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Superspace approach to the M5-brane

Dealing with chiral p-forms in superspace

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Overview

We aim to give a (2-derivative) lagrangian description of the low-energy dynamics of a stack of M5-branes, including all fermion contributions.

The final goal would be the formulation of an action principle in superspace

- First step: Lagrangian of n non-interacting tensor multiplets in flat 6D (4,0) superspace $\checkmark$
- Second step: interaction with 6D supergravity in progress
- Third step: adding self-interactions and embedding in 11D superspace ...

Problematic lagrangian description of 2-forms with self-dual field-strengths (chiral 2-forms)

How to implement the self-duality constraint?

Outline

The model

Chiral (2n)-forms in D=(4n+2) space-time dimensions

6D N=(4,0) rigid supersymmetry in the rheonomic approach

Implementing Sen's prescription (at first order) in the rheonomic approach

Conclusions and outlook

The model

M5-brane - Field content

M5-branes, together with M2-branes, are half-BPS extended objects of M-theory, appearing in its low energy 11D supergravity limit as black p-brane sources of the 3-form potential $A^{(3)}$ and of its magnetic dual $B^{(6)}$.

[Duff; Bershoeff-Sezgin-Townsend '87-'88; Duff-Stelle '91; Güven '91; Gibbons-Townsend '93; Townsend '95; Witten '95-'96;...]

The world-volume theory of the M5 brane is described at low energy by the dynamics of a tensor multiplet in chiral D=6, N=(4,0) superspace (16 supercharges):

$$\left(\phi^{[AB]}, B_{\mu\nu}, \lambda_A\right) \,, \qquad A, B, \ldots \in Sp(4) \simeq SO(5)$$

R-symmetry: $Sp(4)$, and $\Gamma_7 \psi_A = -\psi_A$

Counting of on-shell d.o.f.:

- Fermionic d.o.f.: $\Gamma_7 \lambda_A = \lambda_A \qquad \longrightarrow \qquad \# f = 16/2 = 8$
- Bosonic d.o.f.: $\begin{cases} \phi^{[AB]} : & \mathbb{C}_{AB}\phi^{[AB]} = 0 \\ B_{\mu\nu} : & H_{\mu\nu\rho} \equiv \frac{1}{6}\partial_{[\mu}B_{\nu\rho]} = -\frac{1}{6}\epsilon_{\mu\nu\rho\sigma\lambda\tau}\partial^\sigma B_{\lambda\tau} \end{cases} \qquad \longrightarrow \qquad \# b = 5 + 3 = 8$

Self-duality constraint on $H_{\mu\nu\rho}(B)$: $\quad H^{(3)} = H^{(3)}_{-} \quad \Leftrightarrow \quad$ B is a chiral 2-form

Chiral (2n)-forms in D=(4n+2) space-time

A well known problem: In D=4n+2, the kinetic term of chiral (2n)-forms is zero!

Contrary to the D=4n case, in D=4n+2:

- (anti)self-dual (D/2)-forms $Q_{\pm}^{(2n+1)} = \pm \star Q_{\pm}^{(2n+1)}$ are real
- Any D-forms: $P_{\pm}^{(2n+1)} \wedge Q_{\pm}^{(2n+1)} = 0$ for any $P_{\pm}^{(2n+1)}$, $Q_{\pm}^{(2n+1)}$,

  while $P^{(2n+1)} \wedge Q_{\pm}^{(2n+1)} = P_{\mp}^{(2n+1)} \wedge Q_{\pm}^{(2p+1)} \neq 0$

How to write a lagrangian kinetic term for chiral (2n)-forms in D=(4n+2) space-time dimensions?

The corresponding field-strength $H^{(2n+1)} = dB^{((2n)}$ is self-dual: $H^{(2n+1)} = H_{-}^{(2n+1)}$, so that:

$$\mathcal{L}_K = \frac{1}{2} H_{-}^{(2n+1)} \wedge \star H_{-}^{(2n+1)} = -\frac{1}{2} H_{-}^{(2n+1)} \wedge H_{-}^{(2n+1)} = 0$$

One has to deal with the kinetic term of unconstrained (2n)-forms

...but the self-duality constraint is necessary for SUSY (pairing of d.o.f.)... To be imposed by hand?

