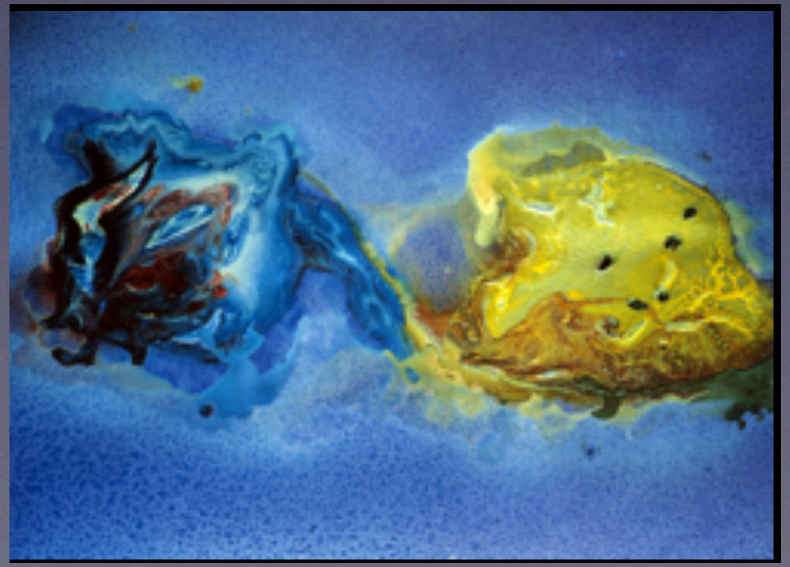
α_s from the QCD STATIC ENERGY and the QCD FORCE



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Nora Brambilla

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it was calculated since the start of QCD

it is an observable (up to an additive constant)

it is very well known in perturbation theory: using effective field theory (pNRQCD) we can obtain it at 3 loops and with NNNLL accuracy

it is now calculated with high precision on the lattice with 2+1 and 2+1+1 flavors

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Deal with the renormalon between the mass and the potential

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Deal with the renormalon between the mass and the potential

—> calculate directly on the lattice the static force which is renormalon free

STATIC ENERGY

Bibliography

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A. Bazavov, N. Brambilla, X. Garcia i Tormo, P. Petrecky, J. Soto, A. Vairo, J. Weber

Determination of the QCD coupling from the static energy and the free energy

Phys. ReV. D 100 (2019), 11, 114511 arXiv:1907.11747

(1) A. Bazavov, N. Brambilla, X. Garcia i Tormo, P. Petreczky, J. Soto and A. Vairo
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Precision determination of $r_0\Lambda_{\overline{\rm MS}}$ from the QCD static energy
Phys. Rev. Lett. 105 (2010) 212001 arXiv:1006.2066

(5) N. Brambilla, X. Garcia i Tormo, J. Soto and A. Vairo
The QCD static energy at NNNLL
Phys. Rev. D80 (2009) 034016 arXiv:0906.1390

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N. Brambilla, Hee Sok Chung, A. Vairo, X. Wang
QCD static force in gradient flow
in press on JHEP arXiv:2111.07811

V. Leino, N. Brambilla, J. Mayer-Steudte, A. Vairo
Static force from generalized Wilson loops using gradient flow
arXiv:2111.10212

N. Brambilla, V.Leino, O. Philipsen, C. Reisinger, A. Vairo, M. Wagner
Lattice gauge theory computation of the static force
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A. Vairo,
Strong coupling from QCD static energy
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PRD 63 (2001) 014023, arXiv 000250

Extraction of alphas from comparing the QCD static energy $E_0(r)$

calculated in perturbative QCD (using pNRQCD)

known at 3 loops and NNNLL accuracy

measured on the lattice

given by the static Wilson loop

Extraction of alphas from comparing the QCD static energy $E_0(r)$

calculated in perturbative QCD (using pNRQCD) — known at 3 loops and NNNLL accuracy

measured on the lattice — given by the static Wilson loop

- If $r\Lambda_{QCD} << 1$ both evaluations should agree
  - Fix Λ_{QCD} from a low enegy observable calculated on the lattice
  - Evaluate E_0 perturbatively in the standard $\overline{MS}$ scheme
  - Get $\Lambda_{\overline{MS}}$ by equating lattice and perturbative expressions
- No lattice to $\overline{MS}$ renormalization scheme change necessary

Extraction of alphas from comparing the QCD static energy $E_0(r)$

alphas extracted in this way gives one of the most precise determinations at a low energy scale (lattice cannot explore too short distances)

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competitive

complementary to high energy determinations

intrinsic value-> add to our understanding of QCD and heavily constrains the running

Static energy of a static quark-antiquark pair located at a distance r

$$E_0(r) \;=\; \lim_{T\to\infty} \frac{i}{T} \ln \langle \square \rangle; \qquad r \overset{T}{\square} = \exp\left\{ ig \oint_{rXT} dz^\mu A_\mu \right\}$$

Perturbation theory describes $E_0(r)$ in the short range ($r\Lambda \ll 1$, $\alpha_s(1/r) < 1$):

$$E_0(r) = \Lambda_s - \frac{C_F \alpha_s}{r}(1 + \#\alpha_s + \#\alpha_s^2 + \#\alpha_s^3 + \#\alpha_s^3 \ln \alpha_s + \#\alpha_s^4 \ln^2 \alpha_s + \#\alpha_s^4 \ln \alpha_s + \dots)$$

- $E_0(r)$ is known at three loops.
- $\ln \alpha_s$ signals the cancellation of contributions coming from different energy scales:

$$\ln \alpha_s = \ln \frac{\mu}{1/r} + \ln \frac{\alpha_s/r}{\mu}$$

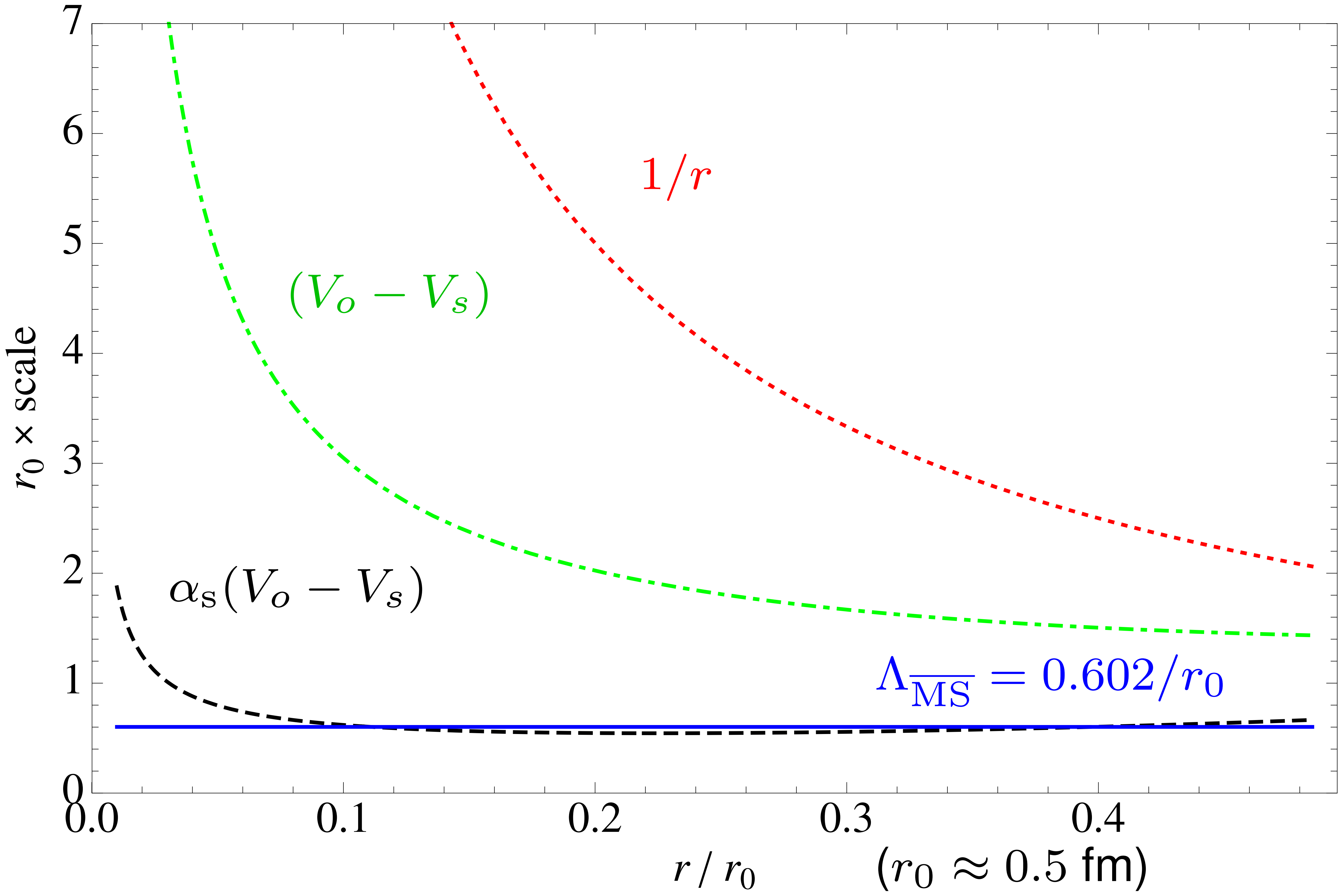
○ Brambilla Pineda Soto Vairo PRD 60 (1999) 091502

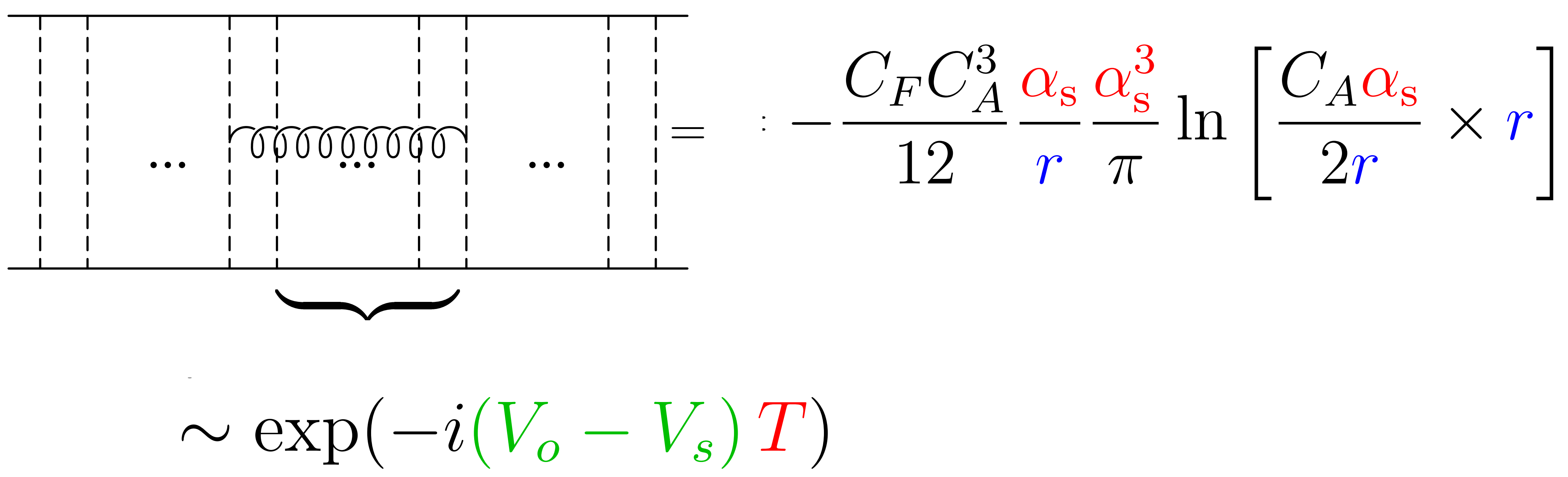
Energy scales

In the short range the static Wilson loop is characterized by a hierarchy of energy scales:

$$1/r \gg V_o - V_s \gg \Lambda; \qquad V_s \approx -C_F \frac{\alpha_s}{r}, \quad V_o \approx \frac{1}{2N}\frac{\alpha_s}{r}$$

The wilson loop calculated order in perturbation theory is divergent from 3 loops on: one needs an EFT to resume and combine contribution from different scales





Appelquist Dine Muzinich 78, Brambilla Pineda Soto Vairo 99

pNRQCD (potential NonRelativistic QCD) EFT for QQbar r<< Lambda_QCD^-1

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \sum_{i=1}^{n_f} \bar{q}_i \, i \not{D} q_i + \int d^3 r \, \mathrm{Tr} \left\{ \mathrm{S}^\dagger \left(i\partial_0 - h_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - h_o \right) \mathrm{O} \right\}$$

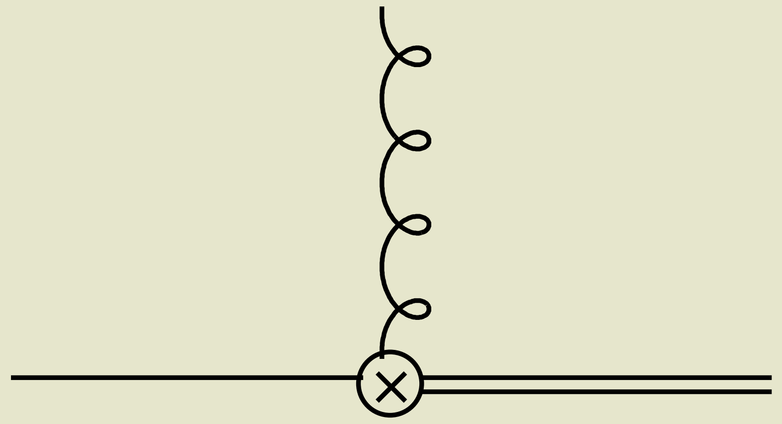
- LO in r

$\theta(T)\, e^{-iTh_s}$ $\quad$ $\theta(T)\, e^{-iTh_o} \left(e^{-i\int dt\, A^{\mathrm{adj}}} \right)$

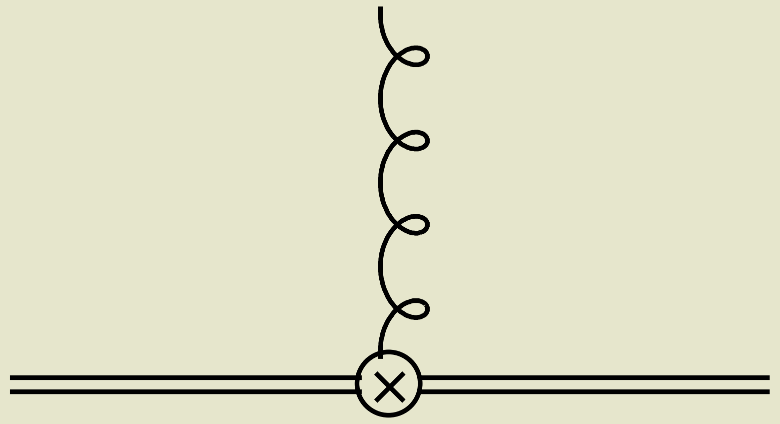
pNRQCD allows to address scale factorization

$$+ V_A \mathrm{Tr} \left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S} + \mathrm{S}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} \right\} + \frac{V_B}{2} \mathrm{Tr} \left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} + \mathrm{O}^\dagger \mathrm{O} \mathbf{r} \cdot g\mathbf{E} \right\}$$

