# Measurements of $\alpha_s$ from spin structure functions

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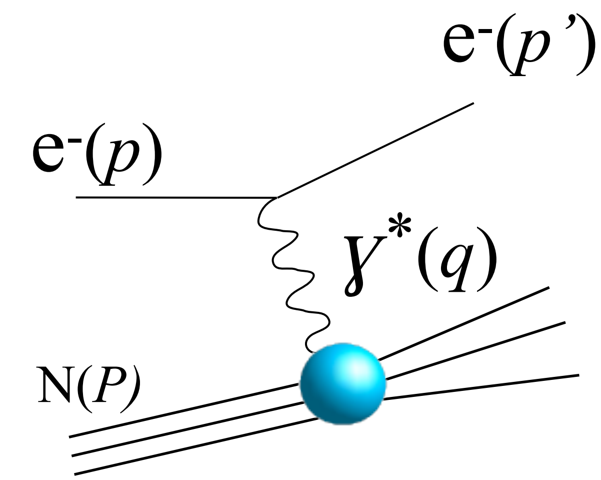
*Exploring the Nature of Matter*

# Reminder: inclusive lepton-nucleon scattering




- $p=(E,\boldsymbol{p})$, $p'=(E-\nu,\boldsymbol{p}-\boldsymbol{q})$, $q=(\nu,\boldsymbol{q})$
- $\gamma^*$ virtual photons: $q^2$
- Since $q^2<0$ here, we use $Q^2 = -q^2$.
- Inclusive experiments: only the scattered electrons are detected: target or target fragments are ignored.
- At high energy, Bjorken scaling variable $x= Q^2/2M\nu$ is more convenient than $\nu$.

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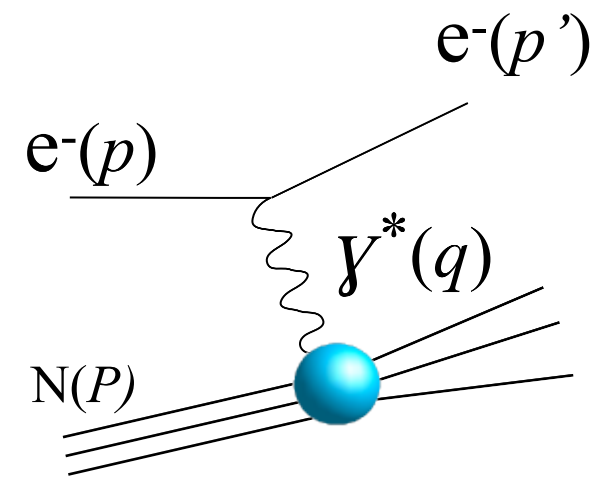


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Cross section: $\sigma = \sigma_{Mott}[\alpha F_1(x,Q^2)+ \beta F_2(x,Q^2)+ \gamma g_1(x,Q^2)+\varpi g_2(x,Q^2)]$

pointlike scattering×(spin independent + spin dependent)

## $F_1$, $F_2$, $g_1$ and $g_2$: structure functions

$F_1$ and $F_2$, are obtained with unpolarized beam and target and varying kinematic factors $\alpha$ and $\beta$.
$g_1$ and $g_2$ are obtained with beam and target both polarized, measuring beam spin asymmetries and varying the target spin direction.

Considering the nucleon inclusive spin structure, $\alpha_s$ can be extracted from:

- $Q^2$-evolution of $g_1(x, Q^2)$. Complex: involves DGLAP global fit, non-perturbative inputs: quark and gluon distributions, possibly higher-twists for low-$Q^2$ / large-$x$ data.

- $Q^2$-evolution of moment $\int_0^1 g_1(x, Q^2)dx$. Simpler: no $x$-dependence, non-perturbative inputs: more-or-less well measured axial charges $a_0$, $a_3$ and $a_8$ (+ possibly higher-twists for low-$Q^2$ data). Issues: unmeasurable low-$x$ contribution, $a_0$ is $Q^2$-dependent and may have contribution from gluon $\Delta G$ pdf (but not the case in $\overline{MS}$).

- $Q^2$-evolution of isovector moment $\int_0^1 g_1^{p-n}(x, Q^2)dx$, i.e Bjorken sum. Simplest. Axial charge $a_3 = g_A$ precisely measured ($g_A = 1.2762 \pm 0.0005$). DGLAP-evolution known to higher order than single nucleon case. No gluon contribution. But low-$x$ issue and demands measurement on polarized p and n.

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# Bjorken sum rule

$$\int g_1^{p-n}dx \equiv \Gamma_1^{p-n} = \frac{1}{6}g_A\left[1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s}{\pi}\right)^3 - 175.7\left(\frac{\alpha_s}{\pi}\right)^4 - \sim 893\left(\frac{\alpha_s}{\pi}\right)^5\right]$$

$$+\frac{M^2}{Q^2}\left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s)\right] + \ldots$$

Nucleon's First spin structure function

Nucleon axial charge. (Value of $\Gamma_1^{p-n}(Q^2)$ in the $Q^2 \to \infty$ limit)

pQCD radiative corrections ($\overline{MS}$ Scheme.)

Non-perturbative $1/Q^{2n}$ power corrections. (+rad. corr.)