The same problem also affects chiral theories in 2D, 10D

Various strategies have been found to face the problem at the lagrangian level

- non Lorentz covariant formulations [Henneaux-Teitelboim '89; Schwarz-Sen '94; Perry-Schwarz '96;...]
- infinite number of auxiliary fields [McClain-Wu-Yu '90; Martin-Restuccia '94; Devecchi-Henneaux '96;...]
- non-polynomial actions [Pasti-Sorokin-Tonin '95-'96; Bandos et al. '97]
- rheonomic approach in superspace [D'Auria-Fré-Regge '83; Castellani-Pesando '93;]

These approaches are effective, each one with its advantages and drawbacks, most of them formulated at the bosonic level

Recently: Sen's approach inspired by string field theory, manifestly Lorentz covariant and with finite number of auxiliary fields, revived the interest in the subject since it evaded a possible no-go theorem:

" *if a manifestly Lorentz invariant superstring field theory could be formulated, then it should allow as low energy limit a 2-derivative lagrangian description including IIB supergravity and its chiral 4-forms!*" [Sen '15-'19]

I will discuss how the rheonomic superspace approach can deal with chiral forms (in this case, chiral 2-forms on the M5-brane world-volume), and how it can be modified to incorporate Sen's prescription at low energy

6D N=(4,0) rigid supersymmetry in the rheonomic approach

The rheonomic approach

This is a geometric superspace approach to build supersymmetric theories.

It can be applied also to theories with chiral p-forms: One deals with unconstraind p-form fields off-shell: the self-duality constraint has not to be imposed by hand, it emerges from the field equations of a superspace Lagrangian and, independently, from closure of the superspace Bianchi identities (BI)

It was applied in the past to the construction of theories with chiral p-forms:

- 6D minimal pure supergravity [D'Auria-Fré-Regge '83]
- 10D IIB chiral supergravity theory [Castellani-Pesando '93]

Advantages: no exotic auxiliary fields are needed

Drawbacks: once the superspace lagrangian is restricted to space-time, the self-duality constraint is lost

This is the approach we followed to build the 6D chiral theory

Short summary of the rheonomic approach

It is a superspace generalization of Einstein-Cartan formalism, where supersymmetry transformations are associated with superdiffeomorphisms in the odd directions of superspace.
The full local (super)symmetry is encoded in the formal definition of the super-fieldstrengths and on their *rheonomic parametrization* on a anholonomic basis of superspace, spanned by the 1-forms (V^a, ψ_α^A)

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The construction of a given SUSY theory in D-dimensions is based on two (equivalent) pillars:

- Consistency of the rheonomic parametrizations with the Bianchi "identities" in superspace.
  It gives constraints on the superfields of the theory (relying on Fierz id.s), which include:
  - the dynamical field equations on space-time
  - the self-duality constraint on the chiral p-forms
  - the SUSY transfromation laws of the fields on space-time
  - all the other contstraints required for SUSY

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  - the SUSY transfromation laws of the fields on space-time
  - all the other contstraints required for SUSY
- Construction of a geometric D-form lagrangian in superspace written in terms of differential form fields, and the exterior d operator (no Hodge operator). Its field equations in superspace are k-form equations such that:
  - their components along V^k are the dynamical field equations on space-time
  - their components along $V..\psi..$ give all the contstraint required for SUSY, including the self-duality constraint on the chiral p-forms

  Its restriction to space-time is invariant (up to total derivative) w.r.t. the SUSY transfromation laws

Our results: The super-fieldstrengths (FDA in superspace)

Rigid SUSY background: $\begin{cases} T^a & \equiv & dV^a - \frac{\mathrm{i}}{2}\overline{\psi}_A\Gamma^a\psi^A = 0 \\ \rho^A & \equiv & d\psi^A = 0 \end{cases}$, with: $\begin{cases} V^a & = \mathrm{d}x^a + \frac{\mathrm{i}}{2}\overline{\theta}_A\Gamma^a\mathrm{d}\theta^A, \\ \psi^A & = -\Gamma_7\psi_A = \mathrm{d}\theta^A. \end{cases}$

