Higgs Vacuum Decay and Primordial Black Holes

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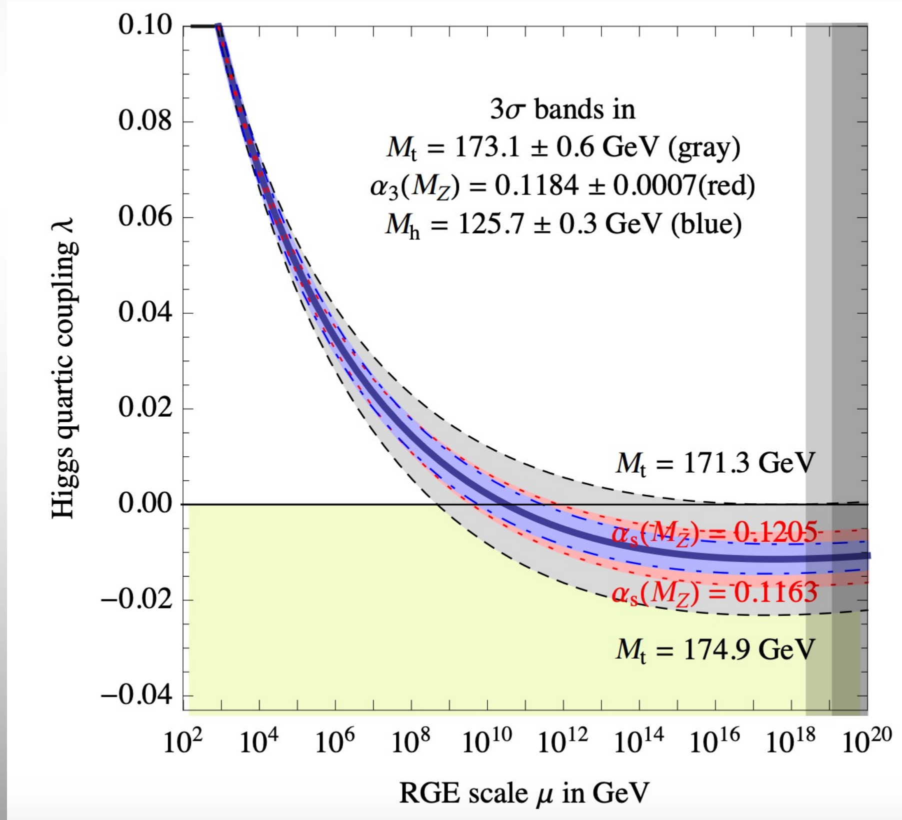
Higgs Vacuum Decay

The big picture from the Standard Model is that our universe may not be entirely stable! At high energies, the Higgs self-coupling becomes negative, opening the possibility of vacuum tunnelling that could destroy the universe as we know it.

$$V(h) = \lambda(h)\frac{|h|^4}{4}$$

But standard calculations indicate a lifetime of the universe of 10^{139} years!



Though see 1904.05237 (CMS) & 1905.02302 (ATLAS).

Degrassi et al. 1205.5497

The Full Story?

But in this talk, I will claim this is not the full story!

Any picture of decay of the universe must take into account gravity – and following that to its logical conclusion – must take into account gravitational impurities, or, BLACK HOLES.

This changes the calculation….

…....enormously!

How Stable is our Vacuum?

Outline:

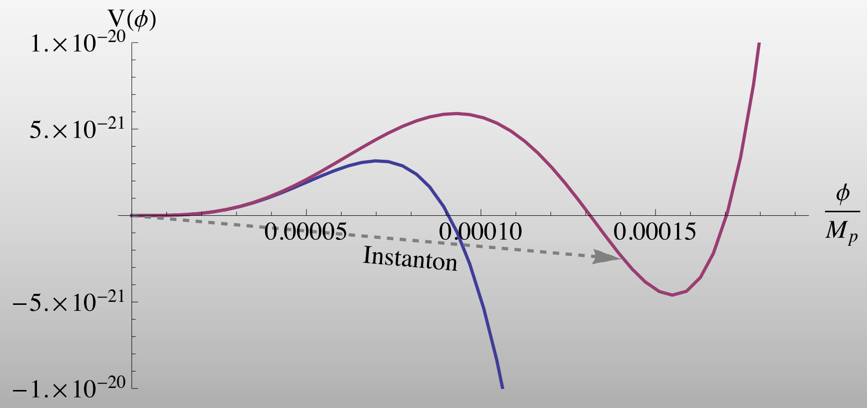
- Review Coleman Method
- Add a Black Hole
- Compare to Evaporation

Quantum Tunnelling

Developed by Coleman and others in the 1970's.

Vacuum understood as an effective state, defined by the minimum of a potential.

The potential itself depends on temperature and scale

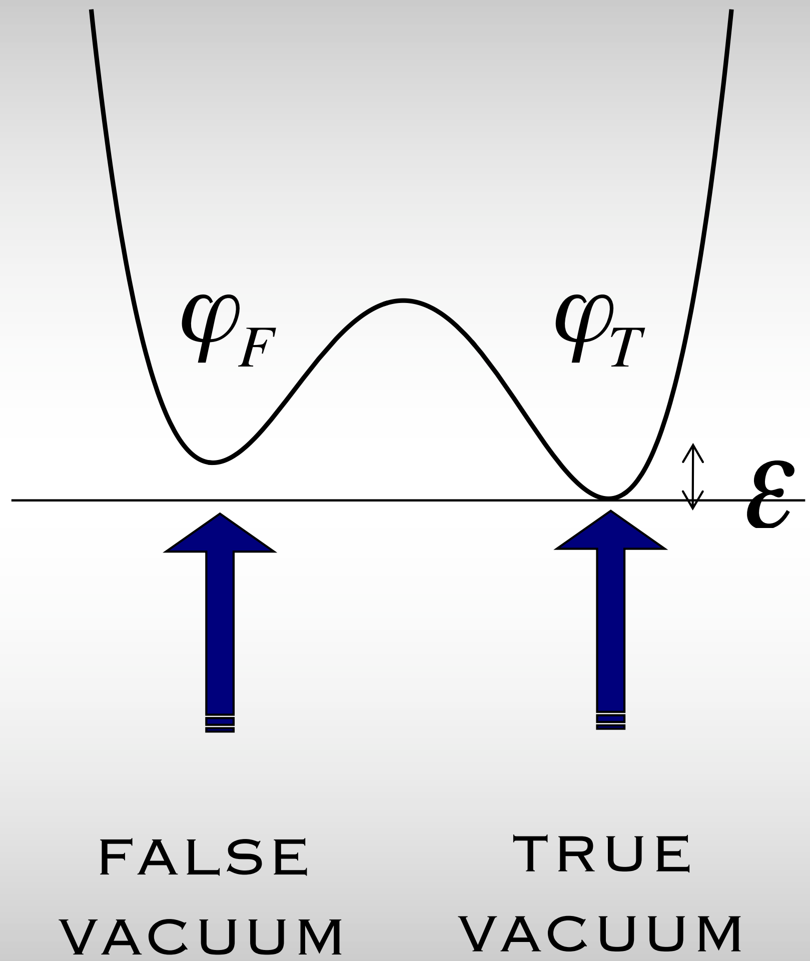


Coleman; Callan & Coleman; Coleman & de Luccia; Kobzarev Okun & Voloshin

FALSE AND TRUE VACUUM

We call such a local – not global – minimum a *false vacuum*, and expect there is a tunneling process to the true minimum / true vacuum.

