Connected Vacua of String Models

Kang-Sin Choi

Ewha Womans University

In collaboration with
Stephen Angus (APCTP), Tatsuo Kobayashi (Hokkaido)

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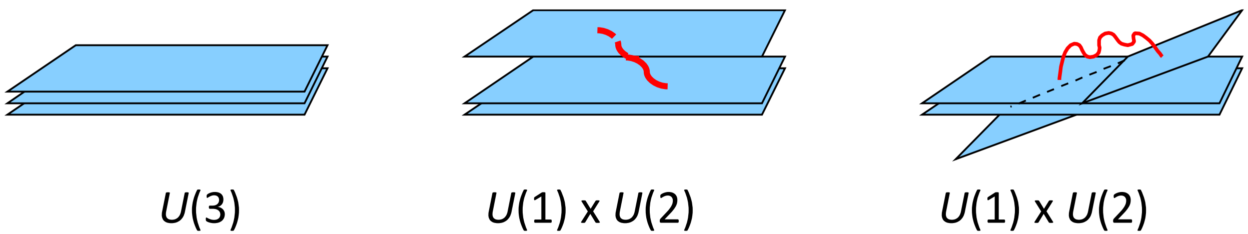
Bifundamental

- Bifundamental nature of the SM fields can be nicely explained by open string en ding on intersecting branes.



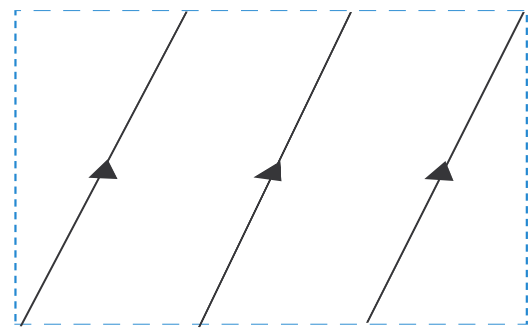
- Geometrical understanding.
  1. Local gauge symm enhancement to $U(1)$x$U(2)$ -> $U(3)$.
     W bosons, (1,-1,0)..., become light.
  2. Chiral matter: Either 2_1 or 2^*_{-1} exclusively survives.
  3. Problems in D-brane construction using perturbative srings.

cf. Not enough for down-quark Yukawa
3,4-pronged strings at string coupling [DeWolfe et al.]
F-theory unifies it with closed string theory [Vafa]

Tilted D-branes

- T-duality along i-th direction: A_i <-> X
- Const A (Wilson line): translation
- Const F: slope

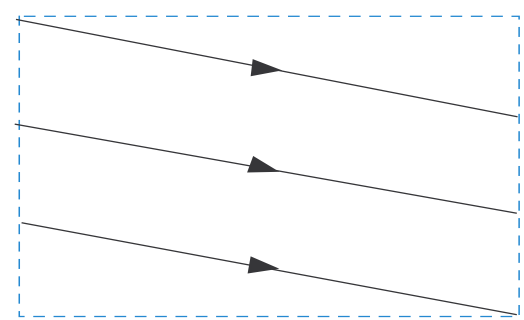
$$F_{12} = \frac{2\pi}{A} \; (3)$$ <->



D-brane wrapping (1,3)-cycle
RR charges 1 and 3

- Toron quantization: closed curve.
- Generalization: homological cycle.

$$F_{12} = \frac{2\pi}{A} \begin{pmatrix} -\frac{1}{3} & & \\ & -\frac{1}{3} & \\ & & -\frac{1}{3} \end{pmatrix}$$ <->



Torons are magnetized brane

- 2-cycles in T^4 induced from 1-cycles of T^2
- (n, m) cycle -> magnetized flux
- 4D

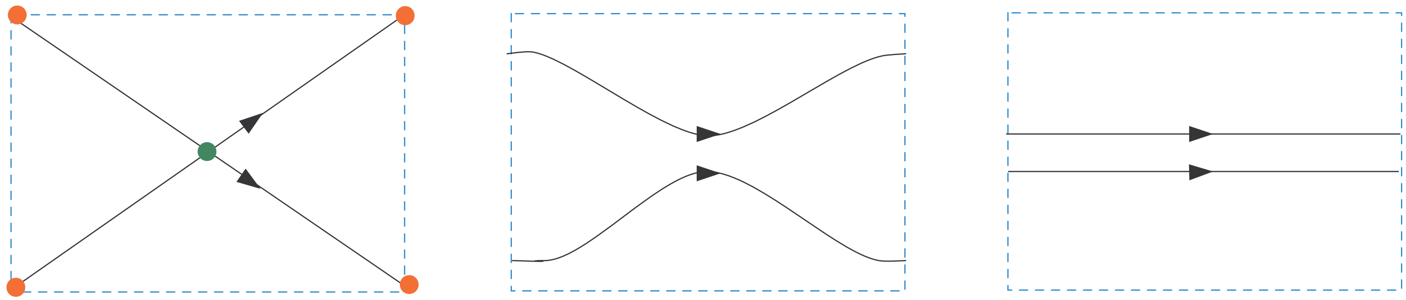
$$\begin{aligned}(n^1, m^1)(n^2, m^2) &= (n^1 n^2, n^1 m^2, m^1 n^2, m^1 m^2) \\ &= (N, c_1^{12}, c_2^{34}, c_2)\end{aligned}$$

$$F_{12} = \frac{2\pi}{A_2} \begin{pmatrix} \frac{m_1^1}{n_1^1} \mathbf{1}_{n_1^1 n_1^2} & & \\ & \ddots & \\ & & \frac{m_k^1}{n_k^1} \mathbf{1}_{n_k^1 n_k^2} \end{pmatrix} \qquad F_{34} = \frac{2\pi}{A_{34}} \begin{pmatrix} \frac{m_1^2}{n_1^2} \mathbf{1}_{n_1^1 n_1^2} & & \\ & \ddots & \\ & & \frac{m_k^2}{n_k^2} \mathbf{1}_{n_k^1 n_k^2} \end{pmatrix}$$

