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Gauge Group Topology and Higher-Form Structures in Consistent Quantum Gravity

Mirjam Cvetič

UNIVERSITY of PENNSYLVANIA

CAMTP University of Maribor

CERN

Univerza v Ljubljani
Fakulteta za matematiko in fiziko

Motivation

- Quantum field theories coupled to consistent quantum gravity should be subject to additional constraints beyond standard QFT consistency ones → Swampland Program

[Vafa '06]

- Globally consistent compactifications of String Theory → automatically include quantum gravity & constraints emerge due to geometry of compactified space → Does String Theory realize all consistent theories of quantum gravity [String Universality]?

- Focus on finding physical conditions, reflecting geometric constraints of consistent quantum gravity (without reference to String Theory)

Long history: [...Kumar, Taylor '09; Adams, DeWolfe, Taylor '
García-Etxebarria, Hayashi, Ohmori, Tachikawa, Yonekura '
Kim, Tarazi, Vafa '19; M.C., Dierigl, Lin, Zhang '20; Mon
Hamada, Vafa '21; Tarazi, Vafa '21;...]

Highlight

- Gauge symmetry topology for N =1 Supergravity in 8D → gauging of one-form symmetries

- Top-down classification via string junctions → all 8D (& 9D) N=1 string vacua

Guiding principles

- Geometry: primarily F-theory compactification
- Physics: global symmetries, including higher-form ones, gauged or broken in consistent quantum gravity [No Global Symmetry Hypothesis]

…[Harlow, Ooguri ’

Based on

- Gauge symmetry topology constraints in 8D

- M.C., M.Dierigl, L.Lin and H.Y.Zhang,
  ``String Universality and Non-Simply-Connected Gauge Gr
in 8d,''
  PRL, arXiv:2008.10605 [hep-th];

- ``Higher-form Symmetries and Their Anomalies in M-/F-theor
  Duality,''
  PRD, arXiv:2106.07654 [hep-th] - 8D/7D & 6D/5D

- ``Gauge group topology of 8D Chaudhuri-Hockney-Lykken
  vacua,''
  PRD, arXiv:2107.04031 [hep-th];

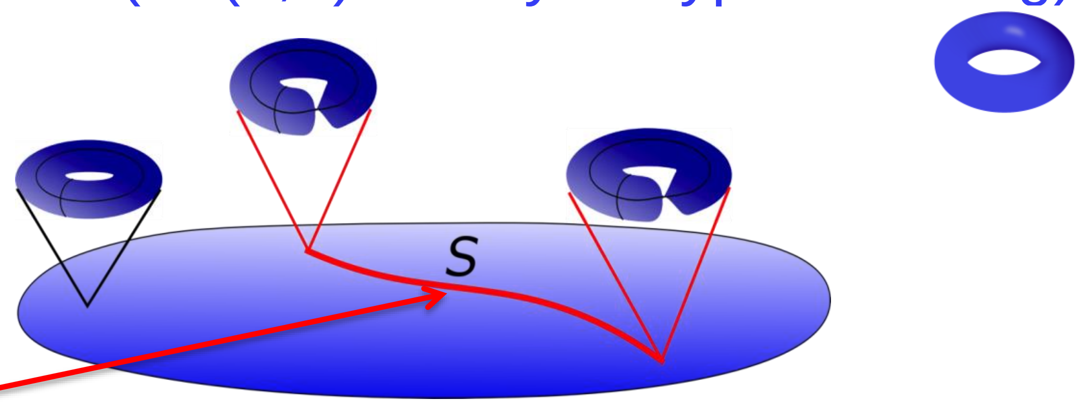
- ``One Loop to Rule Them All: Eight and Nine Dimensional St
  Vacua
  from Junctions,'' arXiv:2203.03644 [hep-th]- String juncti

Digression: [Vafa'96;Morrison,Vafa'96],…review [Weiga

Key features of F-theory compactification

- F-theory, a powerful framework that geometrizes τ =axio-dilaton as a modular parameter of T^2 (SL(2,Z) duality of Type IIB string)

- Compactification on singular, elliptically fibered Calabi-Yau fewfolds



- 7-brane non-Abelian gauge symmetries G, encoded in types of singular T^2 fibration (ADE singularities)

- T^2 (elliptic curve) carries arithmetic structure: Mordell-Weil group of rational points → U(1)'s [Morrison,Park'12; M.C.,Klevers,Piragua'13; Borchmann, Mayrhofer,Palti,Weigand'13;…]
  torsional points → gauge group topology Z→ G/Z

