TACHYONS AND MISALIGNED SUPERSYMMETRY

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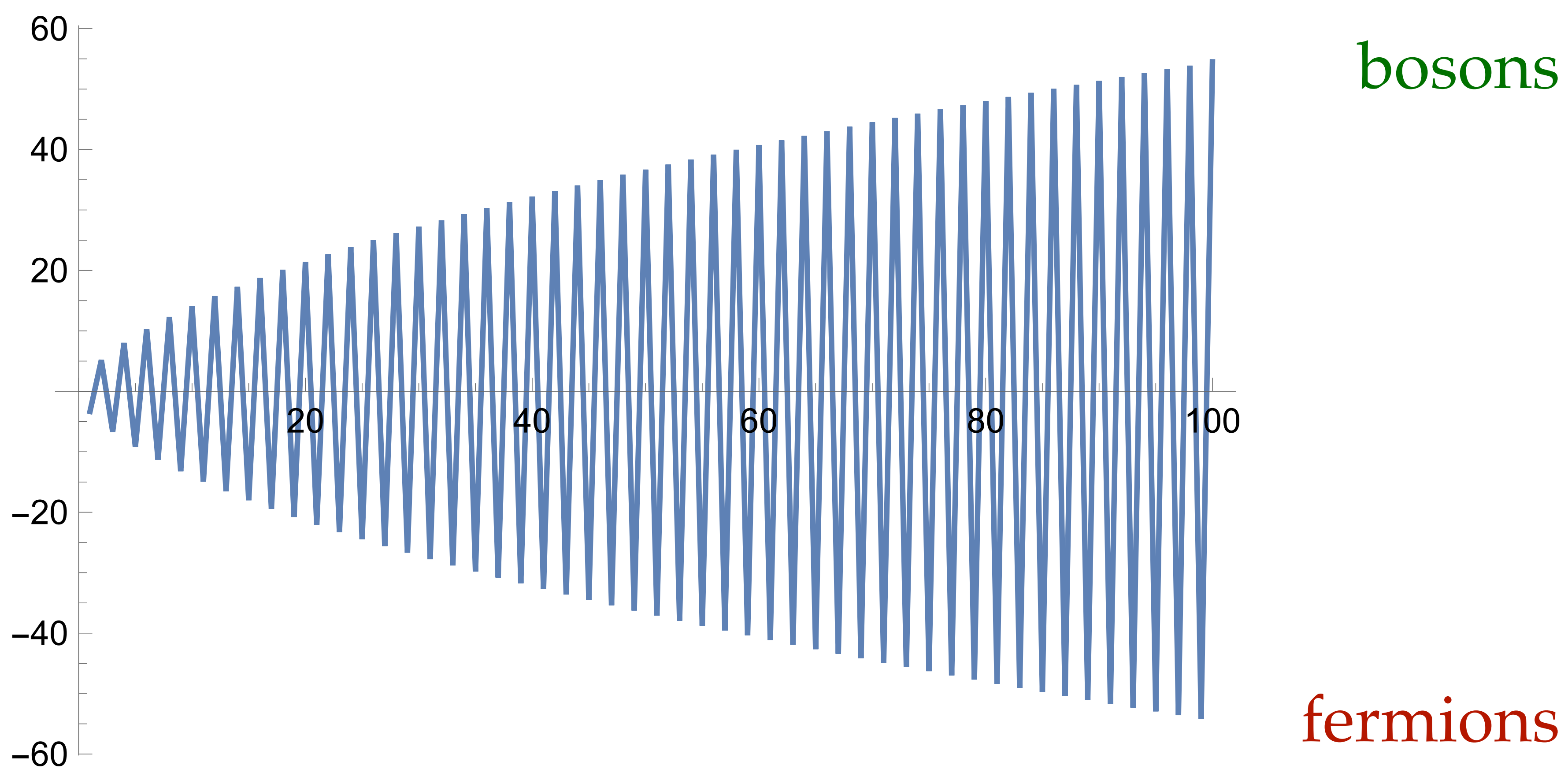
Based on work in progress with Giorgio Leone and Ioannis Florakis
To appear soon

In the mid 90's Keith Dienes conjectured that a hidden ***misaligned supersymmetry*** is present in the string spectrum of non-tachyonic (non-supersymmetric) vacua



He argued that this *misaligned supersymmetry* be responsible for the finiteness of the one-loop vacuum energy of closed strings

The Spectrum of Closed Strings

$$
\begin{aligned}
\mathcal{Z} &= \sum_s \mathrm{tr}_s P_{\mathrm{GSO}}\,(-1)^F\, q^H \bar{q}^{\bar{H}} \\
&= \tau_2^{1-D/2} \sum_{i,j} \bar{\chi}_i(\bar{q})\,\mathcal{N}_{ij}\chi_j(q) \\
&= \tau_2^{1-D/2} \sum_{m,n} a_{mn}\,\bar{q}^m\, q^n
\end{aligned}
$$

Characters of the RCFT

Integer-valued matrix fixed
by modular invariance

a_{nn} are (signed) integers and count the number
of degrees of freedom at the n-th mass level

The Basic Idea of Misaligned Supersymmetry

$$\bar{\chi}_i(\bar{q})\,\chi_j(q) \Rightarrow a_{nn}^{(ij)} \sim An^{-B}\,e^{4\pi C_{\mathrm{tot}}\sqrt{n}}$$

The exponential growth is determined by the central charge of the CFT

$$C_{\mathrm{tot}} = C_{\mathrm{left}} + C_{\mathrm{right}} \equiv \sqrt{\frac{c_{\mathrm{left}}}{24}} + \sqrt{\frac{c_{\mathrm{right}}}{24}}$$

