

# ***Dark Matter candidate particles in SUGRA models***

***Vassilis C. Spanos***



**National and Kapodistrian  
University of Athens**

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# Outline

- **Revisit neutralino DM scenarion in CMSSM**
- **Gravitino DM models, calculation of gravitino thermal density**
- **Discussion of SUGRA models that can produce PBH**
- **Summary**

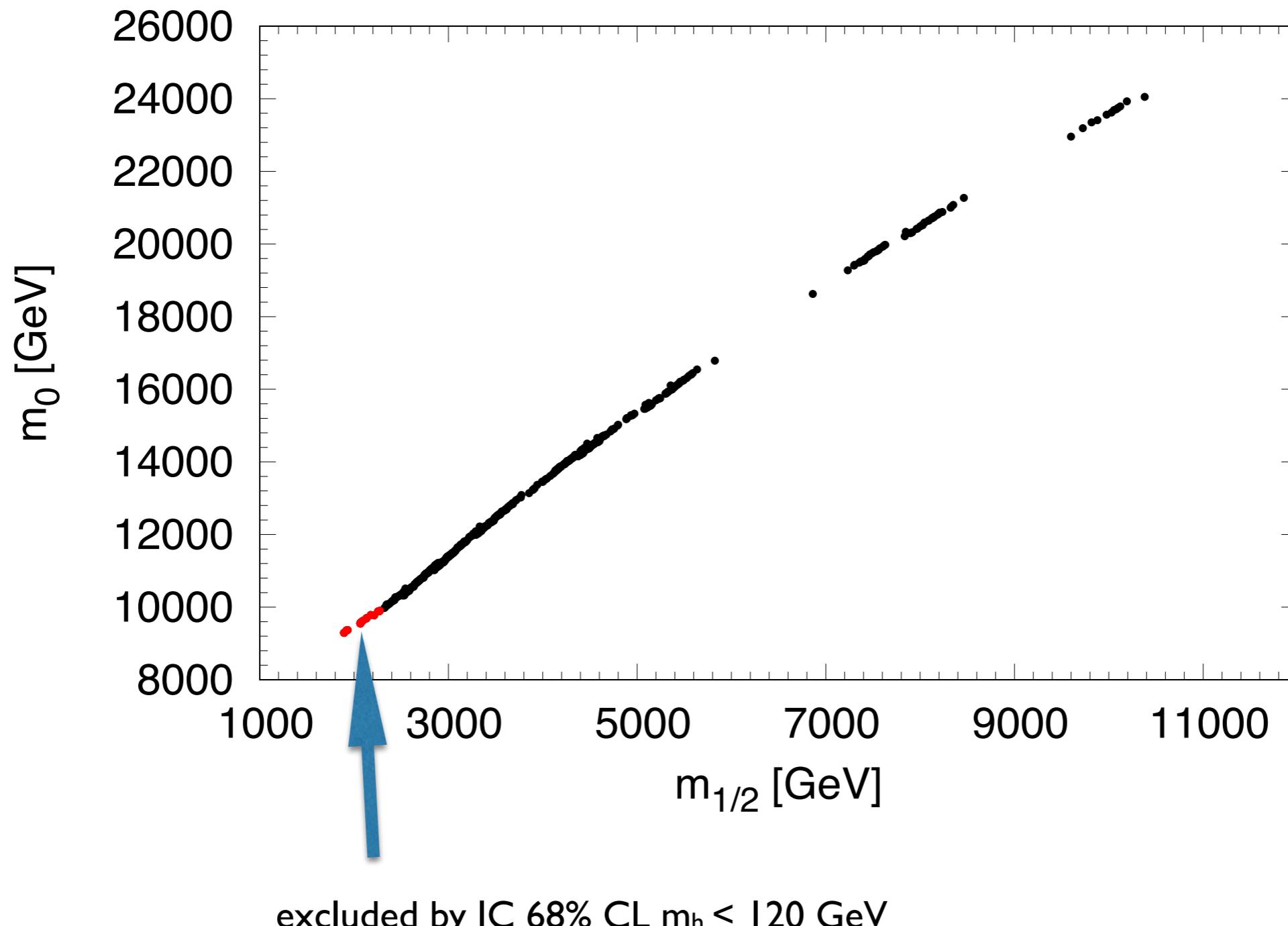
# Neutralino ( $\chi$ ) as DM

- **The most well studied SUSY DM candidate particle, both theoretically and experimentally**
- **Motivated by the so-called WIMP miracle, it has been studied in various SUSY models: CMSSM/mSUGRA, NUHM, pMSSM etc**
- **$\Omega h^2 \sim 0.12$  is achieved in particular regions of the parameter space: coannihilation regions (stop, stau, gauginos), focus point region, A-funnel region**
- **Possible tension in these regions between direct and indirect DM constraints**

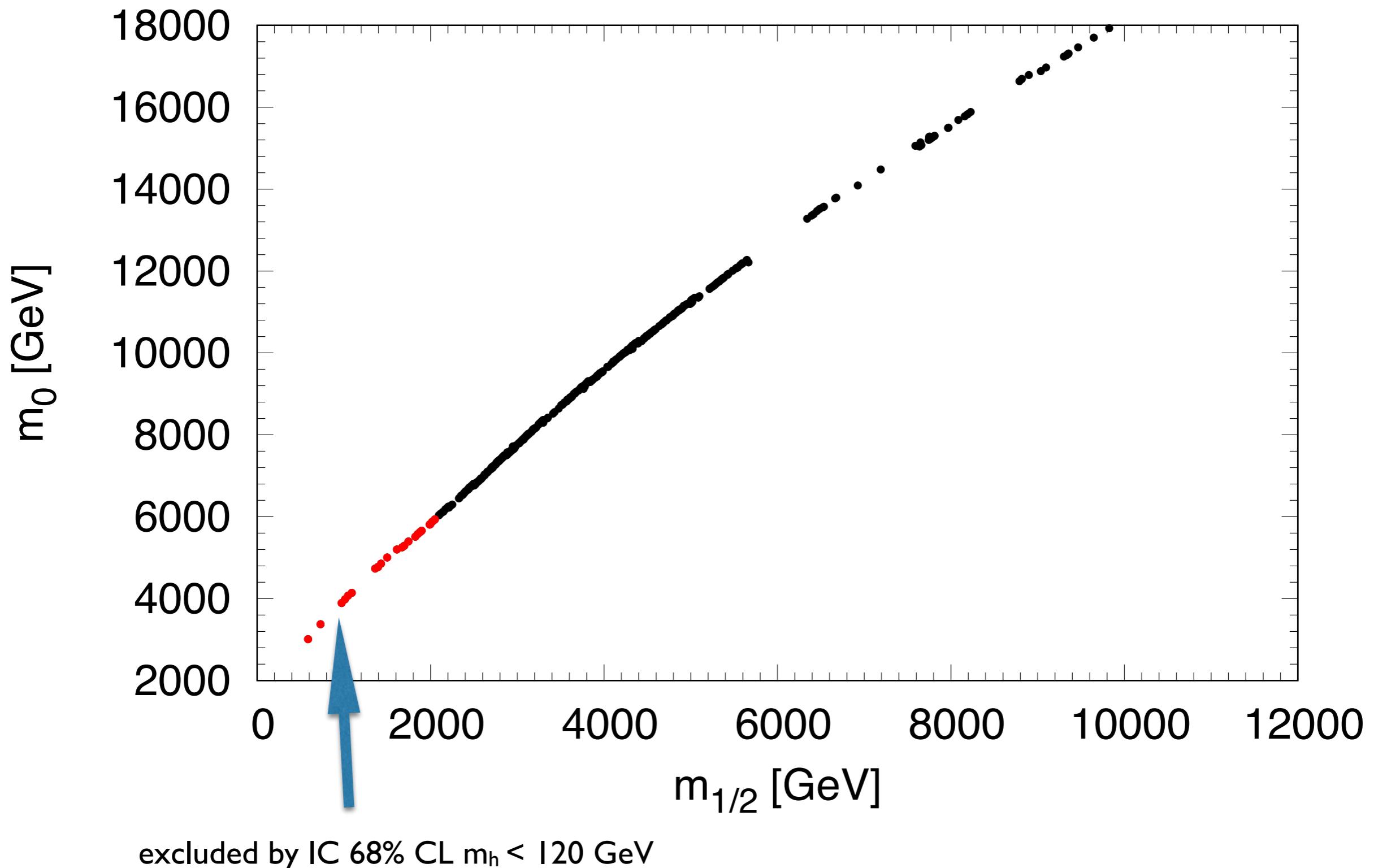
# New study

- Revisit the  $\chi$  DM regions in CMSSM
- Use indirect constraints from neutrino fluxes from Sun (IceCube data) and gammas from dSph (Fermi-LAT data)
- Work in progress [Ellis, Olive, VCS, Stamou (2022) to appear]