This will give a first order phase transition, where we tunnel from one local energy minimum to a region with lower overall energy.



Quantum Tunnelling

The key ingredient of the Coleman method is to Wick rotate the system to Euclidean time then solve the equations of motion. This gives the saddle point of the path integral.

The difference between the action of this Euclidean Instanton solution and the undecayed one is the action for the decay, and this is the leading order part of the amplitude for decay.

“Goldilocks Bubble”

But a more intuitive picture is the “Goldilocks” one: if a bubble fluctuates into existence, we gain energy from moving to true vacuum, but the bubble wall costs energy.

Too small and the bubble has too much surface area – recollapses.

Too large and it is too expensive to form.

“Just Right” means the bubble will not recollapse, but is still “cheap enough” to form.

Balancing the Cost

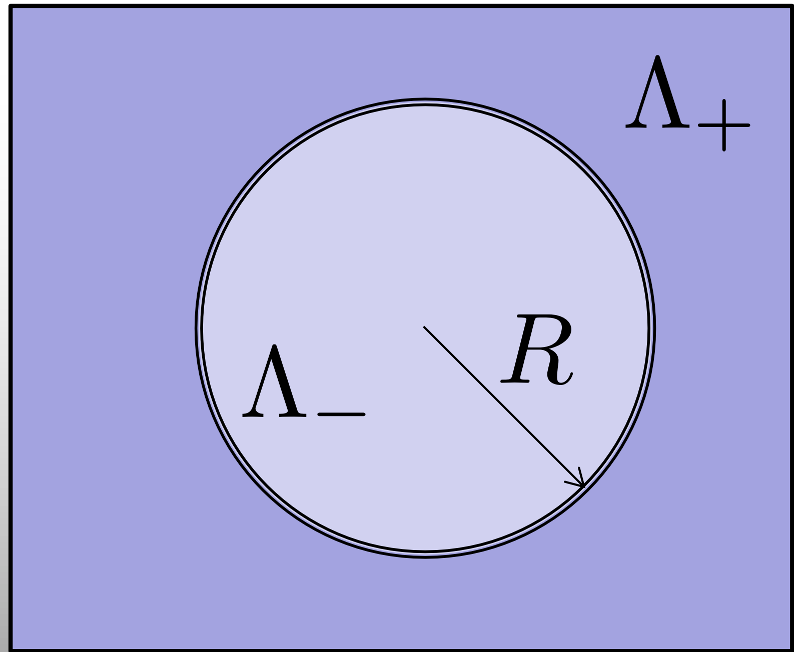
The energy cost of the interface is the surface "area" times the tension, and the energy gain is the difference in vacuum energy times the volume:

ENERGY COST $\quad \sigma \times 2\pi^2 R^3$

ENERGY GAIN $\quad \varepsilon \times \pi^2 R^4/2$

Solution stationary wrt R,

$$\Rightarrow R = 3\sigma/\varepsilon$$



Coleman Bounce

This gives us the bubble radius, and the amplitude for the decay – backed up by full field theory calculations.

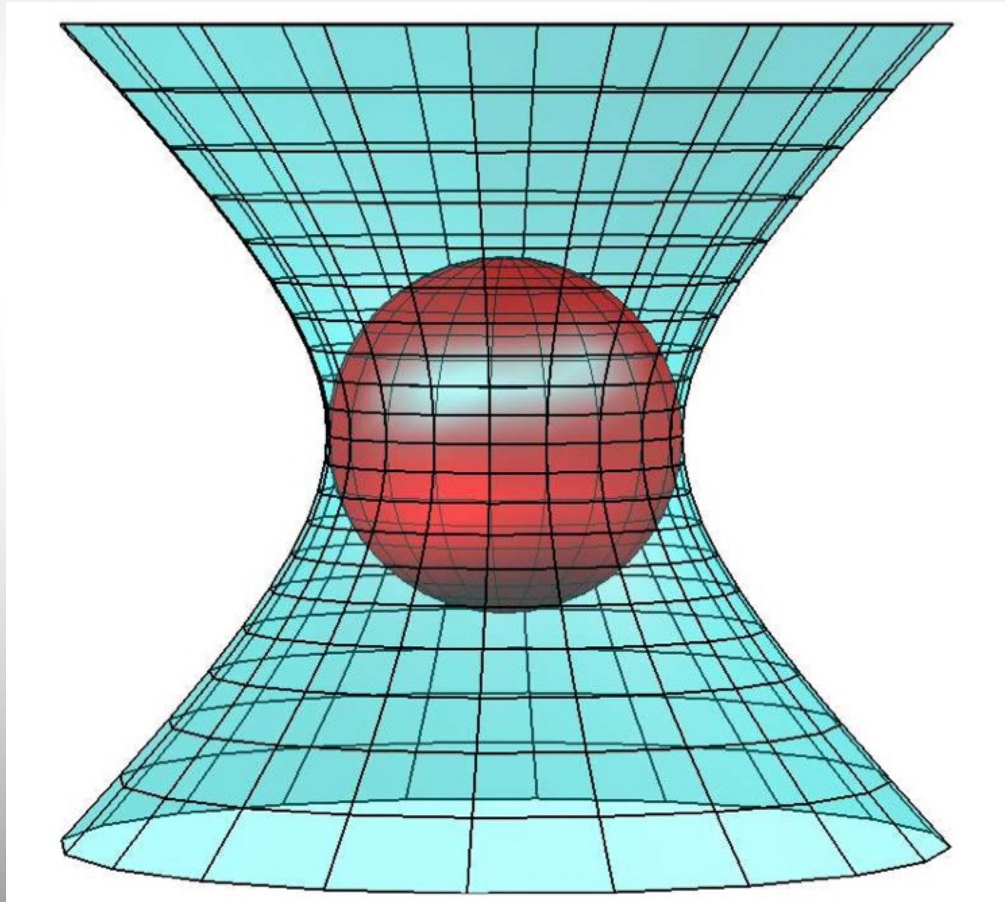
$$\mathcal{B} = \frac{\pi^2 R^3}{2} \left(-\sigma + \varepsilon R\right) \sim \frac{27\pi^2}{2} \frac{\sigma^4}{\varepsilon^3}$$

This gives the leading order or saddle point approximation to the amplitude:

$$\mathcal{P} \sim e^{-\mathcal{B}/\hbar}$$

Does this Euclidean calculation ***mean*** anything real?
Conventional answer is to rotate back to real time: $i\tau \to t$