The Basic Idea of Misaligned Supersymmetry

K. Dienes proved that, when tachyons are absent,

$$\langle a_{nn} \rangle = \sum_{i,j} \mathcal{N}_{ij}\, a_{nn}^{(ij)} \sim e^{4\pi C_{\rm eff}\sqrt{n}}$$

with $C_{\rm eff} < C_{\rm tot}$

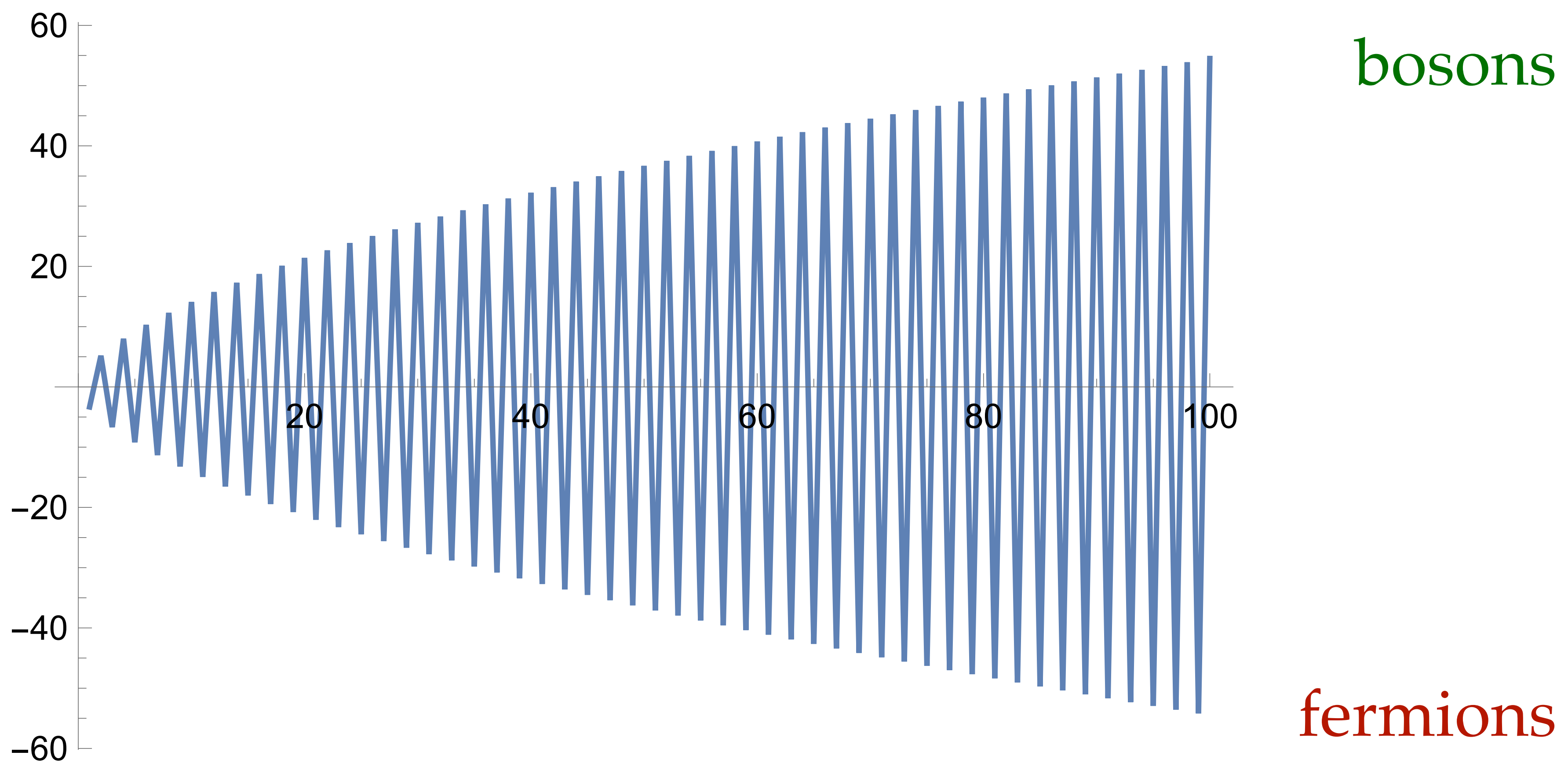
He then conjectured that

$$C_{\rm eff} = 0$$

[Dienes 1994]

The Basic Idea of Misaligned Supersymmetry

He argued that $C_{\text{eff}} < C_{\text{tot}}$ implies oscillations in the string spectrum



What is the right trademark for the absence of tachyons?

What is the right trademark for the absence of tachyons?

To (try to) answer this question, we have studied all non-supersymmetric theories in ten dimensions, and their freely-acting deformation in nine dimensions

A clear pattern emerges from this analysis

NON-SUPERSYMMETRIC STRING VACUA

In D=10 there are 8 non-supersymmetric vacua

$$\begin{aligned}\mathcal{Z}_{16\times 16} = {} & V_8(\bar{O}_{16}\bar{O}_{16} + \bar{S}_{16}\bar{S}_{16}) - S_8(\bar{O}_{16}\bar{S}_{16} + \bar{S}_{16}\bar{O}_{16}) \\ & + O_8(\bar{V}_{16}\bar{C}_{16} + \bar{C}_{16}\bar{V}_{16}) - C_8(\bar{V}_{16}\bar{V}_{16} + \bar{C}_{16}\bar{C}_{16})\end{aligned}$$