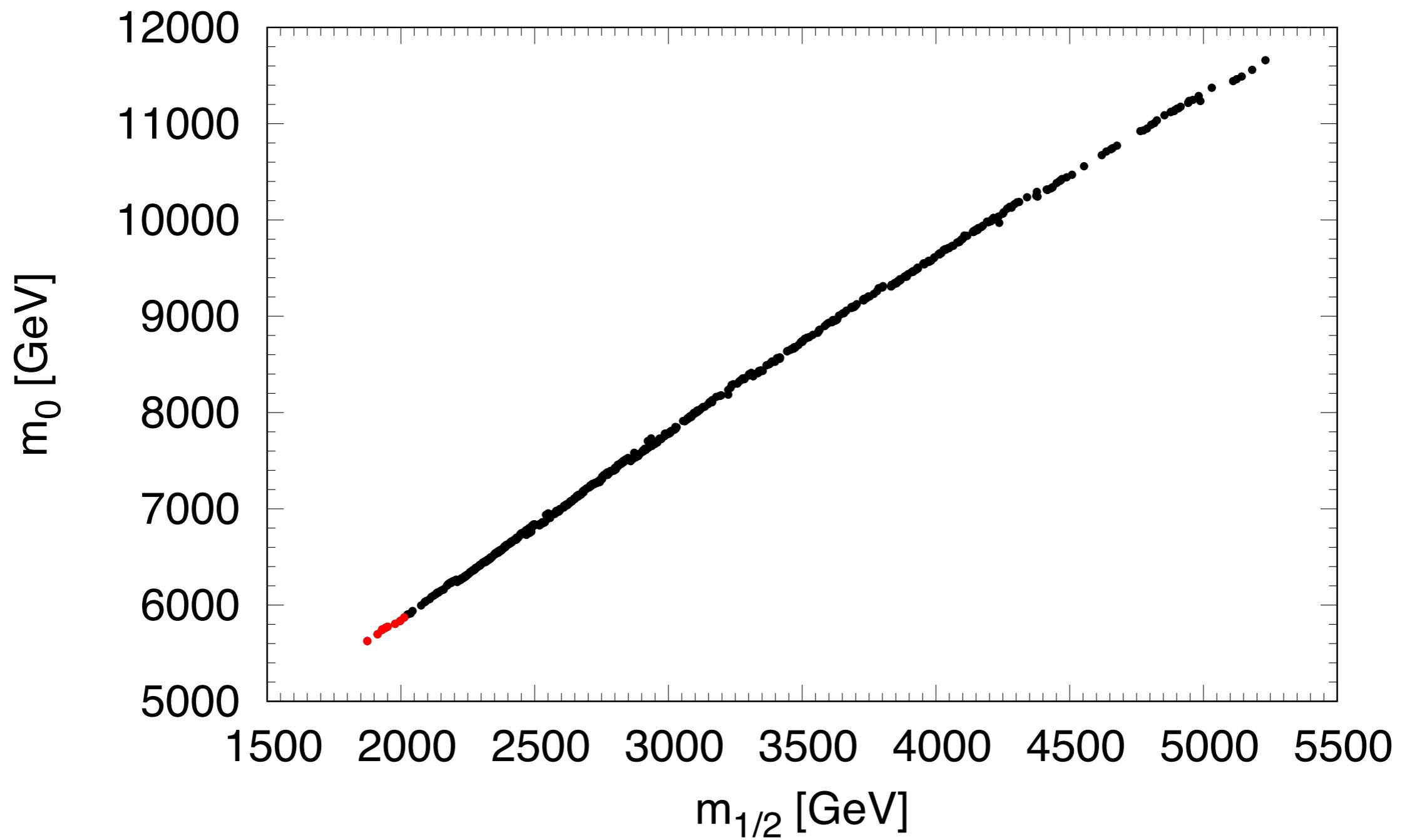
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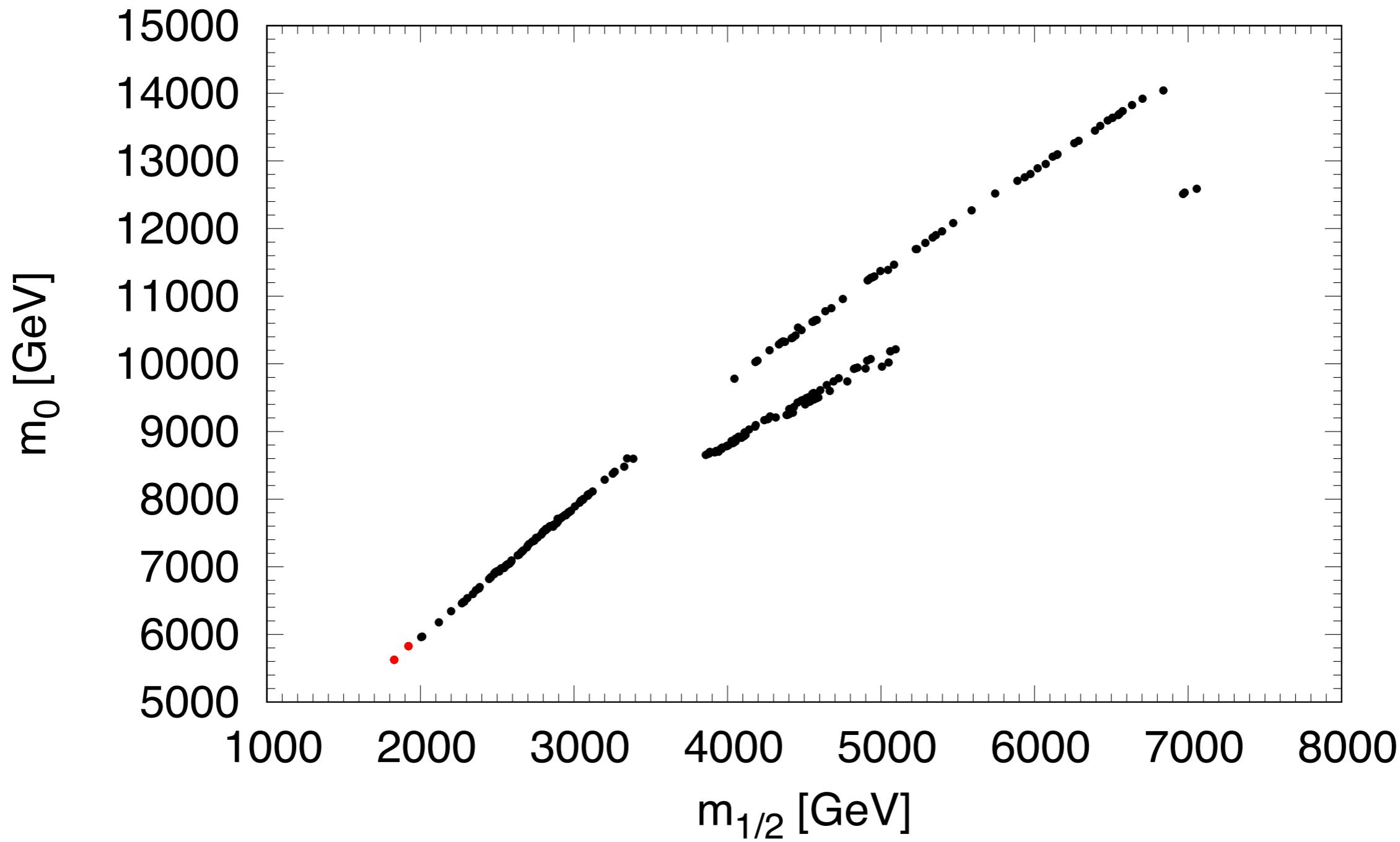
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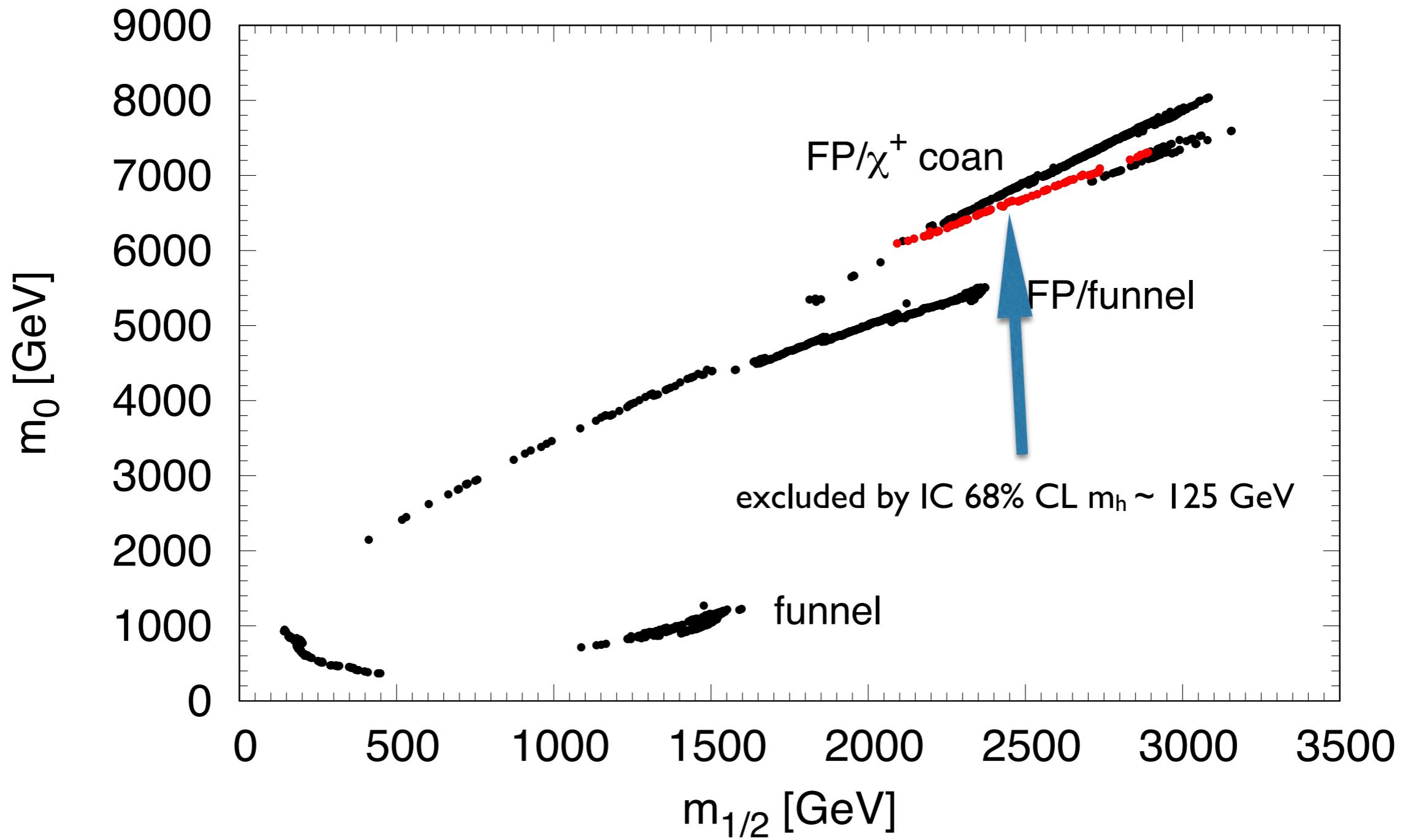
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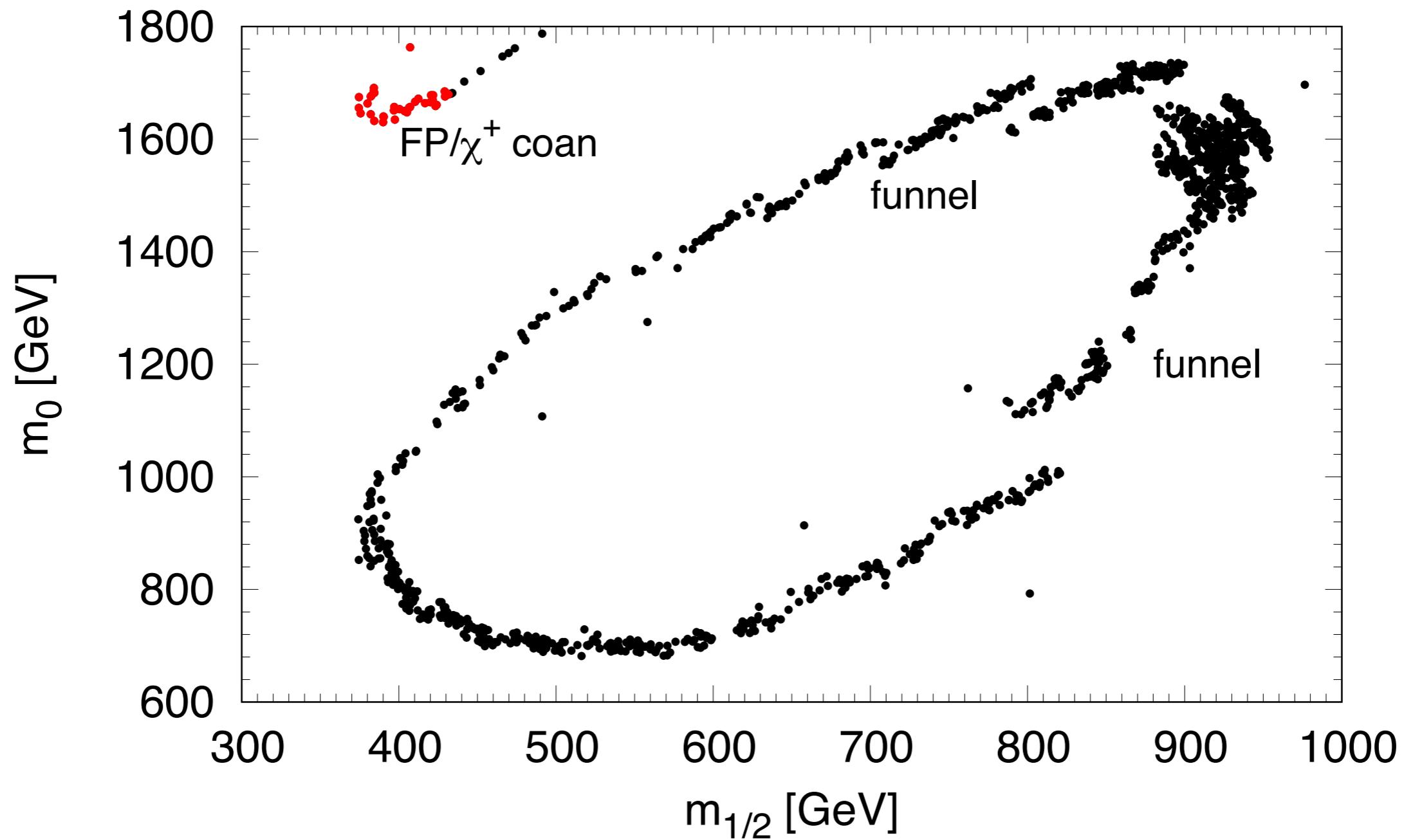
$tb=50, A_0=0$



