Third Family Hypercharge

by
Ben Allanach
(DAMTP, University of Cambridge)

- Anomalies: $b \to s\ell\ell$
- QFT anomalies
- A simple-minded Z' model
- Z' searches

Cambridge Pheno Working Group

Where data and theory collide

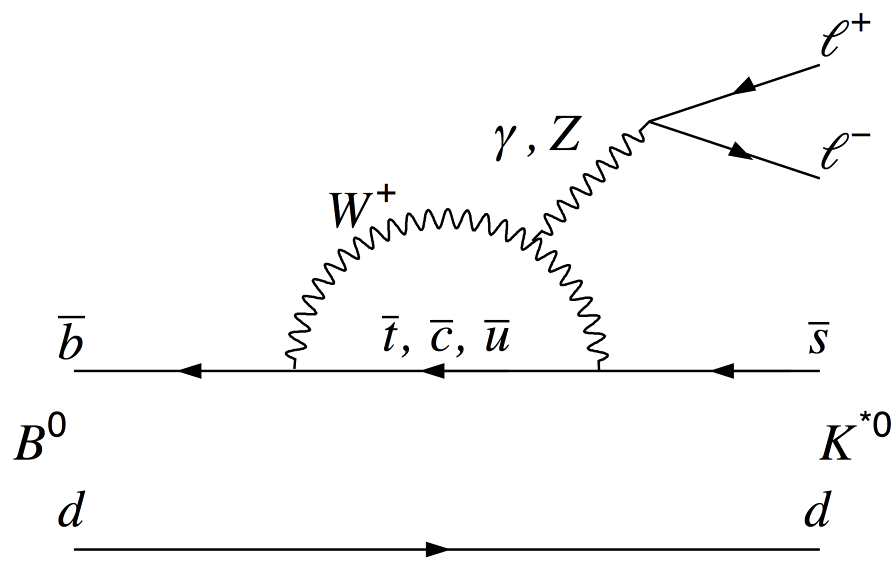
Science & Technology Facilities Council

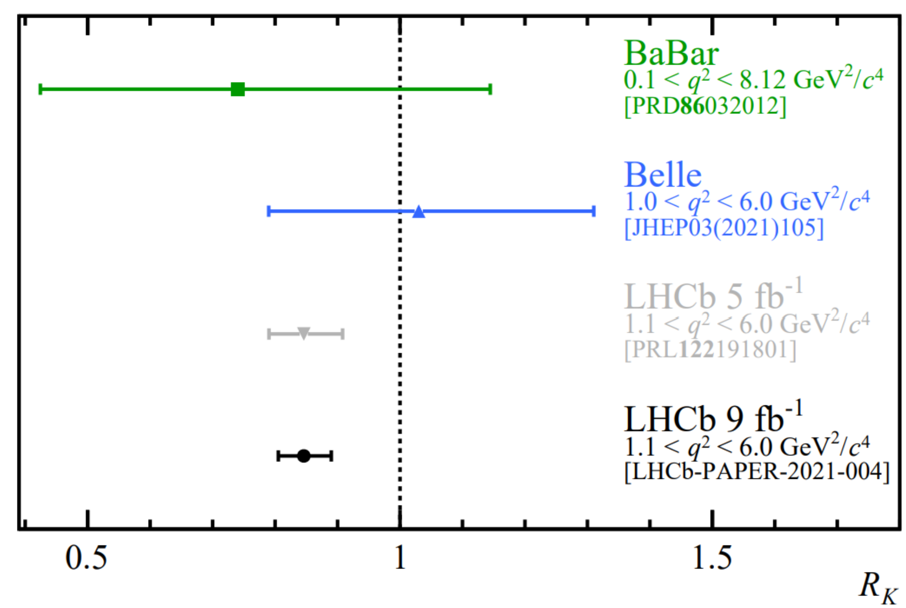
Strange b Activity

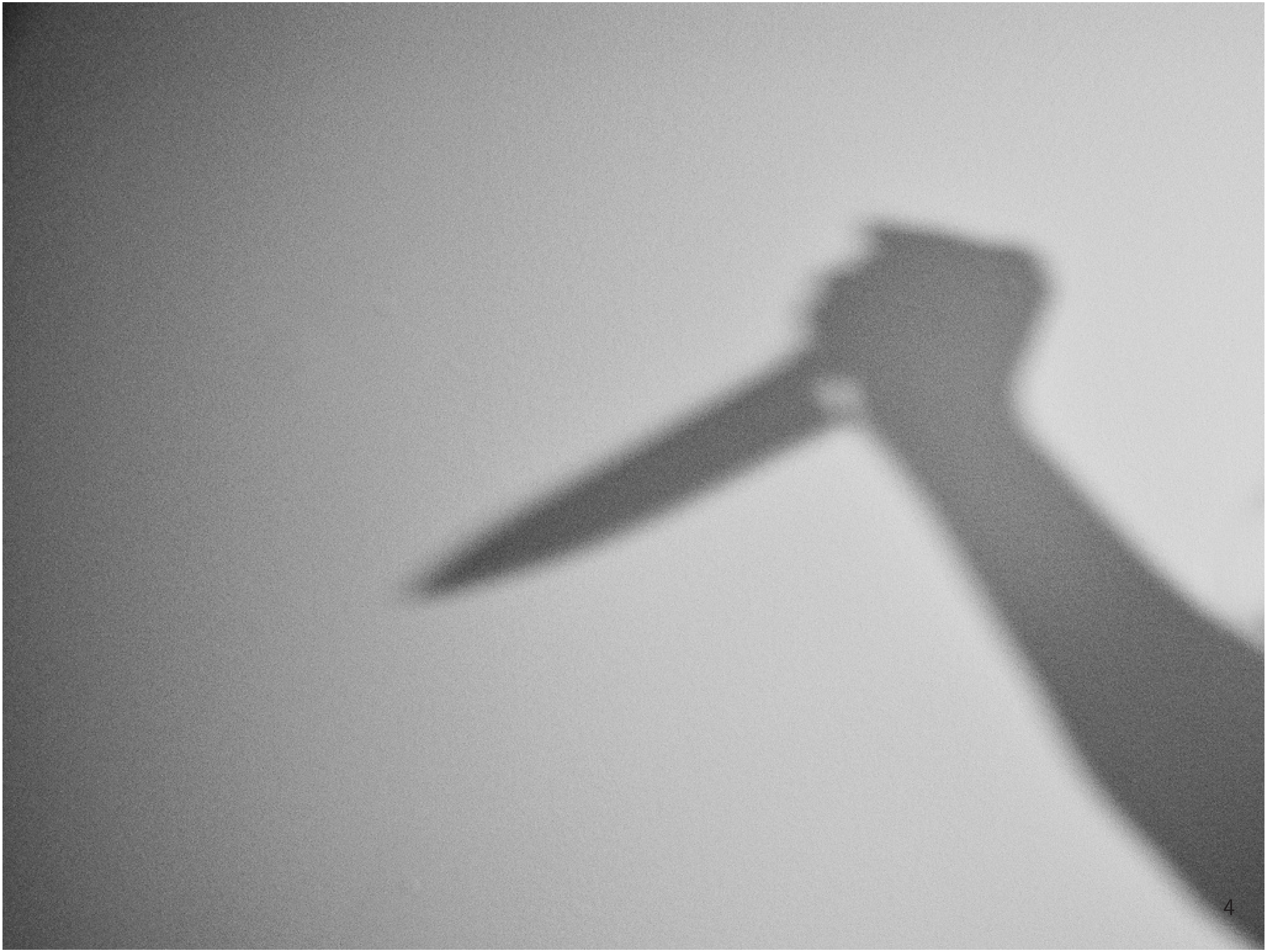
$R_K^{(*)}$ in Standard Model

$$R_K = \frac{BR(B \to K\mu^+\mu^-)}{BR(B \to Ke^+e^-)}, \qquad R_{K^*} = \frac{BR(B \to K^*\mu^+\mu^-)}{BR(B \to K^*e^+e^-)}.$$

These are rare decays (each BR$\sim \mathcal{O}(10^{-7})$) because they are absent at tree level in SM+EW+CKM



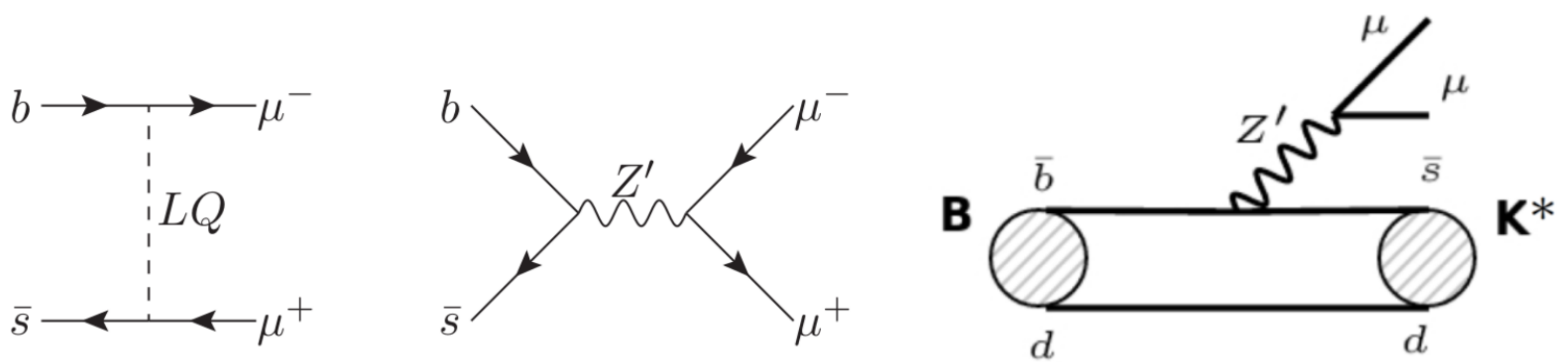




$b \to s\mu\mu$ **Simplified Models**

A good few $2 - 4\sigma$ Discrepancies with SM predictions. Computing with look elsewhere effect implies a 4.3σ discrepancy with the SM (conservative theory errors).[1]

We have tree-level flavour changing new physics options:



[1] Isidori, Lancierini, Owen and Serra, arXiv:2104.05631

Extra $u(1)$ plus SM-singlets

Idea: break SM$\times U(1)_X$ gauge group around a TeV to get Z'. If $U(1)_X$ charges are family non-universal, we should impose quantum field theoretic anomaly cancellation.

- Other uses for Z': dark matter models, axions, fermion masses, ...
- 3 RH neutrinos
- Now, field labels denote the extra $u(1)$ charge
- ACCs become

$$3^2X: \quad 0 = \sum_{j=1}^{3} (2Q_j + U_j + D_j)\,,$$

$$2^2X: \quad 0 = \sum_{j=1}^{3} (3Q_j + L_j)\,,$$

$$Y^2X: \quad 0 = \sum_{j=1}^{3} (Q_j + 8U_j + 2D_j + 3L_j + 6E_j)\,,$$

$$\mathsf{grav}^2X: \quad 0 = \sum_{j=1}^{3} (6Q_j + 3U_j + 3D_j + 2L_j + E_j + N_j),$$

$$YX^2: \quad 0 = \sum_{j=1}^{3} (Q_j^2 - 2U_j^2 + D_j^2 - L_j^2 + E_j^2),$$

$$X^3: \quad 0 = \sum_{j=1}^{3} (6Q_j^3 + 3U_j^3 + 3D_j^3 + 2L_j^3 + E_j^3 + N_j^3).$$

Diophantine Equations

- Since this is $u(1)$, charges are commensurate: looking for compact extensions like the SM
- Thus we are looking for solutions over $\mathbb{Z}^{18}$.
- Any overall real factor in charge can be absorbed in $u(1)_X$ gauge coupling: $\mathcal{L} \supset -g_X \sum_\psi X_\psi \overline{\psi} X_\mu \gamma^\mu \psi$
- General diophantine equations are difficult to solve analytically over the integers
- Number theory state-of-the art for general analytic solution of generic diophantine equations is roughly one cubic in three unknowns

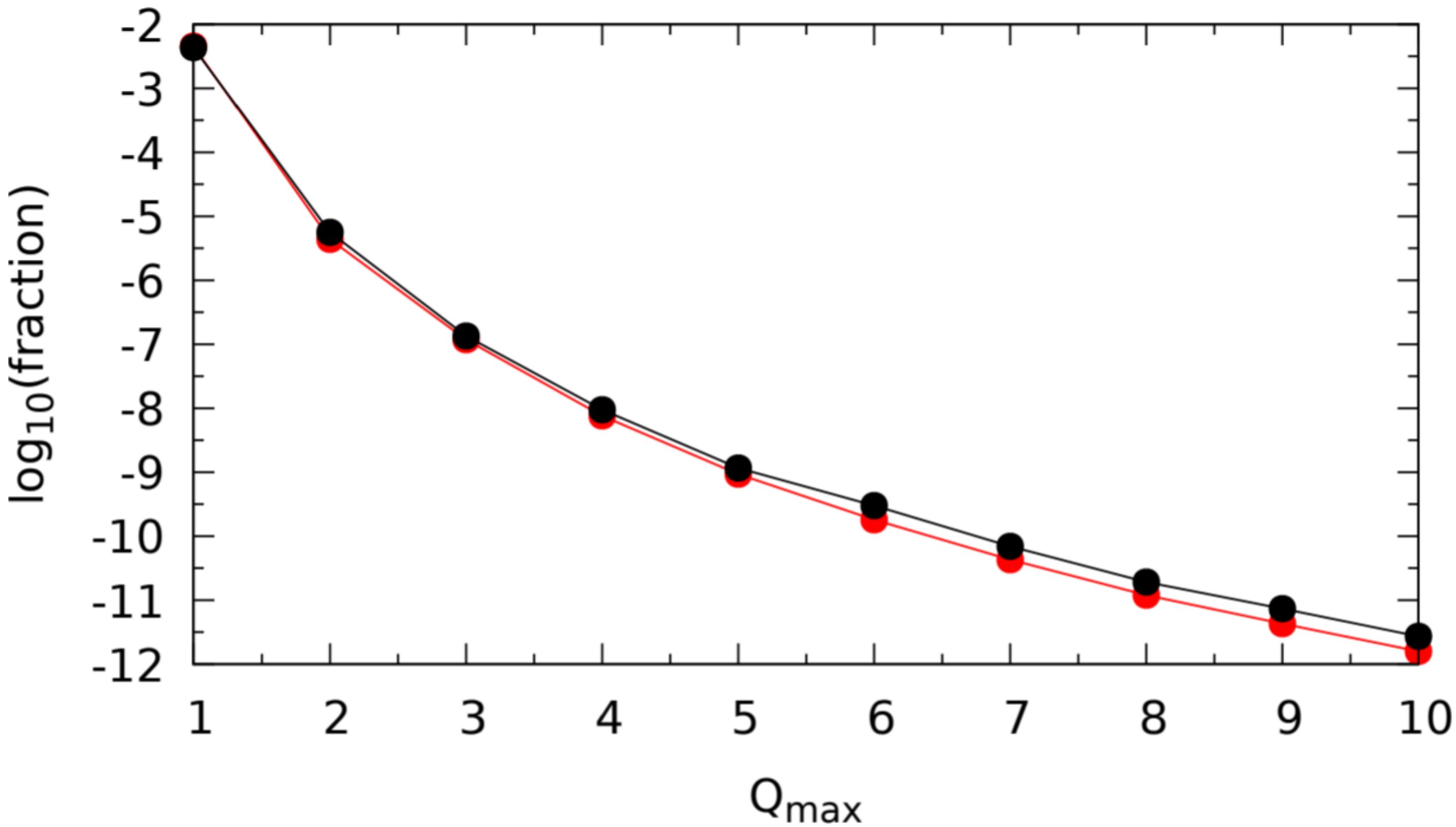
Anomaly-free Atlas

To find solutions for fixed $n \leq 3$ and charges between -10 and 10, we did a numerical scan ($\cancel{21^{18} \sim 10^{24}}$): BCA, Davighi, Melville, arXiv:1812.04602.

An Anomaly-Free Atlas is available for public use: `http://doi.org/10.5281/zenodo.1478085`

Extended to semisimple case (340) in BCA, Gripaios, Tooby-Smith 2104.14555 and MSSM$+3\nu_R \times U(1)_X$ in BCA, Madigan, Tooby-Smith 2107.07926.

Davighi and Tooby-Smith 2206.11271 have investigated which SM$\times U(1)_X$ models fit in semisimple completions.



We begin with 18 charges and 6 anomaly equations reduce these to a 12-dimensional surface of solutions, extending out to infinity, but sparser away from $\mathbf{0}$.

$Q_{\max}$	**Solutions**	Symmetry	Quadratics	Cubics	Time/sec
1	**38**	16	144	38	0.0
2	**358**	48	31439	2829	0.0
3	**4116**	154	1571716	69421	0.1
4	**24552**	338	34761022	932736	0.6
5	**111152**	796	442549238	7993169	6.8
6	**435305**	1218	3813718154	49541883	56
7	**1358388**	2332	24616693253	241368652	312
8	**3612734**	3514	127878976089	978792750	1559
9	**9587085**	5648	558403872034	3432486128	6584
10	**21546920**	7540	2117256832910	10687426240	24748

Inequivalent solutions with 3 RH ν

Known Solutions

	Q_1	Q_2	Q_3	U_1	U_2	U_3	D_1	D_2	D_3	L_1	L_2	L_3	E_1	E_2	E_3	N_1	N_2	N_3
A	0	0	1	0	0	-4	0	0	-2	0	0	-3	0	0	6	0	0	0
B	1	1	1	-1	-1	-1	-1	-1	-1	-3	-3	-3	3	3	3	3	3	3
C	-1	0	1	-1	0	1	-1	0	1	-1	0	1	-1	0	1	0	0	0

- A is TFHM (BCA, Davighi, `arXiv:1809.01158`)
- B is $B - L$, *vector-like*
- C has inter-family cancellation

Analytic Solution

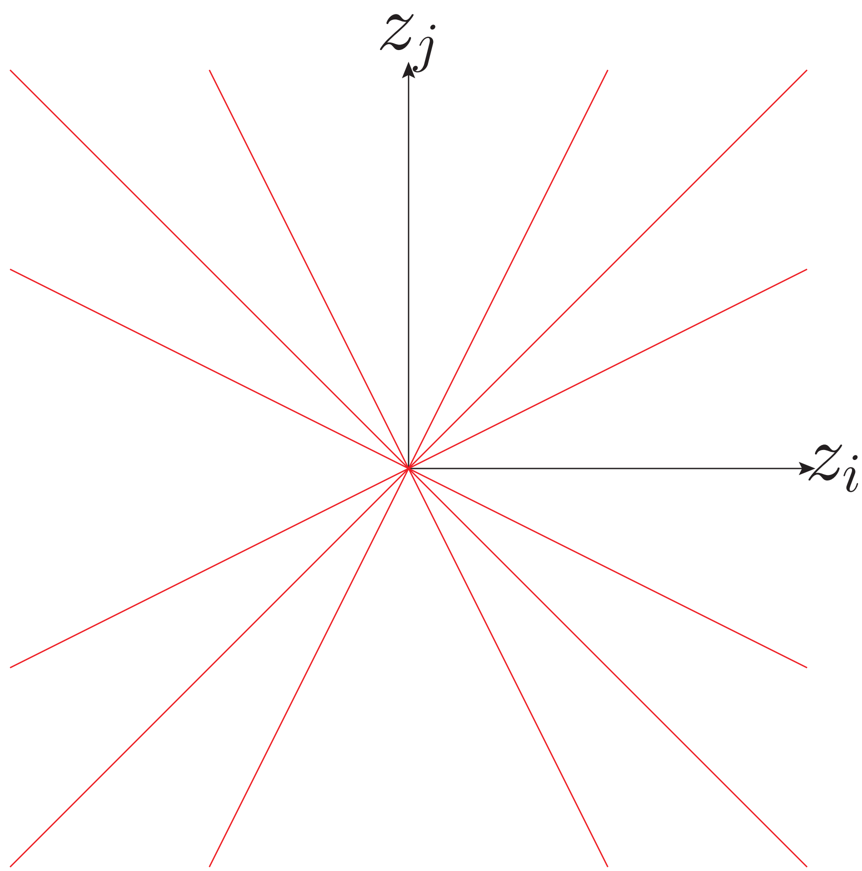
Want a full, general analytic solution for any Q_{max}.

