Sky Meets Laboratory via RGE: Axions, UV-completion and Gravitational Waves

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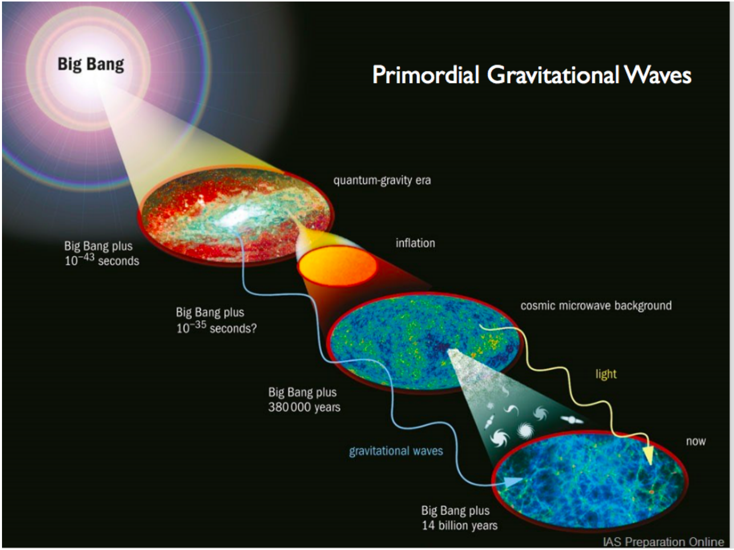
Outline of talk:

- UV & IR physics are connected via RGE.
- Complementarity between Lab versus Cosmic Observables.
- Stochastic Gravitational Waves from Cosmological Phase Transitions
  - Peccei-Quinn Phase Transition & Gravitational Waves.
  - Conformal Invariance & TAF as a direction of UV-completion.
  - Predictions in UV-complete Axion model.
  - Predictions with Conformal Symmetry Breaking.
- Predictions on the GW detectors sensitivity map.
- Recent NanoGrav GW detection.
- Conclusion

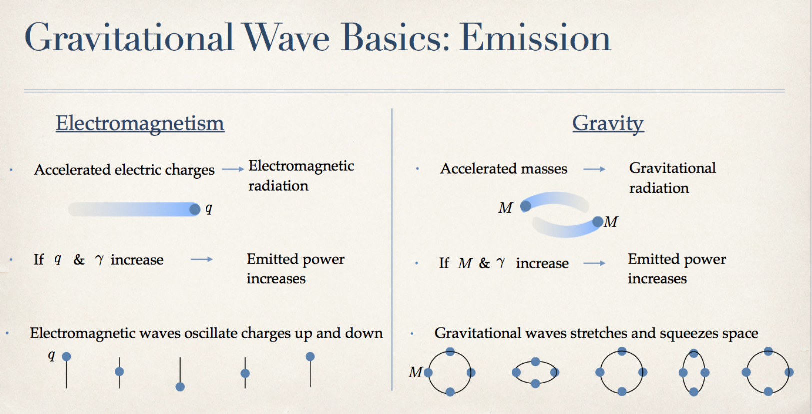
History of the Universe



Gravitational Waves

- Gravitational Waves (GW) first detected in 2016.
- New Window into the Early Universe.
- New Probes of Particle Phenomenology beyond TeV (LHC scale).
- Robust predictions of GW signatures from UV-completion conditions.
- Sources of GW of cosmological origin & corresponding GW spectrum:
    - Inflation: Primordial GW.
    - Inflation: Secondary GW.
    - Strong First-order Phase Transition.
    - Re-heating.
    - Graviton bremsstrahlung.
    - Topological Defects.
    - Oscillon.
    - Primordial BH-induced GW.
- Strong CP Problem dictates $U(1)_{PQ}$ symmetry breaking.
  Peccei-Quinn Phase Transition.

GW - - A Primer


Gravitational Wave Basics: Emission
Electromagnetism
Accelerated electric charges → Electromagnetic radiation
q
If q & γ increase → Emitted power increases
Electromagnetic waves oscillate charges up and down
q
Gravity
Accelerated masses → Gravitational radiation
M
M
If M & γ increase → Emitted power increases
Gravitational waves stretches and squeezes space
M

