α_s from an improved τ vector isovector spectral function

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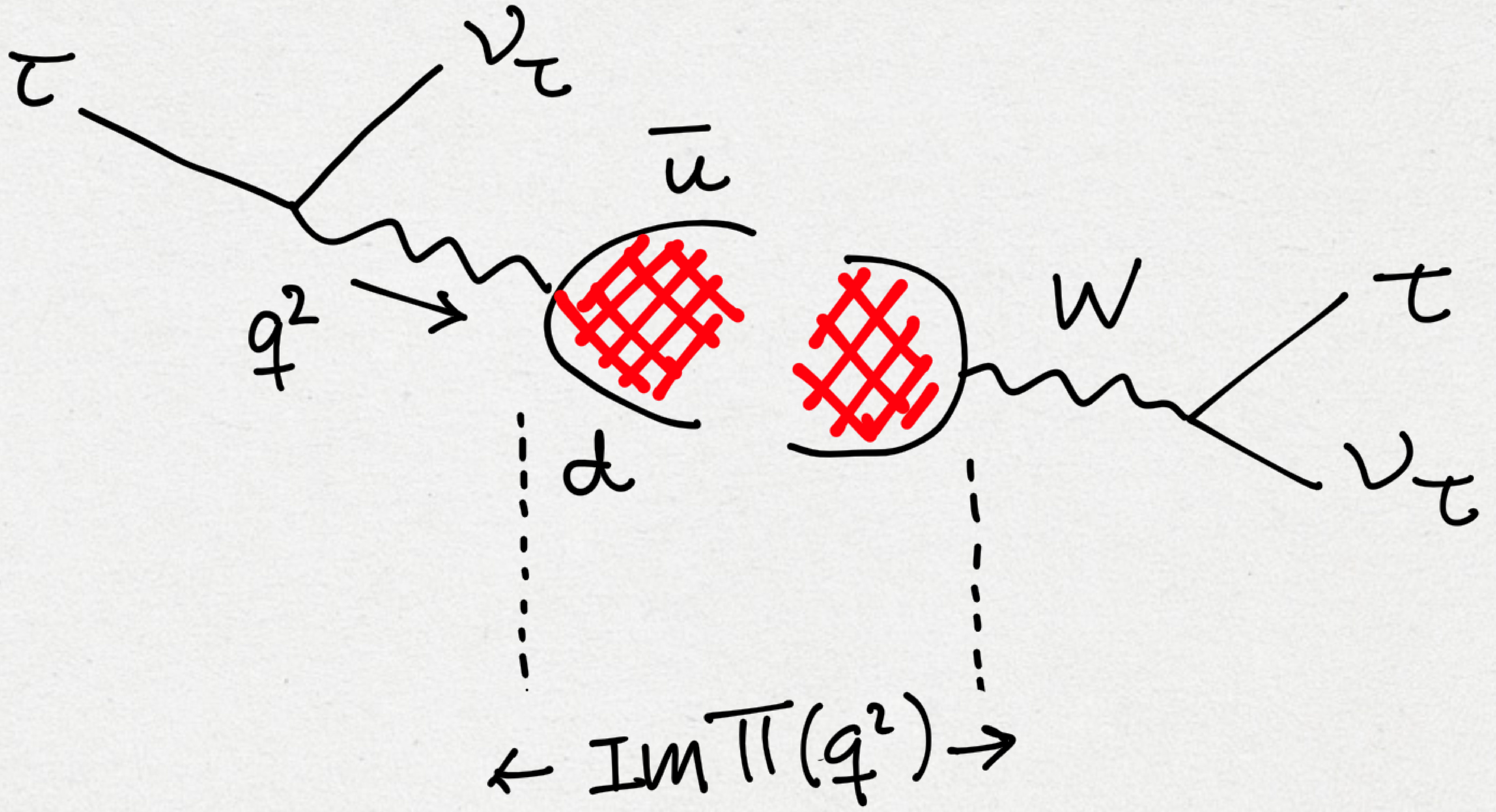
Based on arXiv: 2012.10440 , PRD 103 (2021)

ALPHA_S (2022) @ TRENTO

QCD in τ decay



$$w_T(y) = (1+2y)\boxed{(1-y)^2}$$

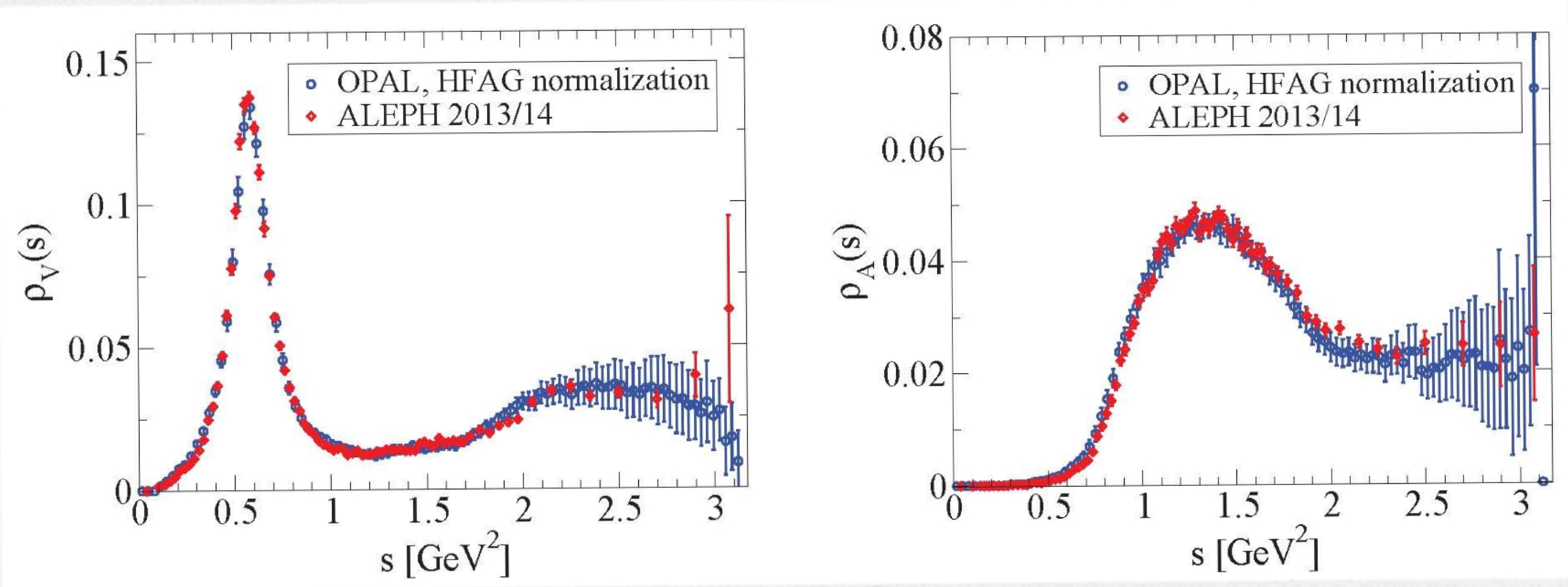
$$w_L(y) = 2y\,\boxed{(1-y)^2}$$

Doubly "pinched"

