Nonthermal Dark Matter Production

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- But: WIMPs getting squeezed (*not* excluded) by negative results from direct and indirect searches
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- Here: out–of–equilibrium of heavy particle Φ!

Basic Mechanism

- Long–lived, massive particle Φ quite generic in Supergravity and Superstring scenarios (Polonyi field, moduli, . . .)

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- Can dominate total energy density
- DM particle can be produced as Φ decay product!

The Simplest Case

Gelmini et al. 2006; Acharya et al. 2009; Kane et al. 2015; Arbey et al. 2018; ...

- Assume Φ decays via Planck–suppressed dim–5 operator:

$$\Gamma_\Phi = \lambda \frac{M_\Phi^3}{M_{\rm Pl}^2} \quad \lambda = {\rm const}$$

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- ρ_Φ dominates energy density, until $T = T_R$ with

$$T_R = \sqrt{\Gamma_\Phi M_{\rm Pl}} \left(\frac{45}{4\pi^3 g_*(T_R)} \right)^{1/4},$$

$M_{\rm Pl} = 1.2 \cdot 10^{19}$ GeV; g_*: eff. number of d.o.f. in radiation

At $T > T_R$:

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- Entropy density s_R is *not* co–moving constant!
- $g_*,\ g_{*s}$ usually *not* constant over the relevant period

Boltzmann Equations:

$$
\begin{aligned}
\frac{d\rho_\Phi}{dt} + 3H\rho_\Phi &= -\Gamma_\Phi \rho_\Phi \,; \\
\frac{ds_R}{dt} + 3H s_R &= \frac{\rho_\Phi \Gamma_\Phi}{T} \,; \\
\frac{dn_\chi}{dt} + 3H n_\chi &= \frac{B_\chi}{M_\Phi} \Gamma_\Phi \rho_\Phi - \langle \sigma v \rangle \left(n_\chi^2 - n_{\chi,EQ}^2 \right) .
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H : Hubble parameter; s_R: entropy density;
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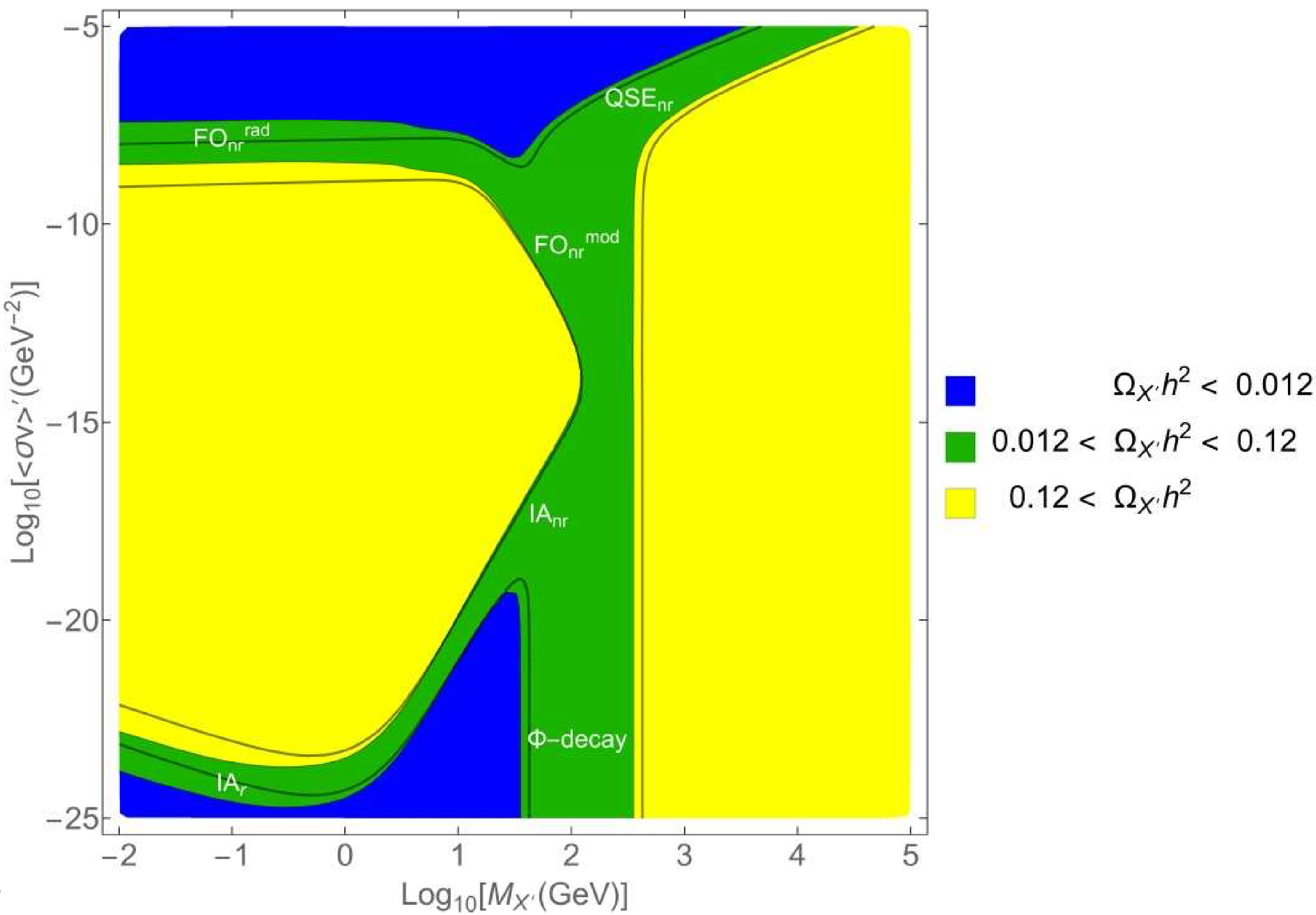
Can open up the allowed parameter space!

