A novel method for computing cross section ratios in perturbative QCD

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High Energy Physics - Phenomenology

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Perturbative QCD predictions in fixed order for cross section ratios

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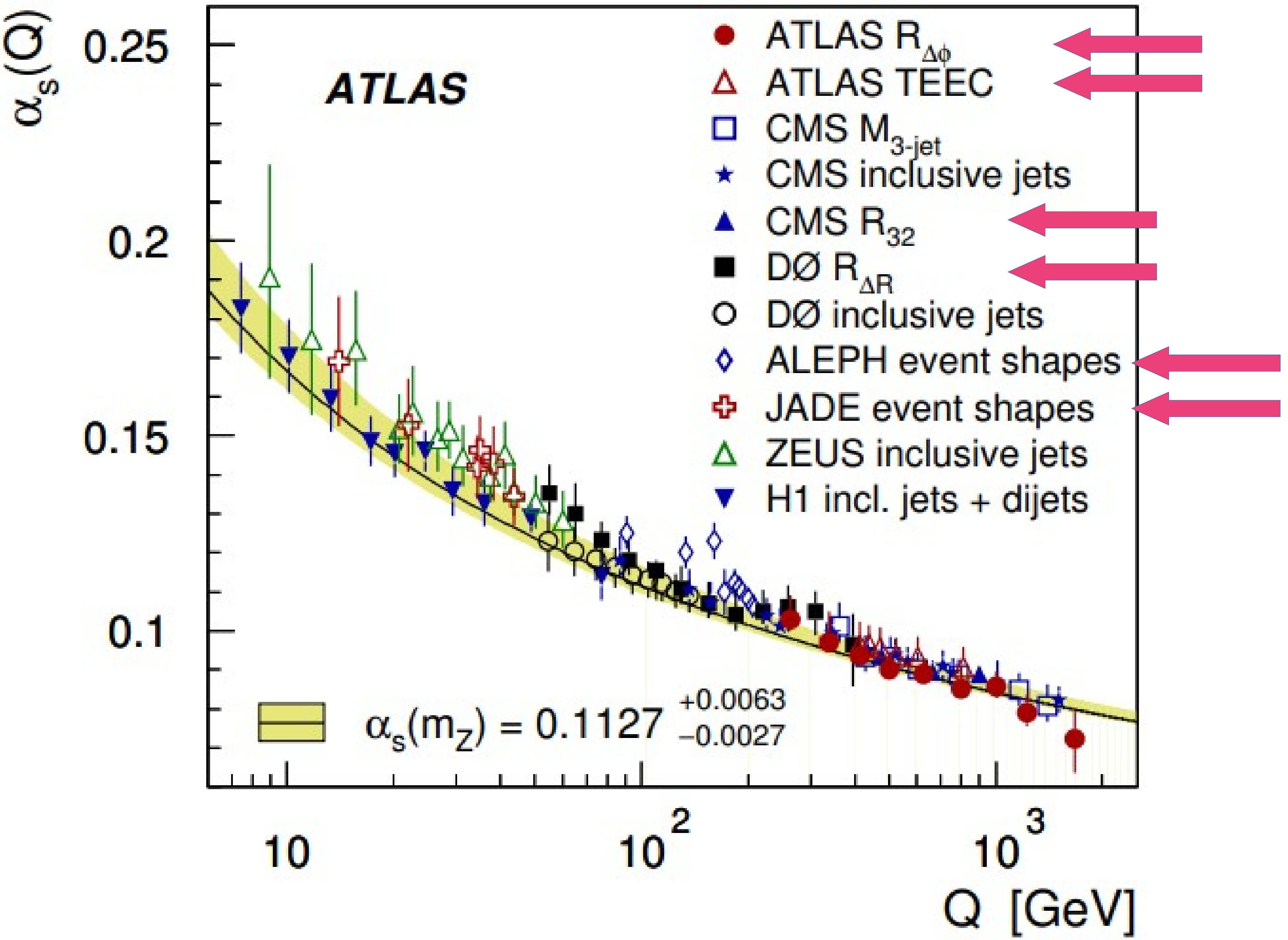
In the standard approach, predictions of perturbative Quantum Chromodynamics for ratios of cross sections are computed as the ratio of fixed-order predictions for the numerator and the denominator. Beyond the lowest order in the perturbative expansion, the result does, however, not correspond to a fixed-order prediction for the ratio. This article describes how exact fixed-order results for ratios of arbitrary cross sections can be obtained. The general method for computations in any order of the perturbative expansion is derived, and results for next-to-leading order and next-to-next-to-leading order calculations are given. The approach is applied to theory predictions for various multi-jet cross section ratios measured at hadron colliders. The two methods are compared with each other and with the experimental data. Recommendations are made how to obtain improved theory predictions with more realistic uncertainty estimates.

Cross Section Ratios → α_s



pQCD $\rightarrow$ "Standard" Method

Compute pQCD prediction for cross section ratio R as

$$R = \frac{\sigma_n \,\text{fixed-order}}{\sigma_d \,\text{fixed-order}}$$

$\rightarrow$ this is **NOT** a fixed-order result for the ratio R

This presentation:

- How to obtain an true fixed-order result for R
- Compare results from both methods for different quantities

Overview

- How to obtain fixed-order prediction for ratios
  - general method
  - formulae for NLO and NNLO pQCD

- Compare fixed-order vs. “standard” methods to five data sets of multi-jet cross section ratios at LHC and Tevatron: R_{32} $R_{\Delta R}$ $R_{\Delta\phi}$

- Identify systematic trends

- Recommendation for future pQCD predictions for x-sect ratios

Now: 5 slides with formulae ……. if you can’t follow, join us again on slide #11

Notation (1) ratio $\leftarrow\rightarrow$ cross sections

Example: R_{32} $n = 3$ $d = 2$

- Measurable quantity $R = \dfrac{\sigma_n}{\sigma_d}$

- Subscripts: n, d denote the order of α_s of the LO contribution to pQCD prediction of σ_n , σ_d $\sigma_{n,LO} \sim \alpha_s^n$ and $\sigma_{d,LO} \sim \alpha_s^d$