$$\rho_{V,A}(s) = \frac{1}{\pi}\,\mathrm{Im}\,\Pi_{V,A}(s)$$

$$s_0 = m_\tau^2$$

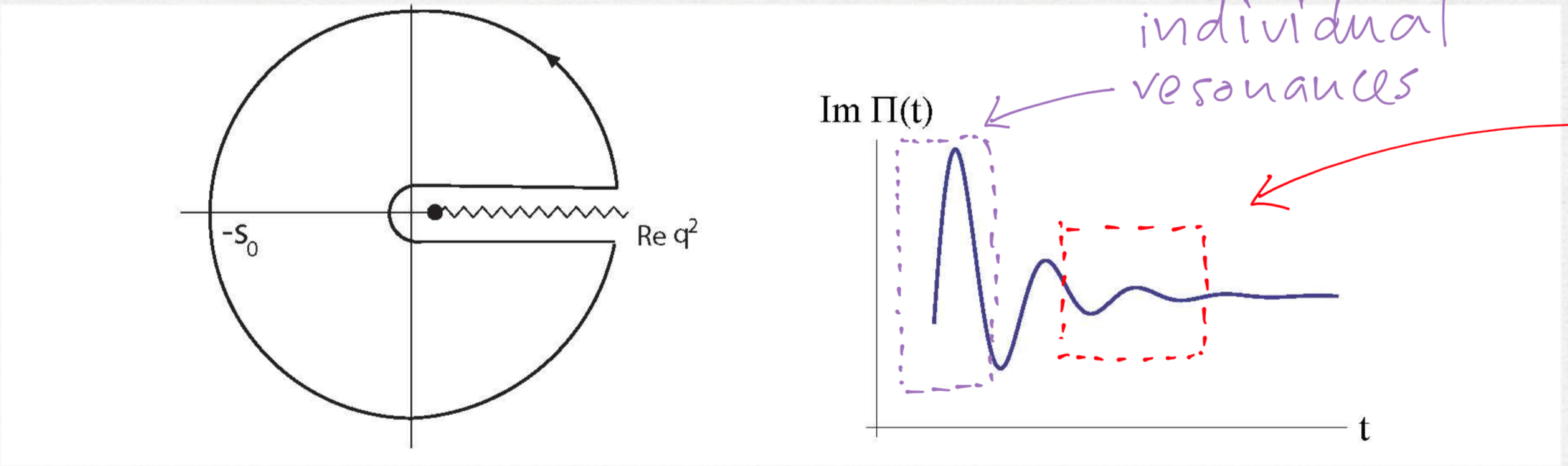
$$\frac{\Gamma(\tau \to \nu_\tau\, \mathrm{had}_{ud}\,(\gamma))}{\Gamma(\tau \to \nu_\tau\, e\, \bar{\nu}_e(\gamma))} = 12\pi^2 |V_{ud}|^2 S_{EW} \int_0^{s_0} \frac{ds}{s_0}\left[w_T\!\left(\frac{s}{s_0}\right) \rho^{(1+0)}_{V+A}(s) - w_L\!\left(\frac{s}{s_0}\right) \rho^{(0)}_{A}(s) \right]$$



Theory : FESRs

Shankar '77; Braaten, Narison, Pich '92

- $\Pi(q^2)$



still wiggles! (resonances overlap)

Poggio, Quinn, Weinberg '76

- "Cauchy's Theorem" ($z = q^2$, $w(z)$ = polynomial):

$$\underbrace{\int_0^{s_0} dt\, w(t) \frac{1}{\pi} \mathrm{Im}\,\Pi(t)}_{\text{exp.}} = -\frac{1}{2\pi} \oint_{|z|=s_0} dz\, w(z) \underbrace{\Pi(z)}_{\Pi_{OPE}+\Pi_{DV}}$$

$$= -\frac{1}{2\pi} \oint_{|z|=s_0} dz\, w(z) \underbrace{\Pi_{OPE}(z)}_{O(\alpha_s^5)\,+\,\text{Condensates}} - \int_{s_0}^{\infty} dt\, w(t) \frac{1}{\pi} \mathrm{Im}\,\Pi_{DV}(t)$$

Cata, Golterman, S.P. '05, '08, '09

FESRs, the OPE and DVs

$$\int_0^{s_0} dt\, w(t) \underbrace{\frac{1}{\pi}\,\mathrm{Im}\,\Pi(t)}_{\text{exp.}} = \frac{-1}{2\pi} \oint_{|z|=s_0} dz\, w(z) \underbrace{\Pi_{OPE}(z)}_{\substack{O(\alpha_s^5) \\ +\text{ Condensates}}} - \int_{s_0}^{\infty} dt\, w(t) \frac{1}{\pi}\,\mathrm{Im}\,\Pi_{DV}(t)$$

- $w(t) \sim t^N \iff \Pi_{OPE} \sim \dfrac{C_{2N+2}}{s_0^{N+1}}$, C_N condensates (up to α_s-suppressed corrections)
- $\Pi(z) = \Pi_{OPE}(z) + \Pi_{DV}(z)$, OPE convergent $\iff \Pi_{DV} = 0$.

  i.e. $\frac{1}{\pi}\,\mathrm{Im}\,\Pi(t) =$ Pert. Theory + DVs
- OPE asymptotic: expect $\frac{1}{\pi}\,\mathrm{Im}\,\Pi_{DV}(t) \sim e^{-\gamma t} \times$ (oscillation), $t \to \infty$