$$r^2 + \tau^2 = R^2 \to r^2 - t^2 = R^2$$



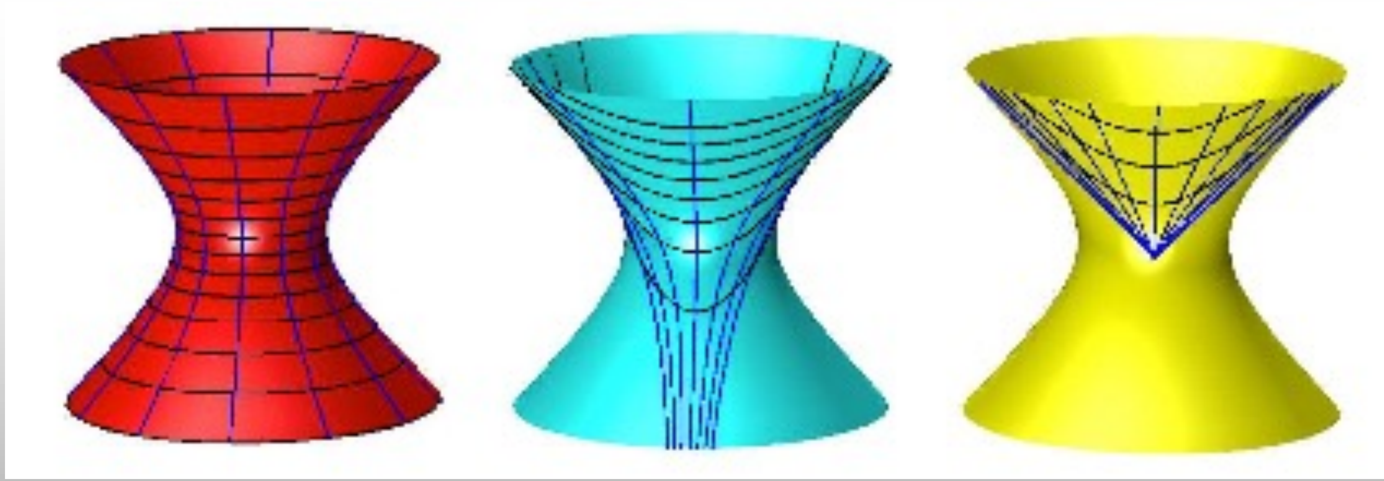
Real time picture is that the bubble expands rapidly.

$$r^2 = R^2 + t^2$$

So it seems the Euclidean picture has some validity.

Gravity and the Vacuum

Vacuum energy gravitates – e.g. a positive cosmological constant gives us de Sitter spacetime – so we must add gravity to this picture



Quantum Effects in Gravity

Although we do not have an uncontested theory of quantum gravity, we do have ideas on how quantum effects in gravity behave below the Planck scale.

Below the Planck scale, we expect that spacetime is essentially classical, but that gravity can contribute to quantum effects through the wave functions of fields, and through the back-reaction of quantum fields on the spacetime.

We use this in black hole thermodynamics, cosmological perturbation theory, and for non-perturbative solutions in field theory, this method is particularly unambiguous, but can we test these ideas in a broader sense?

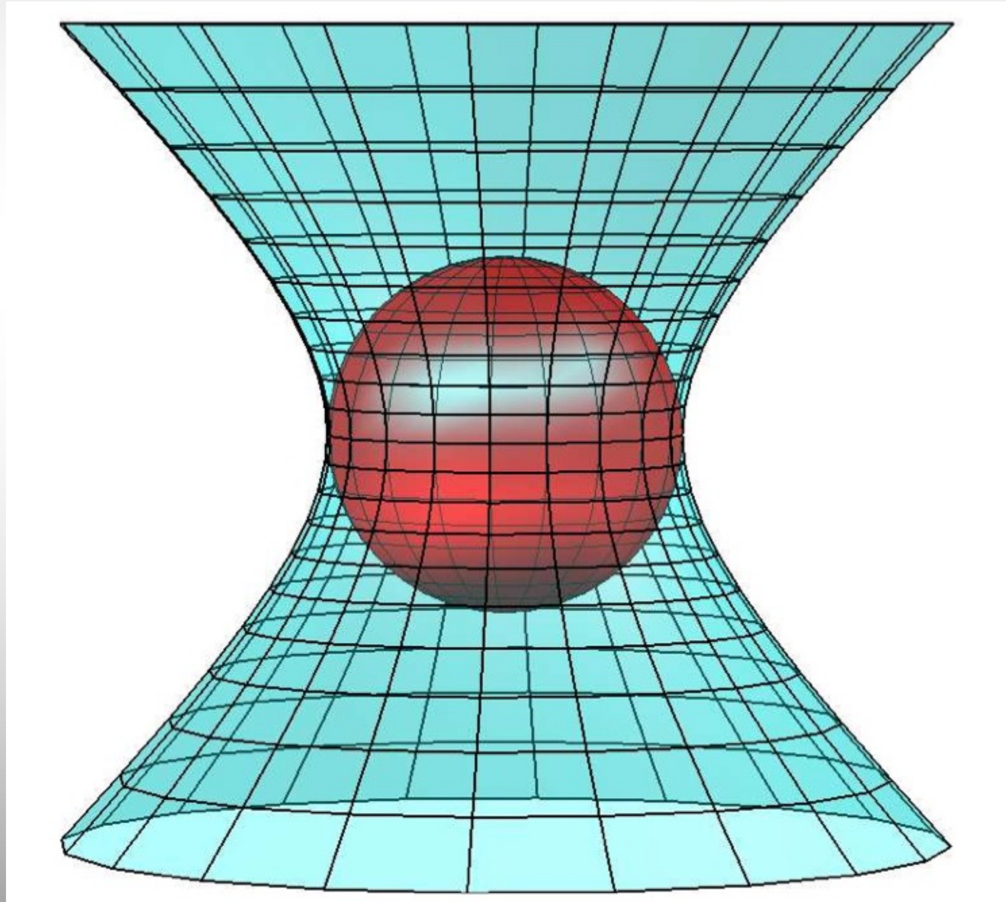
Gibbons-Hawking Euclidean Approach

Extend partition function description to include the Einstein-Hilbert action – at finite temperature we take finite periodicity of Euclidean time.

$$S = -\frac{M_p^2}{2}\int d^4x\sqrt{|g|}R + \int d^4x\mathcal{L}_{SM}$$

Fluctuations treated with caution, but saddle points unambiguous.

De Sitter spacetime has a Lorentzian (real time) and Euclidean (imaginary time) spacetime. The real time expanding universe looks like a hyperboloid and the Euclidean a sphere:

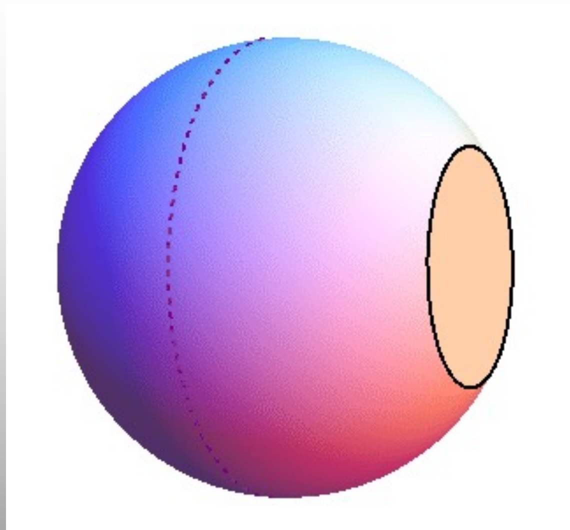


Our instanton must cut the sphere and replace it with flat space (true vacuum).

Coleman de Luccia (CDL)

Coleman and de Luccia showed how to do this with a bubble wall: Euclidean de Sitter space is a sphere, of radius ℓ related to the cosmological constant. The true vacuum has zero cosmological constant, so must be flat.

$$\Lambda = \frac{3}{\ell^2}$$



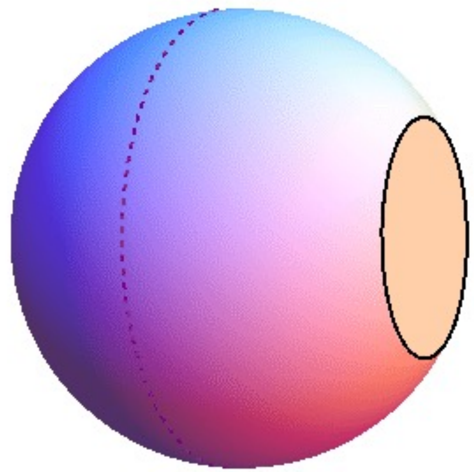
The bounce looks like a truncated sphere.

