Genetic algorithms as a search tool in the string landscape

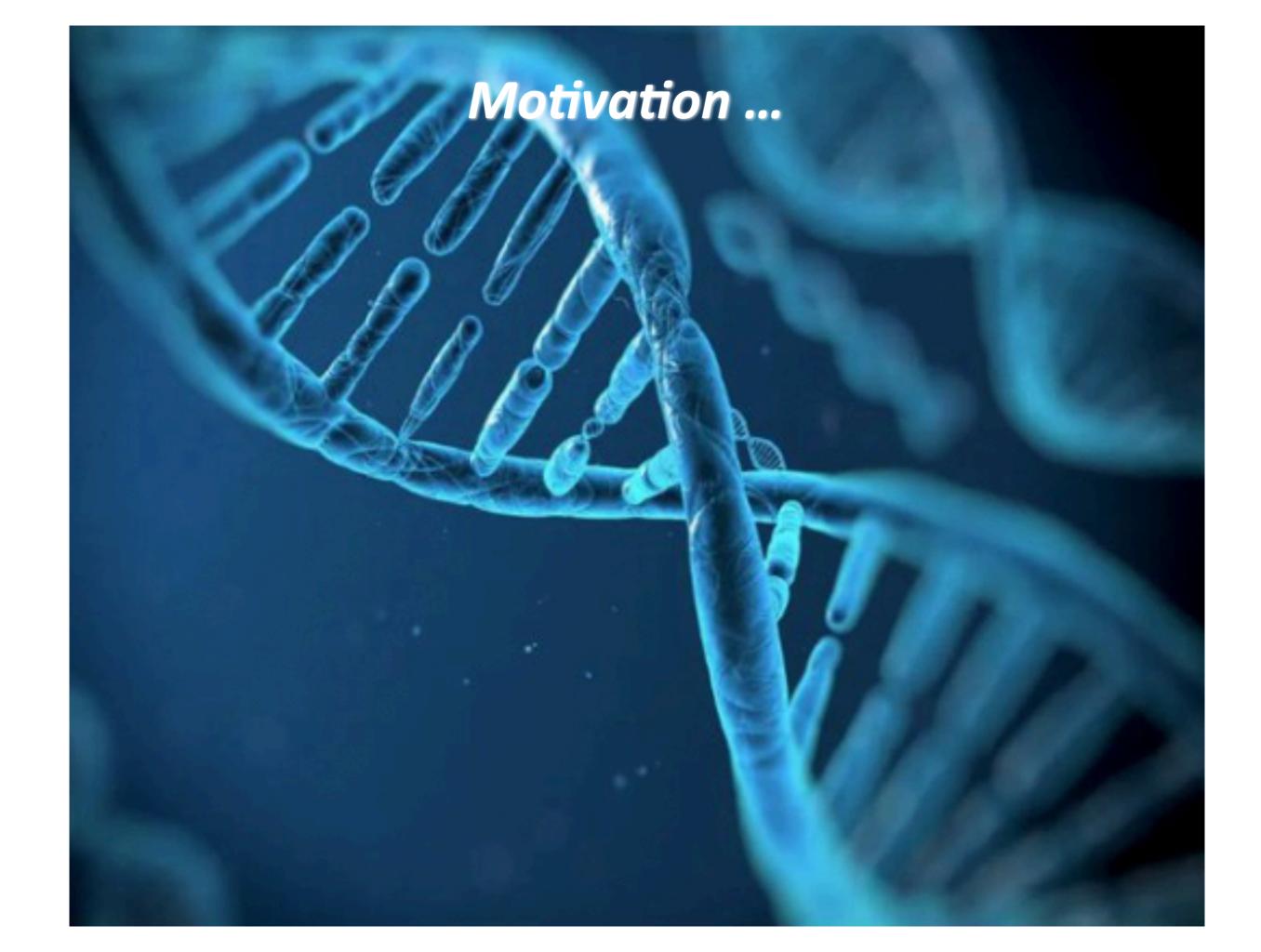
Steven Abel IPPP Durham

SAA, J.Rizos, JHEP 1408 (2014) 010,1404.7359 hep-th SAA, Constantin, Harvey, Lukas, *Fortsch.Phys.* 70 (2022) 5, 2200034 • e-Print: 2110.14029



The XXIX International Conference on Supersymmetry and Unification of Fundamental Interactions (SUSY 2022)

The one follows from everything and everything from the one" Heraclitus







- String theories typically produce vast theory spaces. e.g 10^500 or even 10^(272000)
 Denef, Douglas; Ashøk, Douglas; Denef, Douglas, Greene, Zokowski; Taylor, Wang
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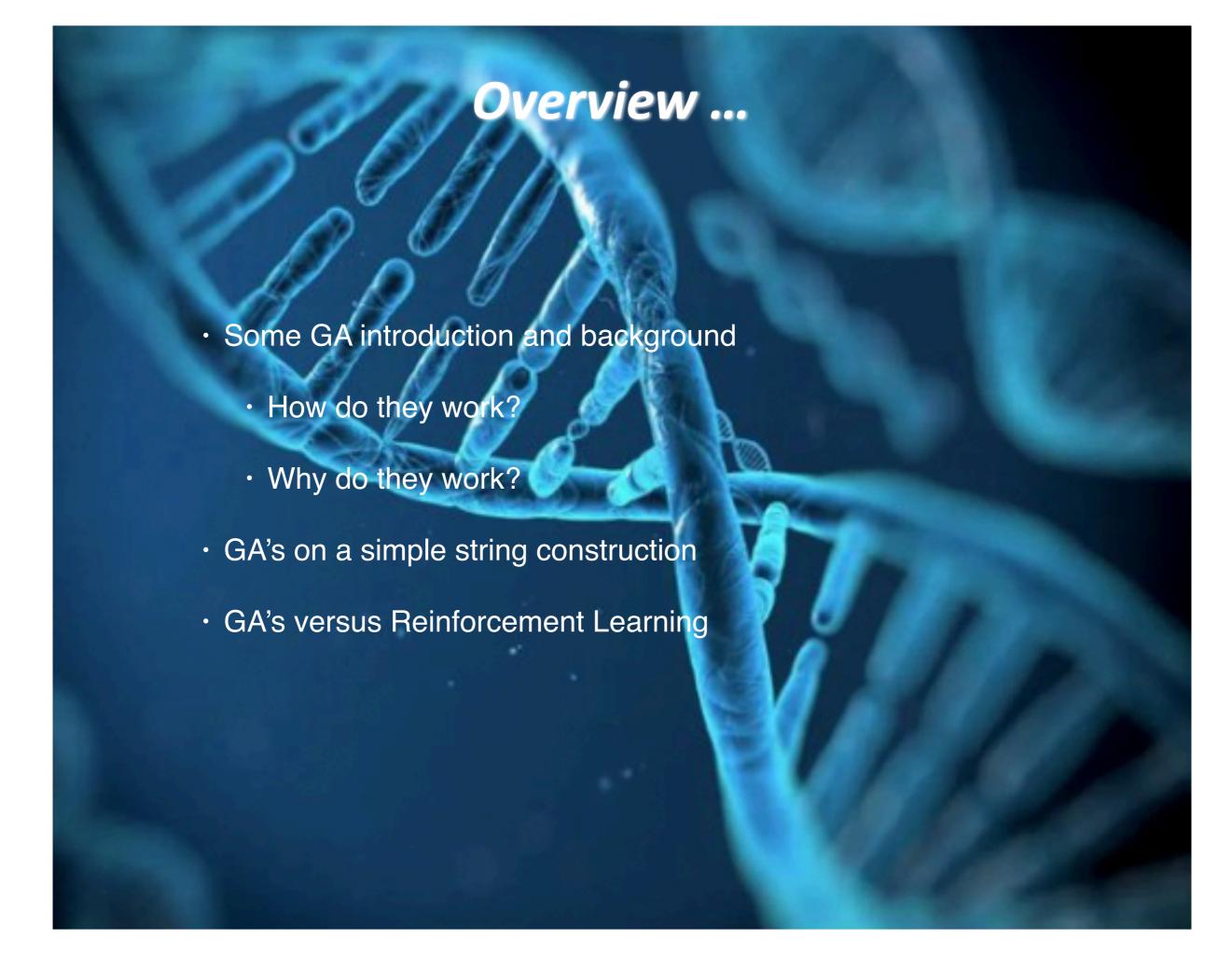
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- In recent years machine learning has been intensely studied as a means of learning about the "local string landscape".
- However there is a venerable technique that doesn't seem to be going away: Genetic
 Algorithms Turing; Barricelli; Fraser, Burnell; Crosby; Bremermann; Holland; Goldberg; Jones

GAs in particle physics ...