$tb=55, A_0=0$



$tb=58, A_0=0$



# Recap for neutralinos

- Neutralino DM is \*well\* studied in CMSSM
- Preliminary results from a new study, with emphasis at the indirect constraints from gammas from dSph and neutrinos form Sun
- The gamma fluxes do not constraint the parameter space of CMSSM. On the other hand the neutrino data from IceCube appear to exclude particular region
- For large value of  $\tan\beta$  these regions are compatible with  $m_h \sim 125$  GeV.
- More work to combine these regions with data from direct DM searches

# Gravitino as DM

- **Gravitino is the s=3/2 superpartner of graviton. Naturally is in the spectrum of any SUGRA model** [Ellis, Hagelin, Nanopoulos, Olive, Srednicki (1983), Khlopov, Linde (1984)]
- **The “classic” freeze-in DM candidate particle**
- **Naturally escapes all the direct and indirect DM searches**
- **Can be produced non-thermally: (i) inflaton decays** [Giudice, Riotto, Tkachev (1999); Kallosh, Kofman, Linde, Van Proeyen (2000); Nilles, Peloso, Sorbo (2001), Endo, Kawasaki, Takahashi, Yanagida (2006)] **(ii) decays from unstable particles, eg NLSP decays in GDM models** [Cyburt, Ellis, Field, Olive, VSC (2006); Kawasaki, Kohri, Moroi, Yotsuyanagi (2008)]
- **In the later case the BBN constraints should be applied** [Cyburt, Ellis, Field, Luo, Olive, VSC (2012)]
- **In any case the thermal gravitino production rate is vital to apply cosmological constraints**

# Background of the calculation

- **Effective theory of light gravitinos, only 1/2 goldstino components**  
[Ellis, Kim, Nanopoulos (1984); Moroi, Murayama, Yamagushi, Kawasaki (1993,1994) ]
- **Use of the Braaten, Pisarski, Yan method including 3/2 components**  
[Ellis, Nanopoulos, Olive, Rey (1996), Bolz, Buchmuller, Plumacher, Brandenburger (1998,2001); Pedlar, Steffen (2007) ]
- **I-loop calculation beyond the HTL approximation** [Rychkov, Strumia (2007)]
- **Our calculation: error corrections and proper parametrization of the result** [Eberl, Gialamas,VCS (2021)]
- **More improvements in the calculation will come** [Eberl, Gialamas,VCS (2022) to appear]

# The setup of the calculation

## The Braaten-Yuan prescription

[Braaten, Pisarski, Yuan (1990,1991)]

$$\gamma = \gamma|_{\text{hard}}^{k^* < k} + \gamma|_{\text{soft}}^{k^* > k}$$

where  $gT \ll k^* \ll T$  assuming  $g \ll 1$

Hard part is calculated from squared matrix elements

$$|\mathcal{M}(a b \rightarrow c \tilde{G})|^2$$

Soft part is calculated from Imaginary part of the gravitino self-energy

$$\gamma|_{\text{hard}}^{k^* < k} = A_{\text{hard}} + B \ln \left( \frac{T}{k^*} \right) \quad \text{and} \quad \gamma|_{\text{soft}}^{k^* > k} = A_{\text{soft}} + B' \ln \left( \frac{k^*}{m_{\text{thermal}}} \right)$$

Thus

$$\gamma_{\text{BY}} = \frac{3\zeta(3)}{16\pi^3} \frac{T^6}{M_P^2} \sum_{N=1}^3 c'_N g_N^2 \left( 1 + \frac{m_{\lambda_N}^2}{3m_{3/2}^2} \right) \ln \left( \frac{k'_N}{g_N} \right)$$

$$c'_N = (11, 27, 72) \quad , \quad k'_N = (1.266, 1.312, 1.271) \quad [\text{Pradler, Steffen (2007)}]$$

Analytical result, but valid only for  $g \ll 1$

where  $\gamma|_{\text{soft}}$  is calculated in the Hard Thermal Loop (HTL) approx

The condition  $g(T) \ll 1$  is not satisfied in the whole temperature range especially if  $g = g_3$

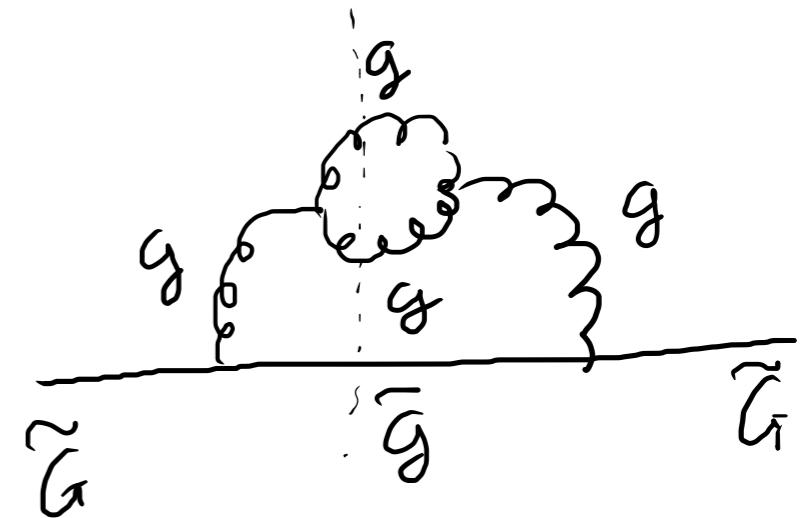
## Beyond the HTL approx

- ▶ **Calculate the full 1-loop gravitino self-energy beyond HTL approximation**
- ▶ **Calculate the so-called subtracted part of the  $|\mathcal{M}|^2$**  [Rychkov, Strumia (2007)]

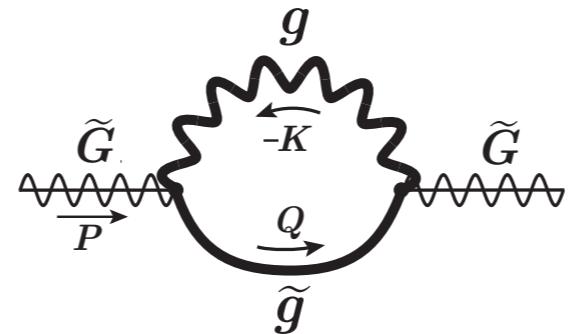
The subtracted part of the squared amplitude is this that cannot be part of the gravitino self-energy

For example if  $X: gg \rightarrow \tilde{g}\tilde{G}$

$$|\mathcal{M}_{X,s}|^2 = \left| \begin{array}{c} \text{Feynman diagram} \\ \text{for } gg \rightarrow \tilde{g}\tilde{G} \end{array} \right|^2 \text{ related to}$$



which is part of



(D-graph)

where thick lines denote resumed thermal propagators

Thus  $\gamma_{3/2} = \gamma_{\text{sub}} + \gamma_D + \gamma_{\text{top}}$

$X$	process	$ \mathcal{M}_{X,\text{full}} ^2$	$ \mathcal{M}_{X,\text{sub}} ^2$
A	$gg \rightarrow \tilde{g}\tilde{G}$	$4C_3(s + 2t + 2t^2/s)$	$-2sC_3$
B	$g\tilde{g} \rightarrow g\tilde{G}$	$-4C_3(t + 2s + 2s^2/t)$	$2tC_3$
C	$\tilde{q}g \rightarrow q\tilde{G}$	$2sC'_3$	0
D	$gq \rightarrow \tilde{q}\tilde{G}$	$-2tC'_3$	0
E	$\tilde{q}q \rightarrow g\tilde{G}$	$-2tC'_3$	0
F	$\tilde{g}\tilde{g} \rightarrow \tilde{g}\tilde{G}$	$8C_3(s^2 + t^2 + u^2)^2/(stu)$	0
G	$q\tilde{g} \rightarrow q\tilde{G}$	$-4C'_3(s + s^2/t)$	0
H	$\tilde{q}\tilde{g} \rightarrow \tilde{q}\tilde{G}$	$-2C'_3(t + 2s + 2s^2/t)$	0
I	$q\tilde{q} \rightarrow \tilde{g}\tilde{G}$	$-4C'_3(t + t^2/s)$	0
J	$\tilde{q}\tilde{q} \rightarrow \tilde{g}\tilde{G}$	$2C'_3(s + 2t + 2t^2/s)$	0