Coleman and de Luccia, PRD21 3305 (1980)

Goldilocks with Gravity

We can play the same "Goldilocks bubble" game – finding the cost of making this truncated sphere, but adding in the effect of gravity.

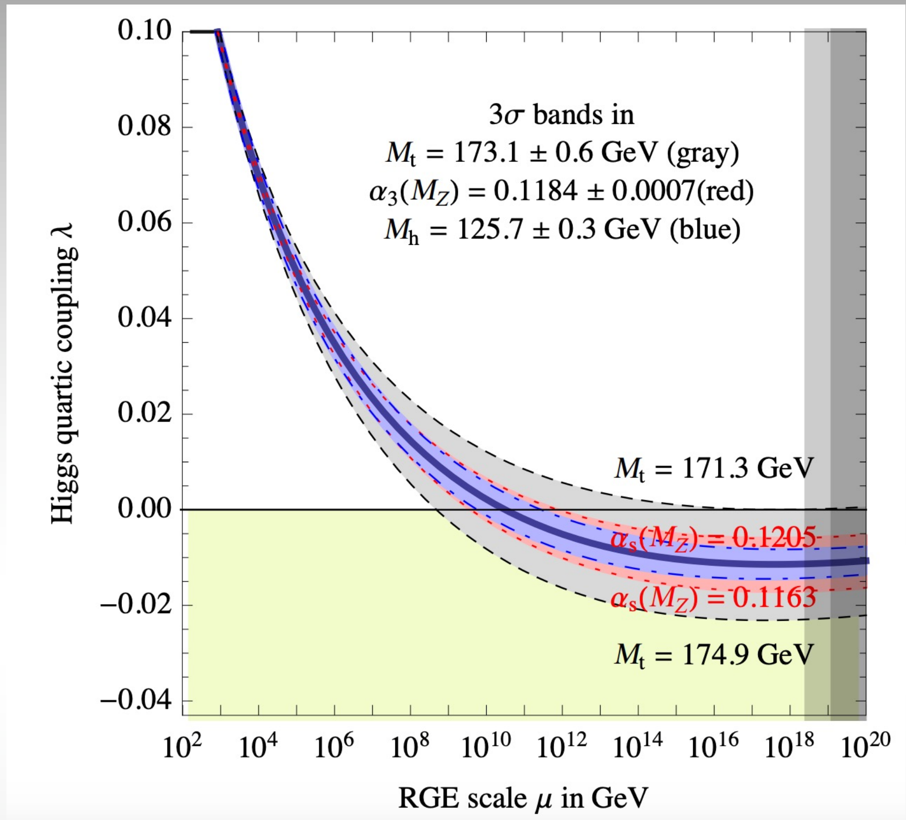
$$\mathcal{B}(R) = \frac{4}{3}\pi^2\varepsilon\ell^4\left[1-\left(1-\frac{R^2}{\ell^2}\right)^{\frac{3}{2}}\right] - 2\pi^2\varepsilon\ell^2R^2 + 2\pi^2\sigma R^3$$



CDL Action

Once again, too small a bubble will recollapse, and large bubbles are harder to make, so there is a "just right" bubble that corresponds to a solution of the Euclidean Einstein equations that we can find either numerically with the full field theory, or analytically if we take our bubble wall to be thin, and we can find our instanton action.

$$
\begin{aligned}
\mathcal{B} &= -\frac{\Lambda}{8\pi G}\int_{\text{int}} d^4x\sqrt{g} - \frac{\sigma}{2}\int_{\mathcal{W}} d^3x\sqrt{h} \\
&= \frac{\pi\ell^2}{4G}(1-\cos\chi_0)^2 = \frac{\pi\ell^2}{G}\frac{16\bar{\sigma}^4\ell^4}{(1+4\bar{\sigma}^2\ell^2)^2}
\end{aligned}
$$



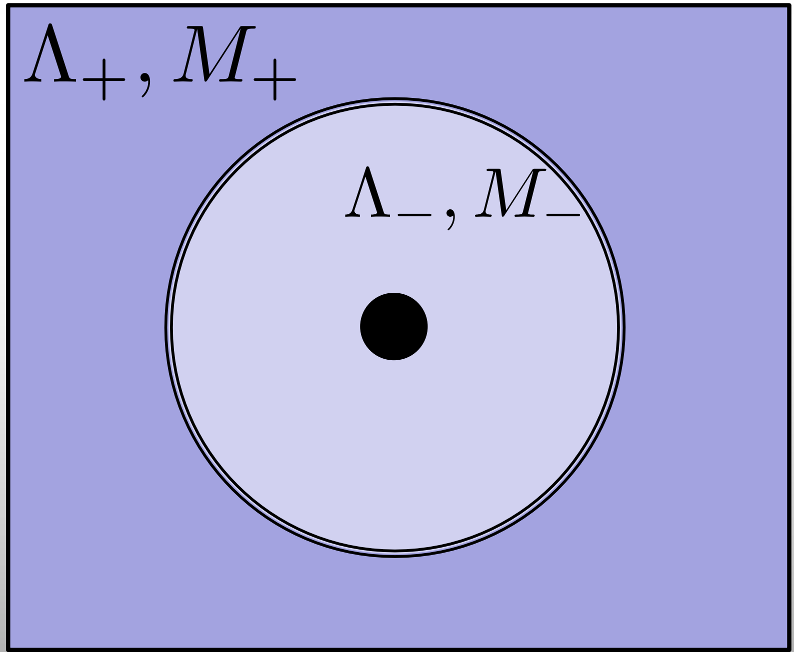
For the Higgs, this gives an insanely long half-life!

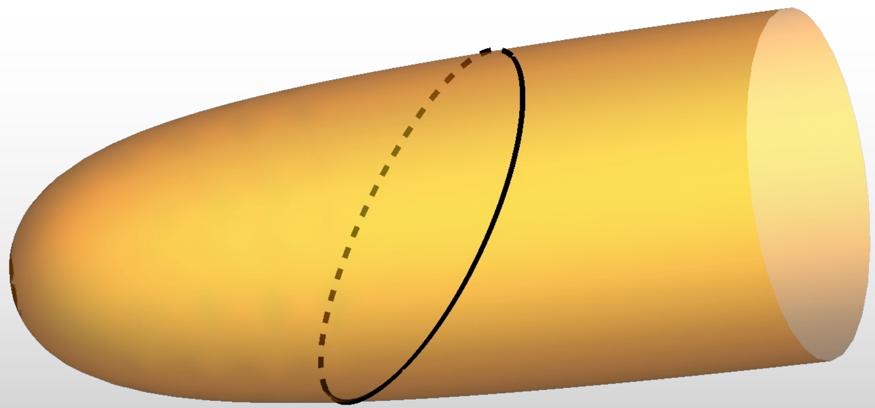
BUT

Most first order phase transitions do not proceed by ideal bubble nucleation, but by seeds.
These calculations are very idealised – an empty and featureless background – what if we throw in a little impurity?

