

Topological Defects, Inflation & Gravity Waves

Qaisar Shafi

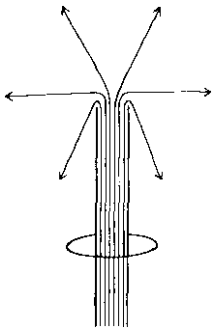
Bartol Research Institute
Department of Physics and Astronomy
University of Delaware



M. Bastero-Gil, G. Dvali, S. King, G. Lazarides, G. Leontaris, R. Maji, N. Okada,
C. Pallis, D. Raut, M. Rehman, N. Senoguz, A. Thapa, F. Vardag, J. Wickman

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Dirac Monopole (1931)



Annu. Rev. Nucl. Part. Sci. 1984.34:461-530



$$eg = \frac{n}{2}$$

Dirac Monopole (1931)

- $A_\theta^U = g(1 - \cos\theta)$, upper hemisphere ($0 \leq \theta \leq \pi/2$)
- $A_\theta^L = -g(1 + \cos\theta)$, lower hemisphere ($\pi/2 \leq \theta \leq \pi$)
- A^U and A^L are connected by gauge transformation at $\theta = \pi/2$.

$$A_\theta^U(\theta = \pi/2) - A_\theta^L(\theta = \pi/2) = 2g = \frac{1}{ie}(\partial_\theta \Omega)\Omega^{-1},$$

where $\Omega(\theta) = \exp[i2eg\theta]$

- For $\Omega(\theta)$ to be single-valued we demand, $eg = \frac{n}{2}$

t'Hooft-Polyakov Monopole (Toy Model)

- Scalar triplet ϕ^a in the adjoint representation of $SU(2)$ breaks $SU(2) \rightarrow U(1)_{em}$.
- We can choose the identity map or "hedgehog" configuration such that $\lim_{r \rightarrow \infty} \phi^a(\vec{x}) = v \hat{r}^a$.
- To ensure a finite energy solution, we require $D_\mu \phi^a(x) = 0$ at the boundary.
- Ansatz for the Higgs and gauge fields,

$$\begin{aligned}\phi^a(\vec{x}) &= v f(r) \hat{r}^a, \\ A_i^a(\vec{x}) &= a(r) \frac{\varepsilon_{aij} \hat{r}^j}{er}.\end{aligned}$$

- Monopole mass $M \sim \frac{M_w}{\alpha}$, core size $\sim M_w^{-1}$.

Magnetic Monopoles in Unified Theories

- Grand Unified Theories based on gauge groups $SU(5)$, $SO(10)$ and E_6 predict the existence of a topologically stable superheavy magnetic monopole that carries one quantum ($2\pi/e$) of Dirac magnetic charge.

This charge is compatible with the Dirac quantization condition because the monopole also carries color magnetic charge which is screened beyond Λ_{QCD}^{-1} .

- If the symmetry breaking to the Standard Model (SM) proceeds via some intermediate step (s), lighter monopoles may appear that carry two or more quanta of the Dirac charge.
- If the SM is embedded in gauge symmetries such as $SU(4)_c \times SU(2)_L \times SU(2)_R$ or $SU(3)_c \times SU(3)_L \times SU(3)_R$, and if we don't require gauge coupling unification, significantly lighter monopoles are predicted, which opens up the possibility of producing them in high energy colliders.

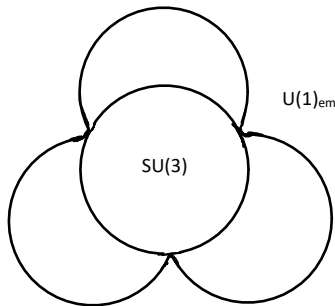
Magnetic Monopoles in Unified Theories

Any unified theory with electric charge quantization predicts the existence of topologically stable ('tHooft-Polyakov) magnetic monopoles. Their mass is about an order of magnitude larger than the associated symmetry breaking scale.

Example :

$SU(5) \rightarrow SM (3-2-1)$

Lightest monopole carries one unit of Dirac magnetic charge even though there exist fractionally charged quarks;



$$\text{monopole mass} \sim \frac{M_G}{\alpha_G}$$

$SU(5)$ Monopole

$$SU(5) \xrightarrow{\text{24-plet Higgs}} SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{\text{5-plet Higgs}} SU(3)_c \times U(1)$$

$$Q_{em} = \begin{pmatrix} 1/3 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$SU(5)$ Monopole

- A 2π rotation with Q_{em} yields:

$$\text{diag} \left(\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{2\pi}{3}, 1, 1 \right)$$

- Next, we perform a $\frac{2\pi}{3}$ rotation with

$$Q_{color} = \text{diag} \left(-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, 0, 0 \right)$$

→ return to identity element.

- The monopole carries one unit of Dirac magnetic charge and color magnetic charge.

$SU(5)$ Monopole

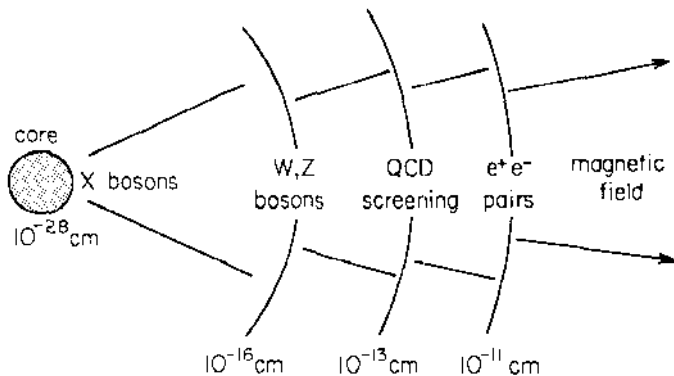


Figure 1 Structure of a grand unified monopole.

SO(10)

Usually broken via one or more intermediate steps to the SM

- $G = SO(10)/Spin(10)$
- $H = SU(3)_c \times U(1)_{e.m.}$
- $\Pi_2(G/H) \cong \Pi_1(H) \Rightarrow$ Monopoles
- $\Pi_1(G/H) \cong \Pi_0(H) = \mathbb{Z}_2 \Rightarrow$ Cosmic Strings (provided $G \rightarrow H$ breaking uses only tensor representations)
- $\mathbb{Z}_2 \subset \mathbb{Z}_4$ (center of $SO(10)$)
[T. Kibble, G. Lazarides, Q.S., PLB, 1982]
- Intermediate scale monopoles and cosmic strings may survive inflation.
- Recent work suggests that this Z_2 symmetry can yield plausible cold dark matter candidates.

[Mario Kadastik, Kristjan Kannike, and Martti Raidal Phys. Rev. D 81 (2010), 015002; Yann Mambrini, Natsumi Nagata, Keith A. Olive, Jeremi Quevillon, and Jiaming Zheng Phys.Rev. D91 (2015) no.9, 095010 ; Sofiane M. Boucenna, Martin B. Krauss, Enrico Nardi Phys.Lett. B755 (2016) 168-17]

○ $SU(4)_c \times SU(2)_L \times SU(2)_R$ (Pati-Salam)

Electric charge is quantized with the smallest permissible charge being $\pm(e/6)$; Lightest monopole carries two units of Dirac magnetic charge;

○ $SO(10) \rightarrow 4-2-2 \rightarrow 3-2-1$

Two sets of monopoles: First breaking produces monopoles with a single unit of Dirac charge.

Second breaking yields monopoles with two Dirac units.