Squared matrix elements for gravitino production in  $SU(3)_c$  in terms of  $g_3^2 Y_3/M_P^2$

$$Y_3 = 1 + m_{\tilde{g}}^2/(3m_{3/2}^2), C_3 = 24 \text{ and } C'_3 = 48$$

$$|\mathcal{M}_{X,\text{full}}|^2 = |\mathcal{M}_{X,s} + \mathcal{M}_{X,t} + \mathcal{M}_{X,u} + \mathcal{M}_{X,x}|^2$$

$$|\mathcal{M}_{X,D}|^2 = |\mathcal{M}_{X,s}|^2 + |\mathcal{M}_{X,t}|^2 + |\mathcal{M}_{X,u}|^2$$

$$|\mathcal{M}_{X,\text{sub}}|^2 = |\mathcal{M}_{X,\text{full}}|^2 - |\mathcal{M}_{X,D}|^2$$

$$\gamma_{\text{sub}}$$

$$\gamma = \frac{1}{(2\pi)^8} \int \frac{d^3 p_a}{2E_a} \frac{d^3 p_b}{2E_b} \frac{d^3 p_c}{2E_c} \frac{d^3 p_{\widetilde{G}}}{2E_{\widetilde{G}}} \, |\mathcal{M}|^2 f_a f_b (1 \pm f_c) \times \delta^4(P_a + P_b - P_c - P_{\widetilde{G}}) \quad f_{B|F} = \frac{1}{e^{\frac{E}{T}} \mp 1}$$

$$|\mathcal{M}_{A,\text{sub}}|^2 + |\mathcal{M}_{B,\text{sub}}|^2 = \frac{g_N^2}{M_P^2} \left( 1 + \frac{m_{\lambda_N}^2}{3m_{3/2}^2} \right) C_N(-s+2t) \quad \text{as taken from the Table with the amplitudes}$$

## Performing numerical integration

$$\gamma_{\text{sub}} = \frac{T^6}{M_P^2} \sum_{N=1}^3 g_N^2 \left( 1 + \frac{m_{\lambda_N}^2}{3m_{3/2}^2} \right) C_N (-\mathcal{C}_{\text{BBF}}^s + 2\mathcal{C}_{\text{BFB}}^t)$$

$$\mathcal{C}_{\text{BBF}}^s = 0.25957 \times 10^{-3}$$

$$\mathcal{C}_{\text{BFB}}^t = -0.13286 \times 10^{-3}.$$

γD

$$\Pi^<(P) = \frac{1}{16M_P^2} \sum_{N=1}^3 n_N \left( 1 + \frac{m_{\lambda_N}^2}{3m_{3/2}^2} \right) \int \frac{d^4 K}{(2\pi)^4} \text{Tr} \left[ \not{P}[K, \gamma^\mu]^* S^<(Q)[K, \gamma^\nu]^* D_{\mu\nu}^<(K) \right]$$

$${}^*S^<(Q) = \frac{f_F(q_0)}{2} \left[ (\gamma_0 - \boldsymbol{\gamma} \cdot \mathbf{q}/q) \rho_+(Q) + (\gamma_0 + \boldsymbol{\gamma} \cdot \mathbf{q}/q) \rho_-(Q) \right]$$

$${}^*D_{\mu\nu}^<(K) = f_B(k_0) \left[ \Pi_{\mu\nu}^T \rho_T(K) + \Pi_{\mu\nu}^L \frac{k^2}{K^2} \rho_L(K) + \xi \frac{K_\mu K_\nu}{K^4} \right]$$

$$\gamma_D = \int \frac{d^3 p}{2p_0(2\pi)^3} \Pi^<(p)$$

$$\begin{aligned} \gamma_D = & \frac{1}{4(2\pi)^5 M_P^2} \sum_{N=1}^3 n_N \left( 1 + \frac{m_{\lambda_N}^2}{3m_{3/2}^2} \right) \int_0^\infty dp \int_{-\infty}^\infty dk_0 \int_0^\infty dk \int_{|k-p|}^{k+p} dq \ k f_B(k_0) f_F(q_0) \\ & \times \left[ \rho_L(K) \rho_-(Q) (p-q)^2 ((p+q)^2 - k^2) + \rho_L(K) \rho_+(Q) (p+q)^2 (k^2 - (p-q)^2) \right. \\ & + \rho_T(K) \rho_-(Q) (k^2 - (p-q)^2) \left( (1 + k_0^2/k^2) (k^2 + (p+q)^2) - 4k_0(p+q) \right) \\ & \left. + \rho_T(K) \rho_+(Q) ((p+q)^2 - k^2) \left( (1 + k_0^2/k^2) (k^2 + (p-q)^2) - 4k_0(p-q) \right) \right], \end{aligned}$$

The diagram shows a closed loop with a wavy boundary. At the top, there is a label  $g$ . On the left side, there is a vertical spring labeled  $\tilde{G}$  above it and  $P$  below it. On the right side, there is another vertical spring labeled  $\tilde{G}$  above it and  $Q$  below it. A curved arrow labeled  $-K$  points from the left side towards the center of the loop.

$\gamma_{\text{top}}$

$$\gamma_{\text{top}} = \frac{T^6}{M_P^2} 72 \mathcal{C}_{\text{BBF}}^s \lambda_t^2 \left( 1 + \frac{A_t^2}{3m_{3/2}^2} \right) \quad \mathcal{C}_{\text{BBF}}^s = 0.25957 \times 10^{-3}$$

$$\rho_{L,T}(K)=2\pi\Bigl\{Z_{L,T}(k)\Bigl[\delta(k_0-\omega_{L,T}(k))-\delta(k_0+\omega_{L,T}(k))\Bigr]+\rho^{\rm cont}_{L,T}(K)\Bigr\}$$

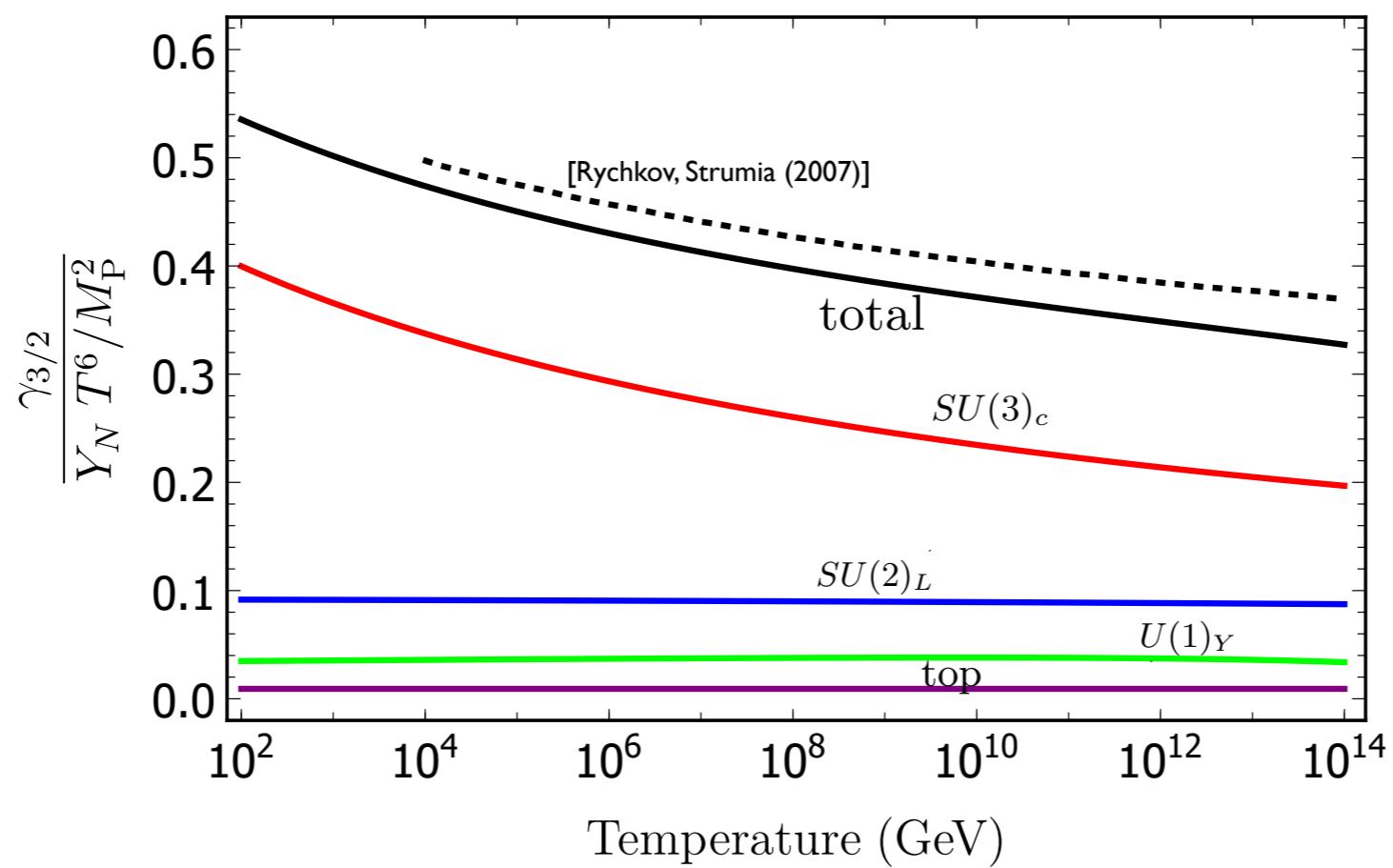
$$\rho_\pm(Q)=2\pi\Bigl\{Z_\pm(q)\delta(q_0-\omega\pm(q))+Z_\mp(q)\delta(q_0+\omega_\mp(q))+\rho^{\rm cont}_\pm(Q)\Bigr\}\,.$$

$$Z_L(k) = \frac{\omega_L(k)(\omega_L^2(k)-k^2)}{k^2(k^2+2m^2-\omega_L^2(k))}~,~ Z_T(k) = \frac{\omega_T(k)(\omega_T^2(k)-k^2)}{2m^2\omega_T^2(k)-(\omega_T^2(k)-k^2)^2}~,~ Z_\pm(q) = \frac{\omega_\pm(q)^2-q^2}{2m_f^2}$$

# Result and cosmological consequences

$$\gamma_{\text{sub}} + \gamma_D = \frac{3\zeta(3)}{16\pi^3} \frac{T^6}{M_P^2} \sum_{N=1}^3 c_N g_N^2 \left( 1 + \frac{m_{\lambda_N}^2}{3m_{3/2}^2} \right) \ln \left( \frac{k_N}{g_N} \right)$$