Tweaking CDL

A black hole is an inhomogeneity, and also exactly soluble:





Hiscock; Berezon, Kuzmin & Tkachev

RG, Moss & Withers, 1401.0017

Goldilocks Black Hole Bubbles

- The bubble with a black hole inside, can have a different mass term outside (seed).
- The solution in general depends on time, but for each seed mass there is a unique bubble with lowest action.
- For small seed masses this is time, but the bubble has no black hole inside it – no remnant black hole.

- For larger seed masses the bubble does not depend on Euclidean time, and has a remnant black hole.

This last case is the relevant one – the action is the difference in entropy (area) between the seed and remnant black holes!

Black Hole Bounces

Balance of action changes because of periodic time:

$$B \sim \sigma \times 4\pi R^2 L - \varepsilon \times \frac{4}{3}\pi R^3 L$$

$$R \sim 2\sigma/\varepsilon$$

$$B \sim \frac{\sigma^3}{\varepsilon^2} L$$

The result is that the action is the difference in entropy of the seed and remnant black hole masses:

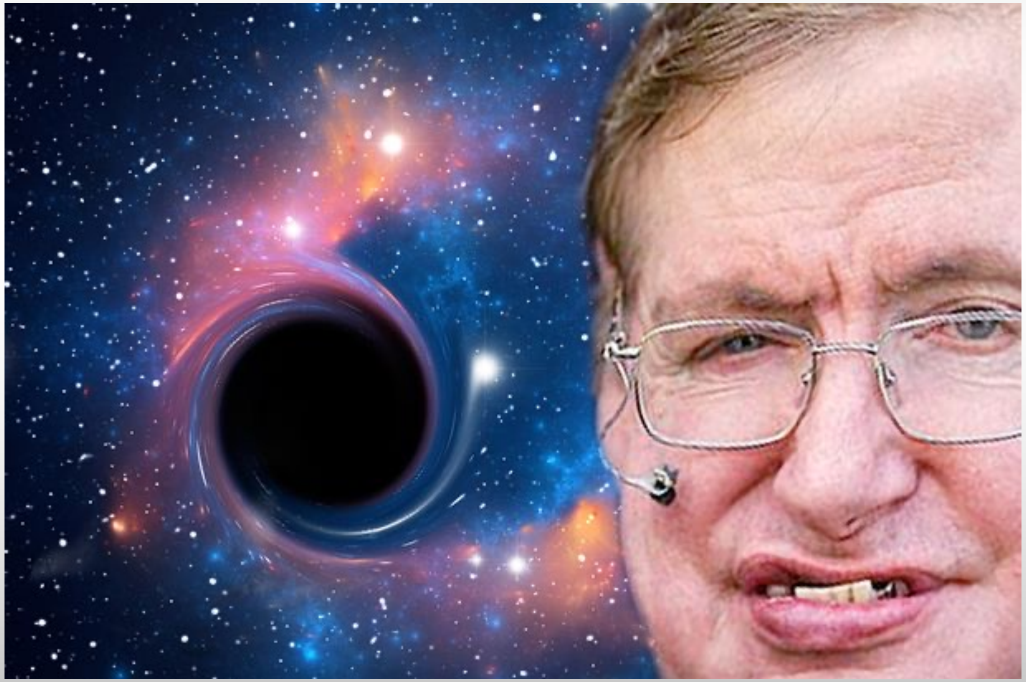
$$B \sim \mathcal{A}_+ - \mathcal{A}_-$$

Seeded tunneling is much more likely than CDL!

The fate of the black hole?

Vacuum decay is not all that can happen! Hawking tells us that black holes are black bodies, and radiate:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}$$



So we must compare evaporation rate to tunneling half-life.

Tunneling v Evaporation

Although we have computed bubble actions in full, we can estimate the dependence of the action on mass using input from our solutions which show that the seed and remnant masses are very close:

$$\begin{aligned} \mathcal{B} &= \pi(r_s^2 - r_r^2) \\ &\sim 4\pi(M_s + M_r)(M_s - M_r) \\ &\sim 8\pi M_s \delta M \end{aligned} \quad \Rightarrow \Gamma_D \propto e^{-8\pi M_s \delta M}$$

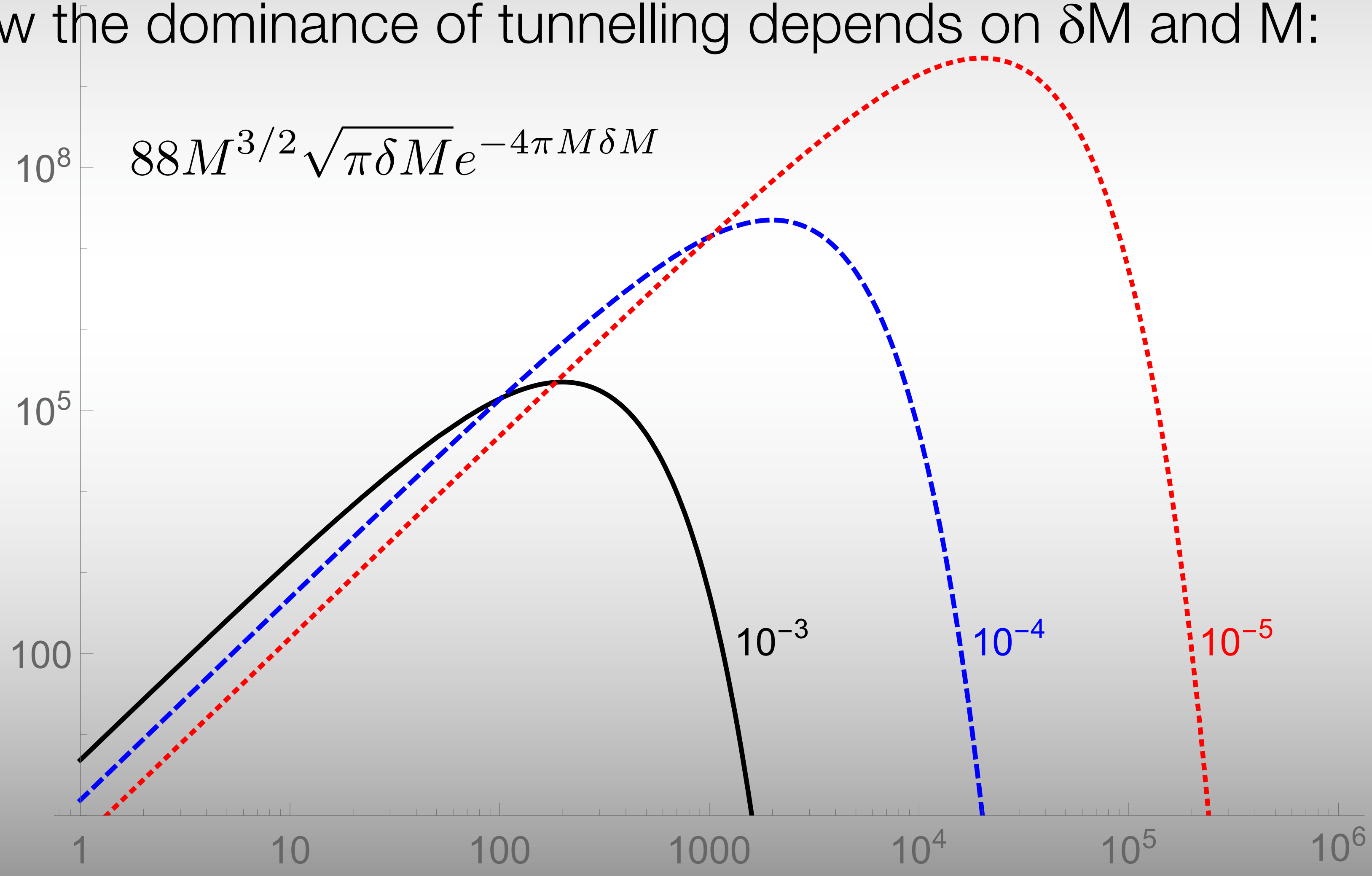
So our decay rate depends on an exponential of M_s, whereas evaporation depends on an inverse power of M – tunneling becomes important for smaller M