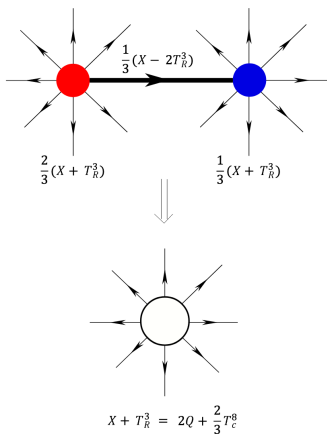
○ E_6 breaking to the SM can yield intermediate mass monopoles carrying three units of Dirac charge.

○ $E_6 \rightarrow SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \rightarrow$
 $SU(3)_c \otimes SU(2)_L \otimes U(1)_{em}$

The discovery of primordial magnetic monopoles would have far-reaching implications for high energy physics & cosmology.

'Schwinger' Monopole

$$\begin{aligned}
 SU(4)_c \times SU(2)_L \times SU(2)_R &\rightarrow SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R \\
 &\rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \quad (1)
 \end{aligned}$$



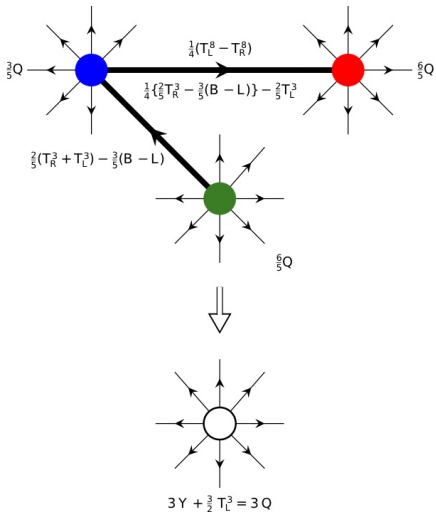
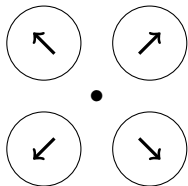


Figure 1: Emergence of the topologically stable triply charged monopole from the symmetry breaking $G \rightarrow SU(3)_c \times SU(2)_L \times U(1)_{Y_L} \times U(1)_{Y_R} \times U(1)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{em}$. An $SU(2)_R$ (green) monopole is connected by a flux tube to an $SU(3)_L$ (blue) monopole which, in turn, is connected to an $SU(3)_R$ (red) monopole by a superconducting flux tube. The constituent monopoles are pulled together to form the triply charged monopole. The fluxes along the tubes and around the monopoles are indicated.

Primordial Monopoles

They are produced via the Kibble Mechanism as $G \rightarrow H$:



Center of monopole has G
symmetry $\langle \phi \rangle = 0$

Initial no. density $\propto T_c^{-3}$. With big bang cosmology such numbers are unacceptable.

$$r_{in} = \frac{N_m}{N_\gamma} \sim 10^{-2}.$$

\Rightarrow Primordial Monopole Problem (Zeldovich & Khlopov, Preskill)

(Need Inflation)

Monopole Searches in Colliders

- Gauge symmetries such as $SU(4)_c \times SU(2)_L \times SU(2)_R$ and $SU(3)_c \times SU(3)_L \times SU(3)_R$ are not truly unified without additional assumptions.

However, electric charge is quantized in these models, and it's plausible that their symmetry breaking scale lies well below the GUT scale.

- If the scale is \sim few TeV or so, the corresponding monopoles may be accessible in HE colliders.
- Monopoles carry two and three quanta of Dirac magnetic charges (respectively).
- In addition, we may find exotic states that are color singlets but carry fractional electric charges, $\pm e/2$ ($\pm e/3$).

Monopole Searches in Colliders

High Energy Physics - Experiment

[Submitted on 22 Jun 2021]

First experimental search for production of magnetic monopoles via the Schwinger mechanism

B. Acharya, J. Alexandre, P. Benes, B. Bergmann, S. Bertolucci, A. Bevan, H. Branza, P. Burian, M. Campbell, Y. M. Cho, M. de Montigny, A. De Roeck, J. R. Ellis, M. El Sawy, M. Fairbairn, D. Felea, M. Frank, O. Gould, J. Hays, A. M. Hirt, D. L. J. Ho, P. Q. Hung, J. Janecek, M. Kalliokoski, A. Korzenev, D. H. Lacarrère, C. Leroy, G. Levi, A. Lioni, A. Maulik, A. Margiotta, N. Mauri, N. E. Mavromatos, P. Mermod, L. Millward, V. A. Mitsou, I. Ostrovskiy, P.-P. Ouimet, J. Papavassiliou, B. Parker, L. Patrizii, G. E. Păvălaș, J. L. Pinfold, L. A. Popa, V. Popa, M. Pozzato, S. Pospisil, A. Rajantie, R. Ruiz de Austri, Z. Sahnoun, M. Sakellariadou, A. Santra, S. Sarkar, G. Semenoff, A. Shaa, G. Sirri, K. Sliwa, R. Soluk, M. Spurio, M. Staelens, M. Suk, M. Tenti, V. Togo, J. A. Tuszyński, A. Upreti, V. Vento, O. Vives

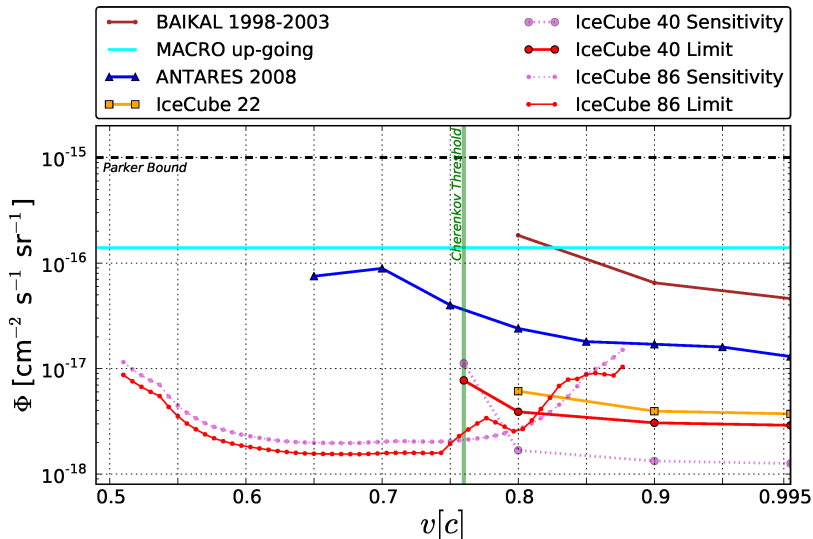
Schwinger showed that electrically-charged particles can be produced in a strong electric field by quantum tunnelling through the Coulomb barrier. By electromagnetic duality, if magnetic monopoles (MMs) exist, they would be produced by the same mechanism in a sufficiently strong magnetic field. Unique advantages of the Schwinger mechanism are that its rate can be calculated using semiclassical techniques without relying on perturbation theory, and the finite MM size and strong MM-photon coupling are expected to enhance their production. Pb-Pb heavy-ion collisions at the LHC produce the strongest known magnetic fields in the current Universe, and this article presents the first search for MM production by the Schwinger mechanism. It was conducted by the MoEDAL experiment during the 5.02 TeV/nucleon heavy-ion run at the LHC in November 2018, during which the MoEDAL trapping detectors (MMTs) were exposed to 0.235 nb^{-1} of Pb-Pb collisions. The MMTs were scanned for the presence of magnetic charge using a SQUID magnetometer. MMs with Dirac charges $1g_D \leq g \leq 3g_D$ and masses up to $75 \text{ GeV}/c^2$ were excluded by the analysis. This provides the first lower mass limit for finite-size MMs from a collider search and significantly extends previous mass bounds.