GW - - A Primer

perturbations of the background metric: $ds^2 = a^2(\tau)(\eta_{\mu\nu} + h_{\mu\nu}(\mathbf{x},\tau))dx^\mu dx^\nu$

scale factor: cosmological expansion ↗ background metric ↑ ↖ GW

governed by linearized Einstein equation $(\tilde{h}_{ij} = a h_{ij}, \text{TT - gauge})$

$$\tilde{h}''_{ij}(\boldsymbol{k},\tau) + \underbrace{\left(k^2 - \frac{a''}{a}\right)}_{\sim a^2H^2}\tilde{h}_{ij}(\boldsymbol{k},\tau) = \underbrace{16\pi G\, a\, \Pi_{ij}(\boldsymbol{k},\tau)}_{\text{source term from } \delta T_{\mu\nu}}$$

source: anisotropic stress-energy tensor

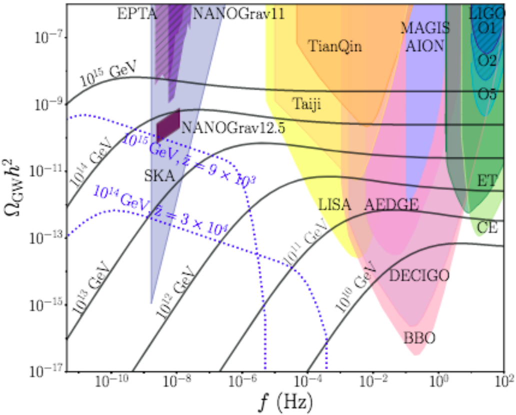
$k \gg aH:\ h_{ij} \sim \cos(\omega\tau)/a\,,\quad k \ll aH:\ h_{ij} \sim \text{const.}$

a useful plane wave expansion: $h_{ij}(\boldsymbol{x},\tau) = \sum_{P=+,\times} \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \int d^2\hat{\boldsymbol{k}}\; h_P(\boldsymbol{k})\; \underbrace{T_k(\tau)}_{\sim a(\tau_i)/a(\tau)}\; e^P_{ij}(\hat{k})\, e^{-ik(\tau - \hat{\boldsymbol{k}}\boldsymbol{x})}$

transfer function , expansion coefficients , polarization tensor $P = +, \times$

GW - - Local Cosmic String

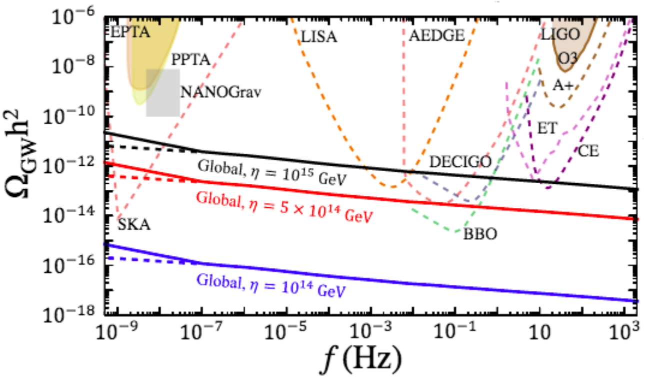
Topological defects like cosmic strings can be formed in early universe when some gauge $U(1)_X$ symmetry is broken in early universe. It give rise to scale invariant GW spectrum. Detection prospects lies on the symmetry breaking scale vev.



King (2020)

GW - - Global Cosmic String

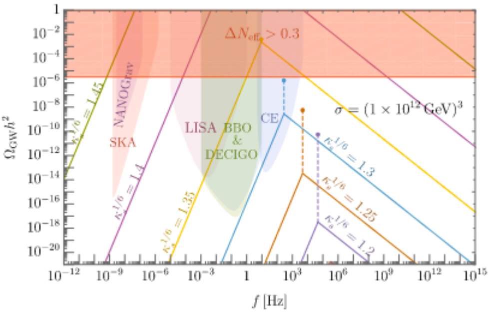
Topological defects like cosmic strings can be formed in early universe when some global $U(1)_X$ symmetry is broken in early universe. Detection prospects lies on the symmetry breaking scale vev which needs to be very high.



Cui (2021)

GW - - Domain Walls

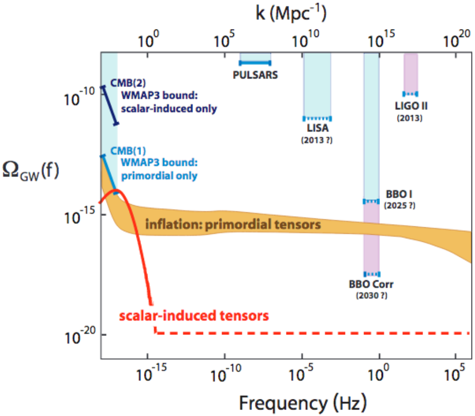
Topological defects like Domain Walls are formed when a discrete symmetry is broken and give rise to GW spectrum may look something like this (still under active research topic). Detection prospects lies of symmetry breaking scale as well as the asymmetry term in the potential, like cubic term.



