α_s from inclusive pp → W, Z cross sections at NNLO

α_s(2022) workshop

ECT* (Trento), 31st Jan.–5th Feb. 2022

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CERN

- *CMS Collaboration: "Determination of α_s from measurements of inclusive W and Z boson cross sections in p-p collisions at 7 and 8 TeV", JHEP 06 (2020) 018; arXiv:1912.04387 [hep-ex]*
- *D. d'Enterria, A. Poldaru: "Extraction of α_s from a combined NNLO analysis of inclusive electroweak boson cross sections at hadron colliders", JHEP 06 (2020) 016; arXiv:1912.11733 [hep-ph]*

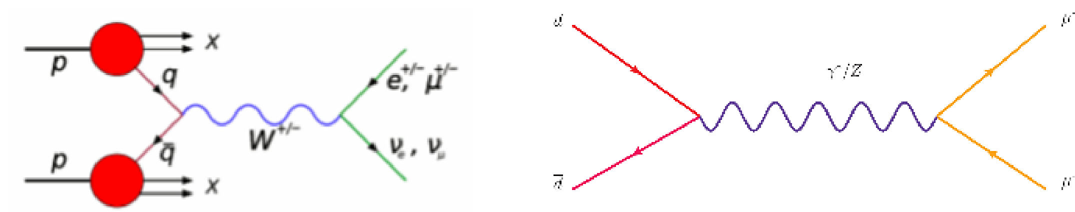
α_s from inclusive σ(W,Z) in hadronic collisions

- Method: Calculate inclusive $W^{\pm}$ and Z production cross sections at NNLO and compare them to the LHC experimental data.

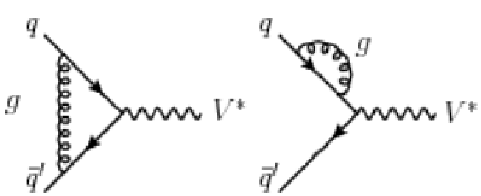
$$\sigma_{\mathrm{pp}\to \mathrm{W,Z+X}} = \iint \mathrm{d}x_1 \mathrm{d}x_2\, f_1(x_1,\mu_F) f_2(x_2,\mu_F)\ \left[\hat{\sigma}_{\mathrm{LO}} + \alpha_{\mathrm{s}}(\mu_R)\hat{\sigma}_{\mathrm{NLO}} + \alpha_{\mathrm{s}}^2(\mu_R)\hat{\sigma}_{\mathrm{NNLO}} + \cdots\right.$$

- The α_s dependency of the cross section comes from higher order effects (virtual & real parton emissions).

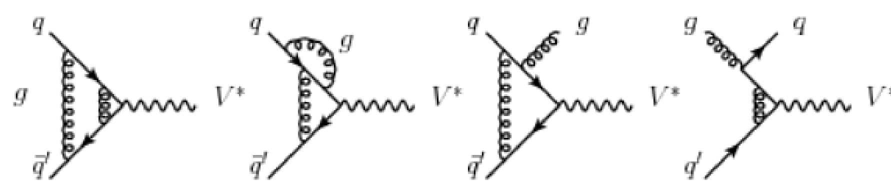
LO diagrams:



NLO diagrams:



NNLO diagrams:



Fiducial cross sections:	CDF	D0	LHCb	ATLAS	CMS
NNLO/LO ratio	1.35	1.35	1.29	1.22	1.33

σ(W,Z) measurements in CMS, LHCb & ATLAS

- Inclusive W & Z x-sections are the most precisely known at the LHC:
  - Experimentally:
    Stat. uncerts. <0.5%
    Syst. uncerts. below ~2% (lumi) in some cases.
  - Theoretically:
    Scale uncertainty: 0.5–1%
    PDF uncertainty: 2–4%
    (that's why α_s extraction is done per PDF set).
- Up to 2019, there were 12+9+7=28 leptonic fiducial measurements by CMS+LHCb+ATLAS.

  We exploited them to carry out a novel α_s extraction.

Measurement	Fiducial cross section
pp at $\sqrt{s} = 7$ TeV [13]	
W_e^+, $p_T^e > 25$ GeV, $\|\eta^e\| < 2.5$	$3404 \pm 12\,(\text{stat}) \pm 67\,(\text{syst}) \pm 136\,(\text{lumi})$ pb $= 3404 \pm 152$ pb
W_e^-, $p_T^e > 25$ GeV, $\|\eta^e\| < 2.5$	$2284 \pm 10\,(\text{stat}) \pm 43\,(\text{syst}) \pm 91\,(\text{lumi})$ pb $= 2284 \pm 101$ pb
Z_e, $p_T^e > 25$ GeV, $\|\eta^e\| < 2.5$, $60 < m_Z < 120$ GeV	$452 \pm 5\,(\text{stat}) \pm 10\,(\text{syst}) \pm 18\,(\text{lumi})$ pb $= 452 \pm 21$ pb
W_μ^+, $p_T^\mu > 25$ GeV, $\|\eta^\mu\| < 2.1$	$2815 \pm 9\,(\text{stat}) \pm 42\,(\text{syst}) \pm 113\,(\text{lumi})$ pb $= 2815 \pm 121$ pb
W_μ^-, $p_T^\mu > 25$ GeV, $\|\eta^\mu\| < 2.1$	$1921 \pm 8\,(\text{stat}) \pm 27\,(\text{syst}) \pm 77\,(\text{lumi})$ pb $= 1921 \pm 82$ pb
Z_μ, $p_T^\mu > 20$ GeV, $\|\eta^\mu\| < 2.1$, $60 < m_Z < 120$ GeV	$396 \pm 3\,(\text{stat}) \pm 7\,(\text{syst}) \pm 16\,(\text{lumi})$ pb $= 396 \pm 18$ pb
pp at $\sqrt{s} = 8$ TeV [14]	
W_e^+, $p_T^e > 25$ GeV, $\|\eta^e\| < 1.44$, $1.57 < \|\eta^e\| < 2.5$	$3540 \pm 20\,(\text{stat}) \pm 110\,(\text{syst}) \pm 90\,(\text{lumi})$ pb $= 3540 \pm 140$ pb
W_e^-, $p_T^e > 25$ GeV, $\|\eta^e\| < 1.44$, $1.57 < \|\eta^e\| < 2.5$	$2390 \pm 10\,(\text{stat}) \pm 60\,(\text{syst}) \pm 60\,(\text{lumi})$ pb $= 2390 \pm 90$ pb
Z_e, $p_T^e > 25$ GeV, $\|\eta^e\| < 1.44$, $1.57 < \|\eta^e\| < 2.5$, $60 < m_Z < 120$ GeV	$450 \pm 10\,(\text{stat}) \pm 10\,(\text{syst}) \pm 10\,(\text{lumi})$ pb $= 450 \pm 20$ pb
W_μ^+, $p_T^\mu > 25$ GeV, $\|\eta^\mu\| < 2.1$	$3100 \pm 10\,(\text{stat}) \pm 40\,(\text{syst}) \pm 80\,(\text{lumi})$ pb $= 3100 \pm 90$ pb
W_μ^-, $p_T^\mu > 25$ GeV, $\|\eta^\mu\| < 2.1$	$2240 \pm 10\,(\text{stat}) \pm 20\,(\text{syst}) \pm 60\,(\text{lumi})$ pb $= 2240 \pm 60$ pb
Z_μ, $p_T^\mu > 25$ GeV, $\|\eta^\mu\| < 2.1$, $60 < m_Z < 120$ GeV	$400 \pm 10\,(\text{stat}) \pm 10\,(\text{syst}) \pm 10\,(\text{lumi})$ pb $= 400 \pm 20$ pb

ATLAS measurement	Fiducial cross section
pp at $\sqrt{s} = 7$ TeV [7]	
W^+, $p_T^\ell > 25$ GeV, $p_T^\nu > 25$ GeV, $\|\eta^\ell\| < 2.5$, $m_T > 40$ GeV	$2947 \pm 1_{(\text{stat})} \pm 15_{(\text{syst})} \pm 53_{(\text{lumi})}$ pb $= 2947 \pm 55$ pb
W^-, $p_T^\ell > 25$ GeV, $p_T^\nu > 25$ GeV, $\|\eta^\ell\| < 2.5$, $m_T > 40$ GeV	$1964 \pm 1_{(\text{stat})} \pm 11_{(\text{syst})} \pm 35_{(\text{lumi})}$ pb $= 1964 \pm 37$ pb
Z, $p_T^\ell > 20$ GeV, $\|\eta^\ell\| < 2.5$, $m_Z = 66$–116 GeV, central	$502.2 \pm 0.3_{(\text{stat})} \pm 1.7_{(\text{syst})} \pm 9.0_{(\text{lumi})}$ pb $= 502.2 \pm 9.2$ pb
pp at $\sqrt{s} = 8$ TeV [8]	
Z, $p_T^\ell > 20$ GeV, $\|\eta^\ell\| < 2.4$, $m_Z = 66$–116 GeV	$537.10 \pm 0.45\%_{(\text{syst})} \pm 2.8\%_{(\text{lumi})}$ pb $= 537.10 \pm 15.23$ pb
pp at $\sqrt{s} = 13$ TeV [9]	
W^+, $p_T^\ell > 25$ GeV, $p_T^\nu > 25$ GeV, $\|\eta^\ell\| < 2.5$, $m_T > 50$ GeV	$4530 \pm 10_{(\text{stat})} \pm 90_{(\text{syst})} \pm 100_{(\text{lumi})}$ pb $= 4530 \pm 130$ pb
W^-, $p_T^\ell > 25$ GeV, $p_T^\nu > 25$ GeV, $\|\eta^\ell\| < 2.5$, $m_T > 50$ GeV	$3500 \pm 10_{(\text{stat})} \pm 70_{(\text{syst})} \pm 70_{(\text{lumi})}$ pb $= 3500 \pm 100$ pb
Z, $p_T^\ell > 25$ GeV, $\|\eta^\ell\| < 2.5$, $m_Z = 66$–116 GeV	$779 \pm 3_{(\text{stat})} \pm 6_{(\text{syst})} \pm 16_{(\text{lumi})}$ pb $= 779 \pm 17$ pb