Subjects: **High Energy Physics - Experiment (hep-ex)**; High Energy Physics - Phenomenology (hep-ph)

Cite as: [arXiv:2106.11933](#) [**hep-ex**]

(or [arXiv:2106.11933v1](#) [**hep-ex**] for this version)

Relativistic Monopoles at IceCube



Source: IceCube Collaboration, Eur. Phys. J. C (2016) 76:133

Strings and Domain Walls

- Other topological structures such as strings, domain walls and composites can also appear in these theories.
- Cosmic strings may emit gravitational waves, which has attracted a great amount of attention in recent years.
- Strings associated with $U(1)_{PQ}$ symmetry breaking emit axion dark matter.
- Strings may be superconducting (Witten), which turns out to be the case for axion strings (Lazarides, QS; Callan, Harvey).
- Walls bounded by strings ($SO(10)$);
Recently discovered in superfluid He3.

Walls Bounded by Strings

- Consider the breaking chain

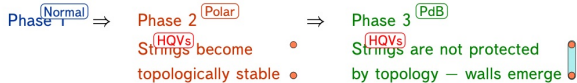
$$\begin{array}{ccc} SO(10) & \xrightarrow{54} & SU(4)_c \times SU(2)_L \times SU(2)_R \\ & & \downarrow \widetilde{126} \\ & & SU(3)_c \times SU(2)_L \times U(1)_Y. \end{array}$$

- The first step leaves unbroken the discrete symmetry ‘C’ (also known as ‘D’) that interchanges left and right, and conjugates the representations.
- The $\widetilde{126}$ vev breaks ‘C’ which produces domain walls
- Thus we end up with walls bounded by strings.
Similar structures also arise in axion models.

HQVs in the PdB phase

KIBBLE-LAZARIDES-SHAFI (KLS) WALL or WALL BOUNDED BY STRINGS

Composite defect suggested in the context of phase transitions in the early Universe:

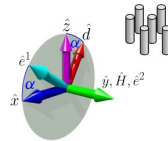
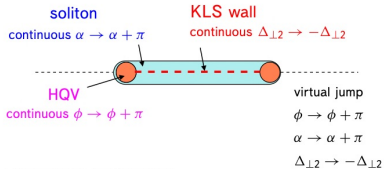


Kibble *et al*,
PRD **26**, 435 (1982)

Polar-distorted B phase:

$$A_{\mu i} = e^{i\phi} (\Delta_{\parallel} \hat{d}_{\mu} \hat{z}_i + \Delta_{\perp 1} \hat{e}_{\mu}^1 \hat{x}_i + \Delta_{\perp 2} \hat{e}_{\mu}^2 \hat{y}_i)$$

$$|\Delta_{\perp 1}| = |\Delta_{\perp 2}| = q |\Delta_{\parallel}|, \quad q < 1$$



$$\hat{\mathbf{d}} = \hat{\mathbf{x}} \cos \alpha - \hat{\mathbf{z}} \sin \alpha$$

$$\hat{\mathbf{e}}^1 = \hat{\mathbf{z}} \cos \alpha + \hat{\mathbf{x}} \sin \alpha$$

$$\hat{\mathbf{e}}^2 = \hat{\mathbf{y}} \parallel \mathbf{H}$$

Successful Primordial Inflation should:

- Explain flatness, isotropy;
- Provide origin of $\frac{\delta T}{T}$;
- Offer testable predictions for n_s, r (gravity waves), $dn_s/d \ln k$;
- Recover Hot Big Bang Cosmology;
- Explain the observed baryon asymmetry;
- Offer plausible CDM candidate;

Slow-roll Inflation

- Inflation is driven by some potential $V(\phi)$:
- Slow-roll parameters:

$$\epsilon = \frac{m_p^2}{2} \left(\frac{V'}{V} \right)^2, \quad \eta = m_p^2 \left(\frac{V''}{V} \right).$$

- The spectral index n_s and the tensor to scalar ratio r are given by

$$n_s - 1 \equiv \frac{d \ln \Delta_{\mathcal{R}}^2}{d \ln k}, \quad r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2},$$

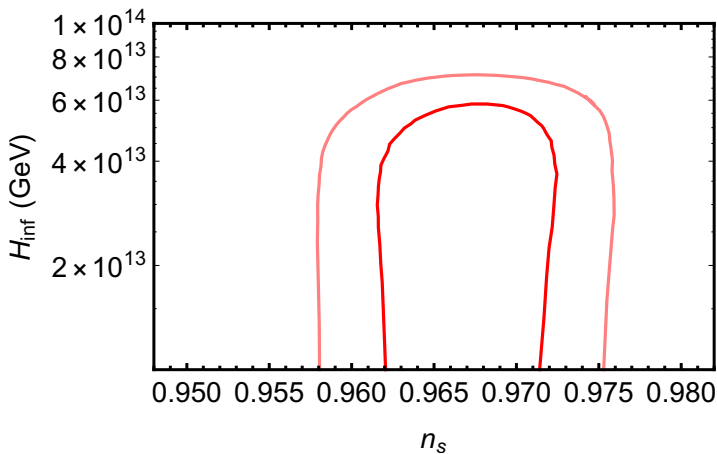
where Δ_h^2 and $\Delta_{\mathcal{R}}^2$ are the spectra of primordial gravity waves and curvature perturbation respectively.

- Assuming slow-roll approximation (i.e. $(\epsilon, |\eta|) \ll 1$), the spectral index n_s and the tensor to scalar ratio r are given by

$$n_s \simeq 1 - 6\epsilon + 2\eta, \quad r \simeq 16\epsilon.$$

Constraint on Inflation

Planck (2018), BK (2015)



SUSY Higgs (Hybrid) Inflation

[Dvali, Shafi, Schaefer; Copeland, Liddle, Lyth, Stewart, Wands '94]

[Lazarides, Schaefer, Shafi '97][Senoguz, Shafi '04; Linde, Riotto '97]

[Buchmüller, Domcke and Schmitz]

- Attractive scenario in which inflation can be associated with symmetry breaking $G \longrightarrow H$
- Simplest inflation model is based on

$$W = \kappa S (\Phi \bar{\Phi} - M^2)$$

S = gauge singlet superfield, $(\Phi, \bar{\Phi})$ belong to suitable representation of G

- Need $\Phi, \bar{\Phi}$ pair in order to preserve SUSY while breaking $G \longrightarrow H$ at scale $M \gg \text{TeV}$, SUSY breaking scale.
- R-symmetry

$$\Phi \bar{\Phi} \rightarrow \Phi \bar{\Phi}, \quad S \rightarrow e^{i\alpha} S, \quad W \rightarrow e^{i\alpha} W$$

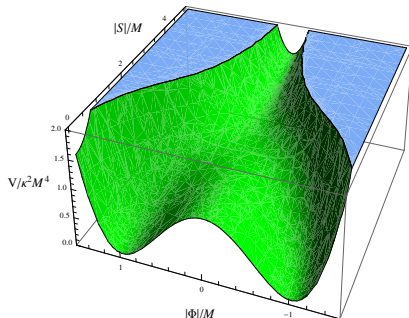
SUSY Higgs (Hybrid) Inflation

- Tree Level Potential

$$V_F = \kappa^2 (M^2 - |\Phi|^2)^2 + 2\kappa^2 |S|^2 |\Phi|^2$$

- SUSY vacua

$$|\langle \bar{\Phi} \rangle| = |\langle \Phi \rangle| = M, \quad \langle S \rangle = 0$$