- Cf. T-branes

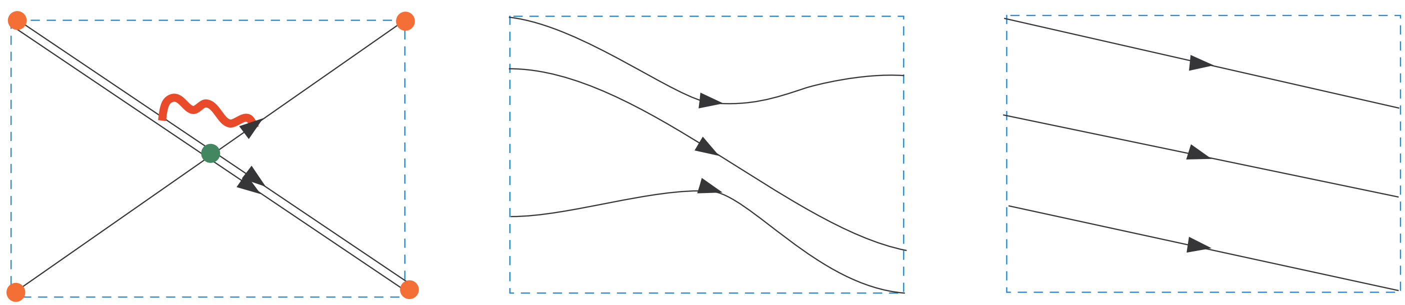
Brane recombination

[Ibanez et al.] [K. Hashimoto, W. Taylor] [Kim, KSC] [KSC]…



(1,1) + (1,-1)
U(1) x U(1)

(1,0) + (1,0)
U(1)xU(1) <-> U(2)



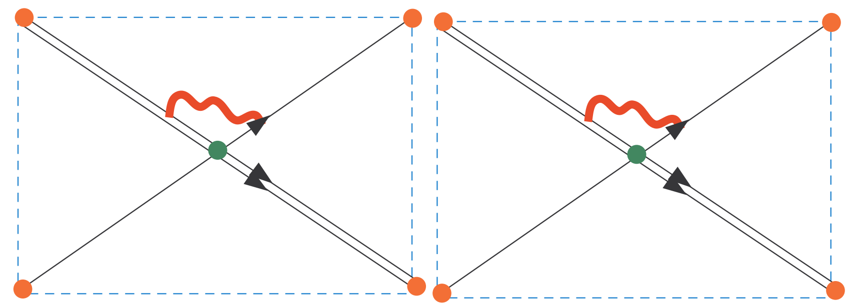
(1,1) + 2(1,-1)
U(1) x U(2) <-> $U(1)^3$

(3,-1)
U(3)

SUSY [Marino, Minahan, Moore, Strominger]

- In 4D SUSY cond: $\theta_{12} \pm \theta_{34} = \text{const.}$

$$f_{12} + f_{34} = \mu(1 - f_{12} f_{34})$$



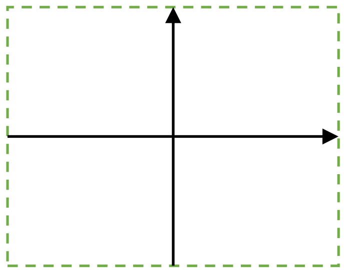
- MMMS equation means projection <-> SUSY cond. of intersecting branes.

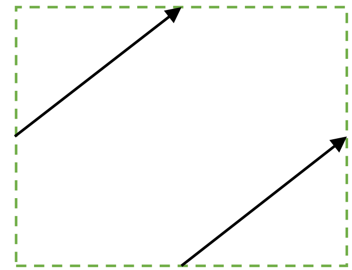
$$\begin{aligned} H &= \tau_9 V_3 V_4 \mathrm{Tr}[(\mathbf{1} + f_{67})(\mathbf{1} + f_{89}^2)]^{1/2} \\ &= \tau_9 V_3 V_4 \mathrm{Tr}[\mathbf{1} + f_{67} + f_{89} + f_{67} f_{89}]^{1/2} \\ \overset{f_{67} = f_{89}}{} &= \tau_9 V_3 V_4 \mathrm{Tr}[\mathbf{1} + f_{67} f_{89}] \\ &= \tau_9 V_3 V_4 [N + c_2] \end{aligned}$$

- DBI action only measures the total Chern numbers.
- 6D: N, $c_{1,45}$, $c_{1,67}$, $c_{1,89}$, c_2.