tachyon free

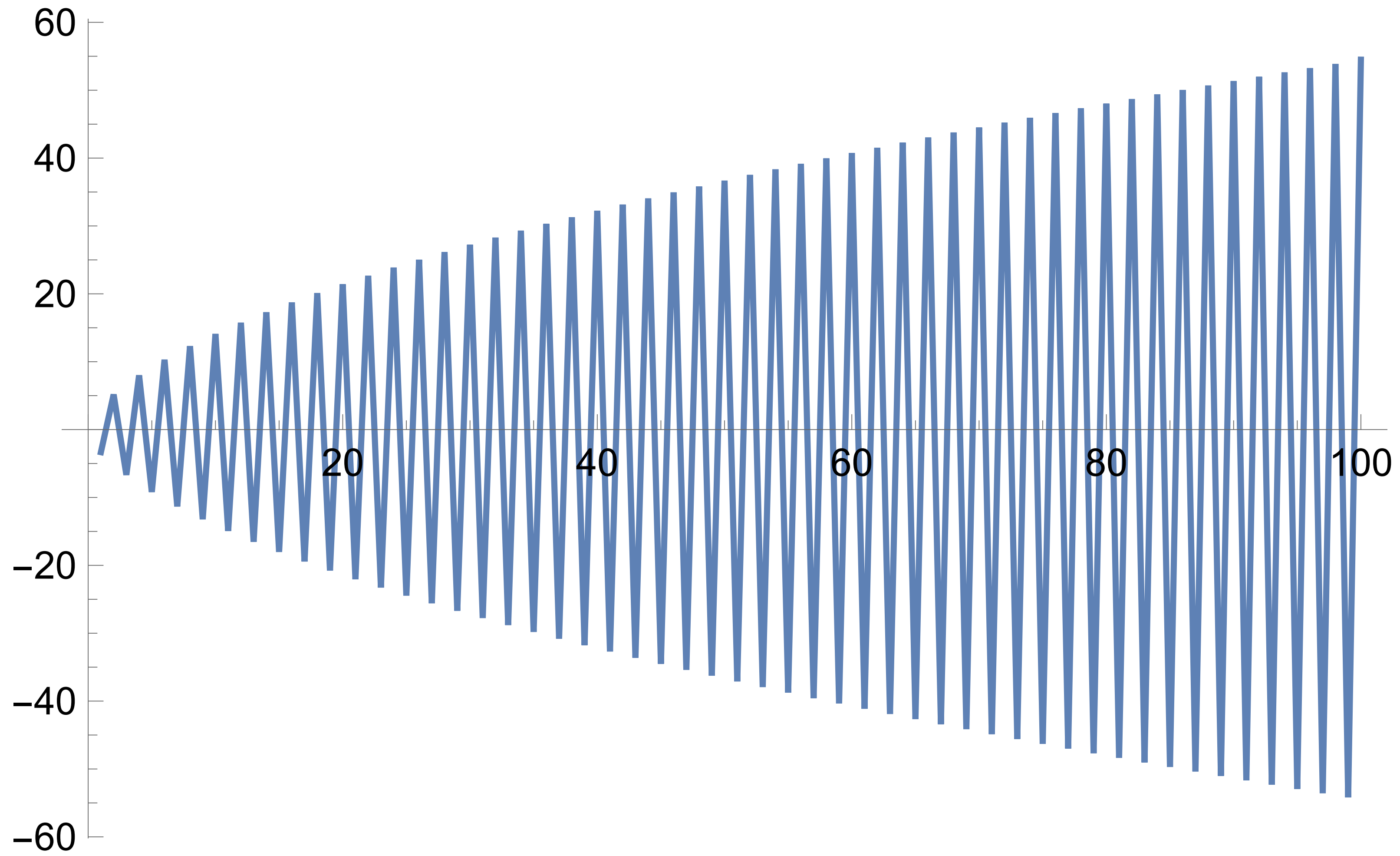
$$\mathcal{Z}_{32} = V_8\,\bar{O}_{32} - S_8\,\bar{S}_{32} + O_8\,\bar{V}_{32} - C_8\,\bar{C}_{32}$$

tachyonic

$SO(24) \times SO(8)$ $\quad$ $SU(2)^2 \times E_7^2$ $\quad$ $SO(16) \times E_8$ $\quad$ $SU(16) \times U(1)$

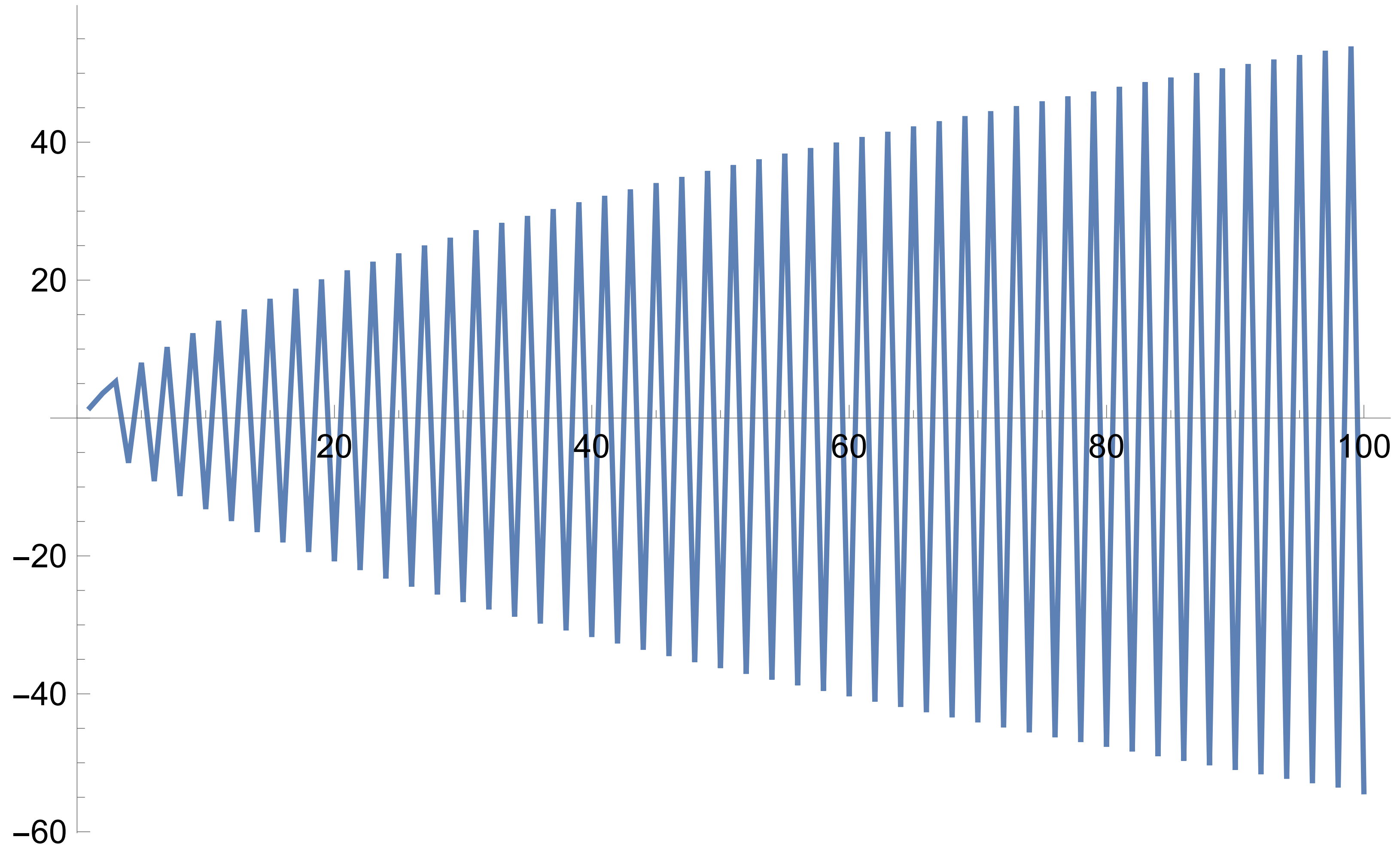
[0A and 0B theories are purely bosonic]