  i.e. "pinching" strategy: polynomial with zero $w(s_0) = 0$ **BUT**

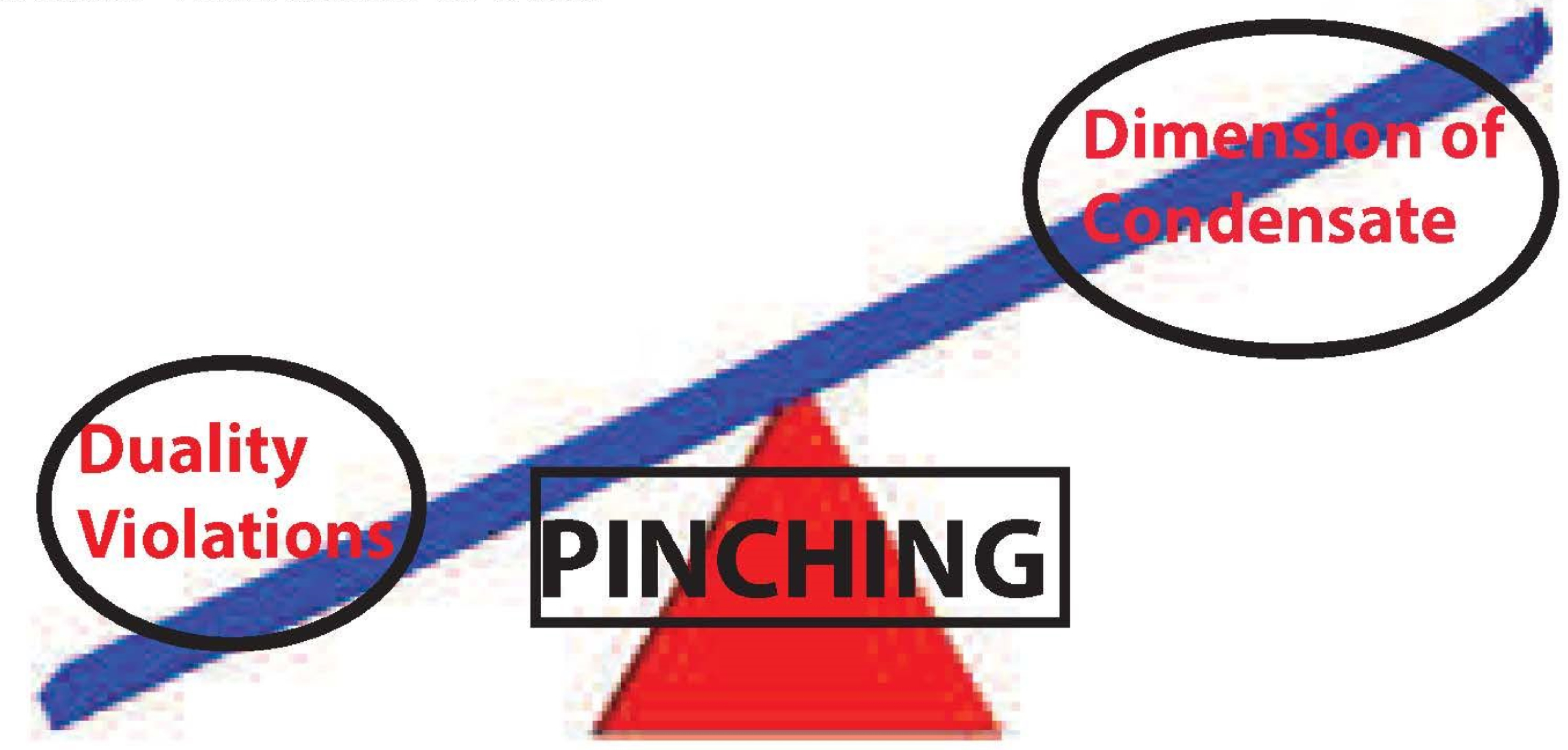
more pinching $\Rightarrow$ higher-degree polynomial $\Rightarrow$ higher condensates contribute

Main Theoretical Message

- with pinching $\exists$ price to pay:

It's **NOT** possible to simultaneously suppress DVs and condensates

★ "**Seesaw**" mechanism at work:



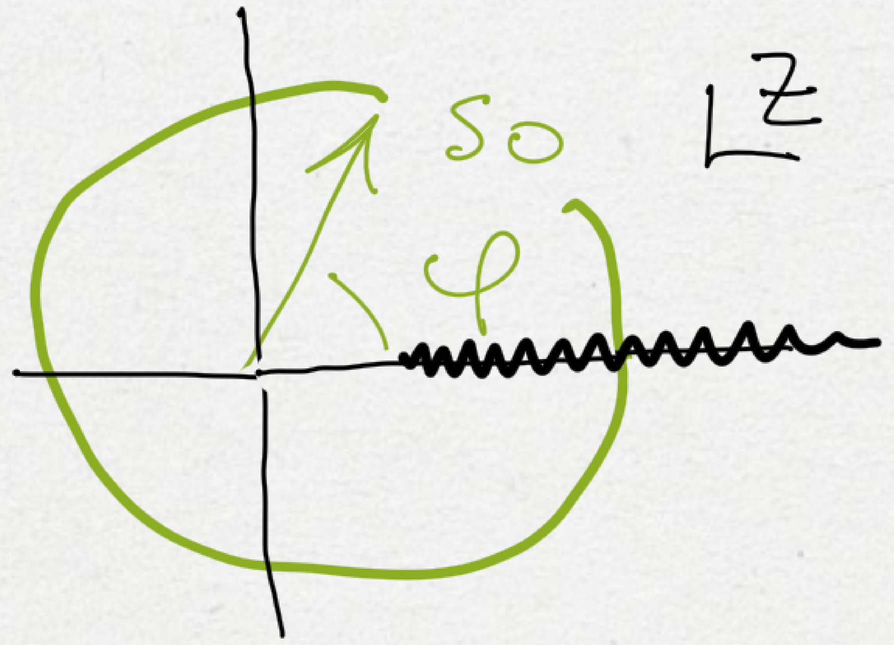
Strategy (I)

$c_{51} = 283\ (\pm 50\%)$ (guess)

• Pert. Theory

Baikov et al. '08

$$D_{\text{Adler}} \equiv -z \frac{d}{dz} C_0(z) = \frac{1}{4\pi^2}\sum_{n=0}^{5}\left(\frac{\alpha_s(\mu)}{\pi}\right)^n \sum_{k=1}^{n+1} k\, c_{nk} \left(\log\frac{-z}{\mu^2}\right)^{k-1}$$



which μ^2?

$\mu^2 = -z = -s_0 e^{i\varphi}$ CIPT

$\mu^2 = s_0$ FOPT ($\overline{MS}$)

• should α_s^{CIPT} be compared with the $\alpha_s^{\overline{MS}}$ extracted from e.g. $\Gamma(Z \to q\bar{q})$?
(where is the circle $|z| = M_Z^2 e^{i\varphi}$ in $\Gamma(Z \to q\bar{q})$?)

• Borel resummation implies mismatch between CIPT and the OPE
(→ Hoang's talk)

Hoang, Regner '20

$\Longrightarrow$ use FOPT

Strategy (II)

(Boito et al. '11, '16 ; Beneke et al '12)

- **OPE** : 4 linearly independent polynomials with degree ≤ 4, no linear term.

$w_0(y) = 1$ $\quad$ $w_2(y) = 1-y^2$ $\quad$ $w_3(y) = (1-y)^2(1+2y)$ $\quad$ $w_4(y) = (1-y^2)^2$

Condensate Dim. $\rightarrow$ (c_0) $\quad$ $(c_0, \underline{c_6})$ $\quad$ $(c_0, \underline{c_6}, c_8)$ $\quad$ $(c_0, \underline{c_6}, c_{10})$

Pert.Th.

- **DVs** : $\frac{1}{\pi} \mathrm{Im}\, \Pi_{DV}(s) \underset{s\to\infty}{=} e^{-\delta-\gamma s} \sin(\alpha+\beta s) \left(1 + \mathcal{O}\left(1/s, 1/N_c, 1/\log s\right)\right)$

Ansatz follows from (asymptotic) Regge behavior at large (but finite) N_c

(Boito et al. '18)

Expected s dependence for OPE asymptotic (transseries).

4 parameters : $\delta, \gamma, \alpha, \beta$.