- Assume: $n \geq d$

- Assume: measure vs. **p** (energy or transverse momentum)
  $\rightarrow R(p) = \sigma_n(p) / \sigma_d(p)$
  such that μ_r = function(p) (**same** for both σ_n , σ_d)
  (from now on: don't denote the p-dependence)

Notation (2) “reduced k-factors”

- The pQCD predictions: $$\sigma_{\mathrm{n}} = \sum_{i=0}^{\infty} \sigma_{n,i} \qquad \text{and} \qquad \sigma_{\mathrm{d}} = \sum_{i=0}^{\infty} \sigma_{d,i}$$

  with: i=0 (LO) i=1 (NLO correct.) i=2 (NNLO correct.) …

- Typically: NLO k-factor $$k_{\mathrm{NLO}} = \frac{\sigma_{n,\mathrm{NLO}}}{\sigma_{n,\mathrm{LO}}} = \frac{\sigma_{n,0} + \sigma_{n,1}}{\sigma_{n,0}} = 1 + \frac{\sigma_{n,1}}{\sigma_{n,0}}$$
- Introduce new variable: $$k_{n,1} = \frac{\sigma_{n,1}}{\sigma_{n,0}} \quad \text{a “reduced NLO } k\text{-factor”}$$
- Extend “reduced k-factor” to all orders: $$k_{n,i} = \frac{\sigma_{n,i}}{\sigma_{n,0}} \quad \text{for} \quad i = 1, 2, 3, \cdots$$
- Then: $$\sigma_{\mathrm{n}} = \sigma_{n,0} \cdot (1 + k_{n,1} + k_{n,2} + \ldots)$$

  and: $$\sigma_{\mathrm{d}} = \sigma_{d,0} \cdot (1 + k_{d,1} + k_{d,2} + \ldots)$$

Computation of ratio - "standard"

- The pQCD prediction for R:
  at LO → uniquely defined
  $$R_{\text{LO}} = \frac{\sigma_{n,\text{LO}}}{\sigma_{d,\text{LO}}} = \frac{\sigma_{n,0}}{\sigma_{d,0}}$$
- Beyond LO → not uniquely
  the "standard" method:
  $$R_{\text{NLO}} = \frac{\sigma_{n,\text{NLO}}}{\sigma_{d,\text{NLO}}} \qquad R_{\text{NNLO}} = \frac{\sigma_{n,\text{NNLO}}}{\sigma_{d,\text{NNLO}}}$$
  compute pQCD result from the ratio of the fixed-order results
  note: this is **NOT** a fixed-order result for the ratio
- Our goal: Obtain a fixed-order result for ratio R

$$R_{\text{fixed-order}} = R_0 + R_1 + R_2 + R_3 + \cdots$$

$$R = R_{\text{LO}} \cdot \left(1 + \sum_{i=1}^{i_{\max}} \alpha_s^i c_i\right)$$

Computation of ratio - at fixed order

Here: at NLO

- Start with: $$R_{\text{NLO}} = \frac{\sigma_{n,\text{NLO}}}{\sigma_{d,\text{NLO}}}$$
- Rewrite as: $$R_{\text{NLO}} = \frac{\sigma_{n,0} \cdot (1 + k_{n,1})}{\sigma_{d,0} \cdot (1 + k_{d,1})} = R_{\text{LO}} \cdot \frac{1 + k_{n,1}}{1 + k_{d,1}} = R_{\text{LO}} \cdot (1 + k_{n,1}) \cdot \left(\frac{1}{1 + k_{d,1}} \right)$$
- Right term
  → Taylor series $$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \cdots \quad \text{with} \quad x = k_{d,1}$$
- Multiply terms in parenthesis and terms from Taylor series & truncate
- Fixed-order result at NLO: $$R_{\text{NLO}} = R_{\text{LO}} \cdot (1 + k_{n,1} - k_{d,1})$$

$$R_{\text{LO}} = \frac{\sigma_{n,\text{LO}}}{\sigma_{d,\text{LO}}} \,, \qquad k_{n,1} = \frac{\sigma_{n,\text{NLO}} - \sigma_{n,\text{LO}}}{\sigma_{n,\text{LO}}} \,, \qquad k_{d,1} = \frac{\sigma_{d,\text{NLO}} - \sigma_{d,\text{LO}}}{\sigma_{n,\text{LO}}}$$

Ratio at fixed-order

- Fixed-order result at NNLO:

$$R_{\text{NNLO}} = R_{\text{LO}} \cdot [1 + (k_{n,1} - k_{d,1}) + (k_{n,2} - k_{d,2}) - k_{d,1}(k_{n,1} - k_{d,1})]$$

→ backup slides/paper: general method for arbitrary orders

Two Methods (here: at NLO)

"Standard" Method

$$R_{\text{standard}} = \frac{\sigma_{n,0} + \sigma_{n,1}}{\sigma_{d,0} + \sigma_{d,1}}$$

Fixed-order result

$$R_{\text{fixed-order}} = R_0 + R_1$$

Up to NLO, the two are identical → the difference is of higher order in α_s

$$R_{\text{standard}} = R_{\text{fixed-order}} + \text{higher orders}$$

Conclude:

Both of these methods are “on the same footing”

They both represent valid NLO predictions

The only differ in terms beyond NLO

None is “better” than the other

(very much like calculations with different renormalization scale choices)

Let’s see how they compare to each other – and to experimental data

- Five data sets: D0: R_{32} $R_{\Delta R}$ $R_{\Delta\phi}$ CMS R_{32} ATLAS $R_{\Delta\phi}$

Please note

All examples here:

Three-jet/two-jet production ratios and in NLO pQCD

But ...