LHCb measurement	Fiducial cross section
pp at $\sqrt{s} = 7$ TeV [10]	
W^+, $p_T^\ell > 20$ GeV, $2. < \eta^\ell < 4.5$	$878.0 \pm 2.1_{(\text{stat})} \pm 6.7_{(\text{syst})} \pm 9.3_{(\text{c.m. en.})} \pm 15.0_{(\text{lumi})}$ pb $= 878.0 \pm 19.0$ pb
W^-, $p_T^\ell > 20$ GeV, $2. < \eta^\ell < 4.5$	$689.5 \pm 2.0_{(\text{stat})} \pm 5.3_{(\text{syst})} \pm 6.3_{(\text{c.m. en.})} \pm 11.8_{(\text{lumi})}$ pb $= 689.5 \pm 14.5$ pb
Z, $p_T^\ell > 20$ GeV, $2. < \eta^\ell < 4.5$, $m_Z = 60$–120 GeV	$76.0 \pm 0.3_{(\text{stat})} \pm 0.5_{(\text{syst})} \pm 1.0_{(\text{c.m. en.})} \pm 1.3_{(\text{lumi})}$ pb $= 76.0 \pm 1.7$ pb
pp at $\sqrt{s} = 8$ TeV [11, 13]	
W_e^+, $p_T^e > 20$ GeV, $2. < \eta^e < 4.25$	$1124.4 \pm 2.1_{(\text{stat})} \pm 21.5_{(\text{syst})} \pm 11.2_{(\text{c.m. en.})} \pm 13.0_{(\text{lumi})}$ pb $= 1124.4 \pm 27.6$ pb
W_e^-, $p_T^e > 20$ GeV, $2. < \eta^e < 4.25$	$809.0 \pm 1.9_{(\text{stat})} \pm 18.1_{(\text{syst})} \pm 7.0_{(\text{c.m. en.})} \pm 9.4_{(\text{lumi})}$ pb $= 809.0 \pm 21.6$ pb
W_μ^+, $p_T^\mu > 20$ GeV, $2. < \eta^\mu < 4.5$	$1093.6 \pm 2.1_{(\text{stat})} \pm 7.2_{(\text{syst})} \pm 10.9_{(\text{c.m. en.})} \pm 12.7_{(\text{lumi})}$ pb $= 1093.6 \pm 18.3$ pb
W_μ^-, $p_T^\mu > 20$ GeV, $2. < \eta^\mu < 4.5$	$818.4 \pm 1.9_{(\text{stat})} \pm 5.0_{(\text{syst})} \pm 7.0_{(\text{c.m. en.})} \pm 9.5_{(\text{lumi})}$ pb $= 818.4 \pm 13.0$ pb
Z_μ, $p_T^\mu > 20$ GeV, $2. < \eta^\mu < 4.5$, $m_Z = 60$–120 GeV	$95.0 \pm 0.3_{(\text{stat})} \pm 0.7_{(\text{syst})} \pm 1.1_{(\text{c.m. en.})} \pm 1.1_{(\text{lumi})}$ pb $= 95.0 \pm 1.7$ pb
pp at $\sqrt{s} = 13$ TeV [12]	
Z, $p_T^\ell > 20$ GeV, $2. < \eta^\ell < 4.5$, $m_Z = 60$–120 GeV	$194.3 \pm 0.9_{(\text{stat})} \pm 3.3_{(\text{syst})} \pm 7.6_{(\text{lumi})}$ pb $= 194.3 \pm 8.3$ pb

Theoretical setup

MCFM v8.0

- MCFM - Monte Carlo for FeMtobarn processes, able to calculate cross sections in defined fiducial vol. at NNLO accuracy.

LHAPDF v6.1.6

- Interface to 4 PDF sets: CT14, HERAPDF2.0, MMHT14 and NNPDF3.0, plus associated eigenvectors/replicas (*)
- PDF sets available for 5 or 7 α_s values: 0.115, 0.116, 0.117, 0.118, 0.119, 0.120, and 0.121.

MCSANC v1.01

- SANC used to calculate electroweak corrections to $W^\pm$ and Z boson production at NLO QCD+EW accuracy.
- For $e^\pm$ final states we recombine the photon with $e^\pm$ if $\Delta R < 0.1$ ("calorimetric" prescription). For $\mu^\pm$ we use the "bare" cross section.
- Multiplicative factor computed to correct the MCFM cross section: $K_{\mathrm{EW}} = \sigma(\mathrm{NLO,EW\,on})/\sigma(\mathrm{NLO,EW\,off})$: Negative 1–4% correction of the overall cross section.

(*) None of these PDFs use in their global fits the **absolute σ(W,Z)** values exploited here.

Experimental data & NNLO computations

A total of 28 different experimental setups considered

$\sqrt{s}$ (TeV)	7	8	13	Total
ATLAS	3	1	3	7
LHCb	3	5	1	9
CMS	6	6	0	12

- The 28 systems represent a total of ~20 000 jobs, taking into account all PDF eigenvalues (101 NNPDF + (56+7) CT14 + (29+14+7) HERA + (51+7) MMHT), and 5 or 7 α_s values (0.115, 0.116, 0.117, 0.118, 0.119, 0.120, and 0.121).
- All these computations are done at the CERN computer cluster with hundreds of jobs in parallel.
- Running at NNLO is slow. Using the longest 2-week queue we get 0.2–0.6% numerical accuracy, comparable to ~1% diffs. to alternative NNLO calculators: FEWZ, DYNNLO (impact on final α_s result assessed at the end)

Examples of NNLO fiducial x-sections per PDF

	System	Fiducial cross section
	$pp \to W^+(\ell^+\nu) + X$, $\sqrt{s} = 7\,\mathrm{TeV}$	ATLAS ($p_T^\ell > 25\,\mathrm{GeV}$, $p_T^\nu > 25\,\mathrm{GeV}$, $\lvert\eta\rvert < 2.5$, $m_T > 40\,\mathrm{GeV}$)
Data:	ATLAS measurement [7]	$2947 \pm 1\,_{(\mathrm{stat})} \pm 15\,_{(\mathrm{syst})} \pm 53\,_{(\mathrm{lumi})}$ pb
CT14:	MCFM (NNLO, CT14)	$2862\,^{+141}_{-74}\rvert_{(\mathrm{PDF})} \pm 28\,_{(\alpha_s)} \pm 36\,_{(\mathrm{scale})} \pm 18\,_{(\mathrm{stat})}$ pb
HERAPDF2.0:	MCFM (NNLO, HERAPDF2.0)	$3019\,^{+41}_{-84}\rvert_{(\mathrm{PDF})} \pm 11\,_{(\alpha_s)} \pm 36\,_{(\mathrm{scale})} \pm 18\,_{(\mathrm{stat})}$ pb
MMHT14:	MCFM (NNLO, MMHT14)	$2880\,^{+156}_{-50}\rvert_{(\mathrm{PDF})} \pm 28\,_{(\alpha_s)} \pm 36\,_{(\mathrm{scale})} \pm 16\,_{(\mathrm{stat})}$ pb
NNPDF3.0:	MCFM (NNLO, NNPDF3.0)	$2827 \pm 61\,_{(\mathrm{PDF})} \pm 25\,_{(\alpha_s)} \pm 36\,_{(\mathrm{scale})} \pm 18\,_{(\mathrm{stat})}$ pb
MCSANC corr.	MCSANC (NLO EW, NNPDF3.0)	-10 pb (-0.4%)
	$pp \to W^-(\ell^-\bar{\nu}) + X$, $\sqrt{s} = 7\,\mathrm{TeV}$	ATLAS ($p_T^\ell > 25\,\mathrm{GeV}$, $p_T^\nu > 25\,\mathrm{GeV}$, $\lvert\eta\rvert < 2.5$, $m_T > 40\,\mathrm{GeV}$)
Data:	ATLAS measurement [7]	$1964 \pm 1\,_{(\mathrm{stat})} \pm 11\,_{(\mathrm{syst})} \pm 35\,_{(\mathrm{lumi})}$ pb
CT14:	MCFM (NNLO, CT14)	$1910\,^{+29}_{-88}\rvert_{(\mathrm{PDF})} \pm 17\,_{(\alpha_s)} \pm 12\,_{(\mathrm{scale})} \pm 8\,_{(\mathrm{stat})}$ pb
HERAPDF2.0:	MCFM (NNLO, HERAPDF2.0)	$1993\,^{+13}_{-74}\rvert_{(\mathrm{PDF})} \pm 7\,_{(\alpha_s)} \pm 12\,_{(\mathrm{scale})} \pm 7\,_{(\mathrm{stat})}$ pb
MMHT14:	MCFM (NNLO, MMHT14)	$1930\,^{+39}_{-55}\rvert_{(\mathrm{PDF})} \pm 20\,_{(\alpha_s)} \pm 12\,_{(\mathrm{scale})} \pm 7\,_{(\mathrm{stat})}$ pb
NNPDF3.0:	MCFM (NNLO, NNPDF3.0)	$1875 \pm 43\,_{(\mathrm{PDF})} \pm 12\,_{(\alpha_s)} \pm 12\,_{(\mathrm{scale})} \pm 7\,_{(\mathrm{stat})}$ pb
MCSANC corr.	MCSANC (NLO EW, NNPDF3.0)	-5.2 pb (-0.3%)
	$pp \to Z(\ell^+\ell^-) + X$, $\sqrt{s} = 7\,\mathrm{TeV}$	ATLAS ($66\,\mathrm{GeV} < m_Z < 116\,\mathrm{GeV}$, $p_T^\ell > 20\,\mathrm{GeV}$, $\lvert\eta\rvert < 2.5$)
Data:	ATLAS measurement [7]	$502 \pm 0.3\,_{(\mathrm{stat})} \pm 2\,_{(\mathrm{syst})} \pm 9\,_{(\mathrm{lumi})}$ pb
CT14:	MCFM (NNLO, CT14)	$482\,^{+10}_{-16}\rvert_{(\mathrm{PDF})} \pm 5\,_{(\alpha_s)} \pm 2\,_{(\mathrm{scale})} \pm 0.9\,_{(\mathrm{stat})}$ pb
HERAPDF2.0:	MCFM (NNLO, HERAPDF2.0)	$499\,^{+8}_{-8}\rvert_{(\mathrm{PDF})} \pm 3\,_{(\alpha_s)} \pm 2\,_{(\mathrm{scale})} \pm 1\,_{(\mathrm{stat})}$ pb
MMHT14:	MCFM (NNLO, MMHT14)	$485\,^{+9}_{-8}\rvert_{(\mathrm{PDF})} \pm 5\,_{(\alpha_s)} \pm 2\,_{(\mathrm{scale})} \pm 1\,_{(\mathrm{stat})}$ pb
NNPDF3.0:	MCFM (NNLO, NNPDF3.0)	$474 \pm 10\,_{(\mathrm{PDF})} \pm 4\,_{(\alpha_s)} \pm 2\,_{(\mathrm{scale})} \pm 1\,_{(\mathrm{stat})}$ pb
MCSANC corr.	MCSANC (NLO EW, NNPDF3.0)	-3.5 pb (-0.7%)
	$pp \to Z(\ell^+\ell^-) + X$, $\sqrt{s} = 8\,\mathrm{TeV}$	ATLAS ($66\,\mathrm{GeV} < m_Z < 116\,\mathrm{GeV}$, $p_T^\ell > 20\,\mathrm{GeV}$, $\lvert\eta\rvert < 2.4$)
Data:	ATLAS measurement [8]	$537 \pm 2\,_{(\mathrm{syst})} \pm 15\,_{(\mathrm{lumi})}$ pb
CT14:	MCFM (NNLO, CT14)	$518\,^{+13}_{-16}\rvert_{(\mathrm{PDF})} \pm 5\,_{(\alpha_s)} \pm 2\,_{(\mathrm{scale})} \pm 1\,_{(\mathrm{stat})}$ pb
HERAPDF2.0:	MCFM (NNLO, HERAPDF2.0)	$537\,^{+12}_{-8}\rvert_{(\mathrm{PDF})} \pm 3\,_{(\alpha_s)} \pm 2\,_{(\mathrm{scale})} \pm 1\,_{(\mathrm{stat})}$ pb
MMHT14:	MCFM (NNLO, MMHT14)	$523\,^{+8}_{-8}\rvert_{(\mathrm{PDF})} \pm 6\,_{(\alpha_s)} \pm 2\,_{(\mathrm{scale})} \pm 1\,_{(\mathrm{stat})}$ pb
NNPDF3.0:	MCFM (NNLO, NNPDF3.0)	$511 \pm 11\,_{(\mathrm{PDF})} \pm 4\,_{(\alpha_s)} \pm 2\,_{(\mathrm{scale})} \pm 1\,_{(\mathrm{stat})}$ pb
MCSANC corr.	MCSANC (NLO EW, NNPDF3.0)	-2.5 pb (-0.5%)