Matter fieldstrengths: $\begin{cases} H^I & \equiv & \mathrm{d}B^I + \frac{\mathrm{i}}{2}\phi^I_{BC}\mathbb{C}^{AC}\overline{\psi}_A\Gamma_a\psi^B V^a \\ D\lambda^I_A & \equiv & \mathrm{d}\lambda^I_A \\ P^I_{AB} & \equiv & \mathrm{d}\phi^I_{AB} \end{cases}$; $\mathrm{d}^2 = 0 \Rightarrow$ BI: $\begin{cases} \mathrm{d}H^I - \frac{\mathrm{i}}{2}\,\mathrm{d}\phi^I_{AB}\mathbb{C}^{AC}\,\overline{\psi}_C\Gamma_a\psi^B\,V^a = 0 \\ \mathrm{d}^2\lambda^I_A = 0 \\ \mathrm{d}^2\phi^I_{AB} = 0 \end{cases}$

Consistency $\Rightarrow$ Rheonomic parametrizations: $\begin{cases} H^I & = & H^I_{abc}V^aV^bV^c + \frac{1}{8}\mathbb{C}^{AB}\overline{\psi}_A\Gamma_{ab}\lambda^I_B V^aV^b \\ D\lambda^I_A & = & \partial_a\lambda^I_A V^a - 2\mathrm{i}P^I_{AB,a}\Gamma^a\psi^B + \mathrm{i}H^I_{abc}\Gamma^{abc}\psi^B\mathbb{C}_{AB}\,, \\ P^I_{AB} & = & P^I_{AB,a}V^a + \overline{\psi}_{[A}\lambda^I_{B]_0} \end{cases}$

+ field equation $\Gamma^a\partial_a\lambda_A = 0$, together with $H^I_{abc} = -\frac{1}{6}\,\epsilon_{abcdef}\,H^{I|def}$

The tensors H^I_{abc}, $P^I_{AB,a}$, $\partial_a\lambda^I_A$ are the so-called *supercovariant fieldstrengths*.

Our results: The supersymmetry transformation laws

SUSY transf. of a generic field ξ are super-diffeomorphisms along odd directions of superspace
$\Rightarrow$ Lie derivatives with spinor parameter ϵ^A: $\delta_\epsilon \xi = \ell_\epsilon \xi = (\imath_\epsilon d + d\imath_\epsilon)\,\xi$

Using the above relations we get:

$$\begin{cases} \delta_\epsilon B^I &= \dfrac{1}{8}\,\mathbb{C}^{AB}\bar{\epsilon}_A \Gamma_{ab}\lambda^I_B V^a V^b - \mathrm{i}\phi^I_{BC}\mathbb{C}^{AC}\bar{\epsilon}_A\Gamma_a\psi^B V^a \\ \delta_\epsilon \lambda^I_A &= -2\mathrm{i} P^I_{AB,a}\Gamma^a\epsilon^B + \mathrm{i}H^I_{abc}\Gamma^{abc}\epsilon^B\mathbb{C}_{AB} \\ \delta_\epsilon\phi^I_{AB} &= \bar{\epsilon}_{[A}\lambda^I_{B]_0} \end{cases}$$

which, restricted to space-time (at $\theta^A = \mathrm{d}\theta^A = 0$, so that $V^a_\mu = \delta^a_\mu$, $\psi^A_\mu = 0$), reduce to:

$$\begin{cases} \delta_\epsilon B^I_{\mu\nu} &= \dfrac{1}{4}\,\mathbb{C}^{AB}\bar{\epsilon}_A\Gamma_{\mu\nu}\lambda^I_B \\ \delta_\epsilon\lambda^I_A &= -2\mathrm{i}\partial_\mu\phi^I_{AB}\Gamma^\mu\epsilon^B + \dfrac{\mathrm{i}}{2}\partial_\mu B^I_{\nu\rho}\Gamma^{\mu\nu\rho}\epsilon^B\mathbb{C}_{AB} \\ \delta_\epsilon\phi^I_{AB} &= \bar{\epsilon}_{[A}\lambda^I_{B]_0} \end{cases}$$

Our results: The superspace lagrangian

The geometric approach allows to derive the following (6|0)-form Lagrangian in superspace, to be integrated on a generic bosonic submanifold of superspace, to be identified with space-time: $\mathcal{A} = \int_{\mathcal{M}^{(6|0)}} \mathcal{L}^{(6|0)}$.