The method can be used for ratios of arbitrary cross sections

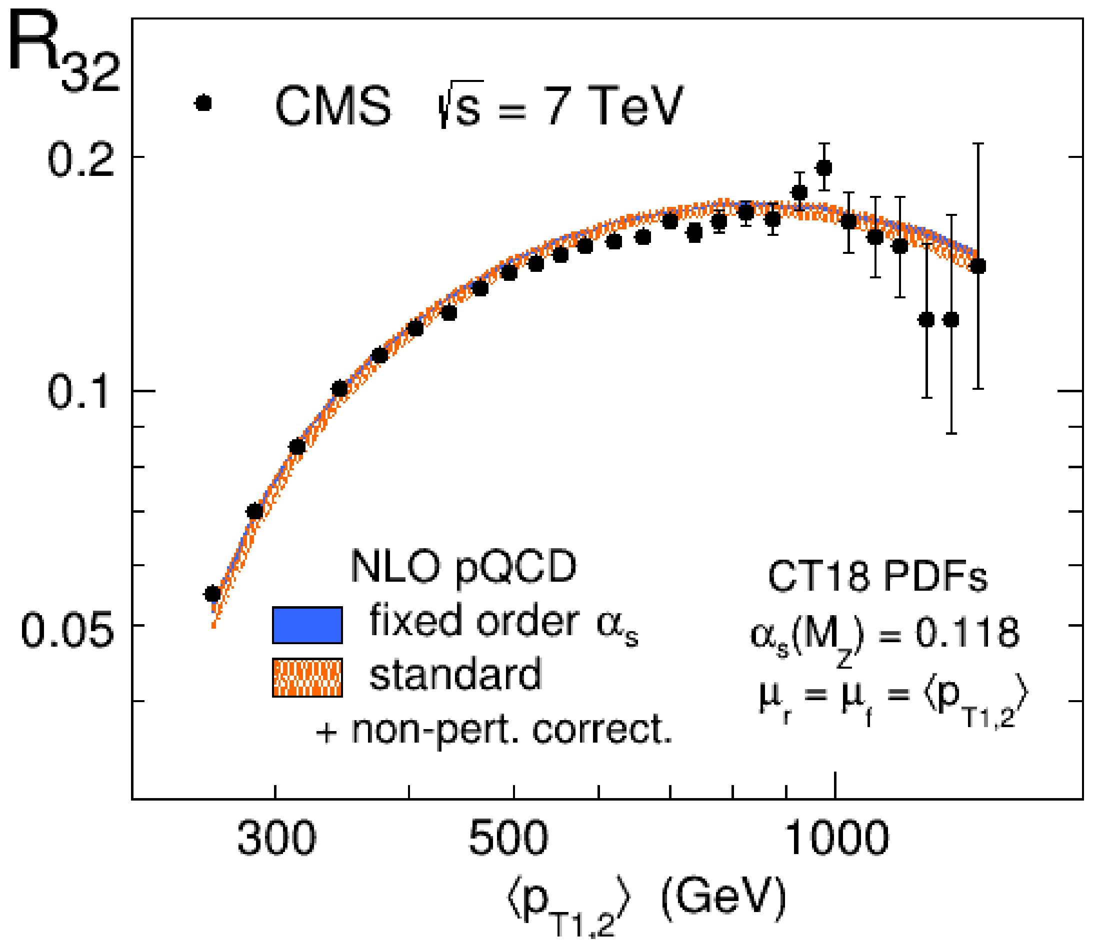
- where the LO contribution can be of **any power** in α_s

e.g.   Z+29 jets /   W+13 jets

It can be used in **any precision** in pQCD

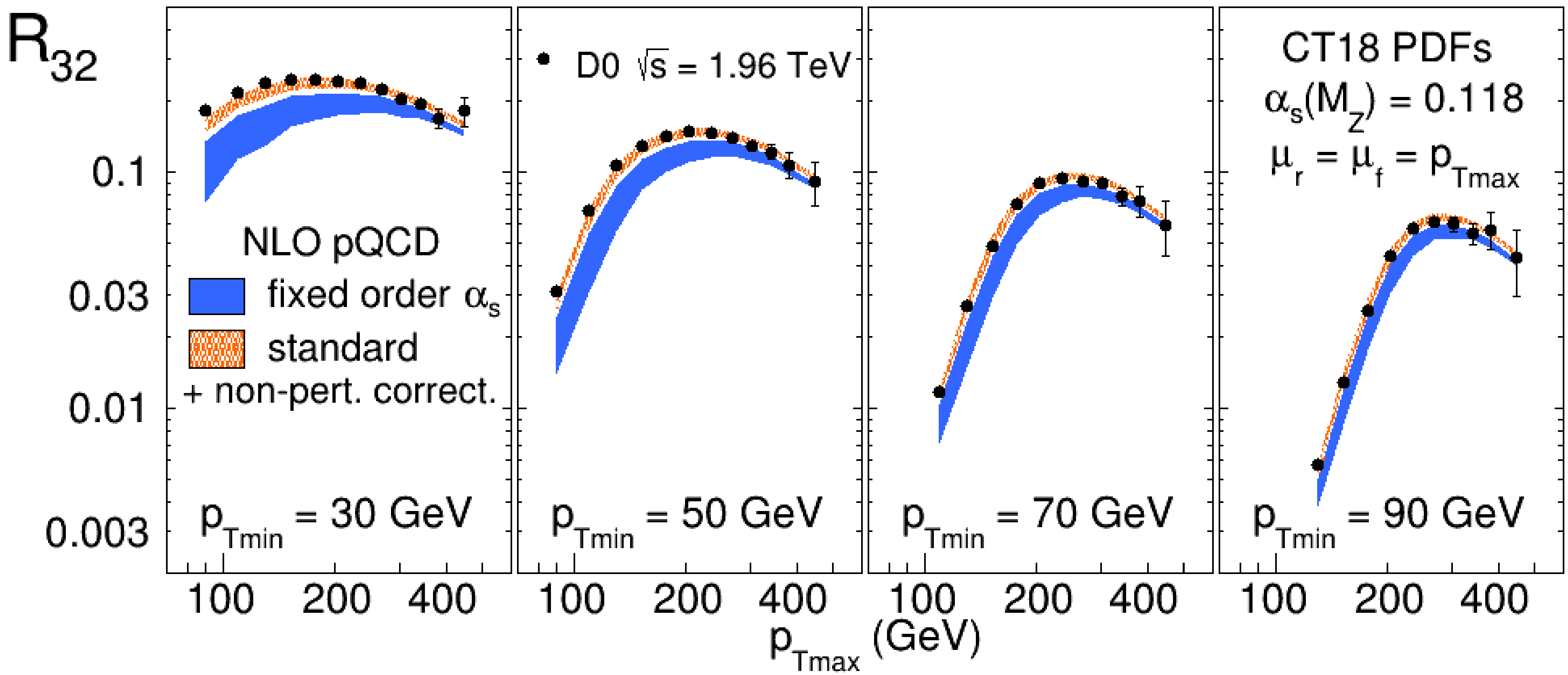
→ NLO, NNLO, ... , NNNNNNNLO, ...

