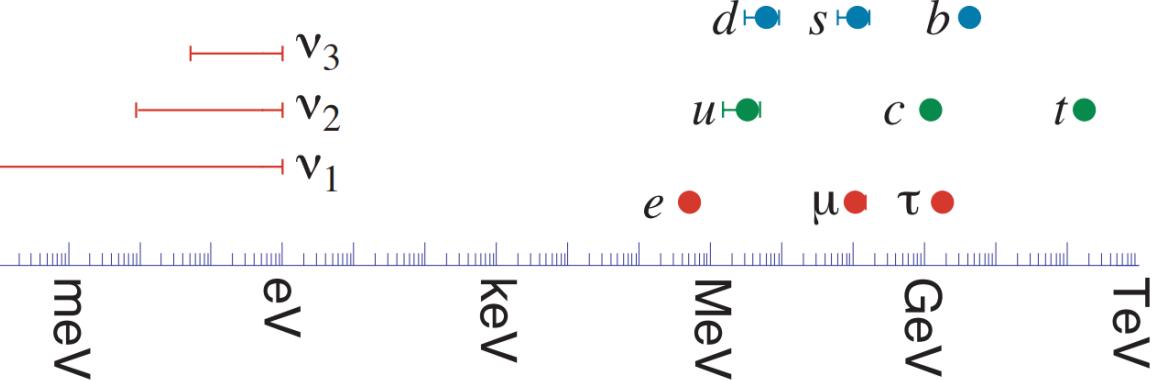
Flavor Problem

➤ Fermion Mass Hierarchy

Q. Mass gaps b.w. the generations?

Q. Small mixings?

$$|V_{CKM}| = \begin{pmatrix} 0.97 & 0.22 & 0.0036 \\ 0.22 & 0.97 & 0.041 \\ 0.0085 & 0.040 & 1.00 \end{pmatrix}$$



[Hewett, et al., 2012]

➤ Neutrino Mystery

Q. Origin of neutrino masses?

Q. Large mixings?

Q. Mass ordering?

$$\left\{ \begin{array}{l} \sin^2 \theta_{12} = 0.307 \\ \sin^2 \theta_{23} = 0.547 \\ \sin^2 \theta_{13} = 0.0218 \end{array} \right.$$

What is the origin of the flavor observables?

Today's Talk

A SUSY standard model with $U(1)_X \supset$ Matter triality (M_3) [AH, 2021]

1. the seesaw mechanism in the RpV scenario
2. the flavor model based on the Froggatt-Nielsen mechanism
3. a light sterile neutrino \Rightarrow the neutrinoless double beta decay

\Rightarrow a unified model of the flavor symmetry and the baryon number conservation in the SSM w/ M_3

Seesaw mechanism in SSM w/ M_3

The Supersymmetric Standard Model with M_3

- An alternative to R-parity \Rightarrow Matter triality M_3 [Ibanez & Ross, 1992]

- ✓ 3 right-handed neutrinos $\hat{\nu}_i^c$
- ✓ B/L violation operators:

	$\leq \text{dim.-4}$	dim.-5
R-parity	$\hat{\nu}_i^c \hat{\nu}_j^c$	$\hat{Q}_i \hat{Q}_j \hat{Q}_k \hat{Q}_l, \hat{u}_i^c \hat{d}_j^c \hat{e}_k^c \hat{e}_l^c, \dots$
M_3	$\hat{\nu}_i^c \hat{\nu}_j^c \hat{\nu}_k^c$	NONE

Superpotential (up to dimension-5)

$$W = y_{ij}^u \hat{u}_i^c \hat{Q}_j \hat{H}_u + y_{ij}^d \hat{d}_i^c \hat{Q}_j \hat{H}_d + y_{ij}^e \hat{e}_i^c \hat{L}_j \hat{H}_d + y_{ij}^\nu \hat{\nu}_i^c \hat{L}_j \hat{H}_u + \frac{1}{3!} \kappa_{ijk} \hat{\nu}_i^c \hat{\nu}_j^c \hat{\nu}_k^c + \mu \hat{H}_u \hat{H}_d,$$

- ✓ the proton stability is ensured instead of R-parity
- ✓ the L violation operator κ_{ijk} \Rightarrow the Majorana mass is spontaneously generated

$$(M_R)_{ij} = \kappa_{ijk} \langle \tilde{\nu}_k^c \rangle$$

Majorana mass & sneutrino VEVs

- The Majorana mass of RHNs comes from the RH-sneutrinos & κ_{ijk}

$$(M_R)_{ij} = \kappa_{ijk} \langle \tilde{\nu}_k^c \rangle$$

- The complex VEVs can be fixed by the extremum conditions

Neutral Scalar potential: $V = V_F + V_D + V_{\text{soft}}$

Extremum conditions: $t_{\phi_I}^0 = \frac{\partial V}{\partial \phi_I} = 0 \quad \Rightarrow \quad \langle \tilde{\nu}_i \rangle = v_{L_i} e^{i\phi_{L_i}}, \langle \tilde{\nu}_i^c \rangle = v_{R_i} e^{i\phi_{R_i}}$

- ✓ LH-sneutrinos : $\langle \tilde{\nu}_i \rangle \sim 0$ ($y^\nu \ll 1$)
- ✓ RH-sneutrinos : $\langle \tilde{\nu}_i^c \rangle \sim m_{\text{soft}}$: **soft SUSY breaking mass**

➔ The Majorana mass $M_R \sim m_{\text{soft}}$ via the RH-sneutrino VEVs!

Neutralino-Neutrino mass matrix

- The RH-sneutrino VEVs ⇒ neutrinos (ν_L, ν_R^c) mix with the MSSM-neutralinos ψ_n

The 10×10 neutral fermion mass (if $y^\nu \ll 1$)

$$\bar{\Psi} \mathcal{M}_n \Psi = (\bar{\psi}_n^c \quad \bar{\nu}_L^c \quad \bar{\nu}_R) \begin{pmatrix} M_N & M_X & 0 \\ M_X^T & 0 & m_D^T \\ 0 & m_D & M_R \end{pmatrix} \begin{pmatrix} \psi_n \\ \nu_L \\ \nu_R^c \end{pmatrix}$$

The mass matrix can be decomposed into the block mass matrix : M_N, M_X, m_D, M_R

- Dirac mass for LHN & Neutralino : $M_X^T = \begin{pmatrix} 0 & 0 & 0 & y_{k1}^\nu \langle \tilde{\nu}_k^c \rangle \\ 0 & 0 & 0 & y_{k2}^\nu \langle \tilde{\nu}_k^c \rangle \\ 0 & 0 & 0 & y_{k3}^\nu \langle \tilde{\nu}_k^c \rangle \end{pmatrix}$ RH-sneutrino VEVs

➔ Seesaw mechanism under M_3 ?