Small toron

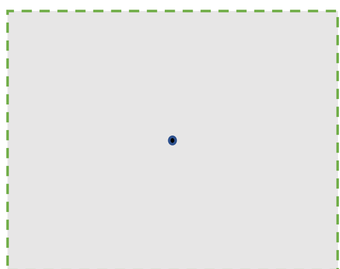
- Brane and magnetized brane





(1,0) + (0,1)
U(1) x U(1)

(1,1)
U(1)



- Small (zero size) toron = D-brane

$$F_{12} = 2\pi\delta^{(2)}(x,y)$$

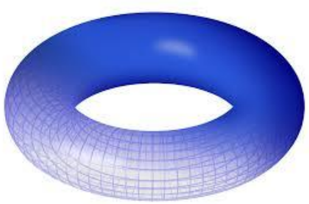
$$F_{12} = \frac{2\pi}{A}$$

- Dual to small instanton transition [Witten] [Aspinwall, Morrison]

Global consistency condition

In string theory,
anomaly cancellation is promoted to global consistency condition
from one-loop diagram.

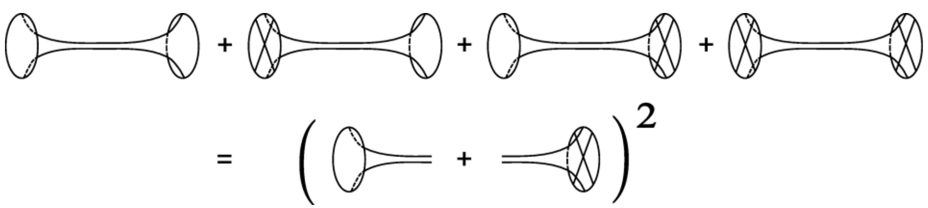
- Closed string:
  - Vacuum-to-vacuum (torus) diagram - modular invariance



  - Condition between gauge symmetry F and geometry R.
  - F, R in the low energy theory.

$$\mathrm{tr}\, R \wedge R - \mathrm{tr}\, F \wedge F = 0$$

- Open string:
  - Cylinder and its twisted variants – RR tadpole cancellation.



  - Constrains the number of D-branes n.

Guaranteeing anomaly free theory.
generations = Indices: local chirality that can change

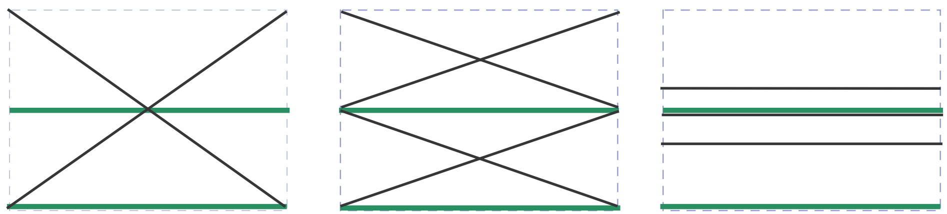
Meaning

- On T^6 compactification with N=1 SUSY in 4D:

The sum of total D-brane charges should be the same as
O9 plane chargeor its T-duals.

- Op planes at the 2^{9-p} fixed points.

- ex. O8s on S/Z_2



- Type I string = type IIB with O9 and 16 D9s. SO(32)
- S-dual to SO(32) heterotic string.
- T-dual to E_8 x E_8 heterotic string.

Reid's fantasy

Math. Ann. 278, 329

My fantasy: The moduli space of string vacua connected
Cf. landscape vs swampland

Examples: intersecting branes, heterotic strings on orbifolds, some F-theory vacua

The Moduli Space of 3-Folds with $K=0$ may Nevertheless be Irreducible

Miles Reid
Mathematical Institute, University of Warwick, Coventry CV4 7AL, UK

To Friedrich Hirzebruch on his sixtieth birthday

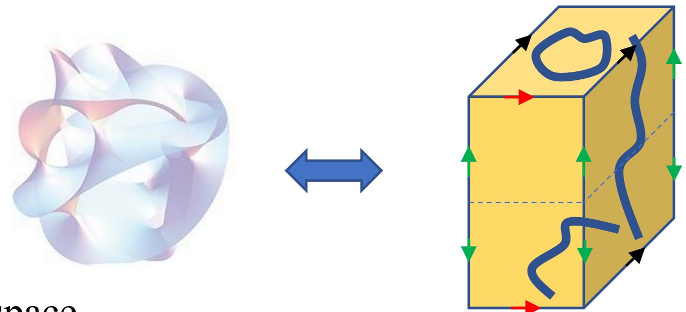
This paper consists mainly of idle speculation. I apologise for having neither the time nor the ability to prepare a proper paper as a birthday tribute to Professor Hirzebruch. I should acknowledge that apart from Hirzebruch's beautiful constructions of many examples of 3-folds with $K=0$, I have been prompted to a large extent by ideas of H. Clemens and R. Friedman.

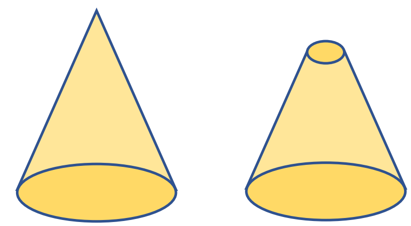
Heterotic string

Orbifold limit of Calabi-Yau

[Dixon, Harvey, Vafa, Witten 85, 86]
[Ibanez, Nilles, Quevedo 87]
….
[KSC, Kim 03]
…

- Singular limit of Calabi-Yau manifold. Ex. K3 = $T^4/\mathbf{Z}_N$.
- Each fixed point: ALE space $\mathbf{R}^4/\mathbf{Z}_N$.