Exp. & theor. uncertainties. χ^2/ndf data-theory

- Experimental & theoretical uncertainties and their correlations:

Source	Uncertainty	Degree of correlation
Experimental:		
Luminosity	2–4% (CMS), 2–3% (ATLAS), 1–4% (LHCb), 3–6% (CDF, D0)	fully correlated per exp. at each $\sqrt{s}$
Systematic	1–3% (CMS), 0.3–2% (ATLAS), 0.6–2% (LHCb), 2–3% (CDF, D0)	partially correlated within each exp.
c.m. energy	0.9–1.3% (LHCb)	fully correlated at each $\sqrt{s}$
Statistical	0.3–2.5% (CMS), $<$ 0.4% (ATLAS), 0.2–0.5% (LHCb), 0.4–2.6% (CDF, D0)	uncorrelated
Theoretical:		
PDF	1–4%	partially correlated within PDF set
Scales	0.4–1.2%	partially correlated among all calculations
Numerical	0.2–0.6%	uncorrelated

Table 4. Typical ATLAS, CMS, LHCb and Tevatron experimental and theoretical uncertainties in the $W^{\pm}$ and Z boson production cross sections, and their degree of correlation

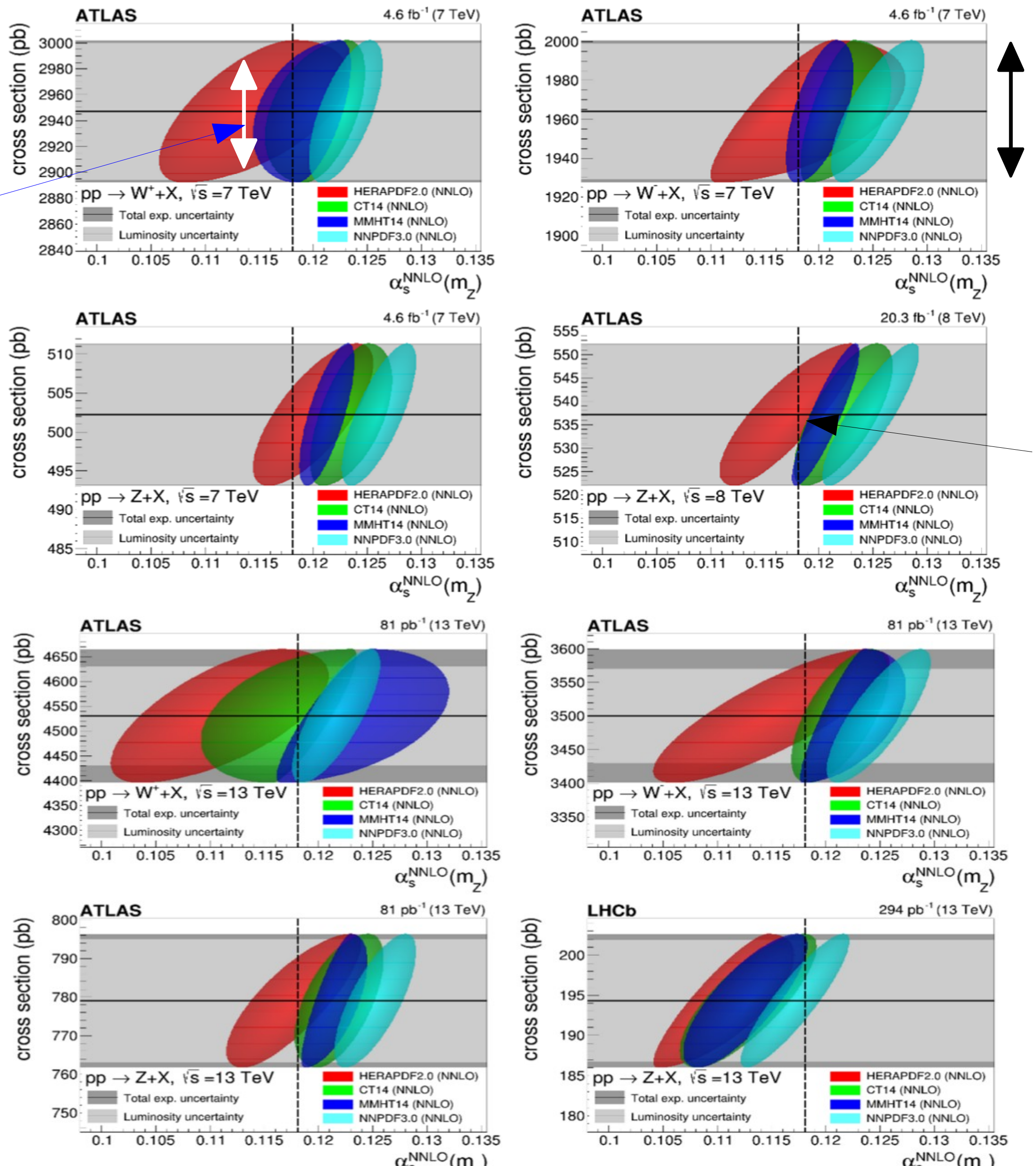
- NNLO-data for default α_s=0.118: Goodness-of-fit ~1 for CT14/MMHT14
~2 for HERAPDF2.0/NNPDF3.0

	CT14	HERAPDF2.0	MMHT14	NNPDF3.0
χ^2/ndf (symmetrized to the largest PDF uncertainty value)	15.8/27	21.8/27	15.7/27	58.8/27
χ^2/ndf (symmetrized to the smallest PDF uncertainty value)	26.3/27	60.4/27	22.7/27	58.8/27

Table 11. Overall goodness-of-fit per number of degrees of freedom, χ^2/ndf, among the 28 LHC experimental measurements of W^+, W^-, and Z boson cross sections and the corresponding theoretical predictions obtained with the four different PDF sets for their default $\alpha_s(m_Z) = 0.118$ value. The first (second) row lists the results obtained symmetrizing the PDF uncertainties of the cross sections obtained with the CT14, HERAPDF2.0, and MMHT14 sets to the largest (smallest) of their respective values.

σ_{NNLO}(W,Z) versus $\alpha_s(m_Z)$ for ATLAS data

Theory uncertainty (mostly PDF)

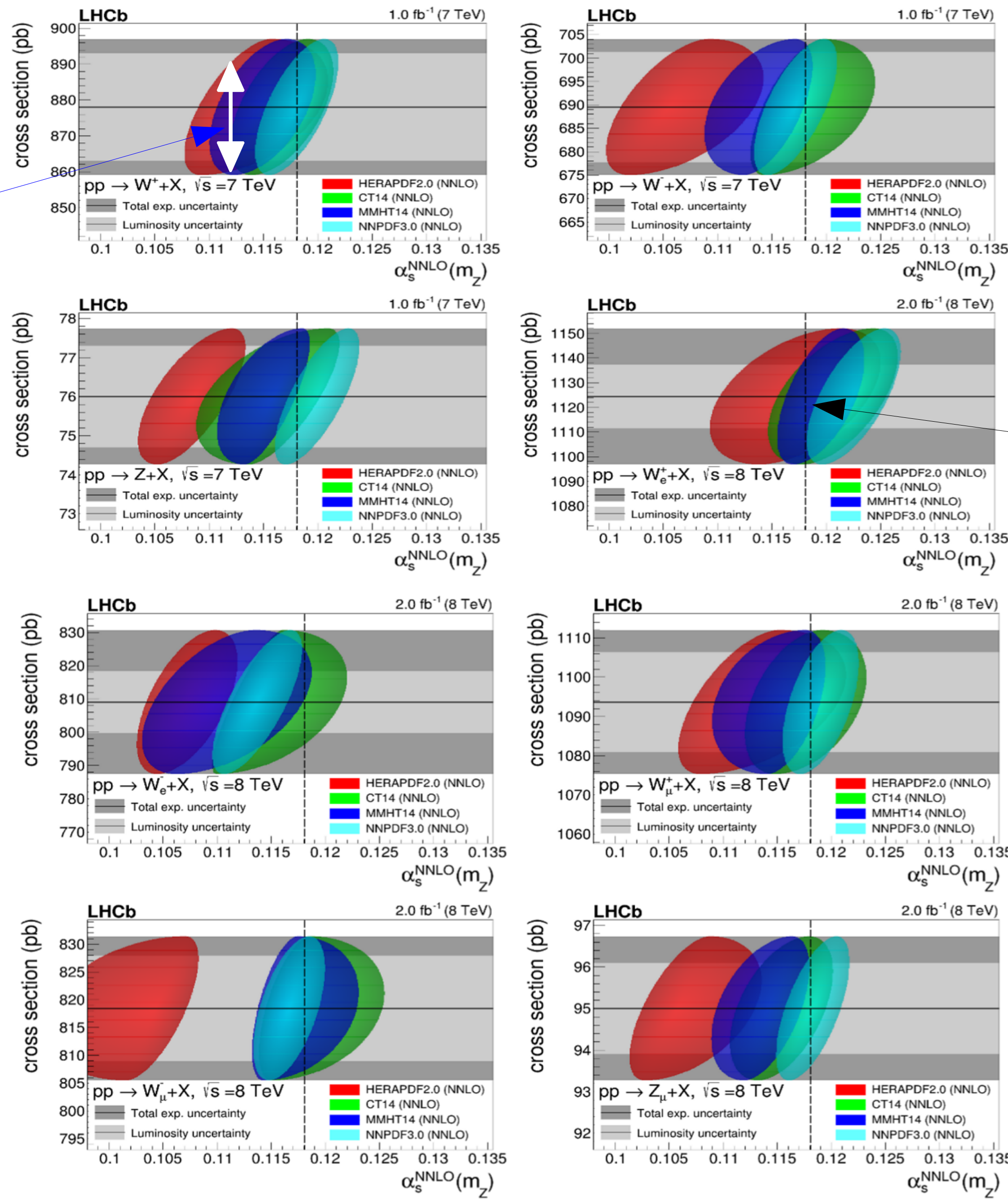


Experimental uncertainty (mostly lumi)