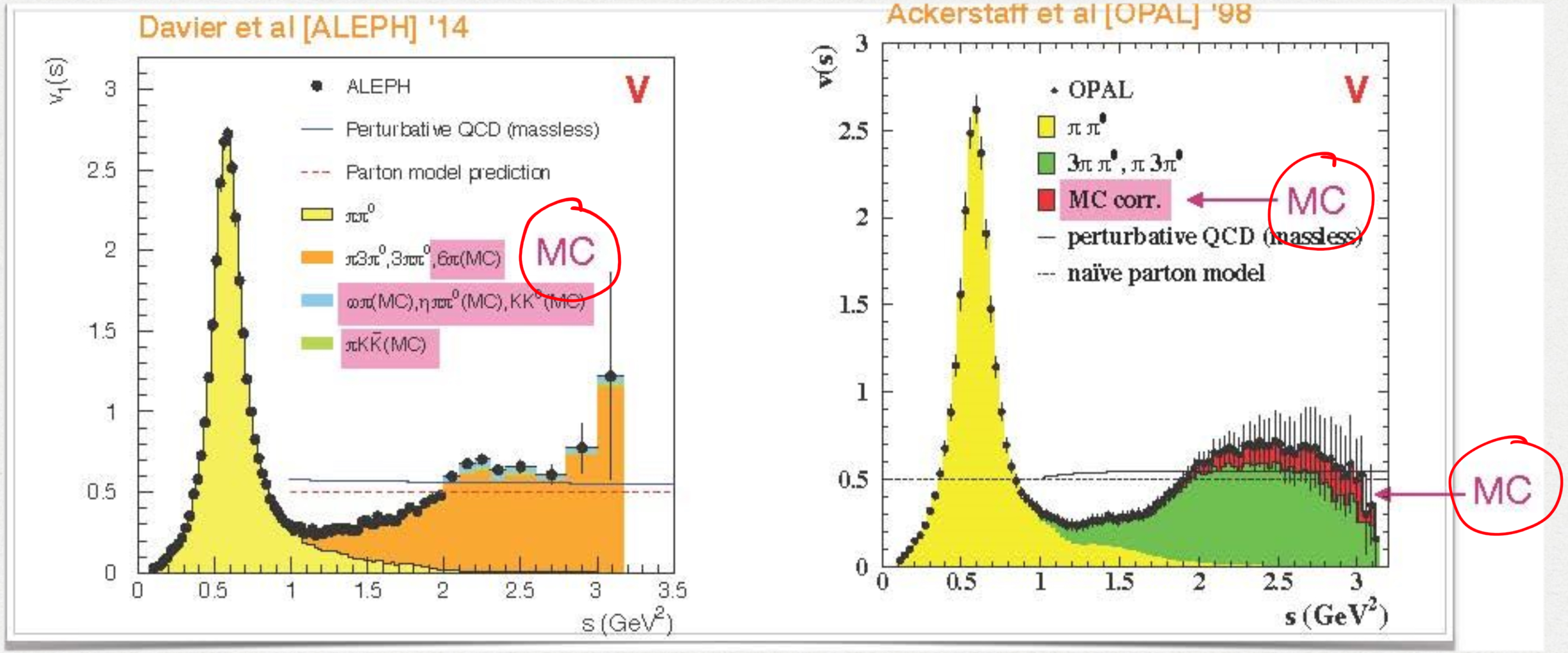
Note:

- different degrees in polynomials ⇒ can crosscheck on values of condensates ⇒ check on OPE ✓
- different degrees of pinching ⇒ can cross check on values of DV parameters ⇒ check on DVs ✓
- no issues with neglecting contributing condensates, ✓ unlike the truncated OPE approach (Pich & Rodriguez-Sanchez '16)

→ Golterman's talk

ALEPH & OPAL Vector Channel Data Sets

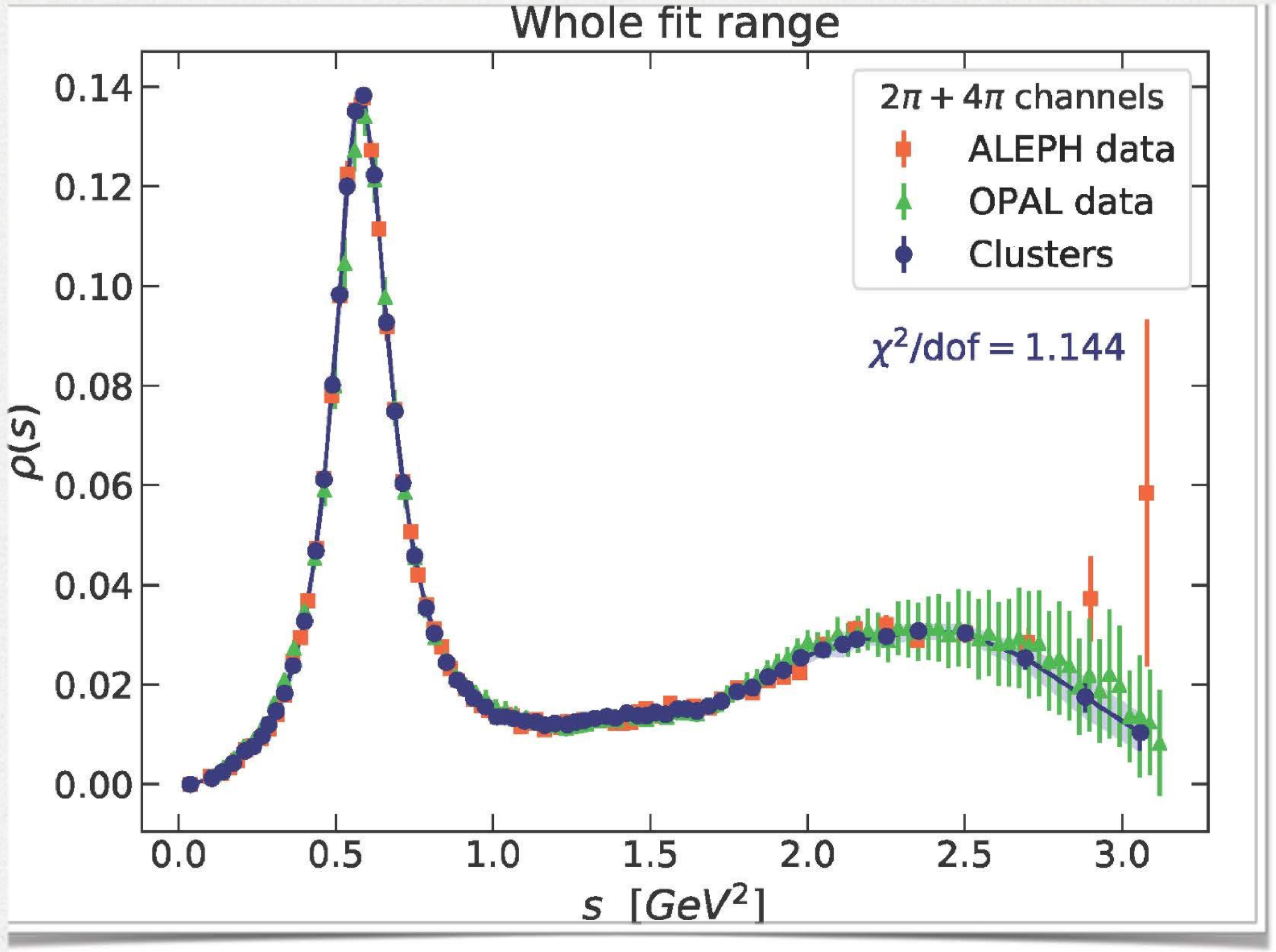
- V channel dominated by $\tau \to \nu_\tau \, {}^{2\pi}_{4\pi}$
- Residual channels subdominant (but important for α_s!)
- $\exists$ Monte Carlo (MC) input for several channels.