Oscillations in the String Spectrum



$O(16) \times O(16)$

[tachyon free]



$SO(32)$

[tachyonic]

A More Quantitative Analysis of the Spectrum

$$\chi_i(q) = \sum_n a_n^{(i)}\, q^{n+H_i} \qquad\qquad H_i = h_i - c/24$$

The *circle method* of Hardy-Ramanujan yields the asymptotic growth

$$a_n^{(i)} = i^k \sum_{\ell=1}^{O([\sqrt{n}])} \sum_{\substack{p=0 \\ \gcd(\ell,p)=1}}^{\ell-1} \sum_j \left(\gamma_{\ell,p}^{-1}\right)_{ij} e^{-\frac{2\pi i}{\ell}((p(n+H_i)-p' H_j)} \frac{2\pi a_0^{(j)}}{\ell} \left(\frac{H_j}{n+H_i}\right)^{\frac{1-k}{2}} J_{k-1}\left(\frac{4\pi}{\ell}\sqrt{H_j(n+H_i)}\right)$$

[Kani, Vafa 1990]

Characters with negative H_j contribute to the exponential growth, $J_{k-1} \to I_{k-1}$

A CFT can have many characters with negative H_j

The most negative one plays the role of the *identity*

A More Quantitative Analysis of the Spectrum

Taking into account only the *tachyonic* contributions (this with H_j<0)

$$a_n^{(i)} \sim e^{4\pi\sqrt{|H_{\rm min}|\,n}} + \sum_{j\neq \rm min} Q_{ij}\, e^{4\pi\sqrt{|H_j|\,n}}$$
$$+ \sum_{\ell} P_i(\ell;n)\, e^{\frac{4\pi}{\ell}\sqrt{|H_{\rm min}|\,n}} + \sum_{j\neq \rm min} P_{ij}(\ell;n)\, e^{\frac{4\pi}{\ell}\sqrt{|H_j|\,n}}$$

$$P(\ell;n+\ell) = P(\ell;n)$$
$$\sum_{n=0}^{\ell-1} P(\ell;n) = 0$$

The contribution of the *identity* is ***universal***

THE GROWTH OF THE DEGREES OF FREEDOM

$$\sum_{i,j} \bar{\chi}_i(\bar{q})\,\mathcal{N}_{ij}\chi_j(q) \qquad \Rightarrow \qquad \langle a_{nn}\rangle \sim \sum_{ij} e^{4\pi\sqrt{|H^{\mathrm{L}}_{\mathrm{min}}|n}}\,\mathcal{N}_{ij}\; e^{4\pi\sqrt{|H^{\mathrm{R}}_{\mathrm{min}}|n}}$$

Therefore, whenever $\sum_{i,j} \mathcal{N}_{ij} = 0$

$$C_{\mathrm{eff}} < C_{\mathrm{tot}}$$

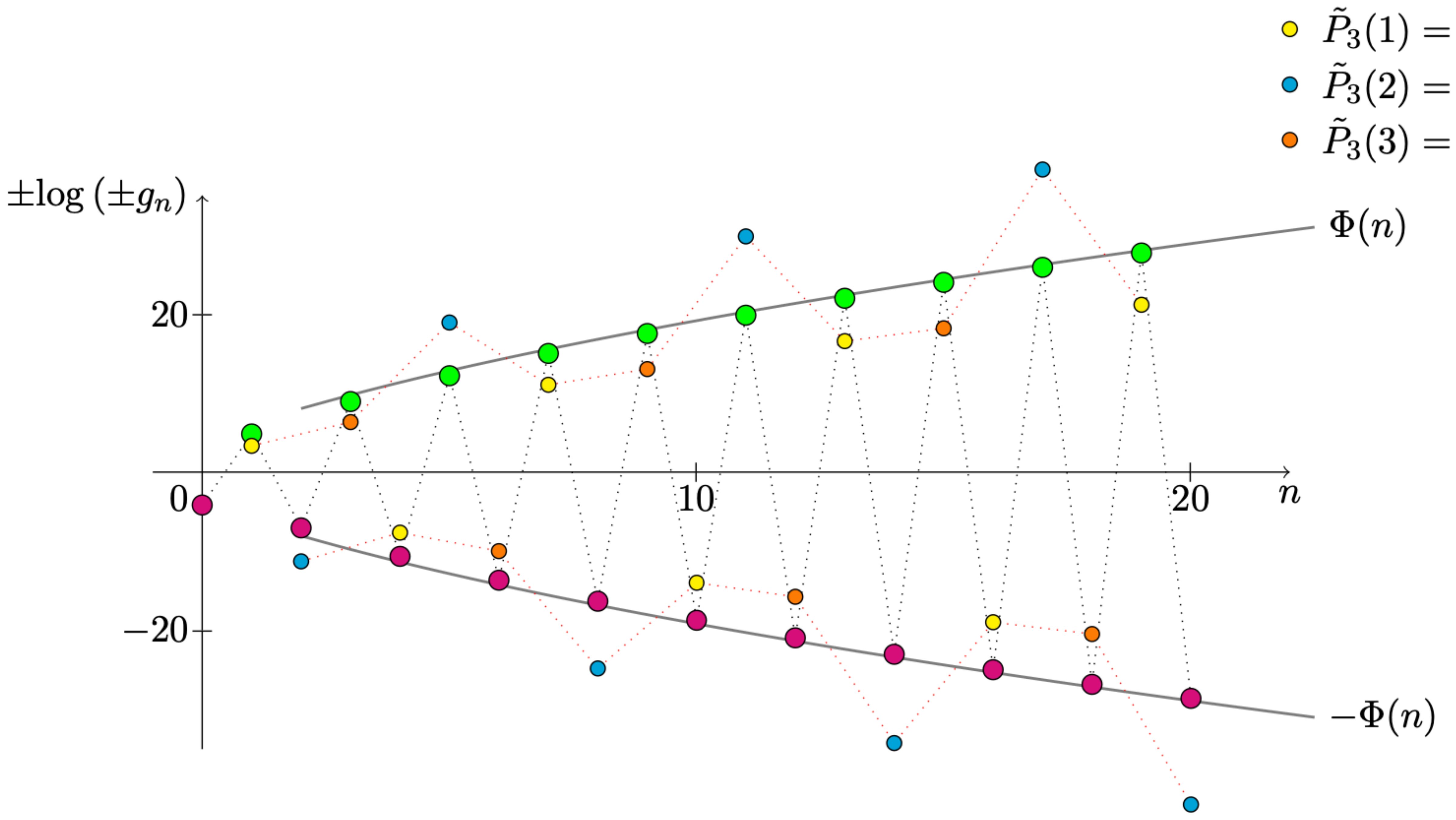
This occurs for ***all*** non-supersymmetric heterotic theories in D=10