SUSY Higgs (Hybrid) Inflation

Take into account radiative corrections (because during inflation $V \neq 0$ and SUSY is broken by $F_S = -\kappa M^2$)

- Mass splitting in $\Phi - \bar{\Phi}$

$$m_{\pm}^2 = \kappa^2 S^2 \pm \kappa^2 M^2, \quad m_F^2 = \kappa^2 S^2$$

- One-loop radiative corrections

$$\Delta V_{1\text{loop}} = \frac{1}{64\pi^2} \text{Str}[\mathcal{M}^4(S)(\ln \frac{\mathcal{M}^2(S)}{Q^2} - \frac{3}{2})]$$

- In the inflationary valley ($\Phi = 0$)

$$V \simeq \kappa^2 M^4 \left(1 + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x) \right)$$

where $x = |S|/M$ and

$$F(x) = \frac{1}{4} \left((x^4 + 1) \ln \frac{(x^4 - 1)}{x^4} + 2x^2 \ln \frac{x^2 + 1}{x^2 - 1} + 2 \ln \frac{\kappa^2 M^2 x^2}{Q^2} - 3 \right)$$

SUSY Higgs (Hybrid) Inflation

Tree level + radiative corrections + minimal Kähler potential yield:

$$n_s = 1 - \frac{1}{N} \approx 0.98.$$

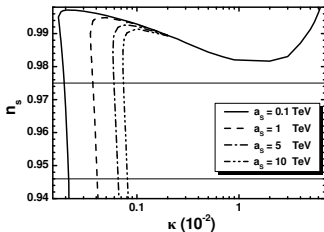
$\delta T/T$ proportional to M^2/M_p^2 , where M denotes the gauge symmetry breaking scale. Thus we expect $M \sim M_{GUT}$ for this simple model. In practice, $M \approx (1 - 5) \times 10^{15}$ GeV

Since observations suggest that n_s lie close to 0.97, there are at least two ways to realize this slightly lower value:

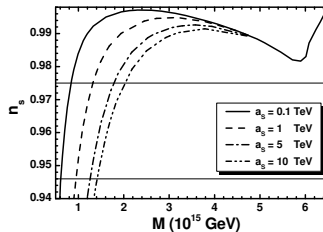
- include soft SUSY breaking terms, especially a linear term in S ;
- employ non-minimal Kähler potential.

SUSY Higgs (Hybrid) Inflation

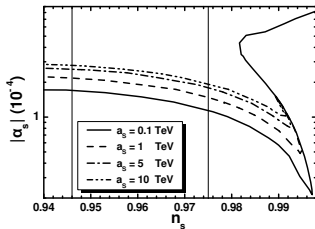
[Pallis, Shafi, 2013; Rehman, Shafi, Wickman, 2010]



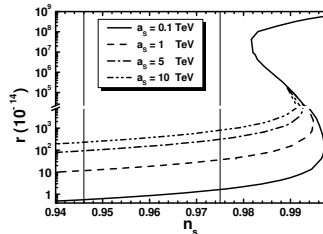
(a)



(b)



(a)

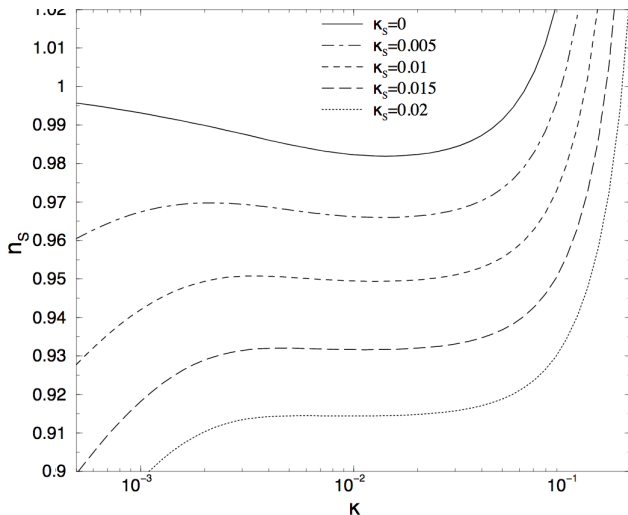


(b)

SUSY Higgs (Hybrid) Inflation

- $K \supset \kappa_s (S^\dagger S)^2$

[M. Bastero-Gil, S. F. King and Q. Shafi, 2006]



Susy Hybrid Inflation

- Some examples of gauge groups G such that

$$G \xrightarrow{\langle \Phi \rangle \neq 0} H \supseteq SM \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$$

where

$$G = SM \times U(1)_{B-L}, \quad (\text{cosmic strings})$$

$$G = SU(5), \quad (\Phi = \bar{\Phi} = 24), \quad (\text{monopoles})$$

$$G = SU(5) \times U(1), \quad (\Phi = 10), \quad (\text{Flipped } SU(5))$$

$$G = SU(4)_c \times SU(2)_L \times SU(2)_R, \quad (\Phi = (\bar{4}, 1, 2)), \quad (\text{monopoles})$$

$$G = SO(10), \quad (\Phi = 16) \quad (\text{monopoles})$$

(Non-minimal) SUGRA Hybrid Inflation

[M. Bastero-Gil, S. F. King, Q. Shafi 2006; M. Rehman, V. N. Senoguz, Q. Shafi 2006]

- The superpotential is given by

$$W = \kappa S [\Phi \bar{\Phi} - M^2]$$

- The Kähler potential can be expanded as

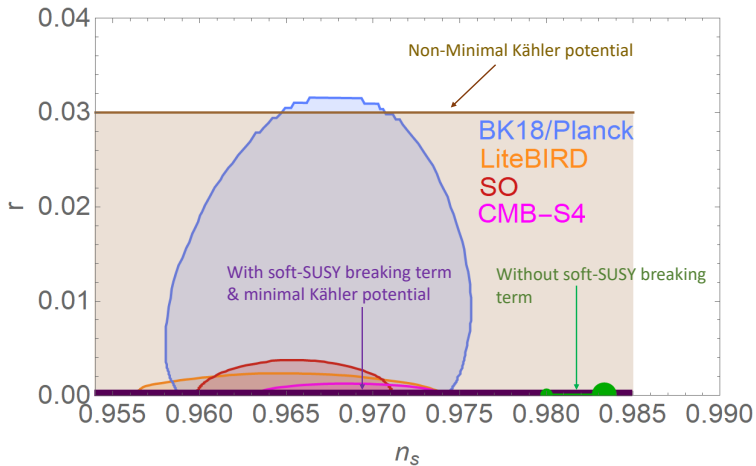
$$\begin{aligned} K &= |S|^2 + |\Phi|^2 + |\bar{\Phi}|^2 \\ &+ \kappa_S \frac{|S|^4}{4 m_P^2} + \kappa_\Phi \frac{|\Phi|^4}{4 m_P^2} + \kappa_{\bar{\Phi}} \frac{|\bar{\Phi}|^4}{4 m_P^2} \\ &+ \kappa_{S\Phi} \frac{|S|^2 |\Phi|^2}{m_P^2} + \kappa_{S\bar{\Phi}} \frac{|S|^2 |\bar{\Phi}|^2}{m_P^2} + \kappa_{\Phi\bar{\Phi}} \frac{|\Phi|^2 |\bar{\Phi}|^2}{m_P^2} \\ &+ \kappa_{SS} \frac{|S|^6}{6 m_P^4} + \dots \end{aligned}$$

- Now including all other corrections potential takes the following form

$$V \simeq \kappa^2 M^4 \left(1 - \kappa_S \left(\frac{S}{m_P} \right)^2 + \frac{\gamma_S}{2} \left(\frac{S}{m_P} \right)^4 \right) + V_{1-loop} + V_{soft}$$

where, $\gamma_S = 1 - \frac{7\kappa_S}{2} - 2\kappa_S^2 - 3\kappa_{SS}$.