CMS measurement of $R_{3/2}$ ($p_{T1,2}$)



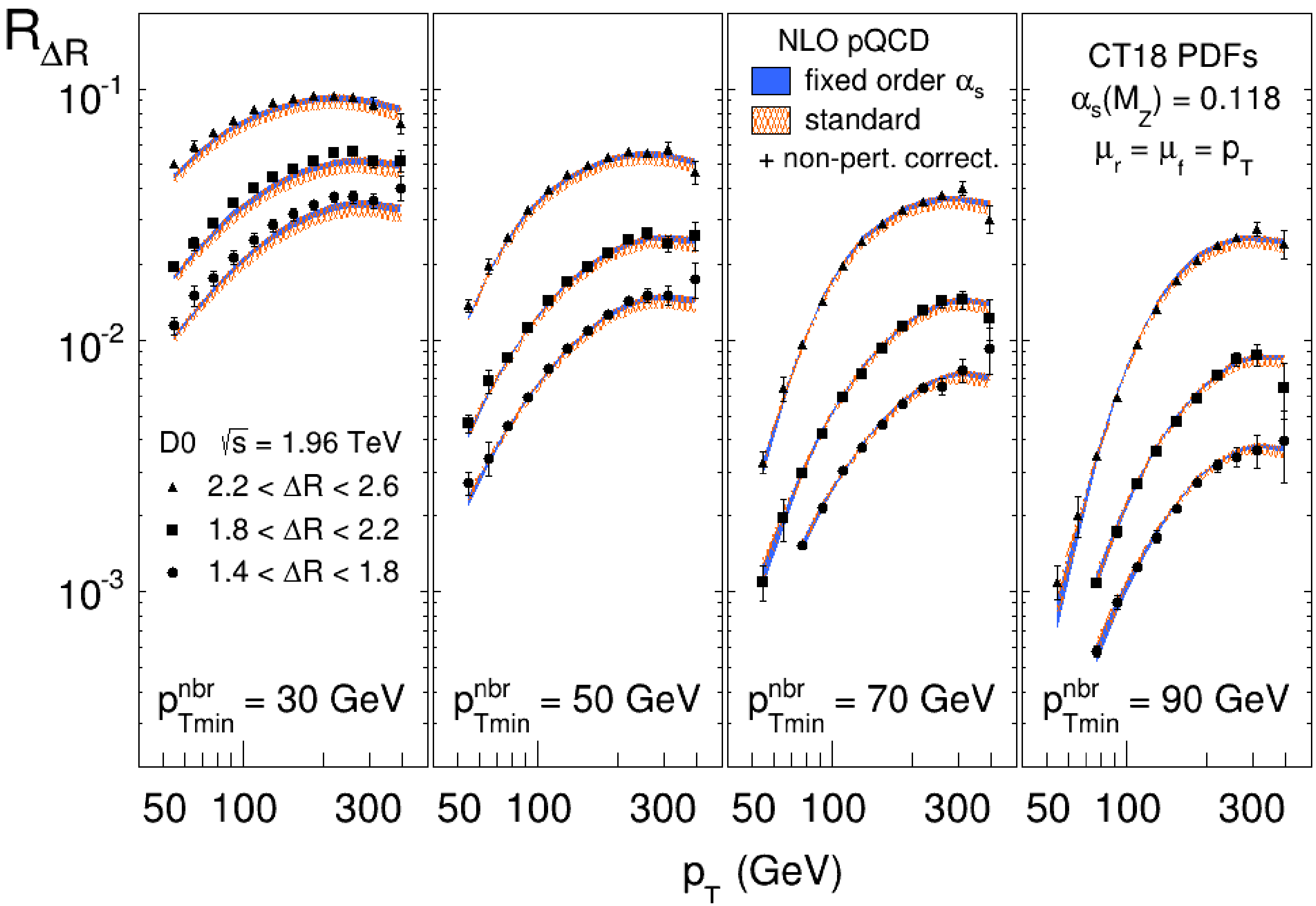
- perfect agreement between fixed-order and “standard” prediction
- fixed-order has much smaller scale dependence

D0 measurement of $R_{3/2}$ (p_{Tmax}, p_{Tmin})



- total disagreement between fixed-order and “standard” prediction
- “standard” method describes the data
- fixed-order has much larger scale dependence