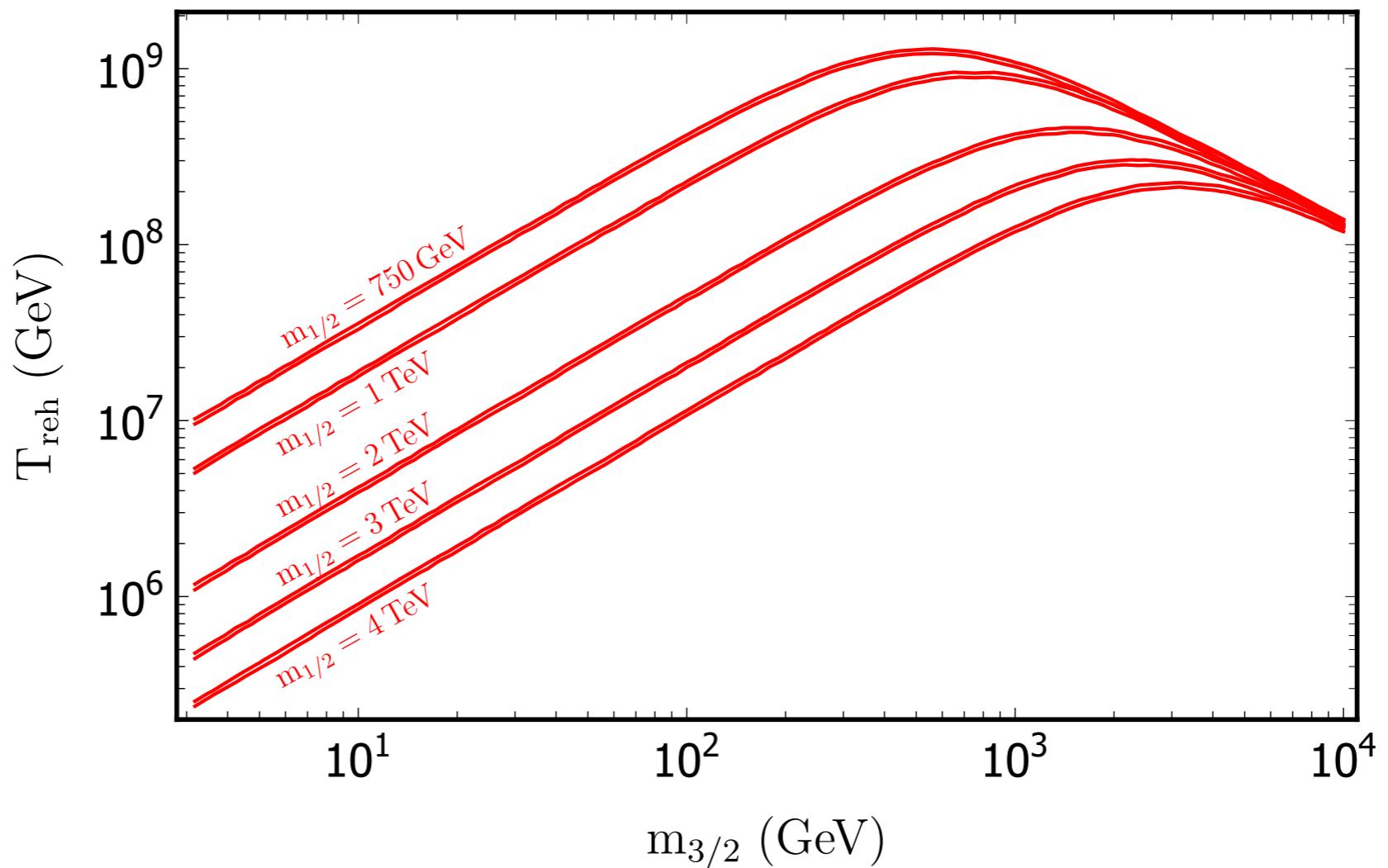
Gauge group	$c_N$	$k_N$
$U(1)_Y$	41.937	0.824
$SU(2)_L$	68.228	1.008
$SU(3)_c$	21.067	6.878



# Gravitino abundance

$$Y_{3/2}(T) \simeq \frac{\gamma_{3/2}(T_{\text{reh}})}{H(T_{\text{reh}}) n_{\text{rad}}(T_{\text{reh}})} \frac{g_{*s}(T)}{g_{*s}(T_{\text{reh}})}$$

$$\Omega_{\text{DM}} h^2 = \frac{\rho_{3/2}(t_0) h^2}{\rho_{\text{cr}}} = \frac{m_{3/2} Y_{3/2}(T_0) n_{\text{rad}}(T_0) h^2}{\rho_{\text{cr}}} \simeq 1.33 \times 10^{24} \frac{m_{3/2} \gamma_{3/2}(T_{\text{reh}})}{T_{\text{reh}}^5}$$



# Recap for gravitinos

- **Gravitino is a natural DM candidate in SUGRA**
- **Thermally produced (Freeze-in mechanism) details explained.**  
**Improvements for this calculation are possible.**
- **No-thermal production (e.g. through inflaton decays) requires a particular inflation model.**
- **Assuming  $m_{1/2} > 750 \text{ GeV}$  (~LHC bound) for  $T_{reh} \sim 10^9 \text{ GeV}$ , we get  $m_{3/2} = 550 \text{ GeV}$ . For  $T_{reh} \sim 10^8 \text{ GeV}$  for the same  $m_{3/2}$ ,  $m_{1/2} \sim 3,4 \text{ TeV}$ .**

# PBH from SUGRA models

- The detection of GW from BH merging by VIRGO/LIGO has sparked a lot in the PBH scenario
- Using as basis the no-scale SUGRA models one can show that adding modifications either to Kaehler potential or to superpotential can create features in the scalar potential, i.e. an inflection point, that can produce a significant enhancement in the power spectrum [Nanopoulos,VCS,Stamou (2020); Stamou (2021); VCS,Stamou (2021)]
- Around the inflection point the slow-roll approximation is not working, thus the numerical solution of the Mukhanov-Sasaki equation
- In each case the models satisfy the Planck constraints for inflation, produce significant amount of DM in form of PBH and GW detectable at LISA, NANOGrav etc.



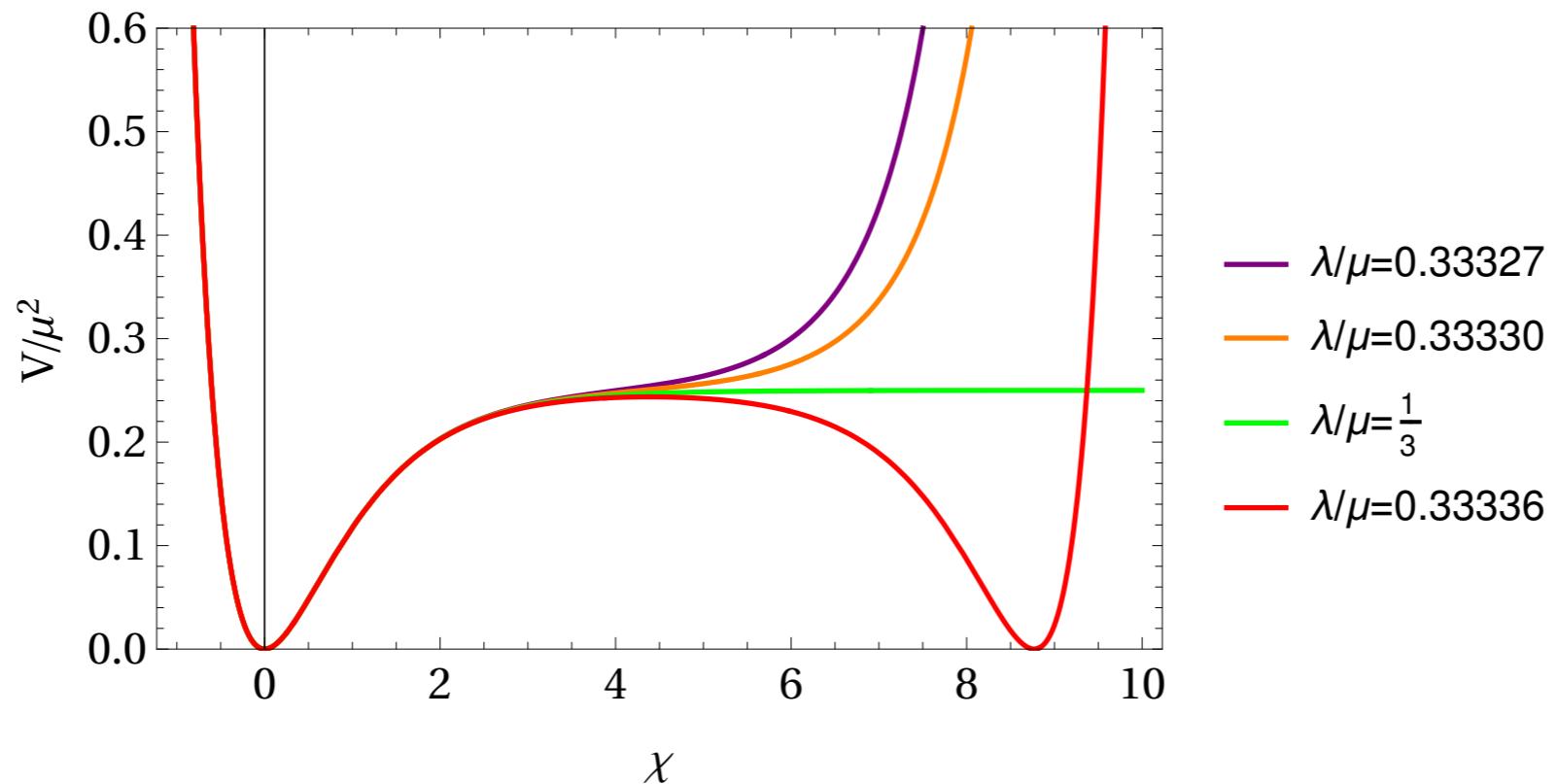
More details from Ioanna Stamou, Nick Mavromatos

## Starobinsky no-scale SUGRA model

$$K = -3 \ln \left( T + \bar{T} - \frac{\varphi \bar{\varphi}}{3} \right) \quad W = \frac{\hat{\mu}}{2} \varphi^2 - \frac{\lambda}{3} \varphi^3$$