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$\Rightarrow$ Two possibilities to extract $\alpha_s(M_Z)$:

- Do an absolute measurement of $\Gamma_1^{p-n}(Q^2)$ and solve the Bj SR for $\alpha_s(Q^2)$.
  - One $\alpha_s$ per $\Gamma_1^{p-n}$ experimental data point.
  - Poor systematic accuracy, typically $\Delta\alpha_s/\alpha_s$ ~10% $\Rightarrow$ Not competitive.
- Measurement of $Q^2$-dependence of $\Gamma_1^{p-n}(Q^2)$.
  - Need several $\Gamma_1^{p-n}$ points. Only one (or a few) value of $\alpha_s$.
  - Good accuracy: 1990's CERN/SLAC data yielded: $\alpha_s(M_Z)$=0.120±0.009

Altarelli, Ball, Forte, Ridolfi, Nucl.Phys. B496 337 (1997)

# $\alpha_s$ from $\Gamma_1^{p-n}(Q^2)$ measurement in JLab Hall B

AD *et al.* PRD 90, 012009 (2014)

EG1dvcs experiment:

- 6 GeV era of JLab.
- CLAS (CEBAF Large Acceptance Spectr.): 18-48$^o$ polar coverage, ~full azimuthal coverage.
- Polarized $NH_3$ and $ND_3$ fixed targets.
- Polarized beam.
- High inclusive statistics (DVCS process meas.): 6 months, 7 nA $\Rightarrow$ $2\times10^{17}$ $e^-$ on target.

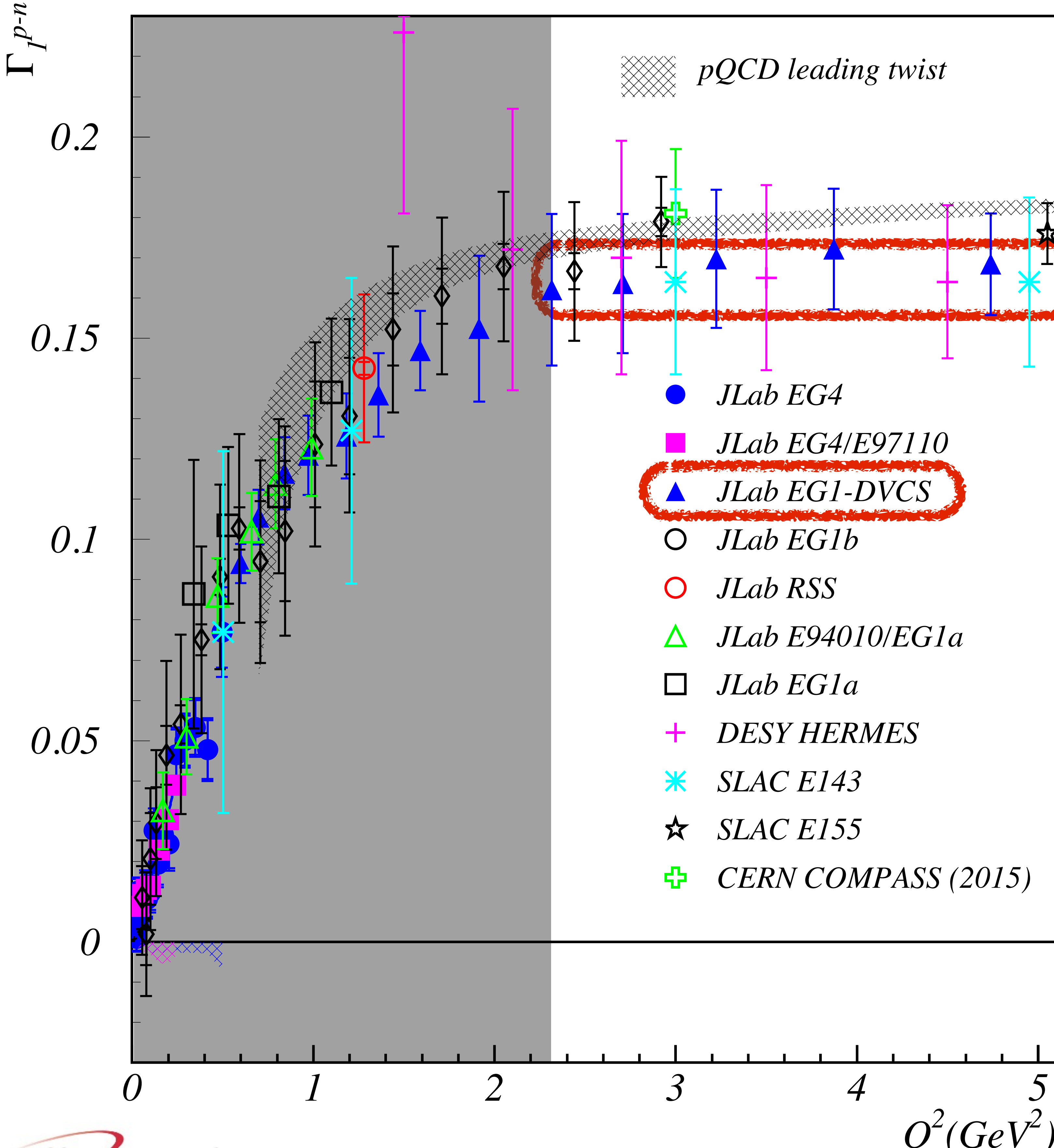
Used EG1dvcs data[1] to avoid uncorrelated systematics between experiments.

- Point-to-point correlated systematics (e.g. polarimetries, nuclear corrections) have minimal impact on uncertainties.
- EG1dvcs data largely dominate world data for statistics.
- Restricted $Q^2$ range: $2.32<Q^2<4.74$ GeV$^2$ rather than $2.<Q^2<10$ GeV$^2$.