$$\Gamma_H \approx 3.6 \times 10^{-4} M^{-3}$$

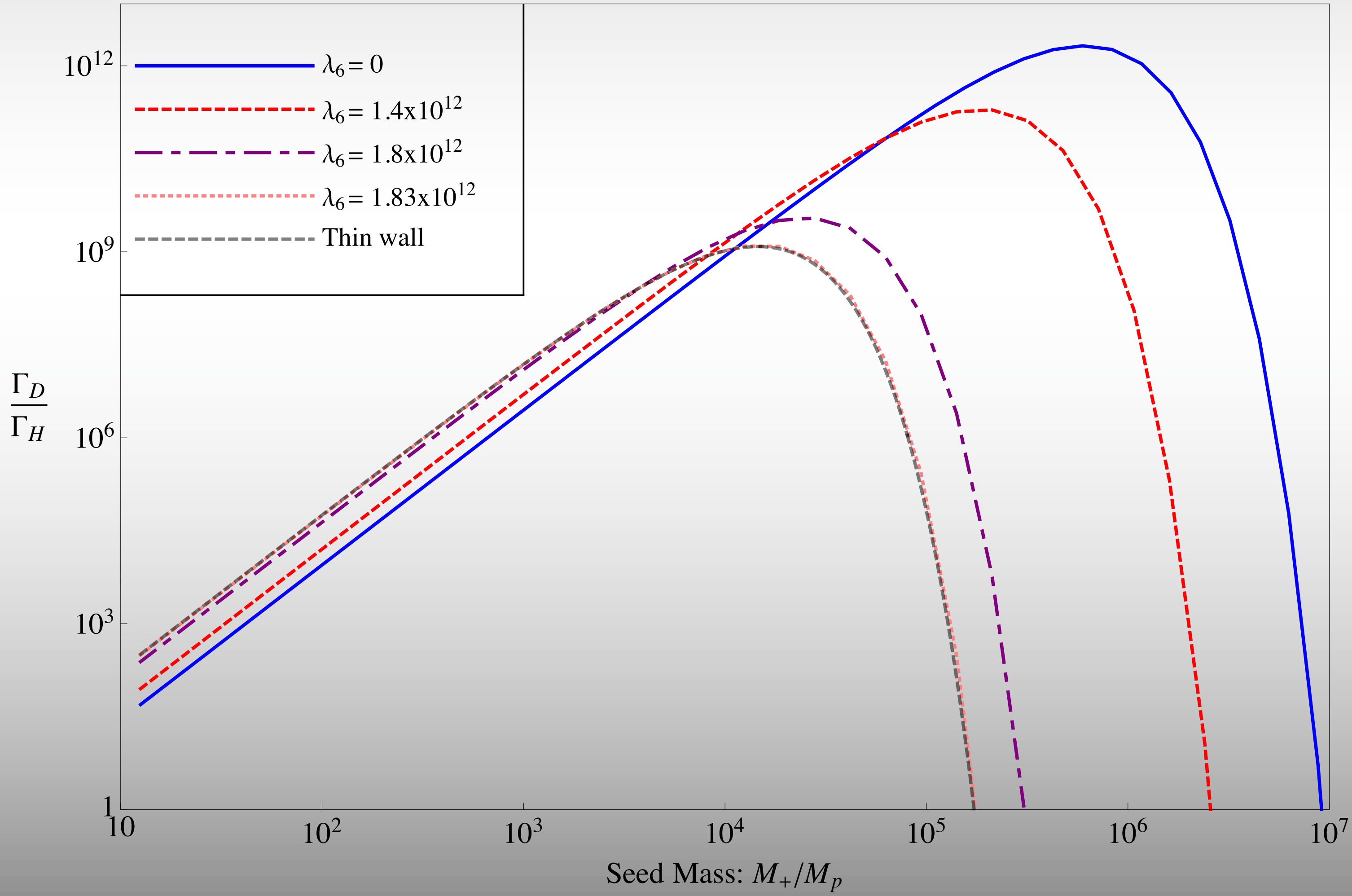
Page, PRD-76

Tunneling v Evaporation

Taking this branching ratio estimate (in Planck units) shows how the dominance of tunnelling depends on δM and M:

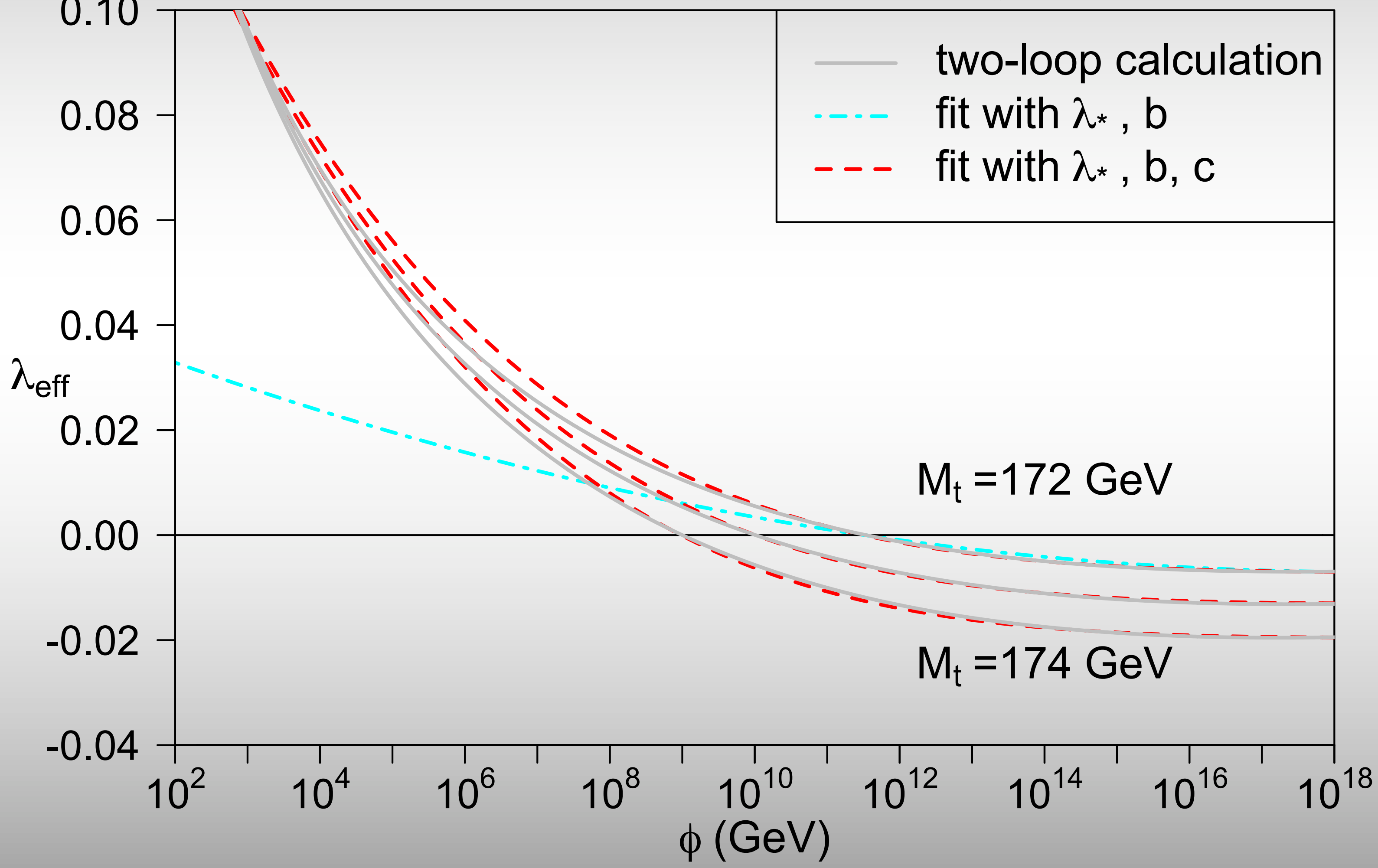


These results are representative with a full (Euclidean) numerical calculation of a fit to the (SM and beyond) potential, but indicate that only Primordial Black Holes can catalyse decay.

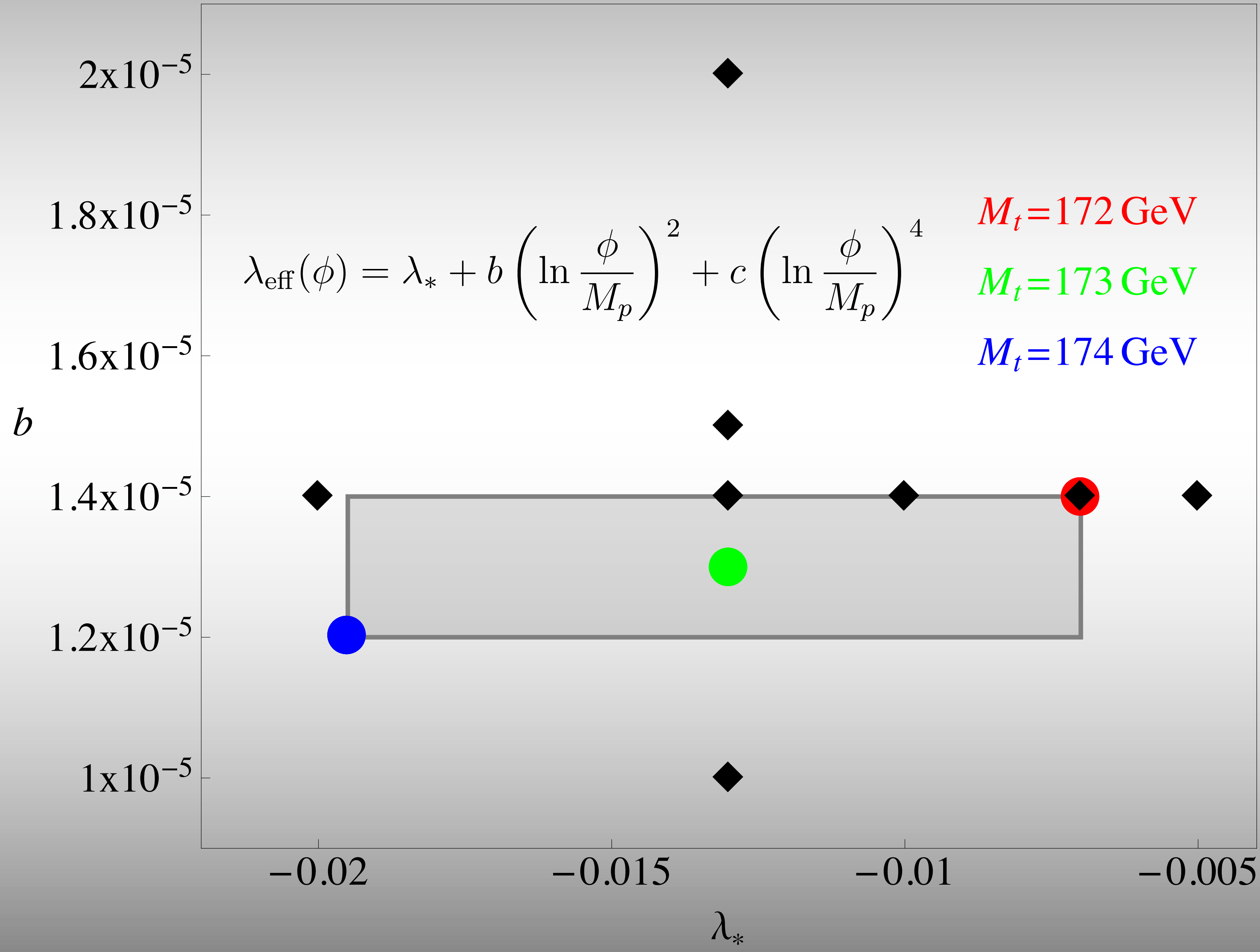


Fitting the Potential

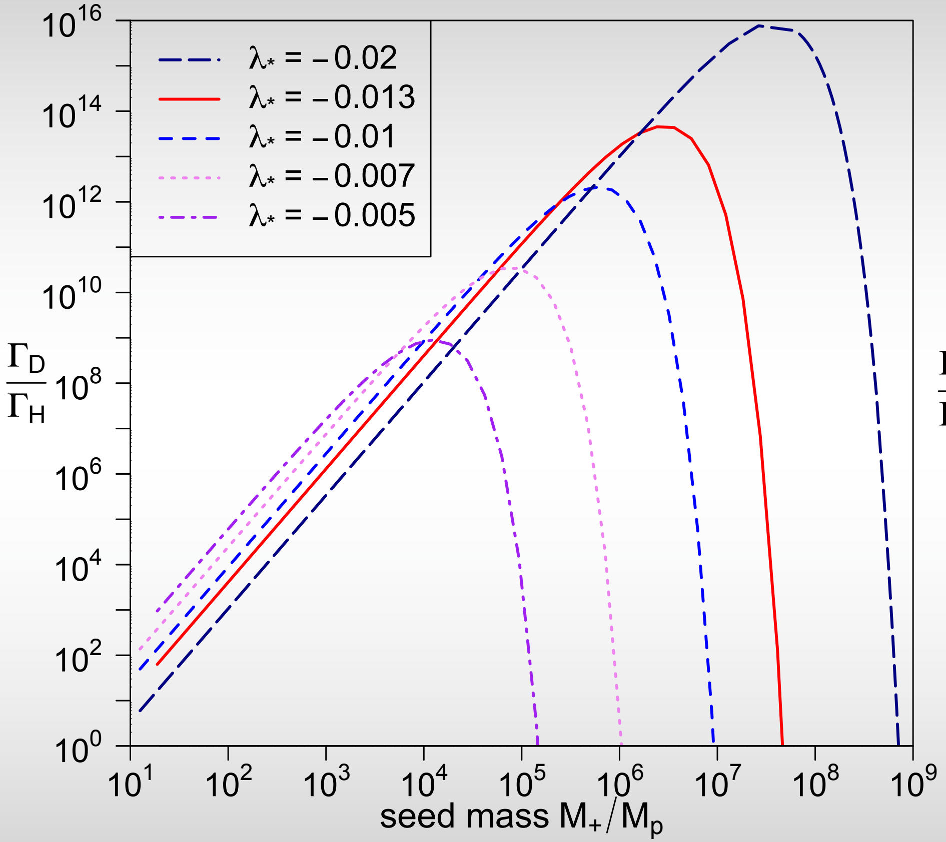
$$\lambda_{\text{eff}}(\phi) = \lambda_* + b\left(\ln\frac{\phi}{M_p}\right)^2 + c\left(\ln\frac{\phi}{M_p}\right)^4$$

