Flavour Anomalies Meet Flavour Symmetry

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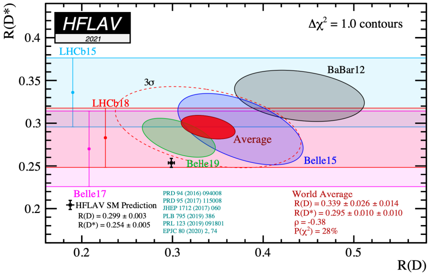
Quo vadis, Lepton-Flavour Universality?



HFLAV 2021, 2206.07501

Neutrino oscillations $\rightarrow$ Flavour structure of nature more complicated than in SM

Hints for LFU violation!
$R(D)$, $R(D^\star)$: $3.4\,\sigma$

Muon AMM:
$\Delta a_\mu = (2.51 \pm 0.59) \times 10^{-9}$
Muon g-2, 2104.03281;

Aoyama, T. et al., 2006.04822

Explain anomalies via scalar LQ $\phi \sim S_1^\dagger$

Predict interaction structure via discrete flavour symmetry!

Contents

- Model Setup
- LQ Coupling Texture
- Symmetry Breaking
- Phenomenology
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  - $R(D)$, $R(D^\star)$, Δa_μ
  - $\tau \to \mu\gamma$
  - $\mu \to e\gamma$, $\mu - e$ conversion in Al
  - $\tau \to 3\mu$
- Conclusion

Model Setup

Extend SM by scalar LQ $\phi \sim S_1^\dagger \sim (3, 1, -1/3)$
Impose baryon-number conservation

$$\mathcal{L}_{\rm LQ}^{\rm int} = \overline{L^c}\, \hat{\boldsymbol{x}}\, Q\, \phi^\dagger + \overline{e_R^c}\, \hat{\boldsymbol{y}}\, u_R\, \phi^\dagger + \text{h.c.}$$

$$\mathcal{L}_{\rm LQ}^{\rm mass} = \overline{(\nu_L^{\rm m})^c}\, \boldsymbol{x}\, d_L^{\rm m}\, \phi^\dagger + \overline{(e_R^{\rm m})^c}\, \boldsymbol{y}\, u_R^{\rm m}\, \phi^\dagger - \overline{(e_L^{\rm m})^c}\, \boldsymbol{z}\, u_L^{\rm m}\, \phi^\dagger + \text{h.c.}$$

$$\boldsymbol{z} = \boldsymbol{x}\, V^\dagger$$

Model Setup

Extend SM by scalar LQ $\phi \sim S_1^\dagger \sim (3, 1, -1/3)$
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$$\mathcal{L}_{\mathrm{LQ}}^{\mathrm{int}} = \overline{L^c}\, \hat{\boldsymbol{x}}\, Q\, \phi^\dagger + \overline{e_R^c}\, \hat{\boldsymbol{y}}\, u_R\, \phi^\dagger + \mathrm{h.c.}$$

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$$\boldsymbol{z} = \boldsymbol{x}\, V^\dagger$$

Formally: Type-II Higgs-Doublet Model

$\sqrt{2}\left(\langle H_u^0\rangle + \langle H_d^0\rangle\right) = v_u^2 + v_d^2 = v^2 \sim (246\,\mathrm{GeV})^2$; Decoupling limit

Hall, Wise, Nucl. Phys. B (1981); Donoghue, Li, Phys. Rev. D (1979);
Haber, Nir: Nucl. Phys. B (1990)

$$\mathcal{L}_{\mathrm{Yuk}} = -\overline{Q_L}\, y_u\, u_R\, H_u - \overline{Q_L}\, y_d\, d_R\, H_d - \overline{L_L}\, y_e\, e_R\, H_d + \mathrm{h.c.}$$

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Extend SM by scalar LQ $\phi \sim S_1^\dagger \sim (3, 1, -1/3)$
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Flavour structure constrained by symmetry group $G_f = D_{17} \times Z_{17}$

- use assignment $\mathbf{2} + \mathbf{1}$ for fermion generations as much as possible
- external Z_n symmetry: mass spectrum; reps of D_n are real; protect $\boldsymbol{y}$

LQ Coupling Texture

Viable texture for $\lambda \approx 0.2$ and $\hat{m}_\phi \equiv \frac{m_\phi}{\text{TeV}} \lesssim 5$ identified in 1704.05849:

$$
\boldsymbol{x} \sim \begin{array}{c} \\ \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{array} \begin{array}{c} \begin{array}{ccc} d_L & s_L & b_L \end{array} \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda^3 & \lambda \\ 0 & \lambda^2 & 1 \end{pmatrix} \end{array}, \quad \boldsymbol{y} \sim \begin{array}{c} \\ e_R \\ \mu_R \\ \tau_R \end{array} \begin{array}{c} \begin{array}{ccc} u_R & c_R & t_R \end{array} \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \lambda^3 \\ 0 & 1 & 0 \end{pmatrix} \end{array}
$$

Basis $\sim d_i$ and e_i mass basis;
$V \sim L_u^\dagger$;
$R_u \sim 1$

$R(D)$, $R(D^\star)$: $x_{33} \sim y_{32} \sim 1$; $\Delta a_\mu \sim 10^{-9}$: $z_{23} y_{23} \sim x_{23} y_{23} \sim \lambda^4$
$B \to K^{(\star)} \nu\nu$, B_s-$\bar{B}_s$ mixing: $x_{i2} x_{i3} \lesssim \lambda^2$; $D^0 \to \mu\bar{\mu}$, $B \to D^{(\star)} \mu\nu$: x_{22}

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ϕ, H_u, H_d, L_3, Q_3, e_{R3}, u_{R2}, u_{R3}, $d_{R3} \sim \mathbf{1}_1$

- $\overline{L_3^c}\, \phi^\dagger\, Q_3$ unsuppressed $\to L_3$, Q_3 in complex conj. reps
- $\overline{e_{R3}^c}\, \phi^\dagger\, u_{R2}$ unsuppressed $\to e_{R3}$, u_{R2} in complex conj. reps
- $\overline{Q_3}\, H_u\, u_{R3}$, $\overline{Q_3}\, H_d\, d_{R3}$, $\overline{L_3}\, H_d\, e_{R3}$: 3rd-gen. charged-fermion masses

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Single spurion $S \sim \mathbf{2}_1$, $\langle S \rangle \sim \lambda$ for LO elements. $L \sim \mathbf{2}_1$, $Q \sim \mathbf{2}_2$, $e_R \sim \mathbf{2}_3$

- $\overline{L^c}\, \phi^\dagger\, Q_3\, S$: one spurion insertion $\to L$ and S in same doublet
- $\overline{L_3^c}\, \phi^\dagger\, Q\, S^2$: two spurion insertions $\to Q$ and S^2 in same doublet
- $\overline{L^c}\, \phi^\dagger\, Q\, S^3$: three spurion insertions $\to \overline{L^c}\, Q$ and S^3 in same doublet
- $\overline{e_R^c}\, \phi^\dagger\, u_{R3}\, S^3$: three spurion insertions $\to e_R$ and S^3 in same doublet

Symmetry Breaking

Symmetry: $D_{17} \times Z_{17}$

$$\boldsymbol{x} = \begin{array}{c} \\ \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{array} \begin{array}{c} \begin{array}{ccc} d_L & s_L & b_L \end{array} \\ \begin{pmatrix} & & \\ & & \\ & & a_{33} \end{pmatrix} \end{array} \qquad \boldsymbol{y} = \begin{array}{c} \\ e_R \\ \mu_R \\ \tau_R \end{array} \begin{array}{c} \begin{array}{ccc} u_R & c_R & t_R \end{array} \\ \begin{pmatrix} & & \\ & & \\ & b_{32} & \end{pmatrix} \end{array}$$