F-theory compactification on elliptically fibered Calabi-Yau fourfolds led, for specific elliptic fibration to D=4 N=1effective theory with

[M.C., Klevers, Peña, Oehlmann, Reut

Standard Model gauge group $\frac{SU(3) \text{ x } SU(2) \text{ x } U(1)}{Z_6}$

with gauge group topology
(geometric - encoded in Shioda Map of MW) [M.C.,Lin '17]

toric geometry techniques
(toric bases B_3)

[M.C., Halverson, Lin, Liu, Tian PRL]

Quadrillion Standard Models (QSMs)
with 3-chiral families & gauge coupling unification

[gauge divisors – in class of *anti-canonical divisor K]*

Current efforts: determination the exact matter spectra
(including # of Higgs pairs) [Bies, M.C., Donagi,(Liu), Ong

Bies, M.C., Donagi, Ong 2205.0

Matter spectra specified by root bundles ($K^{\{\text{frac no}\}}|_{\text{curve}}$) on matter curves:

Identified $O(10^{11})$ F-theory QSM geometries without vector-like matter exotics in the representations of Q_L, q_R, e_R by studying [Caporaso,Casagrande, Cornalb

limit root bundles on nodal matter curves (deformed matter curves)

- Develop algorithm to determine h^0 for all limit root bundles (w/ chirality: $\chi = h^0 - h^1 = 3$)

- For Δ_4 polytope (10^{11} triangulations) 99.995% of root-bundles exactly $h^0 = 3$ → no vector-like exotics
- Statistical analysis for other polytopes → w/ $h^0 = 3$ by far most prevalent

→ Study of Higgs nodal curves [Bies, M.C., Liu, work in pr

Back to the main topic:

I. Gauge group topology in 8D N=1 SG

a) Geometry - String compactification

- G versus G/Z w/ $Z \subset Z(G)$-center
- For simplicity: $G = SU(n_1) \times SU(n_2) \times \ldots$
  w/ $Z(G) = \mathbb{Z}_{n1} \times \mathbb{Z}_{n2} \times \ldots$
- Subgroup Z w/ generators represented as
  $(k_1, k_2, \ldots) \in \Pi_i \mathbb{Z}_{ni}$

- In F-theory compactification Z encoded in the geometry (Mordell-Weil torsion)
  F-theory on elliptically fibered $K3$ → 8D N=1 SG
  → Arithmetic constraint [Miranda, Persson'89]:

$$\boxed{\Sigma_i k^2_i (n_i - 1)/(2n_i) \in \mathbb{Z}}$$

b) Physics - constraints on higher-form symmetries

- *G/Z* has gauged 1-form symmetry [Gaiotto,Kapustin,Seiberg,Willet’14]

G/Z requires no obstruction to gauging the global 1-form!

- For *G*, instanton density fractional [’t Hooft ’81]

w/ 1-form symmetry background C_2 ($F/2\pi \to F/2\pi + C_2$):

$\mathrm{Tr}(F^2)/8\pi^2 \equiv \alpha_G \mathcal{P}(C_2)$ $mod\ \mathbb{Z}$ (``$\mathcal{P}(C_2)$-Pontryagin square”)

w/ α_G fractional, e.g., $\alpha_{SU(n)} = (n-1)/(2n)$

- In 8D N=1 SG, instanton density couples to a tensor in the gravity multiplet, B_4, w/ $B_4 \to B_4 + b_4$ - U(1) large gauge symmetry:

$\mathcal{L} \supset B_4 \wedge \mathrm{Tr}(F^2)/8\pi^2$

- Shown [M.C., Dierigl, Lin, Zhang ’20]: fractional instantons lead [Awada,Townsend ’85] to mixed anomaly between global 1-form Z and gauge U(1):

$\mathcal{A} = \Sigma_i\, k_i^2\, \alpha_{Gi}$

Physics: No anomaly ⟺ Geometry: [Miranda-Persson] constraint

$\Sigma_i\, k^2_i\, (n_i - 1)/(2n_i)$ $mod\ \mathbb{Z}$ $\in \mathbb{Z}$

Classification of allowed gauge groups in 8D N=1 SG

- Anomalies of non-*SU* groups is integer sums of *SU* subgroups

  [Cordova, Freed, Lam, Seiberg '19]

- Solutions to $\Sigma_i k^2_i (n_i - 1)/(2n_i) \in \mathbb{Z}$, subject to $\Sigma_i (n_i - 1) = 18$

[Montero, Vafa '20]

limited. E.g., $G/\mathbb{Z}_\ell$ w/ $\ell > 8$ no anomaly-free solution;

unique solutions $\ell = 7$: $SU(7)^3/\mathbb{Z}_7$; $\ell = 8$: $[SU(8)^2 \times SU(4) \times SU(2)]/\mathbb{Z}_8$

- Also predictions for rank 10 and 2 theories.