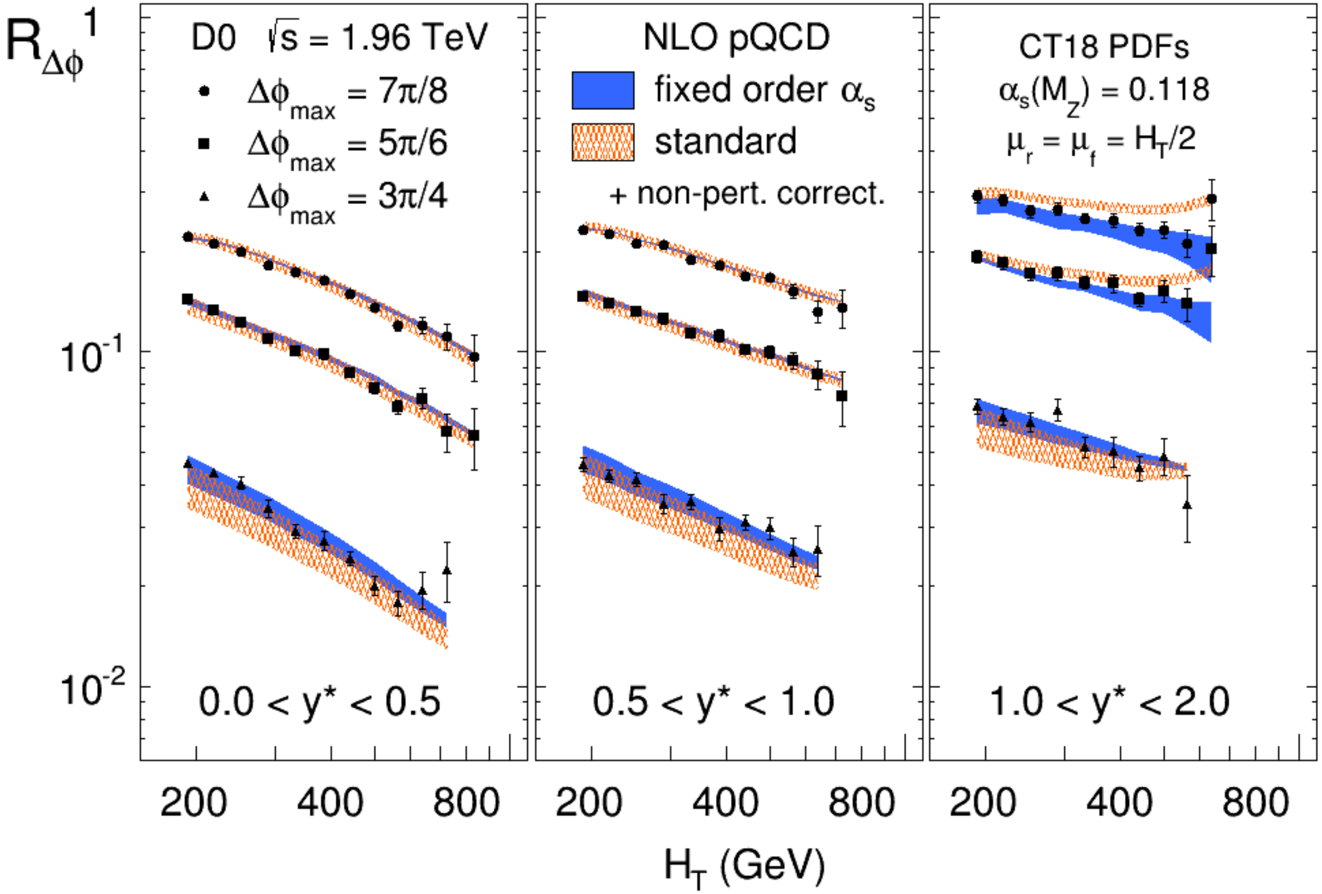
D0 measurement of $R_{\Delta R}$ (p_T, ΔR, $p_{T\,nbr}$)



- perfect agreement between fixed-order and "standard" prediction
- fixed-order has much smaller scale dependence

D0 measurement of $R_{\Delta\phi}$ (H_T, y^*, $\Delta\phi_{max}$)

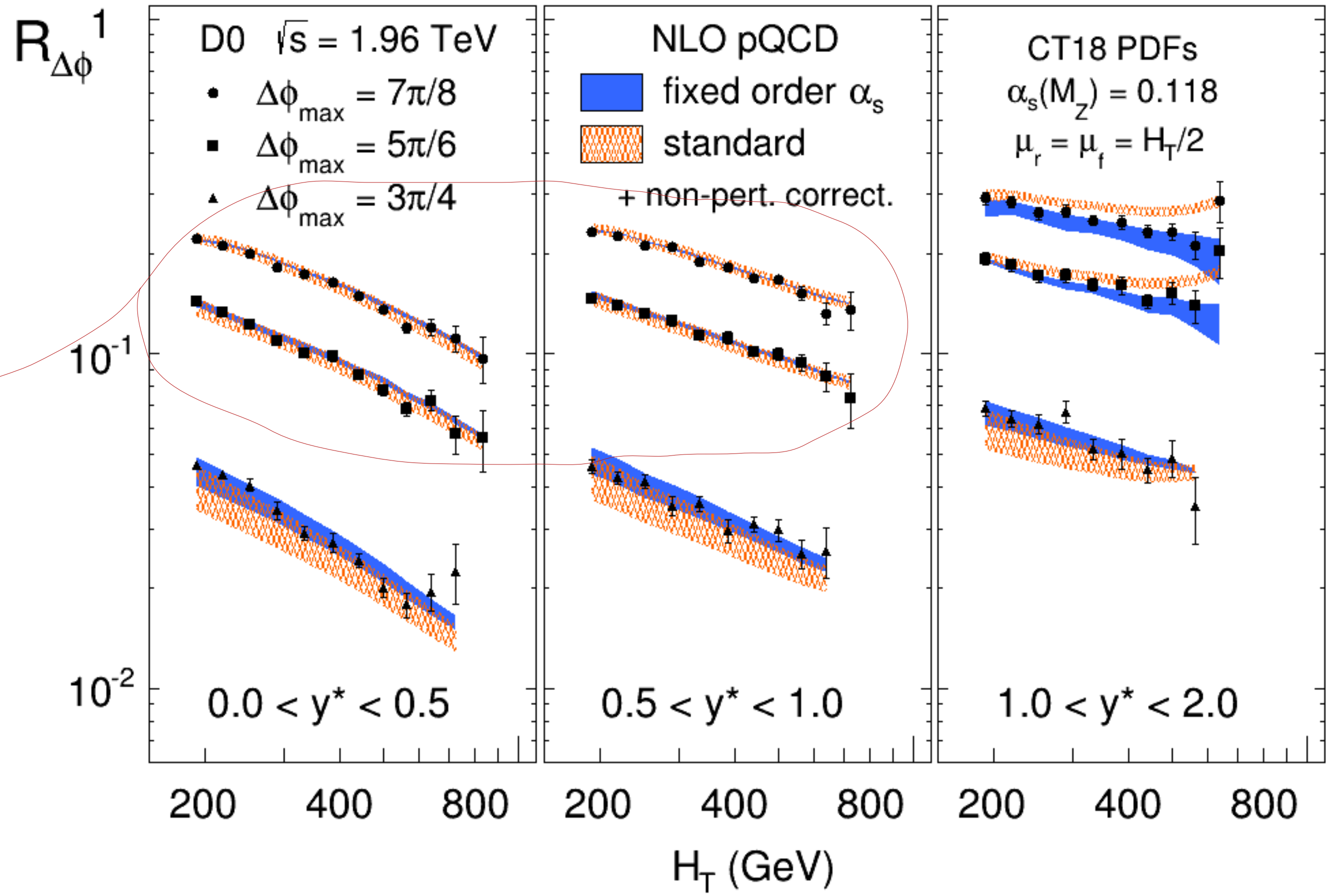
Three cases



D0 measurement of $R_{\Delta\phi}$ (H_T, y^*, $\Delta\phi_{max}$)