Dunsky at. al. (2021)

GW - - Primordial and Scalar Induced Secondary GW

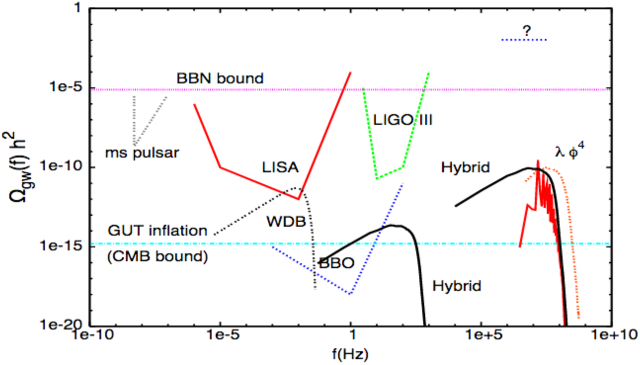
Primary Tensor Perturbations and Secondary Tensor Spectrum induced by first-order scalar perturbation via mixing. Can be tuned to generate high amplitude in high frequency regions.
Acts as natural probes of particle models like Higgs inflation, axion inflation, MSSM inflation, etc.



Baumann (2007)

GW - - (P)-reheating

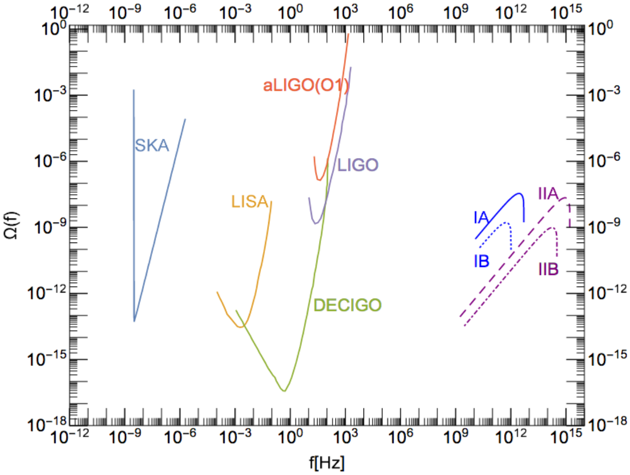
Excitation of tensor perturbations during inflaton oscillating in FRW background. Backreaction and effects of metric fluctuations. Enhancement mechanism: Bose-resonance, tachynic growth, parametric resonance.



Figuera (2007)

GW - - Graviton Bremmstrahlung

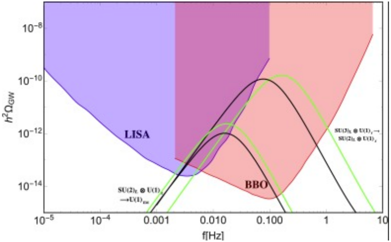
Inflaton radiating away gravitons forming Stochastic GW background.



Nakayama (2018)

GW - - First-order Phase transition

Typical GW spectrum from thermal first-order phase transition:



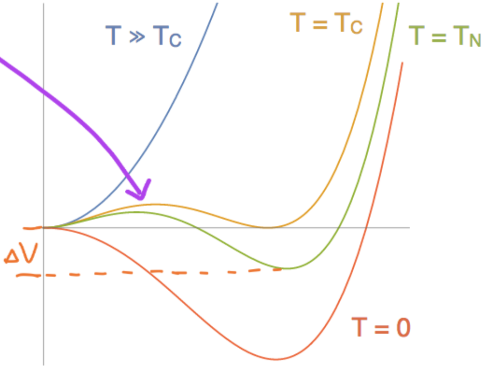
Huang (2018)

Phase Transition

- QFT at finite temperature → symmetry restoration
- For first order PT
  - Need barrier here
- PT occurs at T_N
- Potential energy ΔV

  ↓

  GWs
- Not in SM! Possible in BSM scenarios

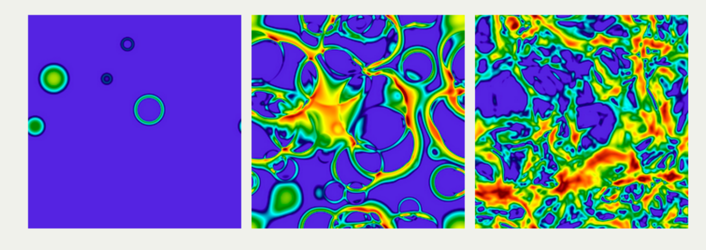


Schwaller (Amsterdam (2019))

Phase Transition

Phase Transitions:

- Bubbles nucleate and grow.
- Expand in plasma.
- Bubbles and fronts collide - - violent process.
- Sound Waves left behind in thermal plasma.
- Turbulence, damping.