In order to be independent of the embedding of $\mathcal{M}^{(6|0)} \subset \mathcal{M}^{(6|16)}$, the bosonic kinetic terms have to be written at first order (no Hodge operator). The Lagrangian reads:

$$
\begin{aligned}
\mathcal{L}^{(6|0)} = \alpha_1 \Big\{ & \left(P^I_{AB} - \overline{\psi}_{[A}\lambda^I_{B]_0}\right) \tilde{P}^a_{I,CD} V^{bcdef} \epsilon_{abcdef}\mathbb{C}^{AC}\mathbb{C}^{BD} - \frac{1}{12}\tilde{P}^I_{AB,l}\tilde{P}^l_{I,CD} V^{abcdef}\epsilon_{abcdef}\mathbb{C}^{AC}\mathbb{C}^{BD} + \\
& + 40\left(H^I(B) - \frac{1}{8}\overline{\psi}_A\Gamma_{lm}\lambda^{IA}V^{lm}\right)\tilde{H}^{abc}_I V^{def}\epsilon_{abcdef} - \tilde{H}^I_{lmn}\tilde{H}^{lmn}_I V^{abcdef}\epsilon_{abcdef} + \\
& - \frac{\mathrm{i}}{4}\overline{\lambda}^I_A\Gamma^a\left(D\lambda^A_I V^{bcdef}\epsilon_{abcdef} + \frac{5\mathrm{i}}{2}\lambda_{BI}\overline{\psi}^A\Gamma^{bcd}\psi^B V_{abcd}\right) + \\
& + \frac{5}{2}P^I_{AB}\left(\overline{\lambda}^A_I\Gamma_{ab}\psi^B V_{cdef}\epsilon^{abcdef} + \frac{4\mathrm{i}}{5}\phi^I_{CD}\mathbb{C}^{DA}\overline{\psi}^B\Gamma_{abc}\psi^C V^{abc}\right) + \\
& -30H^I\left(\overline{\lambda}_{IA}\Gamma_{ab}\psi^A V^{ab} + 4\mathrm{i}\phi_{IAB}\overline{\psi}^A\Gamma_a\psi^B V^a\right) - \frac{5}{4}\overline{\lambda}^I_A\Gamma_{abc}\lambda^I_B\overline{\psi}_C\Gamma_d\psi_D V^{abcd}\left(\mathbb{C}^{AB}\mathbb{C}^{CD} + \frac{3}{2}\mathbb{C}^{AD}\mathbb{C}^{BC}\right)\Big\} \,.
\end{aligned}
$$

The fields $\tilde{P}^I_{AB,a}$, $\tilde{H}^I_{abc}$ are auxiliary. Their equations of motion identify them with the supercovariant field-strengths $P^I_{AB,a}$, H^I_{abc} appearing in the superspace rheonomic parametrizations.

The self-duality contraint follows from the ψ^2 component of $\dfrac{\delta\mathcal{L}^{(6|0)}}{\delta B^I}$

... and its space-time restriction

The restriction to space-time ($\psi_\mu^A = 0$) reduces to the free lagrangian:

$$\mathcal{L}^{\text{s.t.}} = \left(\frac{1}{4}\,\partial_\mu \phi^{IAB} \partial^\mu \phi_{I,AB} + \frac{3}{4}\,\partial_{[\mu} B_{\nu\rho]} \partial^{[\mu} B^{\nu\rho]} + \frac{\mathrm{i}}{8} \overline{\lambda}^{IA} \Gamma^\mu\, D_\mu \lambda_{IA} \right)\, \mathrm{d}^6 x$$

- In the restriction to space-time, the self-duality constraint is projected out
  $\Rightarrow \mathcal{L}^{\text{s.t.}}$ depends on unconstrained $B^I_{\mu\nu}$, and the corresponding field equations do not account for the correct matching of d.o.f.

$\mathcal{L}^{\text{s.t.}}$ is invariant off-shell, up to total derivative, under the SUSY transformations written above:

$$\delta_\epsilon \mathcal{L}^{\text{s.t.}} = \partial_\mu\, K^\mu\, \mathrm{d}^6 x\,, \quad \text{with} \quad K^\mu = \frac{1}{4} \overline{\lambda}^{IA} \Gamma^\nu \Gamma^\mu \epsilon^B \partial_\nu \phi_{IAB} - \frac{1}{16} \overline{\lambda}^I_A \Gamma^{\rho\sigma\tau} \Gamma^\mu \epsilon^A \partial_\rho B_{I\sigma\tau}$$

- it is off-shell SUSY invariant, but SUSY doesn't close on the fields
- in $\delta_\epsilon \mathcal{L}^{\text{s.t.}}$, the "wrong" self-duality components of dB^I enter the total derivative term