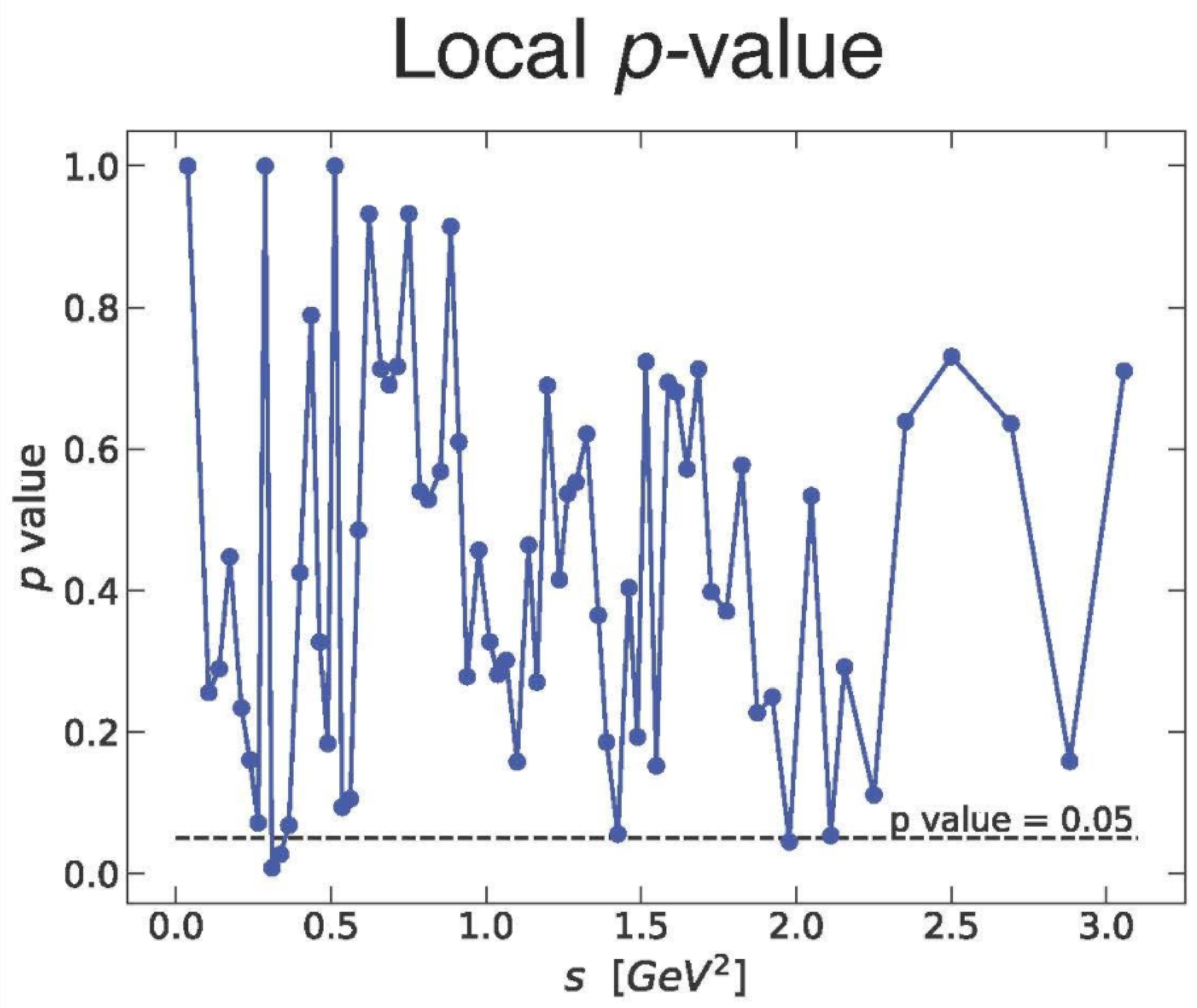


Recent measurements in e^+e^- allow to replace MC.

New Vector Isovector spectral function (I)

- Combined 2π & 4π channels from ALEPH & OPAL with algorithm used in R data for $(g-2)_\mu$ (Keshavarzi, Nomura, Teubner '18)
- No more Monte Carlo: 7 residual channels from e^+e^- using CVC (IB corr's small) and BABAR data for $\tau \to K K_S \nu_\tau$; updated BRs.