First step is to convert it into a problem in geometry by noting that solutions over $\mathbb{Q}$ are equivalent to those over $\mathbb{Z}$ by clearing all denominators. Since $\mathbb{Q}$ is a field, you can define geometry on it.

We start with $\mathbb{Q}^{18}$ solution space.

All solutions where charges z_i differ by a common multiple are physically equivalent so we define an equivalence class to obtain $P\mathbb{Q}^{17}$.

Projective space $P\mathbb{Q}^{17}$



2d surface through origin becomes a line in projective space and a line through origin becomes a point

Preliminaries

4 linear equations restrict $P\mathbb{Q}^{17}$ to a projective subspace isomorphic to $P\mathbb{Q}^{13}$. Within this, we look for the intersection of a quadratic surface

$$0 = \sum_{j=1}^{3} \left(Q_j^2 - 2U_j^2 + D_j^2 - L_j^2 + E_j^2 \right)$$

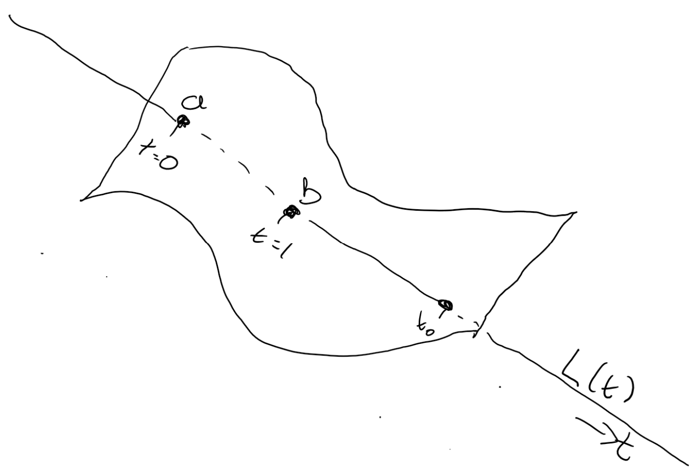
and a cubic surface

$$0 = \sum_{j=1}^{3} \left(6Q_j^3 + 3U_j^3 + 3D_j^3 + 2L_j^3 + E_j^3 + N_j^3 \right)$$

The Method of Chords[2]

"A chord intersecting a rational cubic surface at two known rational points intersects it at 1 other $\mathbb{Q}$ point"

eg Rational cubic $c(z_i) = 0$. Put a line through 2 known intersections a, b: $L(t) = a + t(b - a)$. Along line, $c(L(t)) = kt(t-1)(t-t_0)$, where $k, t_0 \in \mathbb{Q}$.



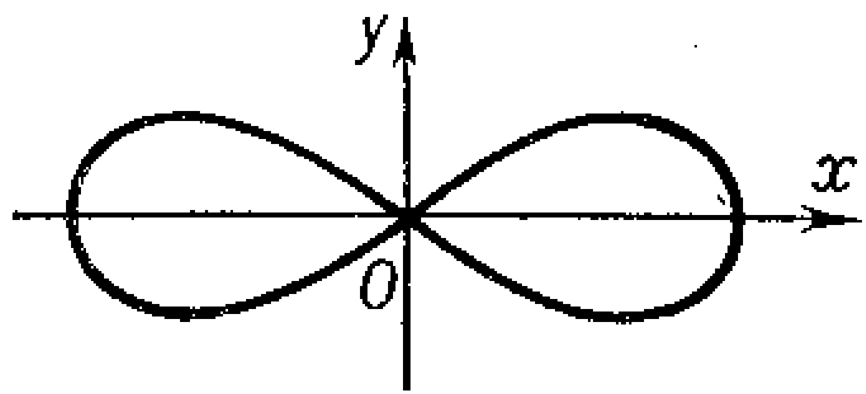
Caveat: It is possible that the line lies *entirely within the cubic surface*, i.e. $c(L(t)) = 0$ irrespective of t.

[2] Newton, Fermat, C17[th]

Double Points

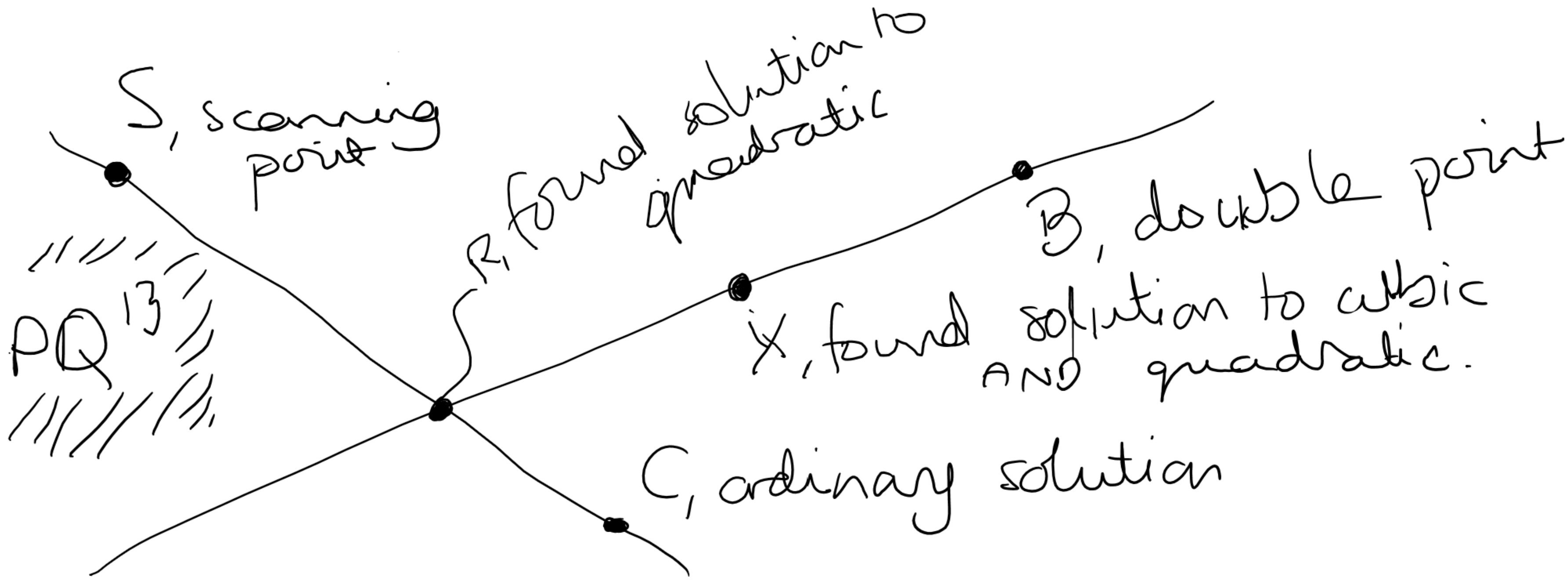
Points which are solutions of multiplicity two. All partial derivatives of the surface vanish there, eg $(x, y) = (0, 0)$ of the curve

$$(x^2 + y^2 + a^2)^2 - 4a^2x^2 - a^4 = 0$$



B is a *double point* of the quadratic and the cubic

Method



- Every solution to quadratic R lies on *some* line SC
- $B - L$ is double point of quadratic $\Rightarrow RB$ in quadratic
- Every solution X lies on a line between some R and B.

The Nitty-Gritty

$$
\begin{aligned}
Q_1 &= \Gamma - \Sigma + \Lambda S_{Q_1}, \\
Q_2 &= \Gamma + \Lambda S_{Q_2}, \\
Q_3 &= \Gamma + \Sigma + \Lambda S_{Q_3}, \\
U_1 &= -\Gamma - \Sigma + \Lambda S_{U_1}, \\
U_2 &= -\Gamma + \Lambda S_{U_2}, \\
U_3 &= -\Gamma + \Sigma + \Lambda S_{U_3}, \\
D_1 &= -\Gamma - \Sigma + \Lambda S_{D_1}, \\
D_2 &= -\Gamma + \Lambda S_{D_2}, \\
D_3 &= -\Gamma + \Sigma + \Lambda S_{D_3}, \\
L_1 &= -3\Gamma - \Sigma + \Lambda S_{L_1}, \\
L_2 &= -3\Gamma + \Lambda S_{L_2}, \\
L_3 &= -3\Gamma + \Sigma + \Lambda S_{L_3}, \\
E_1 &= 3\Gamma - \Sigma + \Lambda S_{E_1}, \\
E_2 &= 3\Gamma + \Lambda S_{E_2}, \\
E_3 &= 3\Gamma + \Sigma + \Lambda S_{E_3}, \\
N_1 &= 3\Gamma + \Lambda S_{N_1}, \\
N_2 &= 3\Gamma + \Lambda S_{N_2}, \\
N_3 &= 3\Gamma + \Lambda S_{N_3},
\end{aligned}
$$

$$
\begin{aligned}
\Gamma &= c(R,R,R) + r\delta_{c(B,R,R),0}\delta_{c(R,R,R),0}, \\
\Sigma &= (-3c(B,R,R) + t\delta_{c(B,R,R),0}\delta_{c(R,R,R),0}) \\
&\quad (q(S,S) + a\delta_{q(S,S),0}\delta_{q(C,S),0}), \\
\Lambda &= (-3c(B,R,R) + t\delta_{c(B,R,R),0}\delta_{c(R,R,R),0}) \\
&\quad (-2q(C,S) + b\delta_{q(S,S),0}\delta_{q(C,S),0}).
\end{aligned}
$$

$$
q(P,P') := \sum_{i=1} \big(Q_i Q_i' - 2U_i {U_i}' + D_i {D_i}' \\
-L_i {L_i}' + E_i {E_i}' \big) ,
$$

$$
c(P,P',P'') := \sum_{i=1}^{3} \big(6Q_i {Q_i}' {Q_i}'' + 3U_i {U_i}' {U_i}'' + 3D_i {D_i}' {D_i}'' \\
+2L_i {L_i}' {L_i}'' + E_i {E_i}' {E_i}'' + N_i {N_i}' {N_i}'' \big) . \qquad (3)
$$

$$
R = q(S,S)C - 2q(C,S)S + \delta_{q(S,S),0}\delta_{q(C,S),0}(aC + bS),
$$

$$
S_{Q_3} = \frac{1}{2}\left[-2S_{Q_1} - 2S_{Q_2} + \sum_{i=1}^{3}(S_{D_i} + S_{N_i}) \right],
$$

$$
S_{U_3} = -\left[S_{U_1} + S_{U_2} + \sum_{i=1}^{3}(2S_{D_i} + S_{N_i}) \right],
$$

$$
S_{L_3} = -\frac{1}{2}\left[2S_{L_1} + 2S_{L_2} + 3\sum_{i=1}^{3}(S_{D_i} + S_{N_i}) \right],
$$

$$
S_{E_3} = -S_{E_1} - S_{E_2} + \sum^{3}(3S_{D_i} + 2S_{N_i}).
$$

Solution Space

Is called a projective *variety*, i.e. *not* a manifold (in $\mathbb{Q}$ anyway, but also there are singular cases of lines within planes where the dimensionality decreases).

Over-parameterisation in terms of 18 integers

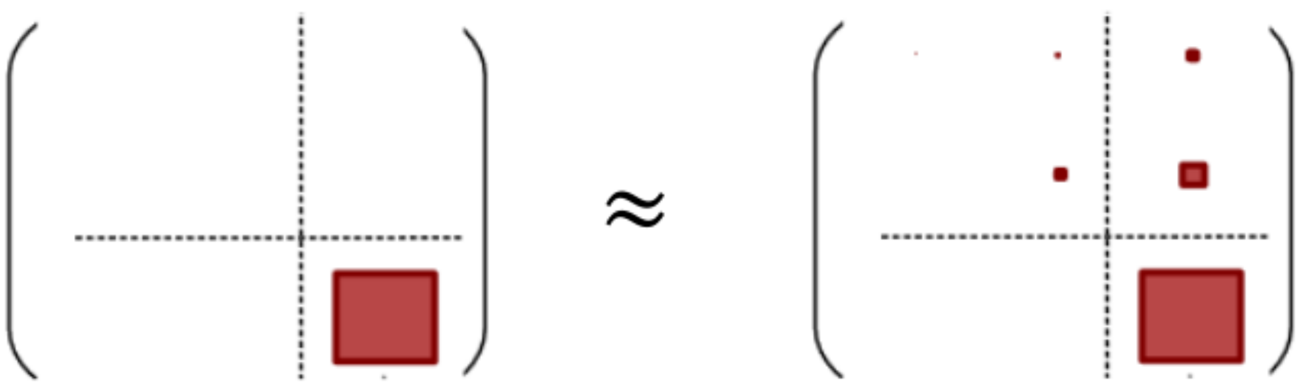
$$S_{Q_1}, S_{Q_2}, S_{U_1}, S_{U_2}, S_{D_1}, S_{D_2}, S_{D_3}, S_{L_1}, S_{L_2}, S_{E_1}, S_{E_2}, \\ S_{N_1}, S_{N_2}, S_{N_3}, a, b, r, t \in \mathbb{Q}$$

It is at most 11-dimensional. $S \cdot C = S \cdot B = 0$. An inverse $(S = T, a = 0, b = 1, r = 0, t = 1)$, was checked against 21 549 920 all Anomaly-free Atlas solns.

Sol: $X = Y_3 + t(B_3 - L_3)$, $t \in \mathbb{Q}$

$X_{Q'_{1,2}} = 0$	$X_{u_{R'_{1,2}}} = 0$	$X_{d_{R'_{1,2}}} = 0$	$X_{L'_{1,2}} = 0$
$X_{e_{R'_{1,2}}} = 0$	$X_H = -1/2$	$X_{Q'_3} = 1/6$	$X_{u'_{R3}} = 2/3$
$X_{d'_{R3}} = -1/3$	$X_{L'_3} = -1/2$	$X_{e'_{R3}} = -1$	$X_\theta \neq 0$

$$\mathcal{L} = Y_t \overline{Q_3{}'_L} H t'_R + Y_b \overline{Q'_{3L}} H^c b'_R + Y_\tau \overline{L_3{}'_L} H^c \tau'_R + H.c.,$$



A Simple Z' Model

BCA, Davighi, 1809.01158: Add complex SM-singlet scalar 'flavon' $\theta_{X\neq 0}$ which breaks gauged $U(1)_X$:

$$\begin{array}{c} SU(3)\times SU(2)_L\times U(1)_Y\times U(1)_X \\ \Big\downarrow \langle\theta\rangle \sim \text{Several TeV} \\ SU(3)\times SU(2)_L\times U(1)_Y \\ \Big\downarrow \langle H\rangle \sim 246 \text{ GeV} \\ SU(3)\times U(1)_{em} \end{array}$$

- SM fermion content
- Zero X charges for first two generations
- Expect SM$\times U(1)_X$ to be subsumed in semisimple model
- Still worry about anomaly cancellation

$$
\begin{aligned}
\mathcal{L}_{X\psi} \;= g_X \; & \Big(\frac{1}{6}\overline{\mathbf{u_L}}\Lambda^{(u_L)}\gamma^\rho \mathbf{u_L} + \frac{1}{6}\overline{\mathbf{d_L}}\Lambda^{(d_L)}\gamma^\rho \mathbf{d_L} - \\
& \frac{1}{2}\overline{\mathbf{n_L}}\Lambda^{(n_L)}\gamma^\rho \mathbf{n_L} - \frac{1}{2}\overline{\mathbf{e_L}}\Lambda^{(e_L)}\gamma^\rho \mathbf{e_L} + \\
& \frac{2}{3}\overline{\mathbf{u_R}}\Lambda^{(u_R)}\gamma^\rho \mathbf{u_R} - \\
& \frac{1}{3}\overline{\mathbf{d_R}}\Lambda^{(d_R)}\gamma^\rho \mathbf{d_R} - \overline{\mathbf{e_R}}\Lambda^{(e_R)}\gamma^\rho \mathbf{e_R} \Big) Z'_\rho,
\end{aligned}
$$

$$
\Lambda^{(I)} \quad \equiv \quad V_I^\dagger \xi V_I, \qquad \xi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

Z' couplings, $I \in \{u_L, d_L, e_L, \nu_L, u_R, d_R, e_R\}$

A simple limiting case

$$V_{u_R} = V_{d_R} = V_{e_R} = 1$$

for simplicity and the ease of passing bounds.

$$V_{d_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & -\sin\theta_{23} \\ 0 & \sin\theta_{23} & \cos\theta_{23} \end{pmatrix}, \qquad V_{e_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$\Rightarrow V_{u_L} = V_{d_L} V_{CKM}^\dagger$ and $V_{\nu_L} = V_{e_L} U_{PMNS}^\dagger$.