The associated Noether current, conserved on-shell: $\partial_\mu \mathcal{J}^\mu_A |_{on-shell} = 0$ reads:

$$\mathcal{J}^\mu_A = -\frac{1}{2} \Gamma^\nu \Gamma^\mu \lambda^{IB} \partial_\nu \phi_{ABI} + \frac{1}{8} \Gamma^{\rho\sigma\tau} \Gamma^\mu \lambda^I_A \partial_\rho B_{\sigma\tau I}.$$

Comments on the above results

The rheonomic superspace lagrangian $\mathcal{L}^{(6|0)}$, for the free case considered, is classically well-defined:

- it is manifestly Lorentz and SUSY invariant
- it includes the self-duality constraint among its EL equations in superspace (but not on the space-time restriction of $\mathcal{L}^{(6|0)}$).

Next step on that side: coupling with supergravity multiplet: $(V^a, \psi^A, 5B^{AB}_{\mu\nu(+)})$ ...under construction...

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Towards the formulation of an action principle, we would like however to modify our superspace Lagrangian into a classically equivalent one, but which would implement Sen's prescription in superspace, and which would include the self-duality constraint also once restricted to space-time.

Implementing Sen’s prescription (at first order) in the rheonomic approach

Sen's prescription

In '15, Sen formulated a prescription to implement the self-duality constraint at the level of the space-time action preserving covariance, at the price of introducing an extra chiral form, with wrong-sign kinetic term but fully decoupled from the physical sector, and an auxiliary self-dual form $Q^{(2n+1)} = Q_-^{(2n+1)}$.

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We aim at extending Sen’s prescription such as to be applied to our geometric superspace lagrangian, starting with the simpler (non interacting) chiral model in rigid 6D superspace.

...and application to our 1st order lagrangian

Complication of our setting: we have a first order lagrangian (no-Hodge operator!)

$\Rightarrow$ two ways, not to loose covariance and SUSY invariance:

- To pass to a second order formulation in superspace, and apply Sen's prescription there in progress
  $\Rightarrow$ This requires extending $\mathcal{L}^{(6|0)} \to \mathcal{L}^{(6|16)}$ through coupling with appropriate integral forms [Castellani-Catenacci-Grassi '14, Cremonini-Grassi '21]

✓ To reformulate Sen's prescription such that it holds at first order, and then to modify our superspace lagrangian $\mathcal{L}^{(6|0)}$ such that:

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  - once restricted to space-time, it is equivalent to Sen's prescription
  - its EL equations in superspace for the physical fields are the same as those from $\mathcal{L}^{(6|0)}$

Short summary of Sen's prescription, in this simplified model (no gravity)

[Sen '15, '19; Lambert '19] Given a theory including chiral n-forms B^I and extra fields Φ, its space-time lagrangian reads (for 10D IIB theory, n=4, and $Y = \tilde{B}^{(2)} \wedge C^{(3)}$, and $\hat{F}^{(5)} \equiv dB^I + Y^I = \hat{F}_-^{(5)}$):

$$\mathcal{L} = \frac{1}{2}\left(dB^I + Y^I(\Phi)\right) \wedge {}^*\left(dB^I + Y^I(\Phi)\right) + Y^I(\Phi) \wedge (dB + Y(\Phi)) + \mathcal{L}_{int}(\Phi) \equiv \mathcal{L}_1 + \mathcal{L}_{int}(\Phi)$$

Prescription:

- replace: $\left(dB^I + Y^I\right) \to {}^*\left(dP^I + Y^I\right)$
- add interaction with auxiliary self-dual $Q_-^I = -{}^*Q_-^I$ (algebraic constraint): ${}^*\left(dP^I + Y^I\right) \wedge Q_-$:

$$\mathcal{L}_1 \to \mathcal{L}_1' = -\frac{1}{2}\left(dP^I + Y^I(\Phi)\right) \wedge *\left(dP^I + Y^I(\Phi)\right) + (Q_-^I + Y^I(\Phi)) \wedge \left(dP^I + Y^I(\Phi)\right)$$

The field equations read (where $\mathbb{P}_\pm(\xi) \equiv \frac{1}{2}(\xi \pm *\xi)$): $\begin{cases} \dfrac{\delta \mathcal{L}_1'}{\delta Q_-^I} = 0 \quad \Rightarrow \quad \mathbb{P}_+(\mathrm{d}P^I + Y^I) = 0\,, \\ \dfrac{\delta \mathcal{L}_1'}{\delta P^I} = 0 \quad \Rightarrow \quad \mathrm{d}\left({}^*\mathrm{d}P^I + Q_-^I\right) = 0 \quad \Rightarrow \quad {}^*\mathrm{d}P^I + Q_-^I = d\Xi \end{cases}$