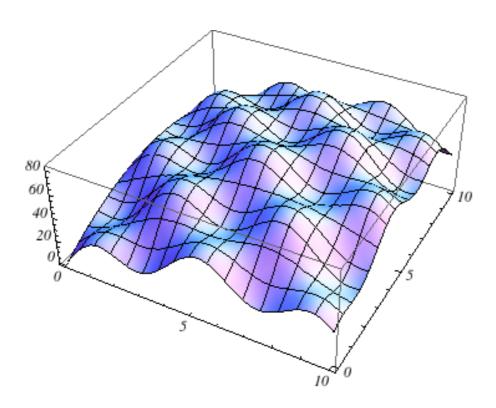
- Yamaguchi and H. Nakajima (2000)
- Allanach, Grellscheid, Quevedo (2004)
- Akrami, Scott, Edsjo, Conrad and Bergstrom (2009)
- Bl°aba¨ck, Danielsson and Dibitetto, (2013)
- SAA, Rizos (2014)
- Ruehle (2017)
- SAA, Cerdeno, Robles (2018) *PMSSM20
- Cole, Schachner, Shiu (2019)
- AbdusSalam, SAA, Cicoli, Quevedo, Shukla (2020)
- Bena, Bl°aba¨ck, Grana, Luest (2021)
- SAA, Constantin, Lukas, Harvey (2021)
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GA's for searching a string sized landscape

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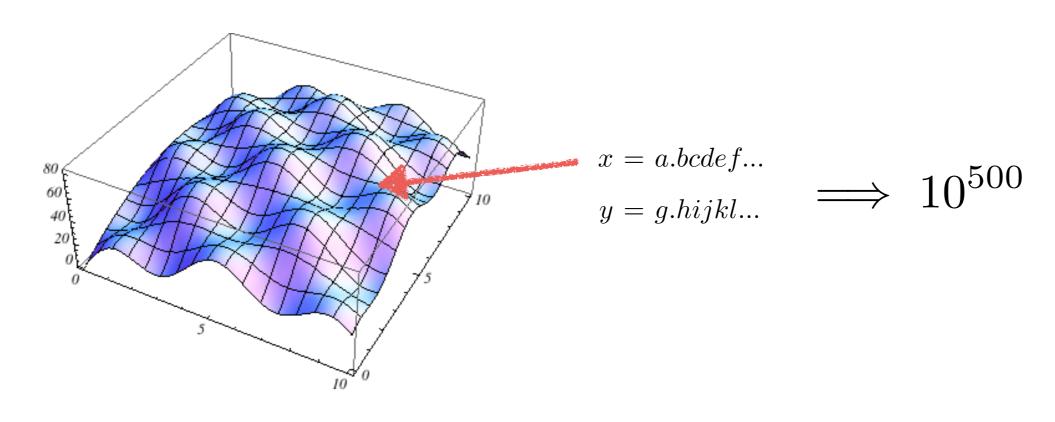
Example landscape task: find global maximum to 250 decimal places without using calculus ...



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GA's for searching a string sized landscape

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Define a "creature" and write out its coordinates => genotype

Terminology: Genotype = data. Phenotype = f(x,y).

$$x = a.bcdef...$$

 $y = g.hijkl...$

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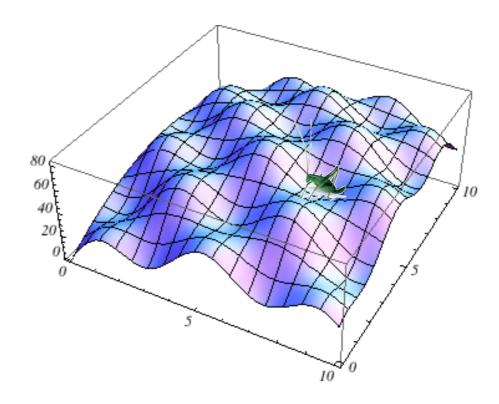
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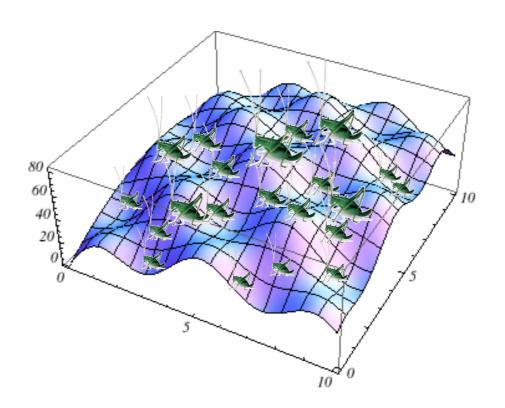
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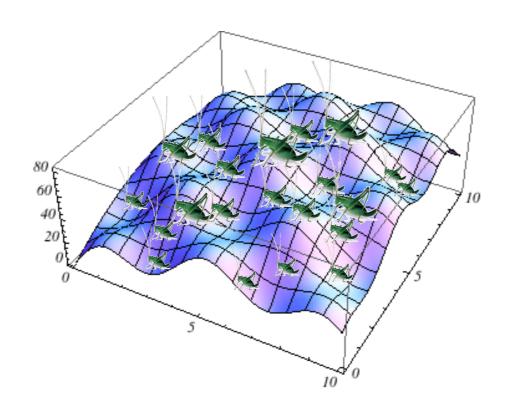
Work with a population of typically ~100 individuals initially sprinkled at random

Step 0: Define fitness function, and work out the fitness F of each individual (e.g. F = f(x,y) in this case).



Step 1: Selection: Select pairs for breeding such that the most fit individuals can breed several times, while unfit ones might not breed at all: e.g. "roulette wheel" based on ranking k, with $P_1=\alpha P_{N_{\mathrm{pop}}}$:

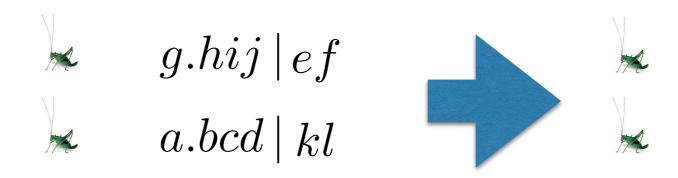
$$P_k = \frac{2}{(1+\alpha)N_{\text{pop}}} \left(1 + \frac{N_{\text{pop}} - k}{N_{\text{pop}} - 1} (\alpha - 1) \right)$$



Step 2: breeding: cut and splice genotypes of breeding pairs somehow (not really crucial how) to make an entirely new population of the same size.

$$g.hij \mid kl$$
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Step 3: Mutation of a randomly chosen small percentage of digits (alleles).