[1] Not fully true: important missing low-x contribution estimated from models fitting world data.

| $Q^2$ (GeV$^2$) | $x-$range (p) | $x-$range (d) | $\Gamma^{p-n}_{1,meas}$ | $\Gamma^{p-n}_{1,meas+hi.x}$ | $\sigma^{syst}_{meas}$ | $\sigma^{syst}_{hi.x}$ | $\Gamma^{p-n}_{1,tot}$ | $\sigma^{syst}$ | $\sigma^{stat}$ | $\Gamma^{p-n}_{1,meas+hi.x}/\Gamma^{p-n}_{1.tot}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 2.316 | 0.263-0.864 | 0.271-0.798 | 0.0523 | 0.0515 | 0.0177 | 0.0001 | 0.1621 | 0.0188 | 0.0008 | 0.317 |
| 2.707 | 0.304-0.825 | 0.326-0.769 | 0.0398 | 0.0388 | 0.0157 | 0.0008 | 0.1636 | 0.0173 | 0.0006 | 0.237 |
| 3.223 | 0.362-0.901 | 0.385-0.799 | 0.0322 | 0.0311 | 0.0152 | 0.0000 | 0.1697 | 0.0171 | 0.0005 | 0.183 |
| 3.871 | 0.438-0.893 | 0.463-0.762 | 0.0227 | 0.0206 | 0.0121 | 0.0002 | 0.1721 | 0.0150 | 0.0004 | 0.120 |
| 4.739 | 0.531-0.909 | 0.663-0.738 | 0.0145 | 0.0113 | 0.0081 | 0.0002 | 0.1684 | 0.0126 | 0.0002 | 0.067 |

# Measurement at 6 GeV in JLab Hall B




EG1dvcs

Error bars: Systematic uncertainties (Polarimetries, low-x,…). Statistical uncertainties smaller than symbols

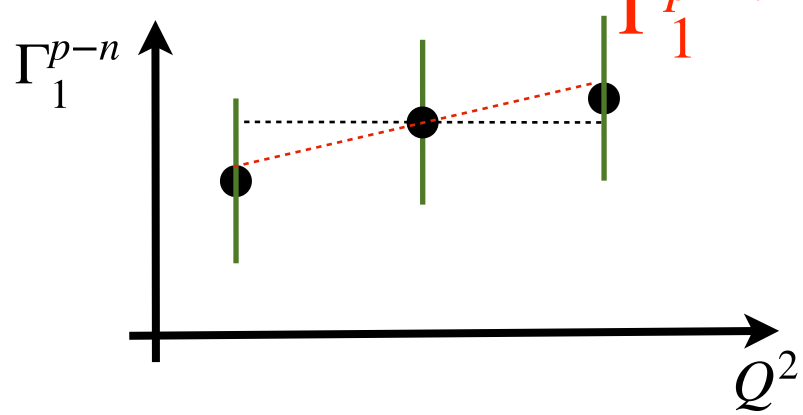
# Uncertainties

Experimental systematic uncertainty: Separate point-to-point correlated and point-to-point uncorrelated parts:

- Fit data with expected pQCD form.
- Use *Unbiased estimate* (i.e. force $\chi^2$=1) to assess point-to-point uncorrelated uncert.
- Point-to-point correlated determined so that correlated $\oplus$ uncorrelated = full experimental syst. uncertainties.

Low-$x$ systematic uncertainties: separate $Q^2$-dependent and independent parts.

Assume Q²-dependent part = $\frac{\text{low-x contribution}}{\Gamma_1^{p-n}} \frac{d\Gamma_1^{p-n}}{dQ^2} \times Q^2$-bin size:




Add the $Q^2$-dependent low-$x$ uncertainty to the point-to-point correlated experimental uncertainty. Use this sum for $\chi^2$ minimization of the fit that yields $\alpha_s$.

$\Rightarrow$ large parts of the low-$x$ and exp. syst. uncertainties are suppressed.

# Uncertainties

Leading uncertainties:

- Point-to-point uncorrelated uncertainties: 4.4%
- Point-to-point correlated uncertainties: 3.3%

$\boxed{5.5\%}$

Negligible uncertainties:

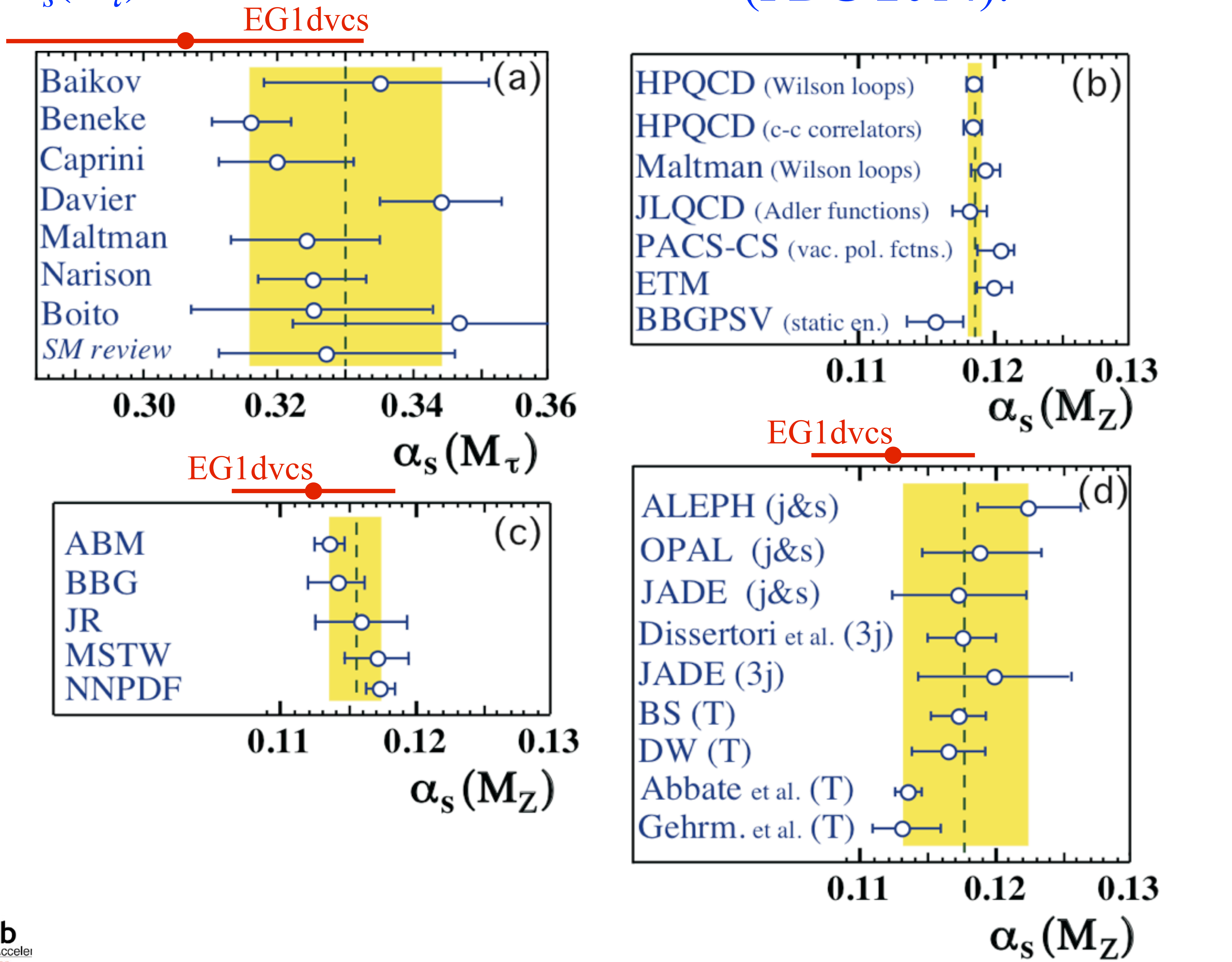
- Statistics.
- Twist-4 contributions: $\frac{M^2}{Q^2}\left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s)\right]$