Seesaw mechanism

- The neutral fermion mass matrix can be diagonalized by Unitary matrices

$$\mathcal{M}_n = \begin{pmatrix} M_N & M_X & 0 \\ M_X^T & 0 & m_D^T \\ 0 & m_D & M_R \end{pmatrix} \rightarrow U^T \mathcal{M}_n U \approx \begin{pmatrix} M_\chi & 0 & 0 \\ 0 & m_{\text{eff}} & 0 \\ 0 & 0 & M_N \end{pmatrix}$$

- The active neutrino mass matrix \Rightarrow the mixing w/ (i)RHNs & (ii)Neutralinos

$$(m_{\text{eff}})_{ij} = -m_D^T M_R^{-1} m_D - M_X^T M_N^{-1} M_X$$

※ In the analogy to the Type-I seesaw mechanism, the unitary matrices are given by

$$U = \begin{pmatrix} \mathbb{I} & 0 & 0 \\ 0 & \mathbb{I} - \frac{1}{2}\theta^*\theta^T & \theta^* \\ 0 & -\theta^T & \mathbb{I} - \frac{1}{2}\theta^T\theta^* \end{pmatrix} \begin{pmatrix} \mathbb{I} - \frac{1}{2}\varepsilon\varepsilon^\dagger & \varepsilon & 0 \\ -\varepsilon^\dagger & \mathbb{I} - \frac{1}{2}\varepsilon^\dagger\varepsilon & 0 \\ 0 & 0 & \mathbb{I} \end{pmatrix} \begin{pmatrix} \mathbb{I} & 0 & -\frac{1}{2}\eta \\ 0 & \mathbb{I} & 0 \\ -\frac{1}{2}\xi & 0 & \mathbb{I} \end{pmatrix}$$

where the mixing angles $\left\{ \begin{array}{l} \theta = m_D^T M_R^{-1} \quad \eta = M_N^{-1} M_X \theta^* \\ \varepsilon = -M_N^{-1} M_X \quad \xi = M_R^{-1} \theta^\dagger M_X^T \end{array} \right\} \quad \theta, \varepsilon, \eta, \xi \ll 1$

$M_3 \subset$ Gauged Flavor $U(1)_X$

Gauged $U(1)_X$ Flavor Model

- Alternatives to R-parity ($M_3, B_3, P_6 \dots$) : remnants of the gauged symmetry
- Identification with the flavor symmetry

R-parity: [Dreiner, Thormeier, Murayama, 2005], B_3, P_6 : [Dreiner, Luhn, Murayama, & Thormeier, Murayama, 2007, 2008]

⇒ $M_3 \supset$ the gauged flavor $U(1)_X$ (cf. Froggatt-Nielsen mechanism) [Froggatt & Nielsen, 1979]

$$\text{X-charge: } X_{\Phi_i} = q_{\Phi} + 3k_{\Phi_i} \Rightarrow M_3 \text{ is a remnant after } U(1)_X \text{ breaking}$$

\nearrow \nwarrow

M_3 charge Generation depending parameter

- The anomaly cancellations for $U(1)_X$ with SM-gauge groups

$$\frac{\mathcal{A}_{CCX}}{k_C} = \frac{\mathcal{A}_{WWX}}{k_W} = \frac{\mathcal{A}_{YYX}}{k_Y}, \quad \mathcal{A}_{YXX} = 0 \quad [\text{Green \& Schwarz, 1984}]$$

the Kac-Moody level: $\begin{cases} k_C = k_W = 1 \\ k_Y \in \mathbb{Q} : \text{parameters} \end{cases}$

The Froggatt-Nielsen mechanism & charge assignments

- We introduce a SM-gauge singlet, **flavon \hat{S}** ($X_\Theta = -3$)

Superpotential:
$$W = g_{ij}^u \hat{u}_i^c \hat{Q}_j \hat{H}_u \left(\frac{\hat{S}}{M_{\text{Pl}}} \right)^{n_{ij}^u} + g_{ij}^d \hat{d}_i^c \hat{Q}_j \hat{H}_d \left(\frac{\hat{S}}{M_{\text{Pl}}} \right)^{n_{ij}^d} + g_{ij}^e \hat{e}_i^c \hat{L}_j \hat{H}_d \left(\frac{\hat{S}}{M_{\text{Pl}}} \right)^{n_{ij}^e}$$
$$+ g_{ij}^\nu \hat{\nu}_i^c \hat{L}_j \hat{H}_u \left(\frac{\hat{S}}{M_{\text{Pl}}} \right)^{n_{ij}^\nu} + \frac{1}{3!} g_{ijk}^\kappa \hat{\nu}_i^c \hat{\nu}_j^c \hat{\nu}_k^c \left(\frac{\hat{S}}{M_{\text{Pl}}} \right)^{n_{ijk}^\kappa} + \mu g^\mu \hat{H}_u \hat{H}_d \left(\frac{\hat{S}}{M_{\text{Pl}}} \right)^{n^\mu}$$

The flavon VEV $\langle S \rangle / M_{\text{Pl}} = \epsilon \sim 0.22 \Rightarrow$ Hierarchical Yukawa Coupling

- The mass hierarchy & mixing angles $\Rightarrow X_{\Phi_i}/k_{\Phi_i}$ are restricted

e.g.) up-type quarks $n_{ij}^u = X_{U_i} + X_{Q_j} + X_{H_u} = \begin{pmatrix} 8 & 7-y & 5-y \\ 5+y & 4 & 2 \\ 3+y & 2 & 0 \end{pmatrix}_{ij} \rightarrow \frac{m_u}{m_t} : \frac{m_c}{m_t} = \epsilon^8 : \epsilon^4$

[Harnik, Larson, Murayama, Thormeier, 1994] [Dreiner, Murayama, Thormeier, 2005]

$y = 0, 1$: the CKM matrix $\Rightarrow k_{\Phi_i}$ is determined except the d.o.f of such parameters

Flavor charge assignments -Neutrino Sector-

- $m_{\text{eff}} \Rightarrow$ the mass differences & PMNS mixings

$$(m_{\text{eff}})_{ij} = -m_D^T M_R^{-1} m_D - M_X^T M_N^{-1} M_X$$

- (i) **RHNs:** $(m_D^T M_R^{-1} m_D)_{ij} \sim \frac{v^2}{m_{\text{soft}}} \sin^2 \beta \epsilon^{2+2k_{H_u} + k_{L_i} + k_{L_j} - k_{N_3}}$
- (ii) **Neutralinos:** $(M_X^T M_N^{-1} M_X)_{ij} \sim -\frac{m_{\text{soft}}}{1 + \tan^2 \beta} \epsilon^{2(k_{H_u} + k_{N_3}) + k_{L_i} + k_{L_j}}$

⇒ Matrix textures can be determined by the flavor charge k_{L_i}

- The dominant contribution is determined by flavor charges $R = \frac{(M_X^T M_N^{-1} M_X)_{ij}}{(m_D^T M_R^{-1} m_D)_{ij}}$