Three cases

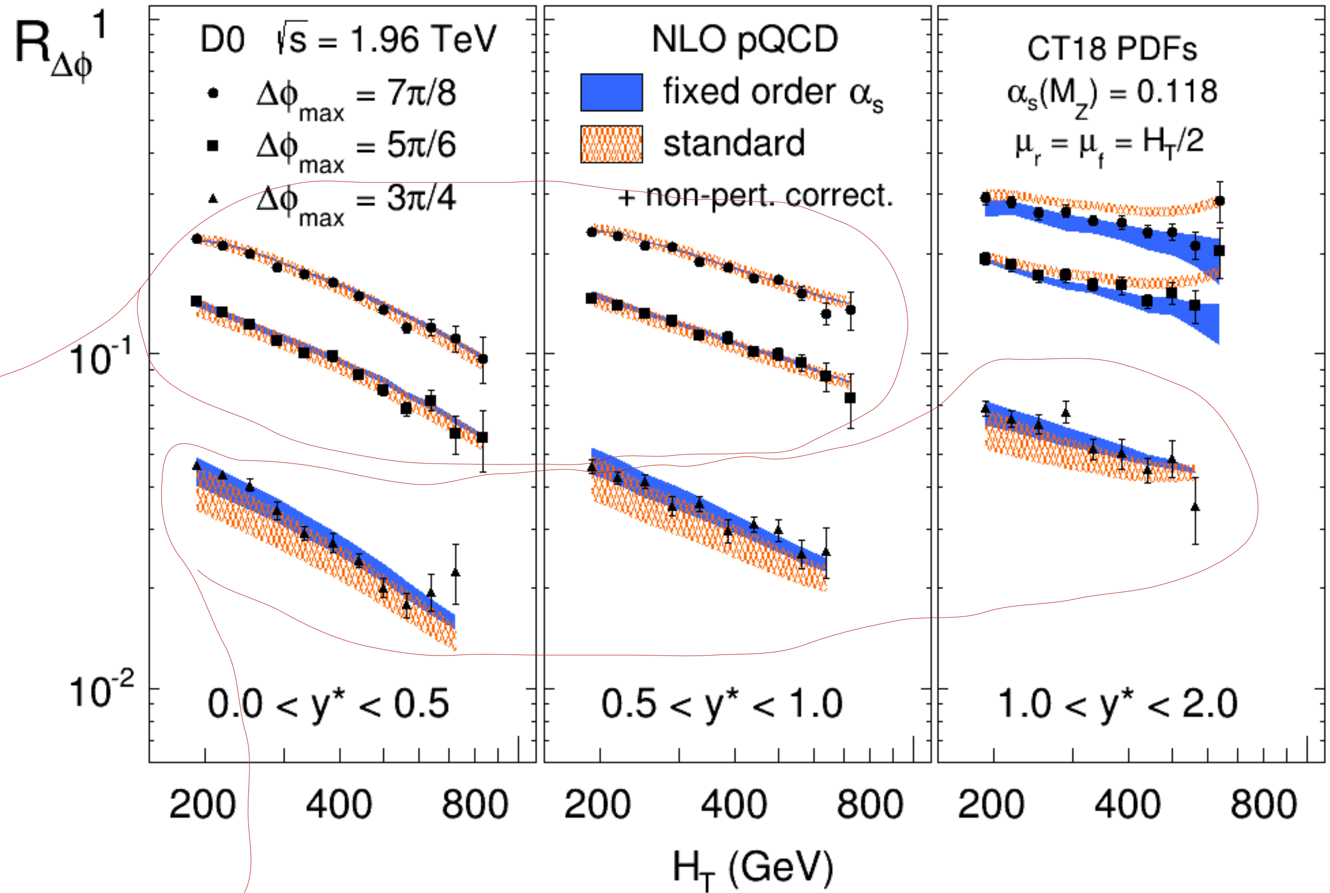
- perfect agreement between fixed-order and “standard” prediction
- fixed-order has much smaller scale dependence



D0 measurement of $R_{\Delta\phi}$ (H_T, y^*, $\Delta\phi_{max}$)

Three cases

- perfect agreement between fixed-order and “standard” prediction
- fixed-order has much smaller scale dependence