Crossing-line point indicates “perfect match” of exp-theory x-sections for the default $\alpha_s(m_Z)$=0.118 of all PDF sets

1σ ellipses are Joint Probability Density Functions (product of data & theory uncertainties)

σ_{NNLO}(W,Z) versus $\alpha_s(m_Z)$ for LHCb data



Theory uncertainty (mostly PDF)

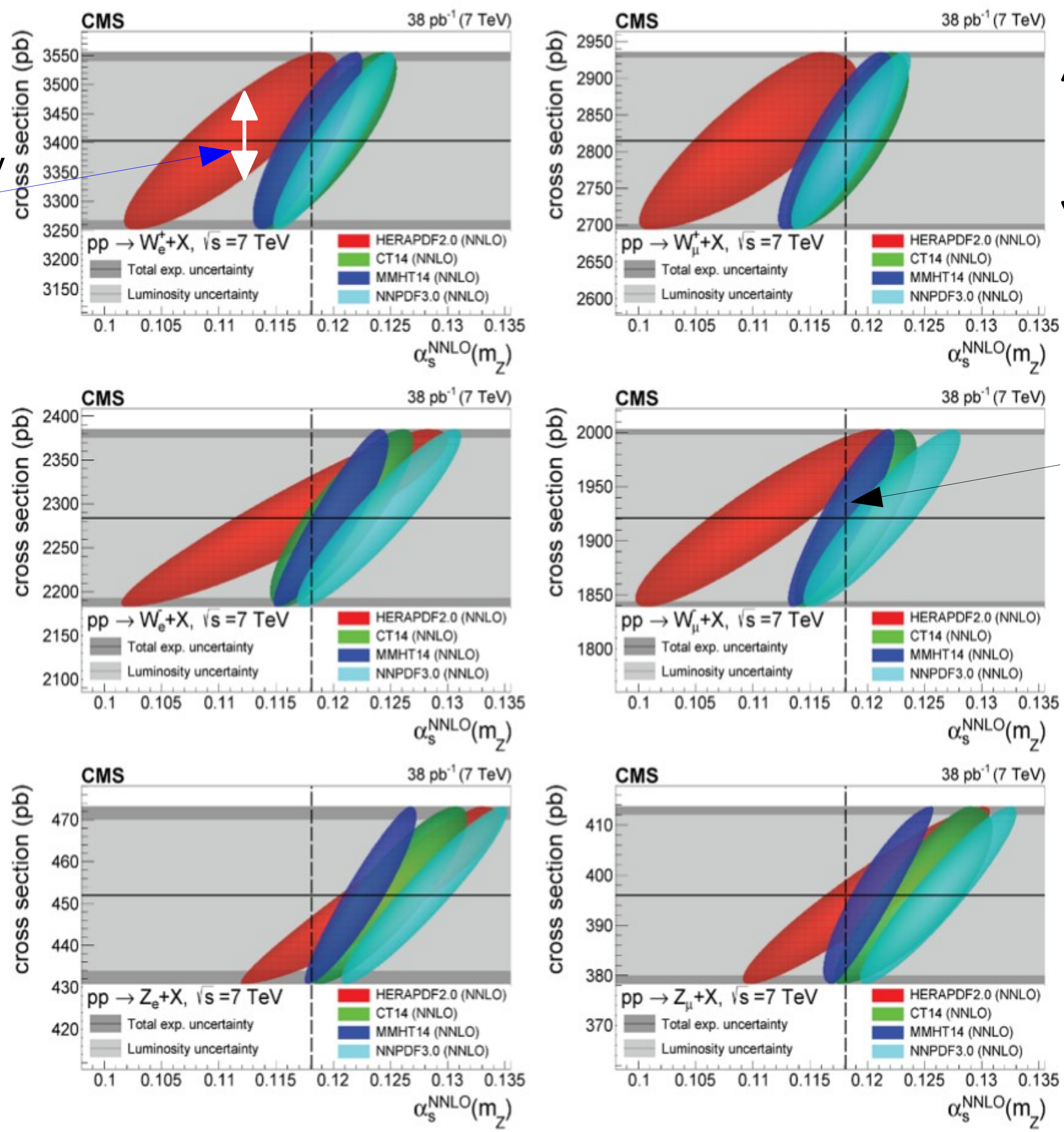
Experimental uncertainty (mostly lumi)

Crossing-line point indicates "perfect match" of exp-theory x-sections for the default $\alpha_s(m_Z)$=0.118 of all PDF sets

1σ ellipses are Joint Probability Density Functions (product of data & theory uncertainties)

$\sigma_{NNLO}(W,Z)$ versus $\alpha_s(m_Z)$ for CMS data

Theory uncertainty (mostly PDF)



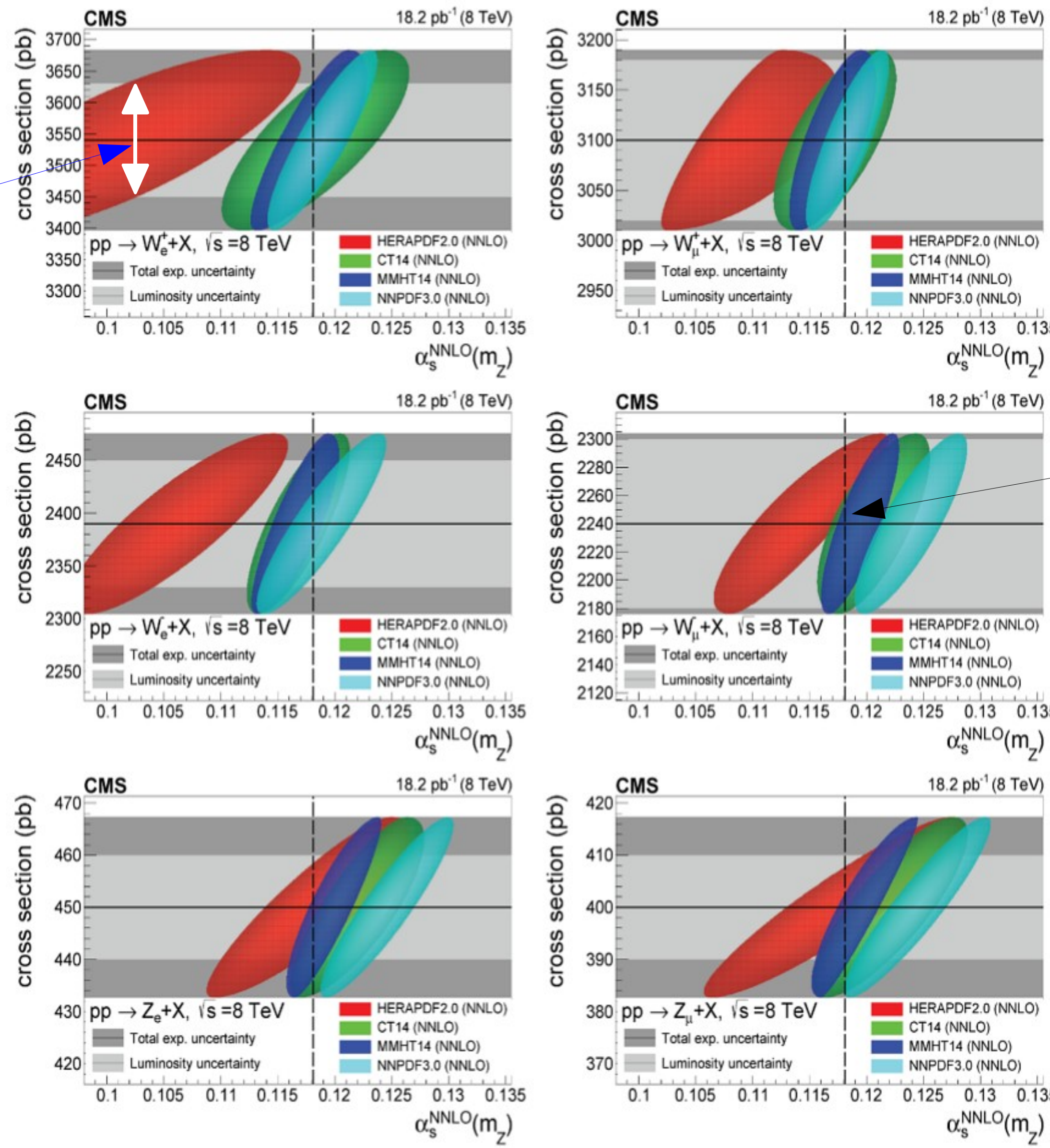
Exp. uncertainty (mostly lumi)

Crossing-line point indicates “perfect match” of exp-theory x-sections for the default $\alpha_s(m_Z)$=0.118 of all PDF sets

1σ ellipses are Joint Probability Density Functions (product of data & theory uncertainties)

σ_{NNLO}(W,Z) versus $\alpha_s(m_Z)$ for CMS data

Theory uncertainty (mostly PDF)

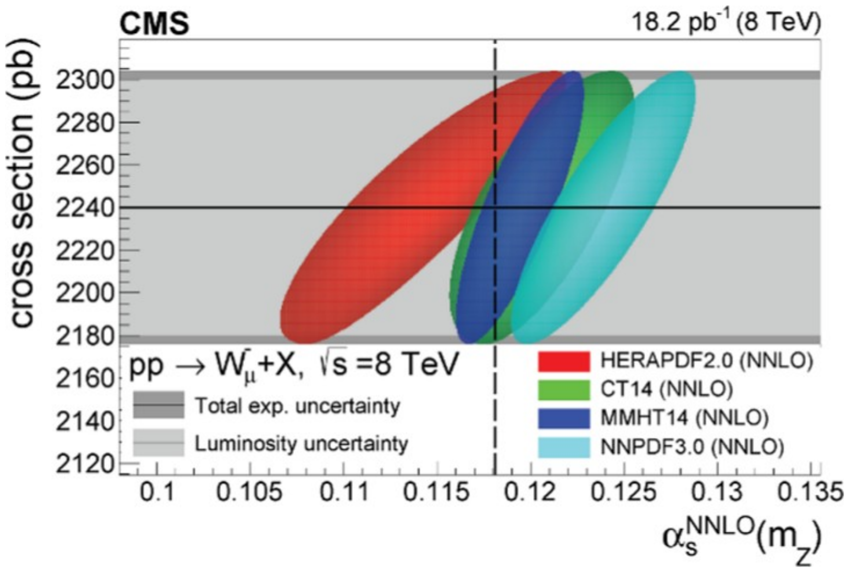


Exp. uncertainty (mostly lumi)