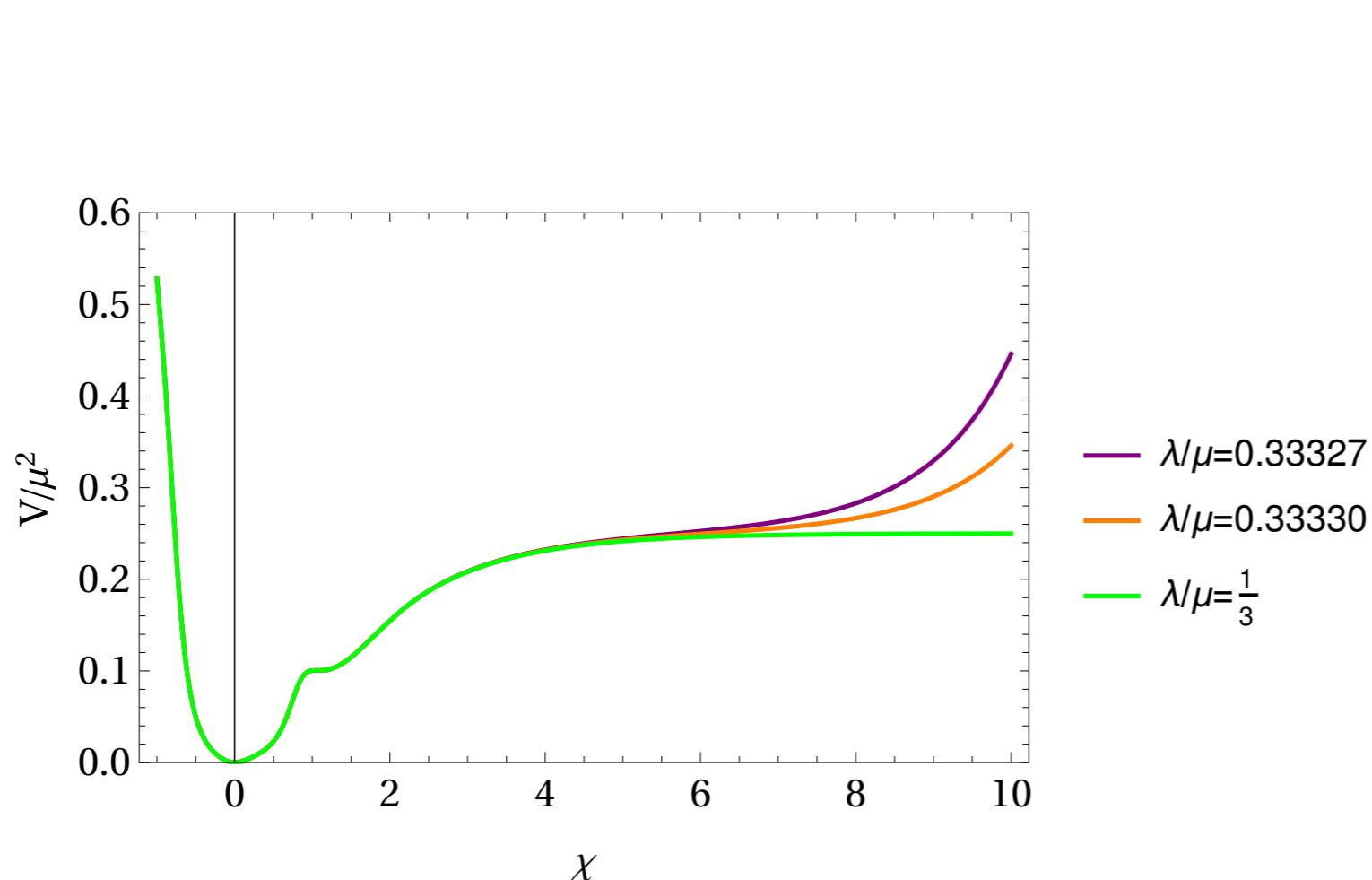
$$T = \bar{T} = \frac{c}{2}, \quad \text{Im}\varphi = 0 \quad \varphi = \sqrt{3c} \tanh \left( \frac{\chi}{\sqrt{3}} \right)$$

$$V(\chi) = \frac{\mu^2}{4} \left( 1 - e^{-\sqrt{\frac{2}{3}}\chi} \right)^2$$



## Modifying the Kaehler potential

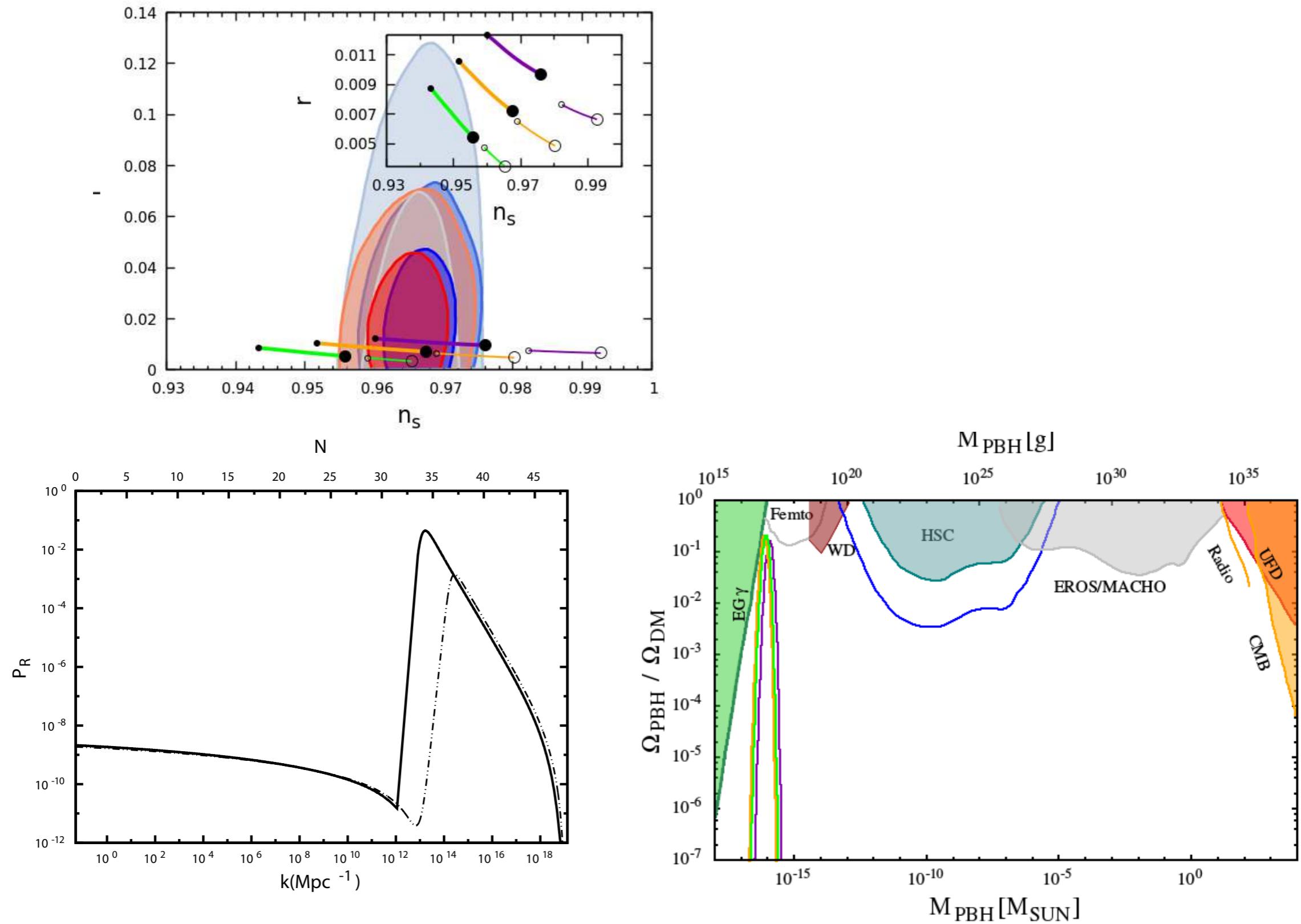
$$K = -3 \ln \left[ T + \bar{T} - \frac{\varphi \bar{\varphi}}{3} + a e^{-b(\varphi+\bar{\varphi})^2} (\varphi + \bar{\varphi})^4 \right]$$