Phase Transition GW - Parameter Dependence

- Total GW energy budget from 3 sources

$$h^2\Omega_{\rm GW} = h^2\Omega_\phi + h^2\Omega_{\rm SW} + h^2\Omega_{\rm MHD}$$

Depends on two important parameters:

- Vacuum energy density: $\alpha = \frac{\rho_{\rm vac}}{\rho^*_{\rm rad}}$ with $\rho^*_{\rm rad} = g_* \pi^2 \frac{T_*^4}{30}$
- (Inverse) Bubble nucleation rate: $\beta / H_* = T \sqrt{\frac{d^2 S_E(T)}{dT^2}} \Big|_{T=T_*}$

- $$h^2\Omega_\phi \propto \left(\frac{\beta}{H_*}\right)^{-2}, \quad h^2\Omega_{\rm SW} \propto \left(\frac{\beta}{H_*}\right)^{-1}, \quad h^2\Omega_{\rm MHD} \propto \left(\frac{\beta}{H_*}\right)^{-1}$$

- The bubble nucleation rate per unit volume at a finite temperature is given by

$$\Gamma(T) = \Gamma_0 e^{-S(T)} \simeq \Gamma_0 e^{-S_E^3(T)/T},$$

-

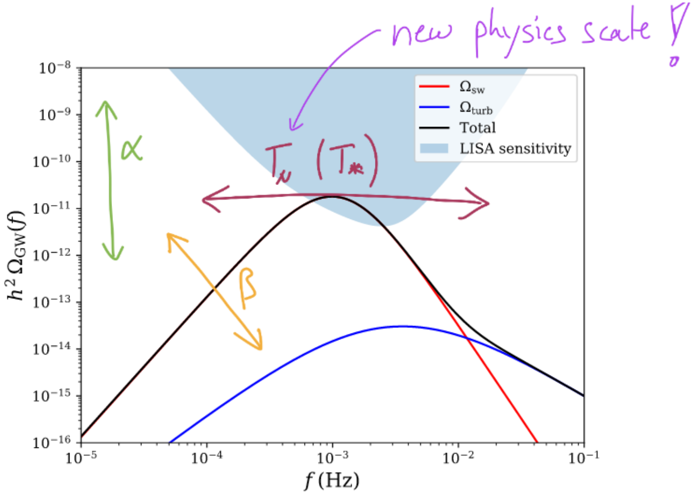
Other important parameter: bubble wall speed v_w, efficiency factors.

Bounce Action S_3:

$$\partial^2 \phi + V'_{\rm eff}(\phi, T) + \sum_i \frac{dm_i^2}{d\phi} \int \frac{d^3 k}{(2\pi)^3\, 2E_i} \delta f_i(\mathbf{k}, \mathbf{x}) = 0$$

- $V'_{\rm eff}(\phi)$: gradient of finite-T effective potential
- $\delta f_i(\mathbf{k}, \mathbf{x})$: deviation from equilibrium phase space density of ith species
- m_i: effective mass of ith species

Phase Transition GW - Parameter Dependence



Schwaller (Amsterdam, 2019)

Axion

Strong CP Problem:

$$\frac{\theta}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \qquad \theta + \mathrm{Arg}[\mathrm{Det}(y_u y_d)] < 10^{-10}$$

Axion solution:

$$\theta \to \frac{a(x)}{f} \qquad \mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

[Peccei-Quinn '77 Weinberg-Wilczek '78]

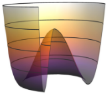
- PQWW axion:
  Axion identified with the phase of the Higgs in a 2HDM
  ($f_a \sim V_{EW}$ was quickly ruled out long ago) [Peccei, Quinn (1977), Weinberg (1978), Wilczek (1978)]

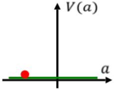
The need to require $f_a \gg V_{EW}$: "invisible axion"

- DSFZ Axion: SM quarks and Higgs charged under PQ.
  Requires 2HDM + 1 scalar singlet. SM leptons can also be charged.
  [Dine, Fischler, Srednicki (1981), Zhitnitsky (1980)]
- KSVZ axion (or QCD axion, or hadronic axion):
  All SM fields are neutral under PQ. QCD anomaly is induced by new quarks, vectorlike under the SM, chiral under PQ.
  [Kim (1979), Shifman, Vainshtein, Sakharov (1980)]