Example of Resulting DM Density

MD, F. Hajkarim, 1711.05007



Contribution from Direct Φ Decay

For negligible χ annihilation:

$$\frac{m_\chi n_\chi(T_R)}{s_R(T_R)} = \frac{m_\chi B_\chi n_\Phi(T_R)}{s_R(T_R)} = \frac{m_\chi B_\chi \rho_\Phi(T_R)}{M_\Phi s_R(T_R)}$$

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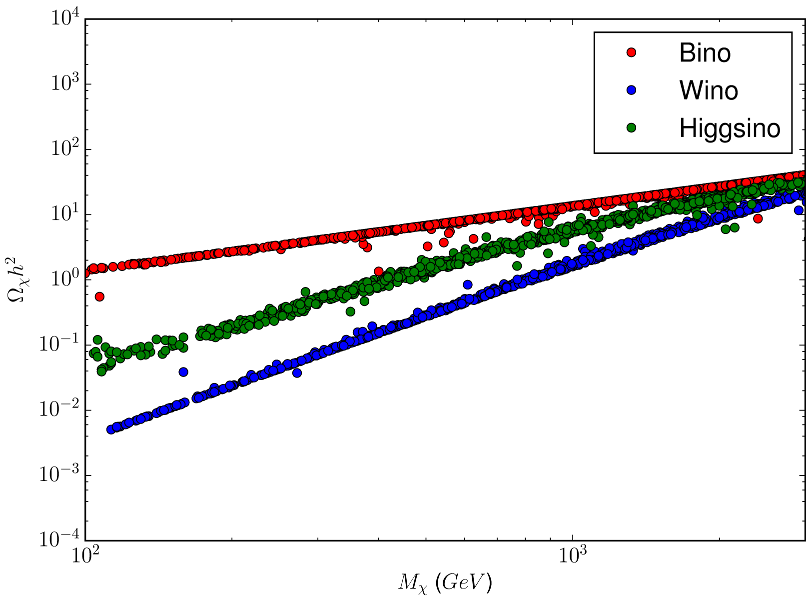
Using $T_R \propto \sqrt{\Gamma_\Phi} \propto \sqrt{\lambda M_\Phi^3}$:

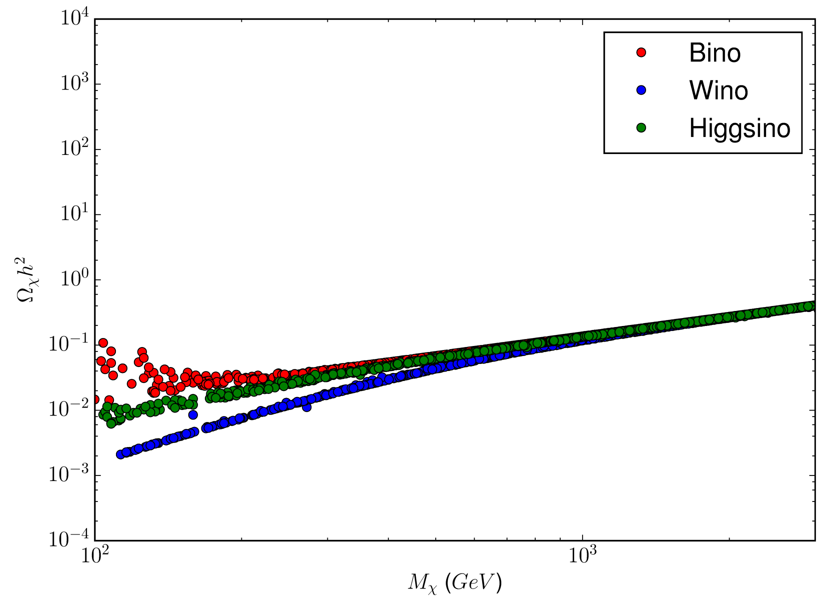
$$\Omega_\chi h^2 \propto m_\chi B_\chi \sqrt{\lambda M_\Phi}$$

Application to SUSY

MD, F. Hajkarim, 1808.05076

Use full (effective) annihilation cross section from `MicrOMEGAs`, supplemented with extended Boltzmann eqs., using $\lambda = 1$:





Good Bino–like DM for:

$$m_{\tilde{B}} \simeq 100 \text{ GeV} \cdot \begin{cases} \frac{1.5\cdot 10^{-4}}{B_\chi} \left(\frac{5\cdot 10^5 \text{ GeV}}{M_\Phi} \right)^{1/2}, & \text{pure nonthermal} \\ \left(\frac{M_\Phi}{5\cdot 10^6 \text{ GeV}} \right)^{3/2} \left(\frac{10^{-13} \text{ GeV}^{-2}}{\langle \sigma v \rangle} \right)^{1/3}, & \text{thermal} \end{cases}$$

First option: $M_\Phi \leq 10^6$ GeV or $\langle \sigma v \rangle$ very small;
Second option: $B_\chi \leq 10^{-5}, \ M_\Phi > 10^6$ GeV: Bino equlibrates at $T \gg T_R$, not at T_R!

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Both assumptions are, strictly speaking, incorrect!

Thermalization

How does an energetic particle, with initial $E \gg T$, thermalize, i.e. turn into $\sim E/T$ particles with energy per particle $\sim T$?

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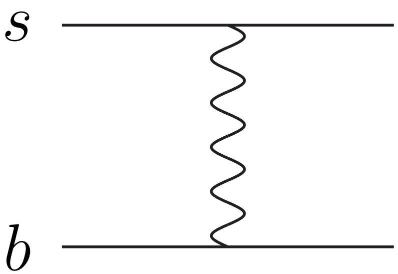
Assume there already is a thermal background, with which the energetic particle can interact!