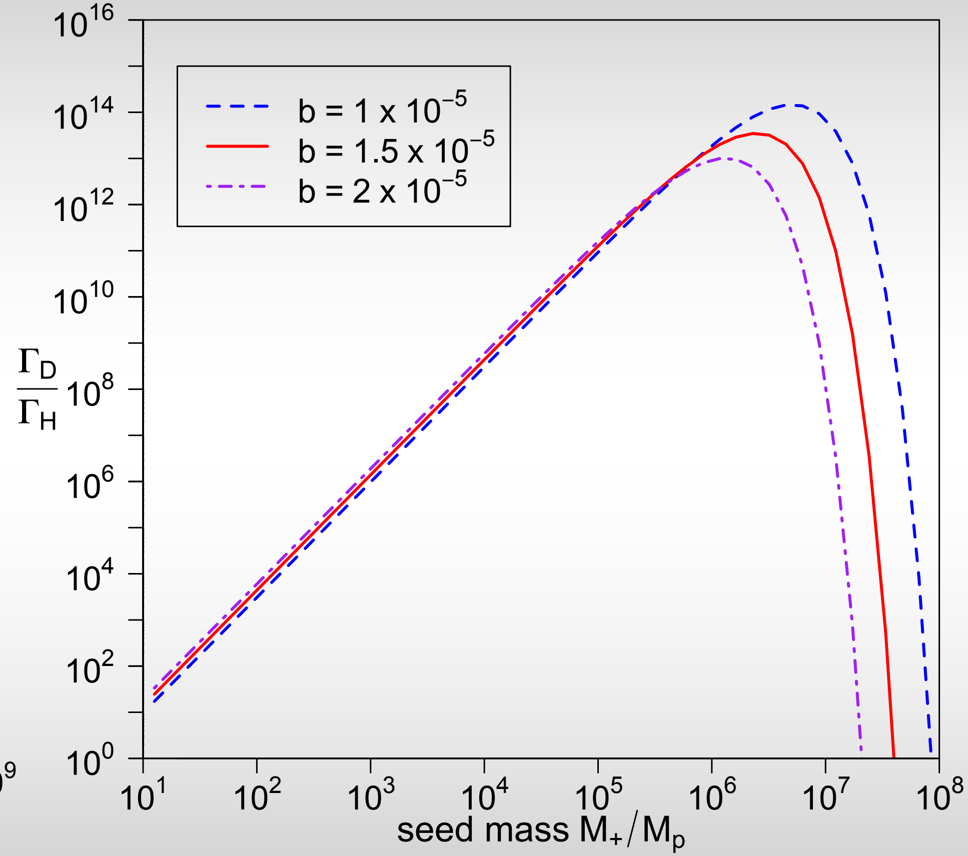


Numerical Integration



Scanning through parameter space for pure SM potential shows main dependence on λ_*:

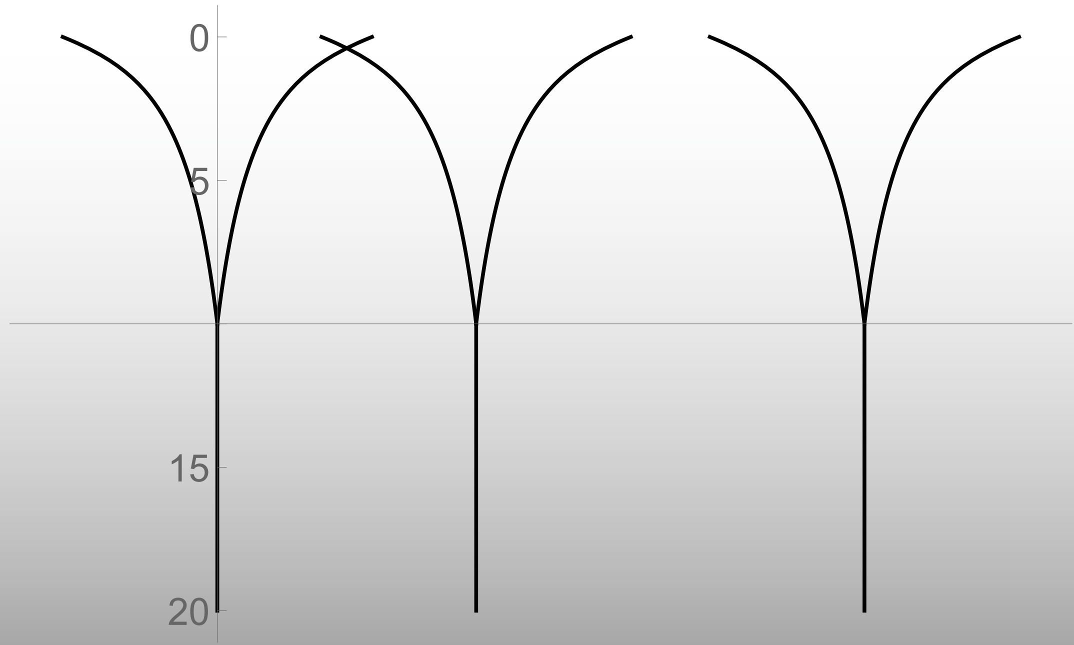




And because we are at such extreme scales, the lifetime of the universe drops to around 10^{-17}s!

PBH Seeded Decay

These results place strong bounds on the allowed mass range of primordial black holes, from the mass at formation, we can calculate the redshift at which these black holes will enter the "danger range" for vacuum decay catalysis, and in essence we cannot have a primordial black hole of less than about 10^{16} g

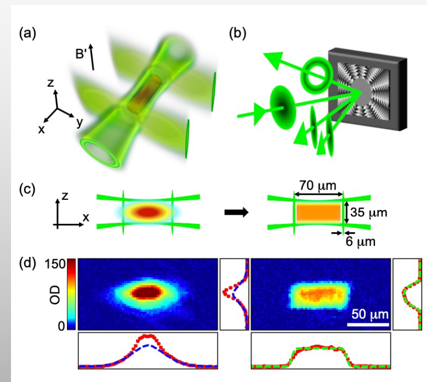


Testing Vacuum Decay

The Euclidean method is a tool – but how much does it capture of the real process? Should we be trying other techniques? QM tunnelling well tested, but QFT tunnelling is another matter.

QSimFP | Quantum Simulators for Fundamental Physics

The false vacuum decay workpackage aims to test the process of relativistic vacuum decay via cold atoms in trap* whose effective theory is a relativistic vacuum.



* A. L. Gaunt et al., Phys. Rev. Lett. 110, 200406 (2013).

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Summary

- Vacuum decay is an example of quantum effects in action with gravity – we have good tools, but they are idealised.
- Tunneling amplitudes significantly enhanced in the presence of a black hole – bubble forms around black hole and can remove it altogether. Important if Higgs vacuum metastable.
- Not a problem for PBH models of dark matter, but if there is a spread of M_F then there may be a constraint.