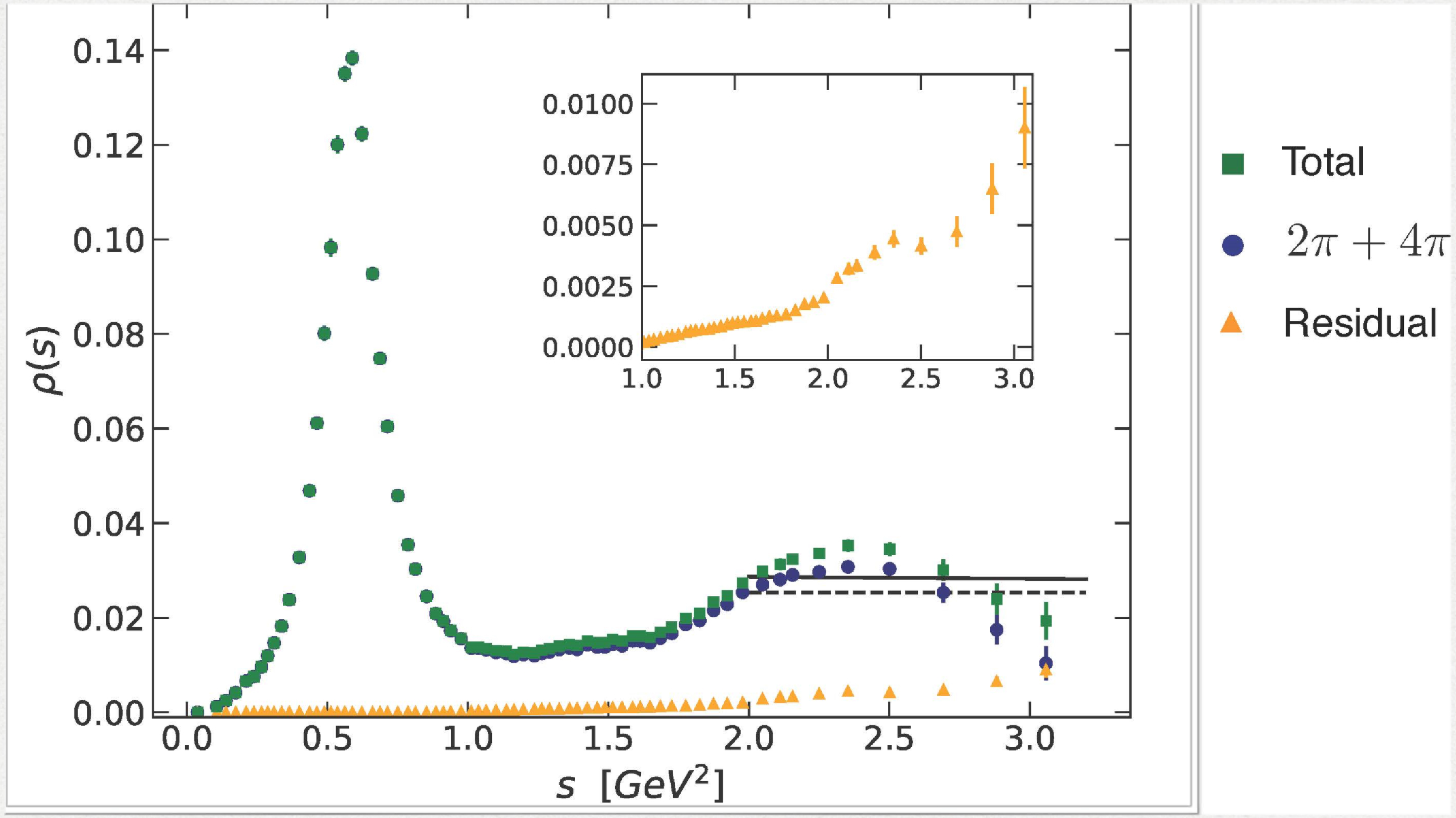




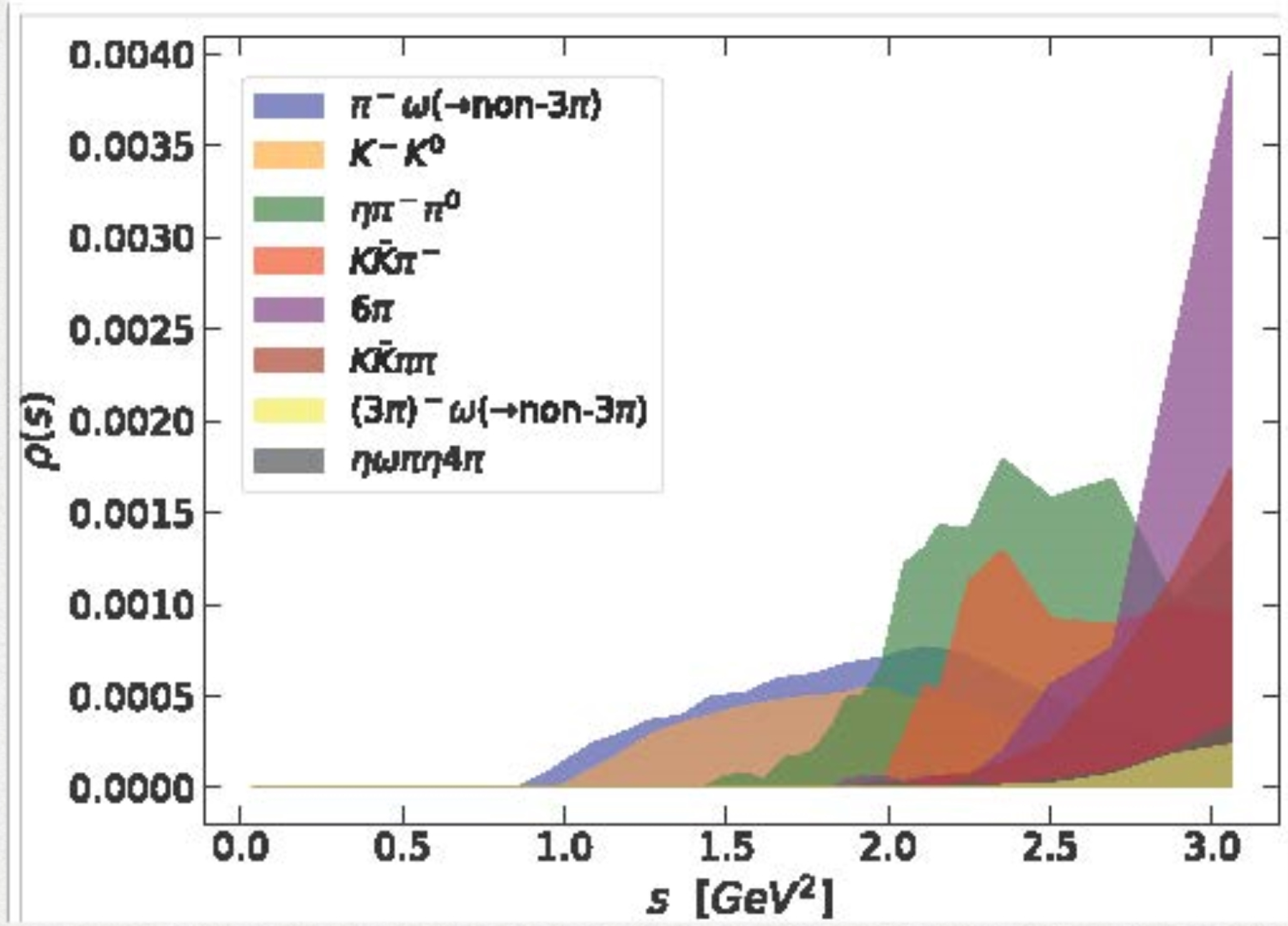
DB, Golterman, Maltman, Peris, Rodrigues and Schaaf, arXiv:2012.10440

- Good χ^2 locally & globally
  no χ^2 inflation needed
- Dramatic error improvement near end point.

New vector isovector spectral function (II)



- Dramatic error improvement near end point.