Low Scale SB SUSY



μ -HYBRID INFLATION

N. Okada and Q. Shafi, 2017

$$W = \kappa S(\Phi\bar{\Phi} - M^2) + \lambda S H_u H_d$$

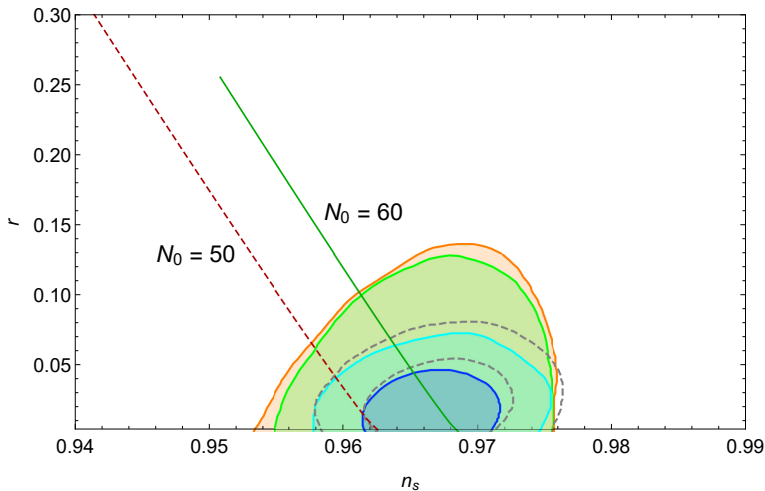
- The S field gets a destabilizing tadpole term $\simeq 2\kappa m_{3/2} M^2 S + h.c.$, and taking account of the term $\simeq 2\kappa^2 M^2 |S|^2$, the resulting vev of S is $\simeq m_{3/2}/\kappa$.
- The vev of S will generate a μ term with

$$\mu = \lambda \langle S \rangle = m_{3/2} (\lambda/\kappa) \simeq (10^2 - 10^3) GeV.$$

$$\Gamma_S(S \rightarrow \tilde{H}_u \tilde{H}_d) = \frac{\lambda^2}{8\pi} m_S,$$

ϕ^4 Inflation with non-minimal coupling to gravity

[Okada, Rehman, Shafi, 2010]



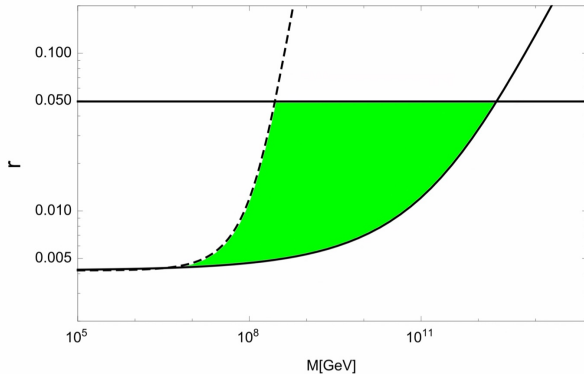
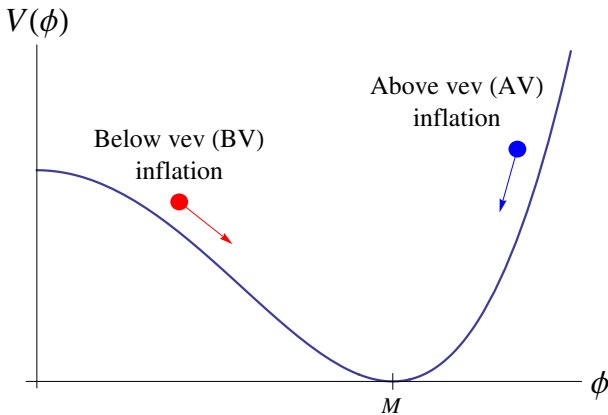


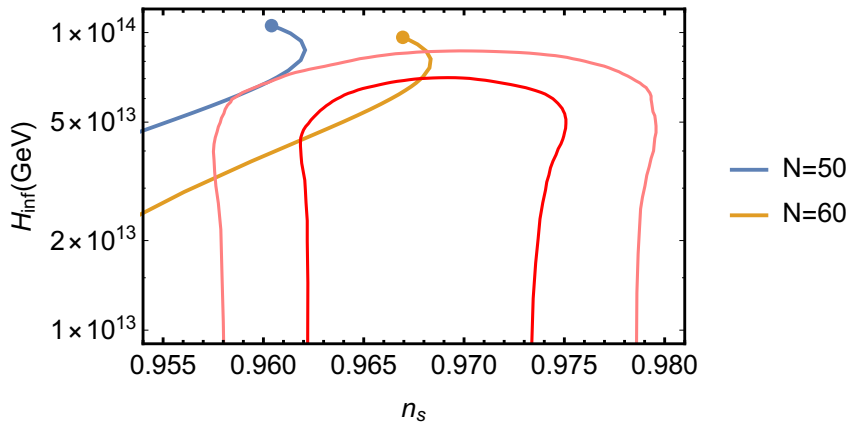
FIG. : Allowed parameter regions in (M, r) -plane. The solid curve corresponds to the solid diagonal line (top), while the dashed curve corresponds to the (left) dashed diagonal line in Figure 2. The horizontal solid line depicts the upper bound from the Planck measurements, $r \leq 0.0496$ for $N_0 = 50$. The shaded region satisfies all the constraints.

Inflation with a CW Higgs Potential



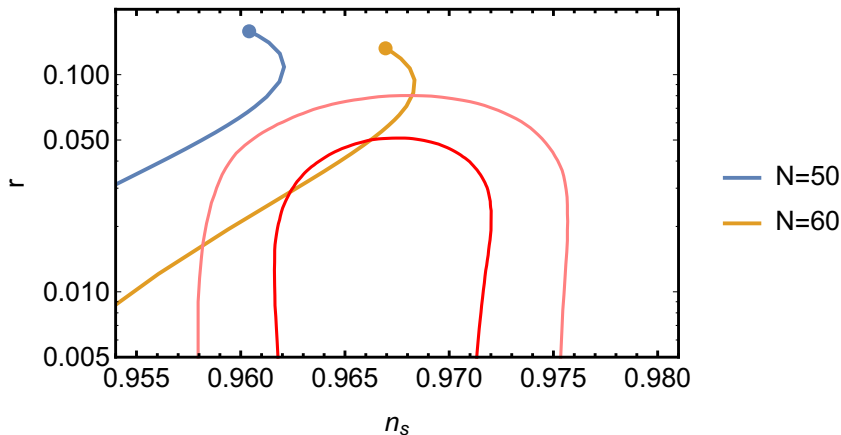
Note: This is for minimal coupling to gravity

Inflation with a CW Higgs Potential



Note: This is for minimal coupling to gravity

Inflation with a CW Higgs Potential



Note: This is for minimal coupling to gravity

Evolution of Intermediate-mass Monopoles

Number density at production, $\xi = \min(H^{-1}, m_{\text{eff}}^{-1})$

Dilution during Inflation

Dilution from Inflaton oscillation

Monopole yield after reheating :

$$Y_M \simeq \frac{\frac{\xi^{-3}}{10} \exp(-3N_M) \left(\frac{\tau}{t_r}\right)^2}{\frac{2\pi^2}{45} g_* T_r^3}$$

Entropy density after reheating

- MACRO bound: $Y_M \lesssim 10^{-27}$.