  $a_2$ (PDF fits) $d_2$ (meas.) $f_2$ (Sidorov-Weiss model, with 50% uncertainty). Twist > 4 neglected.

  Uncertainties: < 0.1%.
- Bjorken twist-2 series truncation: 1.3%
- $\beta$-series truncation (needed it to evolve $\alpha_s$ to $M_Z$): 0.1%

# Result: $\alpha_s(M_Z)$=0.1123±0.0061 $\alpha_s(M_\tau)$=0.306±0.053

# Compared to (then) best world data (PDG 2014):

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# Possible future extractions of $\alpha_s$ from $\Gamma_1^{p-n}(Q^2)$

- JLab: approved experiment EG12 using CLAS12 with 11 GeV. Expected precision similar to EG1dvcs, but with $Q^2$ up to 6 GeV$^2$. Low-x issue reduced.

- Electron Ion Collider (EIC)

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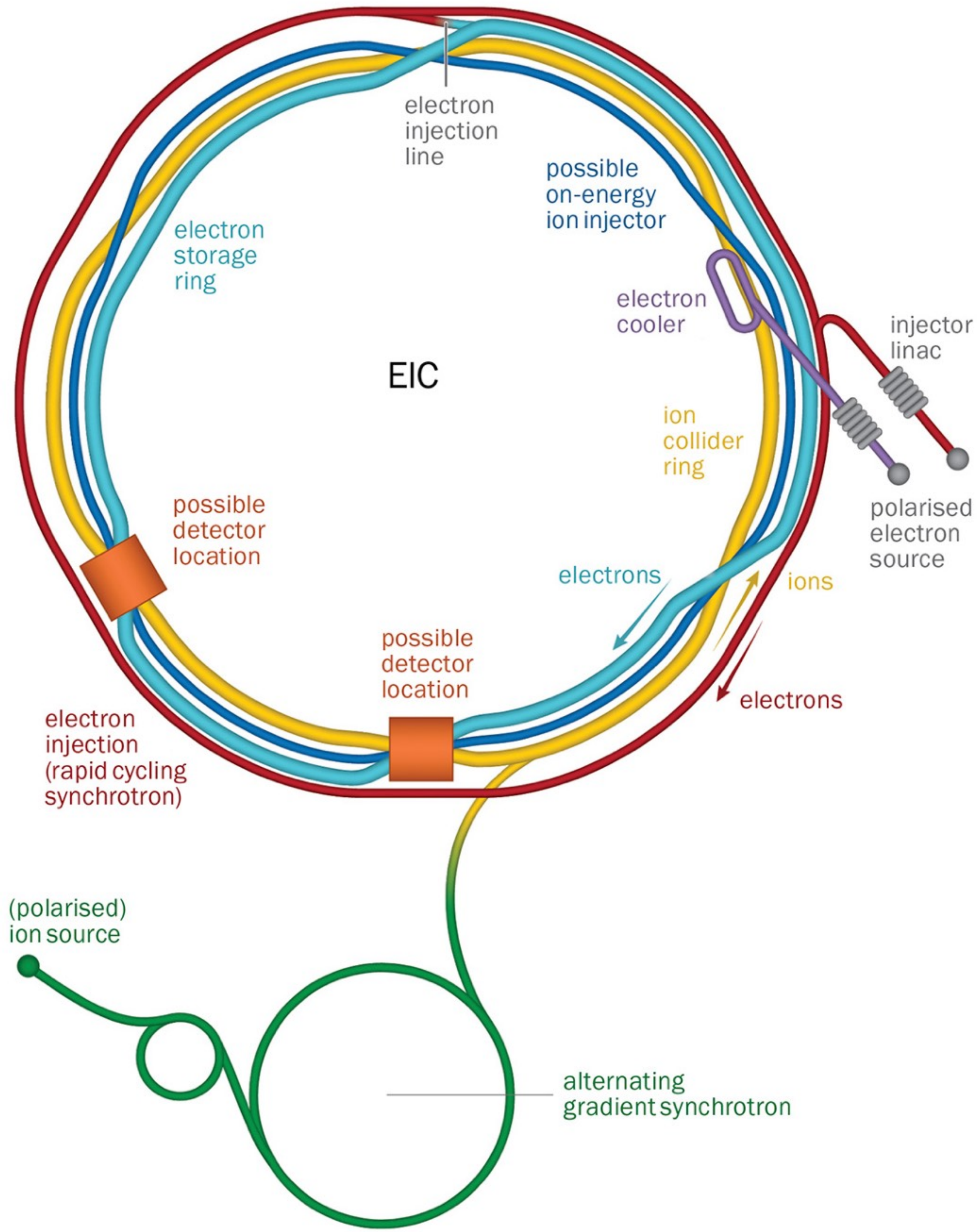
- Electron Ion Collider (EIC)