- NLO in r



$O^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S}$



$O^\dagger \{\mathbf{r} \cdot g\mathbf{E},\, \mathrm{O}\}$

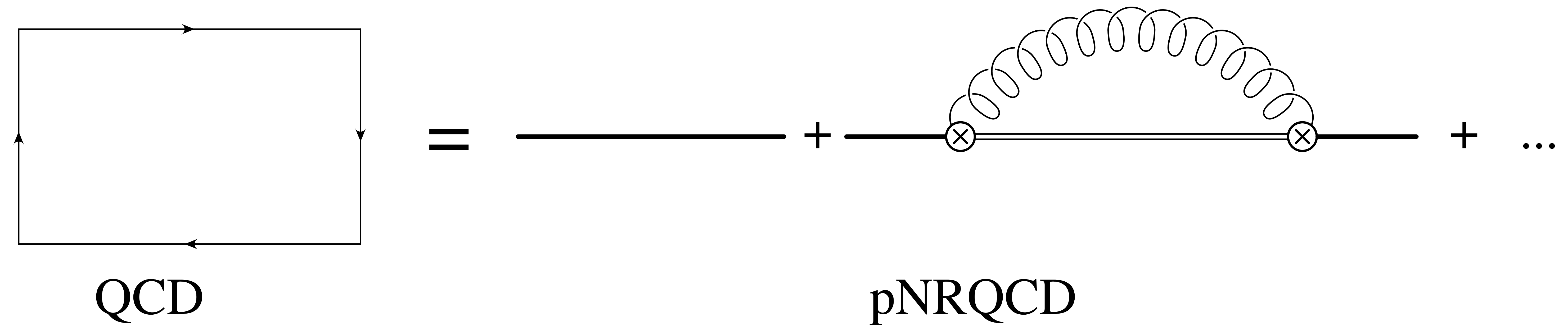
$+ \cdots$

Degrees of freedom: colour singlet S and colour octet O and low energy gluons (multipole expanded)

The potentials are the matching coefficients of pNRQCD : they are calculated via a well defined matching procedure

Effective Field Theories

EFTs allow the factorization of contributions from different energy scales.



$$E_0(r) = \Lambda_s + V_s(r,\mu) - i\frac{g^2}{N} V_A^2 \int_0^\infty dt\, e^{-it(V_o - V_s)} \langle \mathrm{Tr}\, \mathbf{r}\cdot\mathbf{E}(t)\, \mathbf{r}\cdot\mathbf{E}(0)\rangle(\mu) + \dots$$

res. mass — potential — ultrasoft contribution

○ `Brambilla Pineda Soto Vairo NPB 566 (2000) 275`

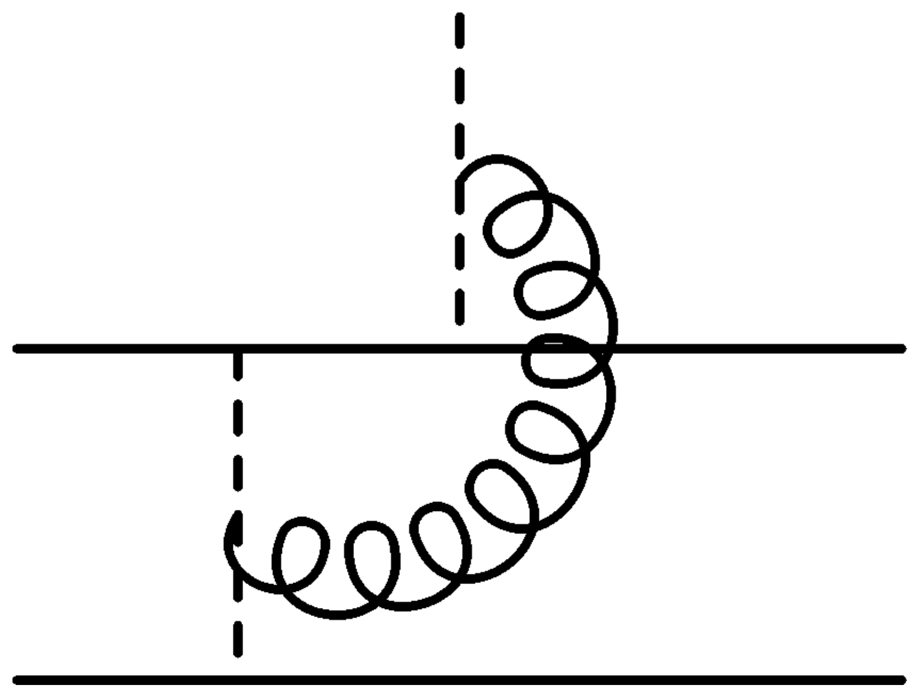
The μ dependence cancels between

$V_s \sim \ln r\mu, \ln^2 r\mu$, ...

ultrasoft contribution $\sim \ln(V_o - V_s)/\mu, \ln^2(V_o - V_s)/\mu, \dots \ln r\mu, \ln^2 r\mu, \dots$

V_A

The first contributing diagrams are of the type:

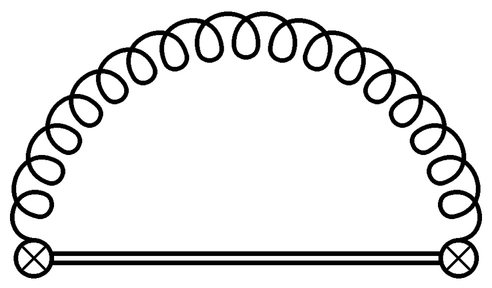


Therefore

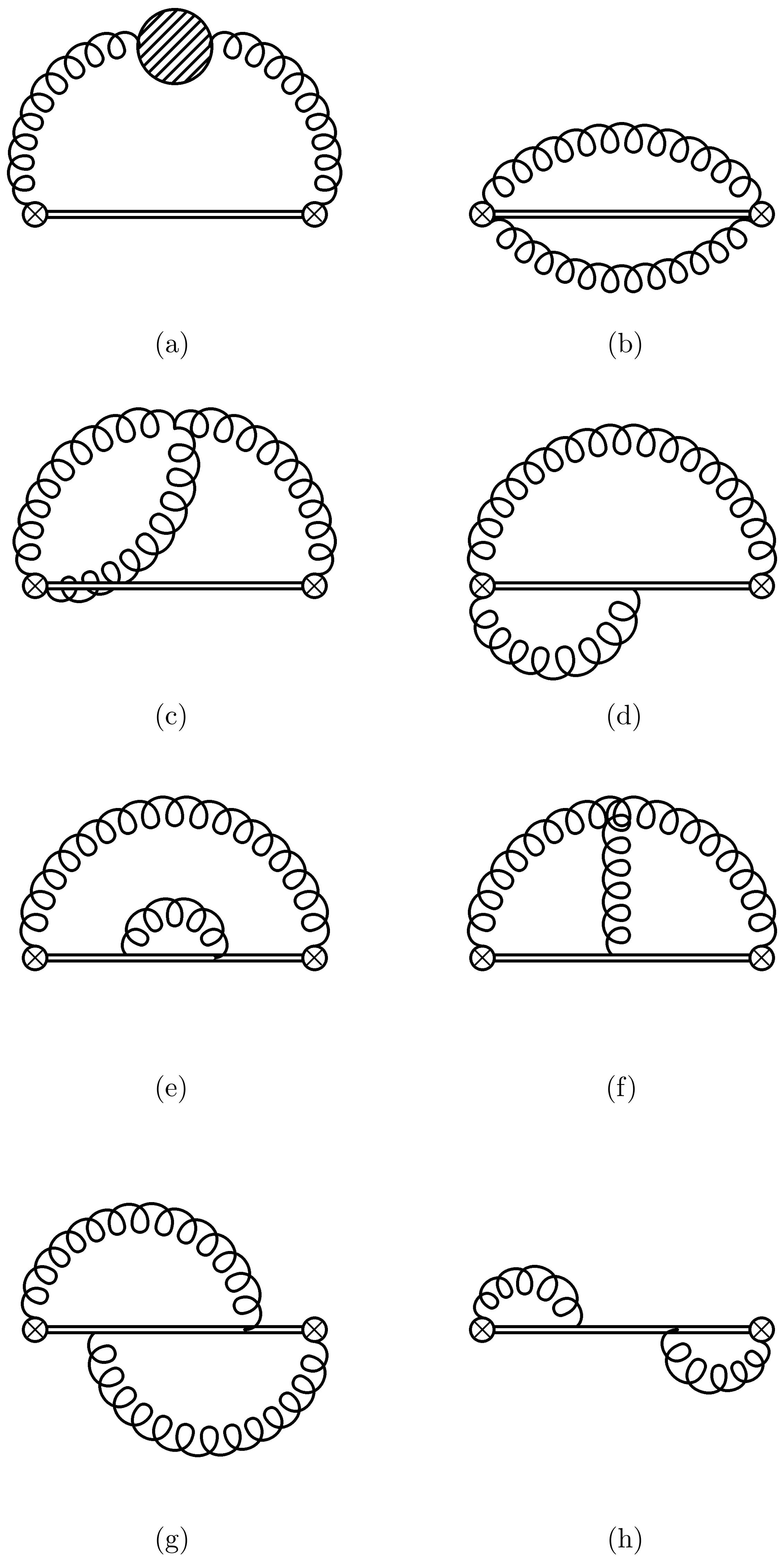
$$V_A(r,\mu) = 1 + \mathcal{O}(\alpha_s^2)$$

Chromoelectric field correlator: $\langle E(t)E(0)\rangle$

Is known at two loops.



LO



NLO

○ Eidemüller Jamin PLB 416 (1998) 415

Static octet potential

$$\lim_{T\to\infty} \frac{i}{T} \ln \frac{\langle \Box \rangle}{\langle \phi_{ab}^{\text{adj}} \rangle} = \frac{1}{2N} \frac{\alpha_s}{r} (1 + \#\alpha_s + \#\alpha_s^2 + \#\alpha_s^3 + \#\alpha_s^3 \ln \mu r + \ldots)$$

Is known at three loops.

- Anzai Prausa A.Smirnov V.Smirnov Steinhauser PRD 88 (2013) 054030

Static singlet potential at N^4LO

$$
\begin{aligned}
V_s(r,\mu) \;=\; & -C_F \frac{\alpha_s(1/r)}{r}\Bigg\{1 + \frac{\alpha_s(1/r)}{4\pi} a_1 + \left(\frac{\alpha_s(1/r)}{4\pi}\right)^2 a_2 \\
& + \left(\frac{\alpha_s(1/r)}{4\pi}\right)^3 \left[\frac{16\,\pi^2}{3} C_A^3 \,\ln r\mu + a_3\right] \\
& + \left(\frac{\alpha_s(1/r)}{4\pi}\right)^4 \left[a_4^{L2} \ln^2 r\mu + \left(a_4^L + \frac{16}{9}\pi^2\, C_A^3 \beta_0(-5 + 6\ln 2)\right) \ln r\mu + \dots\right] \\
& + \qquad \dots \qquad \Bigg\}
\end{aligned}
$$

a_3 from
- Anzai Kiyo Sumino PRL 104 (2010) 112003
  A.Smirnov V.Smirnov Steinhauser PRL 104 (2010) 112002

a_1	Billoire 80
a_2	Schroeder 99, Peter 97
coeff $ln r\mu$	N.B. Pineda, Soto, Vairo 99
a_4^{L2}, a_4^L	N.B., Garcia, Soto, Vairo 06

The constant a_4 at 4 loops is not yet known

Static energy at N^4LO

$$
\begin{aligned}
E_0(r) \quad = \quad & \Lambda_s - \frac{C_F \alpha_s(1/r)}{r} \Bigg\{ 1 + \frac{\alpha_s(1/r)}{4\pi} \left[a_1 + 2\gamma_E \beta_0 \right] \\
& + \left(\frac{\alpha_s(1/r)}{4\pi} \right)^2 \left[a_2 + \left(\frac{\pi^2}{3} + 4\gamma_E^2 \right) \beta_0^2 + \gamma_E \left(4 a_1 \beta_0 + 2\beta_1 \right) \right] \\
& + \left(\frac{\alpha_s(1/r)}{4\pi} \right)^3 \left[\frac{16\pi^2}{3} C_A^3 \ln \frac{C_A \alpha_s(1/r)}{2} + \tilde{a}_3 \right] \\
& + \left(\frac{\alpha_s(1/r)}{4\pi} \right)^4 \left[a_4^{L2} \ln^2 \frac{C_A \alpha_s(1/r)}{2} + a_4^L \ln \frac{C_A \alpha_s(1/r)}{2} + \dots \right] \\
& + \quad \dots \quad \Bigg\}
\end{aligned}
$$

- Brambilla Pineda Soto Vairo PRD 60 (1999) 091502
  Brambilla Garcia Soto Vairo PLB 647 (2007) 185

Renormalization group equations

$$\begin{cases} \mu \dfrac{d}{d\mu} V_s = -\dfrac{2}{3} C_F \dfrac{\alpha_s}{\pi} r^2 V_A^2 \left[V_o - V_s\right]^3 \left(1 + \dfrac{\alpha_s}{\pi} c\right) \\ \mu \dfrac{d}{d\mu} V_o = \dfrac{1}{N} \dfrac{\alpha_s}{\pi} r^2 V_A^2 \left[V_o - V_s\right]^3 \left(1 + \dfrac{\alpha_s}{\pi} c\right) \\ \mu \dfrac{d}{d\mu} V_A = 0 \\ \mu \dfrac{d}{d\mu} \alpha_s = \alpha_s \beta(\alpha_s); \qquad c = \dfrac{-5 n_f + C_A(6\pi^2 + 47)}{108} \end{cases}$$

- Pineda Soto PLB 495 (2000) 323

  Brambilla Garcia Soto Vairo PRD 80 (2009) 034016

Static singlet potential and energy at N^{3}LL

$$
V_s(r,\mu) = V_s(r,1/r) - \frac{C_F C_A^3}{6\beta_0}\frac{\alpha_s^3(1/r)}{r}\left\{\left(1+\frac{3}{4}\frac{\alpha_s(1/r)}{\pi}a_1\right)\ln\frac{\alpha_s(1/r)}{\alpha_s(\mu)}\right.
\left.\left(\frac{\beta_1}{4\beta_0}-6c\right)\left[\frac{\alpha_s(\mu)}{\pi}-\frac{\alpha_s(1/r)}{\pi}\right]\right\}
$$

Summed to the ultrasoft contribution at two loops, it provides the static energy at N^{3}LL.

- Brambilla Garcia Soto Vairo PRD 80 (2009) 034016
  Garcia MPLA 28 (2013) 1330028

Force and mass renormalon

The perturbative expansion of V_s is affected by a renormalon ambiguity of order Λ.
This ambiguity does not affect the slope of the potential (and the extraction of $\alpha_{\rm s}$).