Ambrosio et al. [MACRO Collaboration], EPJC 25, 511 (2002)

- Adopted threshold for observability: $Y_M \gtrsim 10^{-35}$.

Intermediate Mass Monopoles and MACRO

$\frac{V_0^{1/4}}{10^{16}\text{GeV}}$	ϕ_+/m_{pl}	ϕ_-/m_{pl}	H_+	H_-	N_+	N_-	$\log_{10}\left(\frac{M_{I+}}{\text{GeV}}\right)$	$\log_{10}\left(\frac{M_{I-}}{\text{GeV}}\right)$
			(10^{13} GeV)					
1.51	14.41	13.07	3.40	3.91	9.8	16.2	13.30	13.40
1.59	16.04	14.67	3.54	4.10	9.9	16.2	13.30	13.41
1.66	17.91	16.51	3.67	4.28	9.9	16.2	13.31	13.41
1.74	20.05	18.62	3.78	4.45	9.9	16.2	13.31	13.41
1.82	22.51	21.04	3.88	4.59	9.9	16.2	13.31	13.41

Table: Values of the various parameters (indicated by a subscript +) corresponding to the MACRO bound ($Y_M < 10^{-27}$) on the flux of monopoles formed at the scale M_I and their values (indicated by a subscript -) corresponding to the adopted observability threshold ($Y_M > 10^{-35}$) for the monopole flux.

- $$SO(10) \times U(1)_{PQ} \xrightarrow{M_U} SU(2)_L \otimes SU(2)_R \otimes SU(4)_C \times U(1)_{PQ} \xrightarrow{M_I} \langle (1,3,1,-2) \in (1,3,10) \in \overline{126}(-2) \rangle \xrightarrow{M_{II}} SU(2)_L \otimes SU(2)_R \otimes SU(3)_C \otimes U(1)_{B-L} \times U(1)_{PQ} \xrightarrow{f_a} \langle (1,3,1) + (1,1,15) \in 45(4) \rangle \xrightarrow{m_W} SU(3)_C \otimes U(1)_Y \otimes \mathbb{Z}_2 \xrightarrow{m_W} SU(3)_C \otimes U(1)_Q \otimes \mathbb{Z}_2.$$
- $$\psi_{16}^{(i)} (i = 1, 2, 3) \xrightarrow{U(1)_{PQ}} e^{i\theta} \psi_{16}^{(i)}$$
- Residual discrete PQ symmetry is $Z_{12} \Rightarrow$ domain wall problem ($U(1)_{PQ}$ broken after inflation)
- Introduce two $SO(10)$ fermion 10-plets

$$\psi_{10}^{(\alpha)} \rightarrow e^{-2i\theta} \psi_{10}^{(\alpha)} \quad (\alpha = 1, 2)$$
- Residual discrete symmetry is now Z_4 , which coincides with the center Z_4 of $SO(10)$ (Spin(10)) (for the full theory) (Cosmic Strings and Dark Matter)

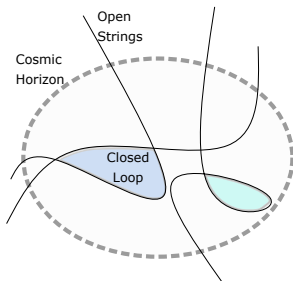
- $\phi_{210} \rightarrow \phi_{210}, \quad \phi_{126} \rightarrow e^{2i\theta} \phi_{126}, \quad \phi_{45} \rightarrow e^{4i\theta} \phi_{45}, \quad \phi_{10} \rightarrow e^{-2i\theta} \phi_{10}$. These transformation properties ensure that the action of the residual PQ symmetry on these fields is identical to that of the center of $SO(10)$.
- Yukawa couplings: $\psi_{16}\psi_{16}\phi_{10}, \psi_{16}\psi_{16}\phi_{126}, \psi_{10}\psi_{10}\phi_{45}$
- Higgs couplings include $\phi_{210}\phi_{126}^\dagger\phi_{126}^\dagger\phi_{45}, \phi_{210}\phi_{126}^\dagger\phi_{10}\phi_{45}, \phi_{210}\phi_{126}\phi_{10}$
- These couplings guarantee that $U(1)_{PQ}$ is the only global symmetry present.

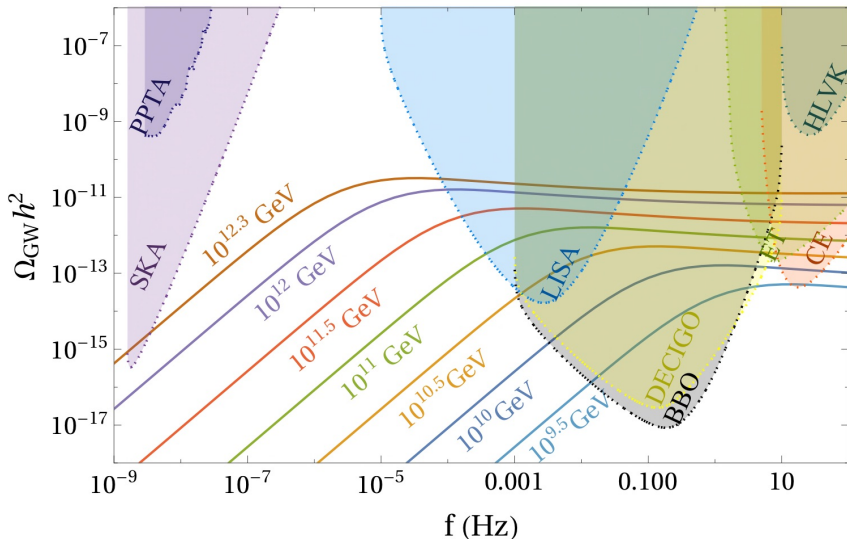
- First breaking produces GUT monopoles that are inflated away.
- Second breaking makes intermediate scale cosmic strings which can appear after inflation \Rightarrow astrophysical test of GUTs.
- Dark Matter:
In addition to axions there could exist WIMP-like DM in this class of models because of the fermion 10-plets.

Cosmic Strings from $SO(10)$

Cosmic Strings arise during symmetry breaking of $G \rightarrow H$ if $\pi_1(G/H)$ is non-trivial. Consider

$SO(10) \xrightarrow{M_{GUT}} SU(4) \times SU(2)_L \times SU(2)_R \xrightarrow{M_I} SM \times Z_2$ Mass
per unit length of string is $\mu \sim M_I^2$, with $M_I \ll M_P$. The strength
of string gravity is determined by the dimensionless parameter
 $G\mu \ll 1$.