Up-type quarks: $\mathcal{L}^u_{\mathrm{Yuk,LO}} = \underbrace{\alpha^u_1 \, \overline{Q_3} \, H_u \, u_{R3}}_{m_t} + \alpha^u_2 \, \overline{Q} \, H_u \, u_{R2} \, W + \alpha^u_3 \, \overline{Q} \, H_u \, u_{R3} \, (S^\dagger)^2 + \alpha^u_4 \, \overline{Q} \, H_u \, u_{R1} \, T^2 \, U \, .$

Down-type quarks: $\mathcal{L}^d_{\mathrm{Yuk,LO}} = \underbrace{\alpha^d_1 \, \overline{Q_3} \, H_d \, d_{R3}}_{m_b} + \alpha^d_2 \, \overline{Q} \, H_d \, d_R \, T + \alpha^d_3 \, \overline{Q} \, H_d \, d_R \, U \, .$

Charged leptons: $\mathcal{L}^e_{\mathrm{Yuk,LO}} = \underbrace{\alpha^e_1 \, \overline{L_3} \, H_d \, e_{R3}}_{m_\tau} + \alpha^e_2 \, \overline{L} \, H_d \, e_R \, T + \alpha^e_3 \, \overline{L} \, H_d \, e_R \, U \, .$

Symmetry Breaking

$$\langle S \rangle = \begin{pmatrix} \lambda \\ 0 \end{pmatrix}$$

Symmetry: $D_{17} \times Z_{17} \xrightarrow{S \to \langle S \rangle} Z_{17}^{\text{diag}}$

$$\boldsymbol{x} = \begin{matrix} & d_L & s_L & b_L \\ \nu_{eL} & & & \\ \nu_{\mu L} & & a_{22}\lambda^3 & a_{23}\lambda \\ \nu_{\tau L} & & a_{32}\lambda^2 & a_{33} \end{matrix} \qquad \boldsymbol{y} = \begin{matrix} & u_R & c_R & t_R \\ e_R & & & \\ \mu_R & & & b_{23}\lambda^3 \\ \tau_R & & b_{32} & \end{matrix}$$

Up-type quarks: $\mathcal{L}^u_{\mathrm{Yuk,LO}} = \underbrace{\alpha^u_1 \, \overline{Q_3} \, H_u \, u_{R3}}_{m_t} + \alpha^u_2 \, \overline{Q} \, H_u \, u_{R2} \, W + \underbrace{\alpha^u_3 \, \overline{Q} \, H_u \, u_{R3} \, (S^\dagger)^2}_{\to \theta_{23}, (\theta_{13})} + \alpha^u_4 \, \overline{Q} \, H_u \, u_{R1} \, T^2 \, U \, .$

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Symmetry Breaking

$$\langle S\rangle = \begin{pmatrix}\lambda\\0\end{pmatrix}, \quad \langle T\rangle = \begin{pmatrix}\lambda^2\\0\end{pmatrix}, \quad \langle U\rangle = \begin{pmatrix}0\\\lambda^4\end{pmatrix}, \quad \langle W\rangle = \begin{pmatrix}\lambda^5\\\lambda^4\end{pmatrix}$$

$$\text{Symmetry: } D_{17}\times Z_{17} \xrightarrow{S\to\langle S\rangle} Z_{17}^{\text{diag}} \xrightarrow{T\to\langle T\rangle,\, U\to\langle U\rangle,\, W\to\langle W\rangle} \text{nil}$$

$$\boldsymbol{x} = \begin{matrix} & d_L & s_L & b_L \\ \nu_{eL} & a_{11}\lambda^9 & a_{12}\lambda^{11} & a_{13}\lambda^9 \\ \nu_{\mu L} & a_{21}\lambda^8 & a_{22}\lambda^3 & a_{23}\lambda \\ \nu_{\tau L} & a_{31}\lambda^8 & a_{32}\lambda^2 & a_{33}\end{matrix} \qquad \boldsymbol{y} = \begin{matrix} & u_R & c_R & t_R \\ e_R & b_{11}\lambda^9 & b_{12}\lambda^9 & b_{13}\lambda^9 \\ \mu_R & b_{21}\lambda^8 & b_{22}\lambda^3 & b_{23}\lambda^3 \\ \tau_R & b_{31}\lambda^5 & b_{32} & b_{33}\lambda^4\end{matrix}$$

Up-type quarks: $\mathcal{L}^u_{\text{Yuk,LO}} = \underbrace{\alpha_1^u\,\overline{Q_3}\,H_u\,u_{R3}}_{m_t} + \underbrace{\alpha_2^u\,\overline{Q}\,H_u\,u_{R2}\,W}_{\to m_c,\theta_C,(\theta_{13})} + \underbrace{\alpha_3^u\,\overline{Q}\,H_u\,u_{R3}\,(S^\dagger)^2}_{\to\theta_{23},(\theta_{13})} + \underbrace{\alpha_4^u\,\overline{Q}\,H_u\,u_{R1}\,T^2\,U}_{\to m_u}\,.$

Down-type quarks: $\mathcal{L}^d_{\text{Yuk,LO}} = \underbrace{\alpha_1^d\,\overline{Q_3}\,H_d\,d_{R3}}_{m_b} + \underbrace{\alpha_2^d\,\overline{Q}\,H_d\,d_R\,T}_{\to m_s} + \underbrace{\alpha_3^d\,\overline{Q}\,H_d\,d_R\,U}_{\to m_d}\,.$

Charged leptons: $\mathcal{L}^e_{\text{Yuk,LO}} = \underbrace{\alpha_1^e\,\overline{L_3}\,H_d\,e_{R3}}_{m_\tau} + \underbrace{\alpha_2^e\,\overline{L}\,H_d\,e_R\,T}_{\to m_\mu} + \underbrace{\alpha_3^e\,\overline{L}\,H_d\,e_R\,U}_{\to m_e}\,.$

Outline of Study

Classification of Observables:

- Primary: $R(D)$, $R(D^\star)$, Δa_μ, $\tau \to \mu\gamma$, $\mu \to e\gamma$, $\tau \to 3\,\mu$, $\tau \to \mu e\bar{e}$, $\mu \to 3\,e$, $\mu - e$ conv. in Al, $B \to K^{(\star)}\nu\bar{\nu}$, g_{τ_A}, $B_c \to \tau\nu$, $c\bar{c} \to \tau\bar{\tau}$
- Secondary: d_μ, g_{μ_A}, $R_D^{\mu/e}$, $R_{D^\star}^{e/\mu}$, $B \to \tau\nu$

$m_\phi \gtrsim 1.2\,\text{TeV}$ at 95% C.L. for $\text{BR}(\phi \to t\tau) \sim \text{BR}(\phi \to b\nu)$ ATLAS, 2108.07665
$\to$ Benchmark LQ masses: $m_\phi = 2, 4, 6\,\text{TeV}$

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1. Primary scan: Primary observables
Coeffs a_{ij}, b_{ij}, c_{ij} (mostly) independently varied $\in [\lambda, 1/\lambda]$ in mass basis

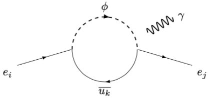
2. Comprehensive scan: All considered observables

- Fit of SM Yukawa parameters to charged-fermion masses, quark mixing
- LQ couplings:
  - Suitable biases derived from primary scan
  - Otherwise independently varied $\in [\lambda, 1/\lambda]$ in interaction basis
  - Coeffs a_{ij}, b_{ij}, c_{ij} in mass basis: Functions of SM Yukawa parameters, LQ coeffs $\hat{a}_{ij}$, $\hat{b}_{ij}$ in interaction basis

Explaining $R(D)$, $R(D^\star)$, Δa_μ

$\Delta a_\mu = [2.51 \pm 0.59\,(0.4)] \times 10^{-9}$
Muon g-2,
2104.03281, 1501.06858;

Aoyama, T. et al., 2006.04822



$$\Delta a_\mu \approx -2 \frac{\mathrm{Re}(b_{23} c_{23}^*)}{\hat{m}_\phi^2} \times 10^{-9}$$

$$\Delta a_\mu = [2.51 \pm 0.59\,(0.4)] \times 10^{-9}$$

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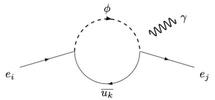