Dominant contribution

- ✓ RHN ($R < 1$)
- ✓ Neutralino ($R > 1$)

Mass Hierarchy:

- ✓ Normal Hierarchy (NH) : $m_1 < m_2 < m_3$
- ✓ Inverted Hierarchy (IH) : $m_3 < m_1 < m_2$

⇒ The charge assignments are classified by the dominant contribution & the mass hierarchy

Flavor charge assignments -Higgs Sector-

- Origin of the supersymmetric Higgs mass term : $\mu \sim \mathcal{O}(m_{\text{EW}})$?

$$W \supset \lambda \left(\frac{\hat{S}}{M_{\text{Pl}}} \right)^{n^\mu} \hat{H}_u \hat{H}_d \Rightarrow \mu = \lambda \epsilon^{n^\mu} M_{\text{Pl}} \quad n^\mu = X_{H_u} + X_{H_d} > 0$$

- The non-minimal coupling in **the Kähler potential** ⇒ effective μ -term

$$K \supset \frac{\hat{Z}}{M_{\text{Pl}}} \left\{ \Theta[-w] \left(\frac{\hat{S}}{M_{\text{Pl}}} \right)^{-n^\mu} + \Theta[w] \left(\frac{\hat{S}}{M_{\text{Pl}}} \right)^{n^\mu} \right\} \hat{H}_u \hat{H}_d + c.c. \Leftrightarrow \mu \sim \epsilon^{n^\mu} m_{\text{soft}}$$

\hat{Z} : anti-chiral superfield
 $\langle F_{\bar{Z}} \rangle / M_{\text{Pl}} = m_{\text{soft}}$

$$\Omega[x] = \begin{cases} 0 & (x < 0) \\ 1 & (x \geq 0) \end{cases}$$

[Giudice & Masiero, 1988]

$$n^\mu = X_{H_u} + X_{H_d} < 0 \Rightarrow \times \mu \sim \epsilon^{n^\mu} M_{\text{Pl}} \quad \circlearrowleft \mu \sim \epsilon^{n^\mu} m_{\text{soft}}$$

- ⇒ The flavor symmetry controls the Higgs mass due to the holomorphy

Soft mass scale: $m_{\text{soft}} \in [1, 10^3] \text{TeV} \Rightarrow n^\mu = -w = -(1 \sim 6)$

Numerical Analysis

flavor charges k_{Φ_i} ($\Phi_i = Q_i, u_i^c, d_i^c, L_i, e_i^c, \nu_i^c, H_{u\alpha}, H_{d\alpha}$)

- Fermion mass hierarchy & mixings $(-15 \leq k_{\Phi_i} \leq 15)$
- Higgs mass : μ -matrix



free parameters $(k_{Q_1}, k_{H_d}, k_{N_3}, x, y, z, p, w, \Omega, \Xi, k_Y)$



131 flavor charge assignments

⇒ the assignments can be divided into 4 classes

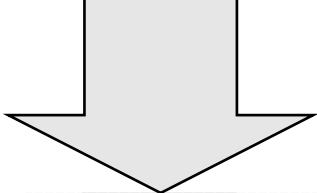
	NH	IH
RHN	26	35
Neutralino	36	39

✓ $k_{N_{1,2}}$: undetermined ⇒ light states of sterile neutrinos

Phenomenological Aspects of the SSM with M_3

Observables in Neutrino Sector

1. Fix the flavor charges / parameters ($k_{Q_1}, k_{H_d}, k_{N_3}, x, y, z, p, w, \Omega, \Xi, k_Y$)



e.g.) RHN-dominated case, NH (model No.4)

- $w = 3 \Rightarrow m_{\text{soft}} \sim \epsilon^{-3} v \sim \mathcal{O}(20\text{TeV})$

2. Fit the VEVs & $\mathcal{O}(1)$ factors to minimize $\chi^2 = \sum_i \left(\frac{x_i - x_i^{\text{obs}}}{\sigma_i} \right)^2$



- $\mathcal{O}(1)$ factor : $\epsilon \leq \mathcal{O}(1) \leq 1/\epsilon$

3. Observables \Rightarrow consistent with the neutrino mixing data

	$\Delta m_{21}^2 \times 10^{-5} [\text{eV}^2]$	$\Delta m_{31}^2 \times 10^{-3} [\text{eV}^2]$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{23}$	$\sin^2 \theta_{13}$	δ_{PMNS}/π	χ^2
bfp \pm err	$7.42^{+0.21}_{-0.20}$	$2.515^{+0.028}_{-0.028}$	$.304^{+0.013}_{-0.012}$	$.573^{+0.018}_{-0.023}$	$.0222^{+0.00068}_{-0.00062}$	$1.078^{+0.2889}_{-0.1389}$	-
result	7.420	2.514	.304	.570	.0222	1.056	0.0

※ Extra-Higgs fields \Rightarrow decoupling from the low energy effective theory

Neutrinoless Double Beta Decay ($0\nu\beta\beta$)

➤ a light sterile neutrino \Rightarrow Neutrinoless double beta decay

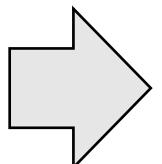
- ✓ The effective Majorana mass from active/sterile neutrinos : $m_{ee} < 61 - 165 \text{ meV}$ [KamLAND-Zen, 2016]

$$m_{ee} = m_{ee}^\nu + m_{ee}^N = \underbrace{\sum_i (U_{\text{PMNS}})_{ei}^2 m_i}_{\text{given}} + \underbrace{\sum_i \Theta_{ei}^2 m_{si} f_\beta(m_{si})}_{\text{Suppression factor}}$$

➤ m_{ee}^N is determined by m_{s1} & $\Theta_{ij} = (m_D^T M_R^{-1})_{ij}$

Sterile neutrino Mass : $m_{s1} \sim (M_R)_{11} = v \epsilon^{-2-w+k_{N_3}+2k_{N_1}}$.

Active-sterile mixing : $\Theta_{e1} \sim (m_D^T M_R^{-1})_{e1} \sim \epsilon^{2+w-k_{N_3}+k_{H_u}+k_{L_1}-k_{N_1}}$