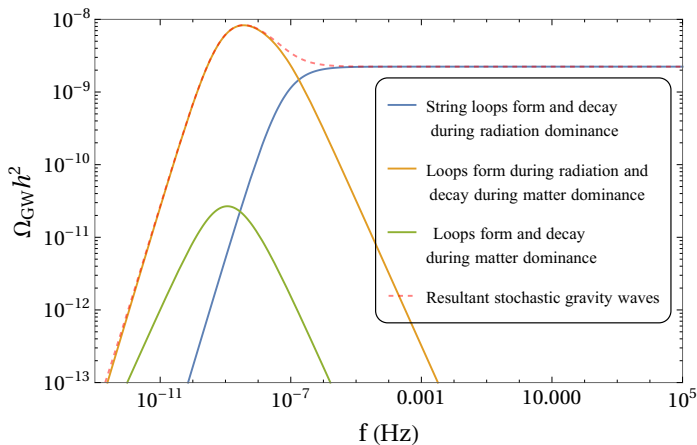


Thank You

Stochastic Gravity Waves from Strings

- Unresolved GWs bursts from string loops at different cosmic era produces the stochastic background.
- Loops that are formed and decay during radiation produce a plateau in the spectrum in the high frequency regime.
- Loops that are produced during radiation dominance but decay during matter dominance generate a sharply peaked spectrum at lower frequencies.
- Loops that are produced and decay during matter domination also generate a sharply peaked spectrum which, however, is overshadowed by the previous case.

Stochastic Gravity Wave Background: Analytic Approximation



Quasi-stable strings & gravitational waves

Consider the symmetry breaking: $G \rightarrow H \times U(1) \rightarrow H$

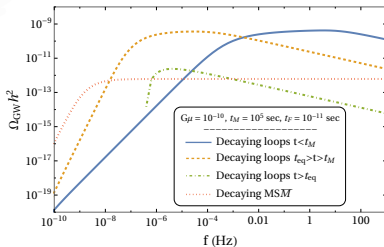
- The first step yields monopoles (& antimonopoles), which then connected to one another by strings from $U(1)$ breaking.
- If there is adequate hierarchy between the two breakings the strings are quantum mechanically stable.

Early Universe:

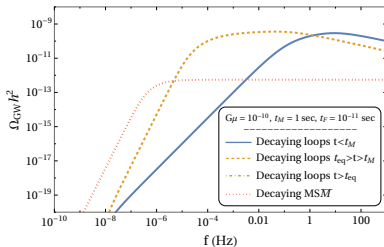
- Suppose monopoles undergo a period of inflation, but are not inflated away.
- Cosmic strings also may experience a period of inflation. As long as the monopoles are absent, we obtain gravitational radiation from the strings in the usual way.
- However, once the monopoles reenter the horizon we obtain strings with ends attached to monopoles antimonopoles. We expect that the long wavelength portion of the gravitational spectrum is modified.
- Models incorporating this scenario can be realized in $SO(10)$ breaking, for instance.

Gravitational Waves from Quasi-stable Strings

The gravitational wave spectra from loops decaying before and after t_M (the horizon reentrance time of the monopoles) during radiation dominance, from loops decaying after the equidensity time t_{eq} , and from the decaying $MS\bar{M}$ structures.



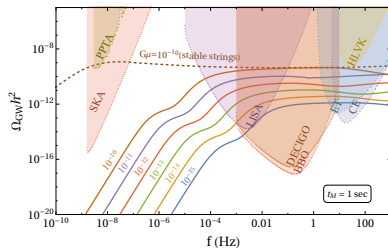
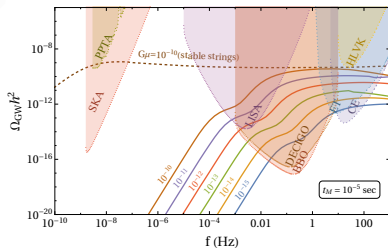
$$m_M = 4.45 \times 10^{15} \text{ GeV}$$



$$m_M = 4.17 \times 10^{15} \text{ GeV}$$

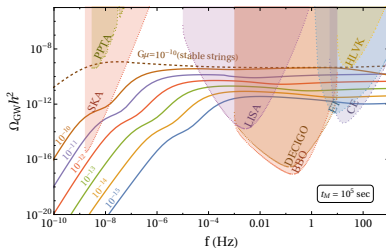
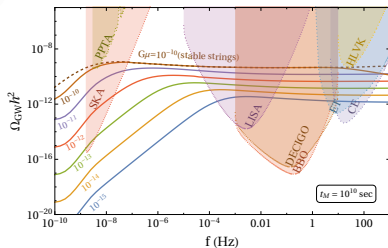
Gravitational Waves from Quasi-stable Strings

The total gravitational wave spectra from quasi-stable cosmic strings with varying $G\mu$ values as indicated and for different horizon reentrance times of the monopole-antimonopole pairs.

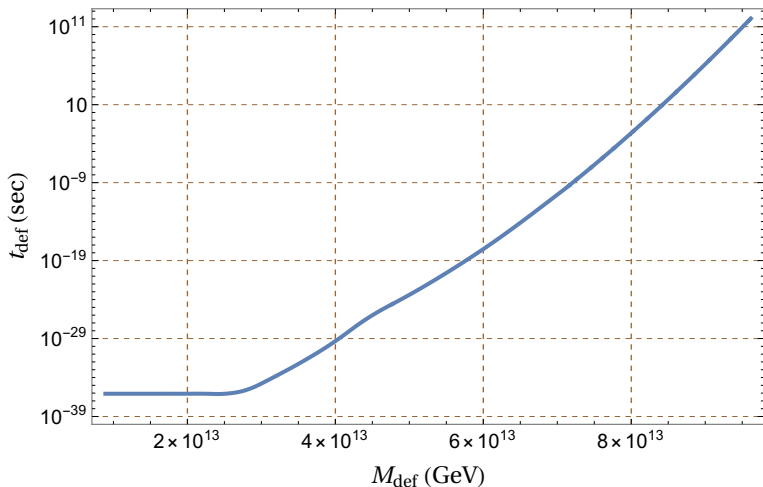


Gravitational Waves from Quasi-stable Strings

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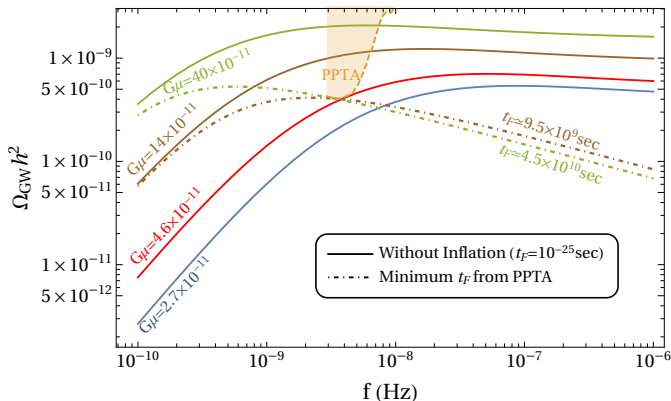


Gravitational Waves from Quasi-stable Strings



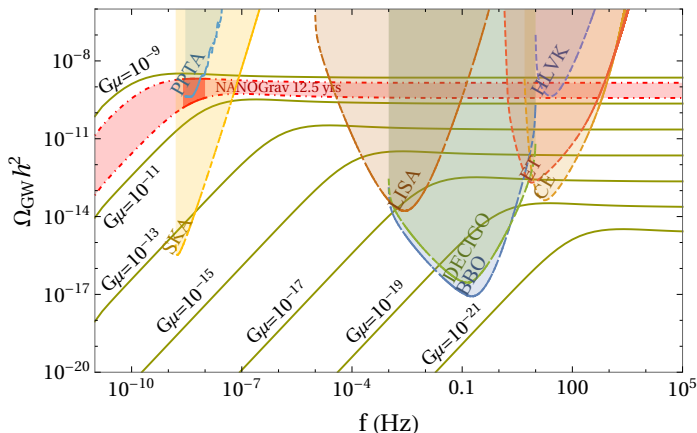
Horizon reentry time t_{def} of the topological defects (monopoles or strings) versus the symmetry breaking scale M_{def} (M_I or M_{II}).