Important Z' Couplings

$$g_X \left[\frac{1}{6} (\overline{d_L}\ \overline{s_L}\ \overline{b_L}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sin^2\theta_{23} & \frac{1}{2}\sin 2\theta_{23} \\ 0 & \frac{1}{2}\sin 2\theta_{23} & \cos^2\theta_{23} \end{pmatrix} \not{Z}' \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \right.$$

$$\left. -\frac{1}{2} (\overline{e_L}\ \overline{\mu_L}\ \overline{\tau_L}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \not{Z}' \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \right]$$

$Z - Z'$ mixing

Because $Y_3(H) = 1/2$, $B - W^3 - X$ bosons mix:

$$\mathcal{M}_N^2 = \frac{1}{4} \begin{array}{cc} \begin{pmatrix} g'^2 v^2 & -g g' v^2 & g' g_X v^2 \\ -g g' v^2 & g^2 v^2 & -g g_X v^2 \\ g' g_X v^2 & -g g_X v^2 & 4 g_X^2 \langle\theta\rangle^2 \left(1 + \frac{\epsilon^2}{4}\right) \end{pmatrix} & \begin{array}{c} -B_\mu \\ -W_\mu^3 \\ -(X)_\mu \end{array} \end{array}$$

- $v \approx 246$ GeV is SM Higgs VEV,
- $\langle\theta\rangle$ ~TeV. $M_{Z'} = g_X \langle\theta\rangle$.
- $g_X = U(1)_X$ gauge coupling
- $\epsilon \equiv v/\langle\theta\rangle \ll 1$

$Z - Z'$ mixing angle

$$\sin\alpha_z \approx \frac{g_X}{\sqrt{g^2+g'^2}}\left(\frac{M_Z}{M'_Z}\right)^2 \ll 1.$$

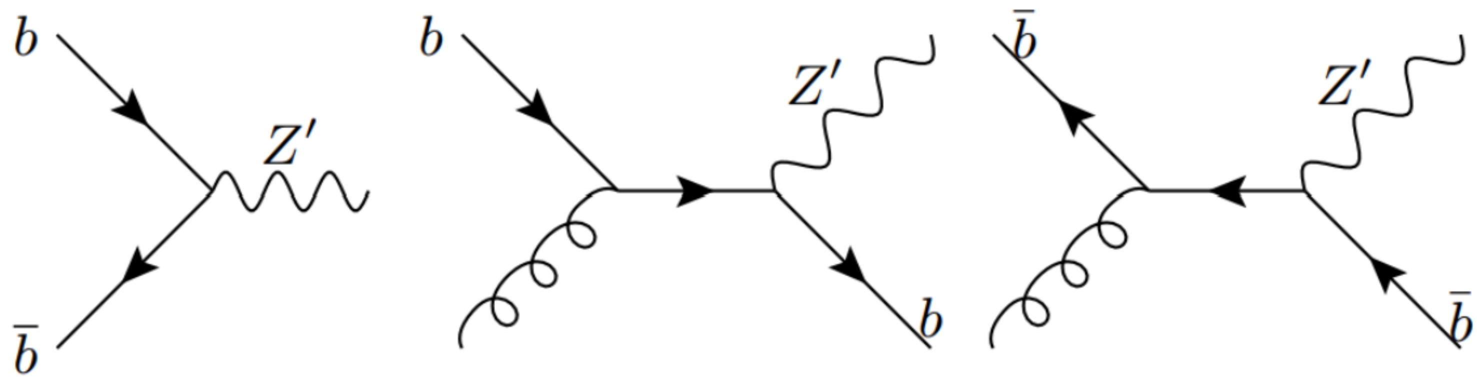
This gives small non-flavour universal couplings to the Z boson propotional to g_X and:

$$Z_\mu = \cos\alpha_z\left(-\sin\theta_w B_\mu + \cos\theta_w W^3_\mu\right) + \sin\alpha_z X_\mu,$$

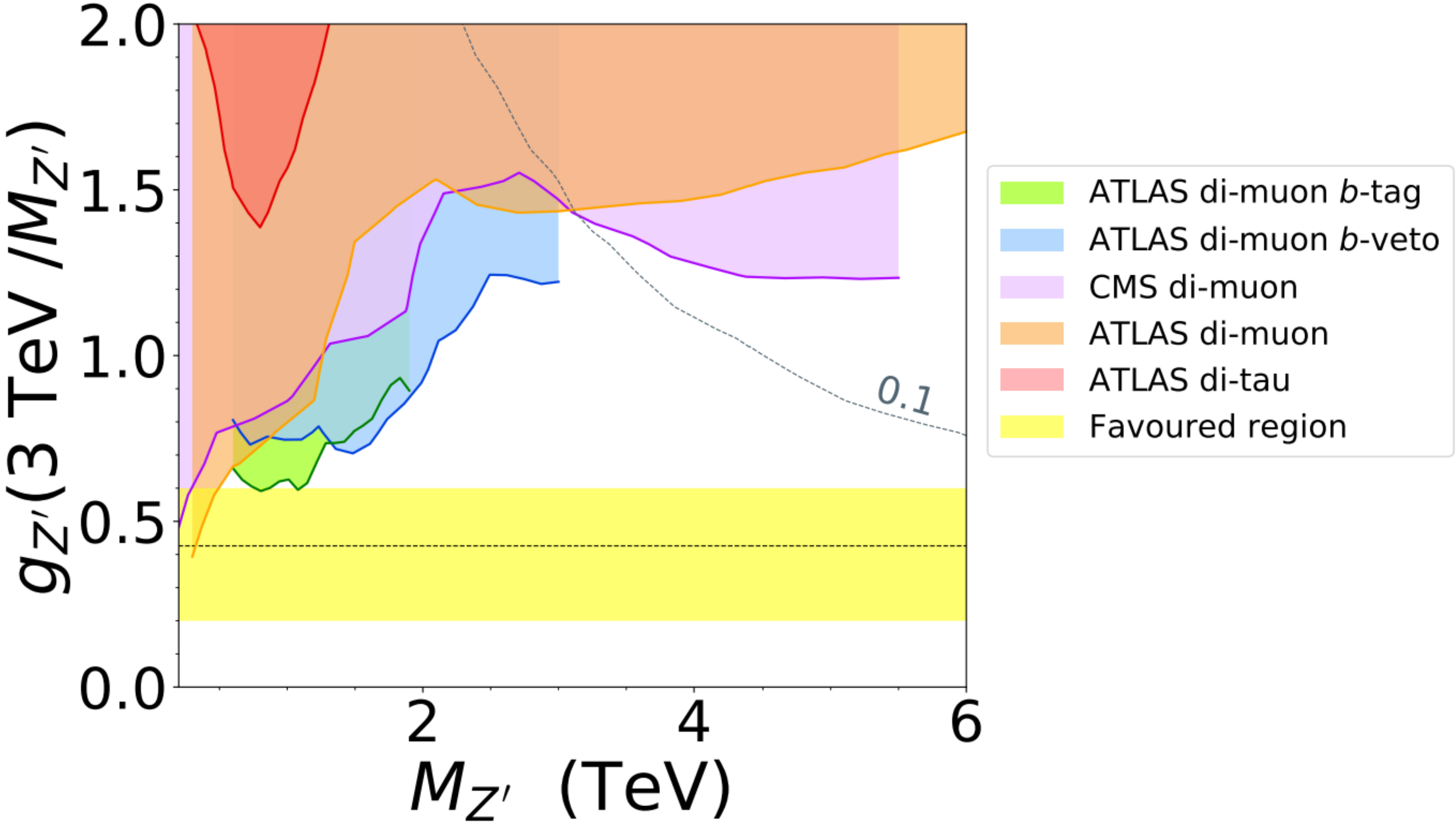
Z' Decay Modes

Mode	BR	Mode	BR	Mode	BR
$t\bar{t}$	0.42	$b\bar{b}$	0.12	$\nu\bar{\nu}'$	0.08
$\mu^+\mu^-$	0.08	$\tau^+\tau^-$	0.30	other $f_i f_j$	$\sim \mathcal{O}(10^{-4})$

LHC Z' Production:

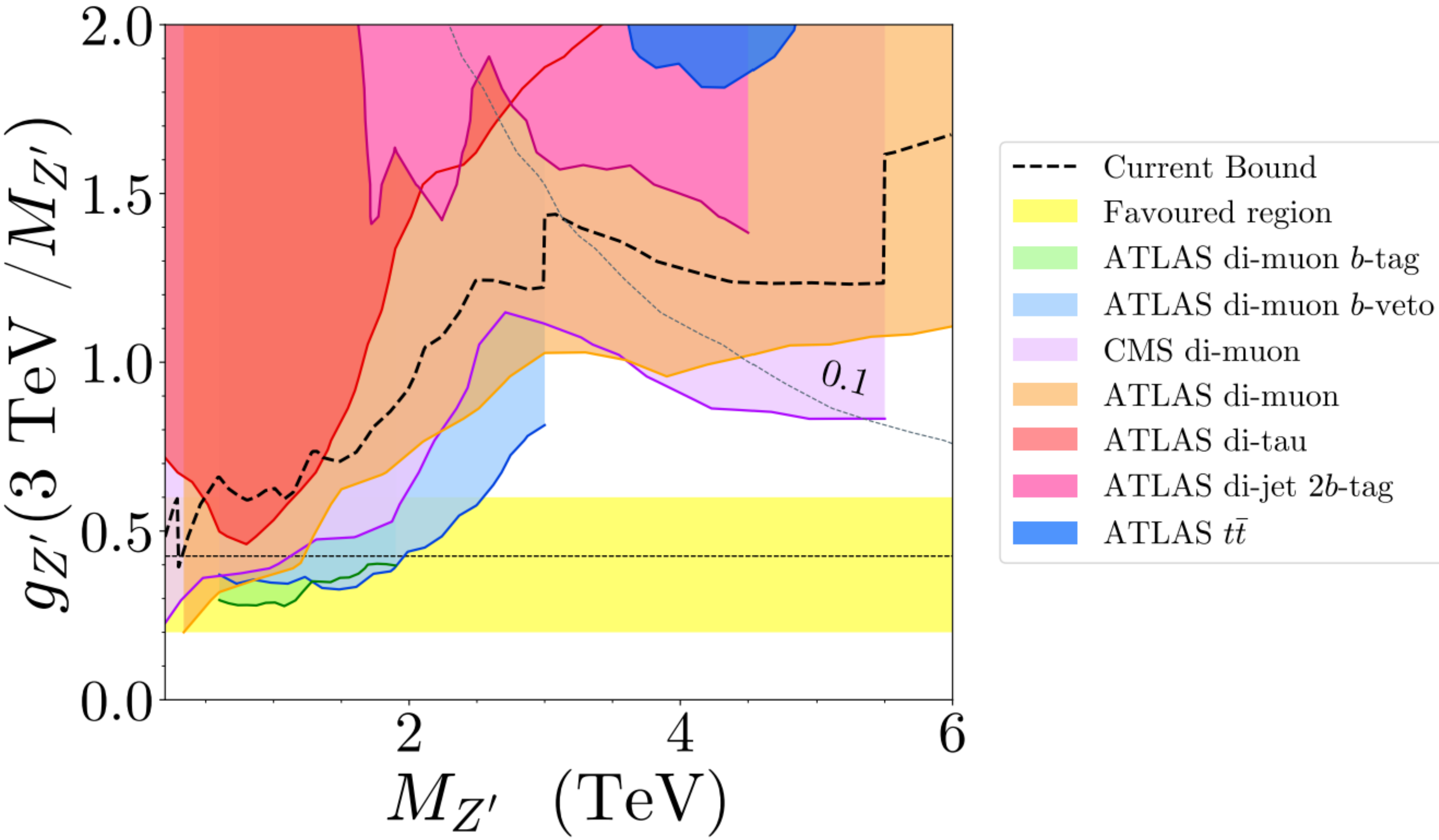


Z' Searches[3]



[3] BCA, Banks, 2111.06691

HL-LHC sensitivity[4]



[4]BCA, Banks, 2111.06691

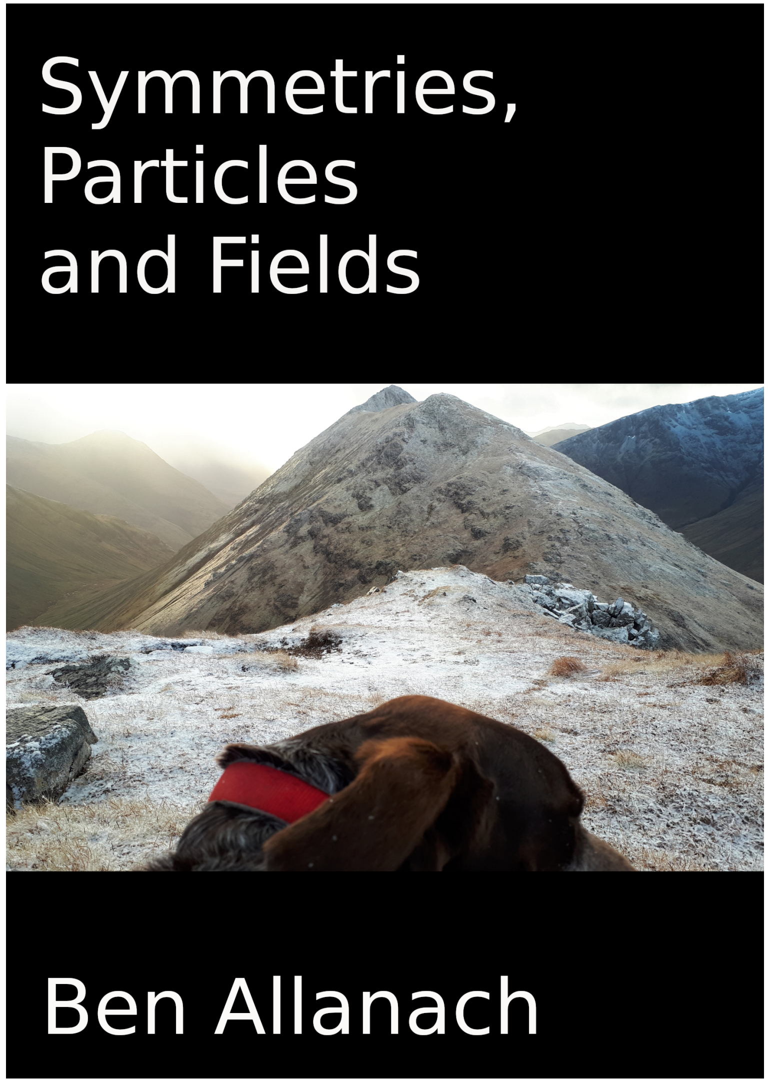
Why $\bar{b}s\mu^+\mu^-$?

If we take these $B-$anomalies seriously, we may ask: why are we seeing the first BSM flavour changing effects particularly in the $b \to s\mu^+\mu^-$ transition, not another one?

Perhaps it's because, in hindsight:

- The largest BSM flavour effects are in heavier generations
- We have many more bs than ts, particularly in LHCb
- Leptons in final states are good experimentally but not (yet) τs: they are too difficult!