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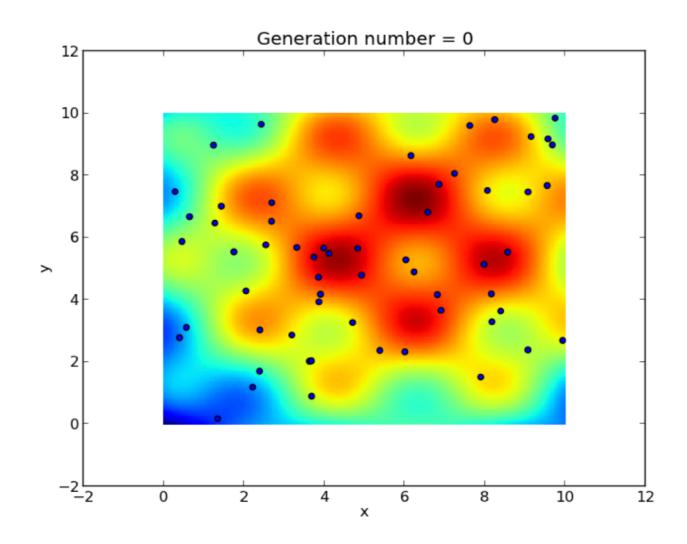
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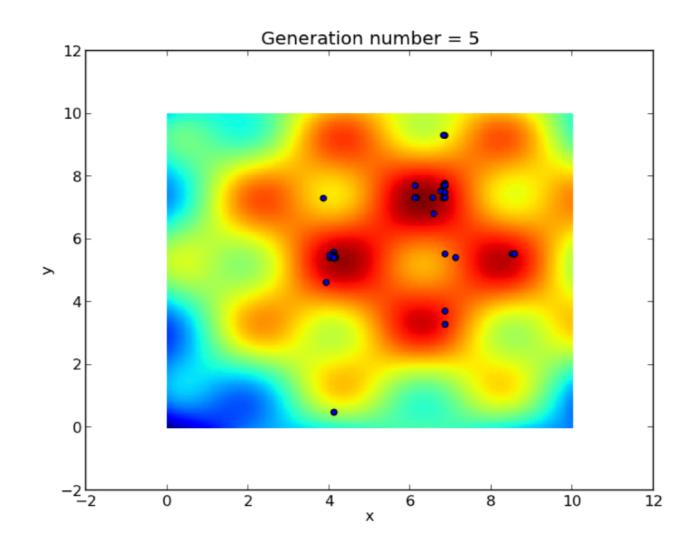
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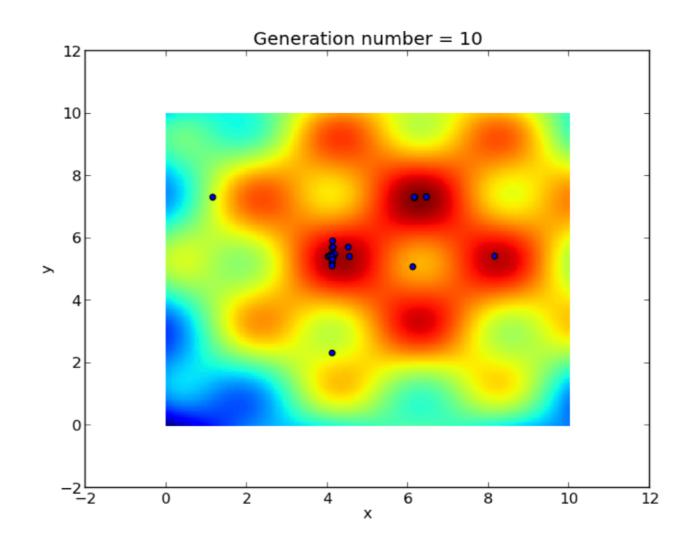
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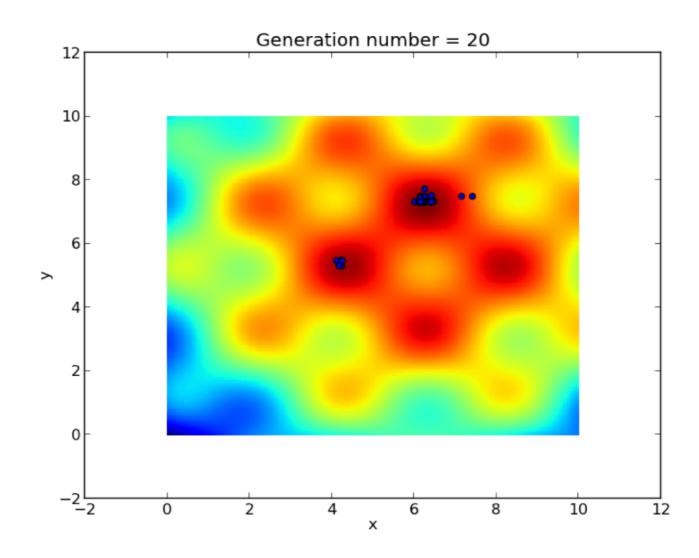


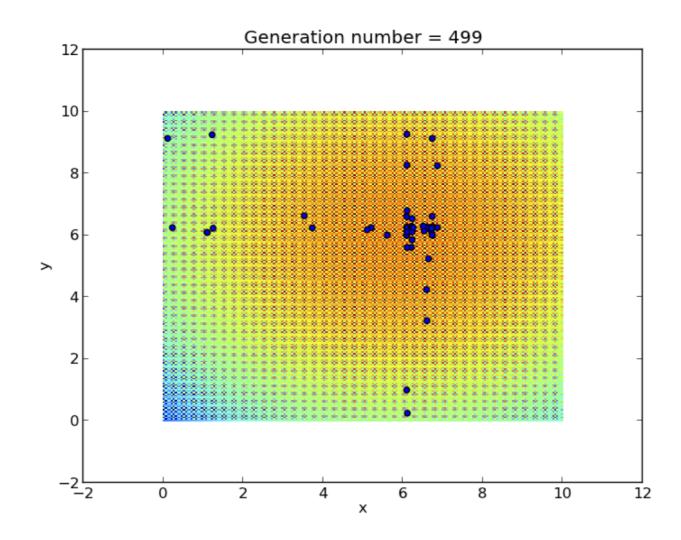
Steps 4 ... **infinity:** rinse and repeat. The population should converge round solutions.











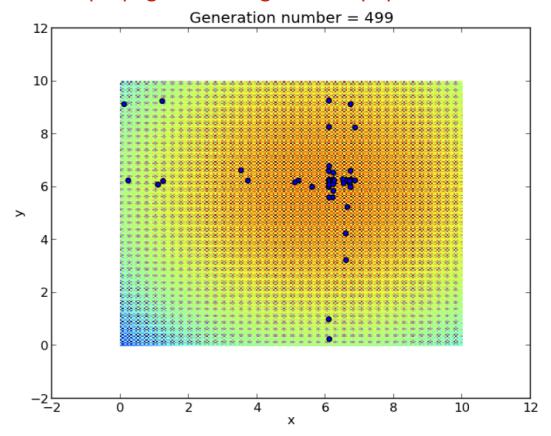
Why do they work?

- Holland proposed a probabilistic explanation for the efficiency of genetic algorithms: based on growth rate of "good" schema S , e.g. here S=61***62***
- Holland argues that initial growth of a good schema in the population is exponential
- Selection pushes towards convergence
- Mutation pushes system away from convergence
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In this example the leading digits of x and y are schemata and get propagated throughout the population



First string example

- Find a phenomenologically attractive Pati-Salam model.
- We will consider the "fermionic string construction". These are general 4D models in which the world sheet degrees of freedom are fermions. Kawai, Lewellyn, Tye; Antoniadis, Bachas, Kounnas
- PS Models are defined in terms of a set of basis vectors (for the experts) Faraggi, Kounnas, Nooij, Rizos

$$v_{1} = \mathbb{1} = \left\{ \psi^{\mu}, \ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8} \right\}$$

$$v_{2} = S = \left\{ \psi^{\mu}, \chi^{1,\dots,6} \right\}$$

$$v_{2+i} = e_{i} = \left\{ y^{i}, \omega^{i} | \bar{y}^{i}, \bar{\omega}^{i} \right\}, \ i = 1,\dots, 6$$

$$v_{9} = b_{1} = \left\{ \chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^{1}, \bar{\psi}^{1,\dots,5} \right\}$$

$$v_{10} = b_{2} = \left\{ \chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^{2}, \bar{\psi}^{1,\dots,5} \right\}$$

$$v_{11} = z_{1} = \left\{ \bar{\phi}^{1,\dots,4} \right\}$$

$$v_{12} = z_{2} = \left\{ \bar{\phi}^{5,\dots,8} \right\}$$

$$v_{13} = \alpha = \left\{ \bar{\psi}^{45}, \bar{y}^{1,2} \right\}.$$

• in addition to a set of GSO projection phases

$$c \begin{bmatrix} v_i \\ v_j \end{bmatrix}, i, j = 1, \dots, n$$

Our genotype will be these phases (think of the string construction as a black box that turns these numbers into phenomenological model)