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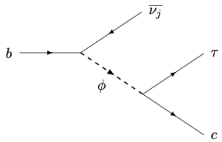
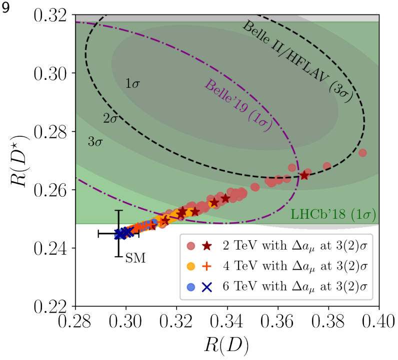




Belle, 1910.05864; LHCB, 1708.08856, 1711.02505

$$\frac{R(D)}{R(D)_{\mathrm{SM}}} \approx 1 + 1.07\,\frac{\mathrm{Re}(a_{33}^* b_{32})}{\hat{m}_\phi^2}$$

$$\frac{R(D^\star)}{R(D^\star)_{\mathrm{SM}}} \approx 1 + 0.36\,\frac{\mathrm{Re}(a_{33}^* b_{32})}{\hat{m}_\phi^2}$$

$$R(D) = 0.339 \pm 0.030\,(0.016)$$

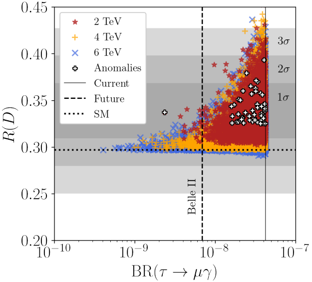
$$R(D^\star) = 0.295 \pm 0.014\,(0.009)$$

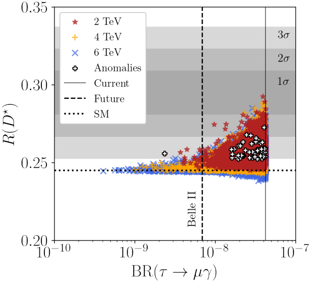
HFLAV 2021, 2206.07501

Belle II, 2203.11349

Straub, 1810.08132; Straub, Stangl, Kirk, Kumar, Niehoff, Gurler et al., 10.5281/zenodo.5543714

Charged-Lepton Flavour Violating Decay $\tau \to \mu\gamma$

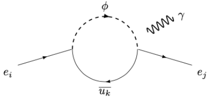




$\mathrm{BR}(\tau \to \mu\gamma)_{\mathrm{exp}}$
$< 4.2\,(0.69) \times 10^{-8}$

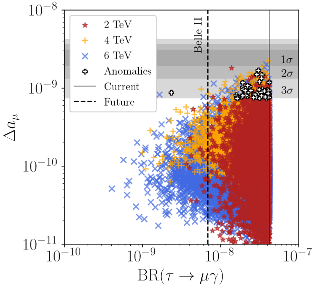
Belle, 2103.12994;

Banerjee, S. et al., 2203.14919



LD γ ($\sim m_t/m_\phi^2$)

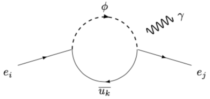
$$\mathrm{BR}(\tau \to \mu\gamma) \sim \frac{|b_{23}c_{33}|^2}{\hat{m}_\phi^4} \times 10^{-5}$$



$\mathrm{BR}(\mu \to e\gamma)_{\mathrm{exp}}$
$< 4.2\,(0.6) \times 10^{-13}$

MEG, 1605.05081;

MEG II, 2107.10767



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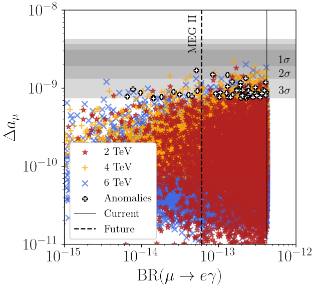
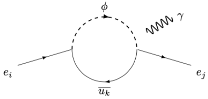


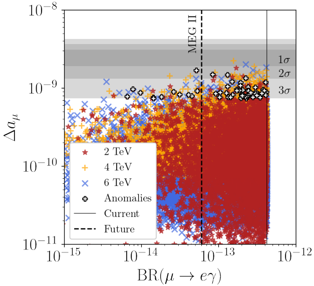
$$\mathrm{BR}(\mu \to e\gamma) \sim \frac{|b_{13} c_{23}|^2}{\hat{m}_\phi^4} \times 10^{-11}$$

$$\mathrm{BR}(\mu \to 3\,e) \approx 0.0069\,\mathrm{BR}(\mu \to e\gamma)$$

$\mathrm{BR}(\mu \to e\gamma)_{\mathrm{exp}}$
$< 4.2\,(0.6) \times 10^{-13}$
MEG, 1605.05081;
MEG II, 2107.10767



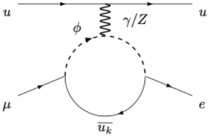
LD γ ($\sim m_t/m_\phi^2$)



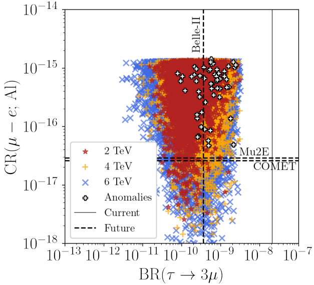
$$\mathrm{BR}(\mu \to e\gamma) \sim \frac{|b_{13}c_{23}|^2}{\hat{m}_\phi^4} \times 10^{-11}$$

$$\mathrm{BR}(\mu \to 3\,e) \approx 0.0069\,\mathrm{BR}(\mu \to e\gamma)$$

Future searches:
$\mathrm{CR}(\mu - e;\ \mathrm{Al})_{\mathrm{exp}}$
$\lesssim 2.6\,(2.9) \times 10^{-17}$
COMET, 1812.09018;
MU2E, 1501.05241



LD γ ($\sim m_t/m_\phi^2$)

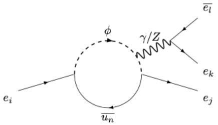


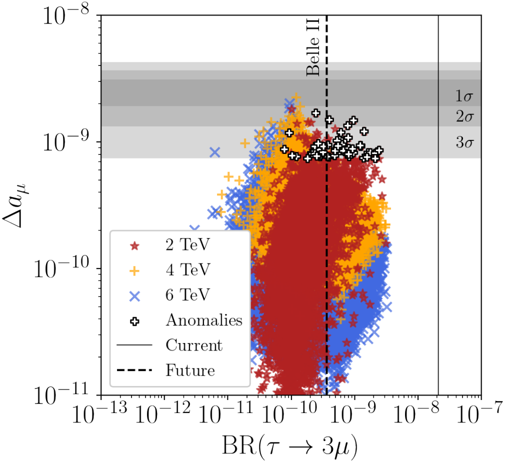
$$\mathrm{CR}(\mu \to e;\,\mathrm{Al}) \sim \left(0.003\,\frac{|c_{13}b_{23}|^2}{\hat{m}_\phi^4} + \frac{|c_{23}b_{13}|^2}{\hat{m}_\phi^2}\right) \times 10^{-13}$$

Trilepton Decay $\tau \to 3\mu$

$\mathrm{BR}(\tau \to 3\,\mu)_{\mathrm{exp}}$
$< 2.1\,(0.036) \times 10^{-8}$
Hayasaka et al., 1001.3221;
Banerjee et al., 2203.14919

Important channels:
LD γ ($\sim m_t/m_\phi^2$)
Z ($\sim m_t^2/m_\phi^2$)





$$\mathrm{BR}(\tau \to 3\,\mu) \sim \frac{|b_{23} c_{33}|^2 + 0.07|c_{23} c_{33}|^2}{\hat{m}_\phi^4} \times 10^{-7}$$

Conclusion

SM extension by scalar LQ $\phi \sim (3, 1, -1/3)$, also involving H_u, H_d.

Flavour structure constrained by symmetry group $G_f = D_{17} \times Z_{17}$.