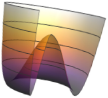
Axion as Dark Matter

- **As long as $\Lambda_{QCD} < T < f_a$:**
  $U(1)_{PQ}$ broken only spontaneously,
  $m_a = 0$, $\langle a_0 \rangle = \theta_0 f_a \sim f_a$

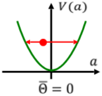




- **As soon as $T \sim \Lambda_{QCD}$:**
  $U(1)_{PQ}$ explicit breaking (instanton effects)
  $m_a(T)$ turns on. When $m_a(T) > H \sim 10^{-9}$ eV,
  $\langle a_0 \rangle \longrightarrow 0$ and starts oscillating undamped



$$\ddot{a} + \cancel{3H\dot{a}} + m_a^2(T) f_a \sin\left(\frac{a}{f_a}\right) = 0$$



- **Energy stored in oscillations behaves as CDM**

Axion Pheno

- Axion or ALP couplings to SM particles are always suppressed by inverse powers of $U(1)_{PQ}$ symmetry breaking scale f_a.
- Phenomenological scalar with complex singlet scalar Φ:

$$\Phi(x) = \frac{1}{\sqrt{2}}(f_a + \phi(x))e^{ia(x)/f_a} \tag{1}$$

- Spontaneous breaking of U(1) may lead to strong first-order phase transition at the f_a scale & generate GW signals to be detected at the current and future detectors.

Phase Transition GW - Finite Temperature

$$\mathcal{V}(\phi,T) = \mathcal{V}_0(\phi) + \mathcal{V}_{\rm CW}(\phi) + \mathcal{V}_T(\phi,T)\,,$$

- Tree-level: $$\mathcal{V}_0 = -\mu^2|H|^2 + \lambda|H|^4 + \kappa|\Phi|^2|H|^2 + \lambda_a\left(|\Phi|^2 - \frac{1}{2}f_a^2\right)^2 .$$
$$= \frac{\lambda_a}{4}\left(\phi^2 - f_a^2\right)^2 + \left[\frac{\kappa}{2}\phi^2 - \mu^2\right]\left(\frac{1}{2}h^2 + \frac{1}{2}G_0^2 + G_+G_-\right) + \lambda\left[\frac{1}{2}h^2 + \frac{1}{2}G_0^2 + G_+G_-\right]^2 .$$
- One-loop: $$\mathcal{V}_{\rm CW}(\phi) = \sum_i (-1)^F n_i \frac{m_i^4(\phi)}{64\pi^2}\left[\log\frac{m_i^2(\phi)}{\Lambda^2} - C_i\right] .$$
- Finite-temperature: $$\mathcal{V}_T(\phi,T) = \sum_i (-1)^F n_i \frac{T^4}{2\pi^2} J_{B/F}\left(\frac{m_i^2(\phi)}{T^2}\right) ,$$
- Temperature-dependent mass terms:
$$\Pi_h(T) = \Pi_{G_{0,\pm}}(T) = \frac{1}{48}\left(9g_2^2 + 3g_1^2 + 12y_t^2 + 24\lambda + 4\kappa\right)T^2\,,$$
$$\Pi_\phi(T) = \frac{1}{3}(\kappa + 2\lambda_a)T^2 .$$

[Dolan, Jackiw (PRD '74); Arnold, Espinosa (PRD '93); Curtin, Meade, Ramani (EPJC '18)]

Phase Transition GW - sensitivity

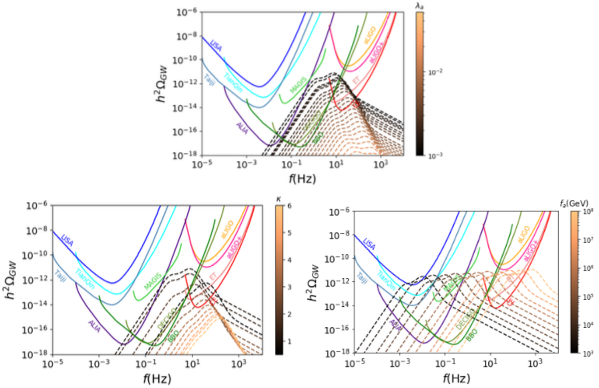
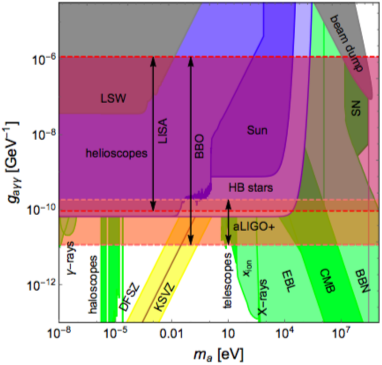
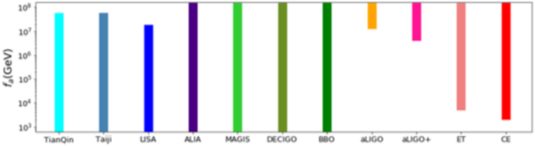