It may be eliminated from the perturbative series

- either by subtracting a (constant) series in $\alpha_{\rm s}$ to V_s and reabsorb it in a redefinition of the residual mass Λ_s,
- or by considering the force:

$$F(r, \alpha_{\rm s}(\nu)) = \frac{d}{dr} E_0(r, \alpha_{\rm s}(\nu))$$

  - The force $F(r, \alpha_{\rm s}(1/r))$ could be directly compared with lattice,
  - or integrated and compared with the static energy

$$E_0(r) = \int_{r_*}^{r} dr'\, F(r', \alpha_{\rm s}(1/r'))$$

up to an irrelevant constant fixed by the overall normalization of the lattice data.
Note that there are no $\ln \nu r$ (ν = renormalization scale).

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$$E_0(r) = \int_{r_*}^{r} dr'\, F(r',\alpha_{\rm s}(1/r'))$$

This is the formula that we used to compare to the lattice data on the static energy

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alphas from the static energies: analysis

2010 extraction of r_0Lambda_MS from quenched data $r_0\Lambda_{\overline{\rm MS}} = 0.637^{+0.032}_{-0.030}$

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2+1 flavour lattice data:

2012 extraction $\alpha_s(M_Z) = 0.1156^{+0.0021}_{-0.0022}$

2014 extraction $\alpha_s(M_Z) = 0.1166^{+0.0012}_{-0.0008}$

2019 extraction $\alpha_s(M_Z) = 0.11660^{+0.00110}_{-0.00056}$

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2014

β	7.373	7.596	7.825
r_1/a	5.172(34)	6.336(56)	7.690(58)
Volume	$48^3 \times 64$	64^4	64^4

2019

		$m_l = m_s/5$		
β	a [fm]	N_σ, N_τ	am_s	$m_\pi L$
8.000	0.035	64^4	0.01299	3.6
8.200	0.029	64^4	0.01071	3.1
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reaches distances as small as 0.0237 fm full analysis will be presented in talk by J. Weber, **in particular lattice artifacts and discretisation errors will be discussed in depth**

2019

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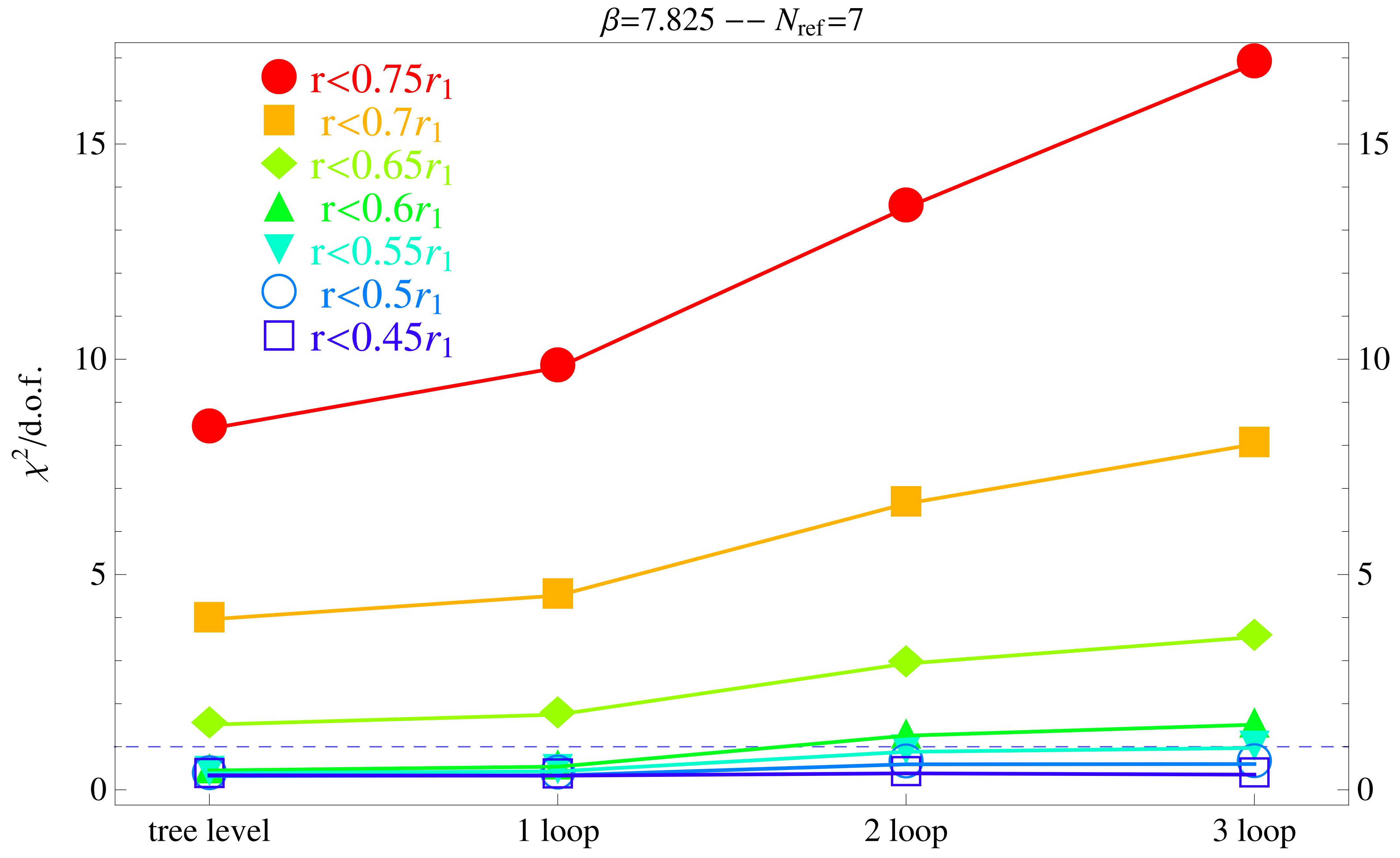
General Procedure (also to make sure that the data have reached the perturbative regime)

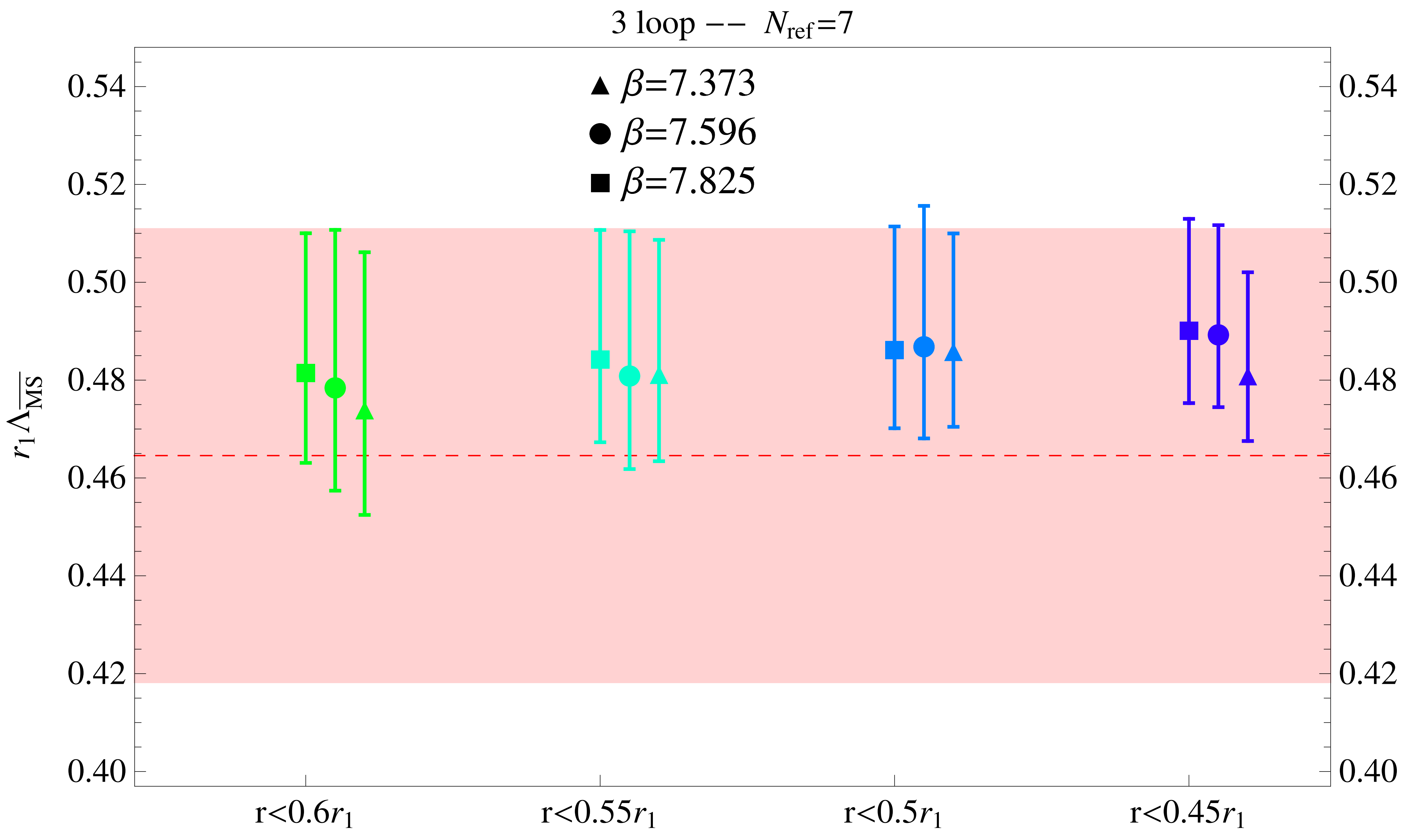
We use data for each value of the lattice spacing separately, and at the end perform an average of the different obtained values of $\alpha_{\rm s}$ with the following procedure.

- Perform fits to the lattice data for the static energy $E_0(r)$ at different orders of perturbative accuracy. The parameter of the fits is $\Lambda_{\overline{\rm MS}}$.
- Repeat the above fits for each of the following distance ranges: $r < 0.75 r_1$, $r < 0.7 r_1$, $r < 0.65 r_1$, $r < 0.6 r_1$, $r < 0.55 r_1$, $r < 0.5 r_1$, and $r < 0.45 r_1$.
- Use ranges where the reduced χ^2 either decreases or does not increase by more than one unit when increasing the perturbative order, or is smaller than 1.
- To estimate the perturbative uncertainty of the result, repeat the fits
  - by varying the scale in the perturbative expansion, from $\nu = 1/r$ to $\nu = \sqrt{2}/r$ and $\nu = 1/(\sqrt{2} r)$,
  - by adding/subtracting a term $\pm (C_F/r^2)\alpha_{\rm s}^{n+2}$ to the expression at n loops.

  Take the largest uncertainty.

Analysis of the energy





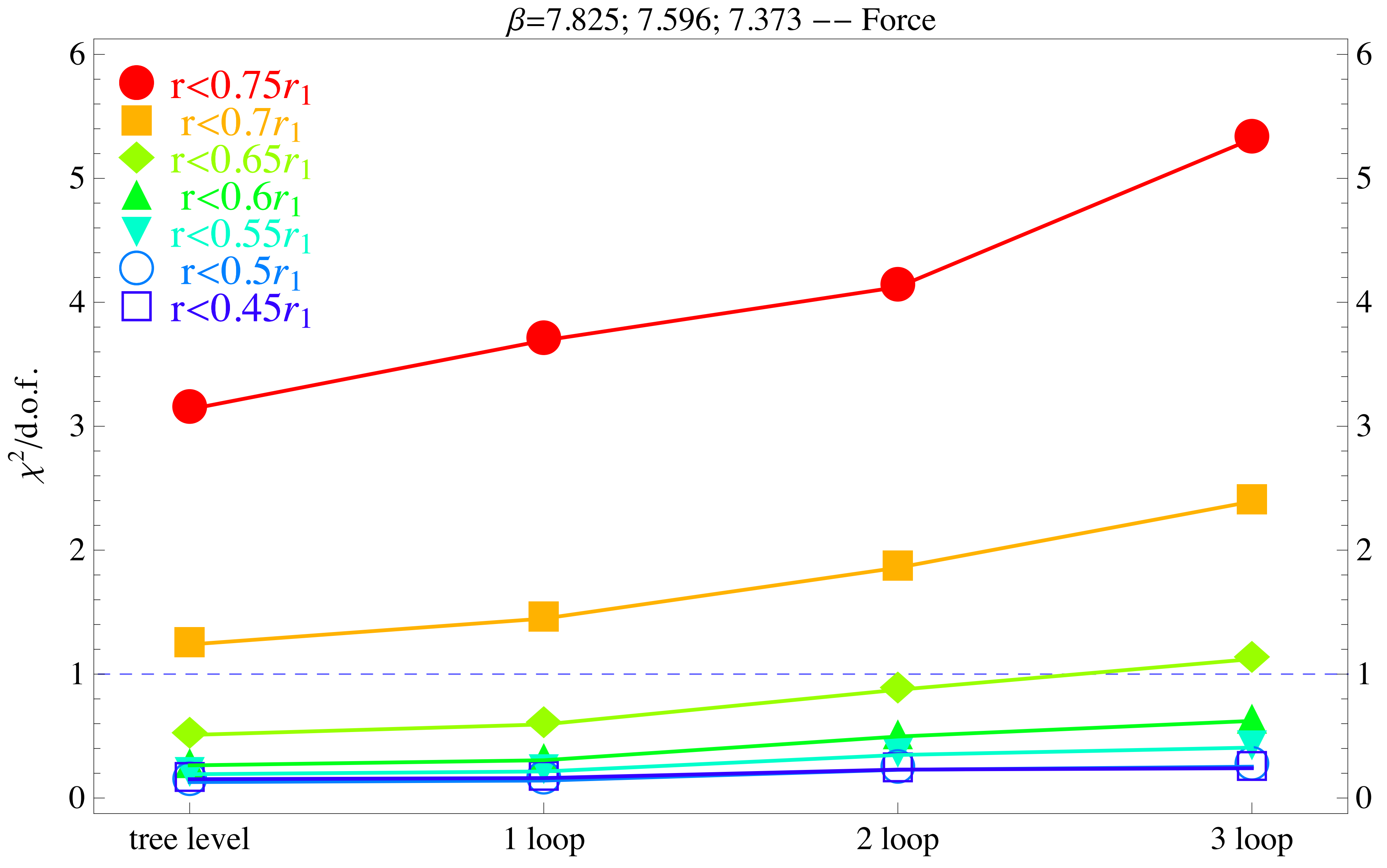
Fits for $r < 0.6r_1$ are acceptable. In the final result we use only fits for $r < 0.5r_1$.