# $\alpha_s$ from $\Gamma_1^{p-n}(Q^2)$ measured at the EIC

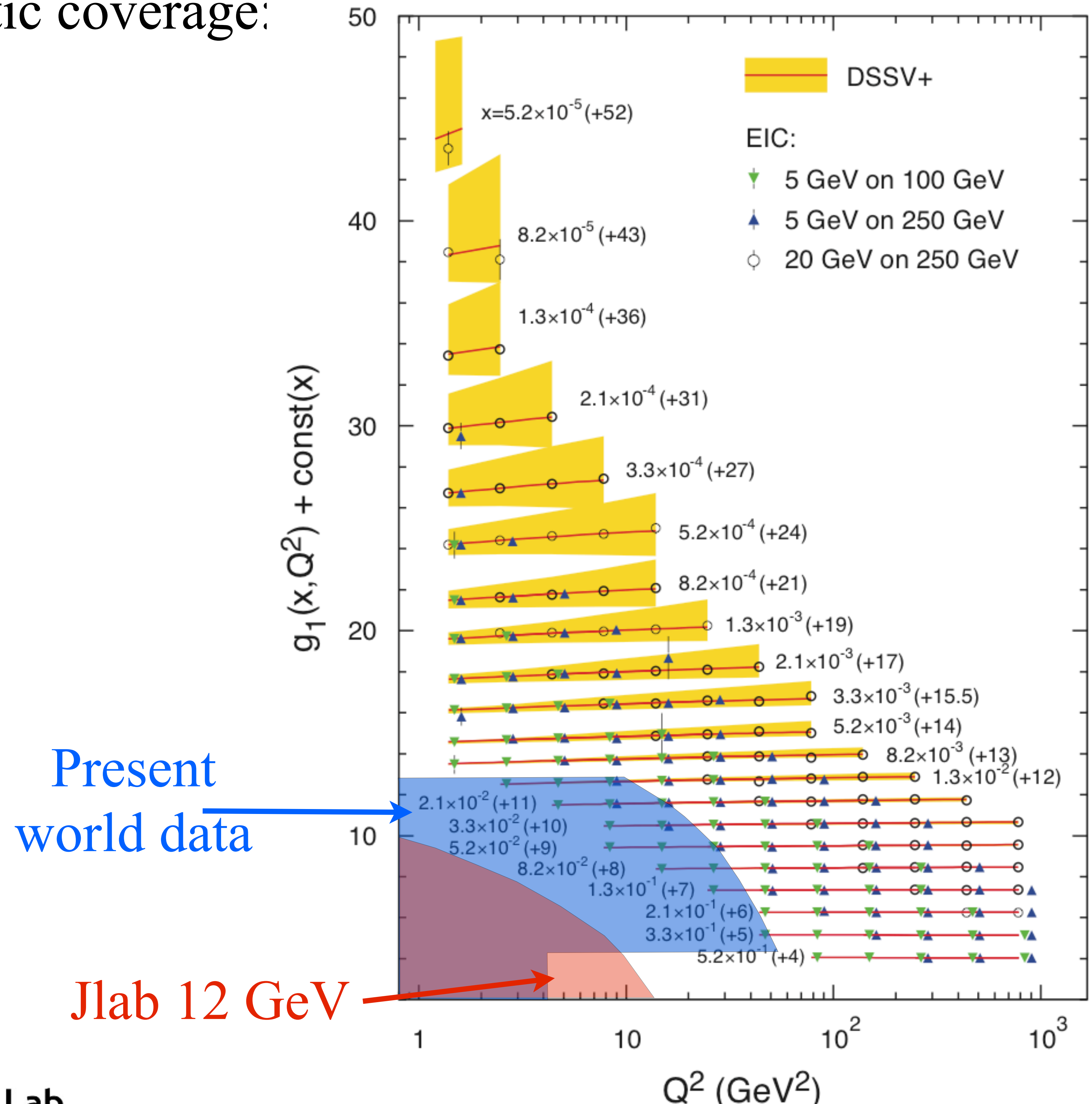
(just a quick exploratory look
to see if it is worth pursuing)

2030s:



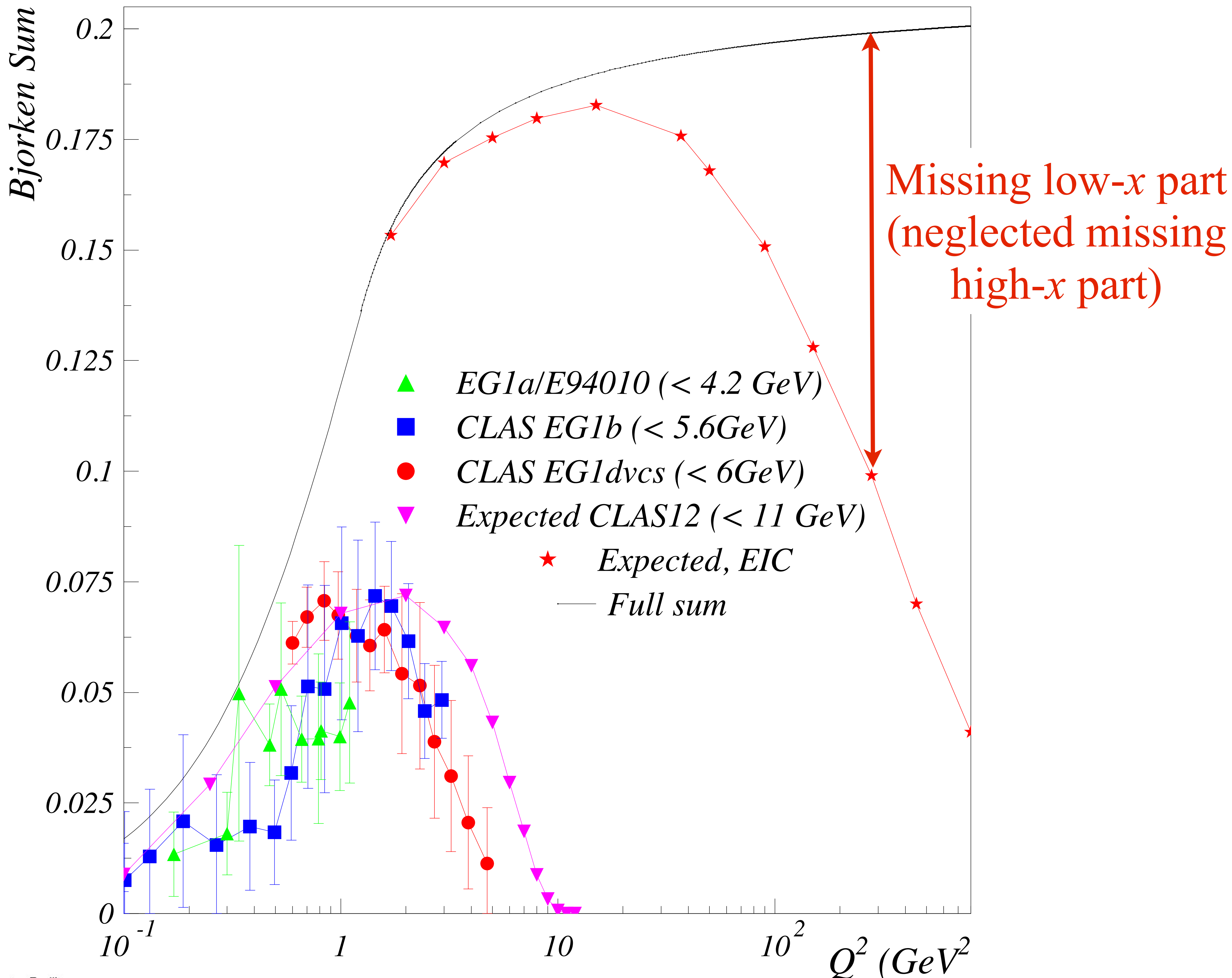
# EIC

Use the 5 GeV on 100 GeV, 5 GeV on 250 GeV and 20 GeV on 250 GeV kinematic coverage:




From the EIC white paper, arXiv:1212.1701

# Measured fraction of the Bjorken sum



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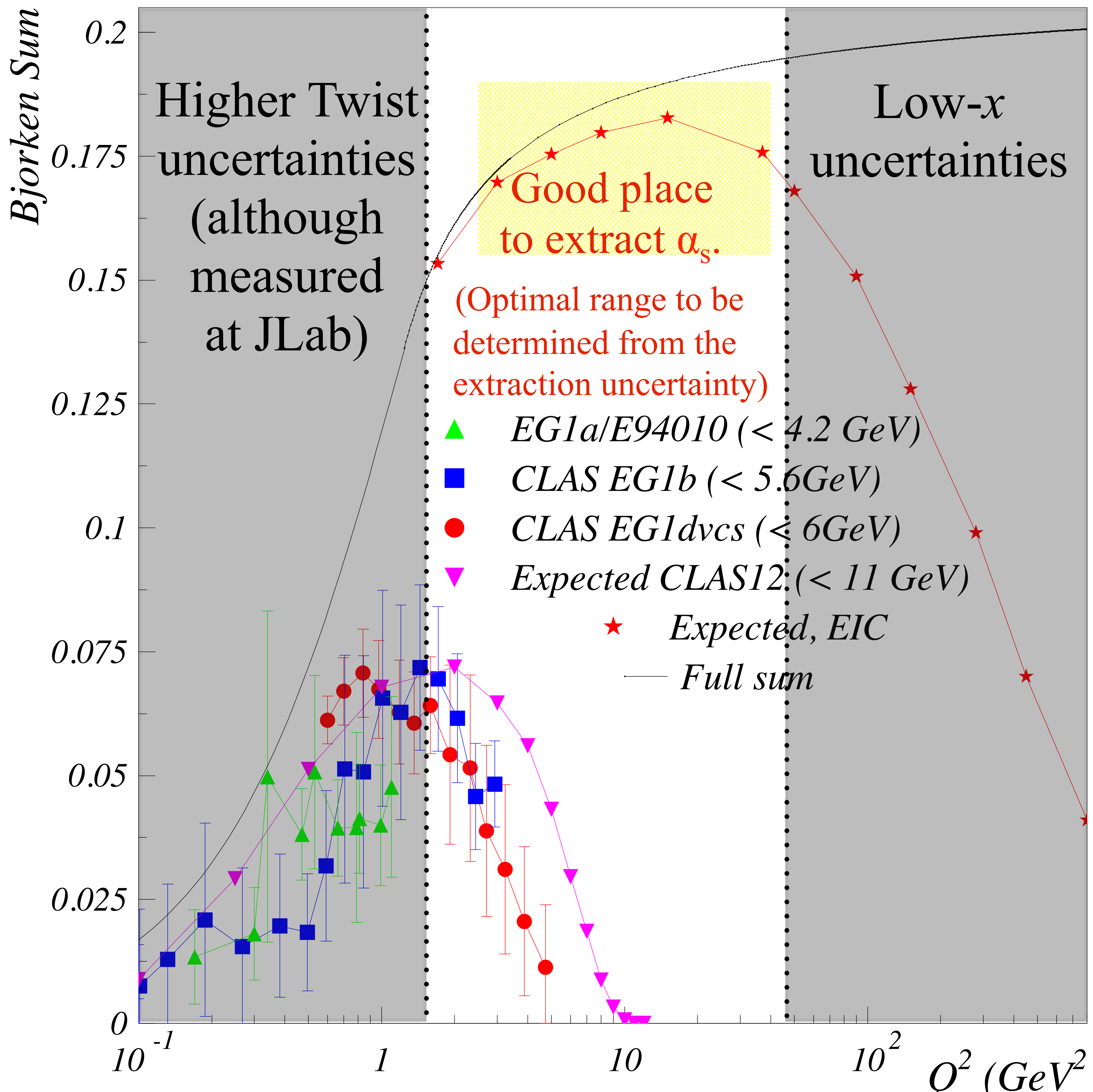