Figure 4. The detection prospects for the GW experiments TianQin [27], Taiji [28], LISA [29, 30], ALIA [31], MAGIS [32], DECIGO [33], BBO [34], aLIGO [37], aLIGO+ [38], ET [36] and CE [35], and the curves of GW strength $h^2\Omega_{\rm GW}(f)$ as functions of the three parameters f_a, κ and λ_a in the ALP model. In the upper panel, we have fixed $f_a = 10^6$ GeV and $\kappa = 1.0$ and varied λ_a from 0.001 to 0.2; in the lower left panel $f_a = 10^6$ GeV and $\lambda_a = 0.001$, with κ varying from 1.0 to 6.00; in the lower right panel $\kappa = 1.0$ and $\lambda_a = 0.001$, with f_a between 10^3 GeV and 10^8 GeV.

Dev et. al. (2019)

Phase Transition GW - Parameter Dependence





Dev et. al. (2019)

KSVZ Axion

– KSVZ axion:

$$U(1)_{\rm PQ}: \qquad X \to e^{i\alpha} X$$

$$\lambda_X (|X|^2 - f^2/2)^2 + (yXQQ^c + h.c.)$$

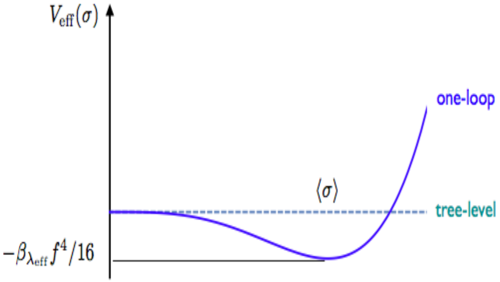
- No massless bosons coupling to X while Peccei-Quinn symmetry is restored.
- Fermion contribution to V_{eff} contributes is negatively.
- Finite temperature corrected potential is of the form $m(T)^{|}X|^2 + \lambda(T)|X|^4$.
- PQ phase transition is of second-order in the minimal case.
- In order of make strong first-order phase transition (PT), and thus enhanced GW, we go to supercooling regime. This requires PT to last long enough.
- This means $\frac{S_3}{T} \sim$ constant $\to$ scale invariant.
- Break PQ symmetry radiatively.
- Or, break non-minimally like strong coupling regime, non-perturbative, extra-dimension etc. (See Delle Rosse (2019) & Von Harling (2019).)

Conformal Symmetry Breaking

Due to conformal symmetry-breaking, the flat direction is lifted at 1-loop when

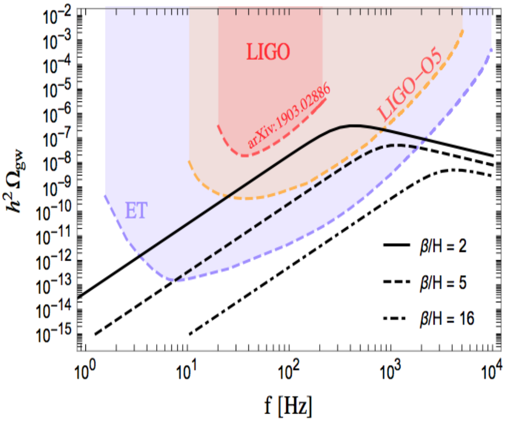
$$V_{eff} = \frac{\beta_\lambda}{4}\sigma^4(log(\frac{\sigma}{f_a} - \frac{1}{4})),$$

where $< \sigma >= f_a$.