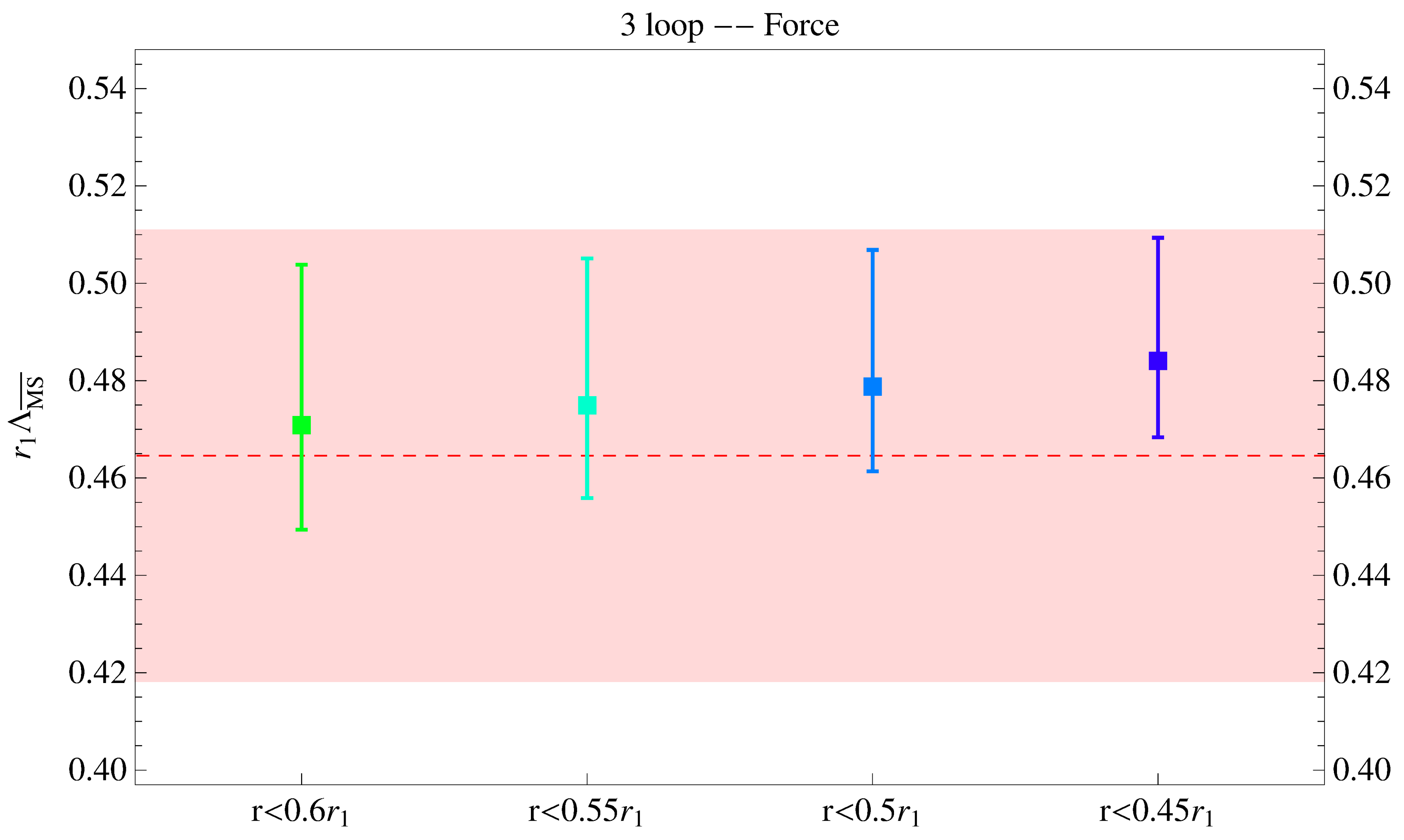
The fitting curve has been normalized on the 7th lattice point.

The band shows an old but similar determination of 2012.

numerically reconstructed from the lattice data
on the static energy interpolated by splines—>bigger error

Analysis with the force



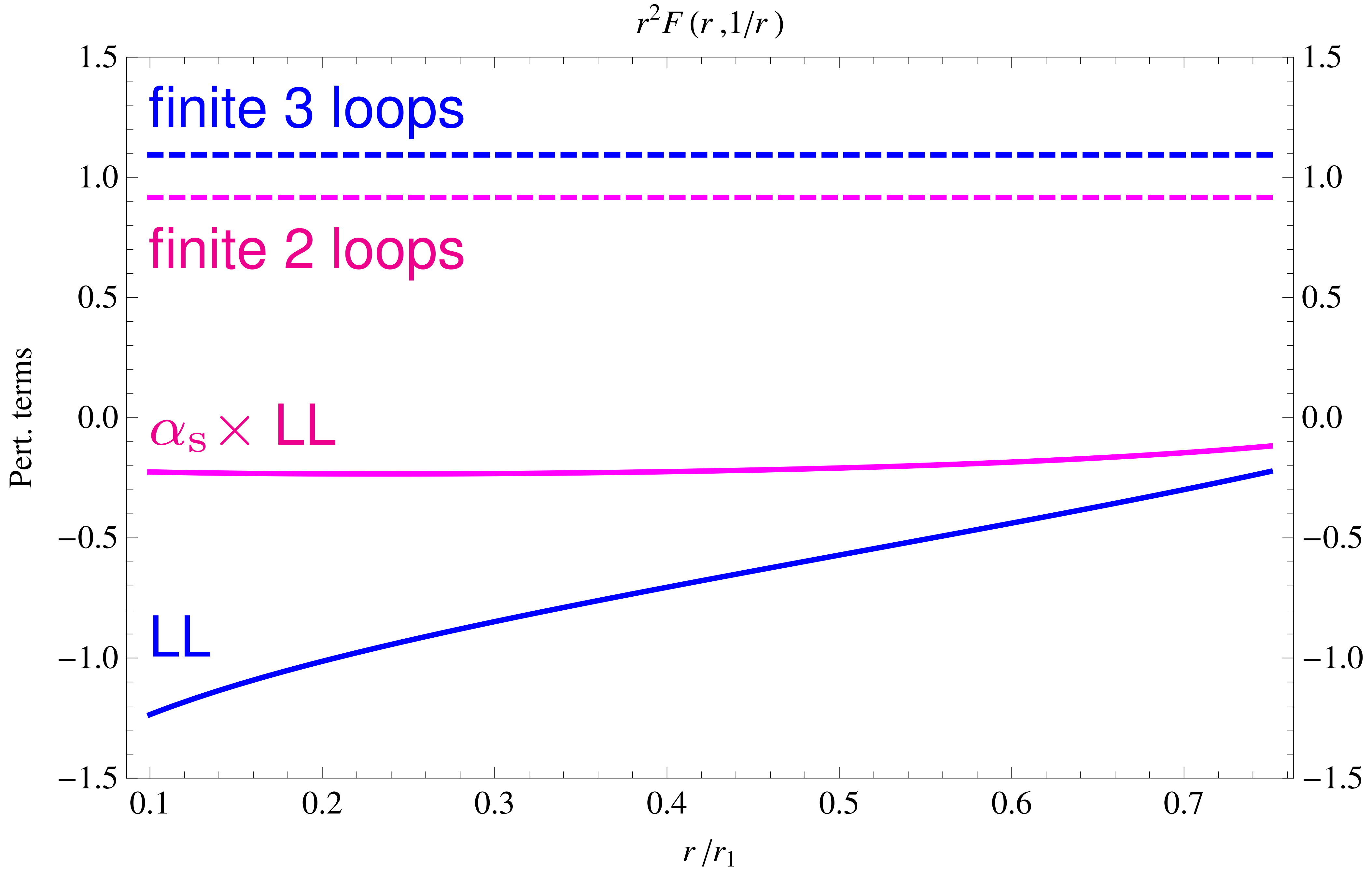


The band shows the determination of 2012.

confirms the extraction from the static energy

The counting of the ultrasoft contributions

finite 3 loops have a alphas^4 factorized out and same for the rest

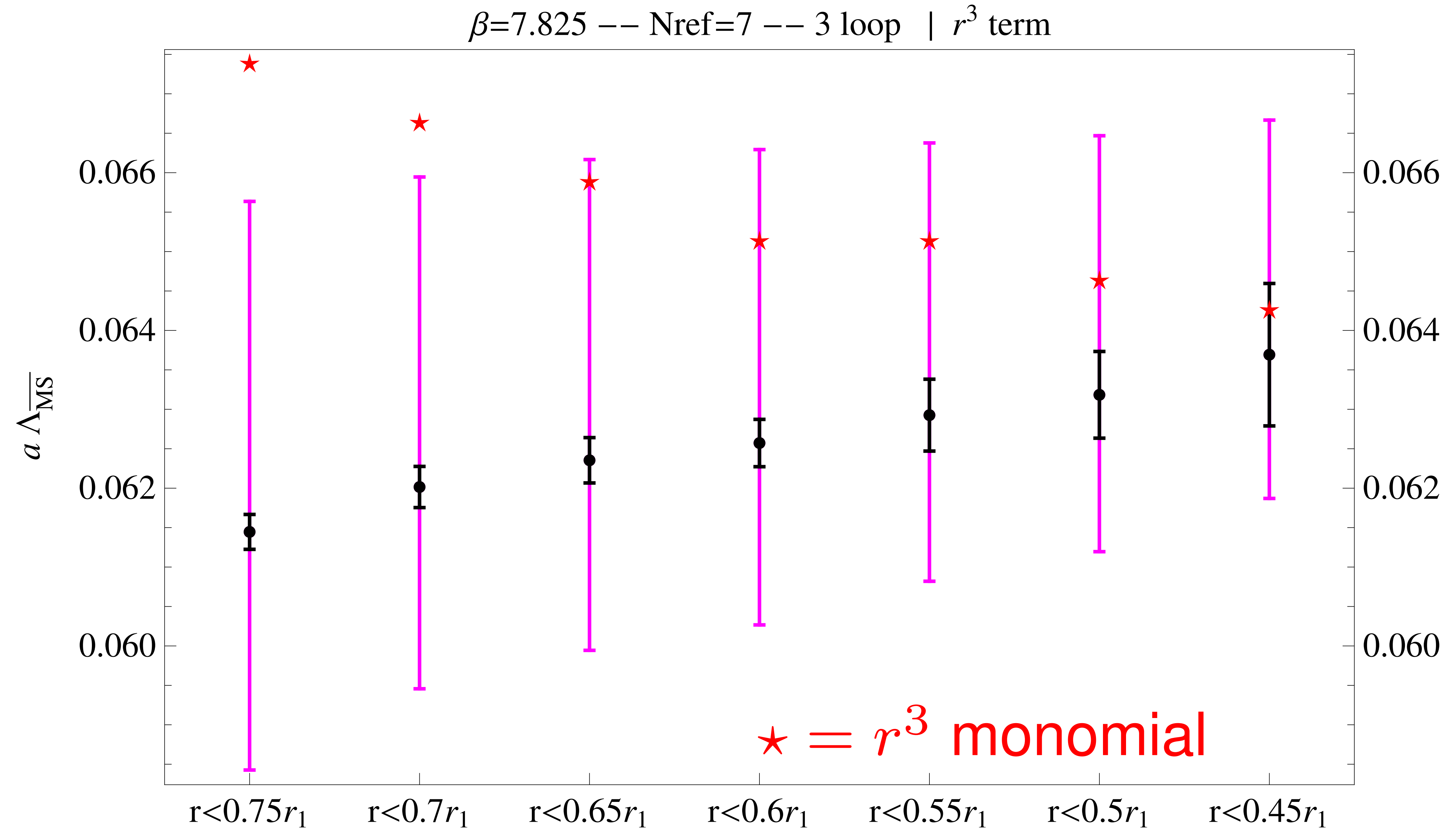


we observe cancellations between the soft and the ultrasoft part at 3 loops, same cancellation may arise at 4 loops but the constant at 4 loops is not known

Given the size of these terms we work at 3 loops and count the US log resummed terms together with the 3 loops: we work at 3 loops plus LL resummation, include in the perturbative error difference between 3 loop and 3 loop plus LL

Leading-ultrasoft resummation included along with the three-loop terms is consistent with the observed size of the terms. This goes in our final result.
We chose $\mu = 1.26 r_1^{-1} \sim 0.8$ GeV, for the ultrasoft factorization scale.
Variations of μ only produce small effects on the results.

Looking for condensates and nonperturbative corrections



By repeating the fits adding a monomial term proportional to r^3 and r^2, which could be associated with gluon and quark local condensates, and also a term proportional to r, we do not find evidence for a significant non-perturbative term at short distances and the value of $\Lambda_{\overline{\rm MS}}$ remains unchanged.

going to smaller distance with lattice data at finite temperature: the free energy

Finite T analysis

Lattice data HOT QCD collaboration

One reason for which it is challenging to reach very fine lattice spacings in lattice QCD with dynamical quarks is that one has to simultaneously maintain the control over finite volume effects coming from the propagation of the lightest hadronic modes at the pion scale. A lattice simulation at high enough temperature avoids this infrared problem, and enables reaching much finer lattice spacings using smaller volumes. In Phys. Rev. D 100 (2019) 114511 we have used finite temperature lattices to reach $a = 0.00848\,\mathrm{fm}$.

We compute the singlet free energy with $N_\sigma/N_\tau = 4$ and $N_\tau = 12$, or 16. These ensembles correspond to the thermal QCD medium at temperatures $T = 1/(aN_\tau)$. The finite temperature ensembles have been generated using lattice parameters that would correspond to the same two pion masses (at zero temperature) in the continuum limit as the zero temperature ensembles.

The perturbative expression of the free energy agrees with the static energy at $T = 0$ plus thermal corrections that have been computed to some accuracy.