$2 \to 2$ scattering



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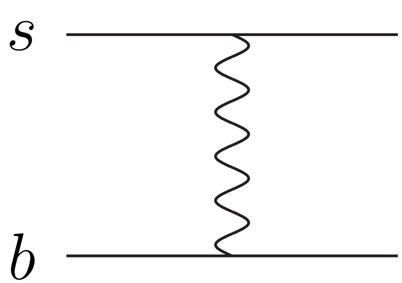

After IR regularizations: $\sigma \sim \frac{\alpha^2}{\alpha T^2} \sim \frac{\alpha}{T^2}$

Typical energy loss per scattering $\Delta E \sim \sqrt{\alpha} T$

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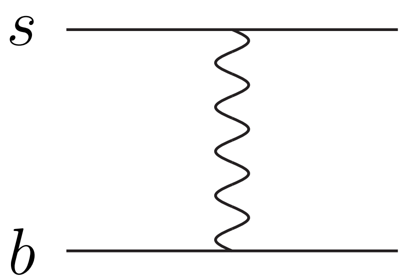

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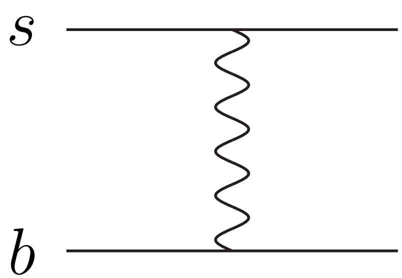

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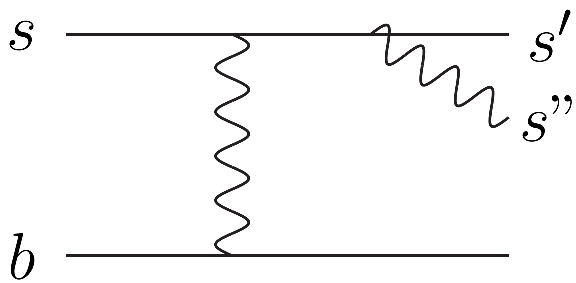
Thermalization time

$$t_{\rm therm} \sim \frac{M_\Phi}{\alpha^{3/2} T^2}$$

$2 \to 3$ scattering



Can have large energy loss, $E_{s'} \sim E_{s''} \sim E_s/2$, without any large virtuality, if emission is colinear!

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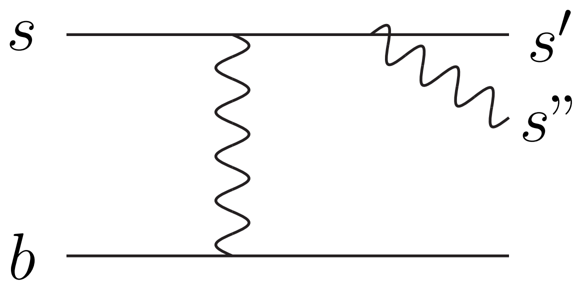

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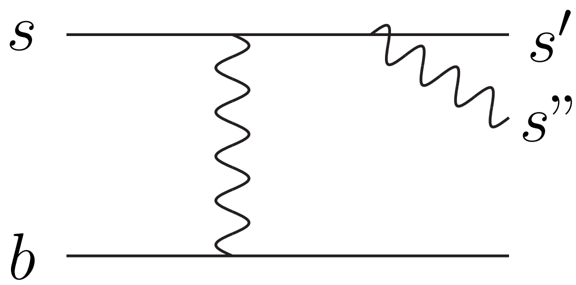
$2 \rightarrow 3$ scattering



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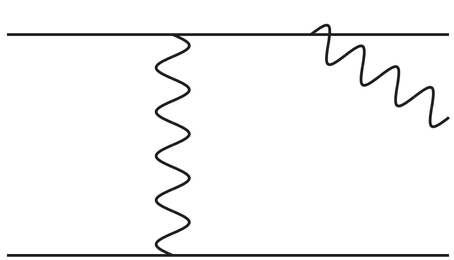
$$\frac{dE_s}{dt} \sim \alpha^2 T \int_0^{E_s/2} E \, dE \, \frac{1}{E} \sim \alpha^2 E_s T$$