- Assignment $\mathbf{1}+\mathbf{1}+\mathbf{1}$ for u_{Ri}, $\mathbf{2}+\mathbf{1}$ for Q_i, d_{Ri}. L_i, e_{Ri} under D_{17}.
- Broken by four spurion fields S, T, U, W, all in $\mathbf{2}$.
- Residual symmetry Z_{17}^{diag} preserved by $\hat{x}$ and $\hat{y}$ at LO.

Simultaneous explanation of $R(D)$, $R(D^\star)$, Δa_μ at $2\,\sigma$ $(3\,\sigma)$ for $\hat{m}_\phi = 2\,(2, 4)$.
Successful fit to charged-fermion masses and quark mixing.

Bigaran, I., Felkl, T., Hagedorn, C. & Schmidt, M.A.;
Flavour Anomalies Meet Flavour Symmetry. arXiv: 2206.XXXXX

Thank you for your attention!

Back-Up

Group theory of D_{17}

D_n non-abelian for $n \geq 3$.

D_{17} contains 34 distinct elements, ten real irreps:
trivial singlet $\mathbf{1}_1$, non-trivial singlet $\mathbf{1}_2$, eight (faithful) doublets $\mathbf{2}_i$.

Two generators a and b with
$$a^{17} = e \, , \quad b^2 = e \, , \quad a\, b\, a = b \, .$$

Representation matrices:
$$a(\mathbf{1}_1) = b(\mathbf{1}_1) = 1 \;\; \text{and} \;\; a(\mathbf{1}_2) = 1 \, , \quad b(\mathbf{1}_2) = -1$$
$$a(\mathbf{2}_i) = \begin{pmatrix} \omega_{17}^{\mathrm{i}} & 0 \\ 0 & \omega_{17}^{17-\mathrm{i}} \end{pmatrix} \;\; \text{and} \;\; b(\mathbf{2}_i) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \, ,$$
where $\omega_{17} = e^{\frac{2\,\pi\,i}{17}}$.

For Kronecker products and Clebsch-Gordan coefficients:
Blum, Hagedorn, Lindner, 0709.3450.

Particle Content: Transformation Properties

Field	D_{17}	Z_{17}
$Q = (Q_1, Q_2)^T$	$\mathbf{2}_2$	1
Q_3	$\mathbf{1}_1$	16
u_{R1}	$\mathbf{1}_2$	13
u_{R2}	$\mathbf{1}_1$	8
u_{R3}	$\mathbf{1}_1$	1
$d_R = (d_{R1}, d_{R2})^T$	$\mathbf{2}_4$	1
d_{R3}	$\mathbf{1}_1$	7
$L = (L_1, L_2)^T$	$\mathbf{2}_1$	2
L_3	$\mathbf{1}_1$	1

Field	D_{17}	Z_{17}
$e_R = (e_{R1}, e_{R2})^T$	$\mathbf{2}_3$	2
e_{R3}	$\mathbf{1}_1$	9
H_u	$\mathbf{1}_1$	15
H_d	$\mathbf{1}_1$	9
ϕ	$\mathbf{1}_1$	0
$S = (S_1, S_2)^T$	$\mathbf{2}_1$	16
$T = (T_1, T_2)^T$	$\mathbf{2}_2$	8
$U = (U_1, U_2)^T$	$\mathbf{2}_2$	8
$W = (W_1, W_2)^T$	$\mathbf{2}_2$	12

Biases from Primary Scan

$\hat{m}_\phi$	$\lvert a_{33}\rvert$	$\lvert b_{32}\rvert$	$\cos[\Delta(a_{33}, b_{32})]$	$\lvert a_{23}\rvert$	$\cos[\Delta(a_{23}, b_{23})]$
2	$[0.2, 0.7]$	$[1.1, 2.6]$	$[0.4, 1.0]$	–	$[-1.0, 0.0]$
4	$[0.2, 1.9]$	$[1.0, 4.5]$	$[0.1, 1.0]$	$[1.6, 4.4]$	$[-1.0, -0.5]$
6	$[0.2, 3.6]$	$[0.8, 4.5]$	$[0.0, 1.0]$	$[1.4, 4.4]$	$[-1.0, -0.3]$

$\Delta(r_{ij}, s_{kl}) \equiv \mathrm{Arg}(r_{ij}) - \mathrm{Arg}(s_{kl})\,.$

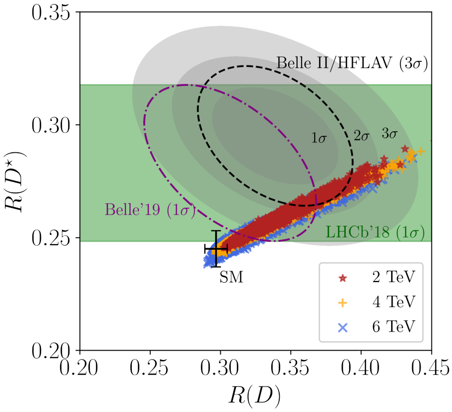
Viable sample points explaining $R(D^{(\star)})$ and/or Δa_μ at $3\,\sigma$ $\rightarrow$ Ranges above.

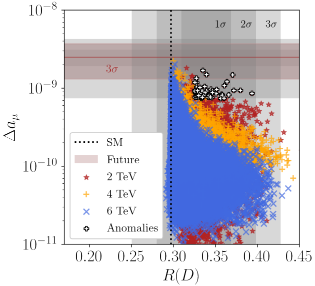
Furthermore impose from $\mu \to e\gamma$ and $\tau \to \mu\gamma$

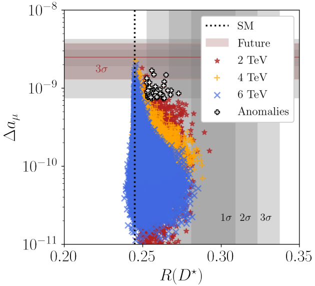
$$|b_{13}| \lesssim \frac{1}{|a_{23}|} \left\{ \begin{array}{ll} 0.41, & \hat{m}_\phi = 2 \\ 1.16, & \hat{m}_\phi = 4 \\ 2.22, & \hat{m}_\phi = 6 \end{array} \right\} \quad \text{and} \quad |b_{23}| \lesssim \frac{1}{|a_{33}|} \left\{ \begin{array}{ll} 0.16, & \hat{m}_\phi = 2 \\ 0.45, & \hat{m}_\phi = 4 \\ 0.86, & \hat{m}_\phi = 6 \end{array} \right\}.$$

Explanation of anomalies prefers $|b_{13}| = \left| \hat{b}_{13} - \hat{b}_{23} \frac{e_{21}}{e_{22}} + \mathcal{O}(\lambda^2) \right|$ smaller than λ.

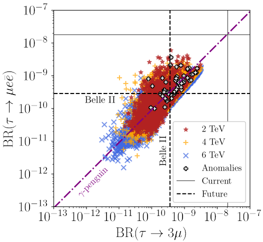
$R(D)$, $R(D^\star)$, Δa_μ

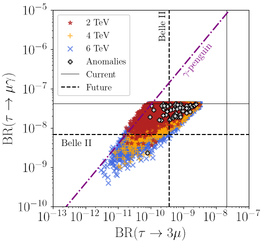




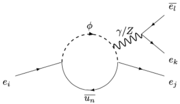


$\tau \to 3\,\mu$, $\tau \to \mu e\bar{e}$



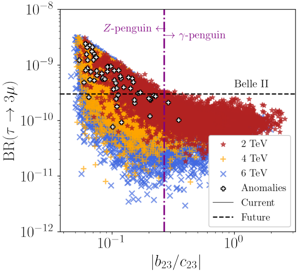


Important channels:
LD γ ($\sim m_t/m_\phi^2$)
Z ($\sim m_t^2/m_\phi^2$)



$\mathrm{BR}(\tau \to 3\,\mu)_{\mathrm{exp}}$
$< 2.1\,(0.036) \times 10^{-8}$
$\mathrm{BR}(\tau \to \mu e\bar{e})_{\mathrm{exp}}$
$< 1.8\,(0.036) \times 10^{-8}$