- Berwein Brambilla Petreczky Vairo PRD 96 (2017) 014025

Singlet free energy of a quark-antiquark static pair : a thermal QCD observable

$$F_S(r,T) = -T \left\langle \ln e^{ig \int_0^{1/T} d\tau A_0(\mathbf{o},\tau)} e^{-ig \int_0^{1/T} d\tau A_0(\boldsymbol{r},\tau)} \right\rangle_T$$

thermal QCD expectation value in Coulomb Gauge

at distances rT <<1 we can use pNRQCD at finite T to write $F_S(r,T) = V_s(r,\mu_{us}) + \delta F_S(r,T,\mu_{us}),$

Thermal effects not visible in the short distance

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The form of the thermal corrections depends on the hierarchy of scales

we consider the hierarchy $1/r \gg \alpha_s/r \gg T \gg m_D \sim gT$ that applies at short distance and then $\mu_{us} \sim \alpha_s/r$ and

$$\delta F_S(r,T,\mu_{us}) = \delta E_{US}(\mu_{us}) + \Delta F_S(r,T)$$

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US T=0 term

We fit the free energy at short distance with the 3 loop plus LL formula that we used for the static energy and we obtain:

- TUMQCD coll PRD 100 (2019) 114511

$\Lambda_{\overline{\mathrm{MS}}} = 310.9^{+13.5}_{-12.3}$ MeV or $\alpha_s(M_Z) = 0.11638^{+0.00095}_{-0.00087}$

Thermal effects not visible in the short distance

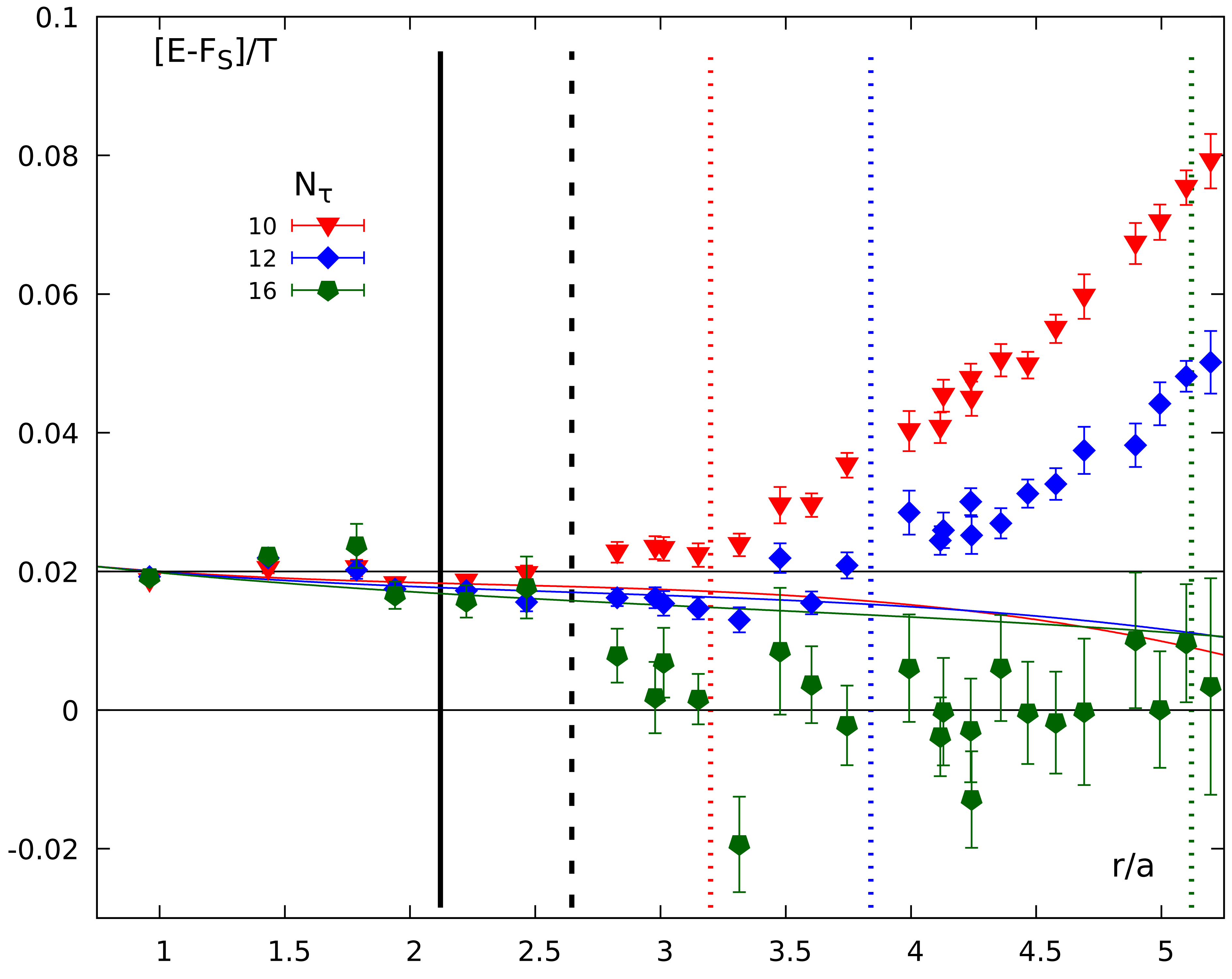
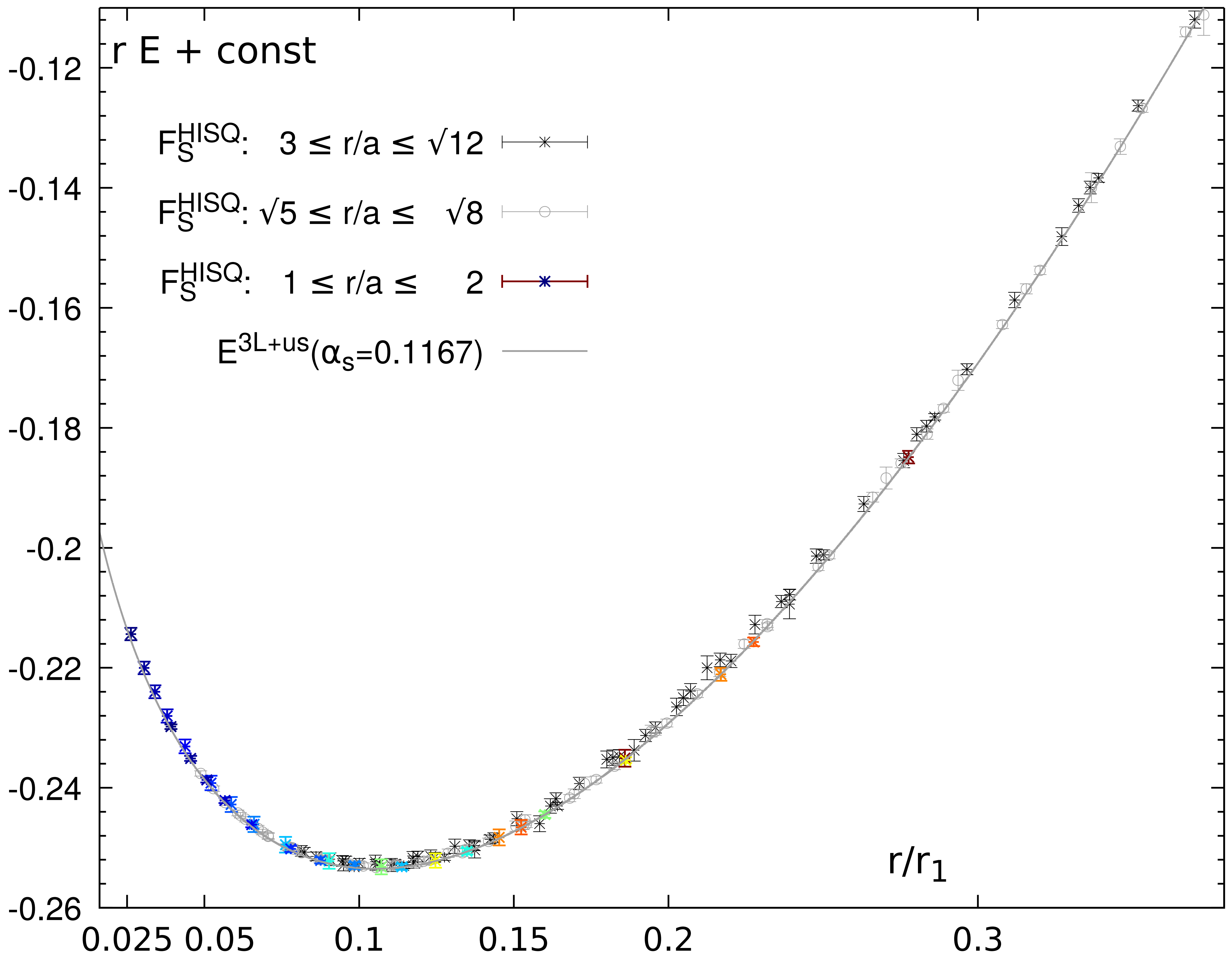


FIG. 9. The difference between the static energy at $T = 0$ and the singlet free energy for $\beta = 8.4$ calculated with $N_\tau = 10$, 12 and 16 in units of the temperature as function of distance in units of the lattice spacing. The lines correspond to the $\delta F(r, T)$ calculated at order g^5. The dotted vertical lines show the boundary $rT = 0.3$ for different N_τ. The solid or dashed vertical lines show the boundaries between $r/a < \sqrt{5}$ and $r/a \geq \sqrt{5}$, or between $r/a < \sqrt{8}$ and $r/a \geq \sqrt{8}$, respectively.



as a matter of fact F_s an be reproduced by E in a range bigger than the one we used for the extraction of alphas

the difference between E and F_s at short distance is a constant times T

alpha_s from the force

The force as a Wilson loop with a chromoelectric field

A direct computation of the force that avoids interpolating the static energy and taking numerically the derivative is possible from the expression of a rectangular Wilson loop, $W_{r\times T}$, with a chromoelectric field insertion on a quark line:

$$F(r) = \frac{d}{dr}E_0(r) = \lim_{T\to\infty} -i \frac{\langle \mathrm{Tr}\{\mathrm{P}\, W_{r\times T}\, \hat{\mathbf{r}}\cdot g\mathbf{E}(\mathbf{r},t^*)\}\rangle}{\langle \mathrm{Tr}\{\mathrm{P}\, W_{r\times T}\}\rangle}$$

An equivalent expression can be written using a Polyakov loop instead of a Wilson loop.
At fixed t^* for $T\to\infty$, the rhs is independent of t^*.
The force is mass renormalon free and finite after charge renormalization.

- Brambilla Pineda Soto Vairo PRD 63 (2001) 014023
  Vairo MPLA 31 (2016) 34, 1630039

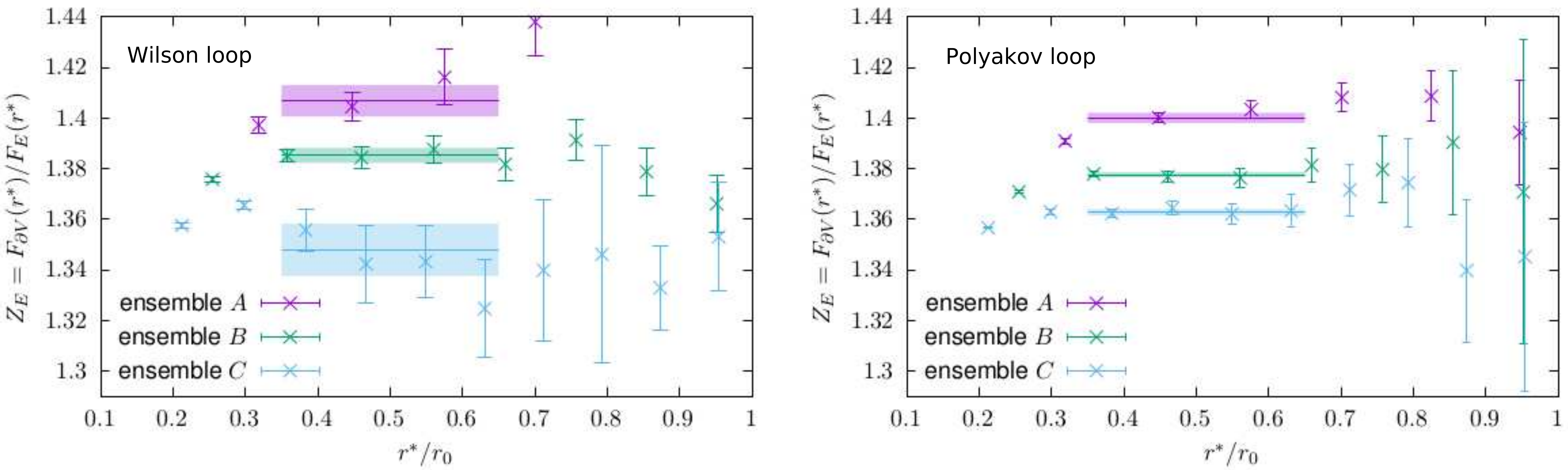
Lattice analysis of 2111.07916

For a study of concept, we have computed the Wilson loop and Polyakov loop with a chromoelectric field on three quenched QCD ($n_f = 0$) ensembles.

ensemble	β	$(L/a)^3 \times T/a$	r_0/a	a in fm
A	6.284	$20^3 \times 40$	8.333	0.060
B	6.451	$26^3 \times 50$	10.417	0.048
C	6.594	$30^3 \times 60$	12.500	0.040

- TUMQCD coll. 2111.07916

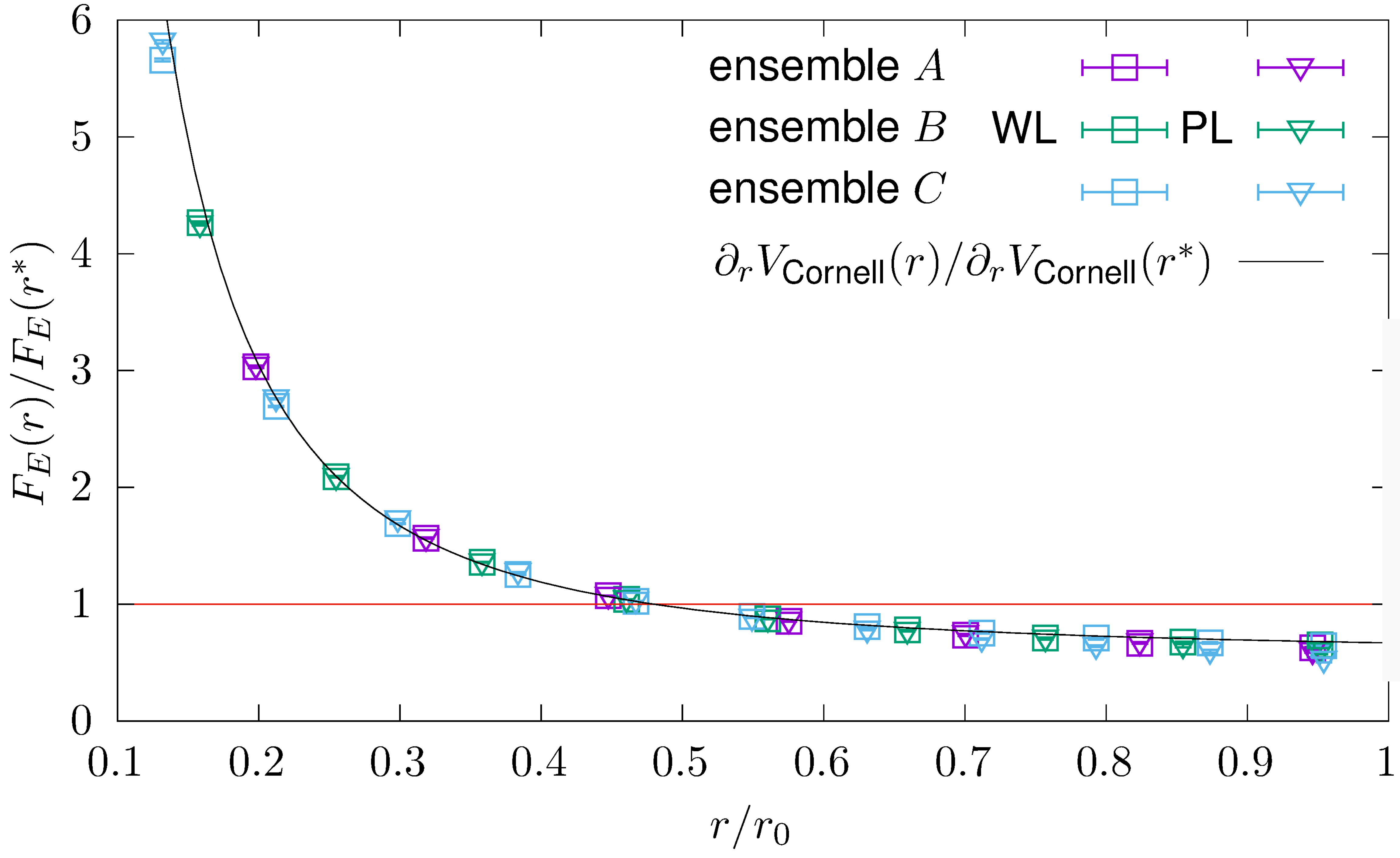
Renormalization constant Z_E



The convergence of the direct force towards the continuum, i.e. the derivative of the static potential, is slow. The ratio of the two determinations is an r independent constant Z_E that may be computed once forever at some fixed (arbitrary) distance r^* ($r_0 \approx 0.5$ fm).

ensemble	a in fm	Z_E from Wilson loops	Z_E from Polyakov loops
A	0.060	$1.4068(63)$	$1.4001(20)$
B	0.048	$1.3853(30)$	$1.3776(10)$
C	0.040	$1.348(11)$	$1.3628(13)$

Direct force vs lattice data



- Remove Z_E by dividing with measurement at $r^* = 0.48 r_0$
- Proof of concept:
  - Both derivative of potential and direct force agree
  - Both Wilson loop and Polyakov loops agree

Once normalized by Z_E the direct force agrees well with the Cornell parameterization based on quenched lattice data of the QCD static energy.
We have chosen $r^* = 0.48\, r_0 \approx 0.24$ fm.