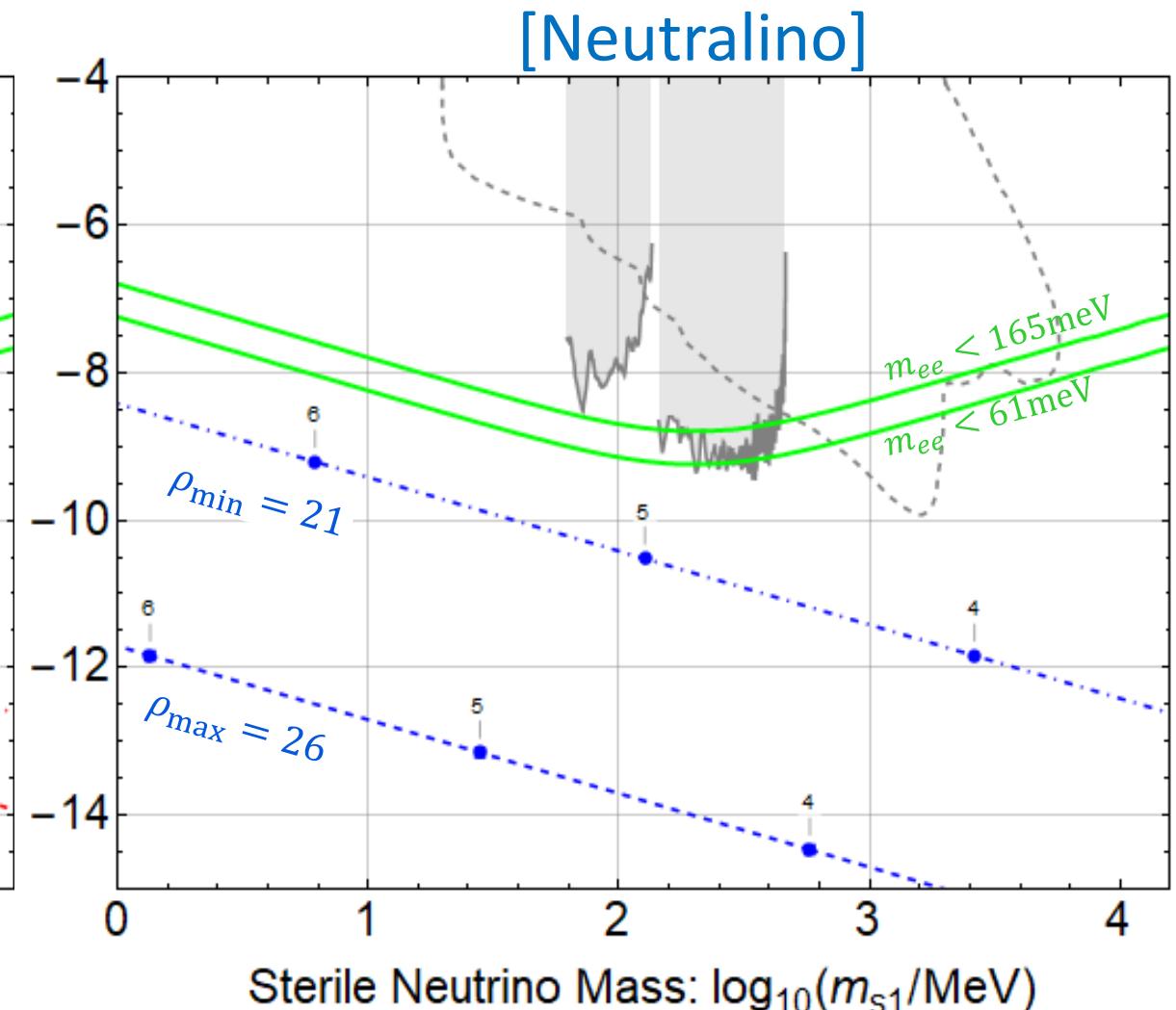
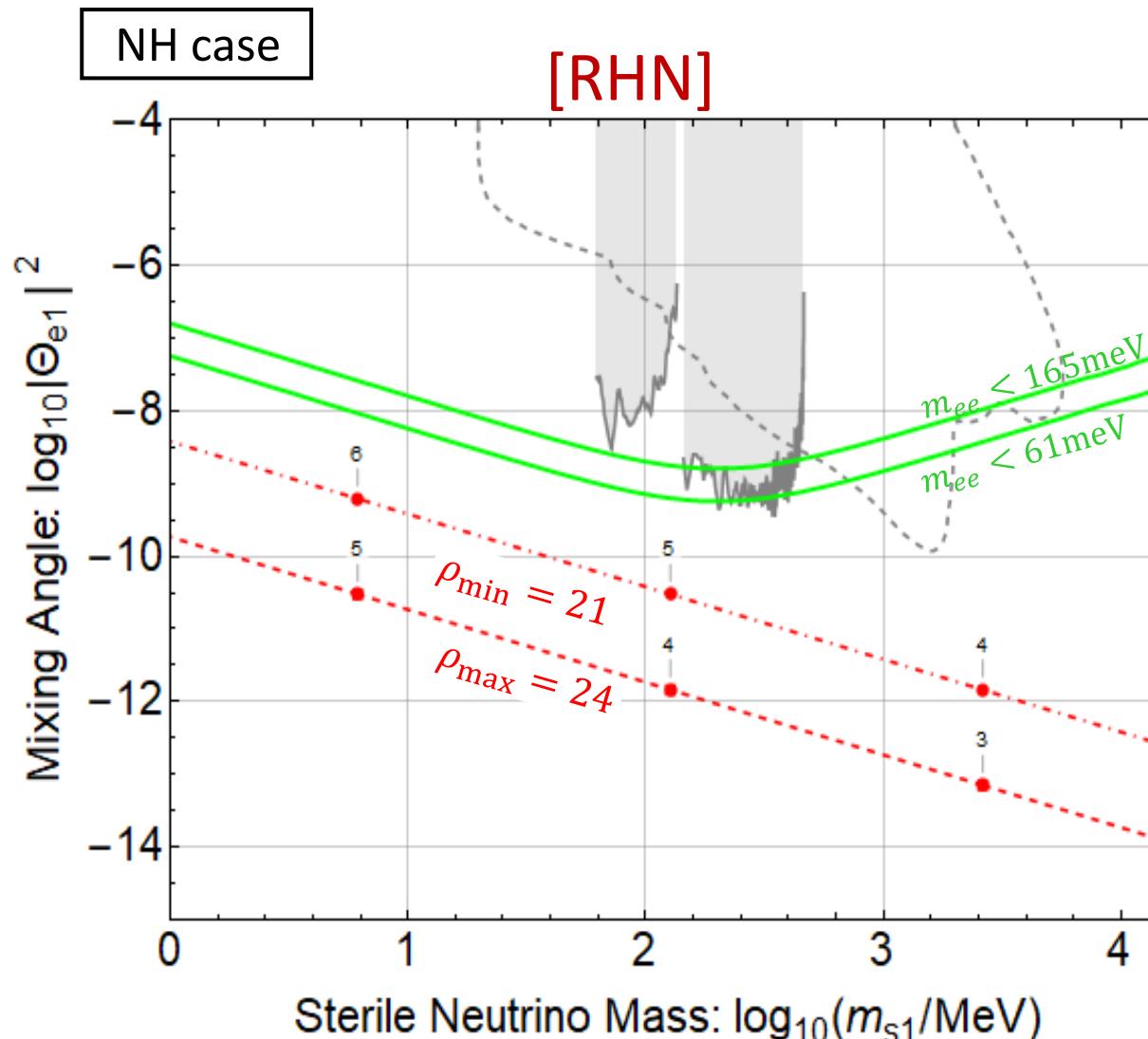


$$m_{ee}^N \sim \sum_i v \epsilon^\rho f_\beta(m_{si}), \quad \rho = 2 + w - k_{N_3} + 2(k_{H_u} + k_{L_1})$$