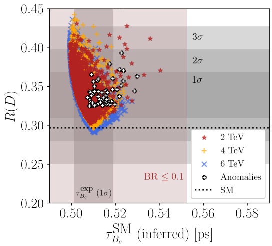
Hayasaka et al., 1001.3221; Banerjee et al., 2203.14919

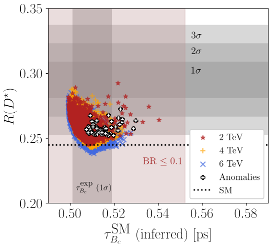


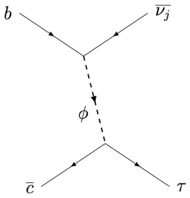
$$\mathrm{BR}(\tau \to 3\,\mu) \sim \frac{|b_{23}c_{33}|^2 + 0.07|c_{23}c_{33}|^2}{\hat{m}_\phi^4} \times 10^{-7}$$

$$\mathrm{BR}(\tau \to \mu e\bar{e}) \sim \frac{|b_{23}c_{33}|^2 + 0.05|c_{23}c_{33}|^2}{\hat{m}_\phi^4} \times 10^{-7}$$

$B_c \to \tau\nu$







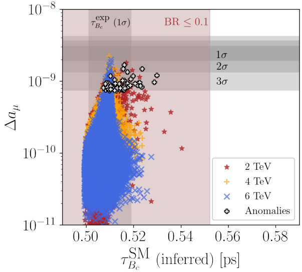
$$\tau_{B_c}^{\text{SM}} = 0.52^{+0.18}_{-0.12}\ \text{ps at } 1\,\sigma$$
$$\tau_{B_c}^{\text{exp}} = 0.510^{+0.009}_{-0.009}\ \text{ps at } 1\,\sigma$$

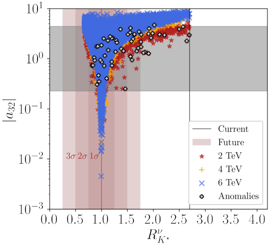
Beneke, Buchalla, hep-ph/9601249,

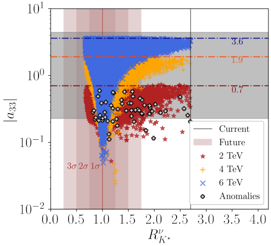
PDG 2022,

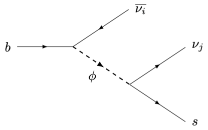
HFLAV 2021.

$$\frac{\tau_{B_c}^{\text{SM}}}{\tau_{B_c}^{\text{exp}}} \approx 1 - 0.13\frac{|a_{33}\,b_{32}|}{\hat{m}_\phi^2}\cos\left(\text{Arg}(a_{33}) - \text{Arg}(b_{32})\right) + 0.19\frac{|a_{33}\,b_{32}|^2}{\hat{m}_\phi^4}$$







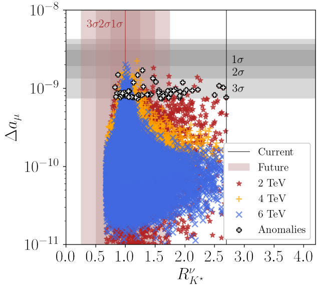


Belle, 1702.03224: $R^{\nu}_{K^\star} < 2.7\,(90\%\,\text{C.L.})$

Belle II, 1808.10567:

$R^{\nu}_{K^\star} = 1.0 \pm 0.25\,(0.1)$ for $5\,(50)\,\text{ab}^{-1}$

$$R^{\nu}_{K^{(\star)}} \approx 1 + 1.69 \frac{|a_{33}a_{32}|}{\hat{m}^2_\phi} \cos\left(\text{Arg}(a_{33}) - \text{Arg}(a_{32})\right) + 2.15 \frac{|a_{33}a_{32}|^2}{\hat{m}^4_\phi}$$



g_{τ_A}, $c\bar{c} \to \tau\bar{\tau}$

Assumption: Lepton flavour conserved for SM couplings
$\to g_{e_A}^{\mathrm{SM}} = g_{e_L}^{\mathrm{SM}} - g_{e_R}^{\mathrm{SM}} \equiv g_A^{\mathrm{SM}} (< 0)$

$$g_{\tau_A}/g_A^{\mathrm{SM}} \approx 1 - \left\{ \begin{array}{ll} 4.5, & \hat{m}_\phi = 2 \\ 1.5, & \hat{m}_\phi = 4 \\ 0.8, & \hat{m}_\phi = 6 \end{array} \right\} |c_{33}|^2 \times 10^{-4}$$

$g_{\tau_A,\mathrm{exp}}/g_A^{\mathrm{SM}}$: 1.00154 ± 0.00128 at 1σ hep-ex/0509008

Angelescu et al., 1808.08179: Reinterpretation of LHC search 1709.07242 for Z' in high-p_T $\tau\bar{\tau}$ tails

From top right in figure 4 in 1808.08179 (LHC does not distinguish between chiralities): $|y_{32}| = |b_{32}| < \hat{m}_\phi + 0.6$

Electric Dipole Moment of the Muon

$|d_\mu| < 1.5 \times 10^{-19}\, e\,\mathrm{cm}$

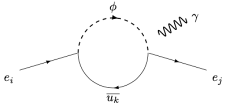
Muon g-2, 0811.1207

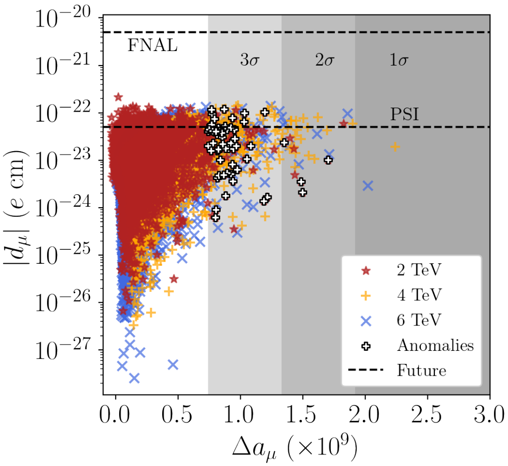
Future searches:

$|d_\mu| < 1000\,(60)$ [1] $\times 10^{-24}\, e\,\mathrm{cm}$

EPJ Web Conf. 118 (2016) 01005; 1506.01465; 2102.08838; hep-ph/0012087;

hep-ph/0307006



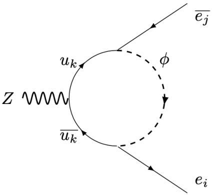


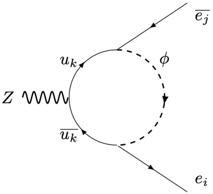
$$|d_\mu| \approx 2\,\frac{|\mathrm{Im}(b_{23} c_{23}^*)|}{\hat{m}_\phi^2} \times 10^{-22}\, e\,\mathrm{cm}$$

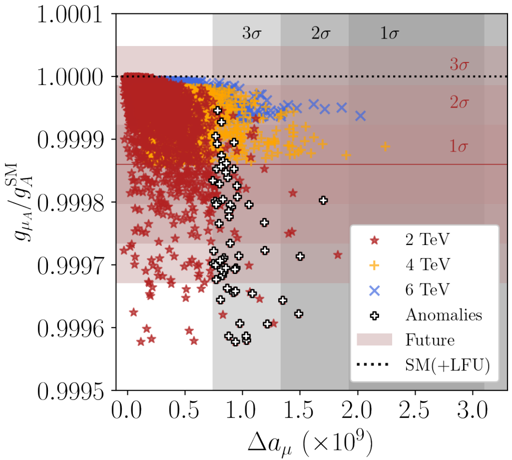
Axial-Vector Z-Boson Coupling to Muons

$g_{\mu_A}/g_A^{\mathrm{SM}} =$
0.99986 ± 0.00108
$(\pm 6.3)[0.63] \times 10^{-5}$

hep-ex/0509008; 2010.06593
1306.6352; Eur. Phys. J. ST
228 (2019) 261







$$g_{\mu_A}/g_A^{\mathrm{SM}} \approx 1 - \left\{ \begin{array}{ll} 2.3, & \hat{m}_\phi = 2 \\ 0.8, & \hat{m}_\phi = 4 \\ 0.4, & \hat{m}_\phi = 6 \end{array} \right\} |c_{23}|^2 \times 10^{-5}\,.$$