○ TUMQCD coll. 2111.07916

Gradient flow

The converge towards the continuum limit may be improved by using gradient flow.

Gradient flow consists in replacing the gluon fields $gA_\mu(x)$ by the flowed fields $B_\mu(x;t)$, where B_μ is defined through the flow equation

$$\begin{aligned} \frac{\partial}{\partial t} B_\mu(x;t) &= D_\nu G_{\nu\mu} + D_\mu \partial_\nu B_\nu \\ G_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu], \quad D_\mu = \partial_\mu + [B_\mu, \cdot] \end{aligned}$$

with the initial condition $B_\mu(x;t=0) = gA_\mu(x)$.

The new theory reduces to QCD in the limit of zero flow time t. But at any finite t it typically shows a much better behaviour than QCD in the ultraviolet (large momenta). We expect that the theory at finite flow time converges faster towards the continuum.

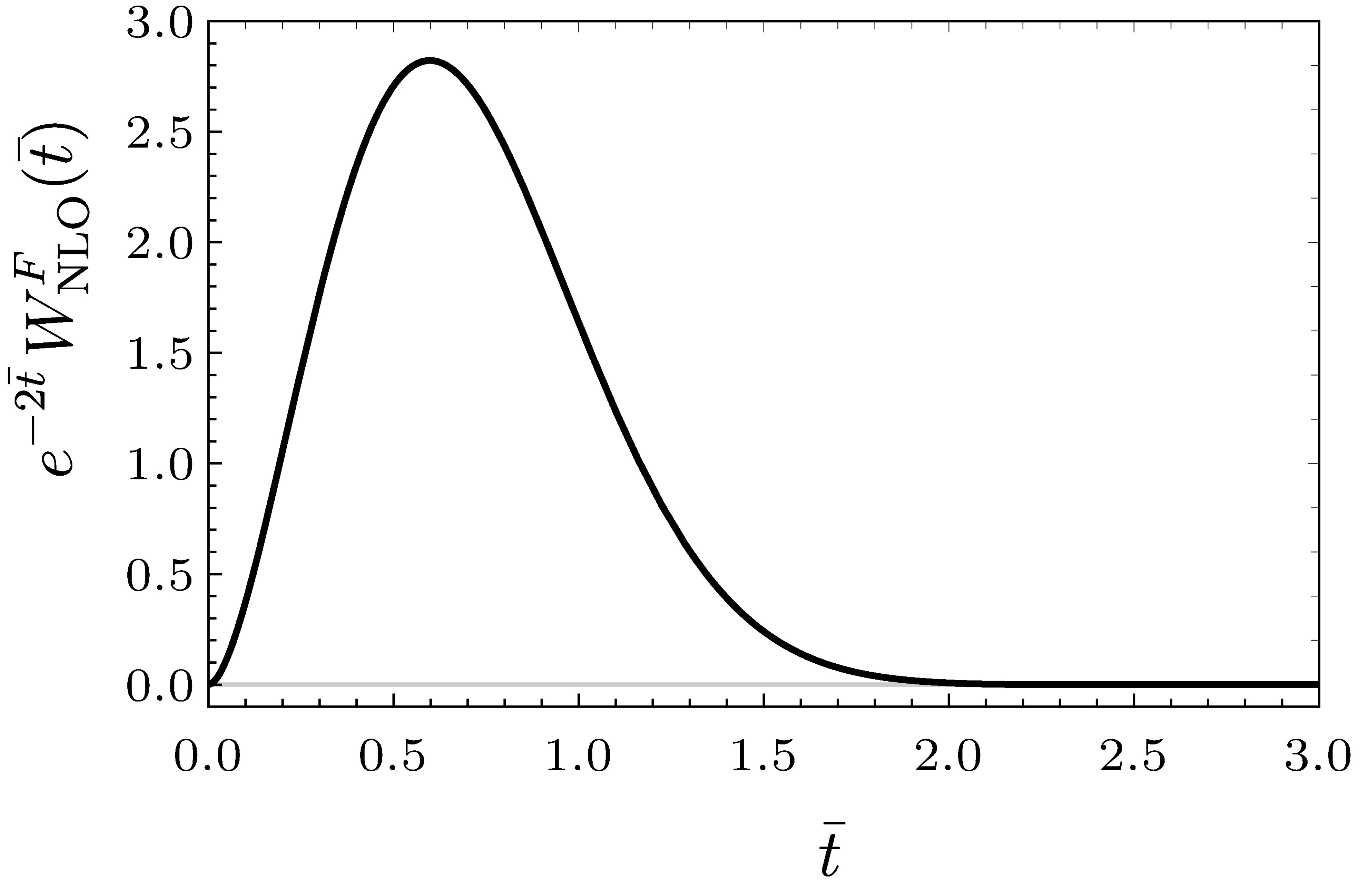
○ Lüscher JHEP 08 (2010) 071, Lüscher Weisz JHEP 02 (2011) 051

The potential from gradient flow up to NLO

In the $\overline{\rm MS}$ scheme, we find at NLO in momentum space ($\bar{t} \equiv \boldsymbol{q}^2 t$)

$$\tilde{V}(\boldsymbol{q};t) = -\frac{4\pi\alpha_{\rm s}(\mu) C_F e^{-2\boldsymbol{q}^2 t}}{\boldsymbol{q}^2}\left\{1 + \frac{\alpha_{\rm s}(\mu)}{4\pi}\left[\beta_0 \log(\mu^2/\boldsymbol{q}^2) + a_1 + C_A\, W^F_{\rm NLO}(\bar{t})\right]\right\}$$

The leading order term decreases like $e^{-2\boldsymbol{q}^2 t}$ for large momentum transfer $\boldsymbol{q}^2$.
Also the NLO one, which is analytically known, decreases exponentially like $e^{-\boldsymbol{q}^2 t}$.



The force from gradient flow at NLO

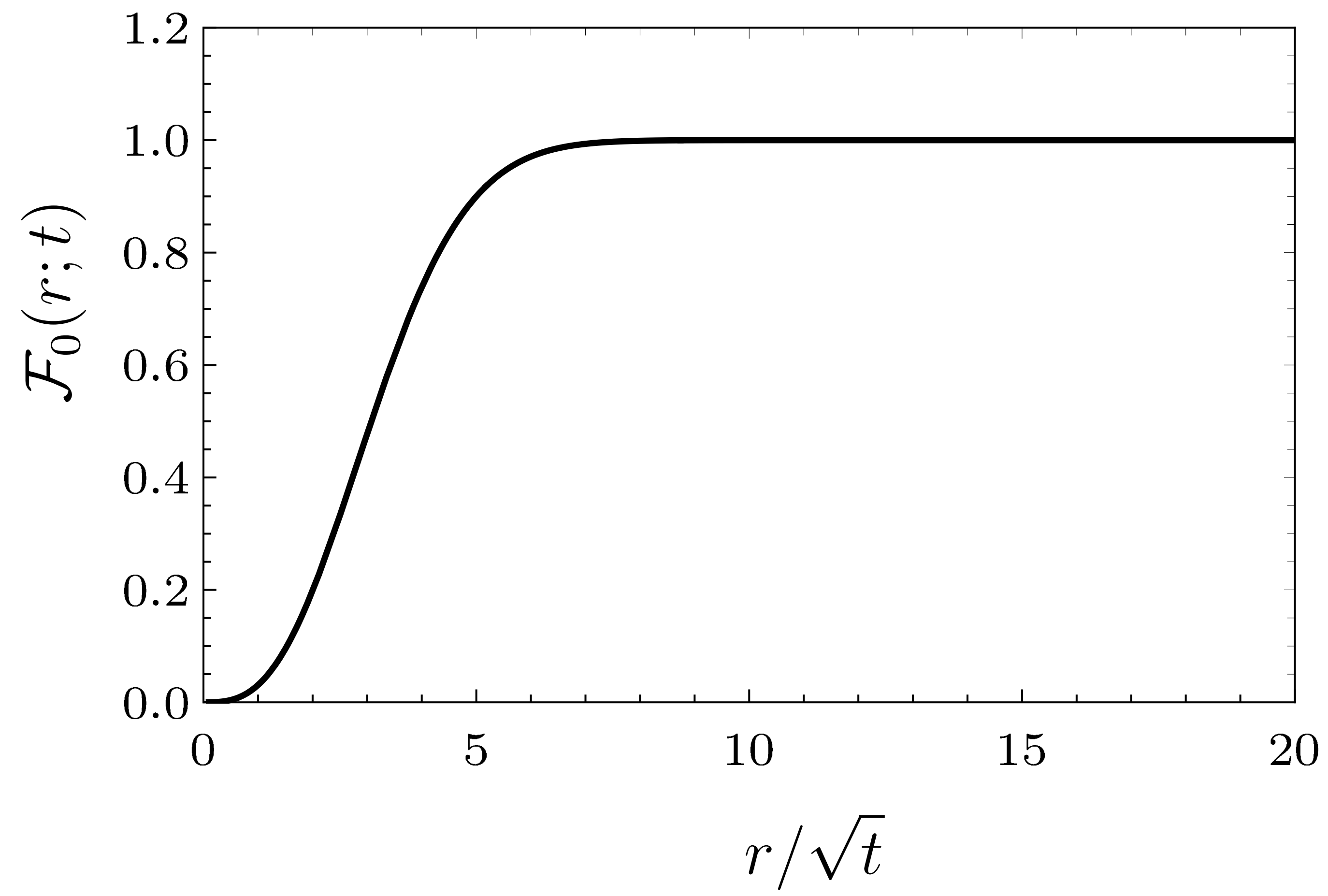
In the $\overline{\mathrm{MS}}$ scheme, we find at NLO in coordinate space

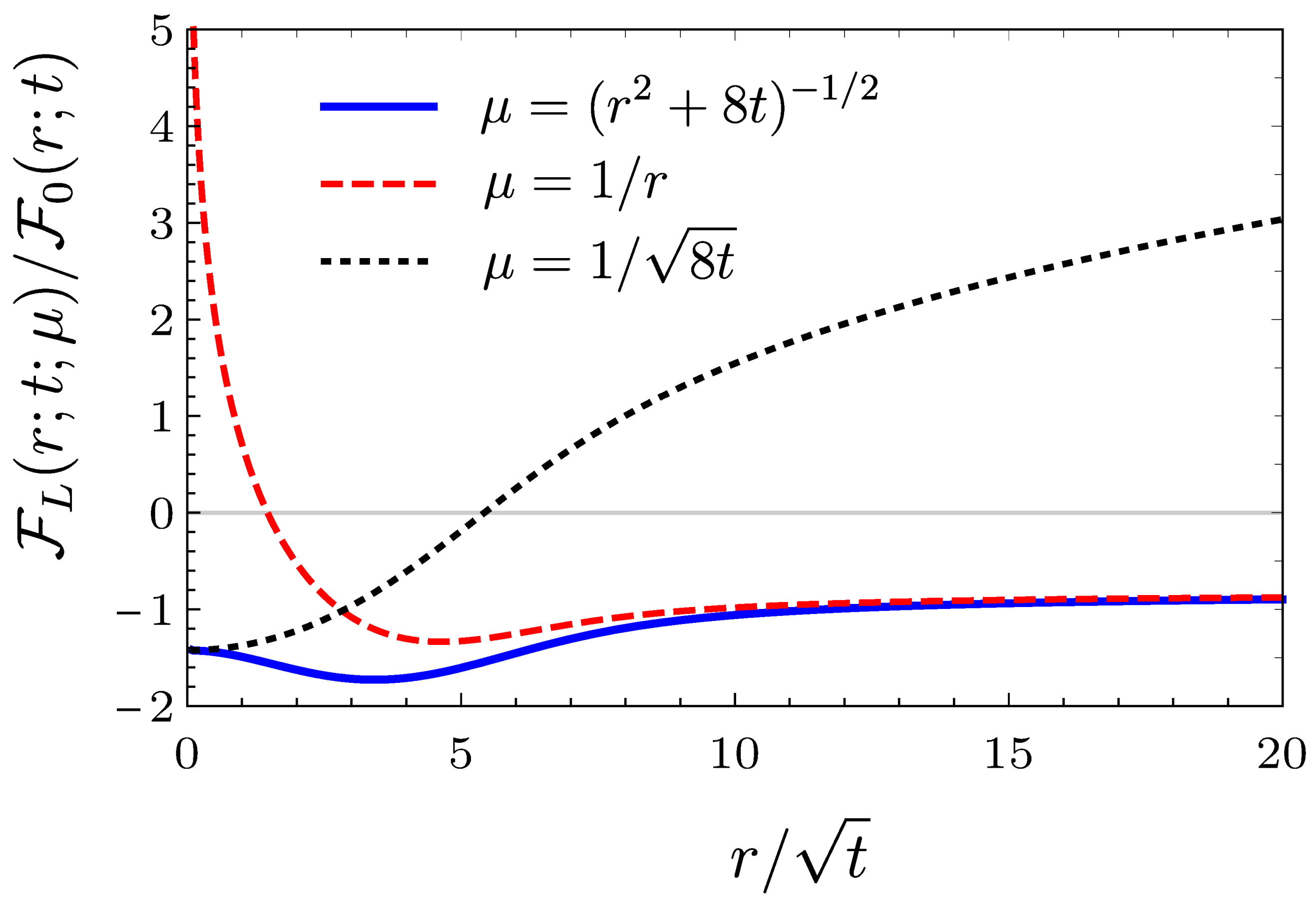
$$F(r;t) = \frac{\alpha_s(\mu)C_F}{r^2}\left[\left(1+\frac{\alpha_s}{4\pi}a_1\right)\mathcal{F}_0(r;t) + \frac{\alpha_s}{4\pi}\beta_0\mathcal{F}^L_{\mathrm{NLO}}(r;t;\mu) + \frac{\alpha_s C_A}{4\pi}\mathcal{F}^F_{\mathrm{NLO}}(r;t)\right]$$