  Confirmed in compactifications of CHL string (rank 10)

  [M.C., Dierigl, Lin, Zha

- Independently quantified by advancing string junction techniques including rank 2 [M.C., Dierigl, Lin, Zhang '2

'22]

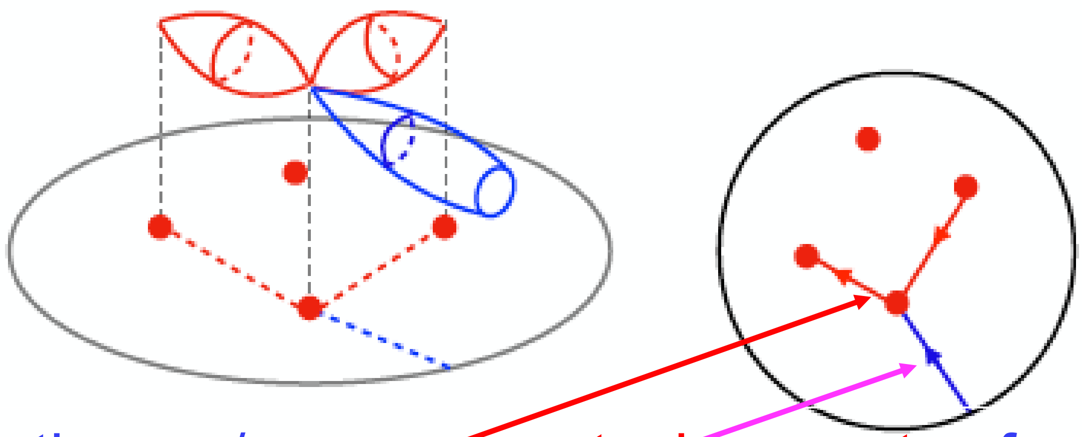
→ Long digression:

[M.C., Dierigl, Lin, Zhang 2203.

String Junctions & All Gauge Groups in 8D String Theory

String junctions between (p,q) 7-branes ⬌ geometry of 2-cycles

[Gabe ... 98]



in perpendicular
2d space

String junctions w/ prongs on stack ⇔ roots of gauge algebra lattice

String junctions w/ external (asymptotic) prongs ⇔ weights

[Magnetic ``junctions'' → 5-branes wrapping the same 2-cycles;
realizes ADE gauge algebras w/ weights = co-weights]

$$\frac{\text{(co-)weights}}{\text{(co-)roots}} \Longleftrightarrow \frac{\text{non-compact}}{\text{compact 2-cycles}} = Z(G)\ !$$

(magnetic) electric higher-form symmetries

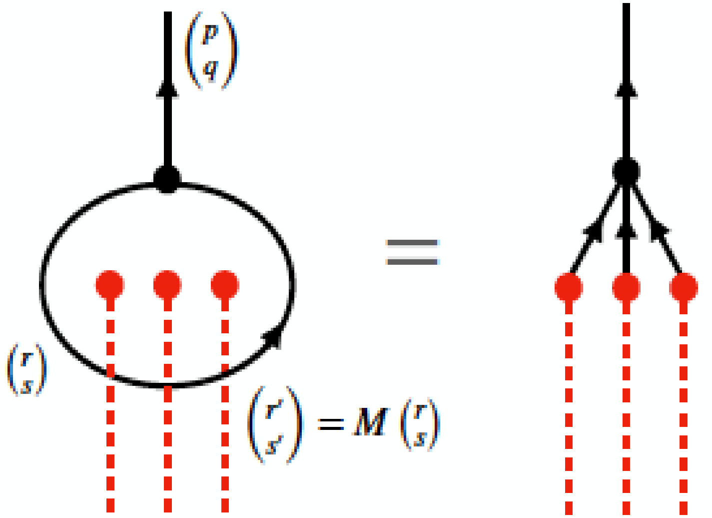
[Morrison,Schäfer-NamekiWillett '20,
Albertini,Del Zotto,García-Etxebarria,Hos

From local (non-compact) gauge group topology

Non-root junctions carry non-zero asymptotic (p,q)-charge

$$\mathbf{j} = \lambda_i \boldsymbol{\alpha}_i + \boldsymbol{\omega}_{(p,q)} \ (\lambda_i \in \mathbb{Q})$$

