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CAUSALITY, NONLINEAR SUSY AND INFLATION

Quentin Bonnefoy (DESY-TH), Gabriele Casagrande (CPHT-Ecole Polytechnique), E.D., [arXiv:2206.13451 [hep-th]]

and also

E.D., M.A.G.Garcia, Y.Mambrini, K.A.Olive, M.Peloso and S.Verner,
Phys. Rev. **D103** (2021), 123519 [arXiv:2104.03749 [hep-th]]

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Outline

- 1) Spin 3/2, potential problems
- 2) Gravitino sound speed in supergravity
- 3) Causality and positivity bounds
- 4) Alternative minimal models of inflation
- 5) Perspectives

1) Spin 3/2, potential problems

SUGRA = SUSY + Gravity

It contains :

- gravity multiplet: Graviton $g_{\mu\nu}$, gravitino ψ_μ
- « matter » fields: (complex) Scalars , Weyl Fermions
chiral superfields

 Φ_i
 ϕ_i
 ψ_i

+ gauge multiplets, etc

Rarita-Schwinger,
spin 3/2



- In supergravity, the gravitino Ψ_μ becomes **massive** by absorbing the **goldstino** G

$$\Psi_\mu \begin{pmatrix} 3/2 \\ - \\ - \\ -3/2 \end{pmatrix} + G \begin{pmatrix} - \\ 1/2 \\ -1/2 \\ - \end{pmatrix} = \Psi_\mu \begin{pmatrix} 3/2 \\ 1/2 \\ -1/2 \\ -3/2 \end{pmatrix}$$

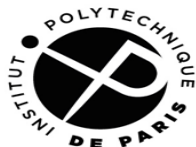


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The consistency of low-energy actions for the spin 3/2 **Rarita-Schwinger field** has a long history :

- 1941: Rarita-Schwinger action
- 1969: Velo-Zwanziger pointed out **potential acausal propagation** for a **charged gravitino** in an e.m. background
- 1977: Deser-Zumino proved that gravitino propagation in minimal supergravity is causal
- 2001: Deser-Waldron proved that gravitino propagation in gauged supergravities is causal
-
- 2021 – **Gravitino swampland conjecture**, gravitino mass conjecture (talk D.Lust)



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History of the subject strongly suggest that **usual supergravities** have no problems with gravitino propagation.

SUSY (linearly realized): nb. bosons = nb. fermions

SUGRA: SUSY is a gauge symmetry, contains gravity

Nonlinear SUSY/SUGRA: nb. bosons \neq nb. fermions

Inflation models in standard SUGRA's have at least one complex scalar field (often several).

Recently, **simple nonlinear** SUSY/SUGRA models were constructed. **More minimal** inflationary models, fewer fields.
(Antoniadis, E.D., Ferrara & Sagnotti; Kallosh, Linde & coll, 2014-)

Even possible to construct **minimal models** with only:
graviton, massive gravitino and inflaton (real scalar)



Simplest nonlinear SUSY's: constrained superfields (see also talk F.

Quevedo).

Example:

- Volkov-Akulov action can be constructed in superspace (Rocek,78) introducing a **constrained, nilpotent** superfield

$$S^2 = 0$$

whose solution is

no fundamental scalar

Superspace fermionic coordinate

$$S = \frac{GG}{2F_S} + \sqrt{2}\theta G + \theta^2 F_S$$

The full VA action is

auxiliary field

$$\mathcal{L}_{VA} = [S\bar{S}]_D + [fS + h.c.]_F$$

2) Gravitino sound speed in supergravity (SUGRA)

The talk deals with the « **speed of sound** » c_s of gravitino in SUGRA, in inflation and more general time-dependent sols

Normally $0 < c_s \leq 1$

Recently, two **potential problematic behaviours** were discussed:

- $c_s = 0$ at particular points on the inflationary trajectory
 ➔ **Large (catastrophic) production** of gravitinos
 (Hasegawa, Terada et al, 2017; Kolb, Long, McDonough, 2021).
- $c_s > 1$ **acausal behaviour** at particular points on the inflationary trajectory in **specific** SUGRA models

The **sound speed** c_s is defined from the dispersion relation

$$\omega^2 = c_s^2 \mathbf{k}^2 + a^2 m^2$$

The transverse spin 3/2 component in a FRW background has a standard dispersion relation with $c_s = 1$

For the longitudinal component:

$c_s < 1$  **Slow gravitino** (Benakli, Darmé, Oz, 2014)

$c_s > 1$ possible for particular nonlinear SUGRA models with
orthogonal constraint

A general expression for **longitudinal gravitino sound speed** is

$$c_s^2 = \frac{\overset{\substack{\downarrow \text{pressure}}}{p - 3m_{3/2}^2}^2}{\underset{\substack{\uparrow \text{energy density}}}{(\rho + 3m_{3/2}^2)}^2} + \frac{\overset{\substack{\downarrow \text{time-derivative}}}{4\dot{m}_{3/2}^2}}{(\rho + 3m_{3/2}^2)^2}$$

The explicit formula in SUGRA is

$$c_s^2 = 1 - \frac{4}{(|\dot{\varphi}|^2 + |F|^2)^2} \left\{ |\dot{\varphi}|^2 |F|^2 - |\dot{\varphi} \cdot F^*|^2 \right\}$$

where $F^i \equiv e^{K/2} K^{ij*} D_{j*} W^*$ in standard SUGRA ,

$$D_i W \equiv \frac{\partial W}{\partial \varphi^i} + \frac{\partial K}{\partial \varphi^i} W$$

and we used the compact notation $|\dot{\varphi}|^2 = \dot{\varphi}^i K_{ij*} \dot{\varphi}^{j*}$,etc

Obs: Cauchy-Schwarz inequality \longrightarrow causality $c_s \leq 1$
 respected in **standard SUGRA's**



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For the (large) majority of SUGRA models we investigated ,
we found **no problems** : $0 < c_s^i \leq 1$

The only models with problems we found is with the
« **orthogonal constraint** » for the inflaton multiplet Φ

$$S(\Phi - \bar{\Phi}) = 0 \quad \longrightarrow$$

Only $Re \phi$ =inflaton is a **dynamical** degree of freedom.