$2 \to 3$ scattering



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$t_{\mathrm{therm}} \sim \frac{\ln(M_\Phi/T)}{\alpha^2 T}$: $2 \to 3$ splittings dominate! (Davidson & Sarkar, 2000)

Complication: LPM Effect

Landau & Pomeranchuk 1953; Migdal 1956: for QED; Harigaya et al. 2013: in present context

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- Particle s after scatter still nearly on–shell, for colinear emission
- Lives a "long time"
- Will undergo multiple scatters: destructive interference
- For splitting $s(p) \to s'(k) s"(p-k)$:
  rate suppressed by $\sqrt{\frac{T}{\min(k, p-k)}}$

$$\Longrightarrow t_{\rm therm} \sim \frac{\sqrt{M_\Phi}}{\alpha^2 T^{3/2}}$$

Still much faster than $2 \to 2$ scattering!

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Gives rise to spectrum of non–thermal particles with

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Can be source of non–thermal relics, through scattering on the thermal background ("hard–soft") or between two non–thermal particles ("hard–hard") R. Allahverdi & MD, hep–ph/0205246

Boltzmann equation

MD, B. Najjari, 2105.01935

Let $\tilde{n}(p) = dn/dp$:

$$\frac{\partial \tilde{n}}{\partial t} - 3Hp\frac{\partial \tilde{n}}{\partial p} = \mathcal{C}_{\rm inj} - \mathcal{C}_{\rm dep}$$

$\mathcal{C}_{\rm inj}$: From Φ decay, and feed–down from $k > p$;
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$t_{\rm therm} \ll 1/H \Longrightarrow$ set $H = 0$;
quickly reach quasi steady–state, where injection and depletion balance! (Depends on T)

Boltzmann equation (cont'd)

$$
\begin{aligned}
2n_\Phi \Gamma_\Phi \delta(p - \frac{M_\Phi}{2}) \quad &+ \quad \int_{p+\kappa T}^{M_\Phi/2} \tilde{n}(k) \frac{d\Gamma^{\text{split}}(k \to p)}{dp} dk \\
&= \quad \int_{\kappa T}^{p/2} \tilde{n}(p) \frac{d\Gamma^{\text{split}}(p \to k)}{dk} dk \,.
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κ: $\mathcal{O}(1)$ IR regulator, does *not* affect result for $p \gg T,\ M_\phi/2 - p \gg T$.

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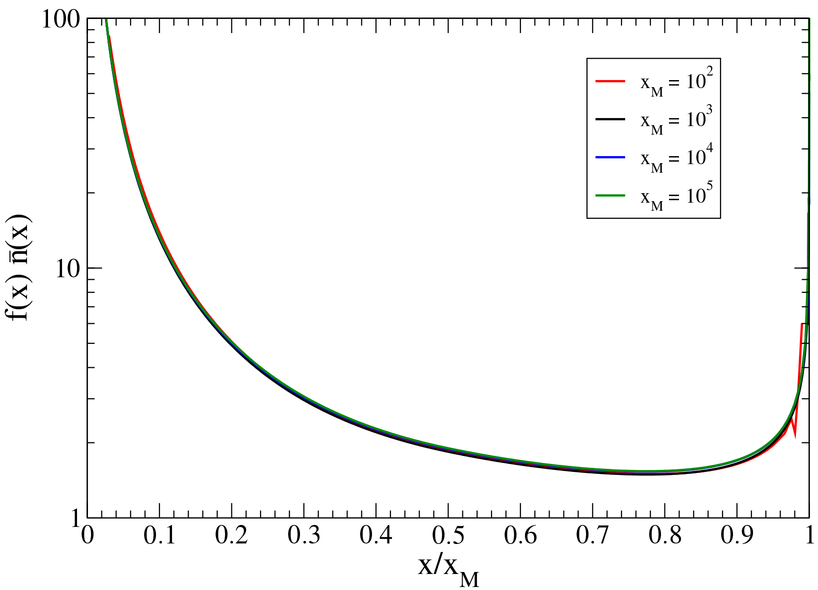
Switch to dimensionless quantities:
$x = p/T, \ x_M = M_\Phi/(2T)$, $\tilde{n}(x) = T\tilde{n}(p = xT)$