ρ characterizes our flavor models from the $0\nu\beta\beta$ -decay

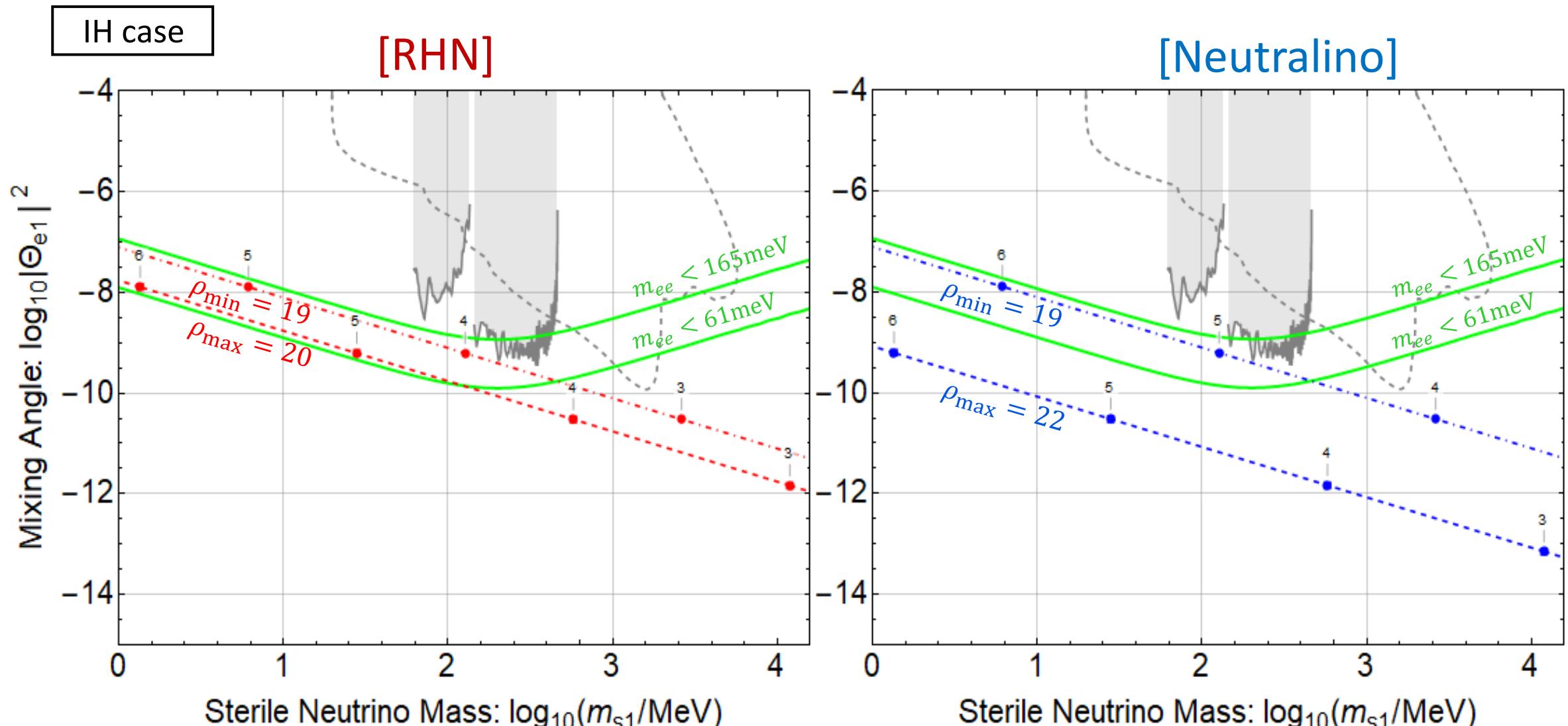
Comparison with the $0\nu\beta\beta$ -decay

HNL search:[SHiP Collaboration, 2018]
 [NA62 collaboration, 2020] [PIENU collaboration, 2018]



Comparison with the $0\nu\beta\beta$ -decay

HNL search:[SHiP Collaboration, 2018]
[NA62 collaboration, 2020] [PIENU collaboration, 2018]



Our flavor models can be characterized by the $0\nu\beta\beta$ decay

Summary & Conclusion

- Matter triality M_3 prohibits B violating operators
 - ✓ The Proton longevity is ensured instead of R-parity
- The seesaw mechanism based on M_3 $(m_{\text{eff}})_{ij} = -m_D^T M_R^{-1} m_D - M_X^T M_N^{-1} M_X$
- $U(1)_X$ controls the Yukawa/Higgs hierarchy
 - ✓ Neutrino observables
 - ✓ Anomaly-free charge assignments can be classified by (RHN, Neutralino) & (NH, IH)
- Flavor models predict the different $m_{s1} \cdot |\theta_{e1}|^2$ ratio
 - ✓ The $0\nu\beta\beta$ -decay distinguishes the flavor charge assignments

Thank you for listening!

Back up slide

Alternative discrete symmetry & B/L violating operators

- The discrete symmetry is derived by the discrete anomaly cancellation

[Ibanez & Ross, 1992]

... Quantum Gravity effects violate the global symmetry

⇒ Baryon triality(B_3), Matter triality(M_3), Proton hexality (P_6)

Table. B/L violating operators under the discrete symmetries

	$\leq \text{dim.-4}$	dim.-5	Comment
R_p	$\hat{\nu}_i^c \hat{\nu}_j^c$	$\hat{Q} \hat{Q} \hat{Q} \hat{L}, \hat{u}^c \hat{d}^c \hat{e}^c \hat{e}^c, \dots$	$(-1)^{3B-L}$
B_3	$\hat{L}_i \hat{Q}_j \hat{d}_k^c, \hat{L}_i \hat{e}_j^c \hat{e}_k^c, \hat{L}_i \hat{H}_u$	$\hat{L}_i \hat{H}_u \hat{L}_j \hat{H}_u, \dots$	w/o RHNs
M_3	$\hat{\nu}_i^c \hat{\nu}_j^c \hat{\nu}_k^c$	NONE	w/ RHNs
P_6	NONE	$\hat{L}_i \hat{H}_u \hat{L}_j \hat{H}_u, \dots$	$R_p \times B_3$

Comparison with the other SSMs

- the next to MSSM : $\mu = \lambda\langle\phi\rangle$, ϕ : a gauge singlet
- $\mu\nu$ SUSY : gauge singlet ϕ is identified with RHN $W = W_{M_3} + \lambda_i \hat{\nu}_i^c \hat{H}_u \hat{H}_d$
[Lopez-Fogliani & Munoz, 2006] [Escudero, Lopez-Fogliani, Munoz, and Ruiz de Austri, 2008]
 - ✓ RH-sneutrino VEVs \Rightarrow Neutrino masses & mixing angles
 - ✓ EW-scale μ -term
- What's the difference the other models?
 - ✓ Motivation: alternative to R-parity? or origin of μ -term?
 - ✓ Origin of μ -term
 - ✓ Light sterile neutrino