Conformal Symmetry Breaking

Strong super-cooling enhances GW signals:



delle Rosse et. al. (2019), von Harling et. al. (2019)

Total Asymptotic Freedom Principle

- No scales are fundamental are nature, all scales that we observe are generated dynamically: 1-loop or via non-perturbative physics.
- Gravitational Corrections not included. For re-normalizable theories of gravity like Quadratic Gravity or non-local gravity, all corrections are softened in the UV.
- Still suffers from Landau poles.
- Total Asymptotic Freedom (TAF) as a direction for UV completion of particle physics. All couplings flow to zero in the UV.
- No Landau poles in theory.
- Theory valid and perturbative upto infinite energy scales.
- For $U(1)_{PQ}$, simplest possibility to replace by $SU(2)_a$.
- Generic conditions for TAF already studied in several places [Giudice (2014), Holdom (2015), Pelaggi (2015)].

Low energy spectrum of the theory contains extra dark photon on top of the SM.
All masses of extra quarks and scalars are expressed in terms of the free parameter f_a.

Phase Transition GW - Parameter Dependence

Renormalization Group Equations of the parameters:

$$\frac{dg^2}{dt} = -bg^4, \qquad b \equiv \frac{11}{3}C_2(G) - \frac{4}{3}S_2(F) - \frac{1}{6}S_2(S),$$

$$b_s = \frac{29}{3} - \Delta, \quad \frac{dy^2}{dt} = y^2\left(\frac{9y^2}{2} - 8g_s^2 - \frac{9g_a^2}{2}\right)$$

and $b_a = \frac{14}{3}$, $t = \frac{ln(\mu^2/\mu_0^2)}{(4\pi)^2}$, where μ_0 is arbitrary energy scale.
Δ is the extra contributions from scalars and fermions in the theory.

$$g_s^2(t) = \frac{\tilde{g}_s^2}{t}, \quad g_a^2(t) = \frac{\tilde{g}_a^2}{t}, \quad y^2(t) = \frac{\tilde{y}^2}{t}, \quad \lambda_i(t) = \frac{\tilde{\lambda}_i}{t},$$

Phase Transition GW - Parameter Dependence

Axion potential:

$$V_A = -m^2 \mathrm{Tr}(A^\dagger A) + \lambda_1 \mathrm{Tr}^2(A^\dagger A) + \lambda_2 |\mathrm{Tr}(AA)|^2,$$

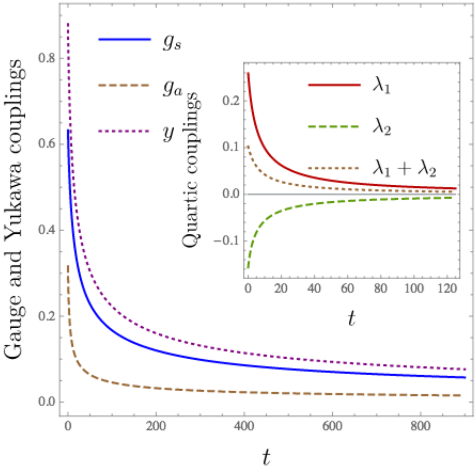
RGEs of λ_1 and λ_2 are $\frac{d\lambda_1}{dt} = \beta_1$, and $\frac{d\lambda_2}{dt} = \beta_2$, where

$$\beta_1(g, y, \lambda) = \frac{9}{2} g_a^4 + \lambda_1 \left(8\lambda_2 + 6y^2 - 12g_a^2\right) + 14\lambda_1^2 + 8\lambda_2^2 - 3y^4$$

$$\beta_2(g, y, \lambda) = \frac{3}{2} g_a^4 + \lambda_2 \left(12\lambda_1 + 6y^2 - 12g_a^2\right) + 6\lambda_2^2 + \frac{3}{2} y^4.$$

X and A are used inter-changeably for denoting the radial part of the axion field.

Phase Transition GW - Parameter Dependence



$t = 100 \equiv \mu = 10^{100}$ GeV.

Phase Transition GW - Parameter Dependence

Some values for satisfying TAF principle.

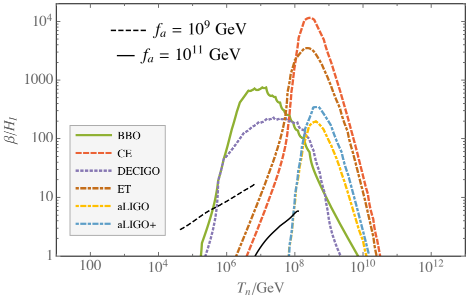
Δ	n_e	unstable vacuum	stable vacuum
28/3	1	(0.219, −3.25)	(1.70, −0.965)
//	2	(0.268, −3.27)	(1.73, −0.986)
//	3	(0.344, −3.30)	(1.77, −1.02)
//	4	(0.469, −3.34)	(1.84, −1.08)
//	5	(0.722, −3.42)	(1.97, −1.20)
//	6	(1.50, −3.49)	(2.34, −1.70)
26/3	1	(0.185, −1.06)	(0.593, −0.362)
//	2	(0.237, −1.07)	(0.619, −0.389)
//	3	(0.314, −1.08)	(0.656, −0.435)
//	4	(0.447, −1.08)	(0.712, −0.528)
8	1	(0.182, −0.601)	(0.365, −0.255)
//	2	(0.236, −0.599)	(0.387, −0.294)
//	3	(0.324, −0.570)	(0.411, −0.376)