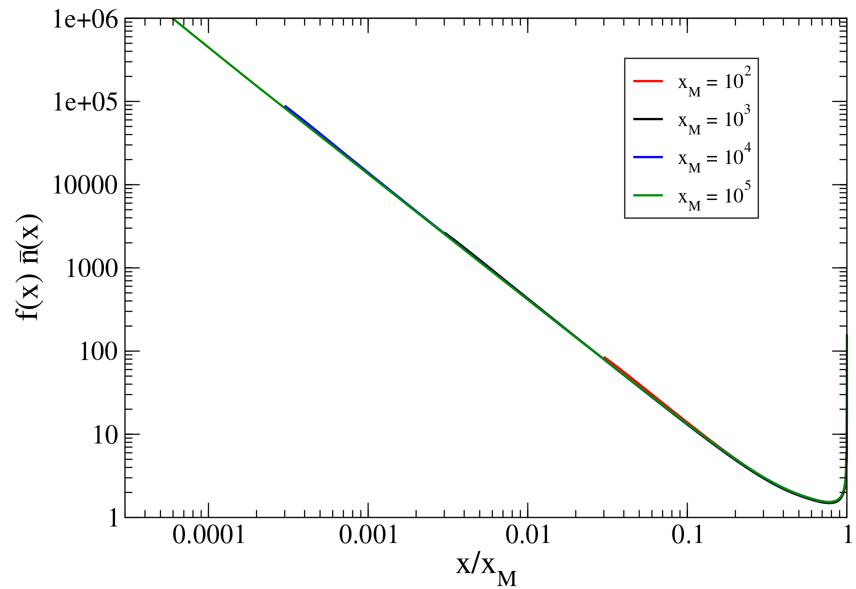
Normalize to $N_M = \frac{2n_\Phi \Gamma_\Phi}{\Gamma^{\text{split}}(M_\Phi/2)}$
$\bar{n}(x) = \tilde{n}(x)/N_M$ is independent of $n_\Phi \Gamma_\Phi$!

Results

For single species cascade (e.g. pure glue):