	$\lambda/\mu$	$b$
1.	0.33327	87.379427
2.	0.33330	87.390563
3.	$1/3$	87.402941

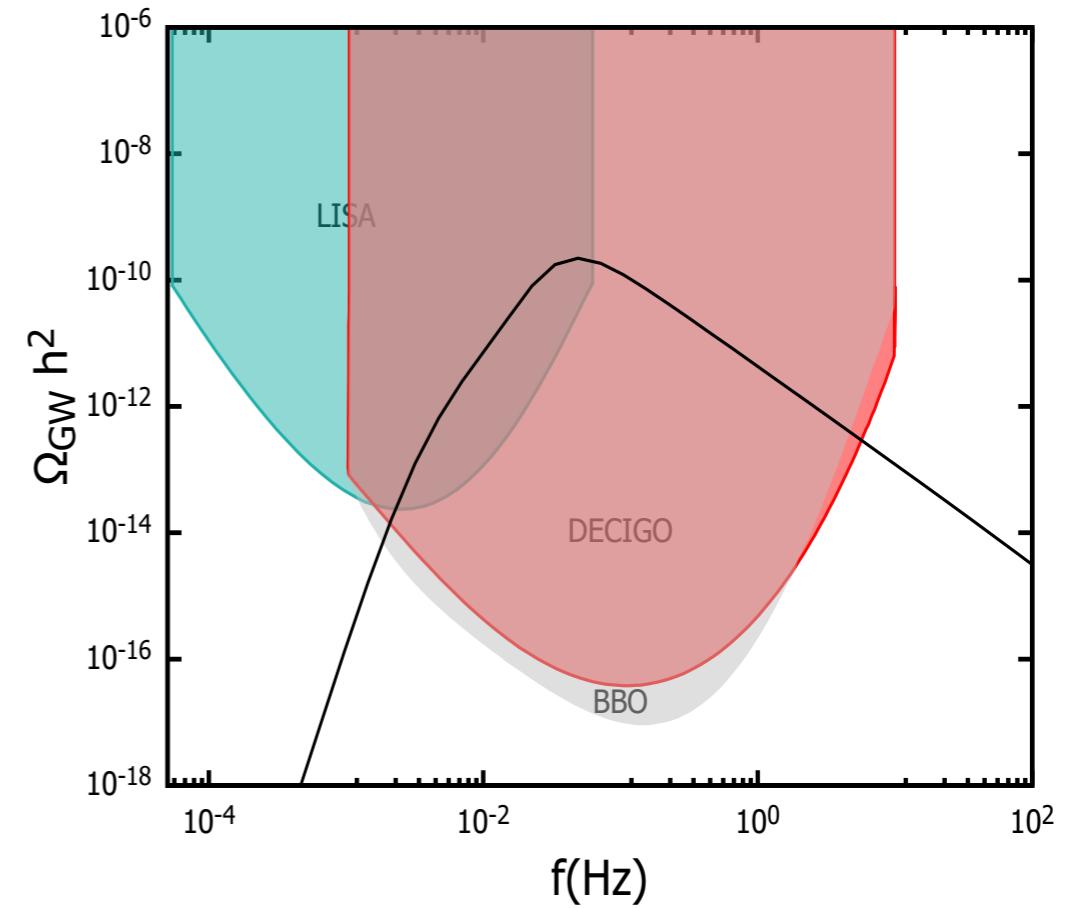
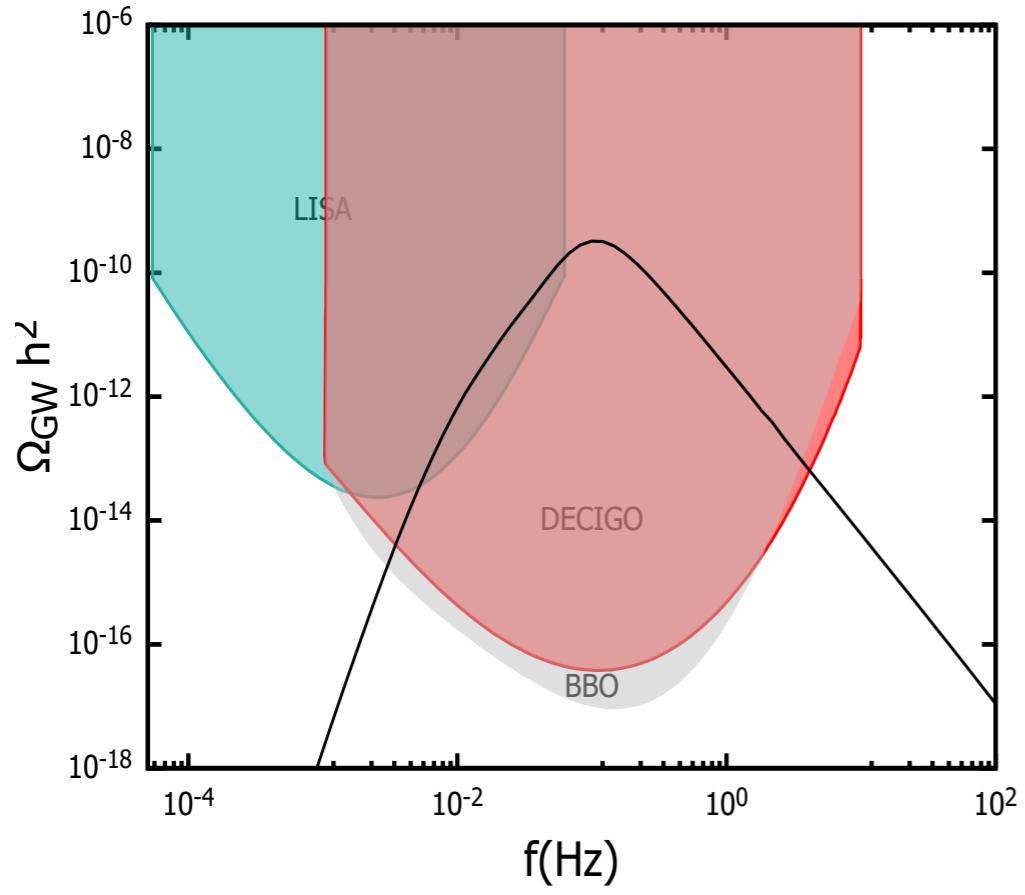
[Nanopoulos, VCS, Stamou (2020)]

# PBH production



[Nanopoulos, VCS, Stamou (2020)]

## GW production



[Stamou (2021); VCS, Stamou (2022)]

# Recap for PBH

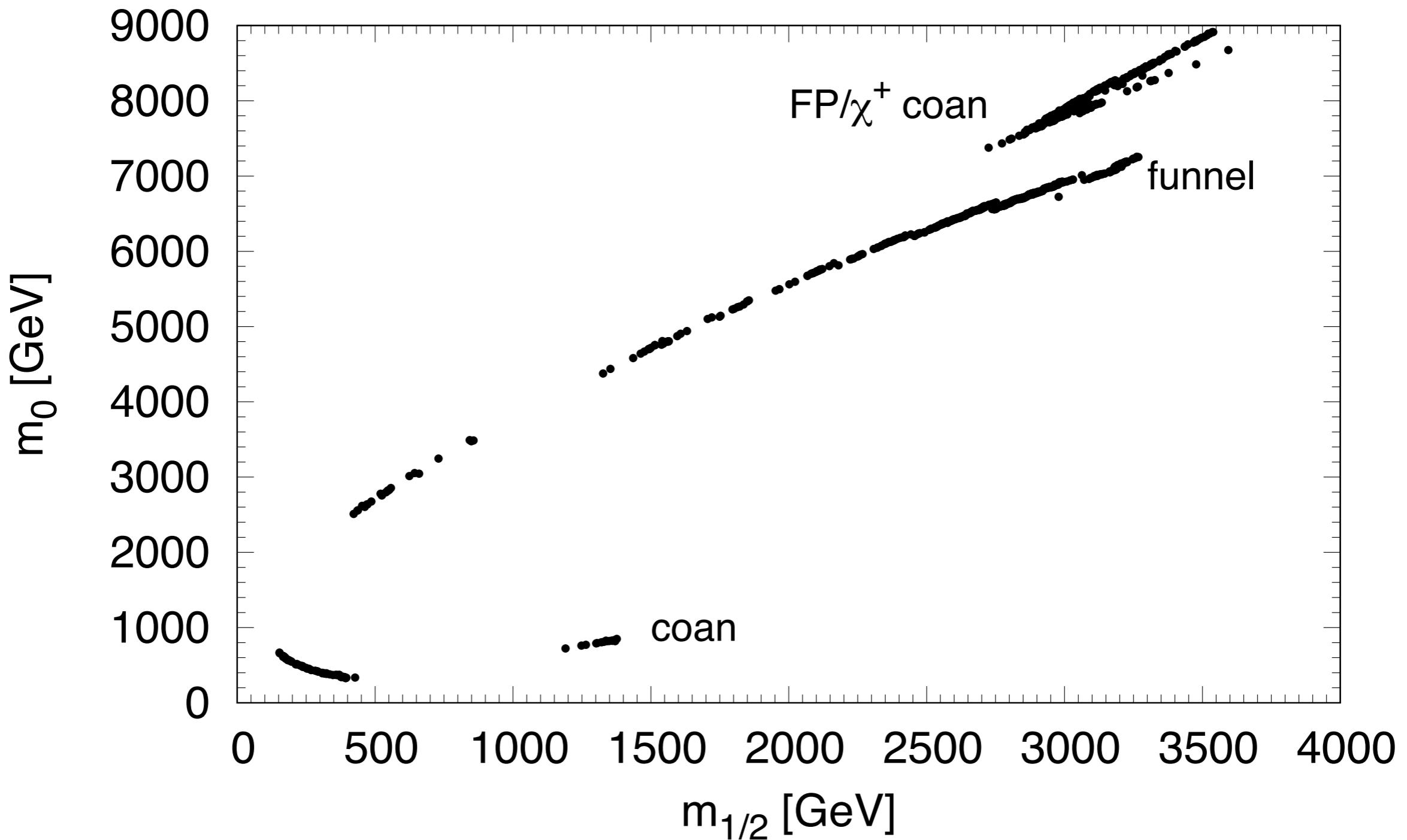
- By modifying the Kaehler potential or the superpotential PBH and GW are produced
- Tuning the parameters of the models inflationary constraints are satisfied and significant amount of DM in the form of PBH is produced, up to 90% or even higher
- In the most of the case sizeable fine tuning is required in order to achieve these
- GW that are produced can be detected in current and future interferometers, like NANOGrav, LISA, Decigo etc