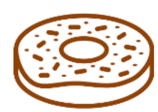




- Blow up → smooth ALE space.
- Flat background connection at "infinity" = shift vector.
- Structure group is $U(1)^r$:
    - r rank of the maximal torus. $SO(16)$x$SO(16)$ or $SO(32)$.
    - $k_U = 3(n_1+n_2)$ ($\mathbf{Z}_3$) for V = 1/3 $(0^{n0},1^{n1},2^{n2})$
    - k_T = # twisted vectors of $SO(n_0)$

[Intriligator 97] [Blum, Intriligator 97] [KSC, Kobayashi 19]

- Gluing $\mathbf{R}^4/\mathbf{Z}_N$'s: shift vector with Wilson lines
- Modular invariance of the partition function
- Instanton number $k_U + k_T = 24$



$$\frac{V^2}{2} - \frac{\phi^2}{2} \equiv 0 \mod \frac{1}{N}$$

$$\mathrm{tr}\, R \wedge R - \mathrm{tr}\, F \wedge F = 0$$

Transitions: example $SO(32)$ het on T^4/Z_3

$U(8)$ x $SO(16)$
V= 1/3 (1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0)

$U: (\mathbf{8}, \mathbf{16}) + (\mathbf{28}, \mathbf{1}) + 2(\mathbf{1}, \mathbf{1})$

$T: 9(\mathbf{28}, \mathbf{1}) + 18(\mathbf{1}, \mathbf{1})$

$k_U = 3 \cdot 8 = 24$
$k_T = 0$

$V = V_1 + V_2$
V_1 = 1/3 (1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0) daughter shift vector
V_2 = 1/3 (0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0) emitted instantons

$U(2)$ x $SO(28)$ x $Sp(18)$
V_1 = 1/3 (1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0)

$U: (\mathbf{2}, \mathbf{28}; \mathbf{1}) + 3(\mathbf{1}, \mathbf{1}; \mathbf{1})$

$T: 9(\mathbf{1}, \mathbf{1}; \mathbf{1}) + 18(\mathbf{1}, \mathbf{1}; \mathbf{1})$

$NP: \frac{1}{2}(\mathbf{1}, \mathbf{28}; \mathbf{36}) + (\mathbf{2}, \mathbf{1}; \mathbf{36})$

$k_U = 3 \cdot 2 = 6$
$k_T = 0$
$n = 3 \cdot 6 = 18$ instantons emitted
and became 5-branes: extra $Sp(18)$
Anomaly free

$U(2)$ x $SO(28)$
V_1 = 1/3 (1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0)

$U: (\mathbf{2}, \mathbf{28}) + 3(\mathbf{1}, \mathbf{1})$

$T: 9(\mathbf{2}, \mathbf{28}) + 18(\mathbf{1}, \mathbf{1}) + 45(\mathbf{1}, \mathbf{1})$

$k_U = 3 \cdot 2 = 6$
k_T = # vectors = 18

Worldsheet CFT with 5-branes

- Every non-perturbative vacua are inherited from perturbative vacua.

$$V = V_1 + V_2$$

Remaining instantons   Emitted instantons → 5-branes

- Worldsheet CFT

$$\frac{1}{2}m_L^2 = \frac{(P+V_1+V_2)^2}{2} + \tilde{N} + E_0 \quad \text{perturbative}$$

$$= \frac{(P+V_1)^2}{2} + \tilde{N} + E_0 + \Delta E_0 \quad \text{non-perturbative}$$

[Aldazabal, Font, Ibanez, Uranga, Violero 98]

- GSO Projection

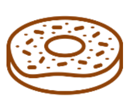
$$e^{2\pi i\left(\tilde{N}-N+(P+V)\cdot V-(s+\phi)\cdot\phi-\frac{1}{2}\left(V^2-\phi^2\right)\right)}$$

$$e^{2\pi i\left(\tilde{\mathrm{N}}-\mathrm{N}+(P+V_1)\cdot V_1-(s+\phi)\cdot\phi-\frac{1}{2}\left(V_1^2-\phi^2\right)+\Delta E_0\right)}$$

$$\Delta E_0 = V_1 \cdot V_2 + \frac{1}{2}V_2^2 = \frac{n}{54}$$

Modified zero point energy

- The same CFT description!