Writing: $\Xi = P + \tilde{P}$ one sees that $\mathbb{P}_+(d\tilde{P}) = 0 \Rightarrow d\tilde{P} = -{}^*d\tilde{P} \Rightarrow d^*d\tilde{P} = 0$ (free field)

Identification: $P^I = B^I + \tilde{P}^I, \quad \Rightarrow \quad \mathbb{P}_+(dB^I + Y^I) = 0, \quad$ and $\quad \delta\Phi \left.\dfrac{\delta \mathcal{L}'}{\delta \Phi}\right|_{Q(P,\Phi)} = \delta\Phi \dfrac{\delta \mathcal{L}}{\delta \Phi}$

First order formulation of Sen's prescription on space-time

To implement Sen's mechanism at first order, we introduce the following (2n+1)-form auxiliary fields:

$$\tilde{H} \equiv \tilde{H}_{abc}\, V^a \wedge V^b \wedge V^c = \tilde{H}_+ + \tilde{H}_- \,, \quad \hat{Q}^l_- = \hat{Q}^l_{-\,abc}\, V^a \wedge V^b \wedge V^c = -{}^*\hat{Q}^l_-$$

and modify the 1st order Lagrangian $(4n+2)$-form in spacetime:

$$\mathcal{L} = (\mathrm{d}B^l + Y^l) \wedge {}^*\tilde{H}_l - \frac{1}{2}\tilde{H}^l \wedge {}^*\tilde{H}_l + \mathrm{d}B^l \wedge Y_l + \mathcal{L}^{\mathrm{st}}_{int}(\Phi)$$

into the following one: $\tilde{\mathcal{L}} = -\left[(\mathrm{d}P_l + Y_l) \wedge \tilde{H} + \left(\tilde{H}^l + Y^l\right) \wedge \hat{Q}_{l-}\right] + \mathcal{L}^{\mathrm{st}}_{int}(\Phi) = \tilde{\mathcal{L}}' + \mathcal{L}^{\mathrm{st}}_{int}(\Phi)$

The field equations read: $\begin{cases} \dfrac{\delta\tilde{\mathcal{L}}'}{\delta\tilde{H}^l} = 0 & \Leftrightarrow \quad \mathrm{d}P^l + Y^l = \hat{Q}^l_- \,, \quad \Rightarrow \quad \mathbb{P}_+(dP^l) = -\mathbb{P}_+(Y^l) \\ \dfrac{\delta\tilde{\mathcal{L}}'}{\delta\hat{Q}^l_-} = 0 & \Leftrightarrow \quad \mathbb{P}_+(\tilde{H}^l) = -\mathbb{P}_+(Y^l)\,, \quad \Rightarrow \quad \mathbb{P}_+(\tilde{H}^l) = \mathbb{P}_+(dP^l) \\ \dfrac{\delta\tilde{\mathcal{L}}'}{\delta P^l} = 0 & \Leftrightarrow \quad \mathrm{d}\left(\tilde{H}^l\right) = 0\,, \quad \Rightarrow \quad \tilde{H}^l = d\Xi^l \end{cases}$

We can define: $B^l \equiv \dfrac{P^l + \Xi^l}{2}\,, \quad \tilde{P} = \dfrac{P^l - \Xi^l}{2}$: B^l are the dynamical interacting fields while $\tilde{P}^l$ are free fields.

Equivalent to Sen's 2d order prescription, with $Q_- = \hat{Q}_- + \mathbb{P}_-(\tilde{H} - Y)$

First order formulation of Sen's prescription in superspace

Our 1st-order superspace lagrangian can be written as:

$$\mathcal{L}^{(6|0)} = (\mathrm{d}B^I + Z^I) \wedge {}^*\tilde{H}_I - \frac{1}{2}\tilde{H}^I \wedge {}^*\tilde{H}_I + \mathrm{d}B^I \wedge Z_I + \mathcal{L}^{(6|0)}_{int}(\Phi)\,,$$

where we have collectively denoted by Φ the scalar and spin-$1/2$ fields, and we have defined:

$$\tilde{H}^I \equiv \tilde{H}^I_{abc}\, V^a \wedge V^b \wedge V^c\,, \quad {}^*\tilde{H} \equiv \frac{1}{6}\epsilon_{abcdef}\tilde{H}^{abc}\, V^d \wedge V^e \wedge V^f\,, \quad \text{(1st order, auxiliary field)}$$

$$Z^I = Z^I(\Phi) \equiv \frac{1}{8}\,\overline{\lambda}^I_A \Gamma_{ab}\psi^A\, V^a \wedge V^b + \frac{\mathrm{i}}{2}\phi^I_{AB}\,\overline{\psi}^A \Gamma_a \psi^B\, V^a\,.$$