Neutral Scalar Potential

$$V_D = \frac{1}{8} (g^2 + g'^2) (v_{L_i} v_{L_i} + v_u^2 + v_d^2)^2$$

$$\begin{aligned} V_F = & \mu^2 v_d^2 + 2\mu v_d v_{R_i} y_{ij}^\nu v_{L_j} \cos(\phi_v - \chi_j + \phi_{R_i}) \\ & + (v_{R_i} y_{ij}^\nu v_{L_j}) (v_{R_l} y_{lm}^\nu v_{L_m}) \cos(\chi_j - \chi_m - \phi_{R_i} + \phi_{R_l}) \\ & + \mu^2 v_u^2 + v_u^2 v_{R_i} (y^\nu y^{\nu T})_{ij} v_{R_j} \cos(\phi_{R_i} - \phi_{R_j}) + v_u^2 v_{L_i} (y^{\nu T} y^\nu)_{ij} v_{L_j} \cos(\chi_i - \chi_j) \\ & + v_u v_{L_m} y_{mi}^{\nu T} \kappa_{ijk} v_{R_j} v_{R_k} \cos(\chi_m + \phi_{R_j} + \phi_{R_k}) \\ & + \frac{1}{4} \kappa_{ijk} \kappa_{ilm} v_{R_j} v_{R_k} v_{R_l} v_{R_m} \cos(\phi_{R_j} + \phi_{R_k} - \phi_{R_l} - \phi_{R_m}) \\ V_{\text{soft}} = & m_{H_u}^2 v_u^2 + m_{H_d}^2 v_d^2 + (m_{\tilde{L}}^2)_{ij} v_{L_i} v_{L_j} \cos(\chi_i - \chi_j) + (m_{\tilde{\nu}}^2)_{ij} v_{R_i} v_{R_j} \cos(\phi_{R_i} - \phi_{R_j}) \\ & + b v_u v_d \cos \phi_v + 2 A_{ij}^\nu v_{R_i} v_{L_j} v_u \cos(\chi_j - \phi_{R_i}) + \frac{1}{3} A_{ijk}^\kappa v_{R_i} v_{R_j} v_{R_k} \cos(\phi_{R_i} + \phi_{R_j} + \phi_{R_k}). \end{aligned}$$

Neutral Scalar Potential

➤ Soft SUSY Breaking Lagrangian

$$\begin{aligned}-\mathcal{L}_{\text{soft}} = & \left(m_{\tilde{Q}}^2\right)_{ij} \tilde{Q}_i^* \tilde{Q}_j + \left(m_{\tilde{u}}^2\right)_{ij} \tilde{u}_i^{c*} \tilde{u}_j^c + \left(m_{\tilde{d}}^2\right)_{ij} \tilde{d}_i^{c*} \tilde{d}_j^c + \left(m_{\tilde{L}}^2\right)_{ij} \tilde{L}_i^* \tilde{L}_j + \left(m_{\tilde{e}}^2\right)_{ij} \tilde{e}_i^{c*} \tilde{e}_j^c \\& + \left(m_{\tilde{\nu}}^2\right)_{ij} \tilde{\nu}_i^{c*} \tilde{\nu}_j^c + m_{H_u}^2 H_u^* H_u + m_{H_d}^2 H_d^* H_d + b [H_u H_d + c.c.] \\& + \left[A_{ij}^u \tilde{u}_i^c \tilde{Q}_j H_u + A_{ij}^d \tilde{d}_i^c \tilde{Q}_j H_d + A_{ij}^e \tilde{e}_i^c \tilde{L}_j H_d + A_{ij}^\nu \tilde{\nu}_i^c \tilde{L}_j H_u + \frac{1}{3!} A_{ijk}^\kappa \tilde{\nu}_i^c \tilde{\nu}_j^c \tilde{\nu}_k^c + c.c.\right] \\& - \frac{1}{2} \left(M_3 \tilde{\lambda}_3 \tilde{\lambda}_3 + M_2 \tilde{\lambda}_2 \tilde{\lambda}_2 + M_1 \tilde{\lambda}_1 \tilde{\lambda}_1 + c.c.\right)\end{aligned}$$

A_{ijk}^κ : A-term for the RHN cubic coupling $(m_{\tilde{\nu}}^2)_{ij}$: soft mass for the RH-sneutrinos

➤ Extremum condition for the RH-sneutrino $\langle \tilde{\nu}_i^c \rangle = v_{Ri} e^{i\phi_{Ri}}$

$$\begin{aligned}t_{R_i}^0 \sim & \kappa_{nij} \kappa_{nlm} v_{R_j} v_{R_l} v_{R_m} \cos(\phi_{R_i} + \phi_{R_j} - \phi_{R_l} - \phi_{R_m}) \\& + 2(m_{\tilde{\nu}}^2)_{ij} v_{R_j} \cos(\phi_{R_i} - \phi_{R_j}) + \frac{1}{3} A_{ijk}^\kappa v_{R_j} v_{R_k} \cos(\phi_{R_i} + \phi_{R_j} + \phi_{R_k}) = 0. \quad \rightarrow \quad v_{Ri} \sim m_{\text{soft}}\end{aligned}$$

Neutral fermion mass matrix

➤ Neutral fermion: $\Psi^T = (\psi_n^T, \nu_{Li}, \nu_{Rj}), \psi_n = (\widetilde{\lambda}_1^0, \widetilde{\lambda}_2^0, \widetilde{H}_d^0, \widetilde{H}_u^0)$

$$\rightarrow \bar{\Psi} \mathcal{M}_n \Psi = (\bar{\psi}_n^c \quad \bar{\nu}_L^c \quad \bar{\nu}_R) \begin{pmatrix} M_N & M_X & 0 \\ M_X^T & 0 & m_D^T \\ 0 & m_D & M_R \end{pmatrix} \begin{pmatrix} \psi_n \\ \nu_L \\ \nu_R^c \end{pmatrix}$$