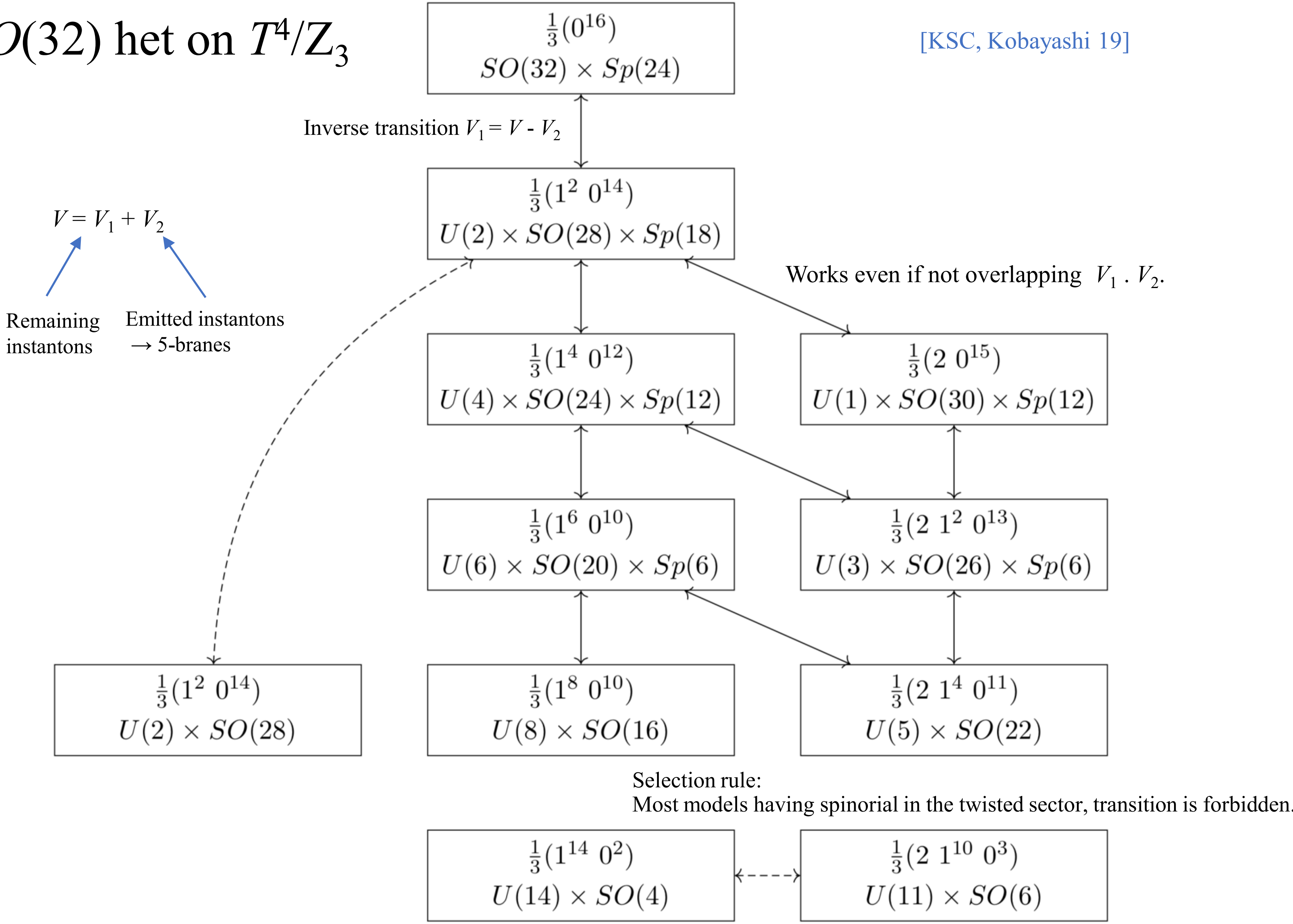
- Modular invariance 

$$\frac{V^2}{2} - \frac{\phi^2}{2} + \Delta E_0 \equiv 0 \mod \frac{1}{N}$$

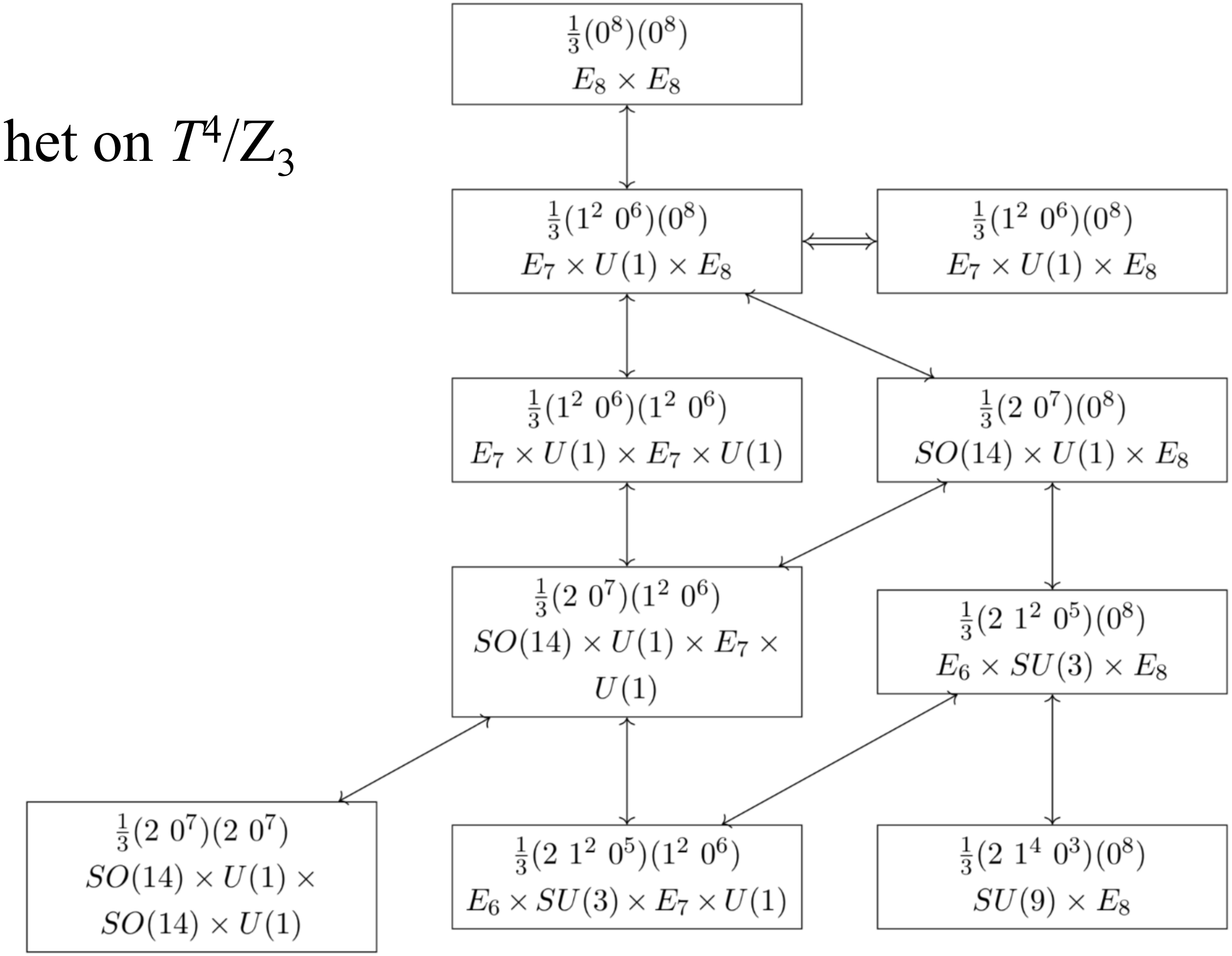
- Instanton # $k_1 + k_2 + n = 24$.