Crossing line point indicates “perfect match” of exp-theory x-sections for the default $\alpha_s(m_Z)$=0.118 of all PDF sets

1σ ellipses are Joint Probability Density Functions (product of data & theory uncertainties)

σ_{NNLO}(W,Z) versus $\alpha_s(m_Z)$ for LHC data: Global trends



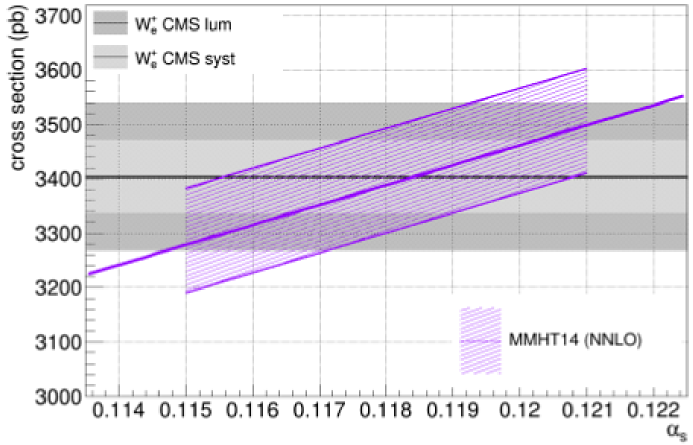
- Theory consistent with data within uncertainties but not systematically for the same $\alpha_s(m_Z)$ value (in particular, HERAPDF2.0 results do not always overlap with any of the others within the 1 std.-dev. region)

- For a fixed $\alpha_s(m_Z)$ value, HERAPDF2.0 (NNPDF3.0) predict larger (smaller) x-sections: HERAPDF2.0 (NNPDF3.0) prefer systematically smaller (larger) $\alpha_s(m_Z)$. MMHT14 and CT14 predictions are in between (and less scattered).

- HERAPDF2.0 (MMHT14) always has the smallest (largest) slope, i.e. HERAPDF2.0 (MMHT14) x-sections are the least (most) sensitive to α_s variations.

Preferred $\alpha_s(m_Z)$ per measurement & propag. uncertainty

- The α_s value for each setup is determined by the intersection point of the fitted theoretical line with the experimental value.



- Each σ uncertainty δ_i propagates into α_s as δ_i/k:
- To motivate this, one can construct a marginalized posterior by multiplying the theoretical and experimental probability densities and integrating over σ:

$$P(\alpha_s) = \int f_{exp}(\sigma) \cdot f_{th}(\sigma|\alpha_s) d\sigma.$$

- Using this the variance of α_s from this single system considering all uncertainty sources δ_i is given by (k is the slope of the fitted line):

$$\text{var}(\alpha_s) = \frac{1}{k} \sum_i \text{var}(\delta_i).$$

Preferred $\alpha_s(m_Z)$ per measurement & per PDF

- Example of preferred $\alpha_s(m_Z)$ per measurement & PDF (total: 28×4 = 112 values):

Cross section	PDF	$\alpha_s(m_Z)$	δ_{α_S} (stat)	δ_{α_S} (lumi)	δ_{α_S} (syst)	δ_{α_S} (PDF)	δ_{α_S} (scale)	δ_{α_S} (num)
ATLAS	CT14	$0.1211^{+0.0056}_{-0.0036}$	0.0000	0.0019	0.0005	$^{+0.0051}_{-0.0027}$	0.0013	0.0007
W^+ (7 TeV)	HERAPDF2.0	$0.1132^{+0.0074}_{-0.0101}$	0.0001	0.0049	0.0014	$^{+0.0038}_{-0.0078}$	0.0034	0.0017
	MMHT14	$0.1205^{+0.0061}_{-0.0030}$	0.0000	0.0019	0.0005	$^{+0.0056}_{-0.0018}$	0.0013	0.0006
	NNPDF3.0	$0.1229^{+0.0036}_{-0.0036}$	0.0000	0.0021	0.0006	$^{+0.0024}_{-0.0024}$	0.0015	0.0007
ATLAS	CT14	$0.1211^{+0.0028}_{-0.0056}$	0.0001	0.0020	0.0006	$^{+0.0017}_{-0.0051}$	0.0007	0.0005
W^- (7 TeV)	HERAPDF2.0	$0.1160^{+0.0060}_{-0.0121}$	0.0001	0.0051	0.0016	$^{+0.0018}_{-0.0107}$	0.0017	0.0010
	MMHT14	$0.1198^{+0.0028}_{-0.0035}$	0.0001	0.0018	0.0006	$^{+0.0020}_{-0.0028}$	0.0006	0.0004
	NNPDF3.0	$0.1254^{+0.0047}_{-0.0047}$	0.0001	0.0030	0.0009	$^{+0.0033}_{-0.0033}$	0.0010	0.0006
ATLAS	CT14	$0.1229^{+0.0032}_{-0.0044}$	0.0001	0.0021	0.0004	$^{+0.0024}_{-0.0038}$	0.0005	0.0002
Z (7 TeV)	HERAPDF2.0	$0.1200^{+0.0055}_{-0.0056}$	0.0001	0.0040	0.0008	$^{+0.0036}_{-0.0038}$	0.0009	0.0005
	MMHT14	$0.1214^{+0.0026}_{-0.0025}$	0.0001	0.0018	0.0003	$^{+0.0018}_{-0.0015}$	0.0004	0.0002
	NNPDF3.0	$0.1260^{+0.0037}_{-0.0037}$	0.0001	0.0025	0.0005	$^{+0.0026}_{-0.0026}$	0.0006	0.0003
ATLAS	CT14	$0.1220^{+0.0044}_{-0.0048}$	0.0000	0.0032	0.0005	$^{+0.0028}_{-0.0035}$	0.0005	0.0002
Z (8 TeV)	HERAPDF2.0	$0.1177^{+0.0068}_{-0.0060}$	0.0000	0.0053	0.0008	$^{+0.0041}_{-0.0027}$	0.0008	0.0004
	MMHT14	$0.1207^{+0.0031}_{-0.0031}$	0.0000	0.0027	0.0004	$^{+0.0014}_{-0.0014}$	0.0004	0.0002
	NNPDF3.0	$0.1247^{+0.0047}_{-0.0047}$	0.0000	0.0038	0.0006	$^{+0.0026}_{-0.0026}$	0.0006	0.0003
ATLAS	CT14	$0.1194^{+0.0102}_{-0.0041}$	0.0003	0.0027	0.0025	$^{+0.0094}_{-0.0009}$	0.0014	0.0008
W^+ (13 TeV)	HERAPDF2.0	$0.1102^{+0.0093}_{-0.0108}$	0.0005	0.0048	0.0043	$^{+0.0059}_{-0.0082}$	0.0024	0.0017
	MMHT14	$0.1208^{+0.0046}_{-0.0113}$	0.0003	0.0031	0.0028	$^{+0.0006}_{-0.0104}$	0.0015	0.0011
	NNPDF3.0	$0.1215^{+0.0042}_{-0.0042}$	0.0002	0.0023	0.0021	$^{+0.0024}_{-0.0024}$	0.0012	0.0009
ATLAS	CT14	$0.1213^{+0.0045}_{-0.0045}$	0.0003	0.0020	0.0020	$^{+0.0032}_{-0.0033}$	0.0009	0.0005
W^- (13 TeV)	HERAPDF2.0	$0.1149^{+0.0107}_{-0.0111}$	0.0009	0.0062	0.0062	$^{+0.0051}_{-0.0058}$	0.0029	0.0014
	MMHT14	$0.1207^{+0.0031}_{-0.0067}$	0.0003	0.0020	0.0020	$^{+0.0005}_{-0.0060}$	0.0009	0.0004
	NNPDF3.0	$0.1248^{+0.0050}_{-0.0050}$	0.0004	0.0027	0.0027	$^{+0.0029}_{-0.0029}$	0.0012	0.0007
ATLAS	CT14	$0.1222^{+0.0044}_{-0.0038}$	0.0004	0.0022	0.0008	$^{+0.0037}_{-0.0028}$	0.0007	0.0003
Z (13 TeV)	HERAPDF2.0	$0.1181^{+0.0065}_{-0.0059}$	0.0009	0.0046	0.0017	$^{+0.0040}_{-0.0028}$	0.0014	0.0006
	MMHT14	$0.1209^{+0.0024}_{-0.0036}$	0.0004	0.0020	0.0007	$^{+0.0010}_{-0.0029}$	0.0006	0.0002
	NNPDF3.0	$0.1251^{+0.0040}_{-0.0040}$	0.0005	0.0026	0.0010	$^{+0.0027}_{-0.0027}$	0.0008	0.0003

- Dominant propagated $\alpha_s(m_Z)$ uncertainties: Luminosity (exp.) & PDF (theory)

Combination of α_s extractions per data-set & PDF-set

- Results are combined using Convino[1] (Neyman χ^2 prescription), taking into account correlations and asymmetric PDF uncertainties (method equivalent to BLUE for symmetric uncertainties).
- 6 uncertainty sources for each estimate considered: EXP: statistical, luminosity, systematic; TH: NNLO scale, PDF, and numerical.
- Uncertainty correlations:
  1. Luminosity at the same $\sqrt{s}$: 1 within one detector, 0. between detectors. 0 between different $\sqrt{s}$.
  2. Stat. uncert.: fully uncorrelated.
  3. Syst. uncert.: see next slide.
  4. TH scale uncert.: partially correlated.
  5. PDF uncertainty: partially correlated within each PDF set (see slides later).
  6. TH Numerical error: fully uncorrelated.

[1] J. Kieseler, Eur. Phys. J. C 77 (2017) 792

Experimental syst. uncertainties

- Typical (%) syst. uncertainties in W^+,W^-, Z x-section measurements (CMS exp.):

Measurement	W_e^+	W_e^-	Z_e	W_μ^+	W_μ^-	Z_μ
7 TeV						
Lepton reconstruction and identification	1.5	1.5	1.8	0.9	0.9	—
Muon trigger inefficiency	—	—	—	0.5	0.5	0.5
Energy scale and resolution	0.5	0.6	0.12	0.19	0.25	0.35
Missing p_T scale and resolution	0.3	0.3	—	0.2	0.2	—
Background subtraction and modelling	0.3	0.5	0.14	0.4	0.5	0.28
8 TeV						
Lepton reconstruction and identification	2.8	2.5	2.8	1.0	0.9	1.1
Energy scale and resolution	0.4	0.7	0.0	0.3	0.3	0.0
Missing p_T scale and resolution	0.8	0.7	—	0.5	0.5	—
Background subtraction and modelling	0.2	0.3	0.4	0.2	0.1	0.4

Table 7. Breakdown of the experimental systematic uncertainties (in percent) for each of the $W^\pm$ and Z boson production cross section measurements at 7 and 8 TeV [13, 14].