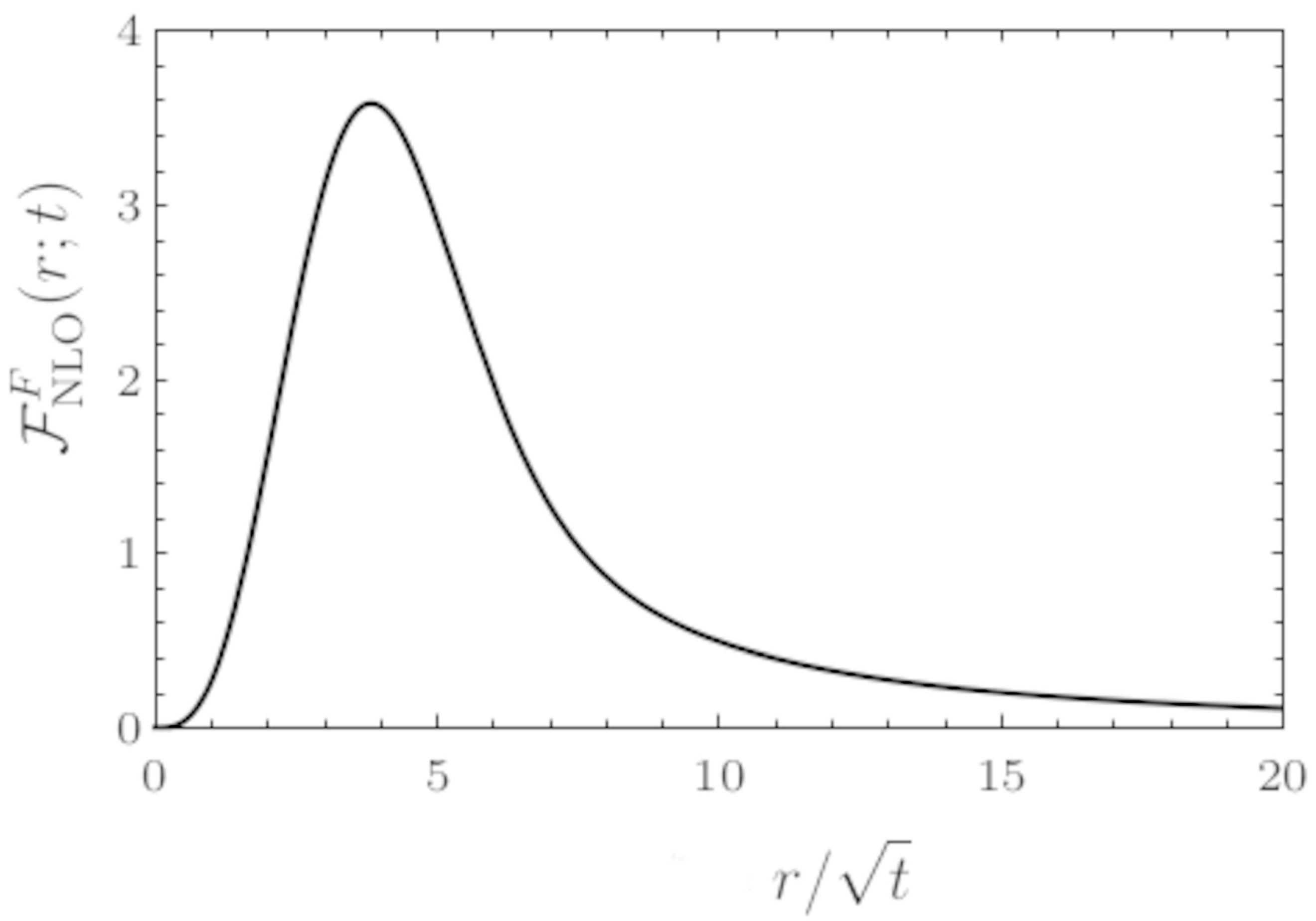
The functions $\mathcal{F}_0(r;t)$, $\mathcal{F}^L_{\mathrm{NLO}}(r;t;\mu)$ and $\mathcal{F}^F_{\mathrm{NLO}}(r;t)$ are analytically known.

- Brambilla Chung Vairo Wang 2111.07811

The force from gradient flow at NLO







Lattice analysis of 2111.10212

For a preliminary study, we have computed the Wilson loop with a chromoelectric field in gradient flow on three quenched QCD ($n_f = 0$) ensembles.

β	$N_\sigma \times N_t$	a[fm]	# configurations
6.284	20×40	0.060	1949
6.481	26×56	0.046	1999
6.594	30×60	0.040	1997

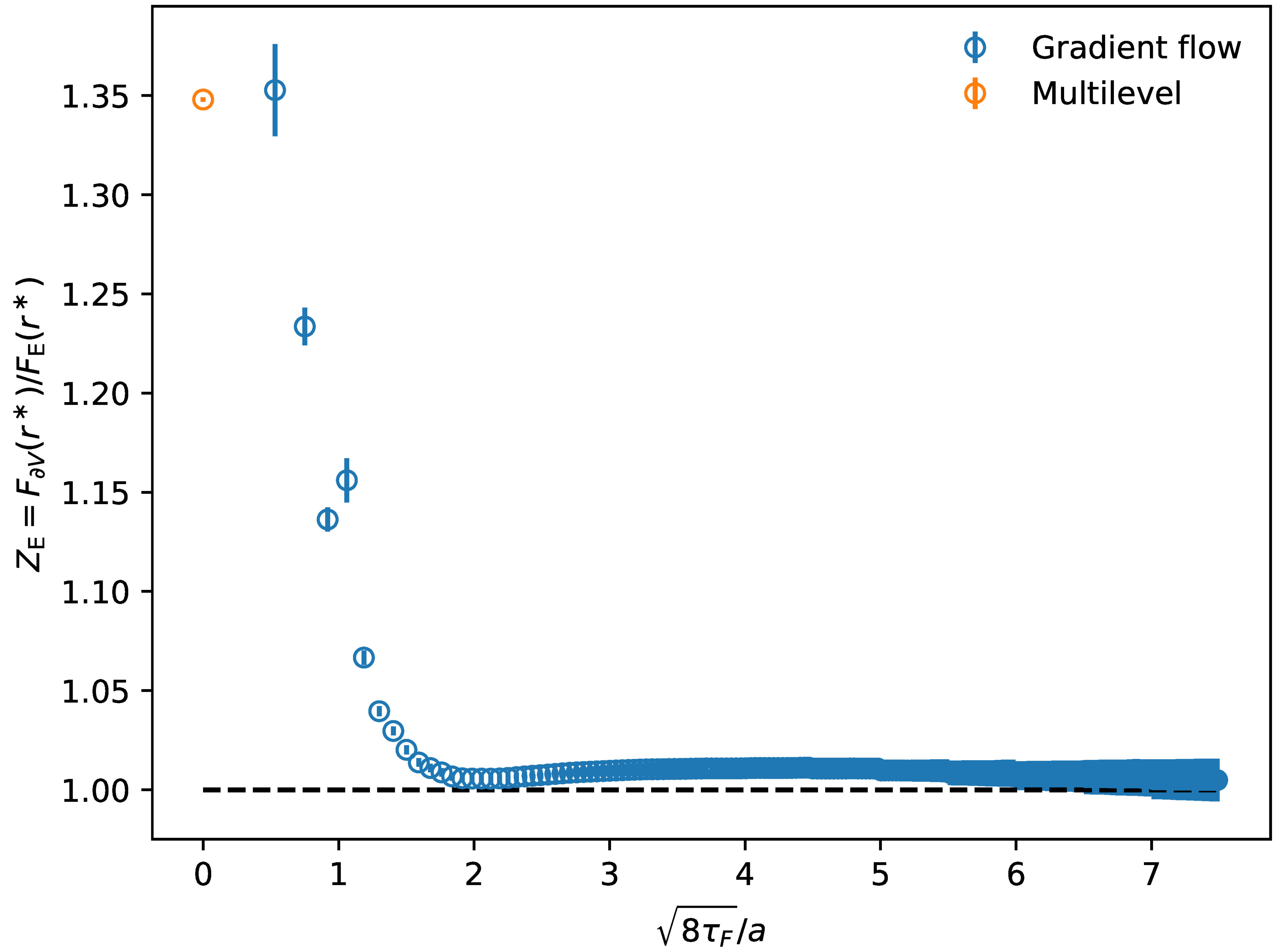
- Brambilla Leino Mayer-Steudte Vairo 2111.10212

Renormalization constant Z_E with gradient flow

at zero flow time we reobtain the previous result for Z_E

At finite flow time the renormalization constant Z_E is about 1.

$$Z_E(a) = \frac{F_{\partial V}(r^*, a)}{F_E(r^*, a)},$$



o Brambilla Leino Mayer-Steudte Vairo 2111.10212

for each flow time find the plateau in r*

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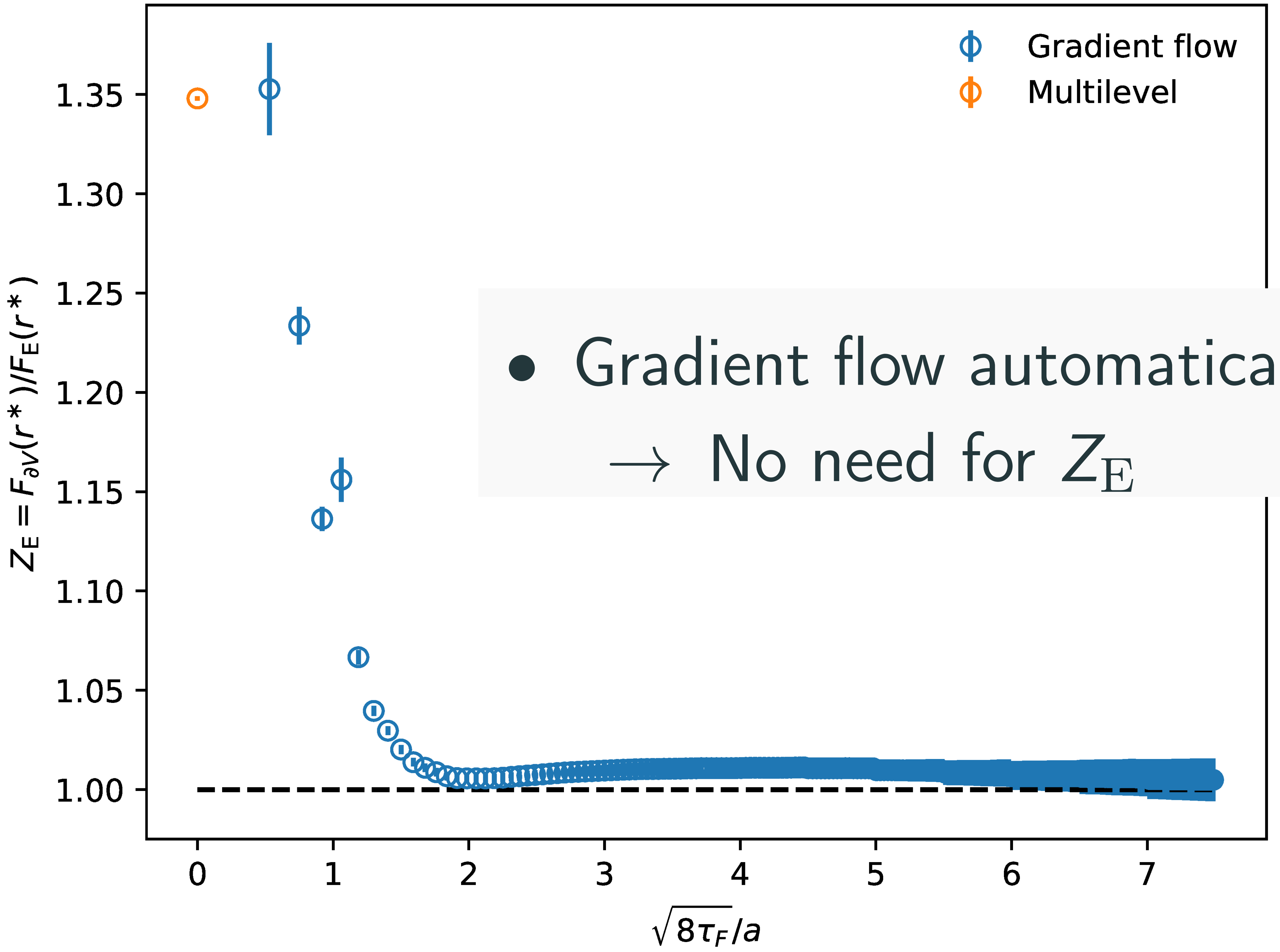
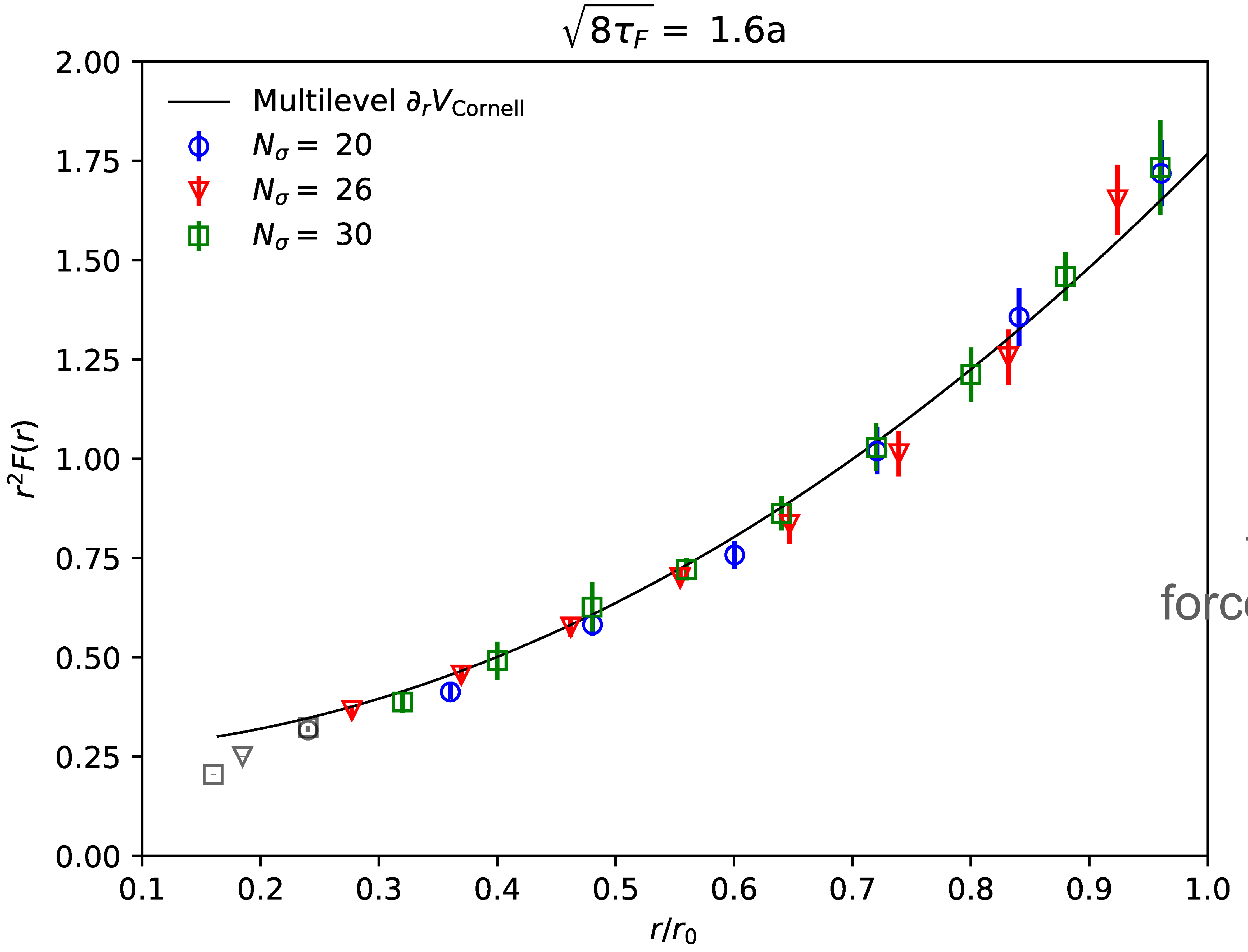


- Gradient flow automatically renormalizes the force at finite flowtime
  $\rightarrow$ No need for Z_E

○ Brambilla Leino Mayer-Steudte Vairo 2111.10212

for each flow time find the plateau in r*

Direct force vs lattice data with gradient flow



Cornell potential from previous lattice data on the Wilson and Polyakov loop

despite the lack of continuum and zero flow time limit the force from gradient flow seems to agree with the force measured from the derivative of the potential calculated previously

- Brambilla Leino Mayer-Steudte Vairo 2111.10212

OUTLOOK

The computation of the static energy and force in QCD has seen remarkable progress in recent years both analytically and numerically resulting in a competitive determination of the strong coupling constant, $\alpha_{\rm s}$.