SM Yukawa sector: Up-Type Quarks (Scenario A)

$$M_u = \begin{pmatrix} f_{11}\,\lambda^8 & f_{12}\,\lambda^5 & f_{13}\,\lambda^8 \\ f_{21}\,\lambda^{10} & f_{22}\,\lambda^4 & f_{23}\,\lambda^2 \\ f_{31}\,\lambda^{12} & f_{32}\,\lambda^4 & f_{33} \end{pmatrix} \langle H_u^0 \rangle$$

$$L_u = \begin{pmatrix} 1 - \frac{f_{12}^2}{2\,f_{22}^2}\,\lambda^2 + \mathcal{O}(\lambda^4) & \frac{f_{12}}{f_{22}}\,\lambda + \mathcal{O}(\lambda^3) & \frac{f_{13}}{f_{33}}\,\lambda^8 + \mathcal{O}(\lambda^9) \\ -\frac{f_{12}}{f_{22}}\,\lambda + \mathcal{O}(\lambda^3) & 1 - \frac{f_{12}^2}{2\,f_{22}^2}\,\lambda^2 + \mathcal{O}(\lambda^4) & \frac{f_{23}}{f_{33}}\,\lambda^2 + \mathcal{O}(\lambda^6) \\ \frac{f_{12}\,f_{23}}{f_{22}\,f_{33}}\,\lambda^3 + \mathcal{O}(\lambda^5) & -\frac{f_{23}}{f_{33}}\,\lambda^2 + \mathcal{O}(\lambda^4) & 1 - \frac{f_{23}^2}{2\,f_{33}^2}\,\lambda^4 + \mathcal{O}(\lambda^8) \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 + \mathcal{O}(\lambda^{10}) & \frac{f_{11}\,f_{12}}{f_{22}^2}\,\lambda^5 + \mathcal{O}(\lambda^6) & \frac{f_{21}\,f_{23} + f_{31}\,f_{33}}{f_{33}^2}\,\lambda^{12} + \wr(\lambda^{12}) \\ -\frac{f_{11}\,f_{12}}{f_{22}^2}\,\lambda^5 + \mathcal{O}(\lambda^6) & 1 + \mathcal{O}(\lambda^8) & \frac{f_{32}}{f_{33}}\,\lambda^4 + \mathcal{O}(\lambda^6) \\ \frac{f_{11}\,f_{12}\,f_{32}}{f_{22}^2\,f_{33}}\,\lambda^9 + \mathcal{O}(\lambda^{10}) & -\frac{f_{32}}{f_{33}}\,\lambda^4 + \mathcal{O}(\lambda^6) & 1 + \mathcal{O}(\lambda^8) \end{pmatrix} .$$

SM Yukawa sector: Up-Type Quarks (Scenario B)

Consider enhancement $(M_u)_{13} = \tilde{f}_{13}\,\lambda^3$ to correctly predict J_{CP}.

$$M_u = \begin{pmatrix} f_{11}\,\lambda^8 & f_{12}\,\lambda^5 & \tilde{f}_{13}\,\lambda^3 \\ f_{21}\,\lambda^{10} & f_{22}\,\lambda^4 & f_{23}\,\lambda^2 \\ f_{31}\,\lambda^{12} & f_{32}\,\lambda^4 & f_{33} \end{pmatrix} \langle H_u^0 \rangle$$

$$L_u = \begin{pmatrix} 1 - \frac{f_{12}^2}{2\,f_{22}^2}\,\lambda^2 + \mathcal{O}(\lambda^4) & \frac{f_{12}}{f_{22}}\,\lambda + \mathcal{O}(\lambda^3) & \frac{\tilde{f}_{13}}{f_{33}}\,\lambda^3 + \mathcal{O}(\lambda^7) \\ -\frac{f_{12}}{f_{22}}\,\lambda + \mathcal{O}(\lambda^3) & 1 - \frac{f_{12}^2}{2\,f_{22}^2}\,\lambda^2 + \mathcal{O}(\lambda^4) & \frac{f_{23}}{f_{33}}\,\lambda^2 + \mathcal{O}(\lambda^6) \\ \left(\frac{f_{12}\,f_{23}}{f_{22}\,f_{33}} - \frac{\tilde{f}_{13}}{f_{33}}\right)\lambda^3 + \mathcal{O}(\lambda^5) & -\frac{f_{23}}{f_{33}}\,\lambda^2 + \mathcal{O}(\lambda^4) & 1 - \frac{f_{23}^2}{2\,f_{33}^2}\,\lambda^4 + \mathcal{O}(\lambda^6) \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 + \mathcal{O}(\lambda^{10}) & \frac{f_{11}\,f_{12}}{f_{22}^2}\,\lambda^5 + \mathcal{O}(\lambda^6) & \frac{f_{11}\,\tilde{f}_{13}}{f_{33}^2}\,\lambda^{11} + \mathcal{O}(\lambda^{12}) \\ -\frac{f_{11}\,f_{12}}{f_{22}^2}\,\lambda^5 + \mathcal{O}(\lambda^6) & 1 + \mathcal{O}(\lambda^8) & \frac{f_{32}}{f_{33}}\,\lambda^4 + \mathcal{O}(\lambda^6) \\ \frac{f_{11}\,f_{12}\,f_{32}}{f_{22}^2\,f_{33}}\,\lambda^9 + \mathcal{O}(\lambda^{10}) & -\frac{f_{32}}{f_{33}}\,\lambda^4 + \mathcal{O}(\lambda^6) & 1 + \mathcal{O}(\lambda^8) \end{pmatrix}.$$

SM Yukawa Sector: Down-Type Quarks

$$M_d = \begin{pmatrix} d_{11}\,\lambda^4 & d_{12}\,\lambda^8 & d_{13}\,\lambda^8 \\ d_{21}\,\lambda^{10} & d_{22}\,\lambda^2 & d_{23}\,\lambda^2 \\ d_{31}\,\lambda^{12} & d_{32}\,\lambda^4 & d_{33} \end{pmatrix} \langle H_d^0 \rangle$$

$$L_d = \begin{pmatrix} 1 - \frac{d_{12}^2}{2\,d_{22}^2}\,\lambda^{12} + \mathrm{o}(\lambda^{12}) & \frac{d_{12}}{d_{22}}\,\lambda^6 + \mathcal{O}(\lambda^{10}) & \frac{d_{13}}{d_{33}}\,\lambda^8 + \mathcal{O}(\lambda^{12}) \\ -\frac{d_{12}}{d_{22}}\,\lambda^6 + \mathcal{O}(\lambda^{10}) & 1 - \frac{d_{23}^2}{2\,d_{33}^2}\,\lambda^4 + \mathcal{O}(\lambda^8) & \frac{d_{23}}{d_{33}}\,\lambda^2 + \mathcal{O}(\lambda^6) \\ L_{d,31}\,\lambda^8 + \mathcal{O}(\lambda^{12}) & -\frac{d_{23}}{d_{33}}\,\lambda^2 + \mathcal{O}(\lambda^6) & 1 - \frac{d_{23}^2}{2\,d_{33}^2}\,\lambda^4 + \mathcal{O}(\lambda^8) \end{pmatrix}$$

with

$$L_{d,31} = \frac{d_{12}\,d_{23} - d_{13}\,d_{22}}{d_{22}\,d_{33}} \,.$$

$$R_d = \begin{pmatrix} 1 + \mathrm{o}(\lambda^{12}) & R_{d,12}\,\lambda^8 + \mathcal{O}(\lambda^{12}) & R_{d,13}\,\lambda^{12} + \mathrm{o}(\lambda^{12}) \\ -R_{d,12}\,\lambda^8 - \mathcal{O}(\lambda^{12}) & 1 + \mathcal{O}(\lambda^8) & \frac{(d_{22}\,d_{23} + d_{32}\,d_{33})}{d_{33}^2}\,\lambda^4 + \mathcal{O}(\lambda^8) \\ \mathcal{O}(\lambda^{12}) & -\frac{(d_{22}\,d_{23} + d_{32}\,d_{33})}{d_{33}^2}\,\lambda^4 + \mathcal{O}(\lambda^8) & 1 + \mathcal{O}(\lambda^8) \end{pmatrix}$$

with

$$R_{d,12} = \frac{d_{11}\,d_{12} + d_{21}\,d_{22}}{d_{22}^2} \quad \text{and} \quad R_{d,13} = \frac{d_{11}\,d_{13} + d_{21}\,d_{23} + d_{31}\,d_{33}}{d_{33}^2} \,.$$