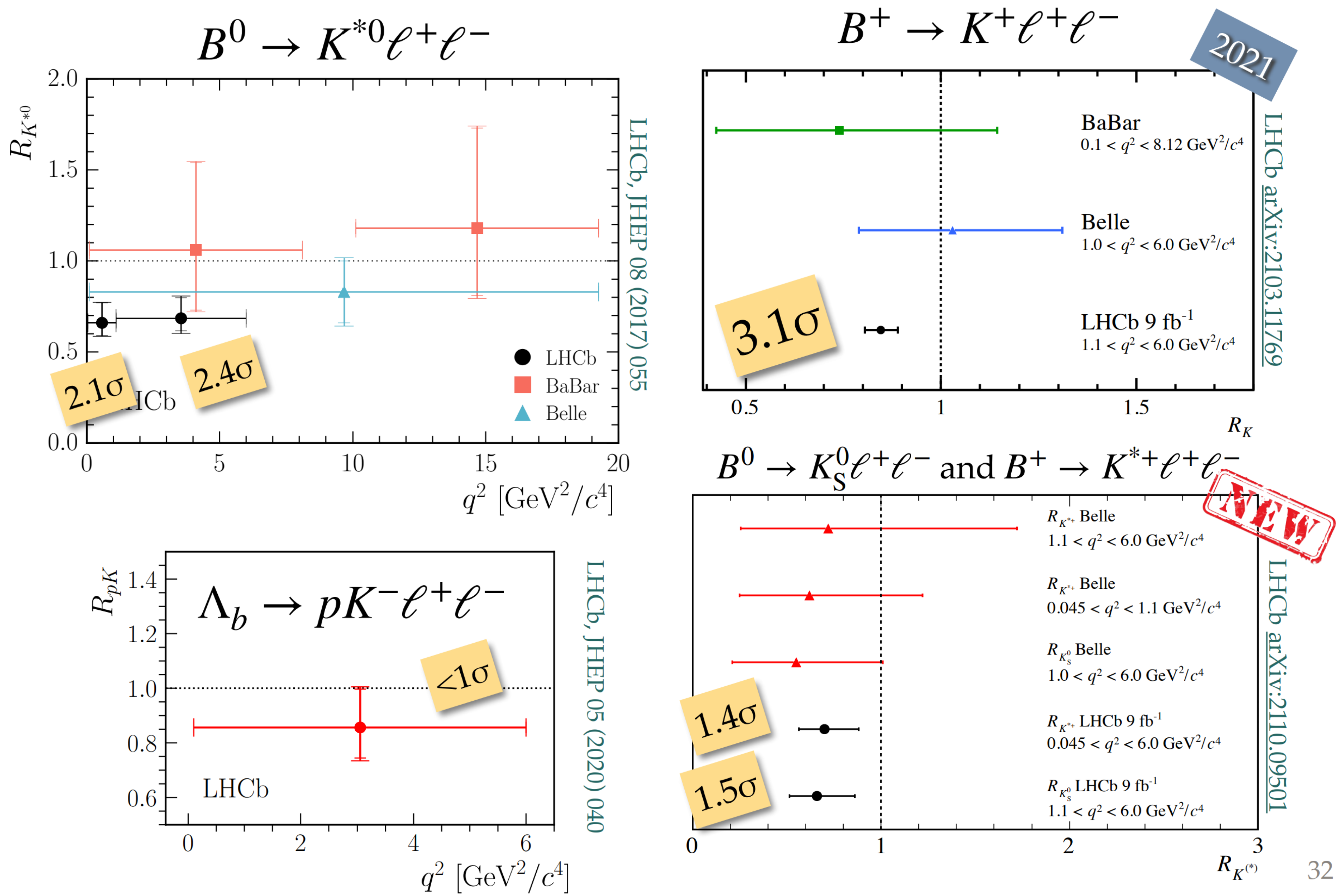
On `amazon.com`∼€20



Summary

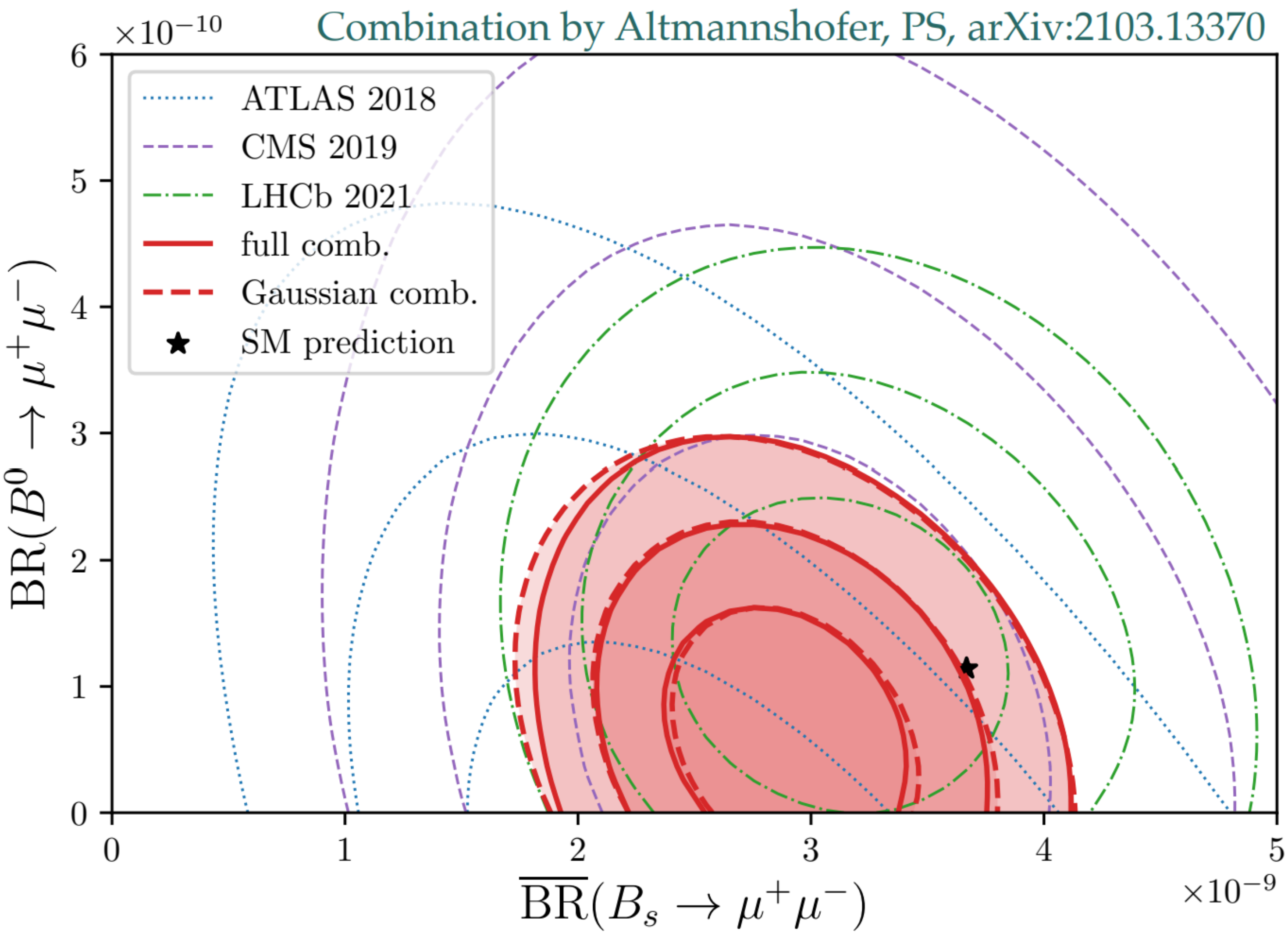
- The $b \to s\mu^+\mu^-$ anomalies look very interesting from a BSM point of view: a consistent picture is emerging.
- Independent check awaited from Belle II in Japan in the coming three years or so: $e^+e^-(10.58\text{ GeV}) \to \Upsilon(4s) \to$ oodles of B mesons.
- Tree-level explanations: leptoquarks and Z's.
- In case a Z' is found directly, measuring its couplings may give us an experimental handle on the fermion mass puzzle.

Backup

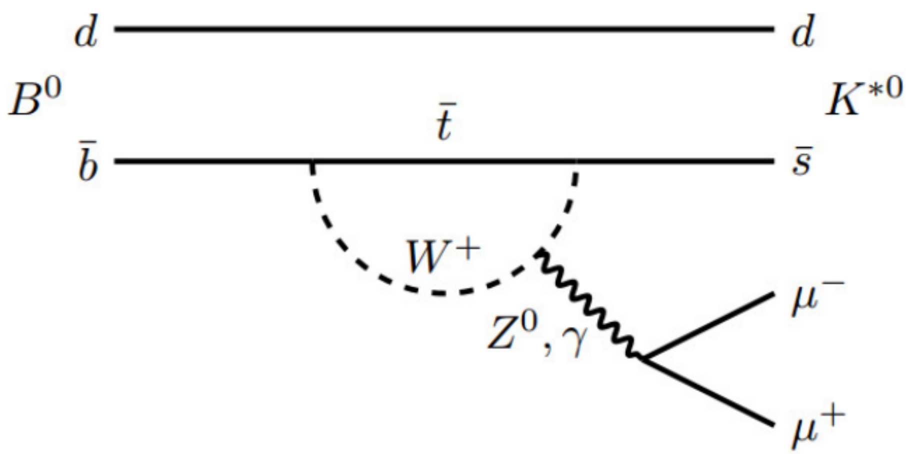


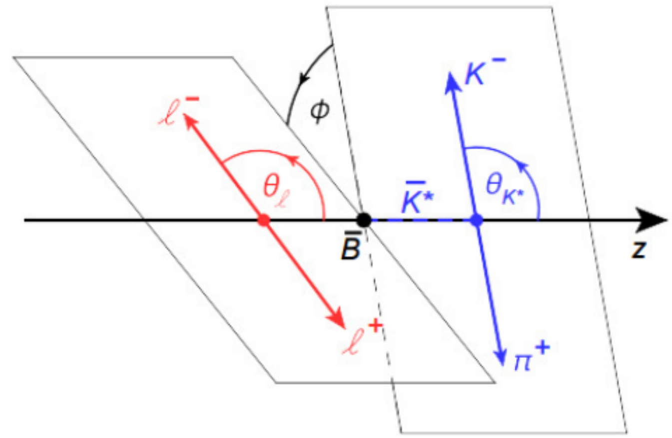
Stolen from Capdevila et al, *Flavour Anomaly Workshop '21*

$BR(B_s \to \mu^+\mu^-)$: $B_s = (\bar{b}s)$, $B^0 = (\bar{b}d)$



$B^0 \to K^{*0}(\to K^+\pi^-)\mu^+\mu^-$

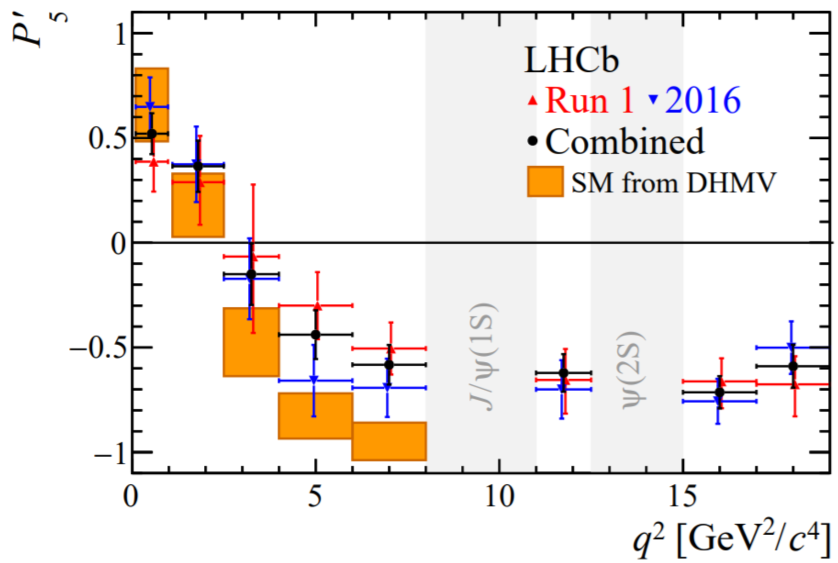


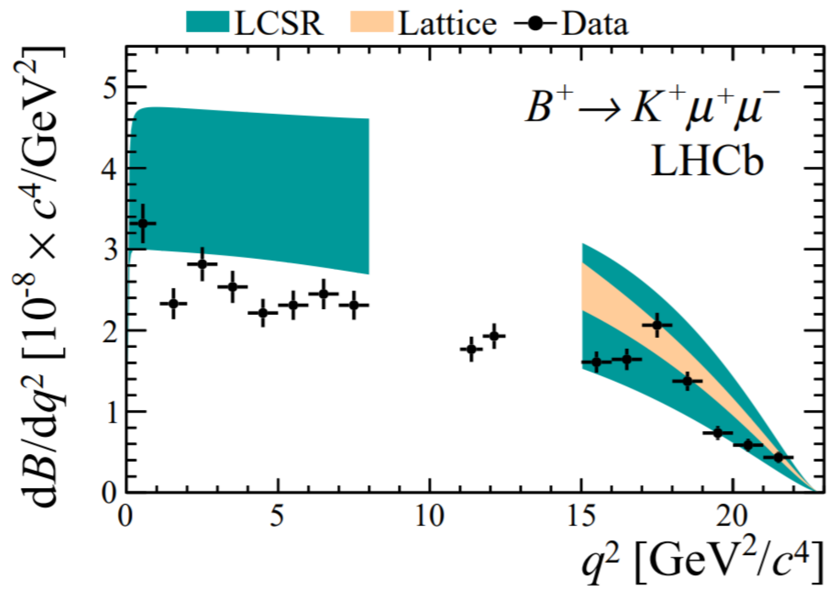


Decay fully described by three helicity angles $\vec{\Omega} = (\theta_\ell, \theta_K, \phi)$ and $q^2 = m^2_{\mu\mu}$

$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2}\frac{\mathrm{d}^3(\Gamma+\bar{\Gamma})}{\mathrm{d}\vec{\Omega}} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_\mathrm{L})\sin^2\theta_K + F_\mathrm{L}\cos^2\theta_K + \tfrac{1}{4}(1-F_\mathrm{L})\sin^2\theta_K\cos 2\theta_\ell$$
$$- F_\mathrm{L}\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi$$
$$+ S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + S_5\sin 2\theta_K\sin\theta_\ell\cos\phi$$
$$+ \tfrac{4}{3}A_\mathrm{FB}\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi$$
$$+ S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big]$$

P_5'

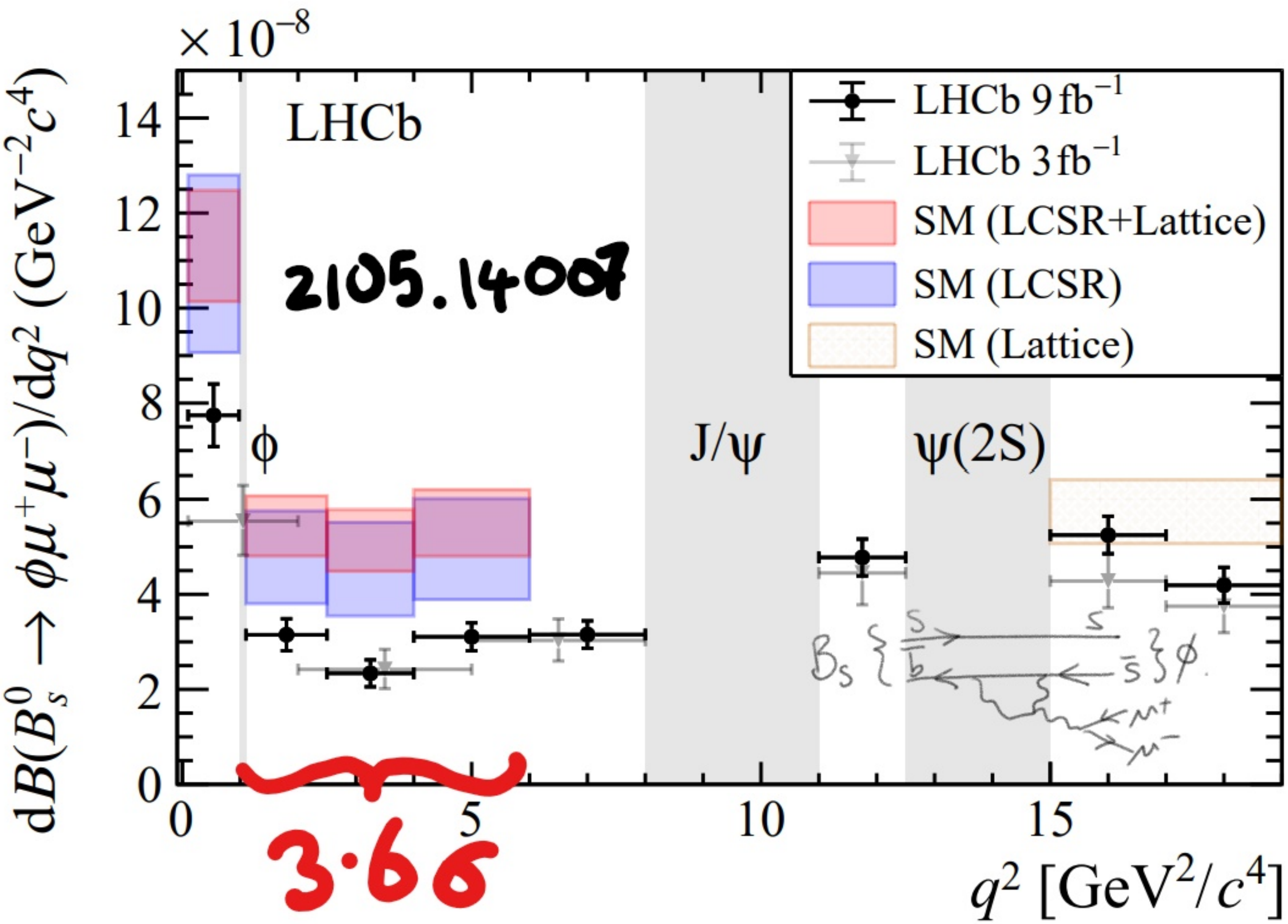




$P_5' = S_5/\sqrt{F_L(1-F_L)}$, leading form factor uncertainties cancel [5]

[5] LHCb, 2003.04831

$B_s \to \phi\mu^+\mu^-$: $\phi = (s\bar{s})$



Theory: uncertainties

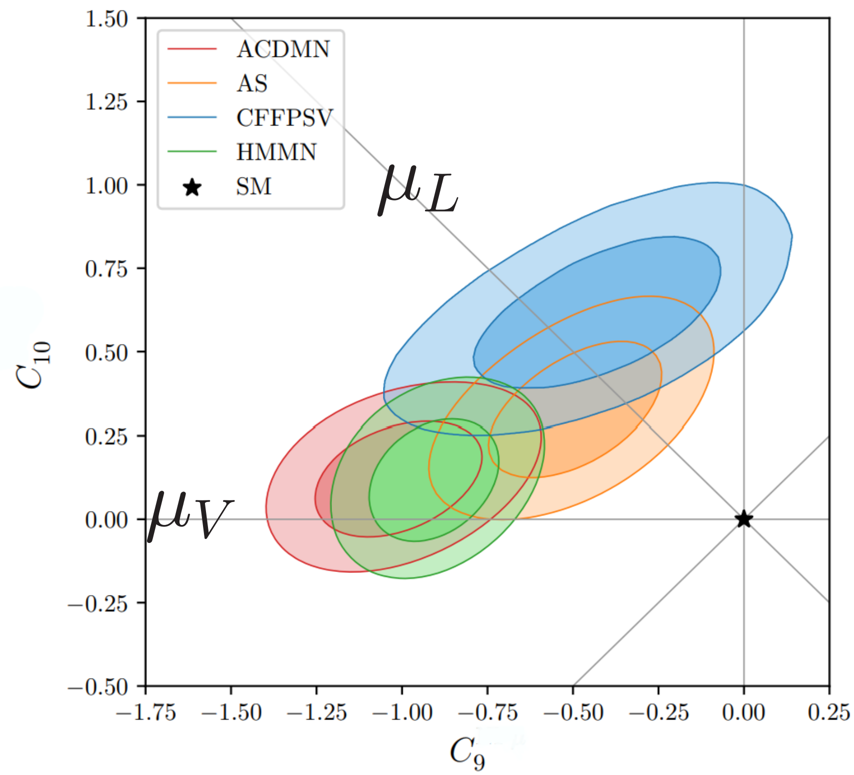
	parametric	form factors	non-local MEs
$BR(B \to Mll)$	yes	large	large
angular	no	small	large
$BR(B_s \to ll)$	yes	small	no
LFU	no	tiny	no

- Parametric uncertainties (eg V_{ts}) easy to deal with
- Large theory uncertainties are taken into account in fits, but one can argue about them