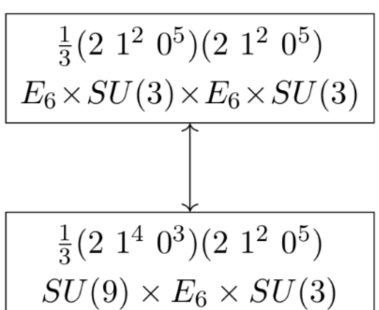
$SO(32)$ het on T^4/Z_3

[KSC, Kobayashi 19]



$E_8 \times E_8$ het on T^4/Z_3



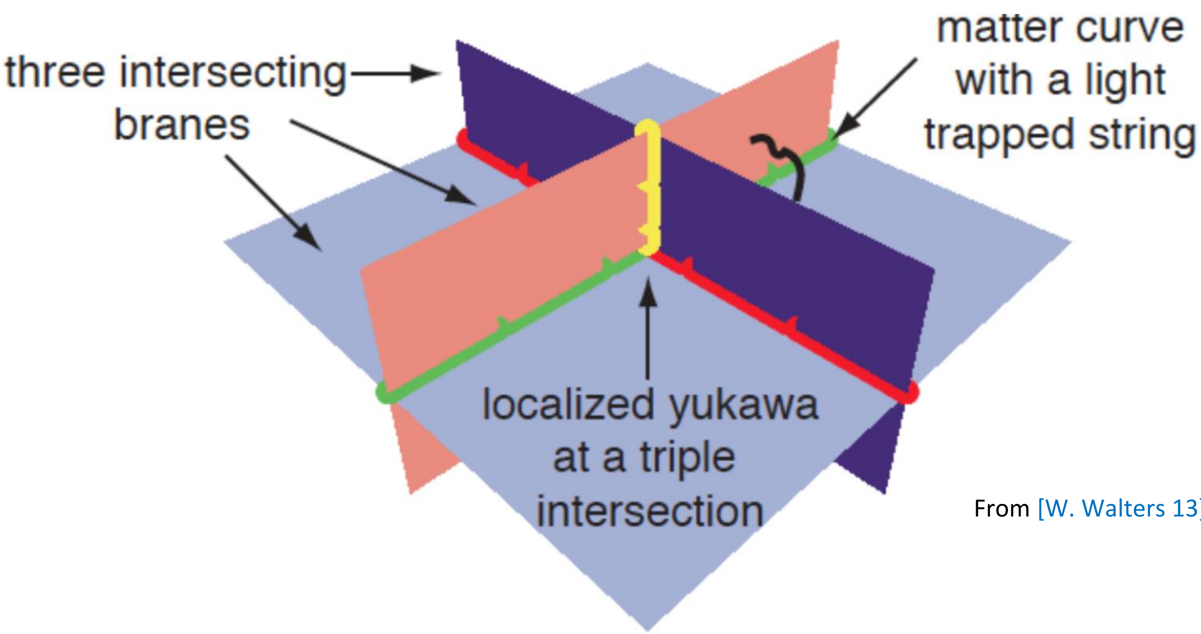


In $E_8 \times E_8$, no extra gauge group from 5-branes

F-theory

F-theory [Vafa] [Beasley, Heckman, Vafa] [Donagi, Wijnholt]…

- 7-branes of type IIB string lift to geometry of Calabi—Yau fourfold
- Intersection (6D): localized matter
- With 4-form flux <G>, magnetic flux is induced on it.
- Yielding to 4D chiral fermion.