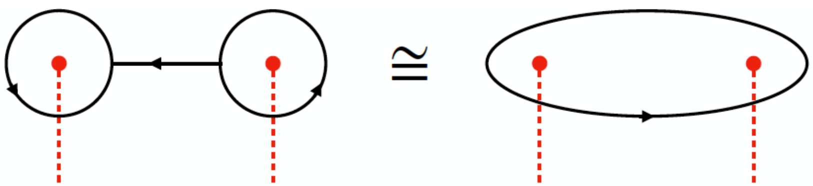
"Fractionality" of $\lambda_i \boldsymbol{\alpha}_i \equiv \mathbf{w}$ encodes charge under *Z(G)* → equivalently captured by *extended weights* $\boldsymbol{\omega}_{(p,q)}$ which *are fractional loop junctions*.



…to global compactification & gauge group topology there

→ no net asymptotic (p,q) charge

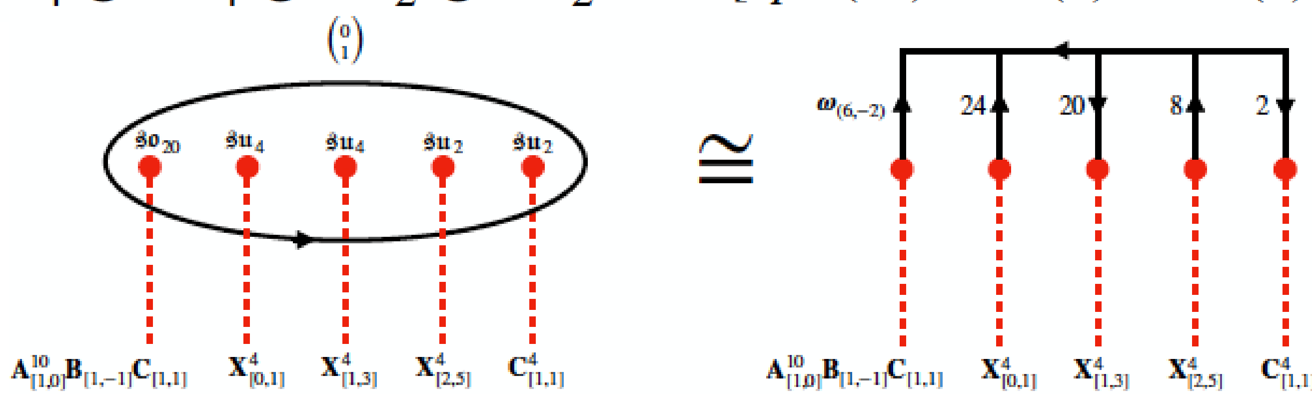
→ restricts allowed junctions in “gluing” local patches
encoded in fractional null junctions of 5-branes (encode *Z*)



alnik ’01]

All rank 18 vacua → Example:

$$\mathfrak{g} = \mathfrak{so}_{20} \oplus \mathfrak{su}_4 \oplus \mathfrak{su}_4 \oplus \mathfrak{su}_2 \oplus \mathfrak{su}_2 \implies [Spin(20) \times SU(4)^2 \times SU(2)^2]/(\mathbb{Z}_2 \times \mathbb{Z}_2)$$



Also for all examples with U(1)’s

Junctions on $O7^+$

- $O7^+$ does not split into (p,q) 7-branes at finite g_s (unlike $O7^-$)
- Same monodromy as $\mathfrak{so}_{16}$ - stack, but w/ "non-trivial flux" that "freezes" singularity in M-/F-theory

[Witten '97, de Boer et al '01, Tachikawa '15]

- Freezing - local: "replacing" one stack [two stacks] with $O7^+$ yields theories of rank 10 [rank 2]

[Hamada, Vafa '21]

- Strings ending on $O7^+$ must have even p and q charges

[Imamura '99, Bergman,Gimon,Sugimoto '01]

[5-brane prongs of any integer (p,q)]

Derived, if configs. with one $O7^+$ are dual to CHL vacua

Analogous constructions w/global topology

w/ one $O7^+$ → all rank 10 vacua

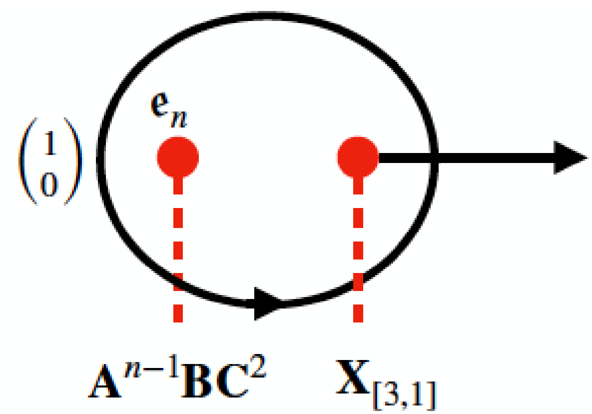
w/ two $O7^+$ → all rank 2 vacua - first construction