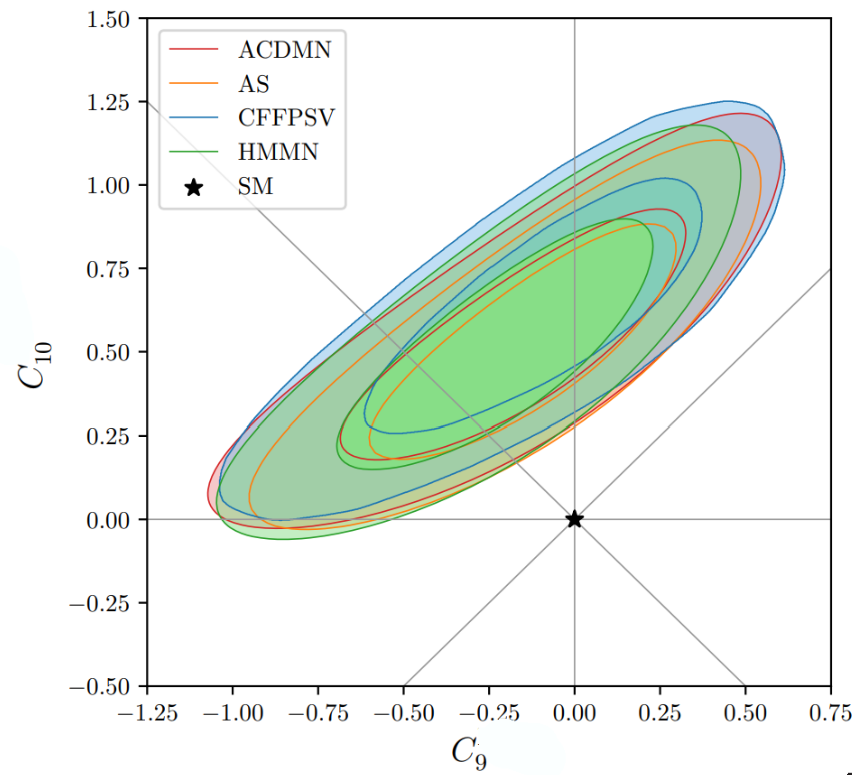
Fits

Alguero *et al*, `2104.08921`; Altmannshofer, Stangl, `flavio 2103.13370`;
Ciuchini *et al*, `HEPfit 2011.01212`; Hurth *et al*, `superIso 2104.10058`

$$\mathcal{L} = N[C_9(\bar{b}_L\gamma^\mu s_L)(\bar{\mu}\gamma_\mu\mu) + C_{10}(\bar{b}_L\gamma^\mu s_L)(\bar{\mu}\gamma^5\gamma_\mu\mu)] + H.c.$$



global fit



fit to LFU observables + $B_s \to \mu\mu$

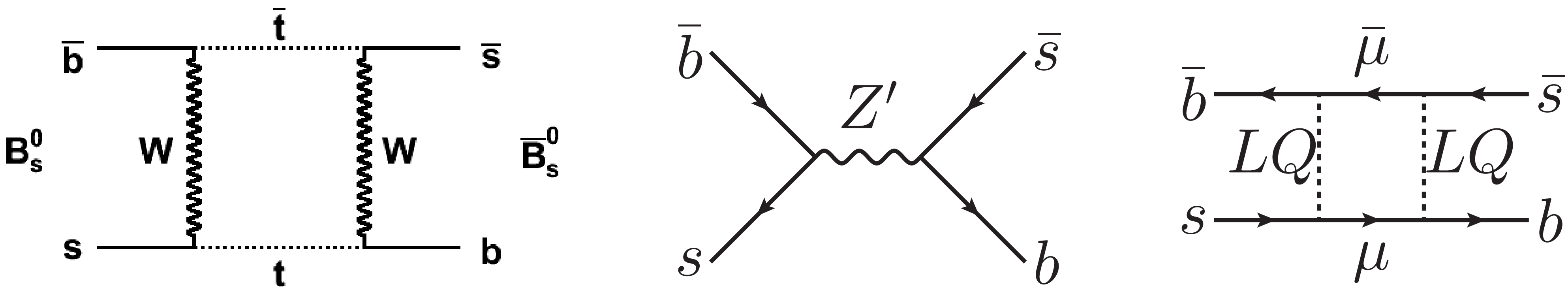
Y_3 Consequences

- Flavour changing TeV-scale Z' to do NCBAs: couples dominantly to EW eigenstates of quarks and leptons of the third family
- First two fermion families massless at renormalisable level
- Their masses and fermion mixings generated by small non-renormalisable operators

This explains the hierarchical heaviness of the third family and small CKM angles

$B_s - \bar{B}_s$ Mixing

Measurement pretty much agrees with SM calculations.



$$g_{sb} \lesssim \frac{M_{Z'}}{194 \text{ TeV}}$$ but uncertain

from QCD sum rules and lattice[6]. Weaker on LQs.

$$M_{Z'} \approx 31 \text{ TeV} \times \sqrt{g_{sb} g_{\mu\mu}}, \qquad M_{LQ} \approx 31 \text{ TeV} \times \sqrt{g_{s\mu} g_{b\mu}}$$

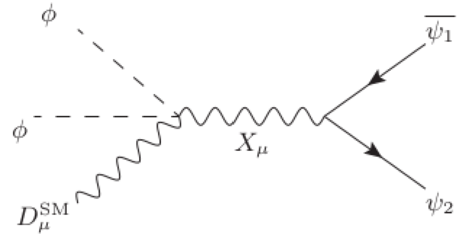
[6] King, Lenz, Rauh, arXiv:1904.00940

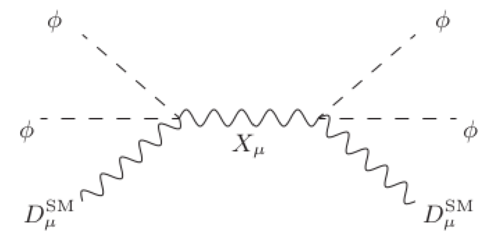
B/EW Observables

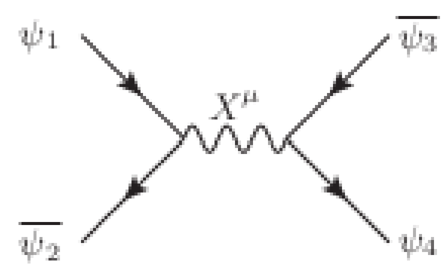
SMEFT$(M_{Z'})$ $\rightarrow$`smelli`$\rightarrow$WET(M_W) $\rightarrow$obs(m_B)

In units of g_X^2/M_X^2:

WC	value	WC	value
C_{ll}^{2222}	$-\frac{1}{8}$	$(\mathbf{C}_{\mathbf{lq}}^{(1)})^{22ij}$	$\frac{1}{12}\Lambda_{\xi\ ij}^{(d_L)}$
$(C_{qq}^{(1)})^{ijkl}$	$\Lambda_{\xi\ ij}^{(d_L)}\Lambda_{\xi\ kl}^{(d_L)}\frac{\delta_{ik}\delta_{jl}-2}{72}$	C_{ee}^{3333}	$-\frac{1}{2}$
C_{uu}^{3333}	$-\frac{2}{9}$	C_{dd}^{3333}	$-\frac{1}{18}$
C_{eu}^{3333}	$\frac{2}{3}$	C_{ed}^{3333}	$-\frac{1}{3}$
$(C_{ud}^{(1)})^{3333}$	$\frac{2}{9}$	C_{le}^{2233}	$-\frac{1}{2}$
C_{lu}^{2233}	$\frac{1}{3}$	C_{ld}^{2233}	$-\frac{1}{6}$
C_{qe}^{ij33}	$\frac{1}{6}\Lambda_{\xi\ ij}^{(d_L)}$	$(C_{qu}^{(1)})^{ij33}$	$-\frac{1}{9}\Lambda_{\xi\ ij}^{(d_L)}$
$(C_{qd}^{(1)})^{ij33}$	$\frac{1}{18}\Lambda_{\xi\ ij}^{(d_L)}$	$(C_{\phi l}^{(1)})^{22}$	$\frac{1}{4}$
$(C_{\phi q}^{(1)})^{ij}$	$-\frac{1}{12}\Lambda_{\xi\ ij}^{(d_L)}$	$C_{\phi e}^{33}$	$\frac{1}{2}$
$C_{\phi u}^{33}$	$-\frac{1}{3}$	$C_{\phi d}^{33}$	$\frac{1}{6}$
$C_{\phi D}$	$-\frac{1}{2}$	$C_{\phi\Box}$	$-\frac{1}{8}$







`smelli` observables

- 167 quarks: P_5', $BR(B_s \to \mu^+\mu^-)$ and others with significant theory errors
- 21 LFU FCNCs: $R_K, R_{K^\star}$, $B \to$di-tau decays
- 31 EWPOs from LEP not assuming lepton flavour universality

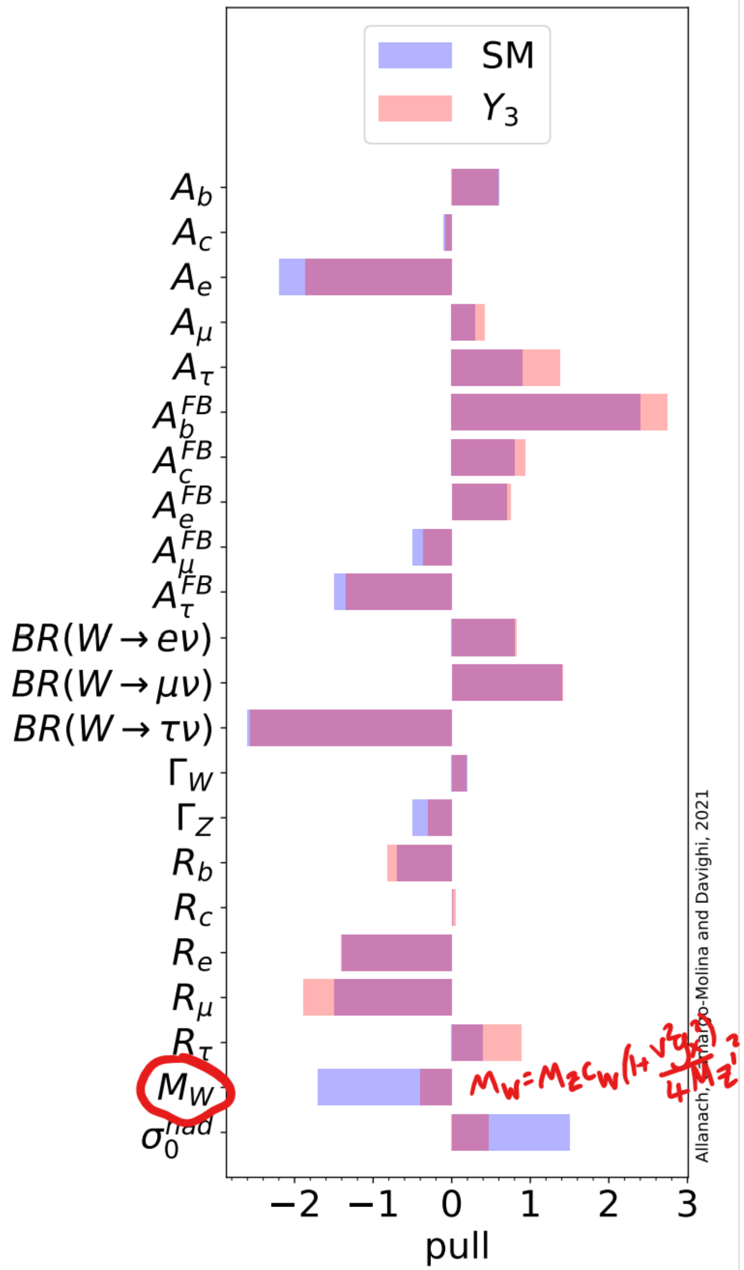
Theory uncertainties modelled as multi-variate Gaussians: approximated to be independent of new physics.

SM:

data set	χ^2	n	$p-$value
quarks	221.6	167	.003
LFU FCNCs	35.3	21	.026
EWPOs	35.7	31	.26
global	292.6	219	.00065

SM

Global Fits $M_{Z'} = 3$ TeV

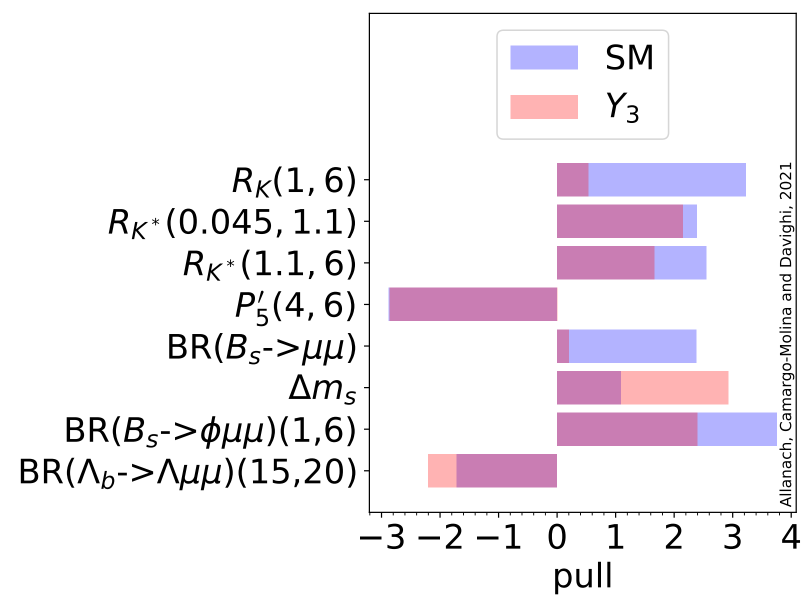


data set	χ^2	n	$p-$value
quarks	221.6	167	.003
LFU FCNCs	35.3	21	.026
EWPOs	35.7	31	.26
global	292.6	219	.00065

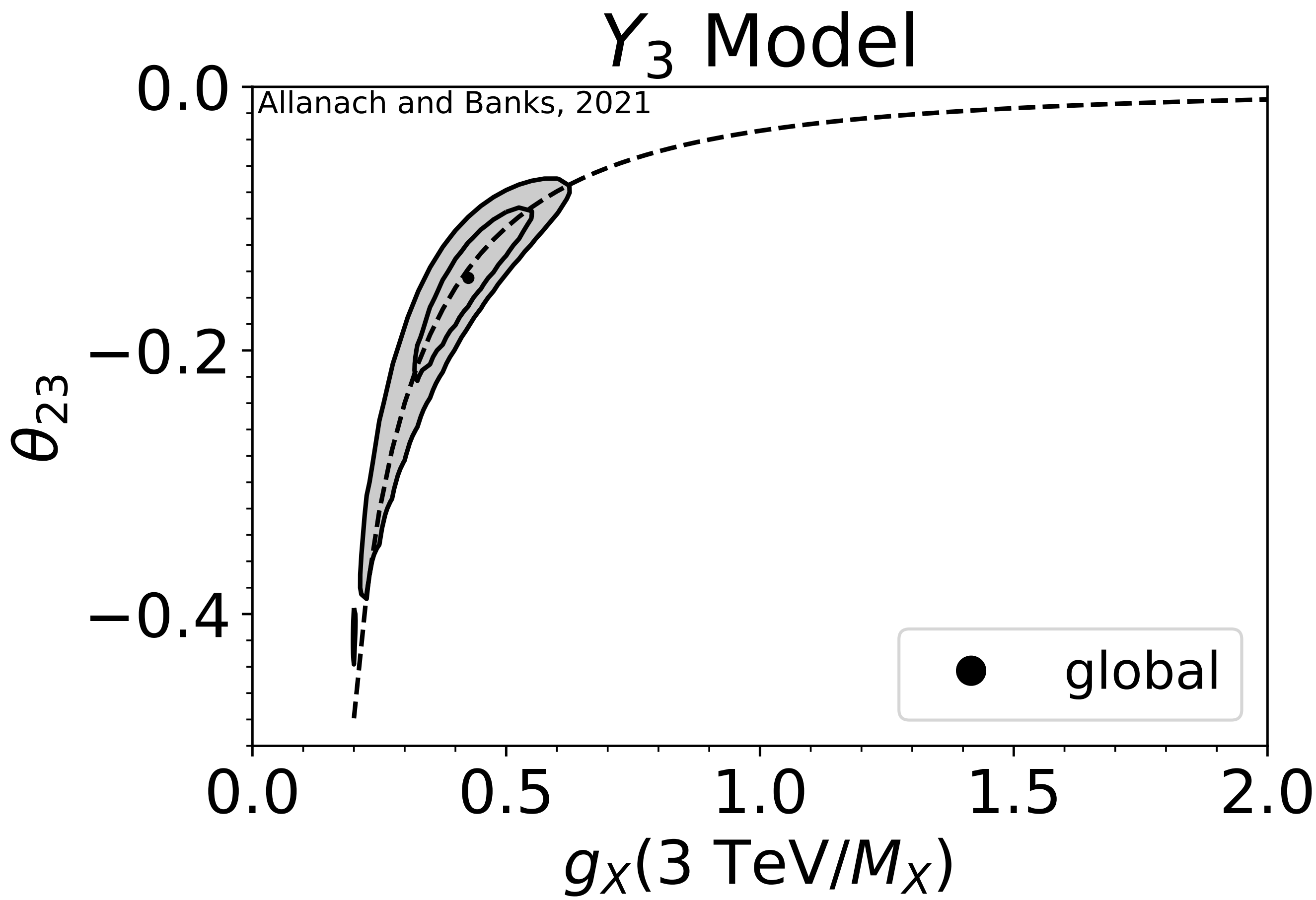
SM

data set	χ^2	n	$p-$value
quarks	192.5	167	.071
LFU FCNCs	21.0	21	.34
EWPOs	36.0	31	.17
global	249.5	219	.064

Y_3



TFHM Fit, 95% CL



Relies on: `smelli-2.2.0` (Aebischer, Kumar, Stangl, Straub, `1810.07698`), `flavio-2.2.0` (Straub, `1810.08132`), `Wilson` (Aebischer *et al*, `1712.05298`)

$Z' \to \mu\mu$ ATLAS 13 TeV 139 fb^{-1}

ATLAS analysis: look for two track-based isolated μ, $p_T > 30$ GeV. One reconstructed primary vertex. Keep only highest scalar sum p_T pair[7]

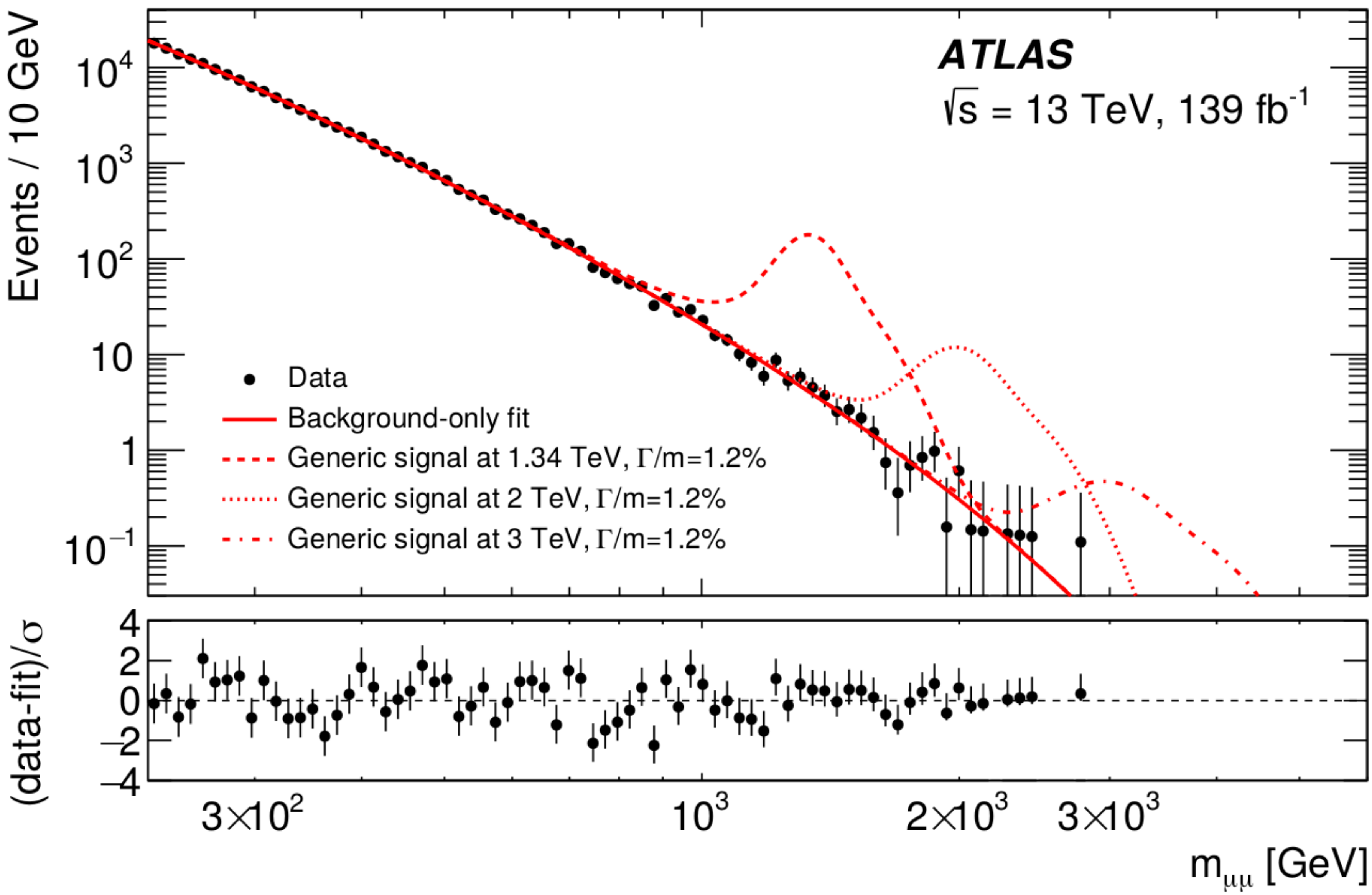
$$m^2_{\mu_1\mu_2} = (p_1^\mu + p_2^\mu)\left(p_{1_\mu} + p_{2_\mu}\right)$$

CMS also have released[8] a 139 fb^{-1} analysis.

