Non-local S-matrix in a solvable model

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Upcoming article:
DK, A. Pilaftsis

Outline

Why non-local S-matrix?

Non-locality is connected to finite volume effects.

[1] J. E. Campagne, Phys. Lett. B **400** (1997), 135-144, A. Ioannisian and A. Pilaftsis, Phys. Rev. D **59** (1999), 053003 [arXiv:hep-ph/9809503 [hep-ph]].

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- Displaced vertex searches. Deeper understanding of how a mediator travels.
- Other possibilities: QFT picture of diffraction, effective interactions (*e.g.* non-local chiral ET[3] or jets), etc.

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$$\mathcal{L}_{int}(x) = \lambda\, S_1(x)\chi_1(x)\ \Phi(x) + g\, S_2(x)\chi_2(x)\ \Phi(x)\ .$$

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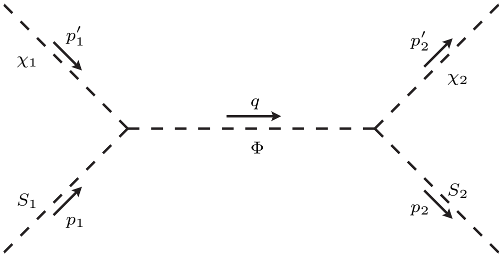

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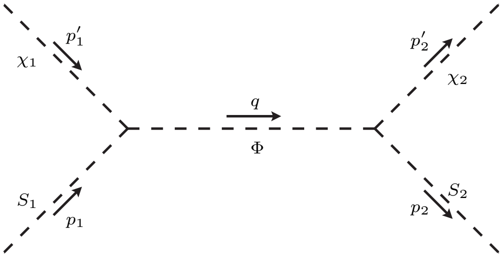

$$T(p,k) = \lambda\, g \int \frac{d^4 q}{(2\pi)^4}\ \frac{1}{q^2 - m_\Phi^2 + i\epsilon}\overbrace{\int d^4x\ d^4y\ e^{-i(p-q)\cdot x} e^{i(k-q)\cdot y}}^{\delta^{(4)}(p-q)\delta^{(4)}(k-q)}\ ,$$

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$$e^{-[(x-\langle x\rangle)\cdot\delta p]^2} \qquad\qquad e^{-[(y-\langle y\rangle)\cdot\delta k]^2}$$

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Simplifications

We can simplify the calculations by assuming *steady state* and *spherical symmetry*; $\delta p^0, \delta k^0 \to 0$, $\delta p^i = 1/\delta x$, and $\delta k^i = 1/\delta y$.

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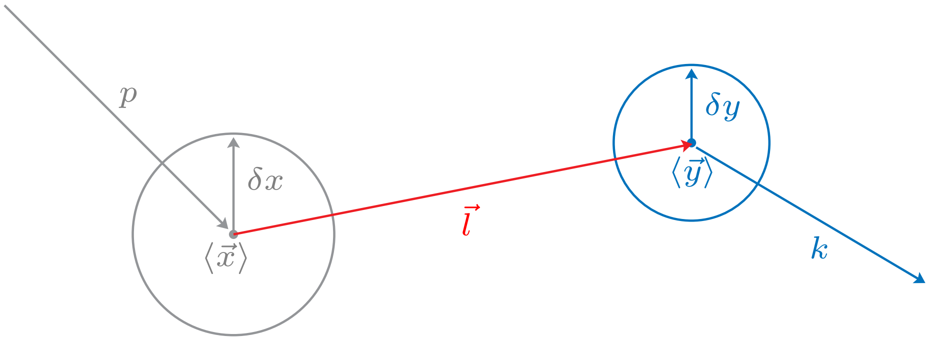
$$\begin{aligned} T(p,k;\vec{l},\delta x,\delta y) =& (2\pi)\ \delta(p^0 - k^0)\ \lambda\, g \\ & \int d^3\vec{x}\ e^{i\vec{p}\cdot\vec{x} - \vec{x}^2/\delta x^2} \\ & \int d^3\vec{y}\ e^{-i\vec{k}\cdot\vec{y} - \vec{y}^2/\delta y^2} \\ & \int \frac{d^3\vec{q}}{(2\pi)^3}\ \frac{e^{-i\vec{q}\cdot(\vec{x}-\vec{y}-\vec{l})}}{\tilde{q}^2 - |\vec{q}|^2 + i\epsilon}\ , \end{aligned}$$

with $\vec{l} = \langle\vec{y}\rangle - \langle\vec{x}\rangle$ and $\tilde{q}^2 = p^{0\,2} - m_\Phi^2$.