$\bar{n}(x) \simeq g(x/x_M)/\sqrt{x_M} + \delta(x - x_M)$





Effect of LPM Suppression

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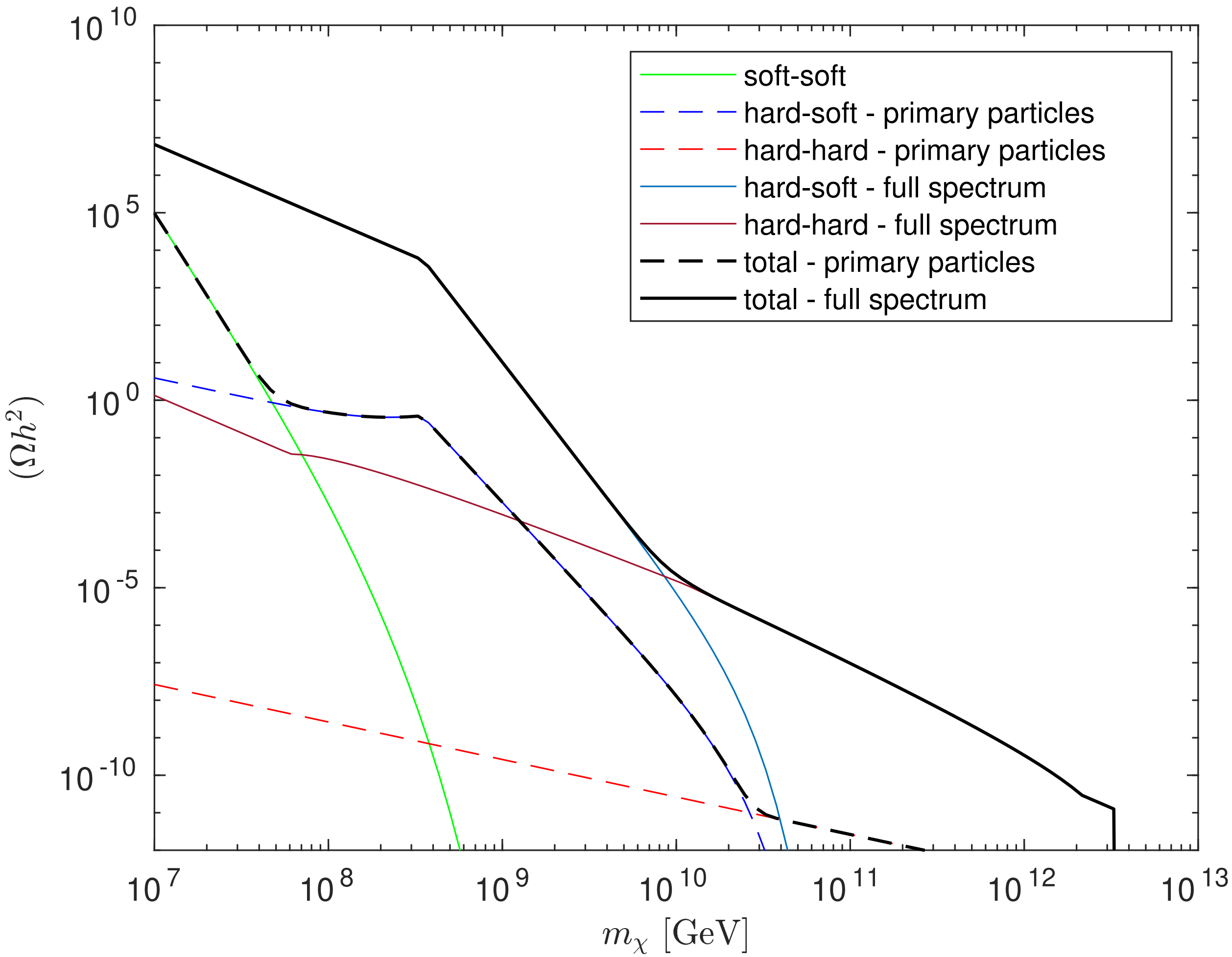
- Depends on energy
  $\Longrightarrow$ changes the shape of the spectrum
- Reduces the thermalization rate
  $\Longrightarrow$ increases normalization of spectrum of non–thermal particles!

Impact on Production of Relics

For $M_\Phi = 10^{13}$ GeV, $\alpha_\chi = 0.01,\ \alpha = 0.05,\ T_R = 10^5$ GeV:



Refinements

MD, B. Najjari, 2205.07741

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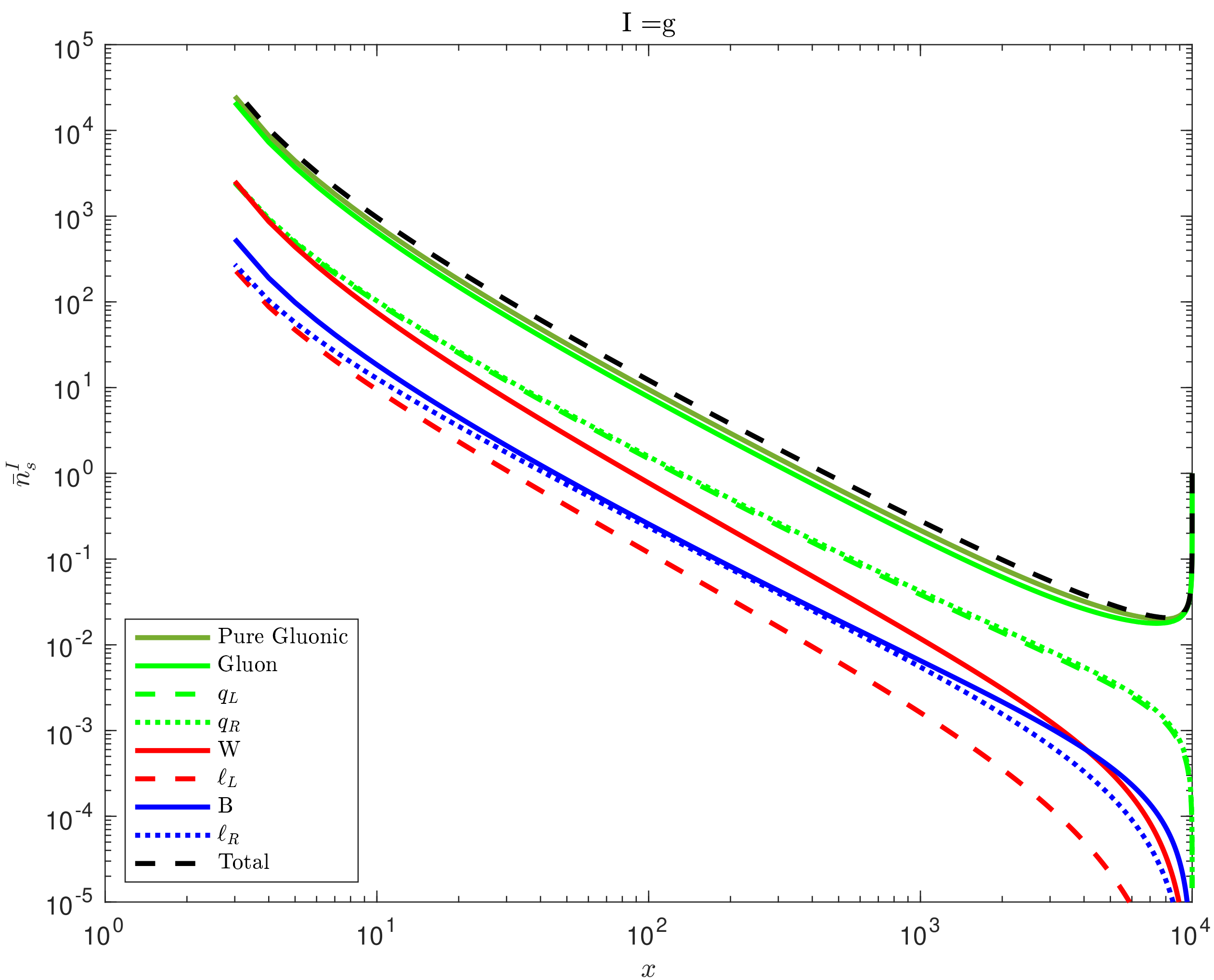
There will be non–thermal χ production whenever $\langle \sigma v \rangle \neq 0$!
E.g $\Phi \to gg$ only, but your relic couples only to ℓ_R:
Need $g \to q \to B \to \ell_R$ splitting cascade!

$x_M = 10^4$, $\Phi \to gg$ only



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  - Proper treatment of $g_*(T)$, hence $T(t)$, $s_R(T)$: done.
  - Spectrum of non–thermal particles: (done)
- Still missing: showering of primary decay products (software exists); Higher–order Φ decays: generally exist if $\langle \sigma v \rangle \neq 0$ R. Allahverdi, MD, hep-ph/0203118