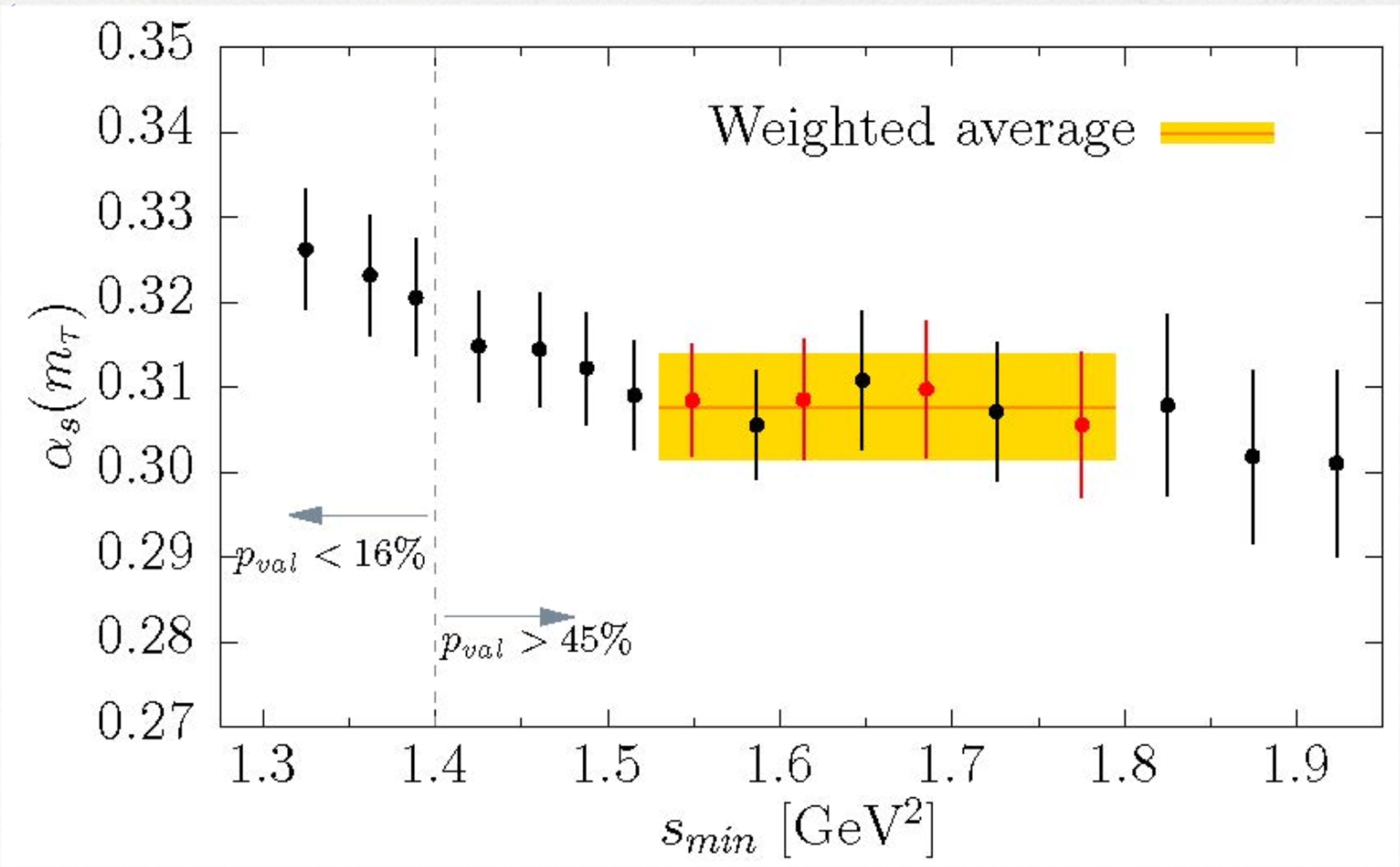


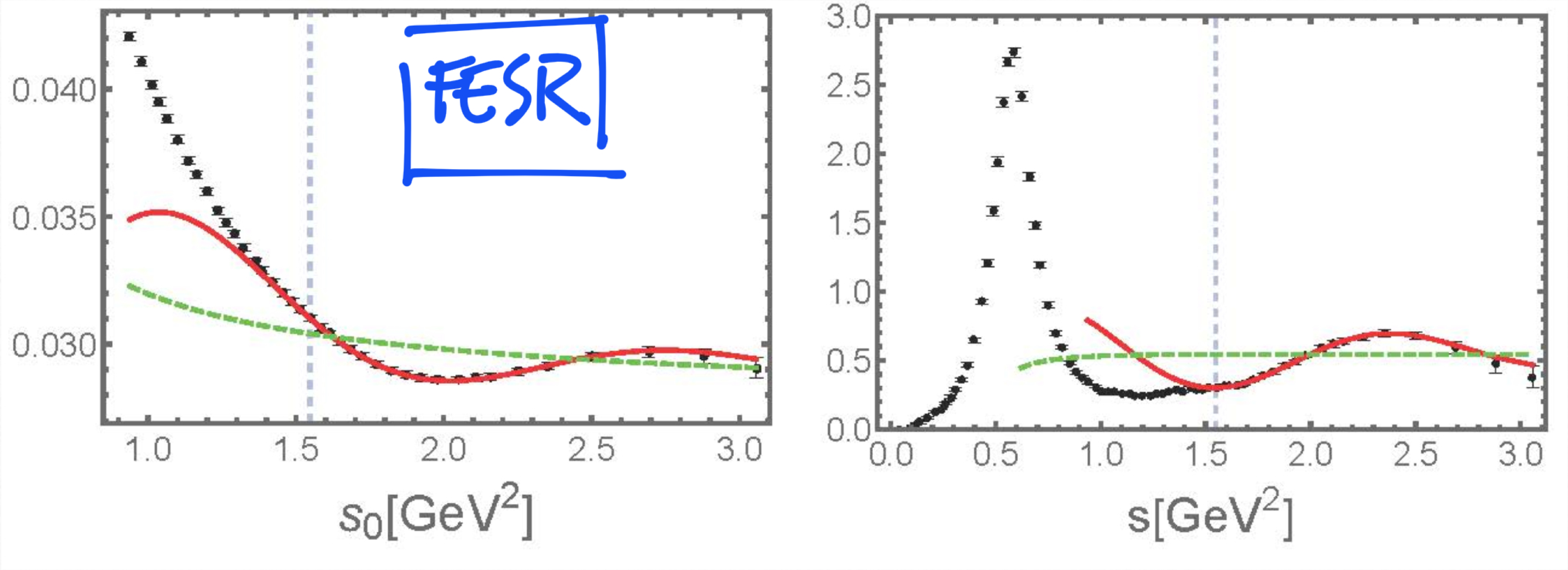
Extraction of α_s (I)

- $w_0 = 1$

$s_0 \in [s_{min}, m_\tau^2]$ $\boxed{\text{FESR}}$

$$\int_0^{s_0} ds\, \frac{1}{\pi} \mathrm{Im}\,\Pi_V(s) = -\frac{1}{2i\pi} \oint_{|z|=s_0} dz\, \Pi_V(z) - \int_{s_0}^{\infty} ds\, \frac{1}{\pi} \mathrm{Im}\,\Pi_{DV}(s)$$





Data

— OPE + DVs

- - - OPE w/o DVs

$w_0 = 1 \longleftrightarrow$ OPE = Pert. Theory

Extraction of α_s (II)

mom.	α_s	$c_6\,[\mathrm{GeV}^6]$
w_0	0.3077(65)	$--$
$w_0\&w_2$	0.3091(69)	$-0.0059(13)$
$w_0\&w_3$	0.3080(70)	$-0.0070(12)$
$w_0\&w_4$	0.3079(70)	$-0.0068(12)$

$w_0 = 1$ $\quad w_3 = (1-y)^2(1+2y)$

$w_2 = 1-y^2$ $\quad w_4 = (1-y^2)^2$

- Several fits, single moments & combined
- Many fit windows $[s_{min}, m_\tau^2]$
- Consistency: α_s, condensates & DV parameters.