$$\mathcal{M}_n = \begin{pmatrix} M_1 & 0 & -A\langle H_d^0 \rangle^* & A\langle H_u^0 \rangle^* & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_2 & B\langle H_d^0 \rangle^* & -B\langle H_u^0 \rangle^* & 0 & 0 & 0 & 0 & 0 & 0 \\ -A\langle H_d^0 \rangle^* & B\langle H_d^0 \rangle^* & 0 & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ A\langle H_u^0 \rangle^* & -B\langle H_u^0 \rangle^* & \mu & 0 & y_{k1}^\nu \langle \tilde{\nu}_k^c \rangle & y_{k2}^\nu \langle \tilde{\nu}_k^c \rangle & y_{k3}^\nu \langle \tilde{\nu}_k^c \rangle & 0 & 0 & 0 \\ 0 & 0 & 0 & y_{k1}^\nu \langle \tilde{\nu}_k^c \rangle & 0 & 0 & 0 & y_{11}^\nu \langle H_u^0 \rangle & y_{21}^\nu \langle H_u^0 \rangle & y_{31}^\nu \langle H_u^0 \rangle \\ 0 & 0 & 0 & y_{k2}^\nu \langle \tilde{\nu}_k^c \rangle & 0 & 0 & 0 & y_{12}^\nu \langle H_u^0 \rangle & y_{22}^\nu \langle H_u^0 \rangle & y_{32}^\nu \langle H_u^0 \rangle \\ 0 & 0 & 0 & y_{k3}^\nu \langle \tilde{\nu}_k^c \rangle & 0 & 0 & 0 & y_{13}^\nu \langle H_u^0 \rangle & y_{23}^\nu \langle H_u^0 \rangle & y_{33}^\nu \langle H_u^0 \rangle \\ 0 & 0 & 0 & 0 & y_{11}^\nu \langle H_u^0 \rangle & y_{12}^\nu \langle H_u^0 \rangle & y_{13}^\nu \langle H_u^0 \rangle & \kappa_{11k} \langle \tilde{\nu}_k^c \rangle & \kappa_{12k} \langle \tilde{\nu}_k^c \rangle & \kappa_{13k} \langle \tilde{\nu}_k^c \rangle \\ 0 & 0 & 0 & 0 & y_{21}^\nu \langle H_u^0 \rangle & y_{22}^\nu \langle H_u^0 \rangle & y_{23}^\nu \langle H_u^0 \rangle & \kappa_{21k} \langle \tilde{\nu}_k^c \rangle & \kappa_{22k} \langle \tilde{\nu}_k^c \rangle & \kappa_{23k} \langle \tilde{\nu}_k^c \rangle \\ 0 & 0 & 0 & 0 & y_{31}^\nu \langle H_u^0 \rangle & y_{32}^\nu \langle H_u^0 \rangle & y_{33}^\nu \langle H_u^0 \rangle & \kappa_{31k} \langle \tilde{\nu}_k^c \rangle & \kappa_{32k} \langle \tilde{\nu}_k^c \rangle & \kappa_{33k} \langle \tilde{\nu}_k^c \rangle \end{pmatrix}$$

where $A = \sqrt{(g^2 + g'^2)/2} \sin \theta_w$ $B = \sqrt{(g^2 + g'^2)/2} \cos \theta_w$

Anomaly Cancellation Conditions

Anomaly coefficients

$$\text{SU}(3)_C - \text{SU}(3)_C - \text{U}(1)_X: \quad \mathcal{A}_{CCX} = \sum_i [2X_{Qi} + X_{Ui} + X_{Di}]$$

$$\text{SU}(2)_L - \text{SU}(2)_L - \text{U}(1)_X: \quad \mathcal{A}_{WWX} = \sum_i [3X_{Qi} + X_{Li}] + X_{H_u} + X_{H_d} + \Omega$$

$$\text{U}(1)_Y - \text{U}(1)_Y - \text{U}(1)_X: \quad \mathcal{A}_{YYX} = \frac{1}{6} \sum_i [X_{Qi} + 8X_{Ui} + 2X_{Di} + 3X_{Li} + 6X_{Ei}] + \frac{1}{2} (X_{H_u} + X_{H_d}) + \Omega$$

$$\text{U}(1)_Y - \text{U}(1)_X - \text{U}(1)_X: \quad \mathcal{A}_{YXX} = \sum_i [X_{Qi}^2 - 2X_{Ui}^2 + X_{Di}^2 - X_{Li}^2 + X_{Ei}^2] - X_{H_u}^2 + X_{H_d}^2 + \Xi$$

➤ Ω & Ξ : the contributions from the extra-Higgs doublets

$$\Omega = \sum_{a=2}^{N_h} 3(k_{H_{ua}} + k_{H_{da}}), \quad \Xi = \sum_{a=2}^{N_h} 3(k_{H_{ua}} + k_{H_{da}})(-2 + 3(k_{H_{ua}} - k_{H_{da}}))$$

➤ the Green-Schwarz mechanism : $\frac{\mathcal{A}_{CCX}}{k_C} = \frac{\mathcal{A}_{WWX}}{k_W} = \frac{\mathcal{A}_{YYX}}{k_Y}, \quad \mathcal{A}_{YXX} = 0$

cf.) SU(5) unification : $k_C = k_W = 3k_Y/5 = 1$

Flavor charge assignments -Quark & Charged-Lepton-

up-quark & down-quark

$$n_{ij}^u = \begin{pmatrix} 8 & 7-y & 5-y \\ 5+y & 4 & 2 \\ 3+y & 2 & 0 \end{pmatrix}_{ij}$$

$$n_{ij}^d = \begin{pmatrix} 4+x & 3-y+x & 1-y+x \\ 3+y+x & 2+x & x \\ 3+y+x & 2+x & x \end{pmatrix}_{ij}$$

charged-lepton

$$n_{ij}^e = \text{diag}(4+z+x, 2+x, x)_{ij}$$

✓ Parameter:

- ✓ $x = 0 \sim 3 : \epsilon^x \sim \left(\frac{m_b}{m_t}\right) \tan\beta$
- ✓ $y = 0, 1$: CKM mixing
- ✓ $z = 0, 1$: charged-lepton mass

[Harnik, Larson, Murayama, Thormeier, 1994]

[Dreiner, Murayama, Thormeier, 2005]

mass hierarchy:

$$\begin{aligned} \frac{m_u}{m_t} : \frac{m_c}{m_t} &= \epsilon^8 : \epsilon^4, \\ \frac{m_d}{m_b} : \frac{m_s}{m_b} &= \epsilon^4 : \epsilon^2, \\ \frac{m_e}{m_\tau} : \frac{m_\mu}{m_\tau} &= \epsilon^{4+z} : \epsilon^2. \end{aligned}$$

CKM mixing angle:

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & \epsilon^{1+y} & \epsilon^{3+y} \\ \epsilon^{1+y} & 1 & \epsilon^2 \\ \epsilon^{3+y} & \epsilon^2 & 1 \end{pmatrix}$$

⇒ Flavor charges can be written by the parameters (x, y, z)