What about the sub-leading exponentials?

THE GROWTH OF THE DEGREES OF FREEDOM

For the $O(16) \times O(16)$ heterotic string in D=10 it was "shown" that

$$C_{\text{eff}} = 0$$ [Cribiori et al 2021]



[plot from Cribiori et al 2021]

The Growth of the Degrees of Freedom

For the $SO(32)$ heterotic string in D=10 instead

$$\underbrace{2\, e^{4\pi\sqrt{n/2}}\, e^{4\pi\sqrt{n/2}}}^{}\quad \ell=(1,1) \qquad \ell=(2,2) \qquad \ell=(3,3)$$

$$\langle a_{nn}\rangle \sim \overset{\ell=(1,1)}{2\, e^{4\pi\sqrt{n/2}}\, e^{4\pi\sqrt{n/2}}} + \overset{\ell=(2,2)}{e^{\frac{4\pi}{2}\sqrt{n/2}}\, e^{\frac{4\pi}{2}\sqrt{n/2}}} + \overset{\ell=(3,3)}{4\, e^{\frac{4\pi}{3}\sqrt{n/2}}\, e^{\frac{4\pi}{3}\sqrt{n/2}}}$$

leading R tachyonic character (O_8)

sub-leading L tachyonic character (V_8)

and thus $\qquad C_{\text{eff}} = 2\sqrt{1/2} < 1 + \sqrt{1/2} = C_{\text{tot}}$

NINE-DIMENSIONAL FREELY ACTING CONSTRUCTIONS

All non-supersymmetric vacua can be built from supersymmetric ones via an involution which contains fermion number

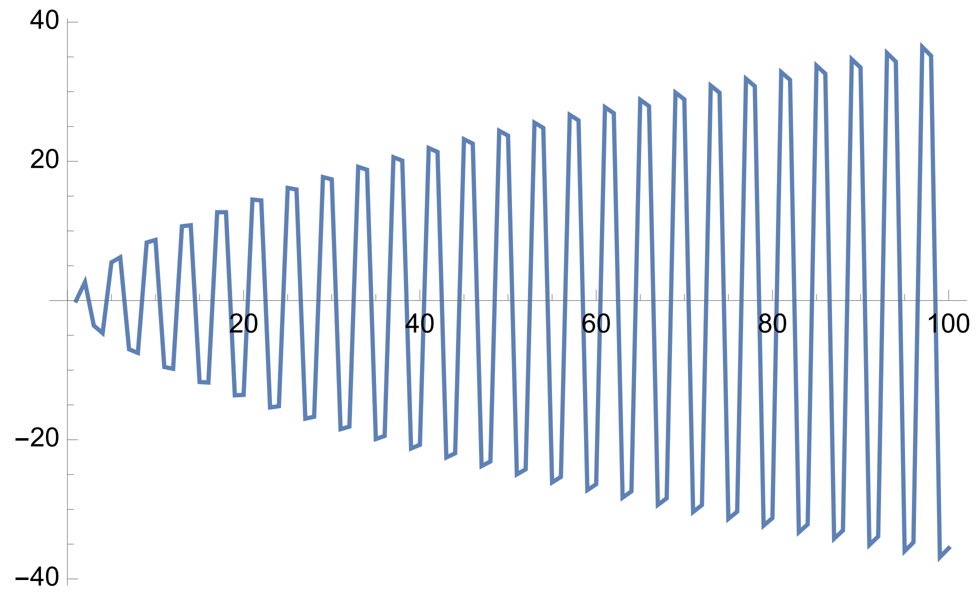
Combining this involution with a shift along a compact direction implies that supersymmetry is broken spontaneously

For instance,

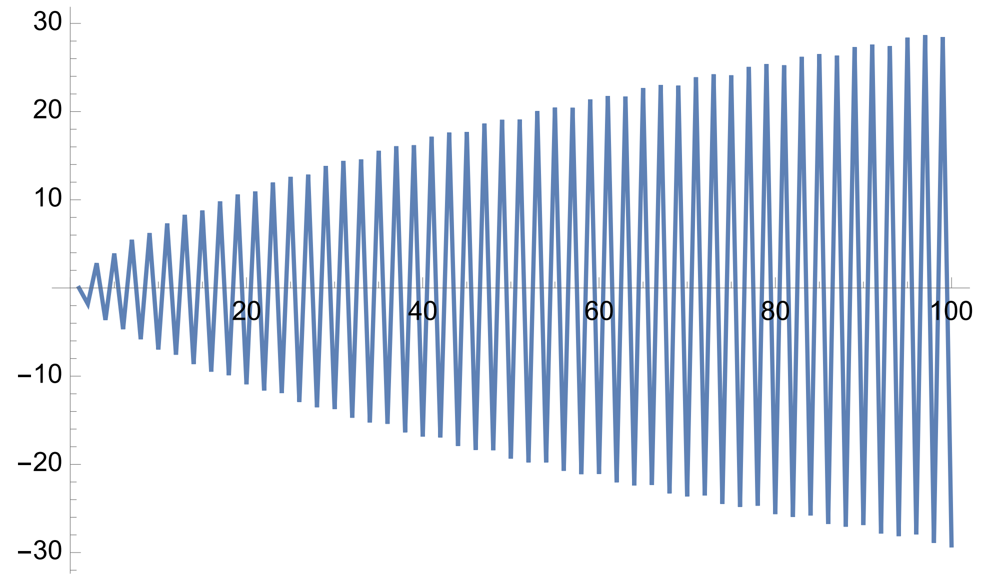
$$\mathcal{Z} = \left[V_8\, \Lambda_{2m,n} - S_8\, \Lambda_{2m+1,n} + O_8\, \Lambda_{2m+1,n+\frac{1}{2}} - C_8\, \Lambda_{2m,n+\frac{1}{2}} \right] (\bar{O}_{32} + \bar{S}_{32})$$