—>the next talk by J. Weber will give all the details and the error budget of the latest alphas extraction of 2019

OUTLOOK

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For the near future: extraction of alphas for the lattice static energy with 2+1+1 flavours

extraction of alphas from the force directly calculated ob the lattice

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For the near future: extraction of alphas for the lattice static energy with 2+1+1 flavours

extraction of alphas from the force directly calculated ob the lattice

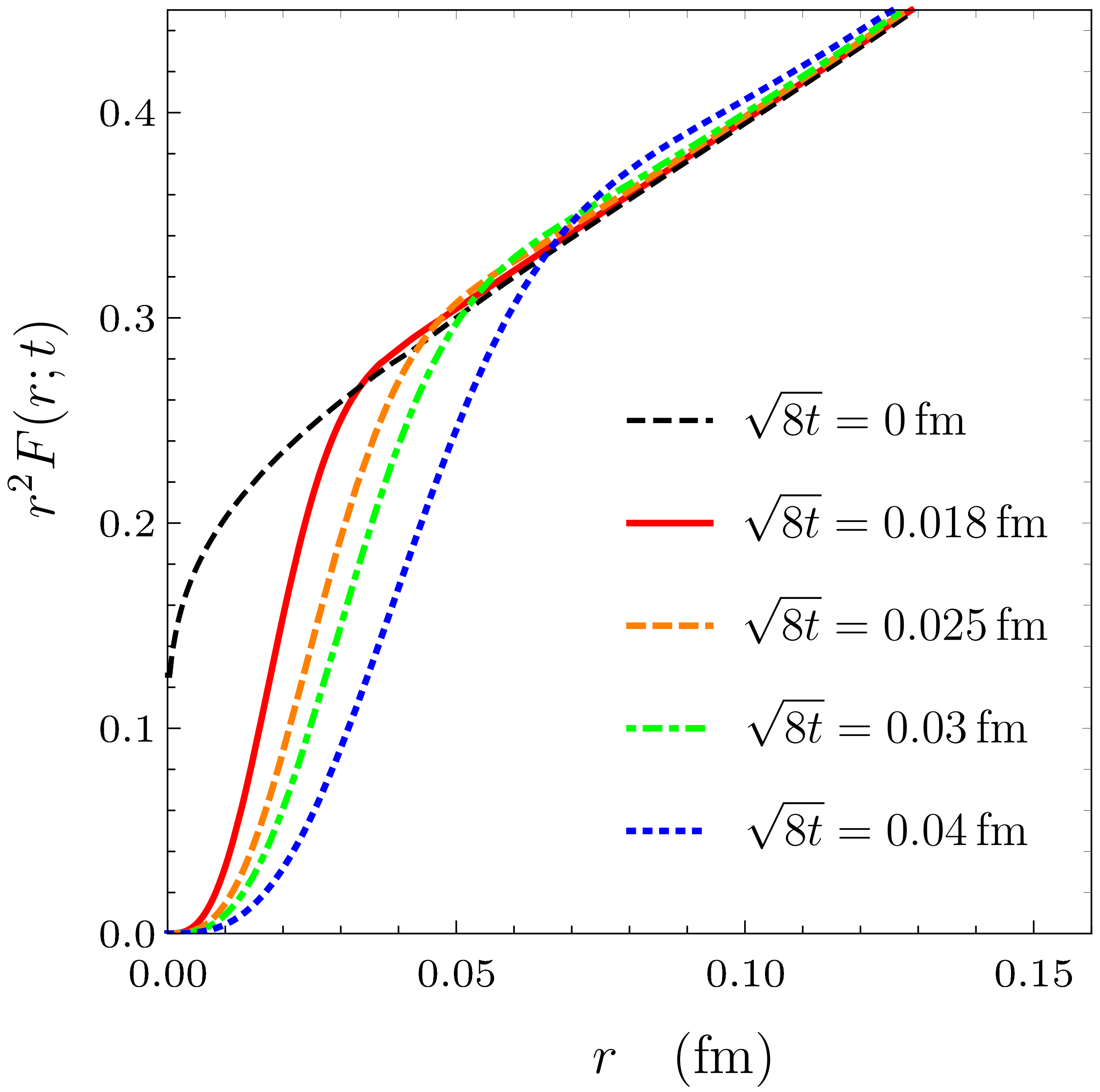
The information about $\alpha_{\rm s}$ is contained in the force. The force may be determined by numerically taking the derivative of the static energy, which requires a precise determination of the static energy. An alternative determination consists in computing a Wilson loop with a chromoelectric field insertion. If this way of determining the force is more or less efficient than the derivative of the static energy remains to be established.

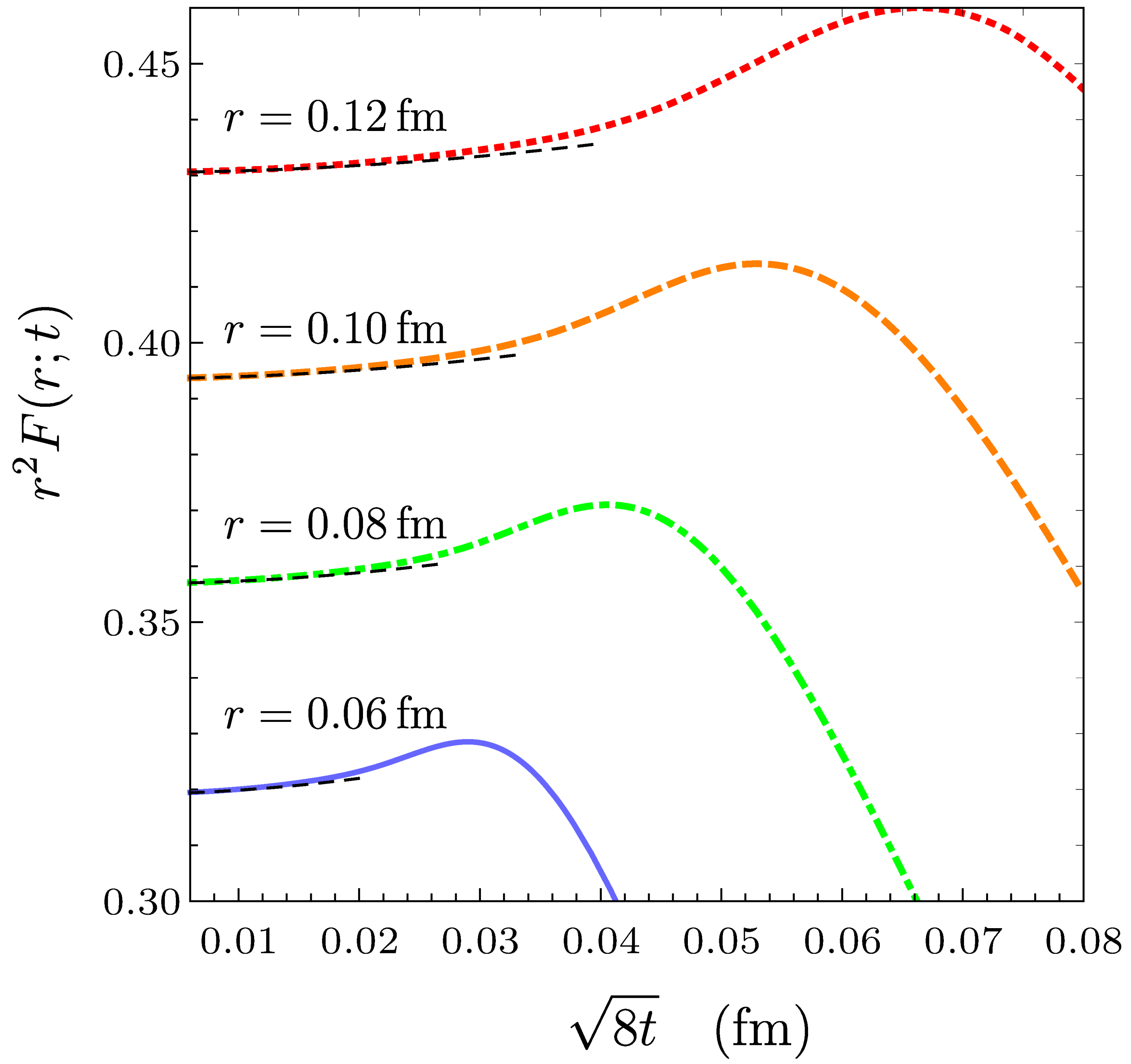
Gradient flow seems to be a promising method for determining the force from a Wilson loop with a chromoelectric field insertion. What remains to be done is a consistent analysis of the zero flow time limit of the lattice data. For this purpose it is certainly of help having the analytical expression of the force in gradient flow at NLO.
Also lattice computations should be extended to full (unquenched) QCD.

backup

Numerical results for the force from gradient flow at NLO

Numerical results for $r^2 F(r;t)$ in QCD with $n_f = 4$ massless quarks.
We have set $\mu = (r^2 + 8t)^{-1/2}$.

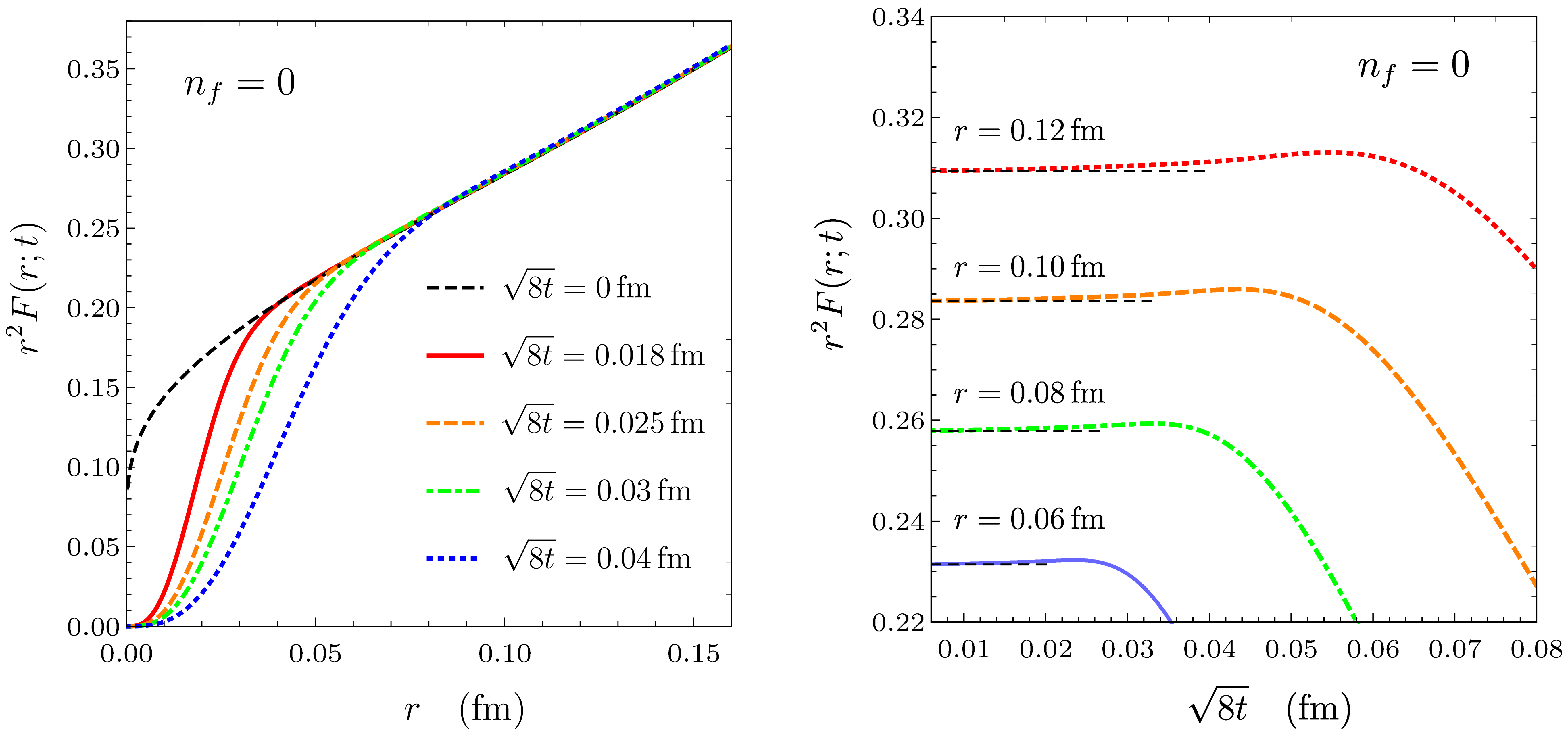




○ Brambilla Chung Vairo Wang 2111.07811

Numerical results for the force from gradient flow at NLO

Numerical results for $r^2 F(r;t)$ in the pure SU(3) gauge theory ($n_f = 0$).
We have set $\mu = (r^2 + 8t)^{-1/2}$.



As a special feature of the quenched case the approach to zero flow time is almost constant (in general it goes like $\dfrac{2\alpha_s^2 C_F n_f}{\pi}\dfrac{t}{r^2}$).

- Brambilla Chung Vairo Wang 2111.07811