Physical representation



Analytical results

First analytical result

After some algebra, the amplitude becomes

$$T(p,k;\vec{l},\delta x,\delta y) \sim \delta(p^0-k^0)\,\frac{\delta x^3\,\delta y^3}{|\vec{L}|}\,e^{-\left[\left(|\vec{p}|^2+\tilde{q}^2\right)\delta x^2+\left(|\vec{k}|^2+\tilde{q}^2\right)\delta y^2\right]/4}$$
$$\left[e^{i\tilde{q}|\vec{L}|}\,\mathrm{Erfc}\left(z_-\right)-e^{-i\tilde{q}|\vec{L}|}\,\mathrm{Erfc}\left(z_+\right)\right]\ ,$$

where $z_\pm = -\dfrac{i}{2}\tilde{q}\,\delta l \pm \dfrac{|\vec{L}|}{\delta l}$, $\vec{L}=\vec{l}-\frac{i}{2}\left(\vec{p}\,\delta x^2+\vec{k}\,\delta y^2\right)$,
$\delta l^2 = \delta x^2+\delta y^2$, and $|\vec{L}| = \sqrt{\vec{L}\cdot\vec{L}}$.

From this form we observe that:

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From this form we observe that:

- Amplitude is finite in the physical region.
- Momentum is not conserved, and the distribution of $\vec{k}$ depends on $\delta x, \delta y$.
- In the limit $\delta x, \delta y \to 0$, $T \sim \delta x^3 \delta y^3 \; e^{i\tilde{q}|\vec{l}|}/|\vec{l}| \; \delta(p^0 - k^0)$.[4]

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Further simpification: Recover conservartion of momentum

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Note:

Momenta at each vertex suffer from uncertainties.
Introduce matrix element as usual

$$M \sim \frac{\delta l^3}{|\vec{L}|} e^{-\delta l^2\left(|\vec{k}|^2+\tilde{q}^2\right)/2} \left[e^{i\tilde{q}|\vec{L}|}\,\mathrm{Erfc}\left(z_-\right) - e^{-i\tilde{q}|\vec{L}|}\,\mathrm{Erfc}\left(z_+\right)\right] .$$

Features

In the far-field (*Fraunhofer*) region we recover the inverse-square law (similar to $\delta l \to 0$). The matrix element becomes:

$$M \sim \delta l^3\ \frac{e^{i\tilde{q}|\vec{l}|}}{|\vec{l}|}\ e^{-\frac{1}{4}\left(\,|\vec{k}|-\tilde{q}\,\hat{l}\,\right)^2 \delta l^2}\ .$$

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Observations:

- Inverse square law.
- Suppressed backwards direction.
- Finite.
- Oscillations of mixed mediators.
- Off-shell mediator may be the most probable outcome.

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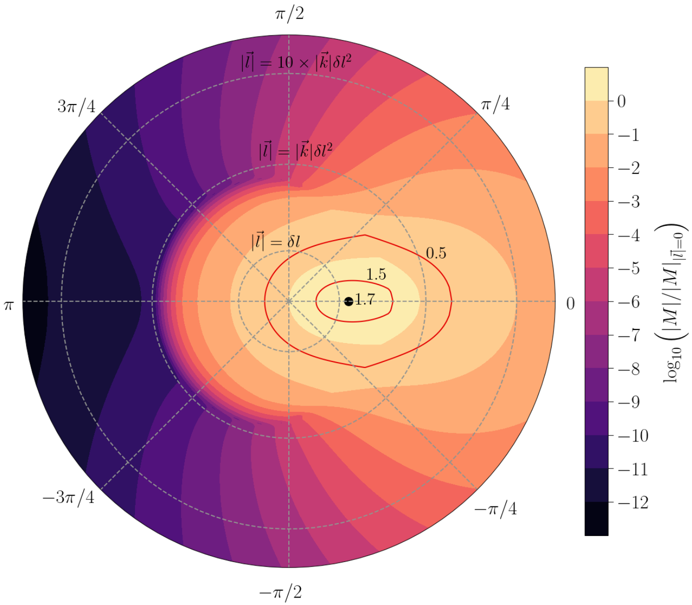
Observations:

- $\delta l \to \infty \Rightarrow M \sim i\pi\ \delta_+(|\vec{k}|^2 - \tilde{q}^2) + \mathcal{P}\left\{\dfrac{1}{|\vec{k}|^2 - \tilde{q}^2}\right\}$.
- Finite.
- Oscillations of mixed mediators.
- Slightly shifted maximum.


$\pi/2$
$3\pi/4$
$\pi/4$
π
0
$-3\pi/4$
$-\pi/4$
$-\pi/2$
$|\vec{l}| = 10 \times |\vec{k}|\delta l^2$
$|\vec{l}| = |\vec{k}|\delta l^2$
$|\vec{l}| = \delta l$
0.5
1.5
1.7
0
−1
−2
−3
−4
−5
−6
−7
−8
−9
−10
−11
−12
$\log_{10}\left(|M|/|M|_{|\vec{l}|=0}\right)$

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- ✓ Distinct features, *i.e.* falsifiable.

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- **Experiments**.

Thank you!