Figure: Values of $(\tilde{\lambda_1}, \tilde{\lambda_2})$ satisfying TAF condition. n_e is the number of vector-like Dirac fermions in the adjoint of $SU(2)_a$.

Salvio et. al. (2020)

Phase Transition GW - Parameter Dependence

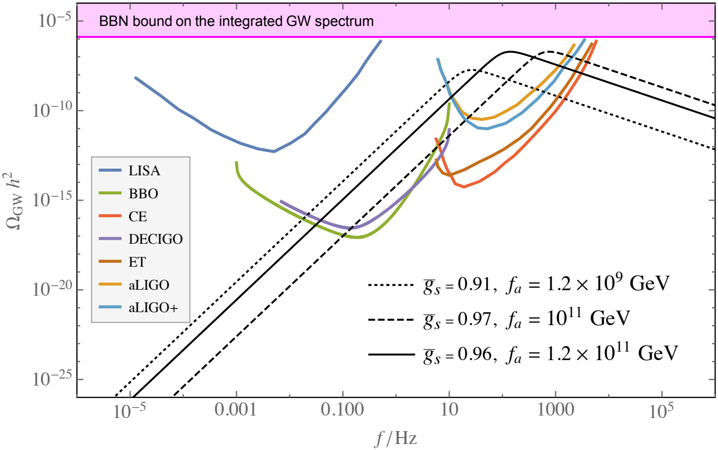
Predictions for some benchmark values.



Imposing conformal symmetry on the axion potential leaves us with only 2 free parameters, thereby very predictive.
Ghoshal et. al. (2020)

Phase Transition GW - Parameter Dependence

Predictions on the GW spectrum



Ghoshal et. al. (2020)

Conclusion: PQ Phase Transition & Gravitational Waves

- Complementarity between the Sky and the Lab via RGE.
- GW detectors will be probing the pre-BBN era.
- UV completion of axion (or any BSM) particle models is insensitive to laboratory or astrophysics searches but predictable in early universe dynamics.
- GW from strong first-order Peccei-Quinn phase transitions will be testable in near future.
- Conformal symmetry breaking makes PQ phase transition very very strong due to supercooling.
- TAF Principle predicts very characteristic & verifiable GW spectrum.
- Gravitational Wave era invites us to dare to imagine, propose and test UV completions of Quantum Field Theory and Gravity !

Percolation Criterion for PT to end

$$P(T) \equiv e^{-I(T)} \lesssim 1/e \Longrightarrow I(T) \gtrsim 1, \tag{D.1}$$

where

$$I(T) = \frac{4\pi}{3} \int_T^{T_c} dT' \frac{\Gamma(T')}{(T' H(T'))^4} \left(\int_T^{T'} \frac{d\tilde{T}}{H(\tilde{T})} \right)^3 . \tag{D.2}$$

One also requires that the physical volume of the false vacuum be decreasing significantly inside of one Hubble time [91, 94–97]

$$\frac{1}{H V_{\text{false}}} \frac{dV_{\text{false}}}{dt} = 3 + T \frac{dI}{dT} \lesssim -1. \tag{D.3}$$

NanoGrav GW Detection

NanoGrav recently detected GW events. Many cosmic sources have been proposed. The GW spectrum nicely fits cosmic strings origin hypothesis.

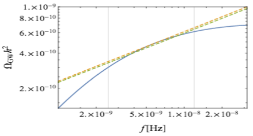
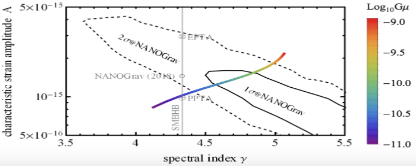


Figure 1. *Cosmic string spectra (solid blue curves) together with our fitted power laws for* $G\mu = 4\times10^{-11}$, *and* $G\mu = 10^{-10}$. *The green dashed lines show the results of numerically fitting the curves, while the orange lines result from the simple logarithmic derivative in Eq. (3.3). The thin grey lines indicate the frequency range of interest that was used in the NANOGrav linear fit.*



Ellis (2020)

Thank You