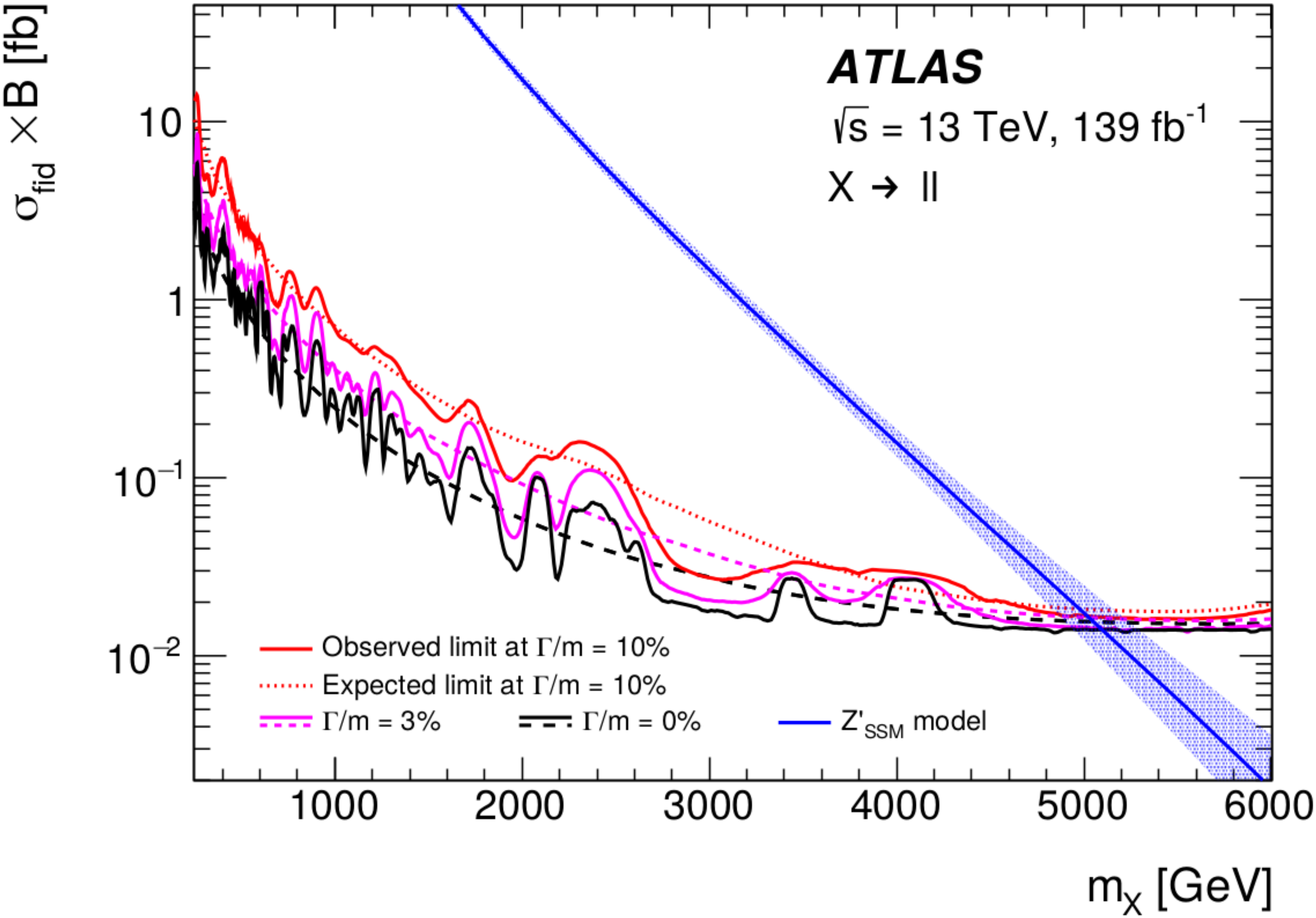
---

[7] `1903.06248`

[8] `2103.02708`


ATLAS
√s = 13 TeV, 139 fb-1
Data
Background-only fit
Generic signal at 1.34 TeV, Γ/m=1.2%
Generic signal at 2 TeV, Γ/m=1.2%
Generic signal at 3 TeV, Γ/m=1.2%
Events / 10 GeV
(data-fit)/σ
m_μμ [GeV]

ATLAS l^+l^- limits



1903.06248

CDF II M_W

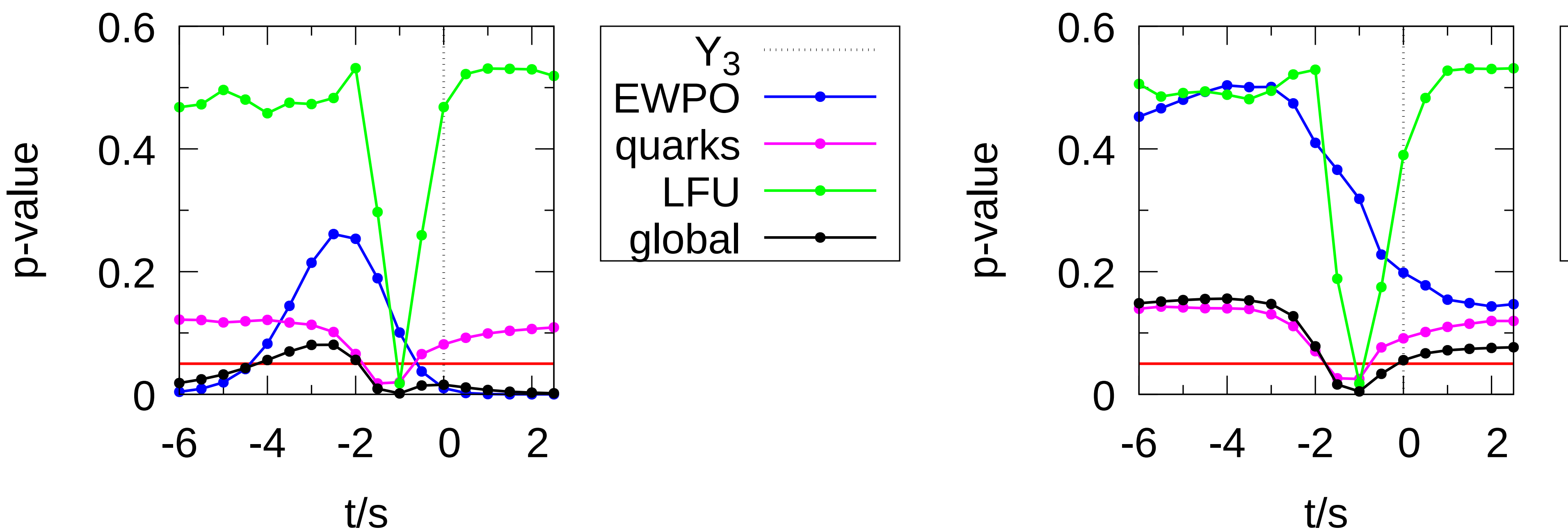
As already noted, $Z - Z'$ mixing implies

$$M_W = \rho_0 M_Z \cos\hat{\theta}_W$$

where

$$\begin{aligned} \rho_0(SM) &= (1.01019 \pm 0.00009), \\ \rho_0(Y_3) &\approx 1 + \frac{X_H^2 g_X^2}{g^2 + g'^2} \frac{M_Z^2}{M_{Z'}^2} > 1. \end{aligned}$$