From [W. Walters 13]

F-theory

- Global consistency condition requires D3 [Sethi, Vafa, Witten]

$$d * dC = \frac{1}{24} c_4(Y) - \frac{1}{8\pi^2} G \wedge G - \sum_{a=1}^{n} \delta^{(8)}(y - y_a)$$

- D3 branes play no role in 4D chiral spectrum…
- Internal YM symmetry is converted to geometry
- F-theory on K3 is dual to heterotic string on torus E

$$\frac{G}{2\pi} = \sum_I F^I \wedge e_I \,, \quad e_I \in H^{1,1}(\mathrm{K3})$$

- Small instanton transition to vertical heterotic 5-brane

$$\frac{1}{2h_{\mathfrak{g}}^{\vee}} \mathrm{Tr}\, F \wedge F|_B = \frac{1}{2h_{\mathfrak{g}'}^{\vee}} \mathrm{Tr}\, F' \wedge F'|_B + \sum_{\text{vertical}} \delta^{(4)}(E)$$

- Small instanton transition
  - Heterotic small instanton -> G-flux in F-theory.
  - Shrinking G-flux –> D3-branes

“G-instanton” transition

- G-instanton $*_Y G = G$ with $J \wedge G = 0$
    - Hermitian YM eq. for holomorphic vector bundle in the heterotic.
- Expansion of G-flux in terms of base curves which is self-dual $C^a = *_B C^a = C_a$

$$\frac{G}{2\pi} = \frac{G'}{2\pi} + P_a \wedge C_a$$

- The coefficient is group theoretical factor

$$P_a = \sum_i c_a^i E_i , \quad c_a^i \in \mathbb{Z} .$$

- Which is translated to geometry in the F-theory side.
- Small ”G-instanton” to D3 transition

$\delta^{(8)}(x - x_a)$

$$\frac{1}{8\pi^2} G \wedge G = \frac{1}{8\pi^2} G' \wedge G' + \frac{1}{2} \pi^* C_a \wedge \pi^* C_a \wedge P_a \wedge P_a .$$

- # D3-branes guided by group theory

$$-\frac{1}{2} \int_{\mathrm{K3}} P_a \wedge P_a = -\frac{1}{2} \int_{\mathrm{K3}} \left(\sum_i (c_a^i)^2 E_i \wedge E_i + 2 \sum_{i<j} c_a^i c_a^j E_i \wedge E_j \right)$$

$$= \sum_i (c_a^i)^2 - \sum_{i<j} c_a^i c_a^j A_{ij}$$

Small "*G*-instanton" transition

- Chiral spectrum
- Ex. SU(5) G-flux on an SU(5) brane

$$G^{SU(5)} \cdot E_i \cdot D_a = 0, \quad i = 1,2,3,4, \quad \alpha^{(i)} \in \Phi_s(SU(5))$$

E_i: root divisors of $SU(5)$ related to the roots $\alpha^{(i)}$
D_a: base diviosor

- SU(5) -> SU(3) x SU(2)

$$\frac{G_{\text{tot}}^{(i)}}{2\pi} = \frac{G_{\lambda'}^{SU(5)}}{2\pi} + 5\Lambda_{(i)} \cdot (ac_1(B') + bB)$$

Λ_i: fundamental weight divisors of $SU(5)$
B': base of elliptic fibration
B: Section of K3-base

$$\begin{aligned} \mathbf{24} &\to (\mathbf{8},\mathbf{1})_0 + (\mathbf{1},\mathbf{3})_0 + (\mathbf{1},\mathbf{1})_0 + (\mathbf{3},\mathbf{2})_{-5/6} + (\bar{\mathbf{3}},\mathbf{2})_{5/6}, \\ \bar{\mathbf{5}} &\to (\bar{\mathbf{3}},\mathbf{1})_{1/3} + (\mathbf{1},\mathbf{2})_{-1/2}, \\ \mathbf{10} &\to (\mathbf{3},\mathbf{2})_{1/6} + (\bar{\mathbf{3}},\mathbf{1})_{-2/3} + (\mathbf{1},\mathbf{1})_1. \end{aligned}$$

Reprs.	Highest weight	Matter surfaces
$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	$\mu_{\bar{\mathbf{5}}}$	$\mathcal{S}_{\bar{\mathbf{5}}}$
$(\mathbf{1},\mathbf{2})_{-1/2},$	$\mu_{\bar{\mathbf{5}}} - \alpha^{(1)} - \alpha^{(2)} - \alpha^{(3)}$	$\mathcal{S}_{\bar{\mathbf{5}}} + (E_1+E_2+E_3)\cdot(8c_1(B') - 5B)$
$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	$\mu_{\mathbf{10}}$	$\mathcal{S}_{\mathbf{10}}$
$(\mathbf{3},\mathbf{2})_{1/6}$	$\mu_{\mathbf{10}} - \alpha^{(1)} - \alpha^{(2)} - \alpha^{(3)}$	$\mathcal{S}_{\mathbf{10}} - (E_1+E_2+E_3)\cdot c_1(B')$
$(\mathbf{1},\mathbf{1})_1$	$\mu_{\mathbf{10}} - \alpha^{(1)} - 2\alpha^{(2)} - 2\alpha^{(3)} - \alpha^{(4)}$	$\mathcal{S}_{\mathbf{10}} - (E_1+2E_2+2E_3+E_4)\cdot c_1(B')$