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51 independent phases in these models: hence search space is

$$2^{51} = 2 \times 10^{15}$$

- (a) 3 complete family generations, $n_g = 3$
- (b) Existence of PS breaking Higgs, $k_R \geq 1$
- (c) Existence of SM Higgs doublets, $n_h \geq 1$
- (d) Absence of exotic fractional charge states, $n_e = 0$
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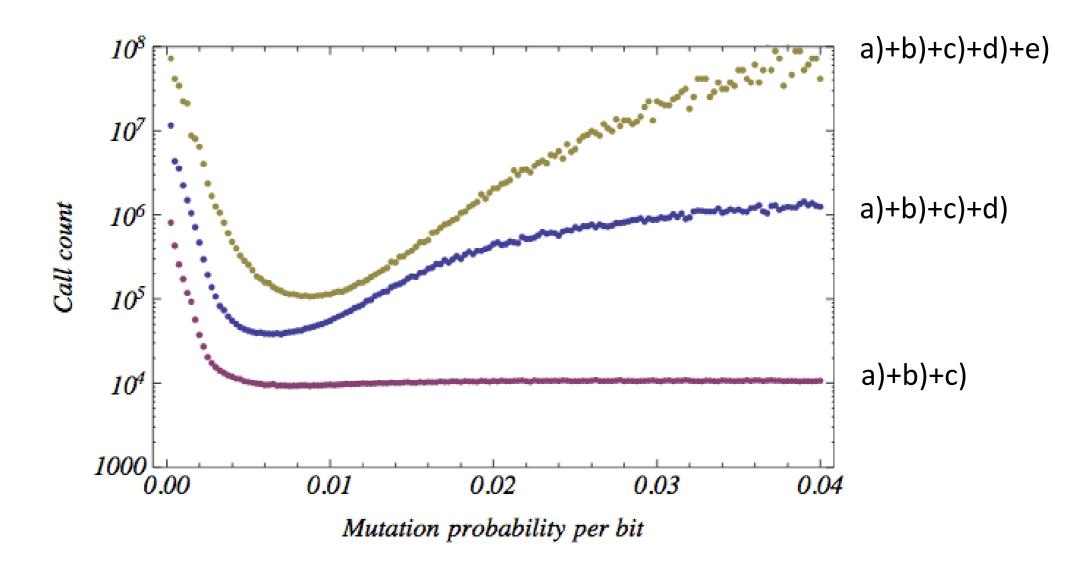
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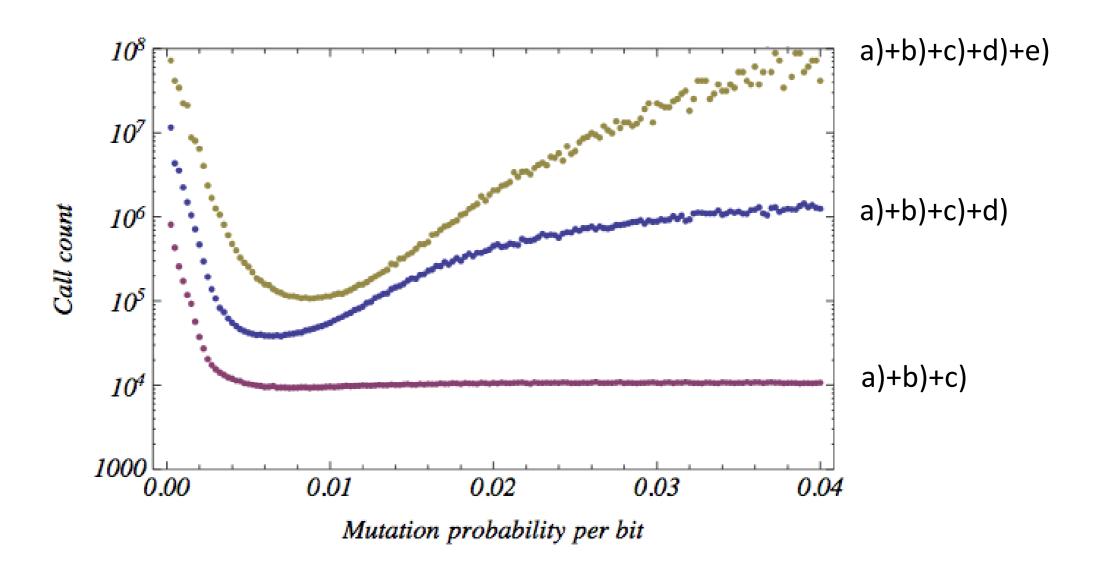
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This search space is (just about) searchable deterministically so we can compare the two methods. Assel, Christodoulides, Faraggi, Kounnas, Rizos

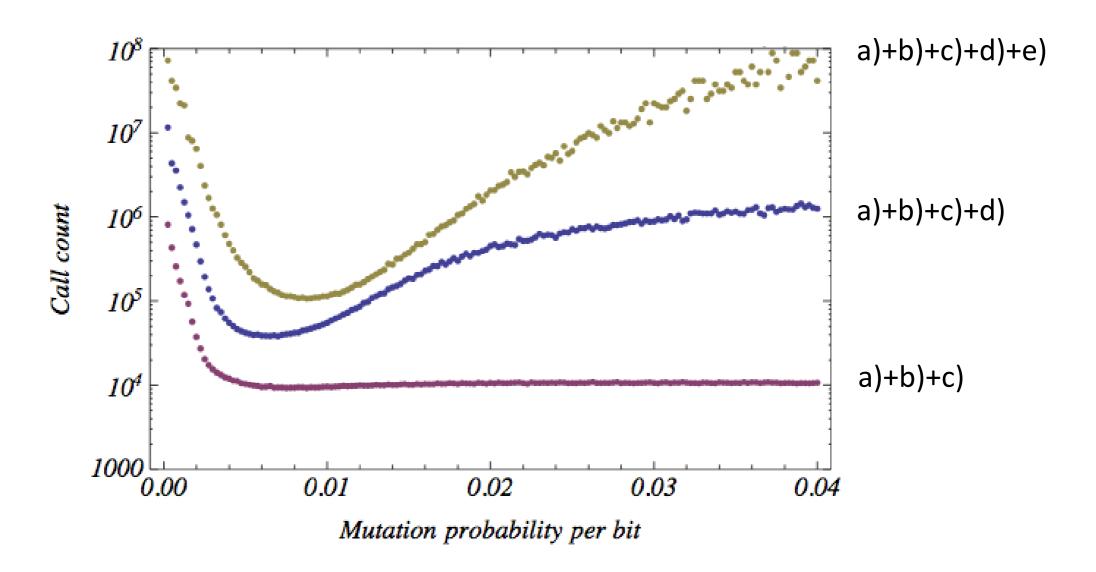
The phases determine the characteristics of the models

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- deterministically we would expect to have to construct 10 billion models to find an example of the latter