Inflation, GWs and PPTA bound



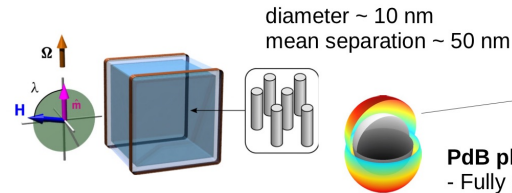
- Partially inflated strings re-enter horizon at a time t_F in post-inflationary universe and can decay via GWs emission.
- Modified GWs spectra from ‘diluted’ strings can satisfy the PPTA bound.

GWs without Inflation and Observational Prospects



- Strongest constraint has come from PPTA: $G\mu \lesssim 10^{-11}$.
- Provisional GWs signal in NANOGrav: $G\mu \sim 10^{-10}$.

Experimental System



We can control:

- Magnetic field
- RF excitation
- Angular velocity
- Temperature
- Pressure

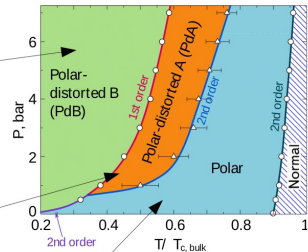
^3He – p-wave superfluid
→ multiple s.f. phases

PdB phase:
- Fully gapped

PdA phase:
- Two node points

Polar phase:
- Nodal line

Phase diagram (on cooling):



Walls Bounded by Strings [J. Makinen, V. Dmitriev, et al. (Nature, 2019)]

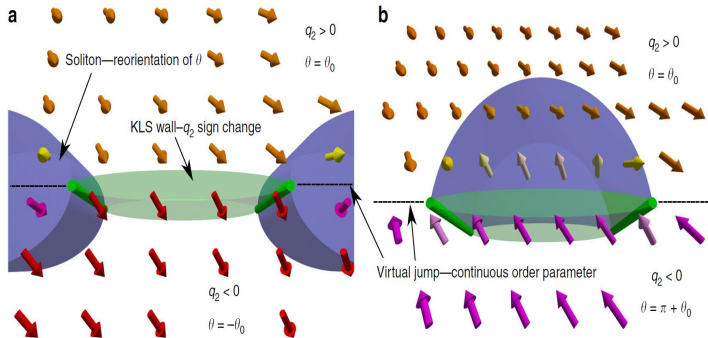


Fig. 4 Kibble-Lazarides-Shafi (KLS) wall configurations in the PdB phase. Each HQV core terminates one soliton—reorientation of the spin part of the order parameter denoted by the angle θ —and one KLS wall. The orientation of the $\hat{\mathbf{d}}$ -vector is shown as arrows where their color indicates the angle θ , based on numerical calculations (Supplementary Figure 2). **a** The KLS wall is bound between a different pair of HQV cores as the soliton. Ignoring the virtual jumps, the angle θ winds by $\pi - 2\theta_0$ across the soliton and by $2\theta_0$ across the KLS wall. The order parameter is continuous across the virtual jumps, where $\phi \rightarrow \phi + \pi$, $\theta \rightarrow \theta + \pi$, and $q_2 \rightarrow -q_2$. **b** The soliton and the KLS wall are bound between the same pair of HQV cores. The total winding of the $\hat{\mathbf{d}}$ -vector is π across the structure. In principle, the KLS wall may lie inside or outside the soliton. Here the KLS wall and the soliton are spatially separated for clarity

ϕ^4 Inflation with non-minimal coupling to gravity

- Consider the following action in the Jordan frame:

$$S_J = \int d^4x \sqrt{-g} \left[\left(\frac{m_P^2 + \xi \phi^2}{2} \right) \mathcal{R} - \frac{1}{2} (\partial\phi)^2 - \frac{\lambda}{4} \phi^4 \right].$$

- In the Einstein frame the potential turns out to be:

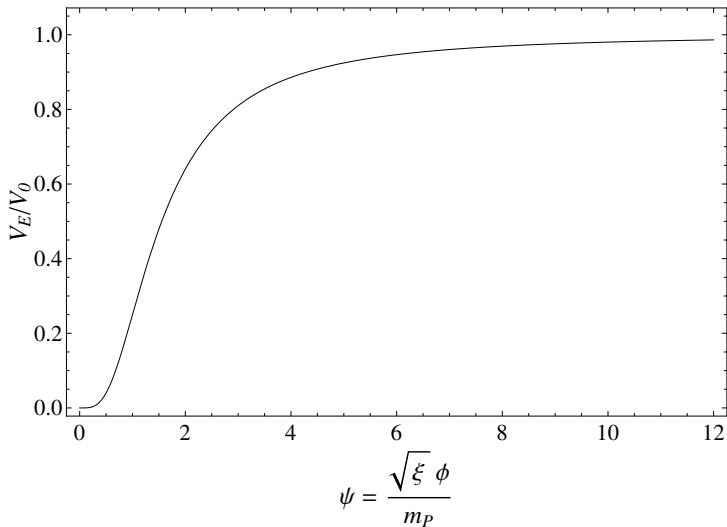
$$V_E(\phi) = \frac{\frac{1}{4} \lambda \phi^4}{\left(1 + \frac{\xi \phi^2}{m_P^2} \right)^2} = V_0 \frac{\psi^4}{(1 + \psi^2)^2},$$

where $V_0 = \left(\frac{\lambda m_P^4}{4 \xi^2} \right)$ and $\psi \equiv \frac{\sqrt{\xi} \phi}{m_P}$.

- The kinetic energy of the scalar field is made canonical with respect to a new field σ as

$$\left(\frac{d\sigma}{d\phi} \right)^{-2} = \frac{\left(1 + \frac{\xi \phi^2}{m_P^2} \right)^2}{1 + (6\xi + 1) \frac{\xi \phi^2}{m_P^2}}.$$

ϕ^4 Inflation with non-minimal coupling to gravity



ϕ^4 Inflation with non-minimal coupling to gravity

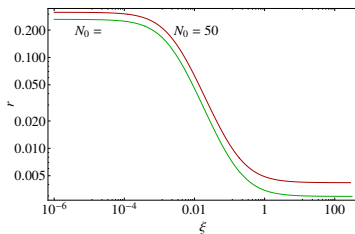
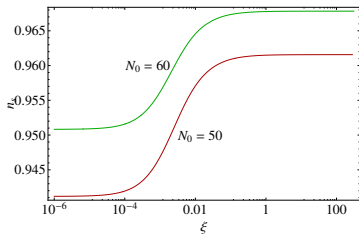
[Okada, Rehman, Shafi, 2010]

- CMB observables in the large ξ limit:

$$n_s \simeq 1 - \frac{2}{N_0} \sim 0.967, \quad r \simeq \frac{12}{N_0^2} \sim 0.003, \quad \text{for } N_0 = 60$$

with

$$A_s \simeq \frac{\lambda}{\xi^2} \frac{N_0^2}{72\pi^2} \Rightarrow \xi \simeq \left(\frac{N_0}{\sqrt{72} \pi A_s} \right) \sqrt{\lambda} \sim 10^4 \text{ for } \lambda \sim 1 \text{ [SM Higgs Inflation?]}$$



ϕ^4 Inflation with non-minimal coupling to gravity

[Okada, Rehman, Shafi, 2010]