tachyonic for $\sqrt{2\alpha'}\sqrt{3-2\sqrt{2}} < R < \sqrt{2\alpha'}\sqrt{3+2\sqrt{2}}$

Nine-Dimensional Freely-Acting Realisation $O(16) \times O(16)$

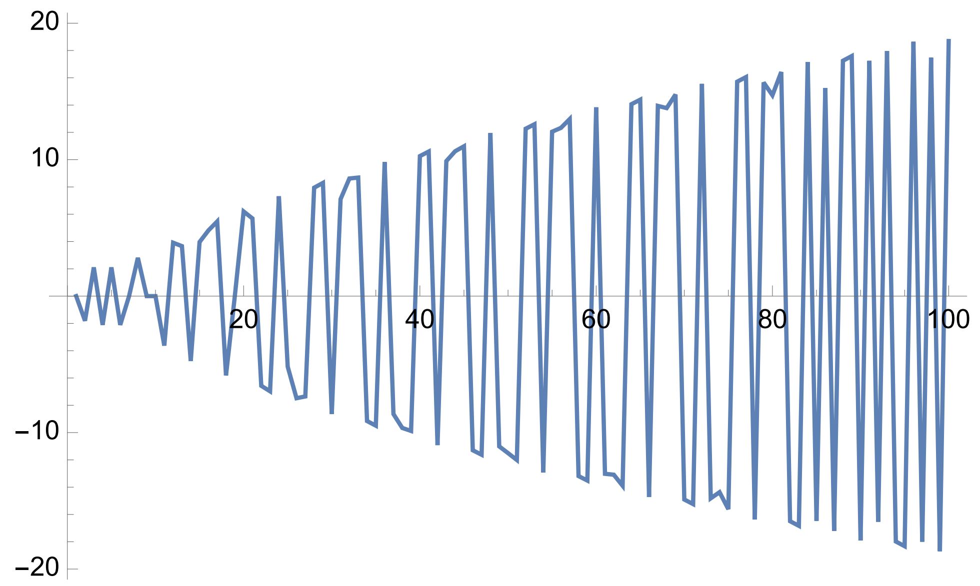


$R = \sqrt{2\alpha'}$

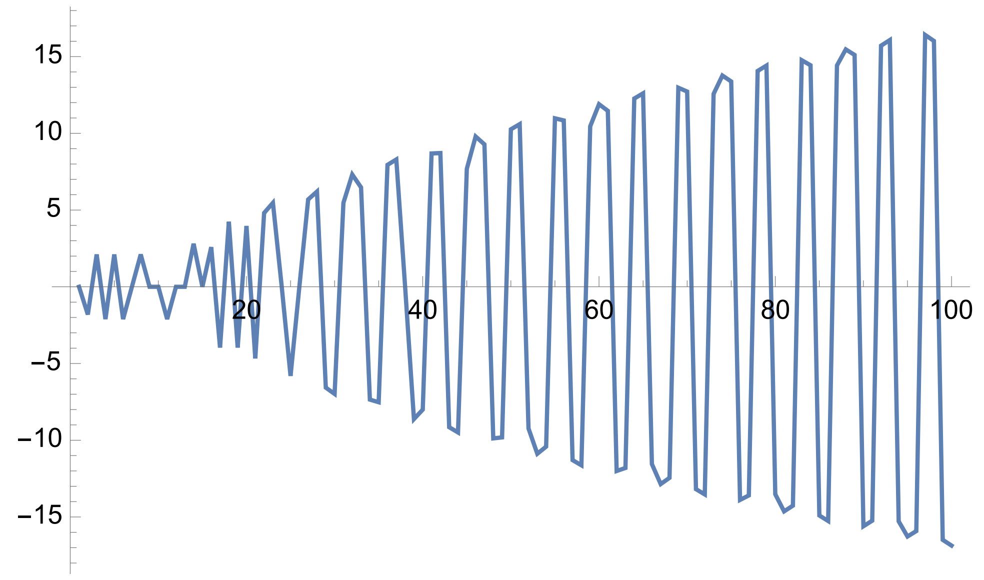


$R = \sqrt{4\alpha'}$

non-tachyonic region



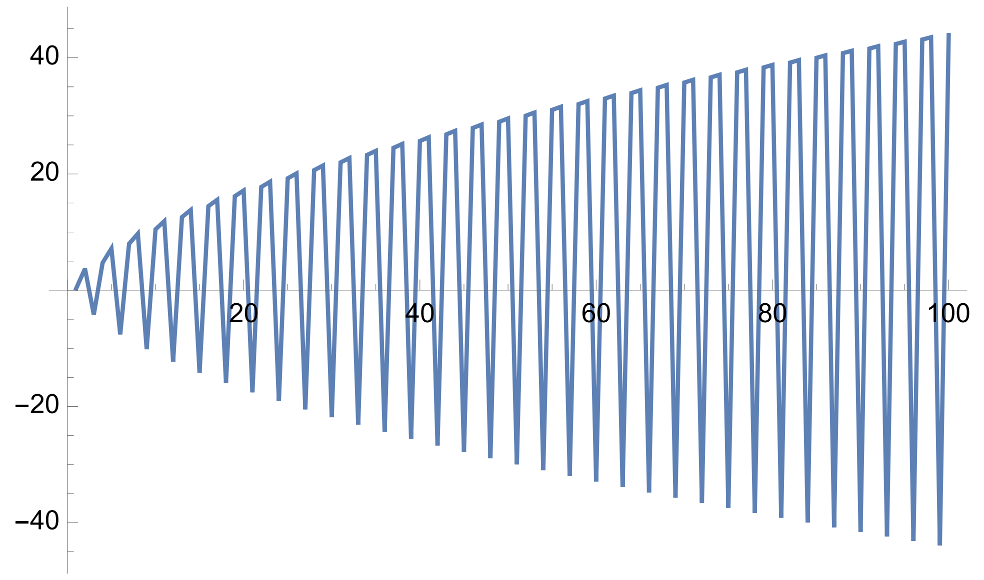
$R = \sqrt{12\alpha'}$



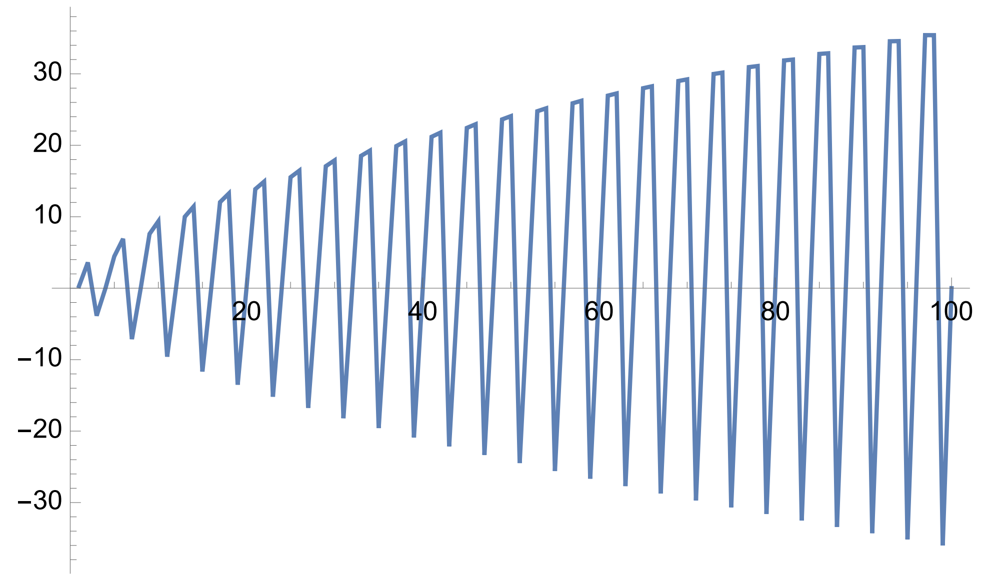
$R = \sqrt{16\alpha'}$

non-tachyonic region

Nine-Dimensional Freely-Acting Realisation $SO(32)$

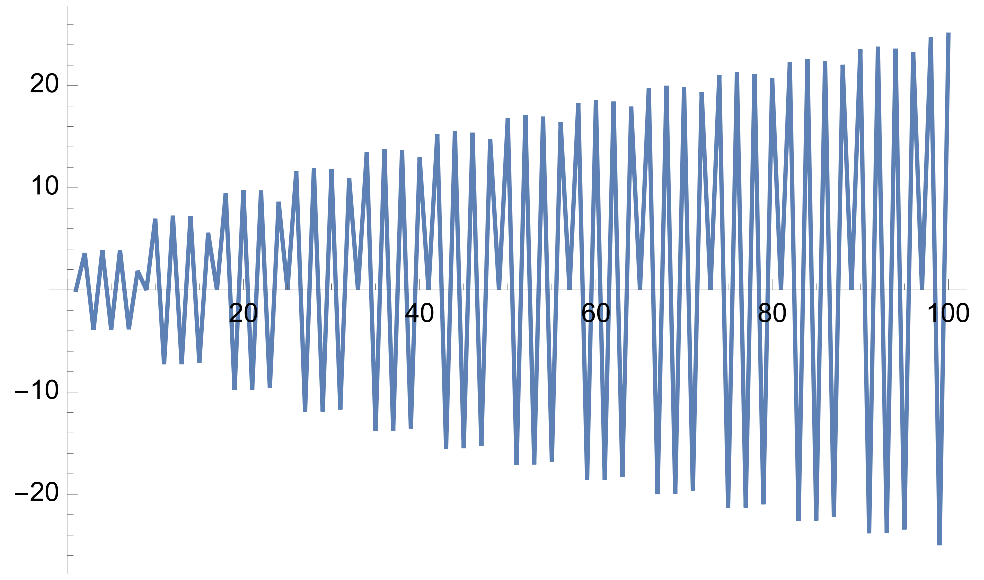


$R = \sqrt{2\alpha'}$

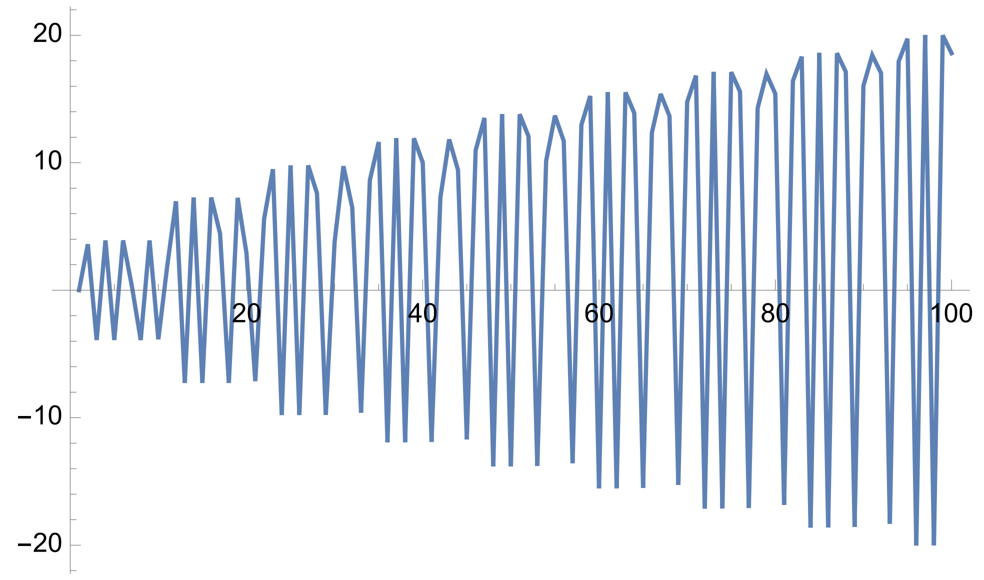


$R = \sqrt{4\alpha'}$

tachyonic region



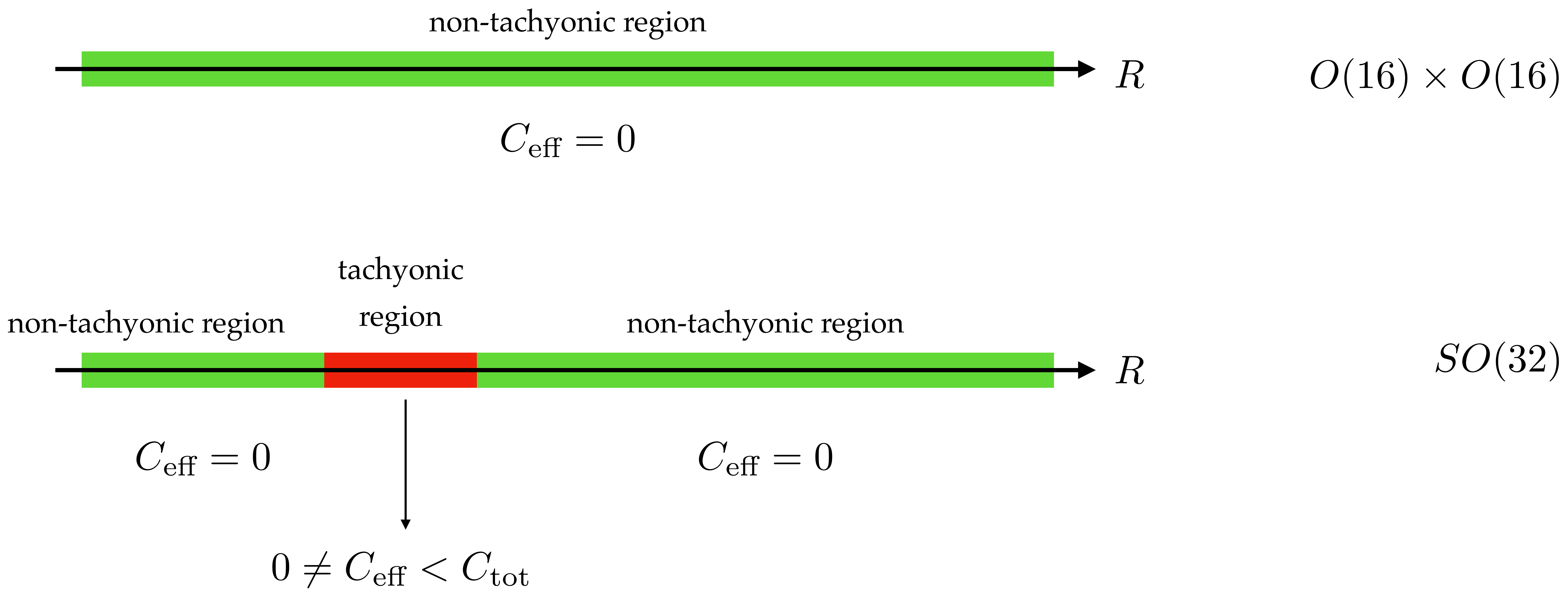
$R = \sqrt{12\alpha'}$



$R = \sqrt{16\alpha'}$

non-tachyonic region

Nine-Dimensional Freely-Acting Realisation