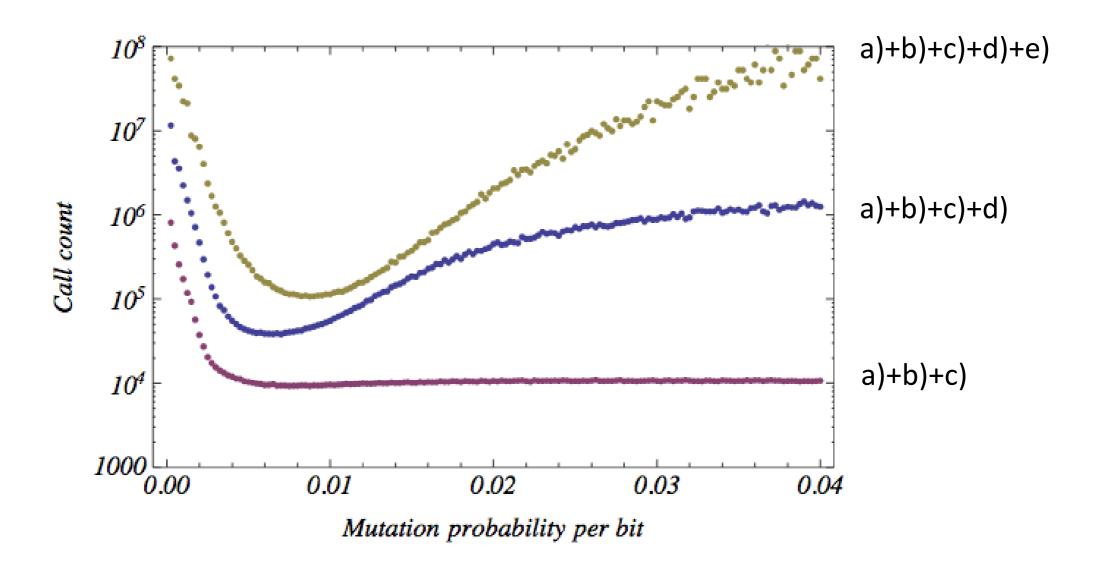




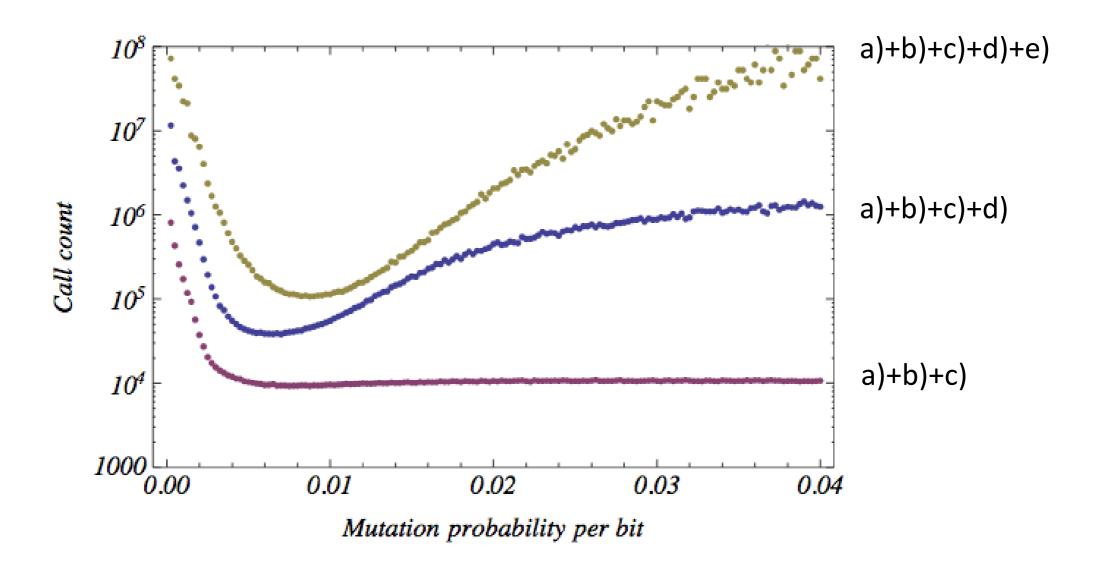
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GAs versus reinforcement learning

SAA, Constantin, Lukas, Harvey cent work by Cole, Schachner, Shiu: Cole,

(c.f. recent work by Cole, Schachner, Shiu; Cole, Krippendorf, Schachner, Shiu; Loges, Shiu)

- First comparison of GA versus RL in string context (NB techniques both work with "environment")
- Consider kind of string construction with a much larger space that has been the subject of intensive ML scrutiny: "Monad bundles on Complete Intersection Calabi Yaus". Distler, Greene; Kachru; Anderson; Anderson, He, Lukas; Anderson, Gray, He, Lukas; He, Lee, Lukas
- Considered the following two kinds of CICY (bi-cubic and triple trilinear respectively) with configuration matrices, where indices are h11, h21, and Euler number (for the experts):

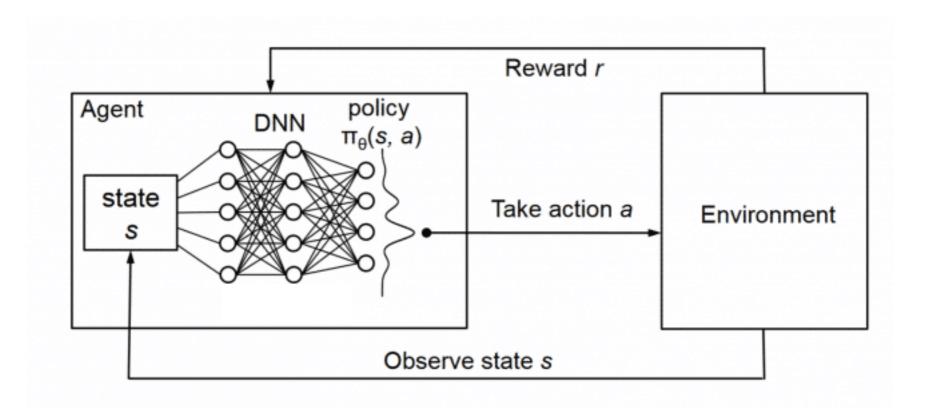
$$\left[\begin{array}{c|c|c} \mathbb{P}^2 & 3 \\ \mathbb{P}^2 & 3 \end{array}\right]_{-162}^{2,83}, \qquad \left[\begin{array}{c|c|c} \mathbb{P}^2 & 1 & 1 & 1 \\ \mathbb{P}^2 & 1 & 1 & 1 \\ \mathbb{P}^2 & 1 & 1 & 1 \end{array}\right]_{-90}^{3,48}$$

• Models constructed by monad bundles on the CICY defining the E8xE8 background: constructed from two line-bundle sums, B and C: in the end boils down to matrix of integers (where k=1,..,h11):

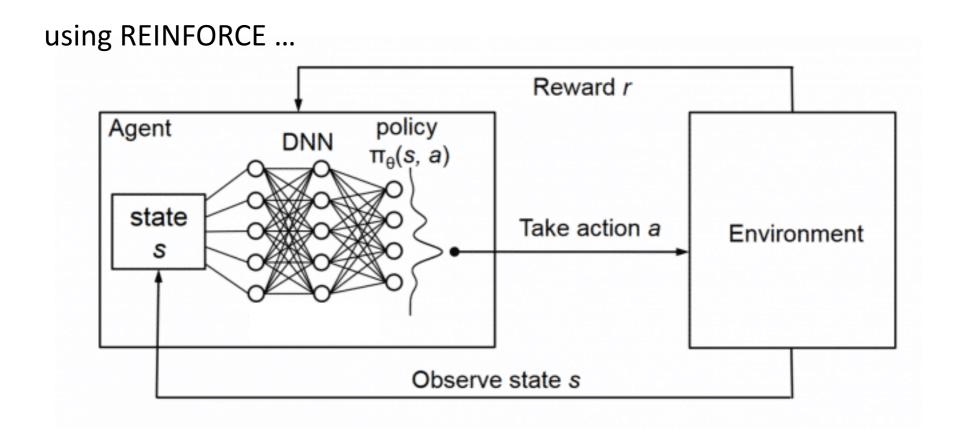
$$\left(b_1^k,\ldots,b_{r_B}^k,c_1^k,\ldots,c_{r_C}^k\right)$$

- As well as various consistency conditions (e.g. anomaly cancellation), all the phenomenological properties (e.g. number of generations) determined by these numbers via (several) index theorems. Constantin, Lukas, Harvey
- Search for "perfect-models" (aka "terminal states"): require SM-like theories (i.e. SO(10) GUT from broken E8, with 3 generations).
- So what is the size of search space? If we take $b_{\min} \leq b_i^k \leq b_{\max}$, $c_{\min} \leq c_a^k \leq c_{\max}$,
- Allowing say 10 values per entry, that is $10^{h^{1,1}(r_B+r_C-1)}$ with say h11=3 it becomes huge very quickly!
- For the GA we simply encode these integers as a single binary string and operate as before. Used quite large population = 250.
- In both RL and GA we use the same function to stand for the reward / fitness, based on the number of criteria that are satisfied.