Exp. uncertainties correlation matrices

- Correlations among W^+,W^-, Z x-section uncertainties sources (CMS exp.):

The following 5 systematic uncertainty components are considered separately with their associated correlation matrices:

Uncertainty source	Correlated
1. Lepton reconstruction & identification	Per lepton
2. Trigger prefiring	Within μ
3. Energy / momentum scale & resolution	Per lepton
4. E_T^{miss} scale & resolution	Within W
5. Background subtraction / modeling	Within W, within Z

These assumption result in very strong correlations (0.9–1.0) of the systematic uncertainty within one lepton.

- Correlation matrices for the W^+,W^-, Z boson x-sections per experiment:

ATLAS, pp at $\sqrt{s} = 7$ TeV

	W^+	W^-	Z
W^+	1.00	0.90	0.36
W^-	0.90	1.00	0.32
Z	0.36	0.32	1.00

LHCb, pp at $\sqrt{s} = 7$ TeV

	W^+	W^-	Z
W^+	1.00	0.56	0.54
W^-	0.56	1.00	0.48
Z	0.54	0.48	1.00

LHCb, pp at $\sqrt{s} = 8$ TeV

	W_e^+	W_e^-
W_e^+	1.00	0.89
W_e^-	0.89	1.00

LHCb, pp at $\sqrt{s} = 8$ TeV

	W_μ^+	W_μ^-	Z_μ
W_μ^+	1.00	0.83	0.61
W_μ^-	0.83	1.00	0.61
Z_μ	0.61	0.61	1.00

ATLAS, pp at $\sqrt{s} = 13$ TeV

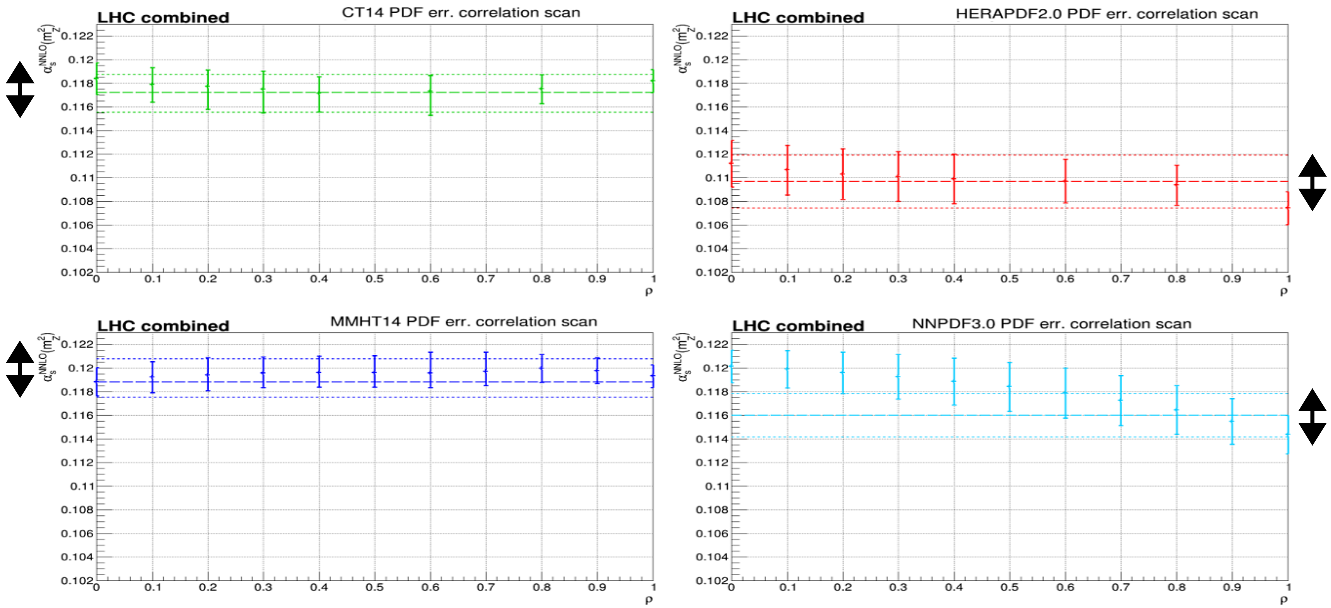
	W_μ^+	W_μ^-	Z_μ
W_μ^+	1.00	0.95	0.21
W_μ^-	0.95	1.00	0.20
Z_μ	0.21	0.20	1.00

Table 12. Correlation matrices among the systematic uncertainties of different W^+, W^-, and Z boson cross section measurements in ATLAS [7–9] and LHCb [10–13] as derived, in some cases using eq. (6.1), from the corresponding experimental studies.

Theoretical uncertainties correlations

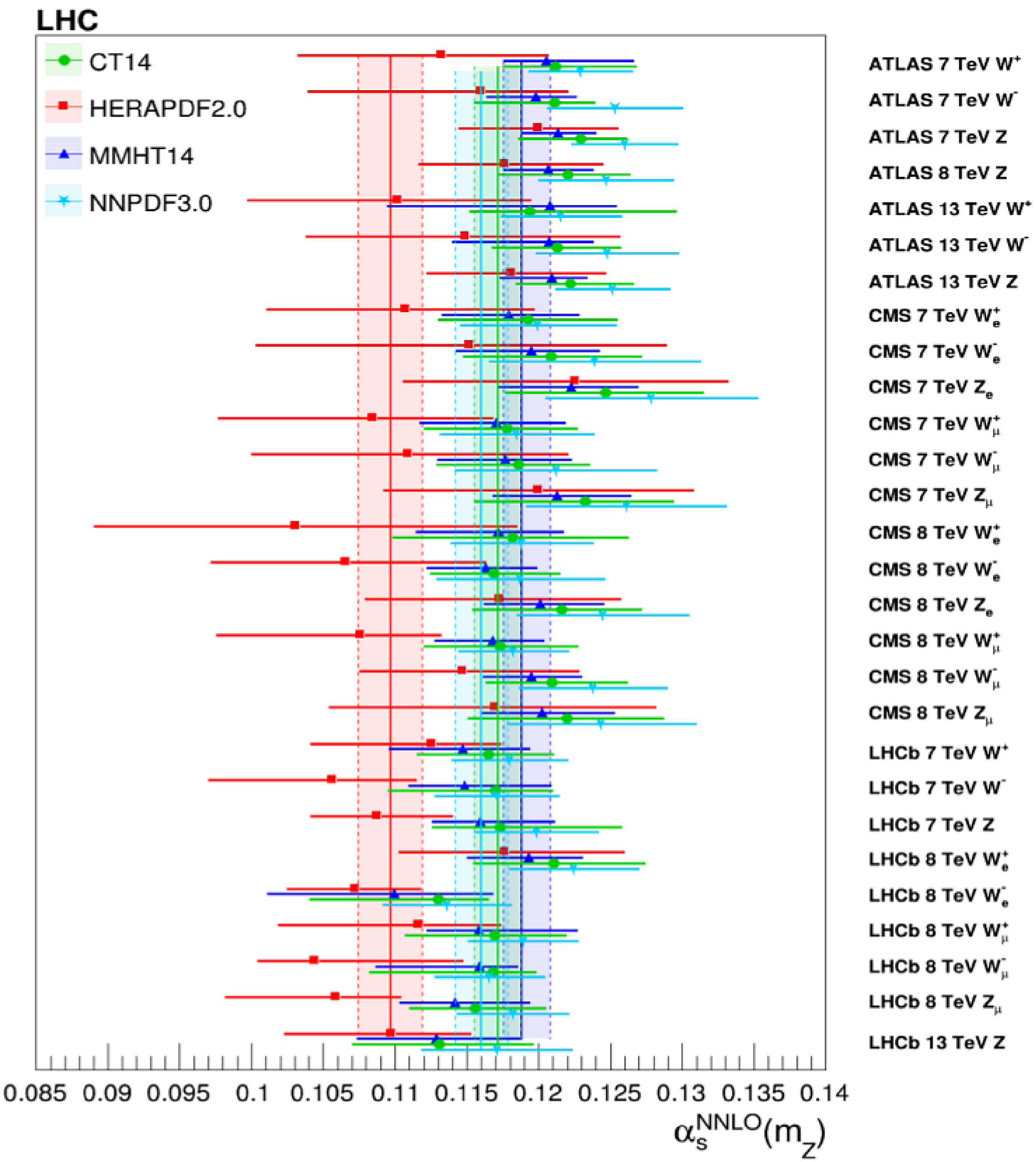
- Degree of correlation of PDF uncertainty for the $\alpha_s(m_Z)$ values extracted with the same PDF set is determined with Pearson correlation coefficients calculated from all x-sections computed with all individual eigenvectors/replicas for each pair of $W^{\pm}$ and Z measurements. Correlations for CT14, MMHT14 HERAPDF2.0, NNPDF3.0 are found to be 0.4–0.8, 0.1–0.6, 0.4–0.8, and 0.8–1.0, respectively. Stability of $\alpha_s(m_Z)$ wrt. assumed degree of correlation:

Pearson's correlat. range



- Correlations between scale uncertainty estimated for each pair of measurements also via Pearson correlation coefficients, vary over 0.0–0.8. When combining $\alpha_s(m_Z)$ estimates, each specific correlation coeff. for every specific pair of estimates is used.