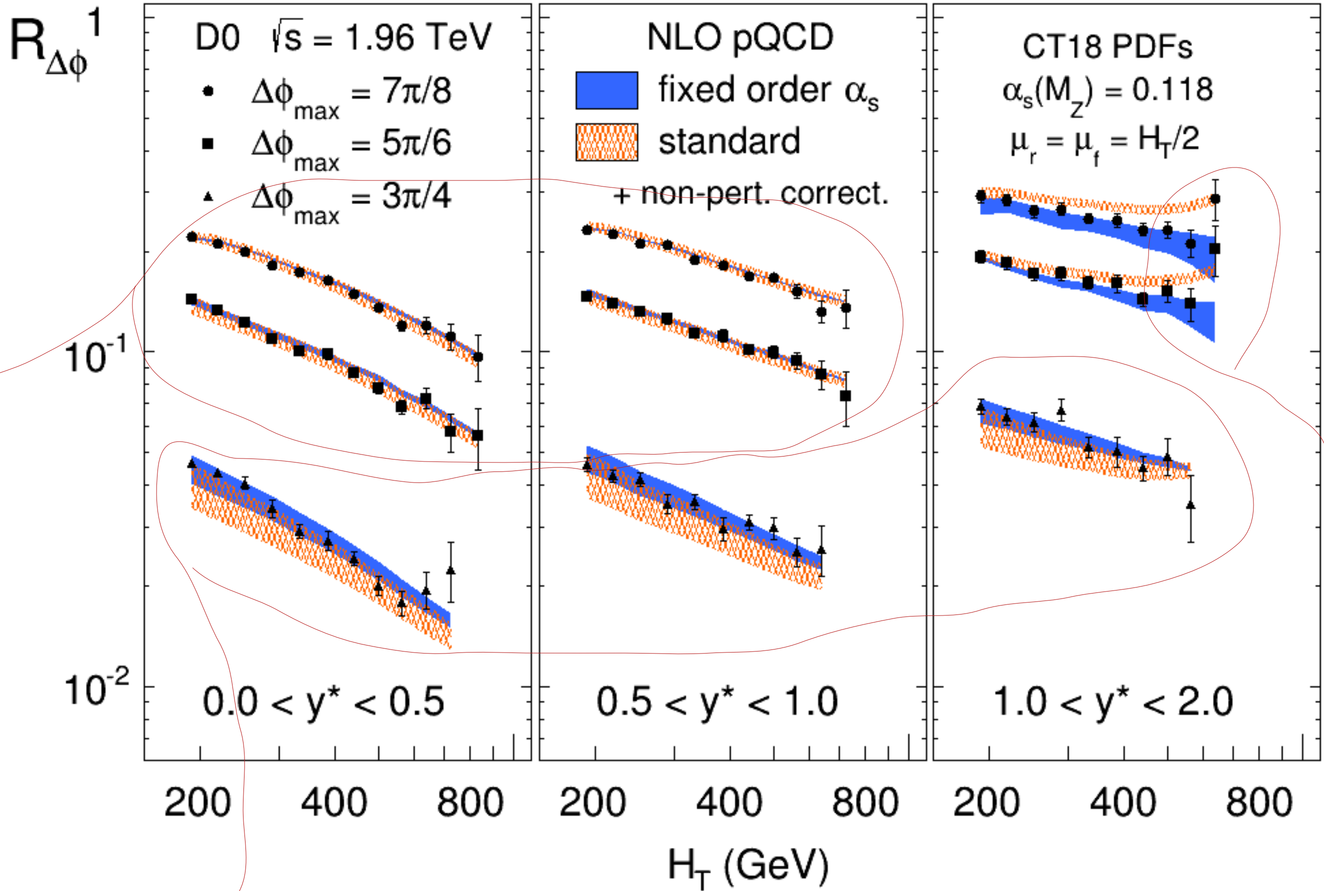


- partial disagreement
- both describes the data at the edges of their uncert.

D0 measurement of $R_{\Delta\phi}$ (H_T, y^*, $\Delta\phi_{max}$)

Three cases

- perfect agreement between fixed-order and “standard” prediction
- fixed-order has much smaller scale dependence

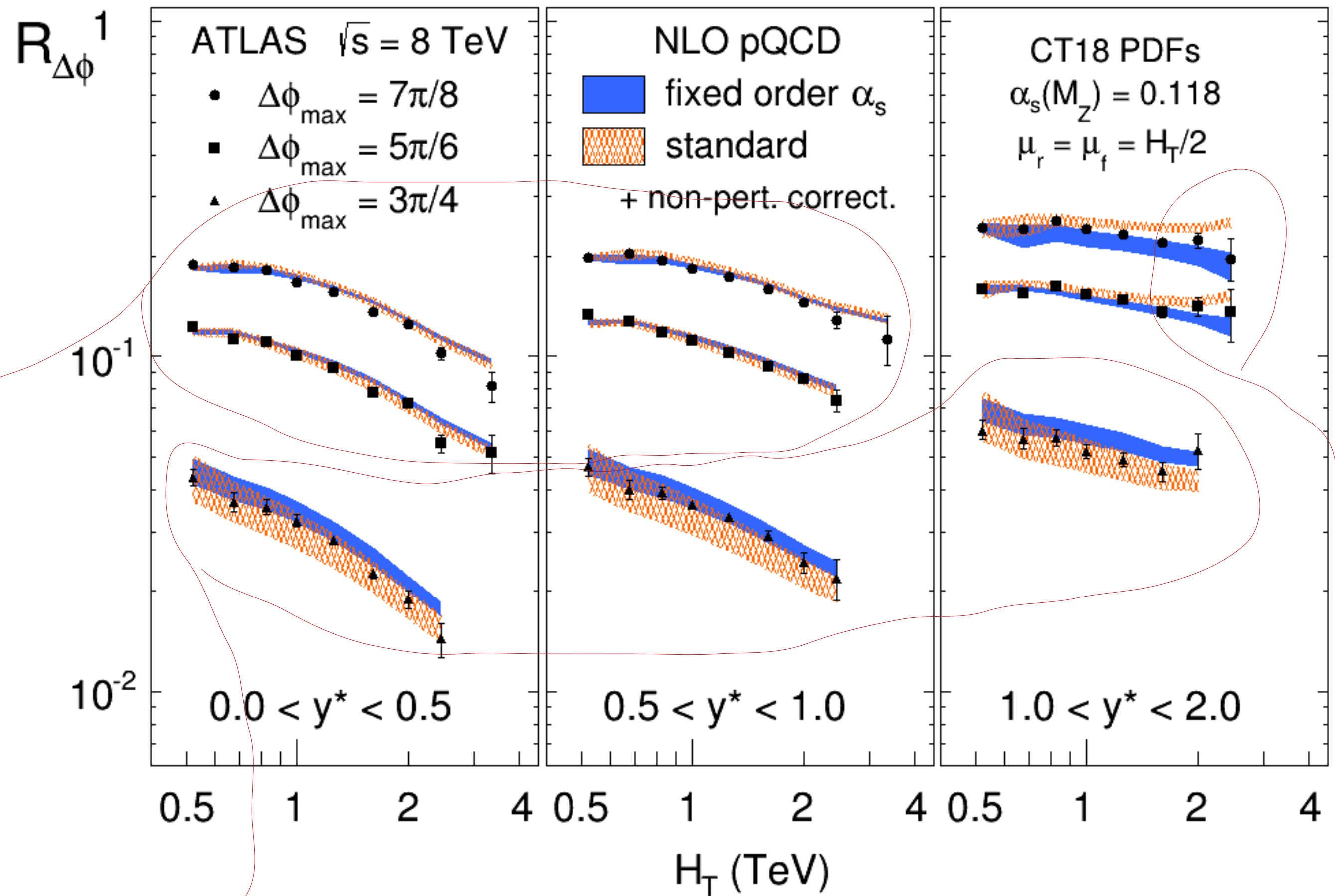


- partial disagreement
- both describes the data at the edges of their uncert.