It looks very similar to Sen's space-time lagrangian. To have it at 1st order in superspace, we write:

$$\tilde{\mathcal{L}} = -\left[(\mathrm{d}P^I + Z^I) \wedge \tilde{\mathbf{H}}_I + \left(\tilde{\mathbf{H}}^I + Z^I\right) \wedge \hat{Q}_{I-}\right] + \mathcal{L}_{int}(\Phi)$$

where $\tilde{\mathbf{H}}^I = \tilde{H}^I + \Delta\tilde{H}^I$ is a 3-form superfield, with $\tilde{H}^I \equiv \tilde{H}^I_{abc} V^a V^b V^c$ its (3,0) component and $\Delta\tilde{H}^I$ the rest (components in odd directions), and $\hat{Q}^I_- \equiv \hat{Q}^I_{abc} V^a V^b V^c$, with ${}^*\hat{Q}^I_- = -\hat{Q}^I_-$ (self-dual on space-time)

Note that its space-time projection is the first-order formulation of Sen's prescription on space-itme, with similar conclusions.

First order formulation of Sen's prescription in superspace

We find: $$\begin{cases} \dfrac{\delta\tilde{\mathcal{L}}}{\delta\tilde{\mathbf{H}}^I} = 0 & \Leftrightarrow \;\; \mathrm{d}P^I + Z^I = \hat{Q}^I_- \,, \\ \dfrac{\delta\tilde{\mathcal{L}}}{\delta\hat{Q}^I_{-\,abc}} = 0 & \Leftrightarrow \;\; \left(V^a\wedge V^b\wedge V^c + \dfrac{1}{6}\,\epsilon^{abcdef}(V_d\wedge V_e\wedge V_f)\right)\wedge(\tilde{\mathbf{H}}^I + Z^I) = 0\,, \\ \dfrac{\delta\tilde{\mathcal{L}}}{\delta P^I} = 0 & \Leftrightarrow \;\; \mathrm{d}\tilde{\mathbf{H}}^I = 0 \end{cases}$$

from which:

$$\tilde{\mathbf{H}}^I = \tilde{H}^I_- - Z^I = \mathrm{d}\Xi^I, \quad B^I \equiv (P^I + \Xi^I)/2, \quad \tilde{P}^I \equiv (P^I - \Xi^I)/2$$

$\Rightarrow$ $P = B + \tilde{P}$, with $\tilde{P}$ chiral, but free and SUSY invariant:

$$\tilde{P} : d\tilde{P} = \frac{1}{2}(\hat{Q}_- - \mathbb{P}_-(\tilde{H})) = (d\tilde{P})_{abc}V^aV^bV^c \Rightarrow d^*d\tilde{P} = 0, \quad \delta_\epsilon\tilde{P} = 0$$

while:

$$H^I \equiv \mathrm{d}B^I + Z^I = \mathbb{P}_-\left(H^I\right) = \frac{1}{2}\left(\hat{H}^I_- + \mathbb{P}_-(\tilde{H}^I)\right); \quad \delta_\Phi\tilde{\mathcal{L}} = \delta_\Phi\mathcal{L}$$

Conclusions and outlook

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- ✓ We have then modified it to implement Sen's prescription (at first order) in superspace and on space-time

Conclusions and outlook

- ✓ We have constructed a SUSY invariant lagrangian in D=6 superspace for tensor multiplets in rigid chiral theory with 16 supercharges whose field equations include the self-duality constraint for the chiral 2-forms.
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Next steps:

- To add interaction with supergravity (for the anomaly free case), and try to apply our extension of Sen's prescription in this more challenging case
- To extend our 6-form lagrangian to a $(6|16)$ form in full superspace by adding appropriate integral forms
- To try to go off-shell (Rheonomic formulation of Harmonic superspace?)

Thank you!