Local chirality change

- Local chiralities change but still anomaly free.
- Ex. SU(5) -> SU(3) x SU(2)

$$
\begin{aligned}
\mathbf{24} &\rightarrow (\mathbf{8},\mathbf{1})_0 + (\mathbf{1},\mathbf{3})_0 + (\mathbf{1},\mathbf{1})_0 + (\mathbf{3},\mathbf{2})_{-5/6} + (\bar{\mathbf{3}},\mathbf{2})_{5/6},\\
\bar{\mathbf{5}} &\rightarrow (\bar{\mathbf{3}},\mathbf{1})_{1/3} + (\mathbf{1},\mathbf{2})_{-1/2},\\
\mathbf{10} &\rightarrow (\mathbf{3},\mathbf{2})_{1/6} + (\bar{\mathbf{3}},\mathbf{1})_{-2/3} + (\mathbf{1},\mathbf{1})_1.
\end{aligned}
$$

$$
\begin{aligned}
-\chi((\bar{\mathbf{3}},\mathbf{1})_{1/3}) &- \chi((\bar{\mathbf{3}},\mathbf{1})_{-2/3}) + 2\chi((\mathbf{3},\mathbf{2})_{1/6}) + 2\chi((\mathbf{3},\mathbf{2})_{-5/6})\\
&= -\chi(\bar{\mathbf{5}}) - \frac{2}{5}\int_B (8c_1 - 3t)\wedge\mathcal{F} - \chi(\mathbf{10}) + \frac{4}{5}\int_B (c_1 - t)\wedge\mathcal{F}\\
&\quad + 2\chi(\mathbf{10}) + 2\cdot\frac{1}{5}\int_B (c_1 - t)\wedge\mathcal{F} + 2\int_B c_1\wedge\mathcal{F}\\
&= 0.
\end{aligned}
$$

Conclusion

- Connected vacua
- Brane recombination dual to small instanton transition

- In heterotic string on orbifolds: non-perturbative correction to CFT, 5-branes
- In F-theory: the role of D3-branes in chirality

- Selection rules given by group theory.

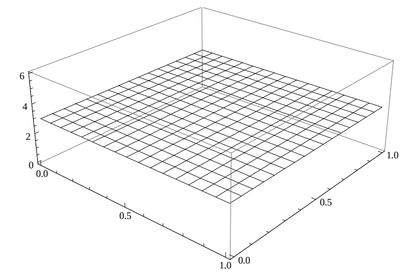
- Outlook:
- Due to quantization condition for the flux, not every decomposition possible.
- The instanton is generalized to stable (Hermitian Yang—Mills) bundle on threefo ld.

generations from extra dimension

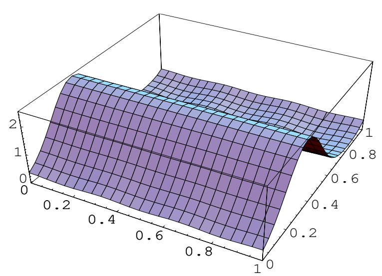
- Dirac equation in higher dimensions with torus

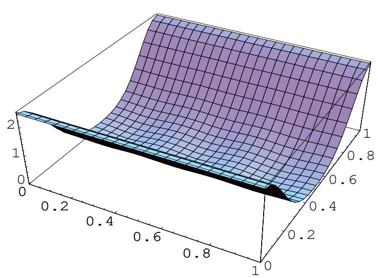
$$D_{(6)} f(x, y^4, y^5) = (D_{(4)} + D_{(2)}) f(x) f(y^4, y^5) = 0$$

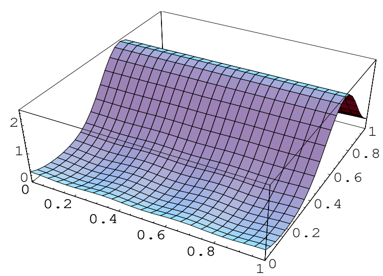
  - Eigenvalue of $D_{(2)}(A_4, A_5)$ looks life 4D mass
  - With magnetic flux F_{45}
  - Torus boundary condition: F_{45} is quantized
  - Zero eigenstates are chiral [Landau]:
    either +1 or -1 representation exclusively survives



- # zero eigenstates of $D_{(2)}$ = # generations in 4D
  - Ex. 3 generations from $F_{45}/2\pi = 3$







generations is topological quantity

- Dirac operator $D_{(2)}$ for the extra dim.
  - ${D_{(2)}}^2 = H$ nonzero eigenstates always pair R and R^*.
  - Chiral: Not necessarily true for zero eigenstates: unpaired R or R^*
- The number of 4D massless field is given by

$$n_R - n_{\overline{R}} = \frac{1}{2\pi} \int d^2x \ \mathrm{tr} F_{45}$$

- Topological index:
  - the index does not depends on smooth deformation of geometry and vector potential.
  - cf. anomaly cancellation
  - A constant field strength F_{45} on torus is quantized due to periodic B.C.

Generalization to higher dimension

- $D\psi(x,y) = \Gamma^\mu \partial_\mu + \Gamma^m (\partial_m - iA_m + \frac{1}{2}\omega_m)\psi = 0$

  bg. gauge geometry

- Vector bundles are generalized to 'characteristic classes' which are integrally quantized:
  - F or R (2D, aka magnetic flux, vortex, monopole or toron number)
  - $F \wedge F = \varepsilon FF$ or $R \wedge R$ (4D aka instanton number)
  - $F \wedge F \wedge F$ or $F \wedge R \wedge R$ (6D)…

$$n_R - n_{R^*} = \frac{1}{3!(4\pi)^3} \int_M [\mathrm{tr}_Q F \wedge F \wedge F - \frac{1}{3} \mathrm{tr}_Q F \wedge \mathrm{tr} R \wedge R].$$

- They classify topology of the internal manifold.