Combined 28 α_s extractions & average α_s per PDF



PDF	$\alpha_s(m_Z)$
CT14	$0.1172^{+0.0015}_{-0.0017}$
HERAPDF2.0	$0.1097^{+0.0022}_{-0.0023}$
MMHT14	$0.1188^{+0.0019}_{-0.0013}$
NNPDF3.0	0.1160 ± 0.0018

- CT14, MMHT14 and NNPDF3.0 scatter around $\alpha_s(m_Z)$=0.117–0.119 world average with ~1% propag. uncertainties.
- HERAPDF2.0 is systematically below, $\alpha_s(m_Z)$~0.110, and ~2% propag. uncertainty

$\alpha_s(m_Z)$ robustness wrt. exp./theor. ingredients

- Stability of extracted $\alpha_s(m_Z)$ values per PDF cross-checked by varying key exp. & theor. ingredients & uncertainties:

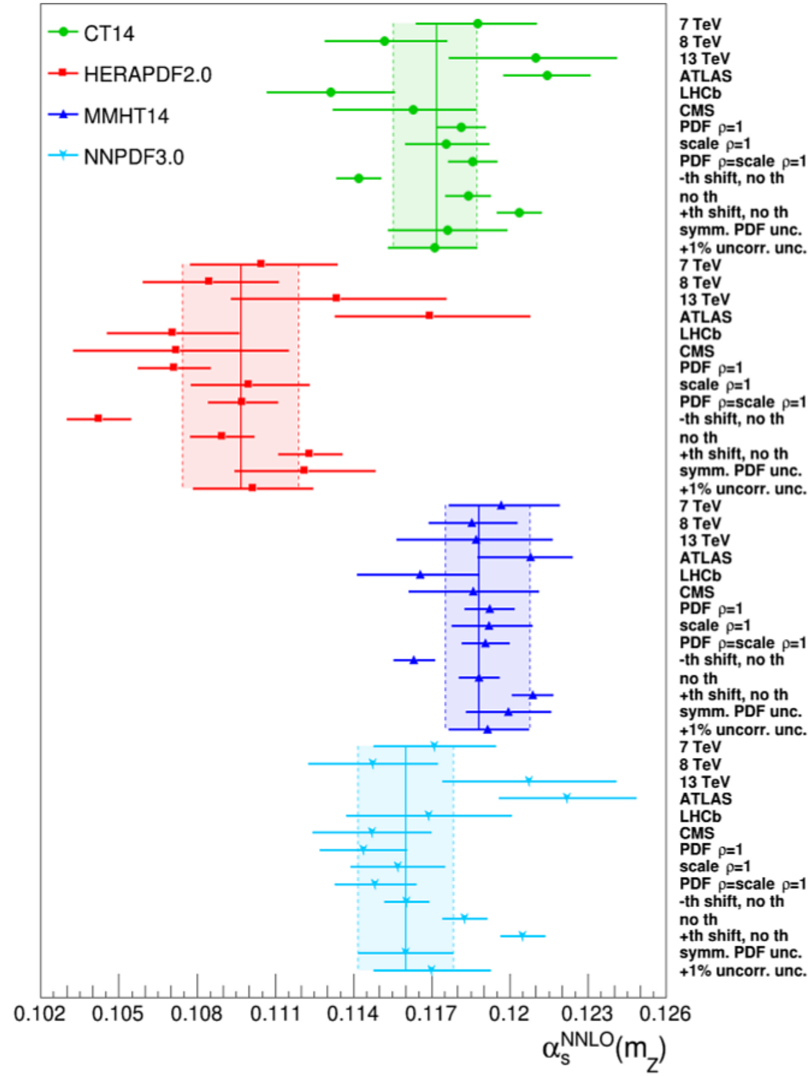
(i) Data subsets: ATLAS/CMS/LHCb or 7, 8, 13 TeV

(ii) Varying ρ=0–1 correlation of PDF/scale uncerts.

(iii) $\pm1\sigma$ TH shift of central values prior to combination

(iv) Adding $\pm1\%$ TH uncert. (diffs. among NNLO calc.)

PDF	$\alpha_s(m_Z)$	$\alpha_s(m_Z)$	$\alpha_s(m_Z)$
	[7 TeV data]	[8 TeV data]	[13 TeV data]
CT14	$0.1188^{+0.0022}_{-0.0024}$	$0.1152^{+0.0024}_{-0.0023}$	$0.1210^{+0.0031}_{-0.0033}$
HERAPDF2.0	$0.1105^{+0.0029}_{-0.0028}$	$0.1085^{+0.0027}_{-0.0025}$	$0.1134^{+0.0042}_{-0.0041}$
MMHT14	$0.1197^{+0.0022}_{-0.0020}$	0.1186 ± 0.0017	$0.1187^{+0.0029}_{-0.0030}$
NNPDF3.0	0.1171 ± 0.0023	0.1147 ± 0.025	0.1207 ± 0.0033
	[ATLAS data]	[LHCb data]	[CMS data] [28]
CT14	0.1214 ± 0.0017	0.1131 ± 0.0024	$0.1163^{+0.0024}_{-0.0031}$
HERAPDF2.0	$0.1170^{+0.0038}_{-0.0037}$	0.1071 ± 0.0021	$0.1072^{+0.0043}_{-0.0040}$
MMHT14	$0.1208^{+0.0016}_{-0.0020}$	$0.1166^{+0.0023}_{-0.0024}$	0.1186 ± 0.0025
NNPDF3.0	0.1222 ± 0.0026	0.1169 ± 0.0032	0.1147 ± 0.0023
	[PDF corr. = 1]	[scale corr. = 1]	[PDF corr. = scale corr. = 1]
CT14	0.1181 ± 0.0010	$0.1176^{+0.0017}_{-0.0016}$	0.1186 ± 0.0009
HERAPDF2.0	0.1071 ± 0.0014	$0.1100^{+0.0023}_{-0.0022}$	0.1098 ± 0.0013
MMHT14	0.1192 ± 0.0009	$0.1192^{+0.0016}_{-0.0014}$	0.1191 ± 0.0009
NNPDF3.0	0.1144 ± 0.0017	0.1157 ± 0.0018	0.1148 ± 0.0016
	[− th. shift, comb. w/o th. unc.]	[comb. w/o th. unc.]	[+ th. shift, comb. w/o th. unc.]
CT14	0.1142 ± 0.0009	0.1184 ± 0.0009	0.1203 ± 0.0009
HERAPDF2.0	0.1042 ± 0.0012	0.1090 ± 0.0012	0.1123 ± 0.00012
MMHT14	0.1163 ± 0.0008	0.1188 ± 0.0008	0.1209 ± 0.0008
NNPDF3.0	0.1160 ± 0.0009	0.1183 ± 0.0009	0.1205 ± 0.0009
	[symm. PDF uncert.]	[+1% uncorr. uncert.]	Largest differences
CT14	0.1176 ± 0.0023	$0.1171^{+0.0016}_{-0.0018}$	(+0.0042, −0.0041)
HERAPDF2.0	0.1121 ± 0.0027	0.1101 ± 0.0023	(+0.0073, −0.0026)
MMHT14	0.1200 ± 0.0016	$0.1191^{+0.0016}_{-0.0015}$	(+0.0020, −0.0022)
NNPDF3.0	0.1160 ± 0.0018	0.1170 ± 0.0022	(+0.0062, −0.0016)

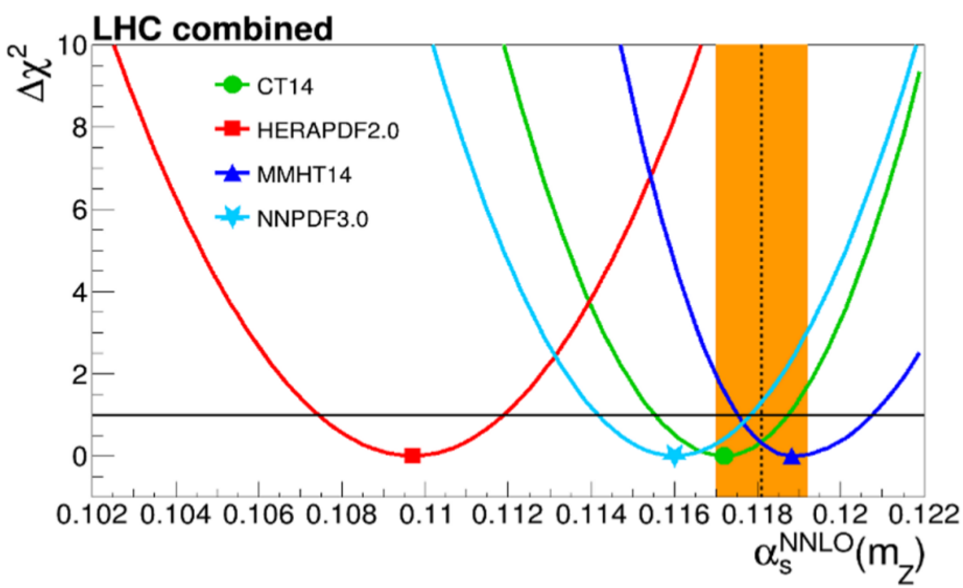


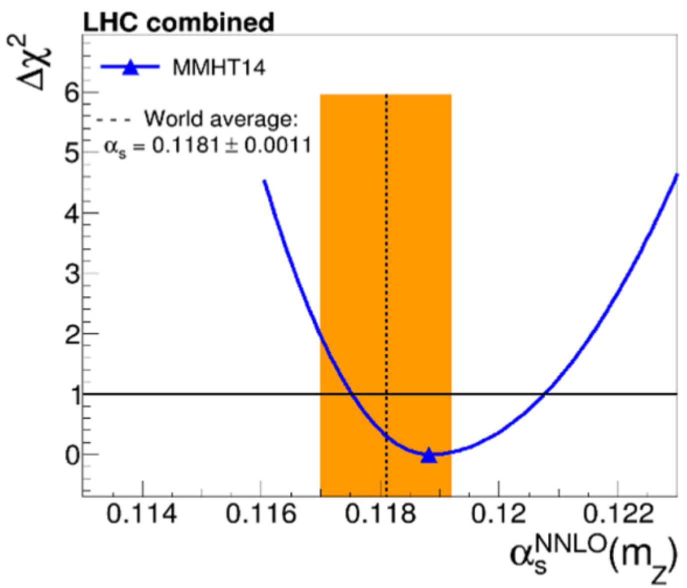
Final combined α_s extractions per PDF set

- Final $\alpha_s(m_Z)$ values per PDF with break-down of uncertainties & χ^2/ndf:

PDF	$\alpha_s(m_Z)$	δ (stat)	δ (lumi)	δ (syst)	δ(PDF)	δ(scale)	δ (num)	χ^2/ndf
CT14	$0.1172^{+0.0015}_{-0.0017}$	0.0003	0.0005	0.0006	$^{+0.0011}_{-0.0013}$	0.0006	0.0003	23.5/27
HERAPDF2.0	$0.1097^{+0.0022}_{-0.0023}$	0.0004	0.0009	0.0009	$^{+0.0015}_{-0.0016}$	0.0007	0.0005	27.0/27
MMHT14	$0.1188^{+0.0019}_{-0.0013}$	0.0002	0.0008	0.0003	$^{+0.0015}_{-0.0007}$	0.0007	0.0002	19.3/27
NNPDF3.0	0.1160 ± 0.0018	0.0006	0.0004	0.0005	0.0013	0.0006	0.0007	56.9/27

- Preferred $\alpha_s(m_Z)=0.1188\pm0.0016$ extraction from MMHT14:
  1) Largest sensitivity (slope) of σ(W,Z) to α_s
  2) Better χ^2/ndf~1 of combined α_s values.
  3) Lowest (symm.) propag. uncert.
  4) Most robust wrt. analysis variations