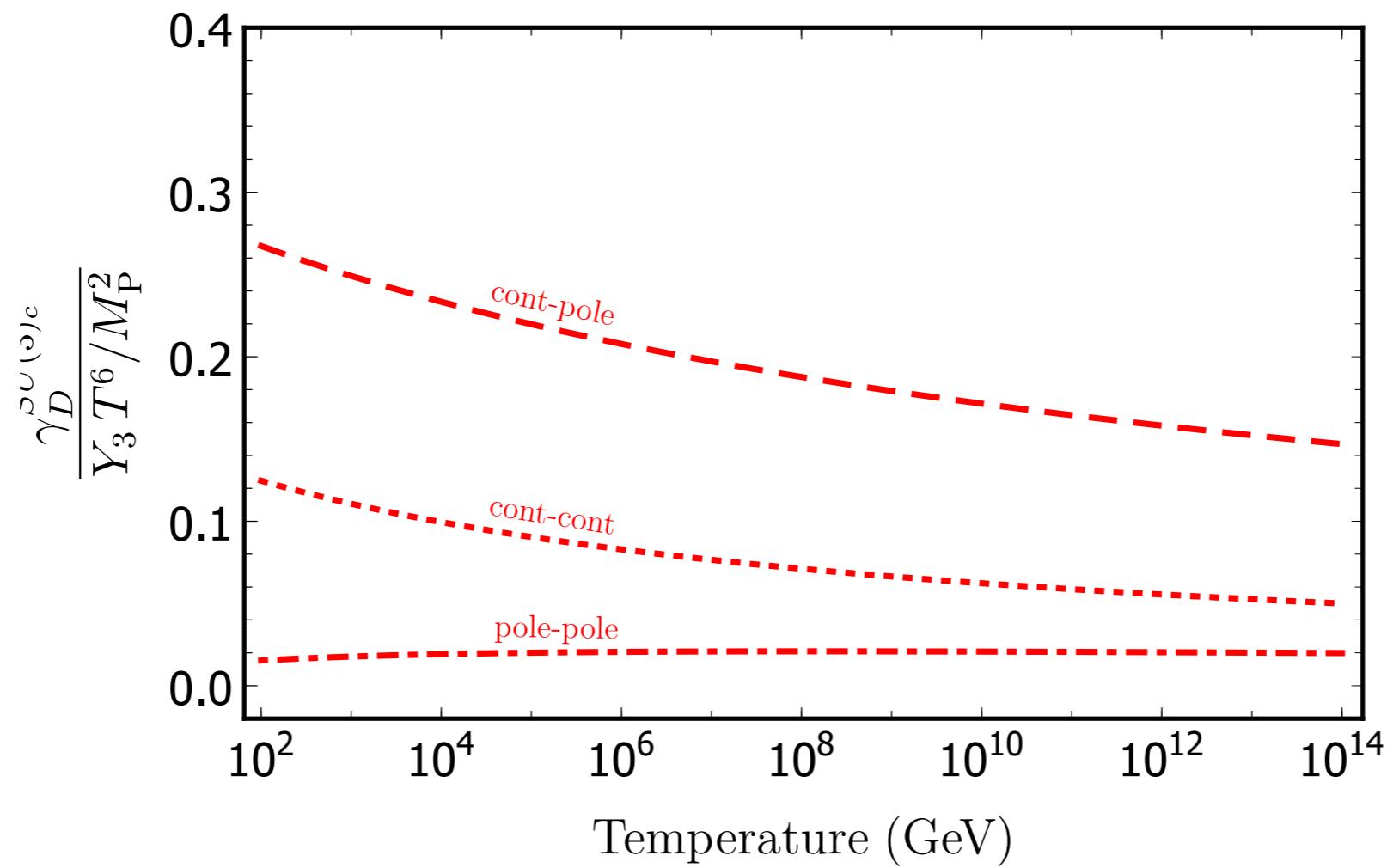
# Summary

- We presented results for neutralino, gravitino and PBH DM in the context of various SUGRA models
- For x work in progress combining direct and indirect data from DM searches
- Gravitino DM scenario not susceptible either to direct or indirect searches, but other constraints, e.g. from BBN should apply
- PBH scenario deserves more attention. Useful conclusions can be obtained, especially in conjunction with studies of potential GW signals at LISA and other interferometers

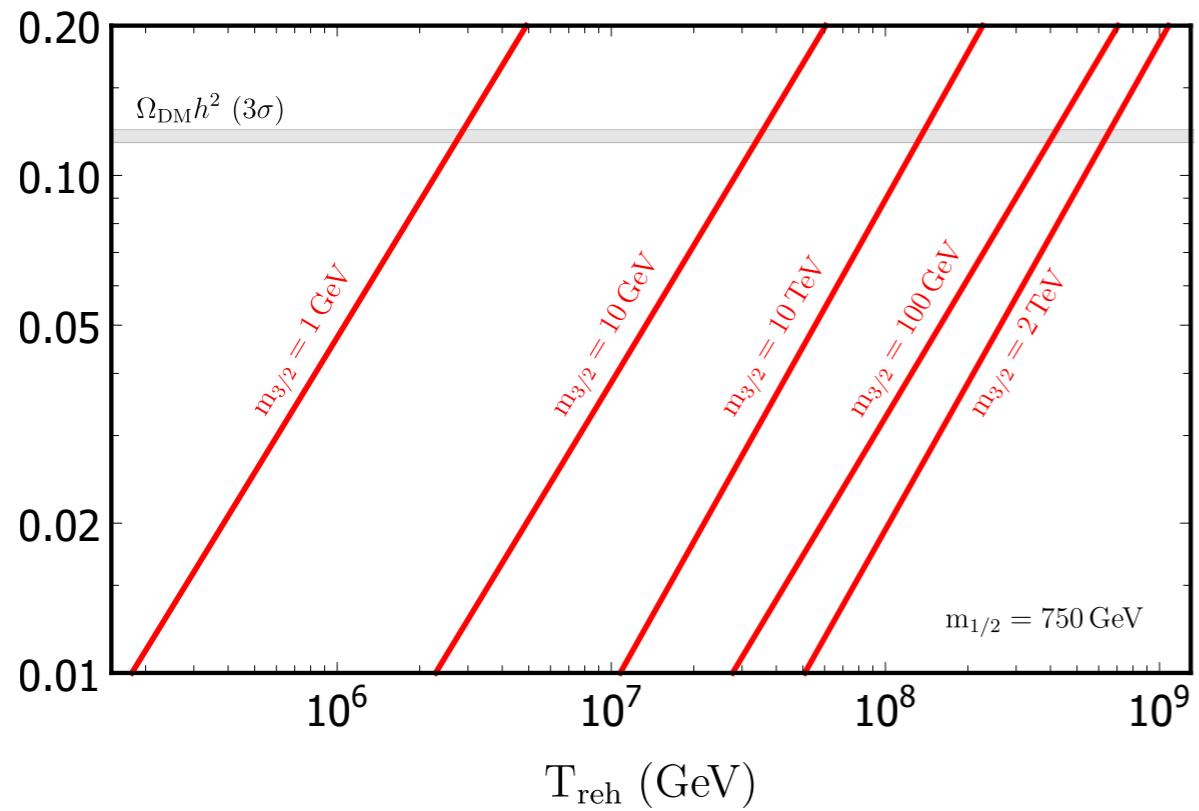
# **Backup slides**

$tb=54, A_0=0$

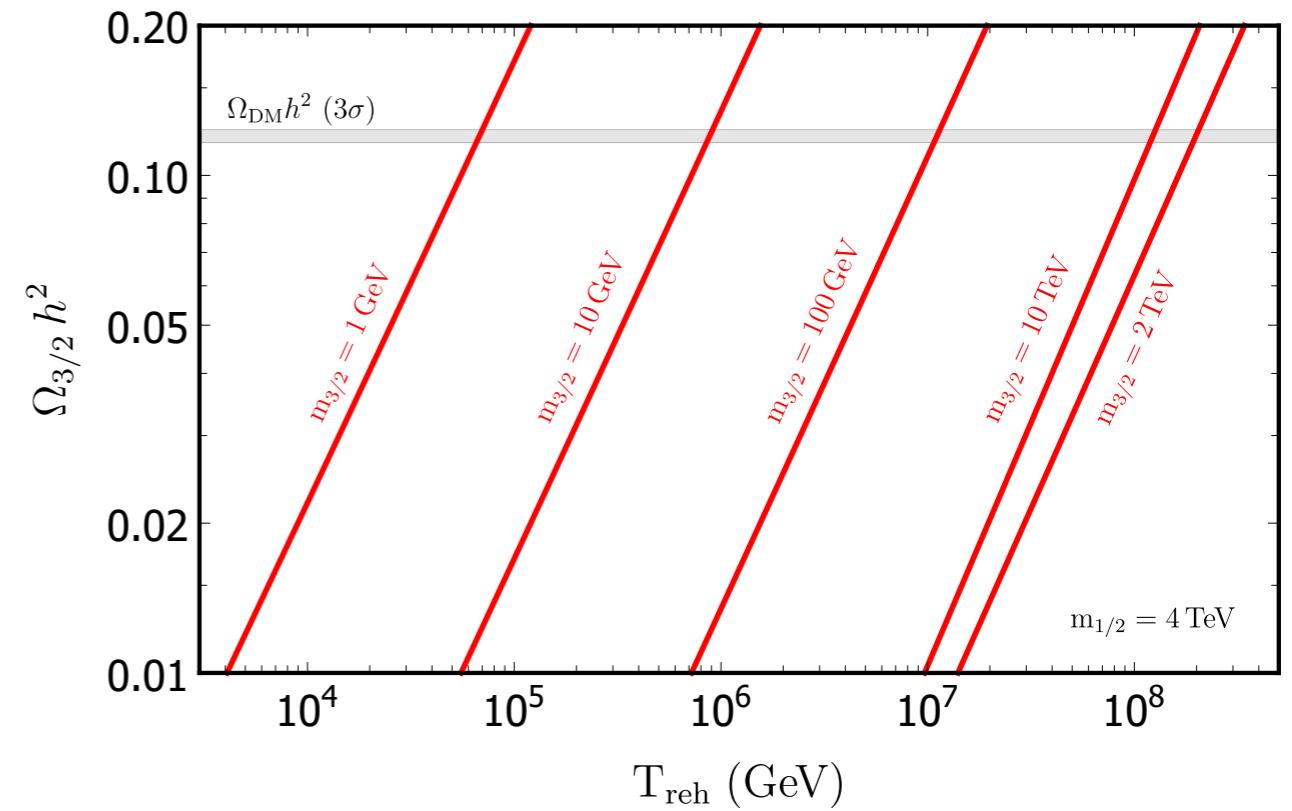




$m_{1/2} = 750 \text{ GeV}$



$m_{1/2} = 4 \text{ TeV}$



$$\Omega_{GW}(k)=\frac{c_g\,\Omega_r}{36}\int_0^{\frac{1}{\sqrt{3}}}\mathrm{d} d\int_{\frac{1}{\sqrt{3}}}^\infty \mathrm{d} s\left[\frac{(s^2-1/3)(d^2-1/3)}{s^2+d^2}\right]^2P_R(kx)P_R(ky)(I_c^2+I_s^2)$$

$$x=\frac{\sqrt{3}}{2}(s+d),\quad y=\frac{\sqrt{3}}{2}(s-d).$$

$$I_c=-36\,\pi\,\frac{(s^2+d^2-2)^2}{(s^2-d^2)^3}\,\Theta(s-1)\\ I_s=-36\,\frac{(s^2+d^2-2)^2}{(s^2-d^2)^2}\Bigg[\frac{(s^2+d^2-2)}{(s^2-d^2)}\,\ln\left|\frac{d^2-1}{s^2-1}\right|+2\Bigg]$$

$$\frac{\Omega_{PBH}}{\Omega_{DM}}(M_{\rm PBH}) = \frac{\beta(M_{\rm PBH})}{8\times 10^{-16}}\left(\frac{\gamma}{0.2}\right)^{3/2}\left(\frac{g_*(T_f)}{106.75}\right)^{-1/4}\left(\frac{M_{\rm PBH}}{10^{-18}\text{ grams}}\right)^{-1/2}$$

$$f_{PBH} = \int \frac{d M_{\rm PBH}}{M_{\rm PBH}} \frac{\Omega_{PBH}}{\Omega_{DM}}$$

$$\sim \\ M_{\rm PBH}(k) = 10^{18} \left(\frac{\gamma}{0.2}\right)\left(\frac{g_*(T_f)}{106.75}\right)^{-1/6}\left(\frac{k}{7\times 10^{13}\,\mathrm{Mpc}^{-1}}\right)^{-2} \mathrm{~in~grams}$$

$$\beta(M_{\rm PBH}) = \frac{1}{\sqrt{2\pi\sigma^2(M_{\rm PBH})}}\int_{\delta_c}^\infty d\delta\,\exp\left(-\frac{\delta^2}{2\sigma^2(M_{\rm PBH})}\right) \qquad \qquad \sigma^2\left(M_{\rm PBH}(k)\right) = \frac{16}{81}\int \frac{dk'}{k'}\left(\frac{k'}{k}\right)^4 P_R(k')\tilde{W}\left(\frac{k'}{k}\right)$$