$Im \phi$, **the inflatino** ψ_ϕ and the auxiliary field F_ϕ
are **determined by the constraint**.

In particular F_ϕ is a bilinear in fermions and **does not appear**
in the scalar potential : $F^\Phi \neq e^{K/2} K^{\Phi\bar{i}} D_{i^*} W^*$



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Consequences:

- There is **no inflatino** \Rightarrow the gravitino sound speed problem $c_s = 0$ **can arise** (model-dependent)
- The Cauchy-Schwarz argument for $c_s \leq 1$ **not valid**. We found examples with $c_s > 1$!

However, the **UV origin** of the orthogonal constraint is **not clear** (Dall'Agata, E.D., Farakos, 2006; Bonnefoy, Casagrande, E.D)

\Rightarrow **Potential pathological behaviour** reminiscent of the **swampland program** ! (Vafa, Ooguri; talks C.Vafa, A. Faraggi...)



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3) Causality and positivity bounds

(Q.Bonnefoy, G. Casagrande & E.D., [arXiv:2206.13451 [hep-th]])

- The potential acausal behaviour concerns the **longitudinal component** of the gravitino.
- Gravitino **equivalence theorem**: at high-energy, gravitino longitudinal component is described by the **goldstino**, with **enhanced couplings** to matter.



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The **general lagrangian** with orthogonal constraint is

$$K = h(\mathcal{A}) \mathcal{B}^2 + S \bar{S}$$

$$W = f(\Phi) S + g(\Phi)$$

where we defined $\Phi = \mathcal{A} + i\mathcal{B}$

Is the acausality found in SUGRA captured by the low-energy lagrangian of the **goldstino** coupled to matter, in the **decoupling limit** $M_P \rightarrow \infty$?

Yes ! The goldstino lagrangian contains a **higher-derivative operator** of the form

$$\frac{1}{f(\varphi)^2} \left(h(\varphi) - \frac{2g'(\varphi)^2}{f(\varphi)^2} \right) (\bar{G} i \gamma^m \partial^n G) \partial_m \varphi \partial_n \varphi$$

The operator is subject to **positivity constraints** from dispersion relation arguments which enforce

$$\frac{h(\varphi)}{2} f(\varphi)^2 \geq g'(\varphi)^2 \quad \longleftrightarrow \quad c_s \leq 1$$

- The issue arises due to the « elimination » of **the auxiliary field** by the orthogonal constraint, **no simple physical interpretation**.



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- **Obs:** SUGRA/inflation subluminality condition valid throughout the inflationary trajectory, positivity constraints valid only in the **ground state**



SUGRA condition is stronger.

- Causality condition of goldstino propagation in **time-dependent solutions** of the goldstino action is equivalent to the SUGRA constraint.

4) Alternative minimal models of inflation

Orthogonal constraint is « **reducible** » \longleftrightarrow three
« **irreducible** » constraints (dall'Agata, E.D, Farakos, 2016)

$$S\bar{S}(\Phi - \bar{\Phi}) = 0, \quad \text{eliminates a scalar}$$

$$S\bar{S}D_{\alpha}\Phi = 0, \quad \text{eliminates the fermion}$$

$$S\bar{S}D^2\Phi = 0, \quad \text{eliminates the auxiliary field}$$



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Simplest alternative with **no potential acausality** problems:
use only

$$S\bar{S}(\Phi - \bar{\Phi}) = 0$$

$$S\bar{S}D_{\alpha}\Phi = 0$$

(Same) minimal spectrum for inflation :
Graviton, massive gravitino, inflaton

Equivalent alternative (Bonnefoy, Casagrande, E.D):
orthogonal constraint, but **higher-derivative UV action**

Comparison

Orthogonal constraint

vs

Alternative constraint

$$V = e^K \{|D_S W|^2 - 3|W|^2\} \quad V = e^K \{|D_S W|^2 + |D_\Phi W|^2 - 3|W|^2\}$$

Example inflation model

$$K = -\frac{1}{2}(\Phi - \bar{\Phi})^2 + \bar{S}S \quad , \quad W = f(\Phi)S + g(\Phi)$$

$$f = \sqrt{3}g \quad g = M^2 \left(\Phi + \frac{1}{a} e^{-a\Phi} \right) + g_0 \quad \longrightarrow$$

$$V(\varphi) = M^4 [1 - e^{-a\varphi}]^2 \quad \text{is the Starobinsky model}$$

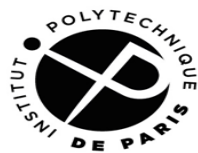


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Conclusions

- Important to check and impose sound speed
 $0 < c_s \leq 1 \longrightarrow$ gravitino swampland conjecture
- Most SUGRA models satisfy it, except **peculiar** models with orthogonal constraint (or similar).
- **Subluminality** constraints captured by goldstino SUSY lagrangians in $M_P \rightarrow \infty$ limit and positivity constraints, but SUGRA condition is **stronger**.
- Alternative minimal inflation models, **no causality issues**
- General interest: **consistency constraints** on nonlinear SUSY/SUGRA, strings with broken SUSY



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THANK YOU !