$sY_3 + t(B_3 - L_3)$

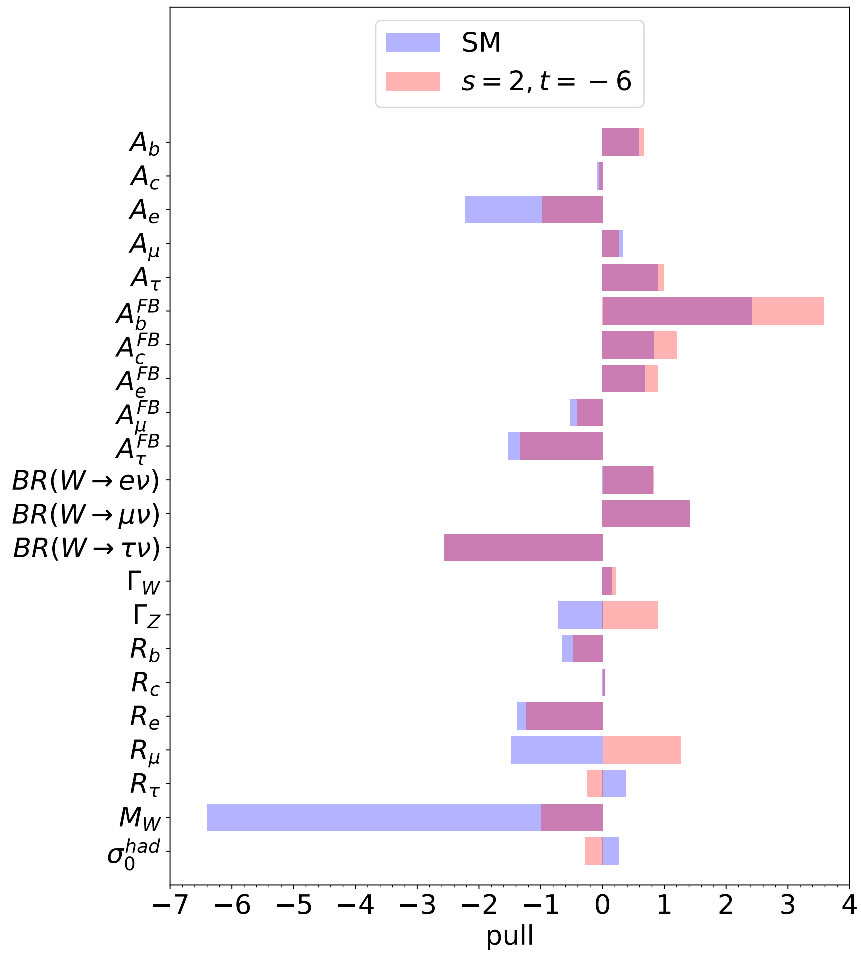


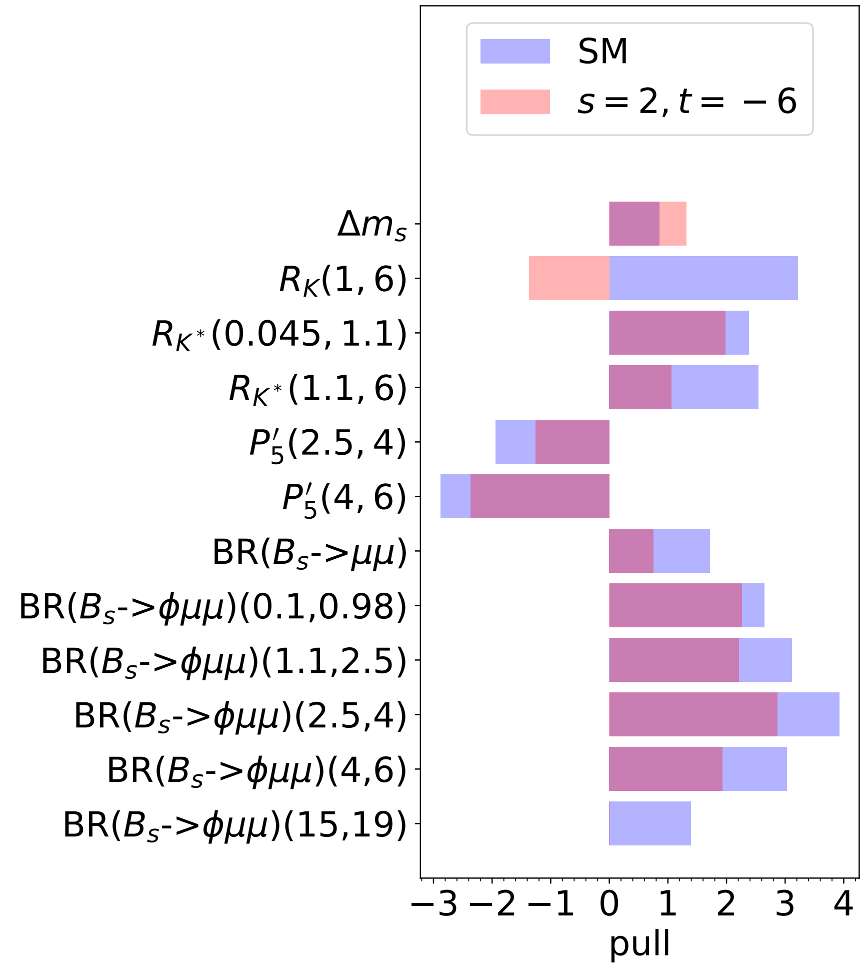
Left incl CDF II M_W, Right excl

BCA, Davighi, 2205.12252

Pick $Y_3 - 3(B_3 - L_3)$ as a well fit example

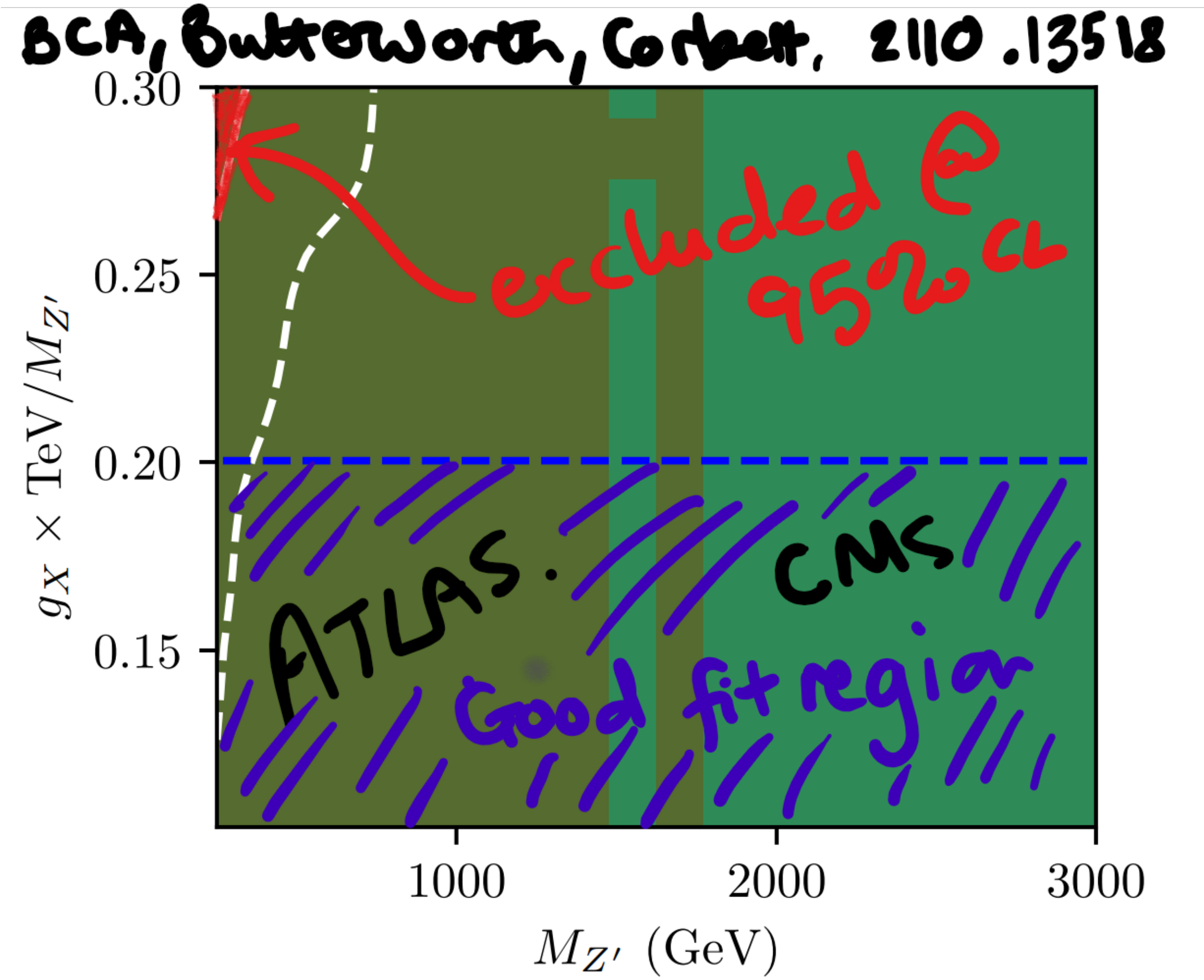
Best-fit point: incl CDF M_W



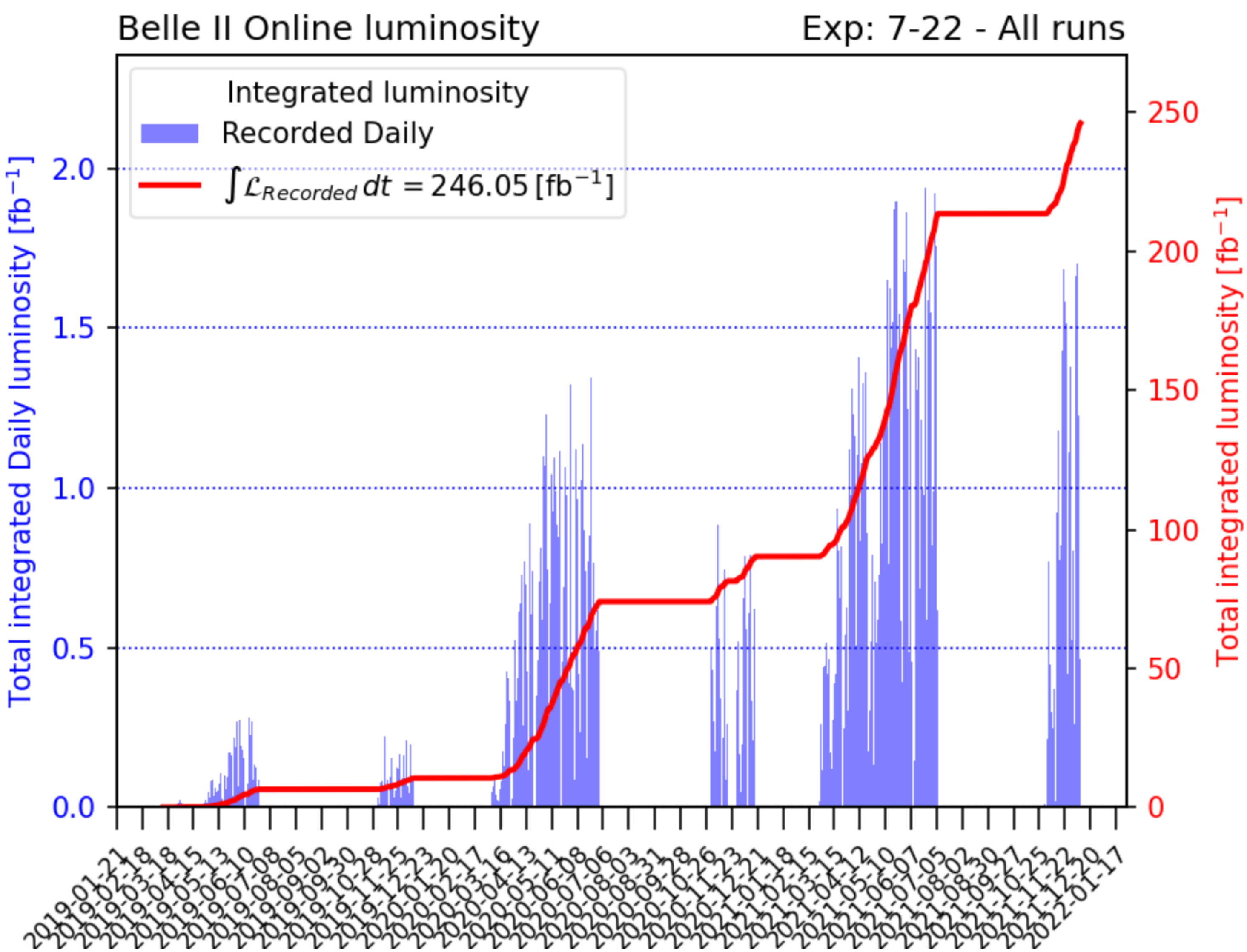


$$g_X = 0.021 \times 1 \text{ TeV}/M_{Z'},\ \theta_{23} = -0.0191,\ p = .08$$

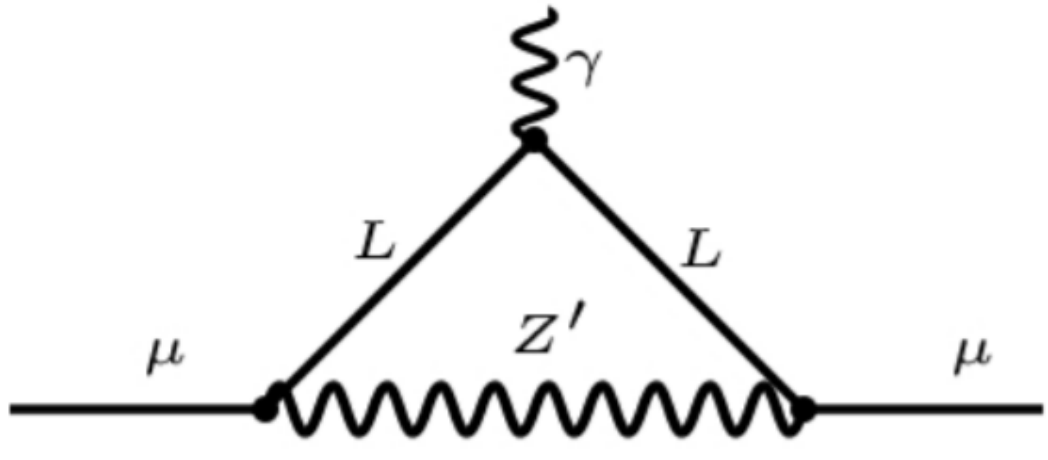
TFHM $Z' \to \mu^+\mu^-$+SM obs

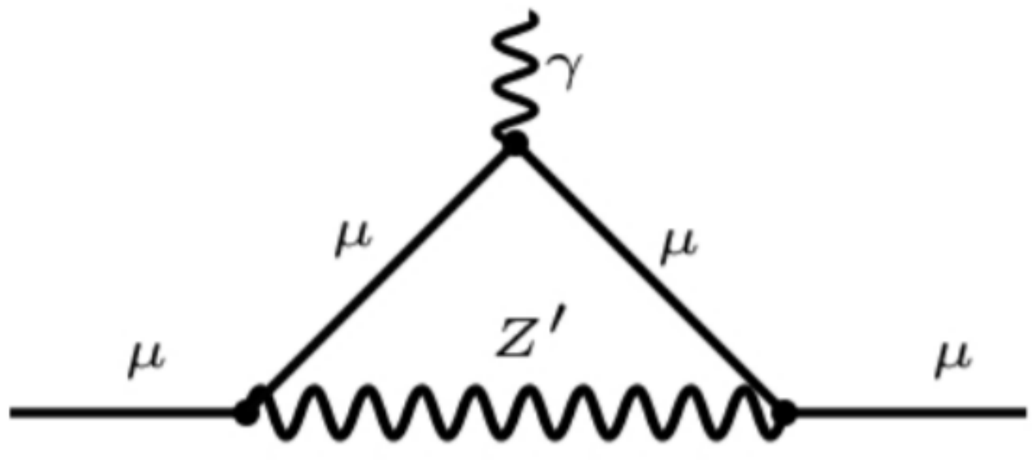


$$\mathbf{1\ fb^{-1}} \approx 10^6 B\bar{B}$$



$(g-2)_\mu$





Trident Neutrino Process

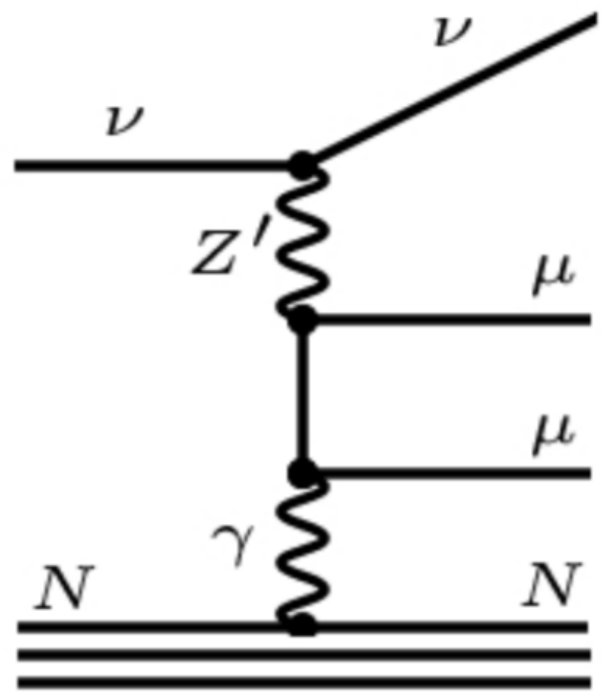
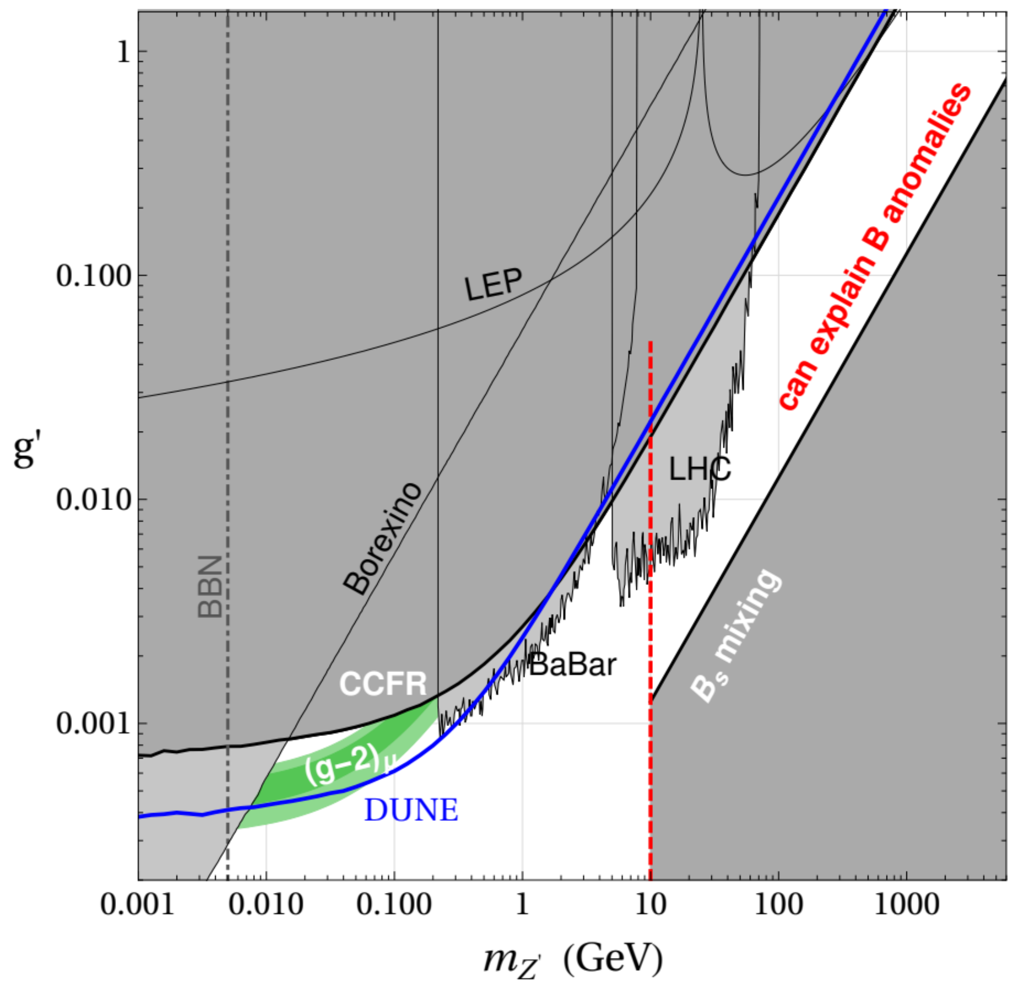


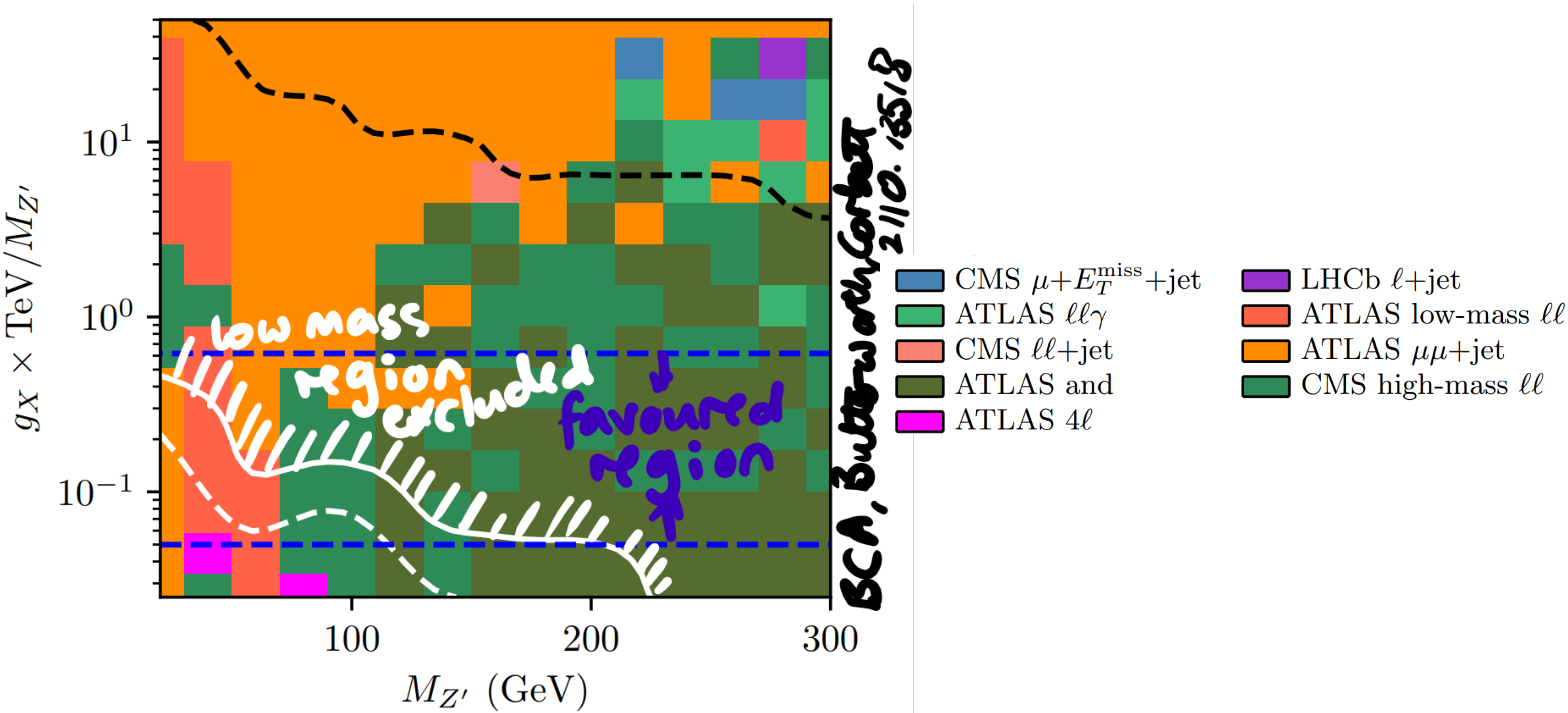
FIG. 10. Neutrino trident process that leads to constraints on the Z^{μ} coupling strength to neutrinos-muons, namely $M_{Z'}/g_{v\mu} \gtrsim 750$ GeV.

Light Z' for $(g-2)_\mu$: $L_\mu - L_\tau$

Altmannshofer, Gori, Martin-Albo, Sousa, Wallbank 1902.06765



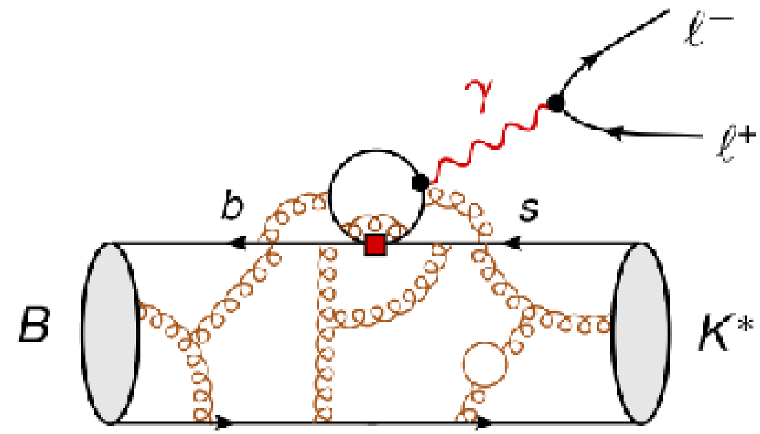
$B_3 - L_2$ model's[9] Z'



[9] Bonilla, Modak, Srivastava, Valle, 1705.00915, Alonso, Cox, Han, Yanagida 1705.03858

Hadronic Uncertainties

- Hadronic effects like charm loop are photon-mediated ⇒ vector-like coupling to leptons just like C_9



- How to disentangle NP $\leftrightarrow$ QCD?
  - Hadronic effect can have different q^2 dependence
  - Hadronic effect is lepton flavour universal ($\to R_K$!)