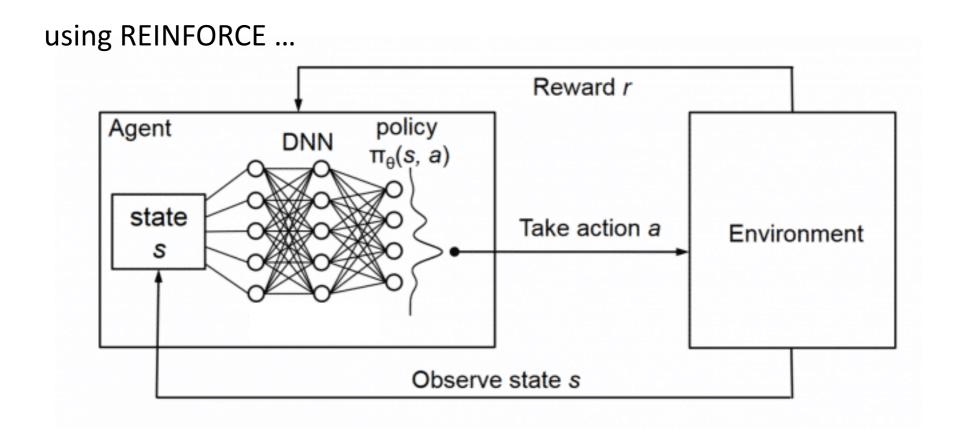
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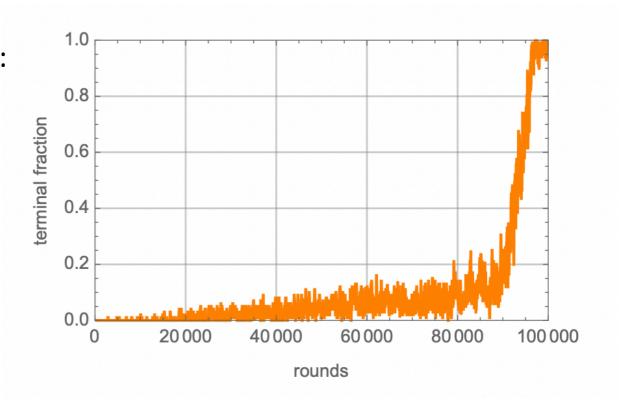


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We find good performance after a long training time: typical run on the (6,2) bi-cubic with

$$b_{\min} = -3, c_{\min} = 0$$
 $b_{\max} = 4, c_{\max} = 7$

for which the search space is $\simeq 4.4 \times 10^{12}$

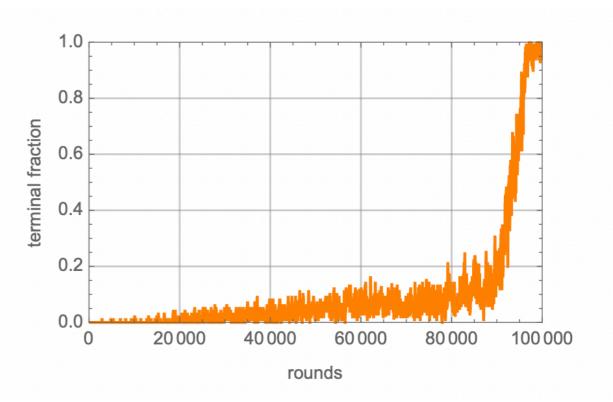


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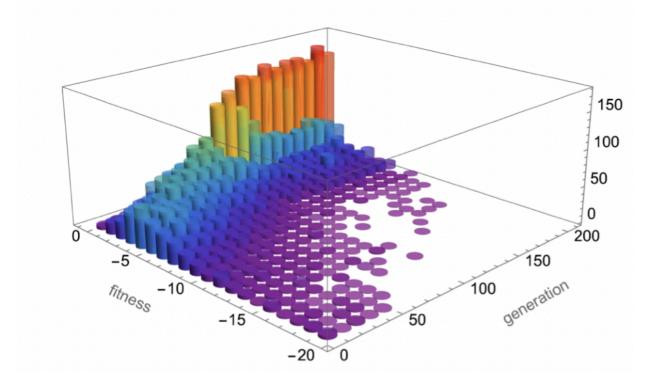
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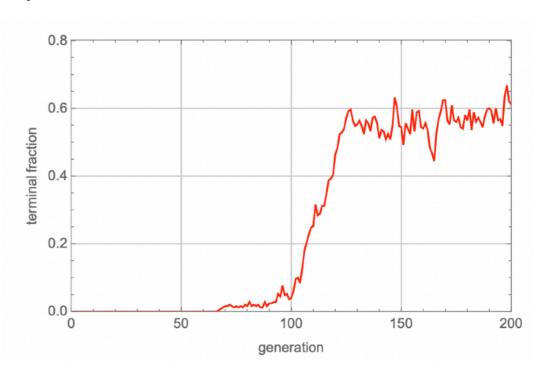
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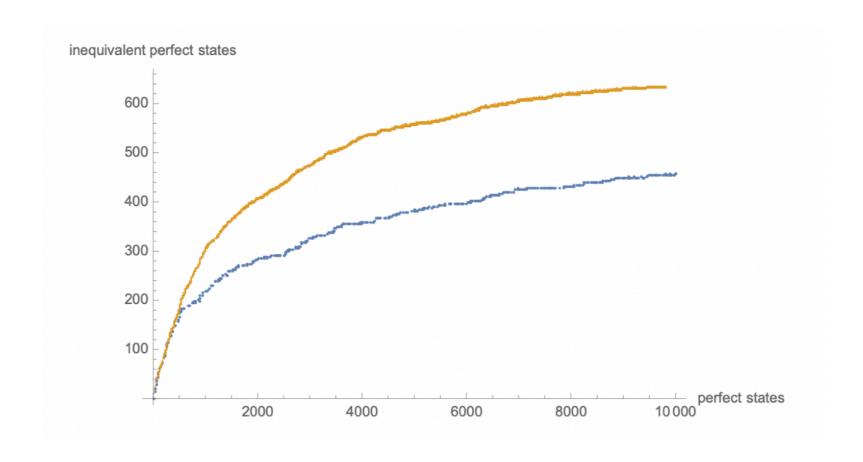


The GA is much faster to the first solutions! Note only 50K states visited:

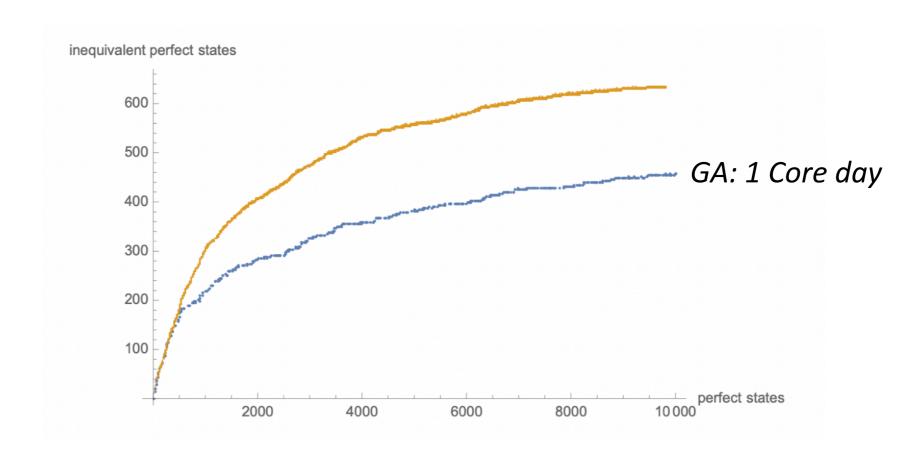




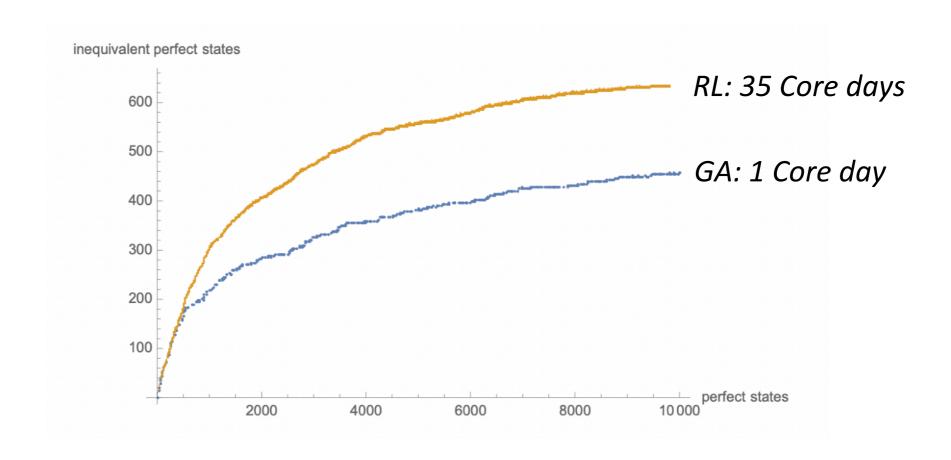
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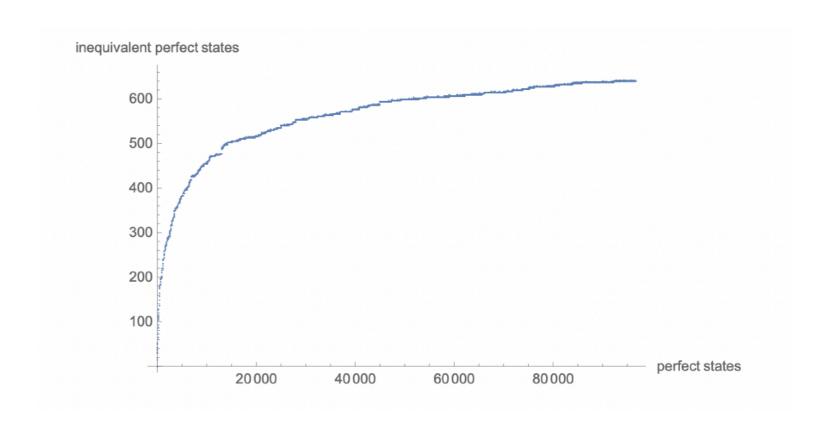


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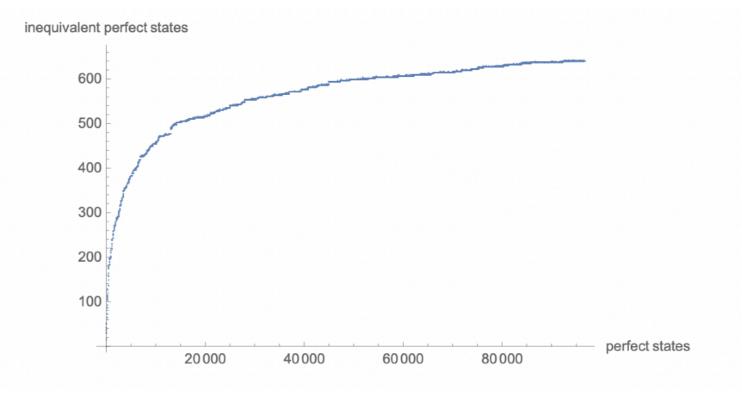
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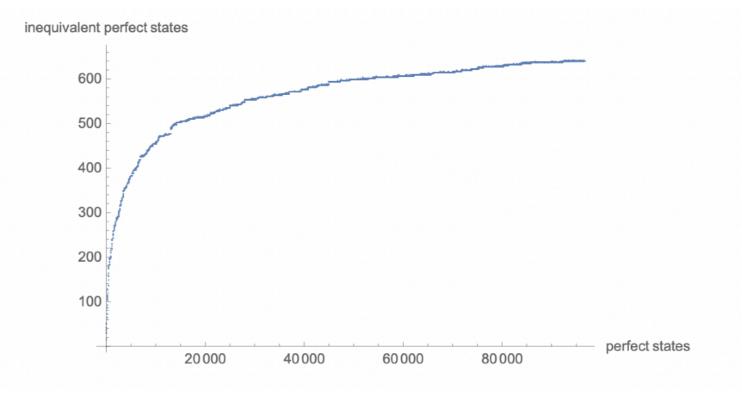
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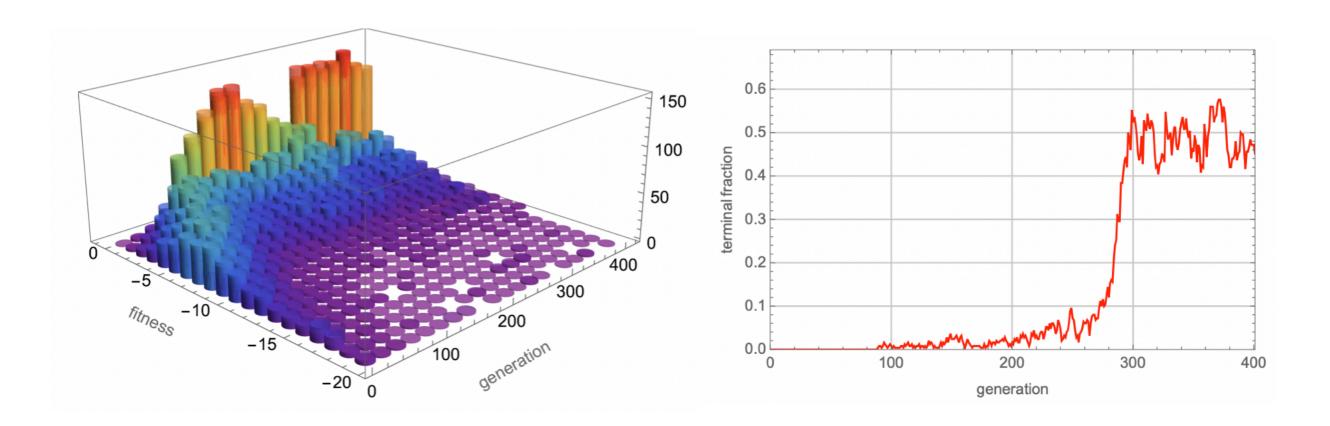
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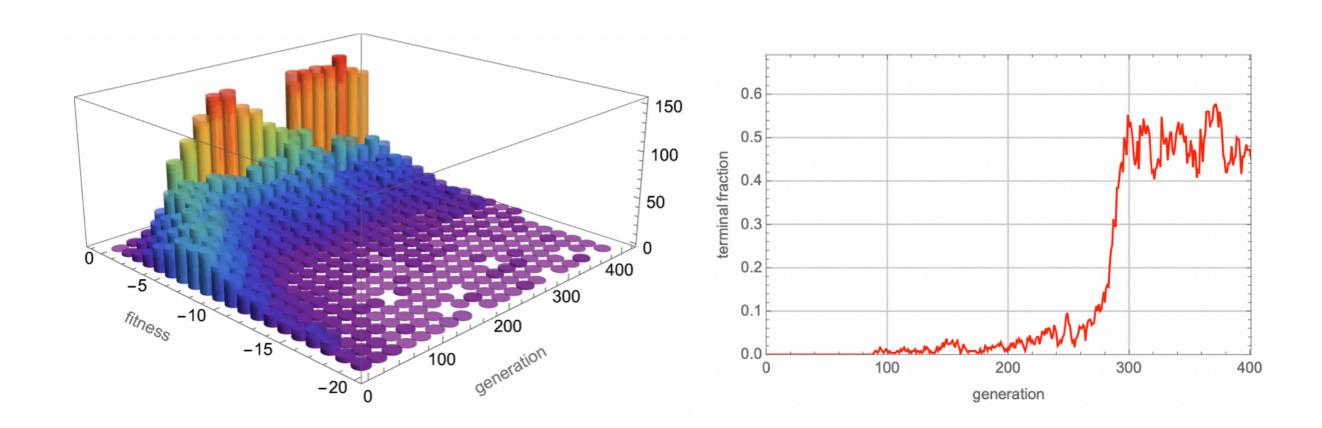