Junctions in 9D uplifts: sharpens swampland distance conjecture

- Suitable infinite distance limits of F-theory in *K3* moduli space describe 9D N=1 theories of rank 17

[Lee, Lerche, Weigand '21]

- Junctions characterized by appearance of singularities associated with affine algebras $\hat{e}_n$:



Two series:

$$\mathfrak{g}_{8d,\infty} = \mathfrak{su}_{18-m-n} \oplus \hat{\mathfrak{e}}_m \oplus \hat{\mathfrak{e}}_n \;\Rightarrow\; \mathfrak{g}_{9d} = \mathfrak{su}_{18-m-n} \oplus \mathfrak{e}_m \oplus \mathfrak{e}_n\,,$$

$$\mathfrak{g}_{8d,\infty} = \mathfrak{so}_{34-2k} \oplus \hat{\mathfrak{e}}_k \;\Rightarrow\; \mathfrak{g}_{9d} = \mathfrak{so}_{34-2k} \oplus \mathfrak{e}_k\,.$$

[Maximal non-Abelian enhancement in D=9 heterotic vacua

9D uplifts with one $O7^+$ → rank 9

- Start with configurations:

$$\mathfrak{g}_{8d,\infty} = \mathfrak{so}_{34-2k} \oplus \hat{\mathfrak{e}}_k \;\Rightarrow\; \mathfrak{g}_{9d} = \mathfrak{so}_{34-2k} \oplus \mathfrak{e}_k$$

&"freeze" $\mathfrak{so}_{16}$, contained in $\mathfrak{so}_{16+2n}$ or $\hat{e}_8$→freezing the latter

- Maximal enhancements $\mathfrak{su}_{10-n} \oplus \mathfrak{e}_n$ or $\mathfrak{so}_{18}$

[CHL: [Mikhailov '98, (Font), Fraiman, (Grana), Parra de Freitas '21]]

9D uplifts with two $O7^+$ → rank 1

- Freezing' of two $\hat{e}_8$: $\mathfrak{g}_{8d,\infty} = \mathfrak{su}_2 \oplus \hat{\mathfrak{e}}_8 \oplus \hat{\mathfrak{e}}_8 \;\Rightarrow\; G_{9d} = SU(2)$

  (dual to M-theory on Klein-bottle)

- 9D, rank 1 has two disconnected moduli branches

[Aharony,Komargodski,Patir '07]

- Shown to be connected through D=8

All 9D string vacua are "emergent" from 8D ones!

Role of 1-form symmetry & Mixed 1-form - gauge anomalies in D≤8

- 8D [Font, Graña ,Fraiman, Freitas ’21] – heterotic
  [M.C., Dierigl, Lin, Zhang ’21, ’22] – string junctions
- 7D [M.C., Dierigl, Lin, Zhang ’21] – F/M-theory duality (torsional boundary G_4)
- 6D [Apruzzi, Dierigl, Lin ’20] – excitations of BPS strings
- 5D [M.C., Dierigl, Lin, Zhang ’21] – F/M-theory duality (torsional boundary G_4)
  [Apruzzi, Bonetti, García-Etxebarria, Hosseini, Schäfer-Nameki ’22]…
- Mixed higher-form - gauge anomalies have important implications also for 6D and 5D SCFTs

Summary

- Physics:

  Employing higher-form symmetries to formulate anomaly condition for gauge group topology

  Gauged 1-form symmetry in 8D

⇕ perfect agreement

- Geometry:

  F-theory/Heterotic string/CHL/string junctions

  Full 8D string theory landscape

Future Directions

- Focused on 8D N=1 and role of 1-form gauge symmetry

- Higher-group structures in D≤6
  0-form & 1-form symmetries → 2-group structures

  – Within SCFT's → geometric origin of higher group structures
  [M. C., Heckman, Hübner , Torres '22]
  [Del Zotto, Etxebarria, Schafer-Nameki '22]

  - Their role in in quantum gravity - string theory on compact spaces
  [M. C., Heckman, Hübner, E. Torres to appear]

Thank you!

Announcement

Geometry and Strings 2023

at UPenn

Date to be fixed