Quark Mixing (Scenario A)

$$V = L_u^\dagger \, L_d$$

$$= \begin{pmatrix} 1 - \frac{f_{12}^2}{2\,f_{22}^2}\,\lambda^2 + \mathcal{O}(\lambda^4) & -\frac{f_{12}}{f_{22}}\,\lambda + \mathcal{O}(\lambda^3) & \frac{f_{12}}{f_{22}}\,V_{32}\,\lambda^3 + \mathcal{O}(\lambda^5) \\ \frac{f_{12}}{f_{22}}\,\lambda + \mathcal{O}(\lambda^3) & 1 - \frac{f_{12}^2}{2\,f_{22}^2}\,\lambda^2 + \mathcal{O}(\lambda^4) & -V_{32}\,\lambda^2 + \mathcal{O}(\lambda^4) \\ V_{31}\,\lambda^8 + \mathcal{O}(\lambda^9) & V_{32}\,\lambda^2 + \mathcal{O}(\lambda^6) & 1 - \frac{1}{2}(V_{32})^2\,\lambda^4 + \mathcal{O}(\lambda^6) \end{pmatrix}$$

with

$$V_{32} \equiv \frac{f_{23}}{f_{33}} - \frac{d_{23}}{d_{33}}$$

and

$$V_{31} \equiv \frac{f_{13}}{f_{33}} - \frac{d_{13}}{d_{33}} - \frac{d_{12}}{d_{22}}\,V_{32}\ .$$

$$J_{\mathrm{CP}} = \mathrm{Im}\,(V_{ud}\,V_{ub}^*\,V_{td}^*\,V_{tb}) \sim \lambda^{11}\,.$$

Quark Mixing (Scenario B)

$$V = L_u^\dagger \, L_d$$

$$= \begin{pmatrix} 1 - \frac{f_{12}^2}{2\, f_{22}^2}\,\lambda^2 + \mathcal{O}(\lambda^4) & -\frac{f_{12}}{f_{22}}\,\lambda + \mathcal{O}(\lambda^3) & \left(\frac{f_{12}}{f_{22}}\,V_{32} - \frac{\tilde{f}_{13}}{f_{33}}\right)\lambda^3 + \mathcal{O}(\lambda^5) \\ \frac{f_{12}}{f_{22}}\,\lambda + \mathcal{O}(\lambda^3) & 1 - \frac{f_{12}^2}{2\, f_{22}^2}\,\lambda^2 + \mathcal{O}(\lambda^4) & -V_{32}\,\lambda^2 + \mathcal{O}(\lambda^4) \\ \frac{\tilde{f}_{13}}{f_{33}}\,\lambda^3 + \mathcal{O}(\lambda^7) & V_{32}\,\lambda^2 + \mathcal{O}(\lambda^6) & 1 - \frac{1}{2}(V_{32})^2\,\lambda^4 + \mathcal{O}(\lambda^6) \end{pmatrix}$$

with

$$V_{32} \equiv \frac{f_{23}}{f_{33}} - \frac{d_{23}}{d_{33}}\;.$$

$$J_{\mathrm{CP}} = \mathrm{Im}\,(V_{ud}\,V_{ub}^*\,V_{td}^*\,V_{tb}) \sim \lambda^6\,.$$

SM Yukawa Sector: Charged Leptons

$$M_e = \begin{pmatrix} e_{11}\,\lambda^4 & e_{12}\,\lambda^{12} & \mathrm{o}(\lambda^{12}) \\ e_{21}\,\lambda^8 & e_{22}\,\lambda^2 & e_{23}\,\lambda \\ e_{31}\,\lambda^9 & e_{32}\,\lambda^3 & e_{33} \end{pmatrix} \langle H_d^0 \rangle$$

$$L_e = \begin{pmatrix} 1+\mathrm{o}(\lambda^{12}) & \frac{e_{11}\,e_{21}}{e_{22}^2}\,\lambda^8+\mathcal{O}(\lambda^{10}) & \mathrm{o}(\lambda^{12}) \\ -\frac{e_{11}\,e_{21}}{e_{22}^2}\,\lambda^8+\mathcal{O}(\lambda^{10}) & 1-\frac{e_{23}^2}{2\,e_{33}^2}\,\lambda^2+\mathcal{O}(\lambda^4) & \frac{e_{23}}{e_{33}}\,\lambda+\mathcal{O}(\lambda^3) \\ \frac{e_{11}\,e_{21}\,e_{23}}{e_{22}^2\,e_{33}}\,\lambda^9+\mathcal{O}(\lambda^{11}) & -\frac{e_{23}}{e_{33}}\,\lambda+\mathcal{O}(\lambda^3) & 1-\frac{e_{23}^2}{2\,e_{33}^2}\,\lambda^2+\mathcal{O}(\lambda^4) \end{pmatrix}$$

$$R_e = \begin{pmatrix} 1-\frac{e_{21}^2}{2\,e_{22}^2}\,\lambda^{12}+\mathrm{o}(\lambda^{12}) & \frac{e_{21}}{e_{22}}\,\lambda^6+\mathcal{O}(\lambda^8) & \frac{(e_{21}\,e_{23}+e_{31}\,e_{33})}{e_{33}^2}\,\lambda^9+\mathcal{O}(\lambda^{11}) \\ -\frac{e_{21}}{e_{22}}\,\lambda^6+\mathcal{O}(\lambda^8) & 1-\frac{1}{2}(R_{e,23})^2\lambda^6+\mathcal{O}(\lambda^8) & R_{e,23}\,\lambda^3+\mathcal{O}(\lambda^5) \\ R_{e,31}\,\lambda^9+\mathcal{O}(\lambda^{11}) & -R_{e,23}\,\lambda^3+\mathcal{O}(\lambda^5) & 1-\frac{1}{2}(R_{e,23})^2\,\lambda^6+\mathcal{O}(\lambda^8) \end{pmatrix}$$

with

$$R_{e,23} \equiv \frac{e_{22}\,e_{23}+e_{32}\,e_{33}}{e_{33}^2} \qquad \text{and} \qquad R_{e,31} \equiv \frac{1}{e_{33}}\left(\frac{e_{21}\,e_{32}}{e_{22}} - e_{31}\right).$$