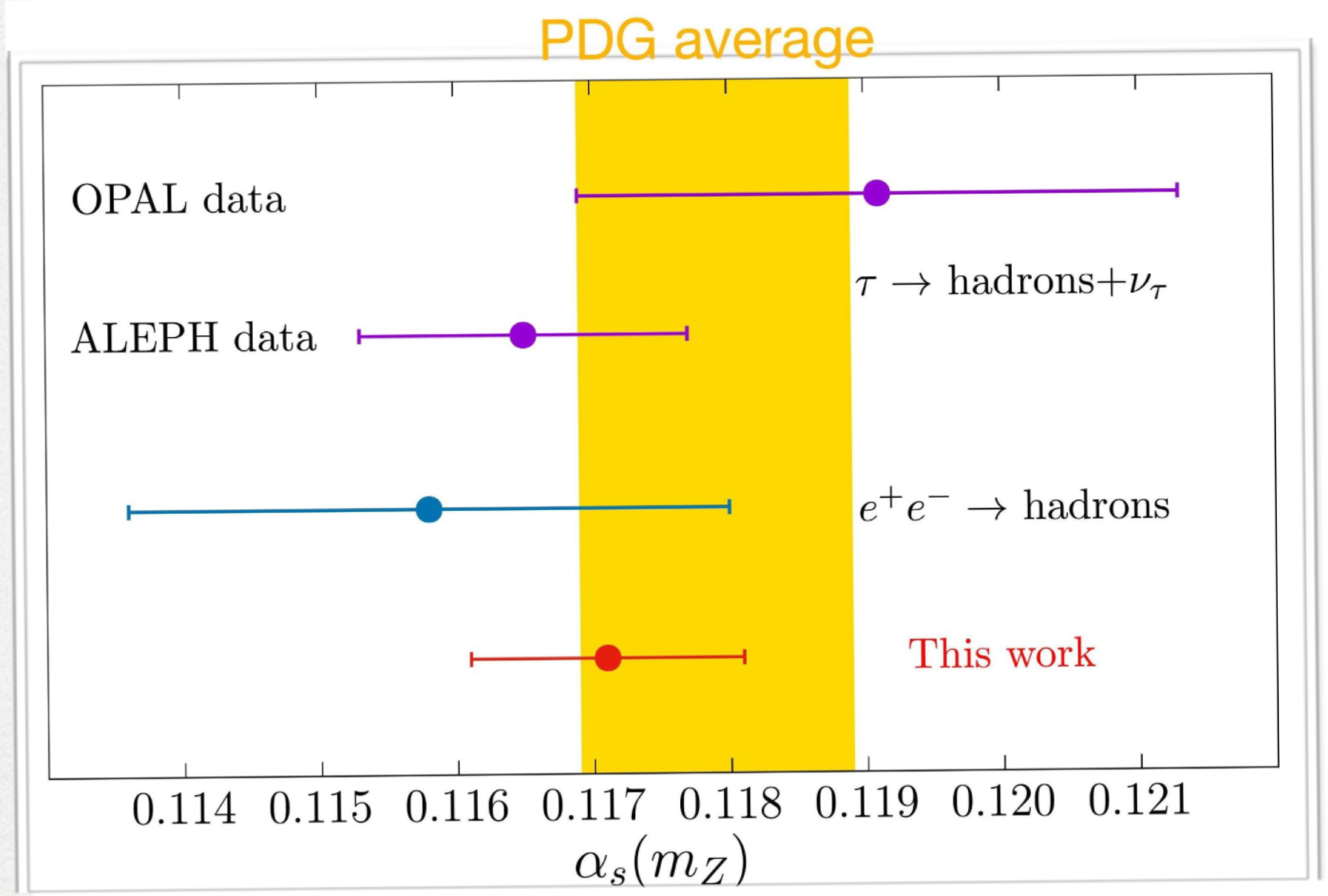
$$\Downarrow$$

$$\alpha_s(m_\tau) = 0.3077 \pm 0.0065_{stat} \pm \boxed{0.0038_{pert.}}$$

$$= 0.3077 \pm 0.0075 \quad (n_f=3,\ \mathrm{FOPT})$$

pert. series truncation & scale variation

SUMMARY & CONCLUSIONS



DB, Golterman, Jamin, Mahdavi, Maltman, Osborne, Peris, '12

DB, Golterman, Maltman, Osborne, Peris, '15

DB, Golterman, Keshavarzi, Maltman, Nomura, Peris, Teubner '18

DB, Golterman, Maltman, Peris, Rodrigues, Schaaf, '21

$$\boxed{\alpha_s(m_\tau) = 0.3077 \pm 0.0075 \quad (n_f=3, \text{FOPT})}$$

$$\Downarrow$$

$$\boxed{\alpha_s(M_Z) = 0.1171 \pm 0.0010 \quad (\overline{MS}, n_f=5)}$$

(For the record

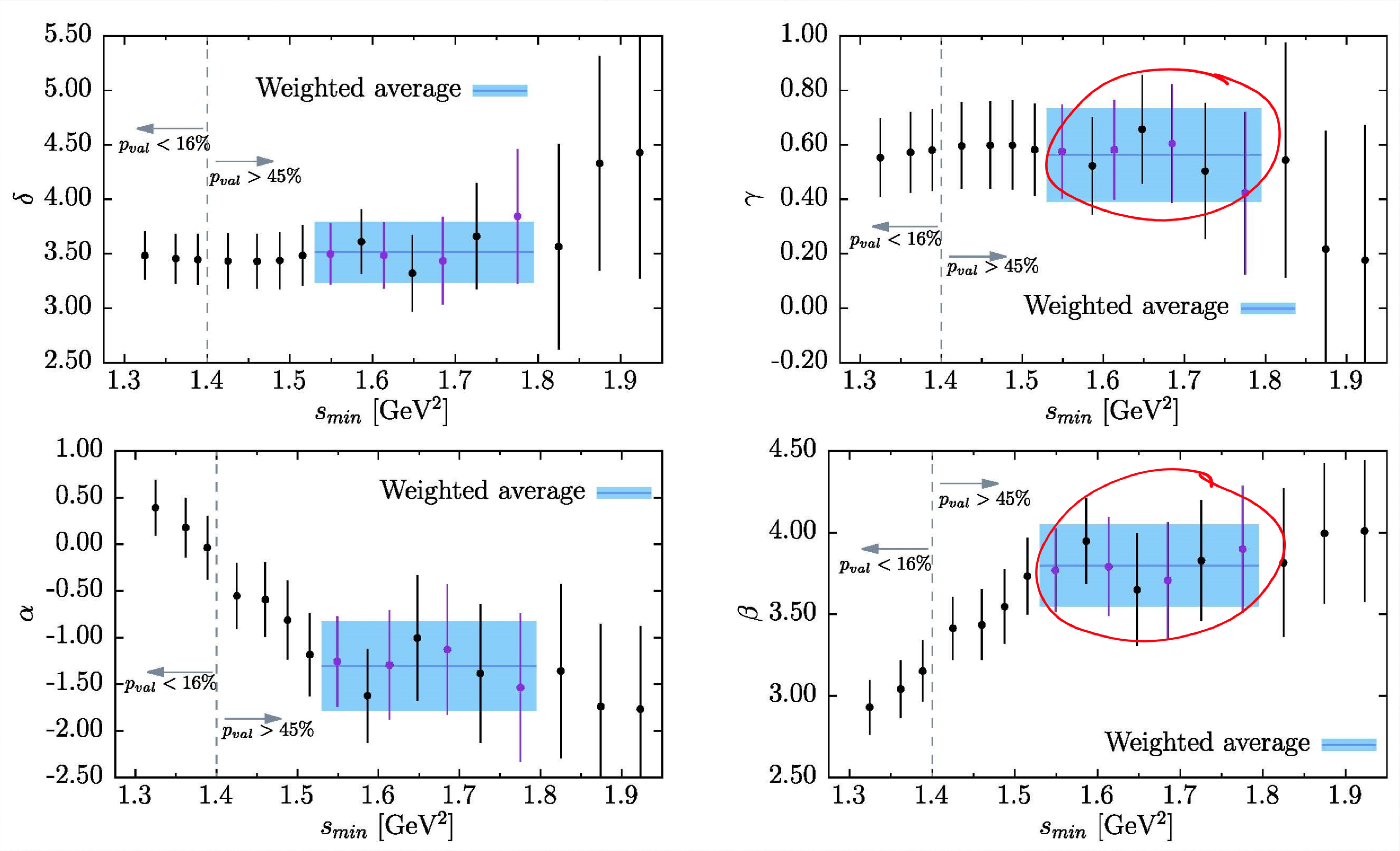
$$\alpha_s(M_Z)_{\text{CIPT}} = 0.1191 \pm 0.0012$$
)

- new V isovector spect. function w/o Monte Carlo input.
- more precise data on $2\pi/4\pi$ channels could improve results (BELLE II?)
- no such possibility for axial channel.
- FESRs dominated by Pert. Theory but non perturbative effects significant.

BACK-UP

SLIDES

DV parameters



agreement with Regge/large N_c expectations