NINE-DIMENSIONAL FREELY-ACTING REALISATION

The analysis is much more complicated because of the presence of many tachyonic characters in the RCFT

For instance, at the point $R = \sqrt{16\alpha'}$ there are 32 characters

Left sector	Right sector
$H_{12} = -\frac{7}{64}$ $\quad H_{14} = -\frac{23}{64}$	$H_1 = -1$ $\quad H_2 = -\frac{63}{64}$ $\quad H_3 = -\frac{15}{16}$ $\quad H_4 = -\frac{55}{64}$ $\quad H_5 = -\frac{3}{4}$
$H_{16} = -\frac{31}{64}$ $\quad H_{18} = -\frac{31}{64}$	$H_6 = -\frac{39}{64}$ $\quad H_7 = -\frac{7}{16}$ $\quad H_8 = -\frac{15}{64}$ $\quad H_{26} = -\frac{15}{64}$ $\quad H_{27} = -\frac{7}{16}$
$H_{20} = -\frac{23}{64}$ $\quad H_{22} = -\frac{7}{64}$	$H_{28} = -\frac{39}{64}$ $\quad H_{29} = -\frac{3}{4}$ $\quad H_{30} = -\frac{55}{64}$ $\quad H_{31} = -\frac{15}{16}$ $\quad H_{32} = -\frac{63}{64}$

Conclusions and Outlook

Oscillations are a universal feature of generic string vacua

Absence of tachyons (classical stability) seems to imply $C_{\text{eff}} = 0$

What is the connection with the results of Kutasov and Seiberg and C.A., Cardella, Elitzur and Rabinovici?

[Kutasov, Seiberg 1991]

[C.A., Cardella, Elitzur, Rabinovici 1991]

Conclusions and Outlook

What is the connection with the results of Kutasov and Seiberg and C.A., Cardella, Elitzur and Rabinovici?

$$\sum_{\{m^2\}} d(m^2) e^{-4\pi m^2/\Lambda^2} = \frac{3}{\pi}\Lambda^{2-D}\Omega + \sum_{\zeta^*(\rho)=0} C_\rho\, \Lambda^{\rho-D}$$

degrees of freedom of the physical spectrum at mass m^2

(finite) vacuum energy

oscillations

$$C_m\, \Lambda^{\beta_m - D}\, \cos(\gamma_m \log \Lambda + \phi_m)$$

$$\rho_m = \beta_m + i\gamma_m$$

[C.A., Cardella, Elitzur, Rabinovici 1991]

THANK YOU