- total disagreement
- fixed-order describes data
- fixed-order has much larger scale dependence

ATLAS measurement of $R_{\Delta\phi}$ (H_T, y^*, $\Delta\phi_{max}$)

Three cases



- perfect agreement between fixed-order and "standard" prediction
- fixed-order has much smaller scale dependence

- partial disagreement
- both describes the data at the edges of their uncert.

- total disagreement
- fixed-order describes data
- fixed-order has much larger scale dependence

All: Same as for D0 $R_{\Delta\phi}$

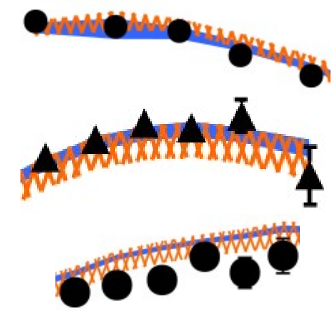
Summary: fixed-order vs. “standard”

Perfect agreement with each other

CMS R_{32} D0 $R_{\Delta R}$ and some kin. regions of D0 & ATLAS $R_{\Delta\phi}$

→ good agreement with data

→ fixed-order has a much smaller scale dependence

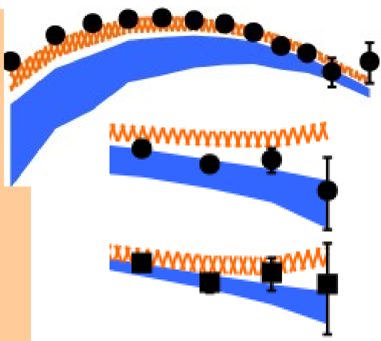


Disagree with each other

Dzero R_{32} and some kin. regions of D0 & ATLAS $R_{\Delta\phi}$

→ different ones agree with data

→ fixed-order has a much larger scale dependence

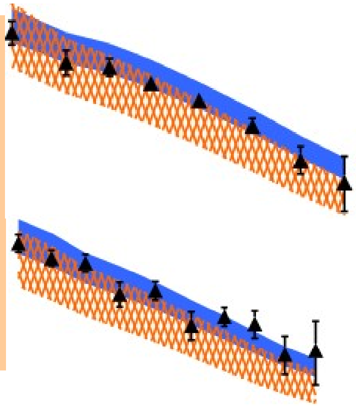


Agree “**on the edge**” of their uncertainties

some kinematic regions of D0 & ATLAS $R_{\Delta\phi}$

→ both (somehow) agree with data

→ both have slightly larger scale dependence



Reminder

None of the two predictions is “better” / “preferable”

→ both on the same footing!

→ Any difference must be treated as uncertainty

Our Recommendation

Compute both, the fixed-order and the “standard” pQCD predictions

→ Central value: maybe best chi^2 to data / by eye

→ Uncertainty: envelope of both bands

If the two methods agree:

Central values are anyway close → make no real difference

Uncertainty band is dominated by “standard” approach → no change

If they disagree

Choice of central value will improve agreement

Increased uncertainty band gives a more realistic estimate

→ Better agreement | Uncertainties more realistic :: larger only if “in trouble”

How would this affect the 5 analyses?

CMS R_{32} no change

D0 R_{32} same central values

significantly increased scale uncertainty

→ correctly addresses the large difference between methods

D0 $R_{\Delta R}$ no change

D0 & ATLAS $R_{\delta\phi}$

- no change in “central” analysis regions
- better description of data at large y*
- better description of data at small $\Delta\phi_{max}$

→ all good!!

Backup slides

General Method (at any order pQCD)

$$R_{\text{fixed-order}} = R_{\text{LO}} \cdot \left(1 + \sum_{i=1}^{i_{\max}} k_{n,i}\right) \cdot \frac{1}{1 + \sum_{i=1}^{i_{\max}} k_{d,i}} \,, \tag{2.11}$$

and expand the right term in a Taylor series

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \cdots \quad \text{with} \quad x = \sum_{i=1}^{i_{\max}} k_{d,i} \,. \tag{2.12}$$

The terms of this series are then multiplied with the terms in parenthesis in (2.11). The resulting terms are then sorted in their powers of α_s and the infinite series is truncated at the corresponding order at which σ_{n} and σ_{d} were computed.[1] This can be done at any order pQCD. To compute a corresponding fixed-order pQCD result for R requires, of course, to compute both σ_{n} and σ_{d} at the same corresponding relative order in α_s (e.g. NLO, NNLO, ...).

[1] For the sorting, it is helpful to notice that the $k_{n,i}$ and $k_{d,i}$ are both proportional to α_s^i, which follows directly from their definition in (2.3).

Estimates of Higher-Order Contributions

$$\sigma_n(\mu_1) = \sigma_n(\mu_2) + \text{higher order}$$

$$R_{n\,\text{new}} = R_{n\,\text{old}} + \text{higher orders}$$

Comparing the two methods

- "**Standard**" method: $$R_{\mathrm{NLO}} = \frac{\sigma_{n,\mathrm{NLO}}}{\sigma_{d,\mathrm{NLO}}}$$ → ratio of fixed orders

  or: $$R_{\mathrm{NLO}} = R_{\mathrm{LO}} \cdot \frac{1 + k_{n,1}}{1 + k_{d,1}}$$

- **Fixed-order** for ratio: $$R_{\mathrm{NLO}} = R_{\mathrm{LO}} \cdot (1 + k_{n,1} - k_{d,1})$$

- The **difference** of the two is due to the truncation of the Taylor series
  → compute the leading term:

$$R_{\mathrm{NLO,fixed-order}} - R_{\mathrm{NLO,standard}} = R_{\mathrm{LO}} \cdot [k_{d,1} \cdot (k_{n,1} - k_{d,1})] + \text{higher orders}$$

→ difference depends on $k_{d,1}$ (k-factor of denominator)
and $(k_{n,1} – k_{d,1})$ (difference of the two k-factors)
→ not on $k_{n,1}$ alone!!