Wilson Coefficients c^l_{ij}

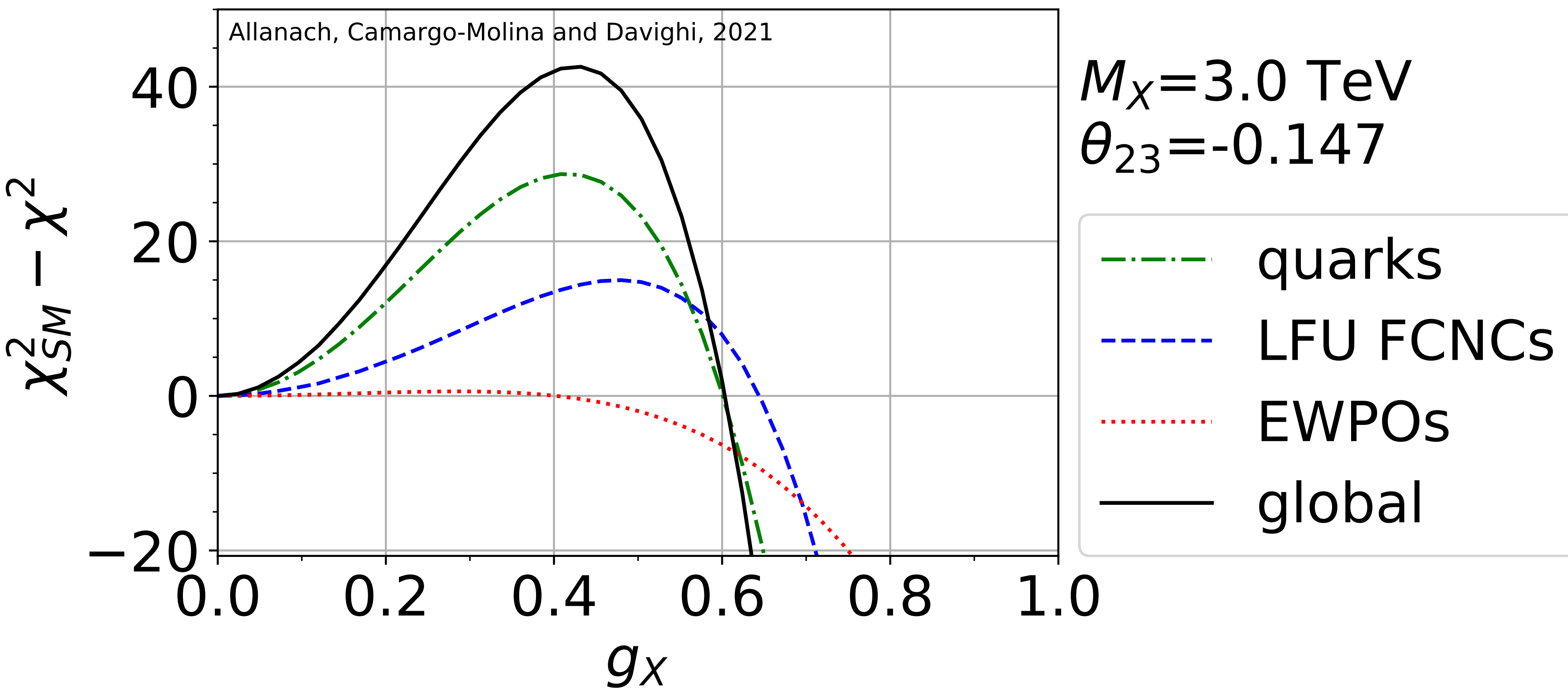
In SM, can form an EFT since $m_B \ll M_W$:

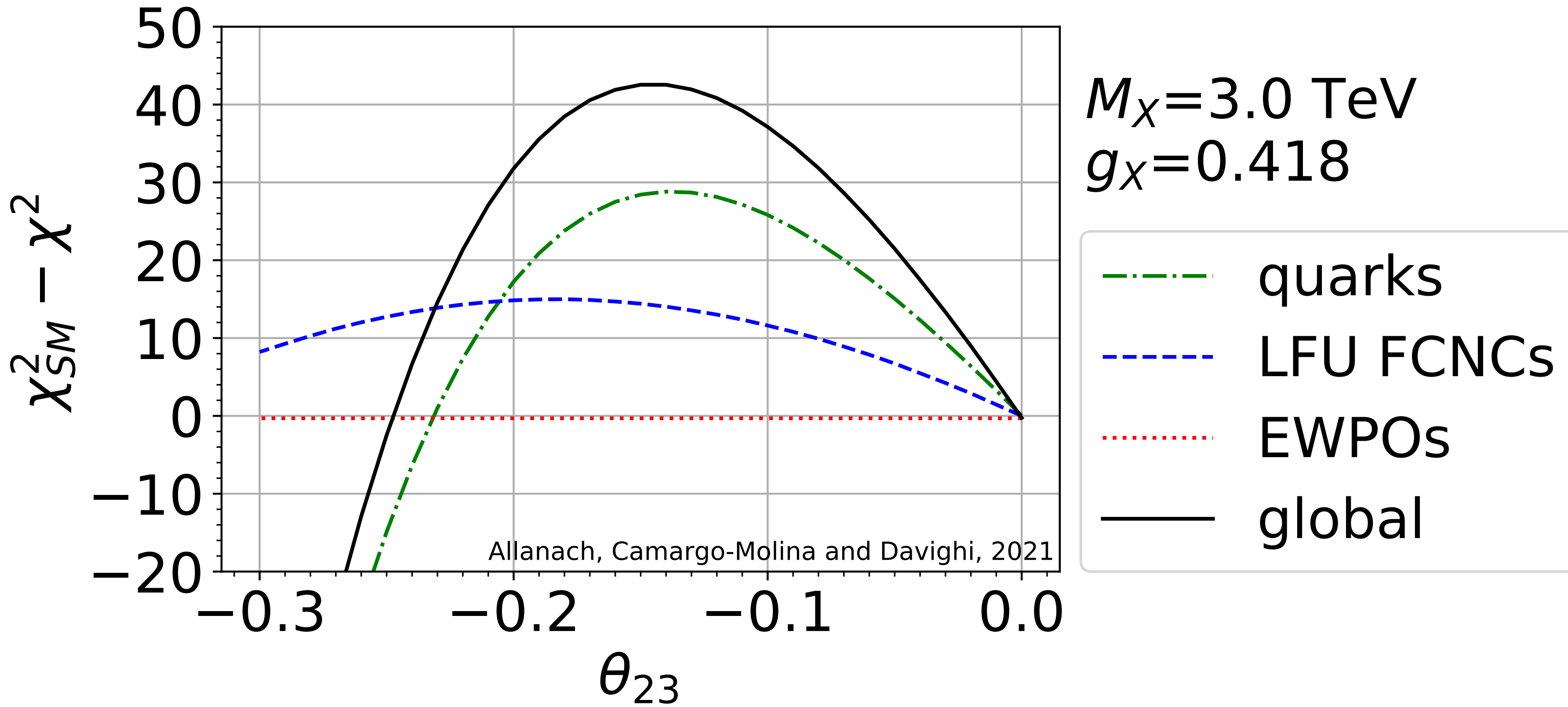
$$\mathcal{L}_{\mathrm{eff}} = \frac{1}{(36\ \mathsf{TeV})^2} c^l_{ij} (\bar{s}\gamma^\mu P_i b)(\bar{l}\gamma_\mu P_j l) \tag{1}$$

One loop weak interactions give $c^l_{ij} \sim \pm\mathcal{O}(1)$ in SM. $(1/36\ \mathsf{TeV})^2 = V_{tb}V^*_{ts}\alpha/(4\pi v^2)$.

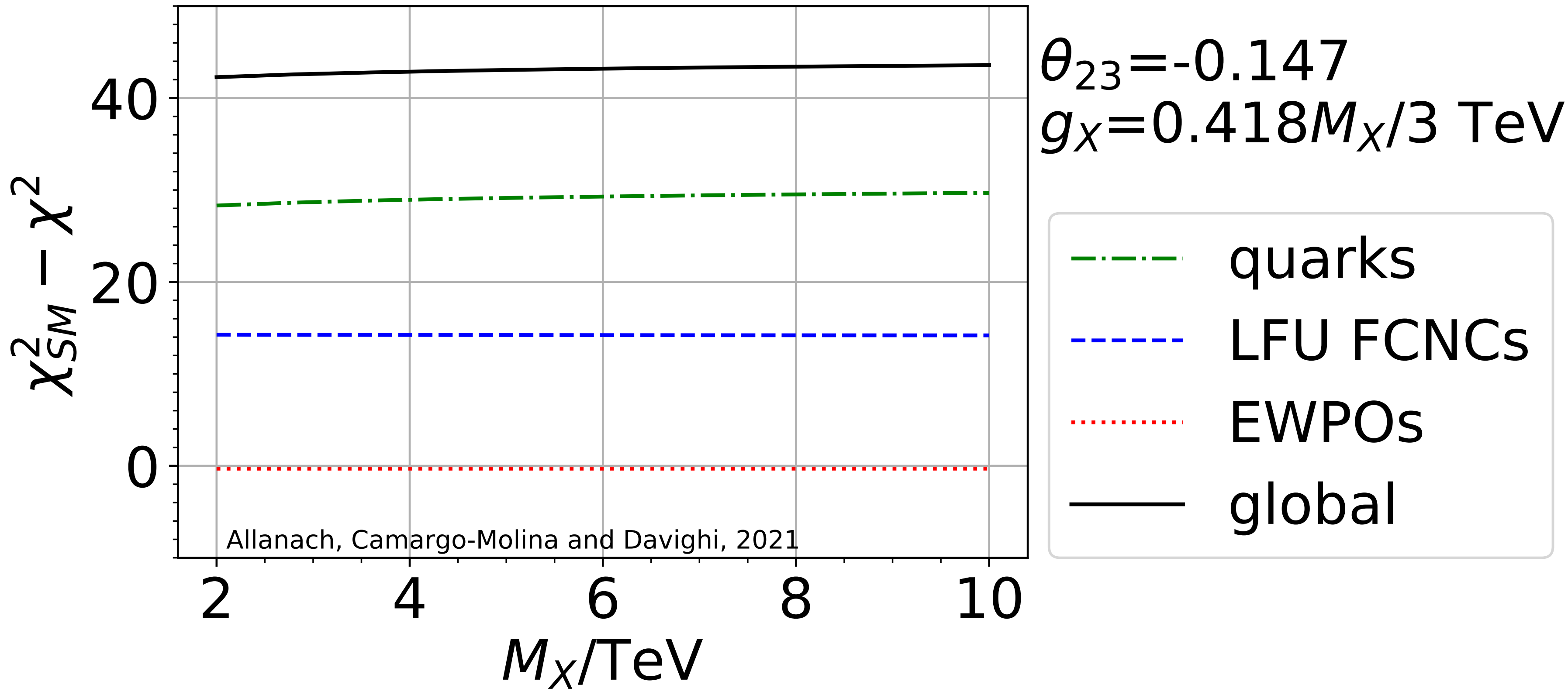
From now on, c^l_{ij} refer to *beyond* SM contribution.

TFHM Near best-fit point





50
40
30
20
10
0
−10
−20
$\chi^2_{SM} - \chi^2$
−0.3
−0.2
−0.1
0.0
θ_{23}
M_X=3.0 TeV
g_X=0.418
quarks
LFU FCNCs
EWPOs
global
Allanach, Camargo-Molina and Davighi, 2021



θ_{23}=-0.147
g_X=0.418M_X/3 TeV
$\chi^2_{SM} - \chi^2$
M_X/TeV
quarks
LFU FCNCs
EWPOs
global
Allanach, Camargo-Molina and Davighi, 2021

Which Ones Work?

Options *for a single* *BSM* *operator*:

- c^e_{ij} operators fine for $R_{K^{(*)}}$ but are disfavoured by global fits including other observables.
- c^μ_{LR} disfavoured: predicts *enhancement* in both R_K and R_{K^*}
- c^μ_{RR}, c^μ_{RL} disfavoured: they pull R_K and R_{K^*} in *opposite directions*.
- $c^\mu_{LL} = -1.06$ fits well globally[10].

[10] D'Amico et al, 1704.05438; Aebischer et al 1903.10434.

Invisible Width of Z Boson

$\Gamma_{\text{inv}}^{(\text{exp})} = 499.0 \pm 1.5$ MeV, whereas $\Gamma_{\text{inv}}^{(\text{SM})} = 501.44$ MeV.

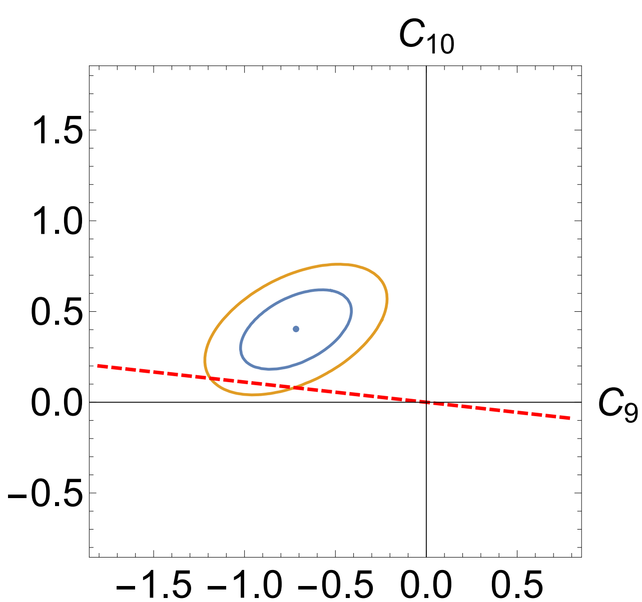
$$\Rightarrow \Delta\Gamma^{(\text{exp})} = \Gamma_{\text{inv}}^{(\text{exp})} - \Gamma_{\text{inv}}^{(\text{SM})} = -2.5 \pm 1.5 \text{ MeV}.$$

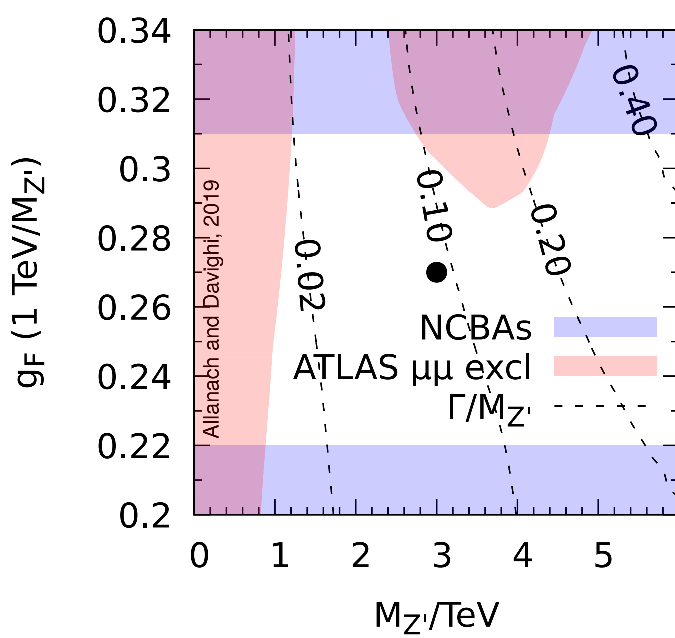
$$\begin{aligned}
\mathcal{L}_{\bar{\nu}\nu Z} \;=\; & -\frac{g}{2\cos\theta_w}\overline{\nu'_{Le}}\not{Z} P_L \nu'_{Le} \\
& -\overline{\nu'_{L\mu}}\left(\frac{g}{2\cos\theta_w} + \frac{5}{6} g_F \sin\alpha_z\right)\not{Z}\nu'_{L\mu} \\
& -\overline{\nu'_{L\tau}}\left(\frac{g}{2\cos\theta_w} - \frac{8}{6} g_F \sin\alpha_z\right)\not{Z}\nu'_{L\tau}.
\end{aligned}$$

Deformed TFHM

$$
\boxed{\begin{array}{cccc}
F_{Q'_i} = 0 & F_{u_{R'_i}} = 0 & F_{d_{R'_i}} = 0 & F_H = -1/2 \\
F_{e_{R'_1}} = 0 & F_{e_{R'_2}} = 2/3 & F_{e_{R'_3}} = -5/3 & \\
F_{L'_1} = 0 & F_{L'_2} = 5/6 & F_{L'_3} = -4/3 & \\
F_{Q'_3} = 1/6 & F_{u'_{R3}} = 2/3 & F_{d'_{R3}} = -1/3 & F_\theta \neq 0
\end{array}}
$$

$$
\mathcal{L} = Y_t \overline{Q'_{3L}} H t'_R + Y_b \overline{Q'_{3L}} H^c b'_R + H.c.,
$$





Neutrino Masses

At dimension 5:

$$\mathcal{L}_{SS} = \frac{1}{2M}({L'_3}^T H^c)(L'_3 H^c),$$

but if we add RH neutrinos, then integrate them out

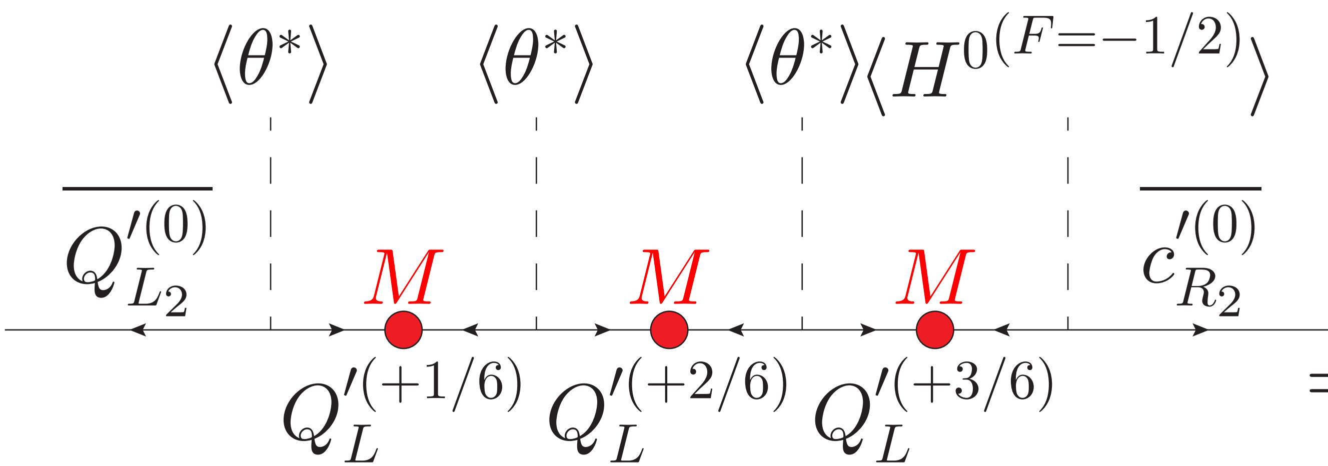
$$\mathcal{L}_{SS} = 1/2 \sum_{ij} (L'_i H^c)(M^{-1})_{ij}(L'_j H^c),$$

where now $(M^{-1})_{ij}$ may well have a non-trivial structure. If $(M^{-1})_{ij}$ are of same order, large PMNS mixing results.

Froggatt Neilsen Mechanism[11]

A means of generating the non-renormalisable Yukawa terms, e.g. $X_\theta = 1/6$:

$$Y_c \overline{Q'_{L2}}^{(F=0)} H^{(F=-1/2)} c'^{(F=0)}_R \sim \mathcal{O}\left[\left(\frac{\langle\theta\rangle}{M}\right)^3 \overline{Q'_{L2}} H c'_R\right]$$



eg $\left(\frac{\langle\theta\rangle}{M}\right) \sim 0.2$

$\Rightarrow Y_c/Y_t \sim 1/100$

[11] C Froggatt and H Neilsen, NPB**147** (1979) 277