Flavor charge assignments -Neutrino Sector-

- **seesaw formula:** $(m_{\text{eff}})_{ij} = -m_D^T M_R^{-1} m_D - M_X^T M_N^{-1} M_X$
 - k_{L_i} determine the neutrino mass & mixing angle
 - e.g.) RHN-dominated : $(m_D^T M_R^{-1} m_D)_{ij} \sim \frac{v^2}{m_{\text{soft}}} \sin^2 \beta \epsilon^{2+2k_{H_u} + k_{L_i} + k_{L_j} - k_{N_3}}$
- The matrix texture of m_{eff} is determined by k_{L_i} & we require ...
- Mass Hierarchy: $\epsilon^{2(k_{L_2} - k_{L_3})} = \frac{m_2}{m_3} \sim \frac{\sqrt{\Delta m_{21}^2}}{\sqrt{\Delta m_{21}^2 + \Delta m_{32}^2}}$ (NH)
 - Leptonic mixing: $\sin \theta_{ij} \sim \epsilon^{k_{L_i} - k_{L_j}}$
 - Mass scale: $(m_D^T M_R^{-1} m_D) \sim v \sin^2 \beta \epsilon^{2+w-k_{N_3}+2(k_{H_u}+k_{L_3})} \sim 0.05 \text{eV}$

Parameter list

Parameters	Implication	Range
x	$\tan\beta = \frac{m_t}{m_b} \epsilon^x$	$0 \sim 3$
y	CKM mixing	0,1
z	C-lepton mass hierarchy	0,1
p	PMNS mixing	0,1
w	$m_{\text{soft}} \sim v \epsilon^{-w}$	$1 \sim 6$
Ω, Ξ	Ex. Higgs contribution to anomaly	$k_{\Phi_i} \in [-15, 15]$
k_Y	Hypercharge normalization	Positive rational #

Higgs sector

➤ The Higgs fields $H_{u\alpha}$ & $H_{d\beta}$ ($\alpha, \beta = 1 \sim N_h$) acquire the μ -term coming from

- **Superpotential** : $\mu_{\alpha\beta} \sim \epsilon^{n_{\alpha\beta}^\mu} M_{\text{Pl}}$ if $n_{\alpha\beta}^\mu = X_{H_{u\alpha}} + X_{H_{d\beta}} > 0$

- **Kähler potential** : $\mu_{\alpha\beta} \sim \epsilon^{|n_{\alpha\beta}^\mu|} m_{\text{soft}}$

cf.) $\mu_{\alpha\beta} \sim \begin{pmatrix} m_{\text{soft}} \\ M_{\text{Pl}} \end{pmatrix}$

SM Higgs
Ex. Higgs

\Rightarrow The Higgs hierarchy is controlled

➤ The extra Higgs Contribution to the anomaly cancellation conditions:

The parameter: $\Omega = \sum_{a=2}^{N_h} 3(k_{H_{ua}} + k_{H_{da}})$, $\Xi = \sum_{a=2}^{N_h} 3(k_{H_{ua}} + k_{H_{da}})(-2 + 3(k_{H_{ua}} - k_{H_{da}}))$

$\Rightarrow (\Omega, \Xi)$ determine some examples of the flavor charges ($N_h \leq 5$)

Observables in Neutrino Sector

e.g.) Model (No.4) **RHN-dominated, NH**

- Parameters : $(k_{Q_1}, k_{H_d}, k_{N_3}, x, y, z, w, p, \Omega, \Xi, k_Y) = (2, -7, 3, 1, 0, 1, 3, 0, 30, -1860, 11/10)$
 - ✓ $x = 1 \rightarrow \tan\beta \sim \frac{m_t}{m_b} \epsilon^x \sim 10$
 - ✓ Soft SUSY breaking scale: $m_{\text{soft}} \sim \epsilon^{-3} v \sim \mathcal{O}(20\text{TeV})$
 - ✓ $k_Y \neq 5/3$: non-standard gauge coupling unification
- $k_{N_1} = k_{N_3} + 1$ & $k_{N_2} = k_{N_3}$
- Complex VEVs : $v_{Ri} \sim m_{\text{soft}}, \phi_{u,d,R_i} \in [0, 2\pi]$

□ Sterile neutrino masses $(m_{s1}, m_{s2}, m_{s3}) = (661, 6.28 \times 10^3, 1.82 \times 10^4) \text{ GeV}$

□ The Higgs mass (1+3 gene.) $\mu_{ij} \sim \begin{pmatrix} \epsilon^3 m_{\text{soft}} & 0 & 0 & 0 \\ 0 & \epsilon^2 M_{\text{Pl}} & \epsilon^3 M_{\text{Pl}} & \epsilon^3 M_{\text{Pl}} \\ 0 & \epsilon^2 M_{\text{Pl}} & \epsilon^3 M_{\text{Pl}} & \epsilon^3 M_{\text{Pl}} \\ 0 & \epsilon^2 M_{\text{Pl}} & \epsilon^3 M_{\text{Pl}} & \epsilon^3 M_{\text{Pl}} \end{pmatrix}_{ij} \rightarrow \text{Decoupling}$

Neutrinoless Double Beta Decay ($0\nu\beta\beta$)

- A sterile neutrino w/ $m_{s1} \sim \mathcal{O}(100\text{MeV})$ induces the $0\nu\beta\beta$ decay

✓ The half lifetime of the $0\nu\beta\beta$ decay : $\tau_{1/2}^{-1} = G |\mathcal{M} m_{ee}|$

✓ The effective Majorana mass from **active/sterile** neutrinos :

$$m_{ee} = m_{ee}^\nu + m_{ee}^N = \sum_i (U_{\text{PMNS}})_{ei}^2 m_i + \sum_i \Theta_{ei}^2 m_{si} f_\beta(m_{si}),$$

Suppression factor: $f_\beta(m_{si}) = \frac{\Lambda_\beta^2}{m_{si}^2 + \Lambda_\beta^2}$ (Fermi momentum : $\Lambda_\beta = 200\text{MeV}$)

cf.) $\tau_{1/2} > 1.07 \times 10^{26} \text{yr}$ @ 90% CL [KamLAND-Zen, 2016] $\Rightarrow m_{ee} < 61 - 165\text{meV}$

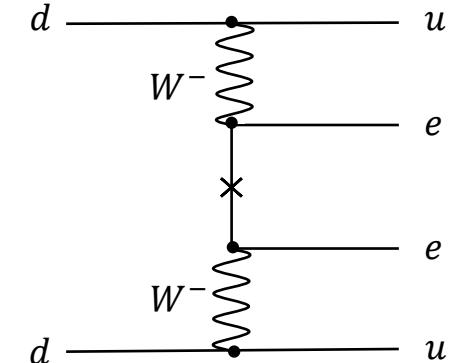


Fig. the $0\nu\beta\beta$ decay

- The RH-sneutrinos : loop suppressed contribution

✓ Sparticle contributions are suppressed

[Hirsch, Klapdor-Kleingrothaus & Kovalenko, 1997]

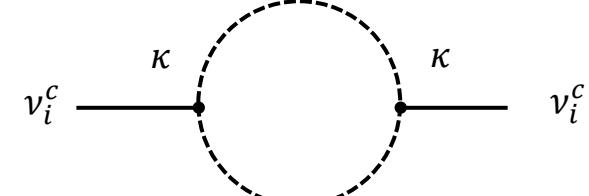


Fig. Majorana mass @ 1-loop