Leptoquark Coupling x

$$x = L_e^T \begin{pmatrix} \hat{a}_{11}\,\lambda^9 & \hat{a}_{12}\,\lambda^{12} & \mathrm{o}(\lambda^{12}) \\ \hat{a}_{21}\,\lambda^8 & \hat{a}_{22}\,\lambda^3 & \hat{a}_{23}\,\lambda \\ \hat{a}_{31}\,\lambda^8 & \hat{a}_{32}\,\lambda^2 & \hat{a}_{33} \end{pmatrix} L_d = \begin{pmatrix} a_{11}\,\lambda^9 & a_{12}\,\lambda^{11} & a_{13}\,\lambda^9 \\ a_{21}\,\lambda^8 & a_{22}\,\lambda^3 & a_{23}\,\lambda \\ a_{31}\,\lambda^8 & a_{32}\,\lambda^2 & a_{33} \end{pmatrix}$$

$$\begin{aligned}
a_{11} &= \hat{a}_{11} + \mathrm{o}(\lambda^3)\,, \\
a_{12} &= -\frac{\hat{a}_{22}\,e_{11}\,e_{21}}{e_{22}^2} + \frac{\hat{a}_{23}\,d_{23}\,e_{11}\,e_{21}}{d_{33}\,e_{22}^2} + \frac{\hat{a}_{32}\,e_{11}\,e_{21}\,e_{23}}{e_{22}^2\,e_{33}} - \frac{\hat{a}_{33}\,d_{23}\,e_{11}\,e_{21}\,e_{23}}{d_{33}\,e_{22}^2\,e_{33}} + \mathcal{O}(\lambda)\,, \\
a_{13} &= -\frac{\hat{a}_{23}\,e_{11}\,e_{21}}{e_{22}^2} + \frac{\hat{a}_{33}\,e_{11}\,e_{21}\,e_{23}}{e_{22}^2\,e_{33}} + \mathcal{O}(\lambda^2)\,, \\
a_{21} &= \hat{a}_{21} + \mathcal{O}(\lambda)\,, \\
a_{22} &= \hat{a}_{22} - \frac{d_{23}}{d_{33}}\left(\hat{a}_{23} - \frac{\hat{a}_{33}\,e_{23}}{e_{33}}\right) - \frac{\hat{a}_{32}\,e_{23}}{e_{33}} + \mathcal{O}(\lambda^2)\,, \\
a_{23} &= \hat{a}_{23} - \frac{\hat{a}_{33}\,e_{23}}{e_{33}} + \mathcal{O}(\lambda^2)\,, \\
a_{31} &= \hat{a}_{31} - \frac{\hat{a}_{32}\,d_{12}}{d_{22}} - \frac{\hat{a}_{33}\,d_{13}}{d_{33}} + \frac{\hat{a}_{33}\,d_{12}\,d_{23}}{d_{22}\,d_{33}} + \mathcal{O}(\lambda)\,, \\
a_{32} &= \hat{a}_{32} - \frac{\hat{a}_{33}\,d_{23}}{d_{33}} + \mathcal{O}(\lambda^2)\,, \\
a_{33} &= \hat{a}_{33} + \mathcal{O}(\lambda^2)\,.
\end{aligned}$$

Leptoquark Coupling z

$$z = L_e^T \, \hat{x} \, L_u = \begin{pmatrix} c_{11}\,\lambda^9 & c_{12}\,\lambda^{10} & c_{13}\,\lambda^9 \\ c_{21}\,\lambda^4 & c_{22}\,\lambda^3 & c_{23}\,\lambda \\ c_{31}\,\lambda^3 & c_{32}\,\lambda^2 & c_{33} \end{pmatrix}$$

$$\begin{aligned}
c_{11} &= \hat{a}_{11} + \mathcal{O}(\lambda^2) \, , \\
c_{12} &= \frac{\hat{a}_{11}\, f_{12}}{f_{22}} + \mathcal{O}(\lambda) \, , \\
c_{13} &= -\frac{\hat{a}_{23}\, e_{11}\, e_{21}}{e_{22}^2} + \frac{\hat{a}_{33}\, e_{11}\, e_{21}\, e_{23}}{e_{22}^2\, e_{33}} + \mathcal{O}(\lambda^2) \, , \\
c_{21} &= -\frac{f_{12}}{e_{33}\, f_{22}\, f_{33}} \left(\hat{a}_{33}\, e_{23}\, f_{23} - \hat{a}_{23}\, e_{33}\, f_{23} - \hat{a}_{32}\, e_{23}\, f_{33} + \hat{a}_{22}\, e_{33}\, f_{33}\right) \\
&\quad - \frac{\tilde{f}_{13}}{f_{33}} \left(\hat{a}_{23} - \frac{\hat{a}_{33}\, e_{23}}{e_{33}}\right) + \mathcal{O}(\lambda^2) \, , \\
c_{22} &= \hat{a}_{22} - \frac{\hat{a}_{32}\, e_{23}}{e_{33}} - \left(\hat{a}_{23} - \frac{\hat{a}_{33}\, e_{23}}{e_{33}}\right) \frac{f_{23}}{f_{33}} + \mathcal{O}(\lambda^2) \, , \\
c_{23} &= \hat{a}_{23} - \frac{\hat{a}_{33}\, e_{23}}{e_{33}} + \mathcal{O}(\lambda^2) \, , \\
c_{31} &= \frac{f_{12}(\hat{a}_{33}\, f_{23} - \hat{a}_{32}\, f_{33})}{f_{22}\, f_{33}} - \frac{\tilde{f}_{13}}{f_{33}}\, \hat{a}_{33} + \mathcal{O}(\lambda^2) \, , \\
c_{32} &= \hat{a}_{32} - \frac{\hat{a}_{33}\, f_{23}}{f_{33}} + \mathcal{O}(\lambda^2) \, , \\
c_{33} &= \hat{a}_{33} + \mathcal{O}(\lambda^2) \, .
\end{aligned}$$

Leptoquark Coupling $\boldsymbol{y}$

$$\boldsymbol{y} = R_e^T \begin{pmatrix} \hat{b}_{11}\,\lambda^9 & \hat{b}_{12}\,\lambda^9 & \hat{b}_{13}\,\lambda^9 \\ \hat{b}_{21}\,\lambda^9 & \hat{b}_{22}\,\lambda^3 & \hat{b}_{23}\,\lambda^3 \\ \hat{b}_{31}\,\lambda^{12} & \hat{b}_{32} & \hat{b}_{33}\,\lambda^4 \end{pmatrix} R_u = \begin{pmatrix} b_{11}\,\lambda^9 & b_{12}\,\lambda^9 & b_{13}\,\lambda^9 \\ b_{21}\,\lambda^8 & b_{22}\,\lambda^3 & b_{23}\,\lambda^3 \\ b_{31}\,\lambda^5 & b_{32} & b_{33}\,\lambda^4 \end{pmatrix}$$

$$\begin{aligned}
b_{11} &= \hat{b}_{11} + \mathrm{o}(\lambda^3)\,, \\
b_{12} &= \hat{b}_{12} - \frac{\hat{b}_{22}\,e_{21}}{e_{22}} - \frac{\hat{b}_{32}\,e_{31}}{e_{33}} + \frac{\hat{b}_{32}\,e_{21}\,e_{32}}{e_{22}\,e_{33}} + \mathcal{O}(\lambda^2)\,, \\
b_{13} &= \hat{b}_{13} - \frac{\hat{b}_{23}\,e_{21}}{e_{22}} + \mathcal{O}(\lambda^2)\,, \\
b_{21} &= -\frac{\hat{b}_{22}\,f_{11}\,f_{12}}{f_{22}^2} + \frac{\hat{b}_{32}\,e_{22}\,e_{23}\,f_{11}\,f_{12}}{e_{33}^2\,f_{22}^2} + \frac{\hat{b}_{32}\,e_{32}\,f_{11}\,f_{12}}{e_{33}\,f_{22}^2} + \mathcal{O}(\lambda)\,, \\
b_{22} &= \hat{b}_{22} - \frac{\hat{b}_{32}(e_{22}\,e_{23} + e_{32}\,e_{33})}{e_{33}^2} + \mathcal{O}(\lambda^2)\,, \\
b_{23} &= \hat{b}_{23} + \mathcal{O}(\lambda^4)\,, \\
b_{31} &= -\frac{\hat{b}_{32}\,f_{11}\,f_{12}}{f_{22}^2} + \mathcal{O}(\lambda)\,, \\
b_{32} &= \hat{b}_{32} + \mathcal{O}(\lambda^6)\,, \\
b_{33} &= \hat{b}_{33} + \frac{\hat{b}_{32}\,f_{32}}{f_{33}} + \mathcal{O}(\lambda^2)\,.
\end{aligned}$$