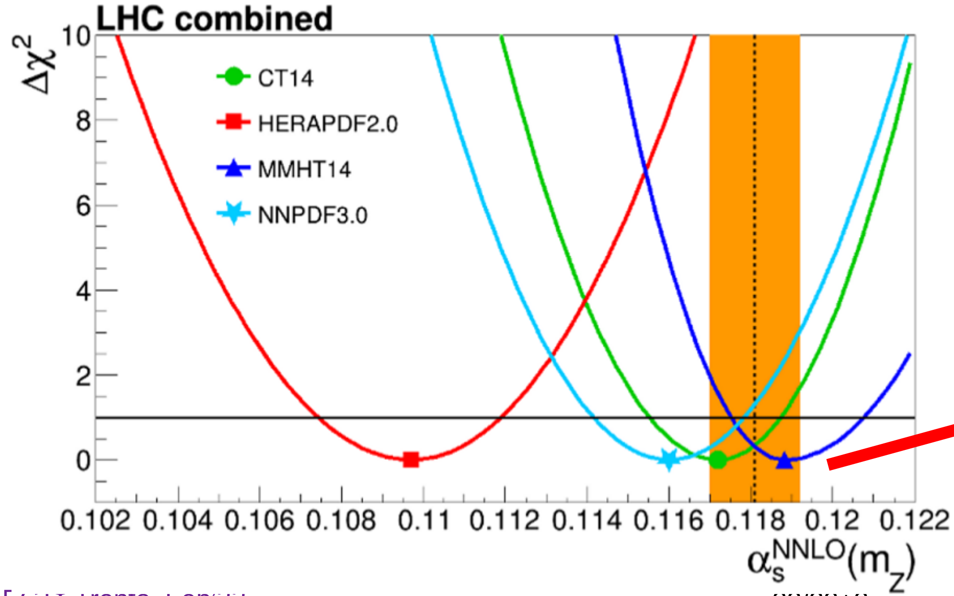


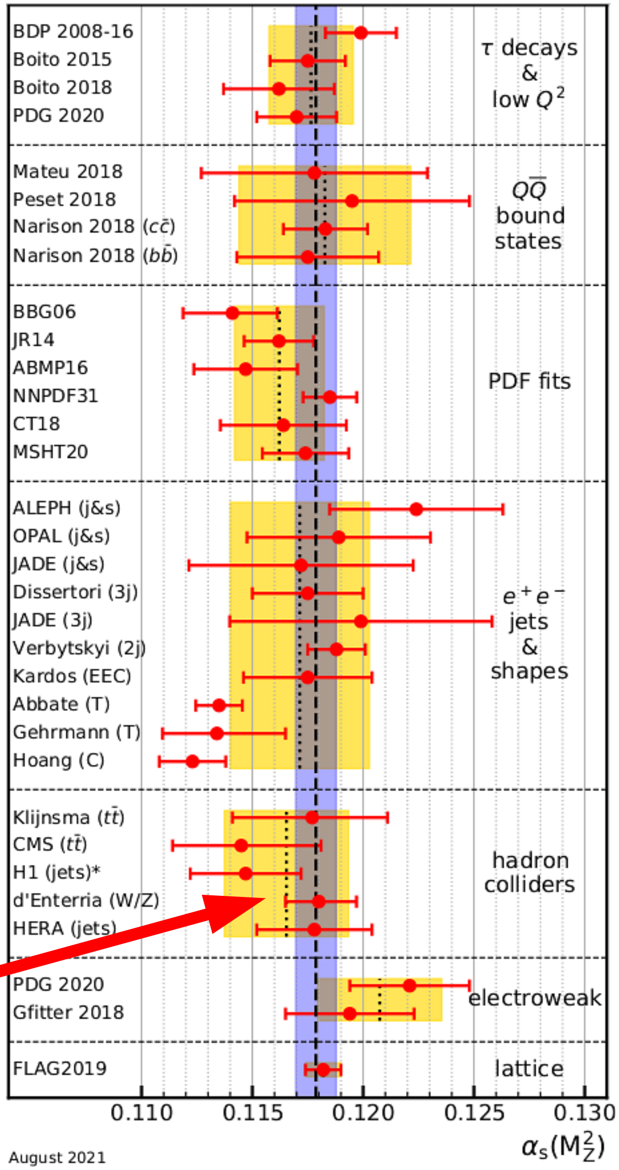
Summary

- New $\alpha_s(m_Z)$ extraction from high-precision inclusive EW boson x-sections at LHC (28 datasets up to 2019).
- 112 individual NNLO $\alpha_s(m_Z)$ combined per PDF accounting for propagated exp./theory uncertainties:

PDF	$\alpha_s(m_Z)$
CT14	$0.1172^{+0.0015}_{-0.0017}$
HERAPDF2.0	$0.1097^{+0.0022}_{-0.0023}$
MMHT14	$0.1188^{+0.0019}_{-0.0013}$
NNPDF3.0	0.1160 ± 0.0018

- $\alpha_s(m_Z)$=0.1188±0.0016 (MMHT14) incorporated to PDG:





Discussion

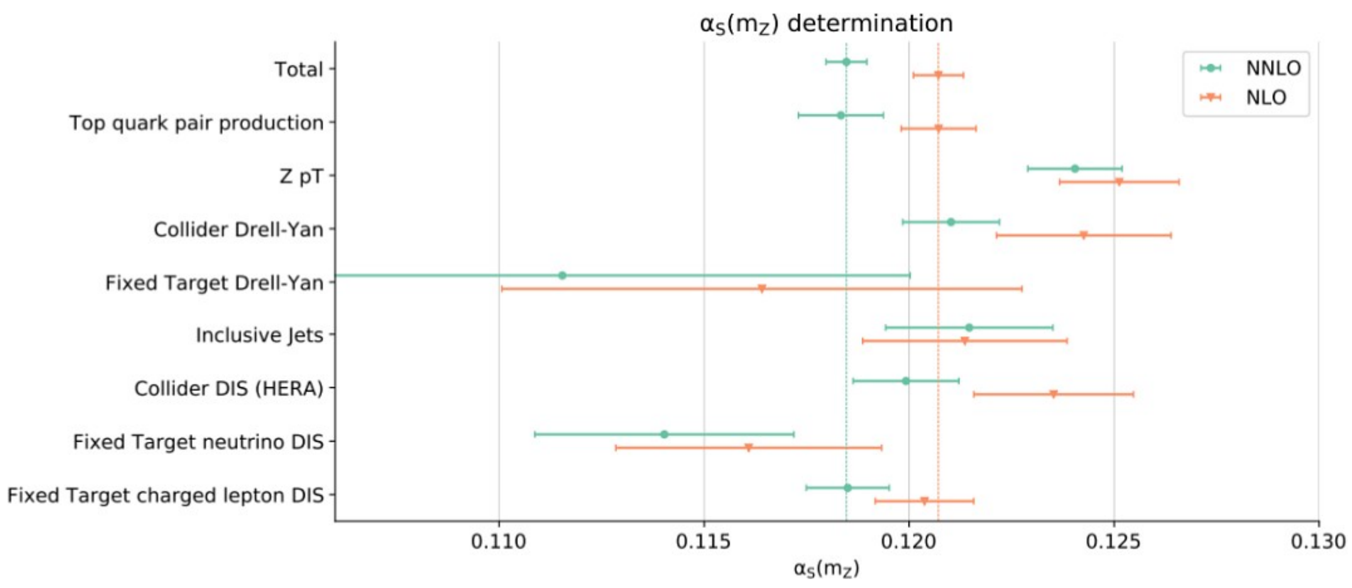
- The σ(W,Z)-based $\alpha_s(m_Z)$ extraction has been "criticized" by Forte & Kassabov (also J.Rojo today):

Why α_s Cannot be Determined from Hadronic Processes without Simultaneously Determining the Parton Distributions

2001.04986 [hep-ph]

Stefano Forte[1] and Zahari Kassabov[2]

0) Nobody(?) cares quantitatively about the "$\alpha_s(m_Z)$ preferred" by a given observable. We want to know "What would be the global $\alpha_s(m_Z)$ fitted by a given PDF global analysis if this observable was incorporated?"



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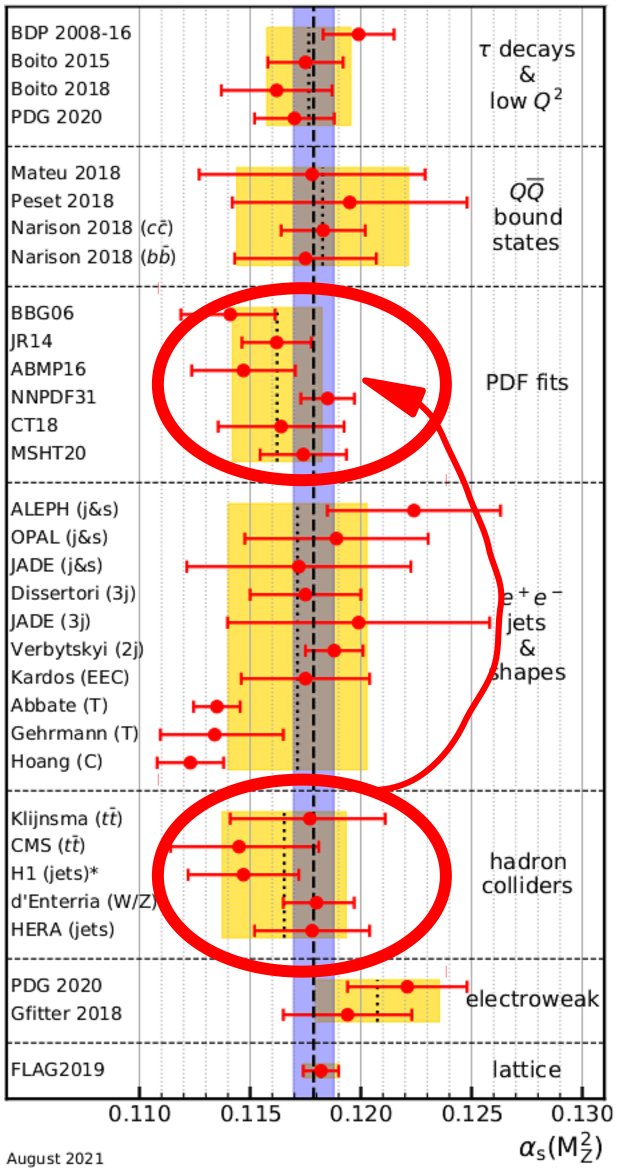
1) Doesn't this concern apply also to the σ(ttbar)-based and HERA jets-based $\alpha_s(m_Z)$ extractions?

2) The 28 (40+ now) inclusive σ(W,Z) measurements are NOT directly included today into the global PDF fits.

3) The global-PDF fits have an intrinsic theory (MHOU) uncertainty, so this independent $\alpha_s(m_Z)$ extraction provides a useful cross-check.

- Wouldn't it be more accurate/appropriate/consistent to incorporate the observables of the "hadron collider" PDG category into the PDF-fits and redo the $\alpha_s(m_Z)$ extraction? Can this be done? How?



Backup slides