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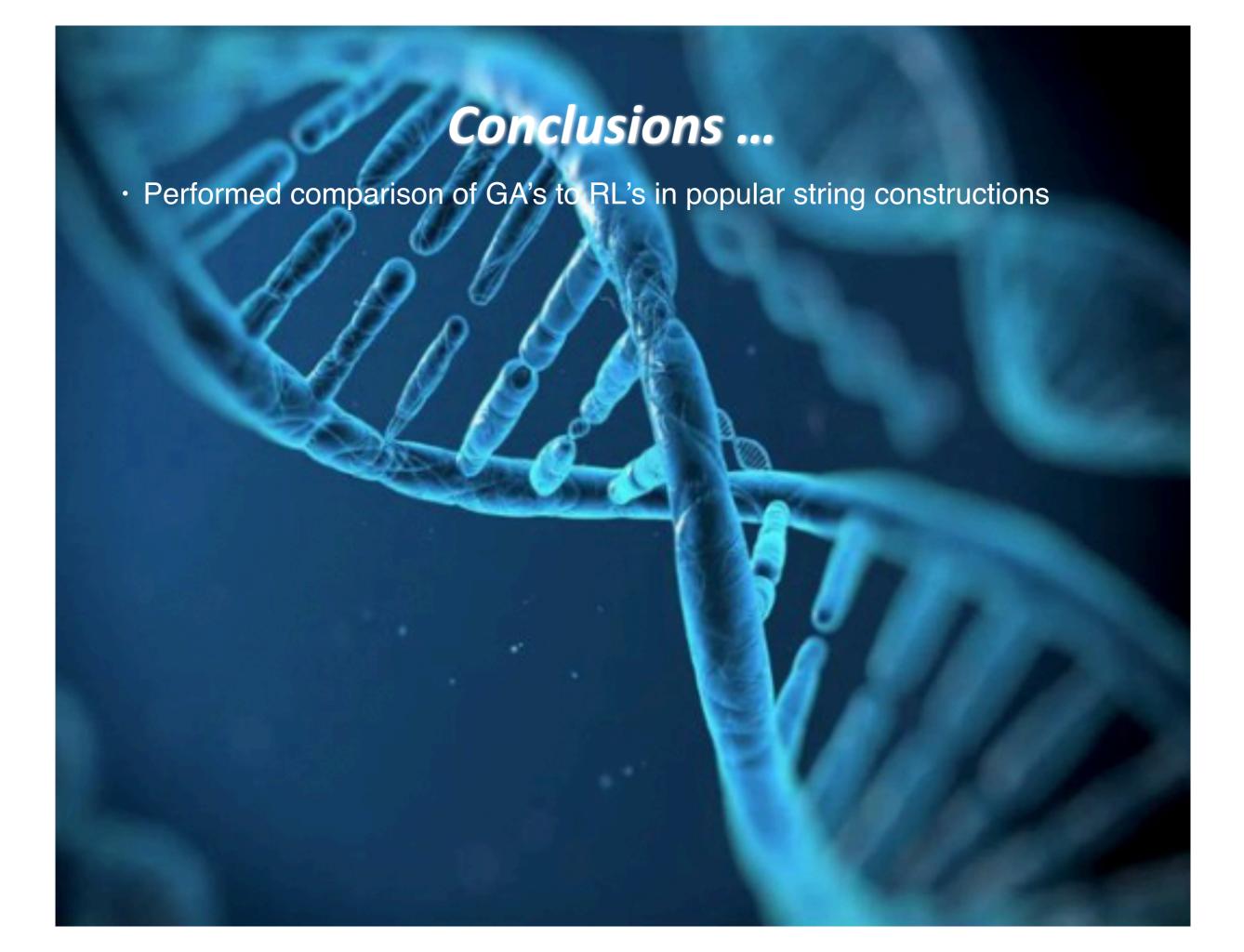


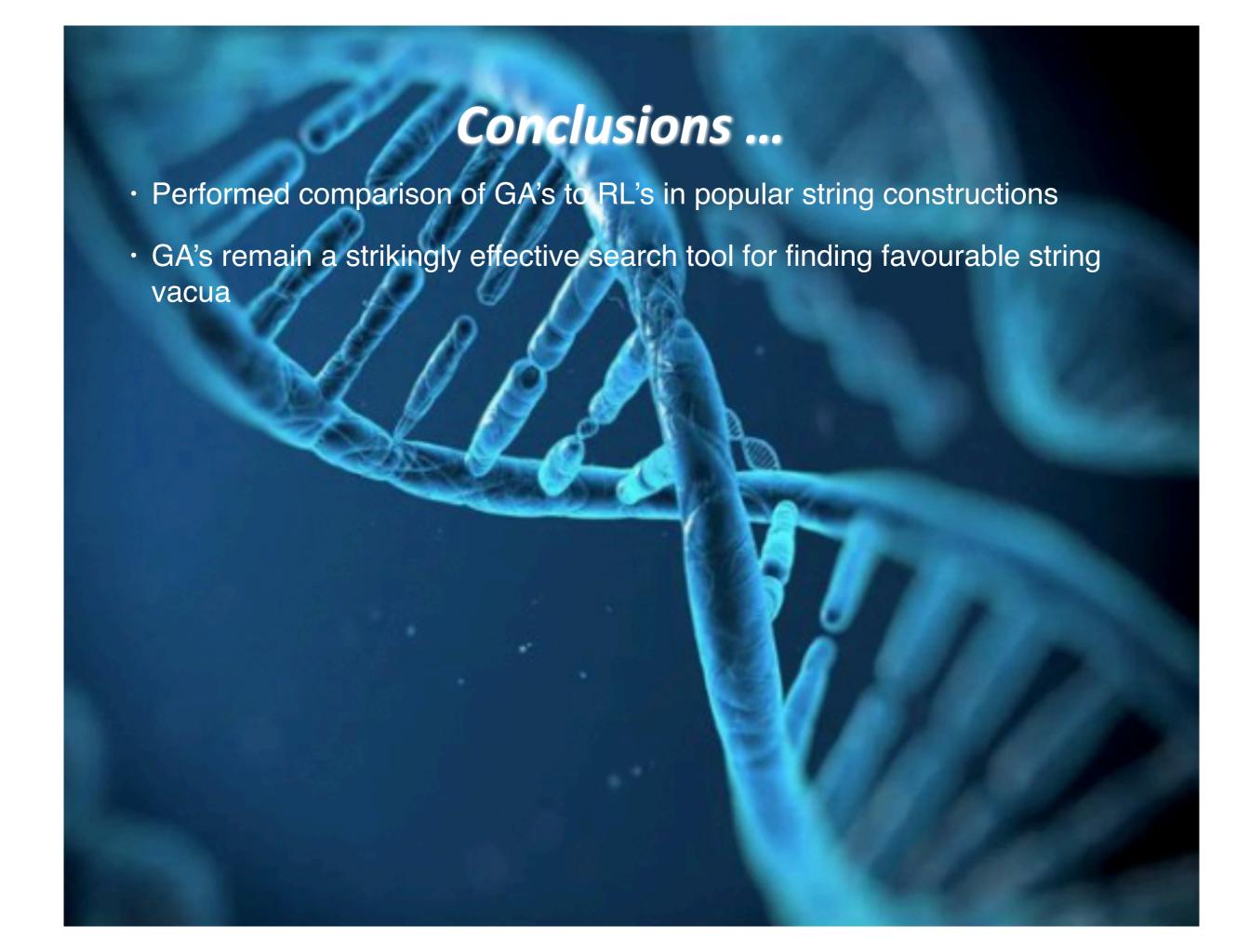
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GA in a given run takes only twice as many generations to reach the saturated fitness.









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- Search difficulty does not seem to be increasing proportionally to difficulty (scaling behaviour remains to be determined)

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- Note we did not yet work hard to optimise the GA in the RL/GA study. (Just single point cross-over, not particularly optimising mutation rate, no creep mutation etc)
- Results suggest estimates of string-landscape size are less meaningful than fitness-distance correlation (i.e. SM is not a needle in a haystack)
- A combined GA+RL approach can indicate when a portion of the landscape has been comprehensively searched for SM models. This is not possible with either method alone or by doing any kind of scan (for NP-hard).
- Search difficulty does not seem to be increasing proportionally to difficulty (scaling behaviour remains to be determined)