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# Uncertainty budget

## Statistics:

•Assume $\Delta\Gamma_1^{p-n} = 0.5\,\%$ ($Q^2 = 3$ GeV$^2$) to $\Delta\Gamma_1^{p-n} = 0.05\,\%$ ($Q^2 = 15$ GeV$^2$), not counting other world data (JLab@6&12 GeV, SLAC, CERN, DESY)

EIC:

- Luminosity: $2\times10^{33}$/s,
- $P_e$-$P_N$: 0.5-0.6
- Dilution factor: none
- Duration: a year.
- $Q^2$ range for $\alpha_s$ fit: $1.5<Q^2<15$ GeV$^2$

Statistics assumed twice better than those of CLAS EG1b experiment:

- Luminosity: $10^{34}$/s,
- *PbPt*: 0.2-0.6
- Dilution factor: ~80%
- Duration: a few months.
- $Q^2$ range for $\alpha_s$ fit: $1<Q^2<3$ GeV$^2$

# Uncertainty budget

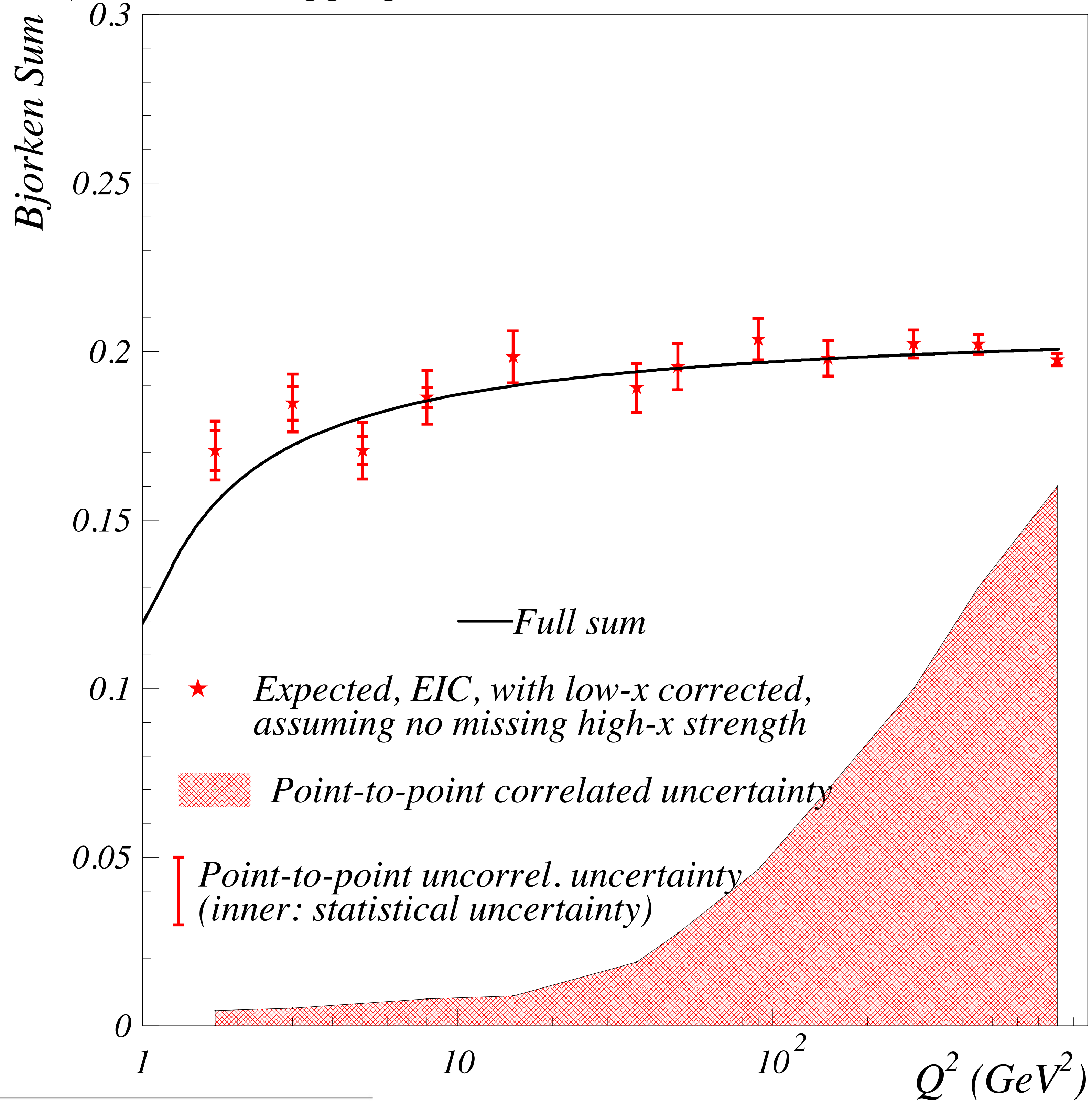
## Assumed systematics:

- Nuclear corrections (D): 4% or negligible if we can tag the ~spectator proton.
- Missing low-$x$ part: Assume 100% uncertainty on it.
- Polarimetries: Assume $\Delta P_e$-$\Delta P_N = 3\%$.
- Radiative corrections: Lower energy data exist. Assume 6%.
- $F_1$ to form $g_1$ from $A_1$: 2.5% (assumed $F_2$: 2% for proton and neutron. $R$: 10%.)
- $g_2$ contribution to longitudinal asym: Measured with transverse pol. ion beam.
- Dilution/purity: 0
- Contamination from particle miss-identification: Assumed negligible.
- Detector/trigger efficiencies, acceptance, beam currents: Neglected (asym).

# Extraction of $\alpha_s(M_Z)$

For CLAS EG1dvcs, 60% of syst. is point-to-point uncorrelated (excluding the low-$x$ error) $\Rightarrow$ add to stat. uncert.

$\Rightarrow$ data may look like (assume no tagging, i.e. include nuclear correction uncertainty):



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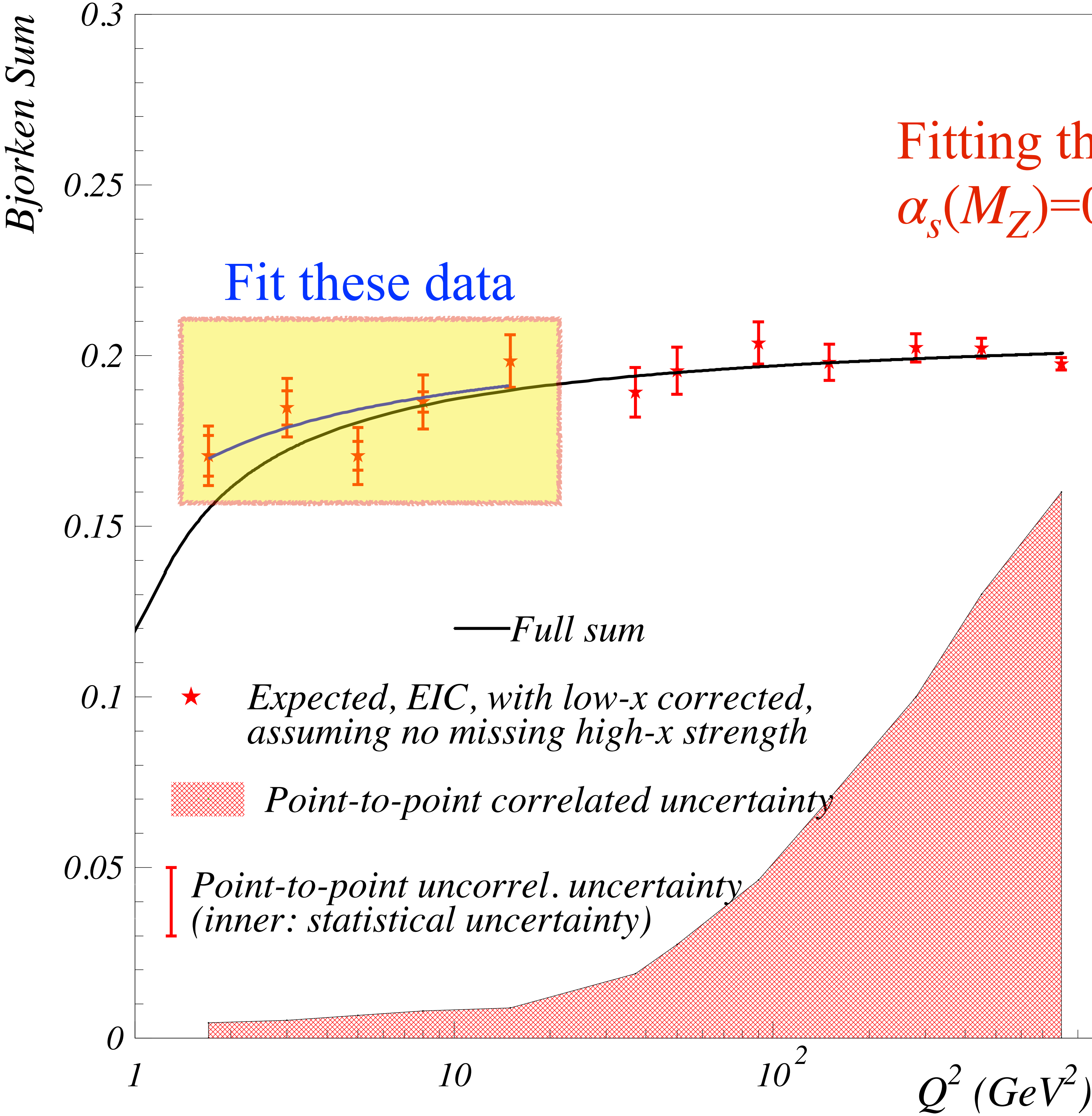
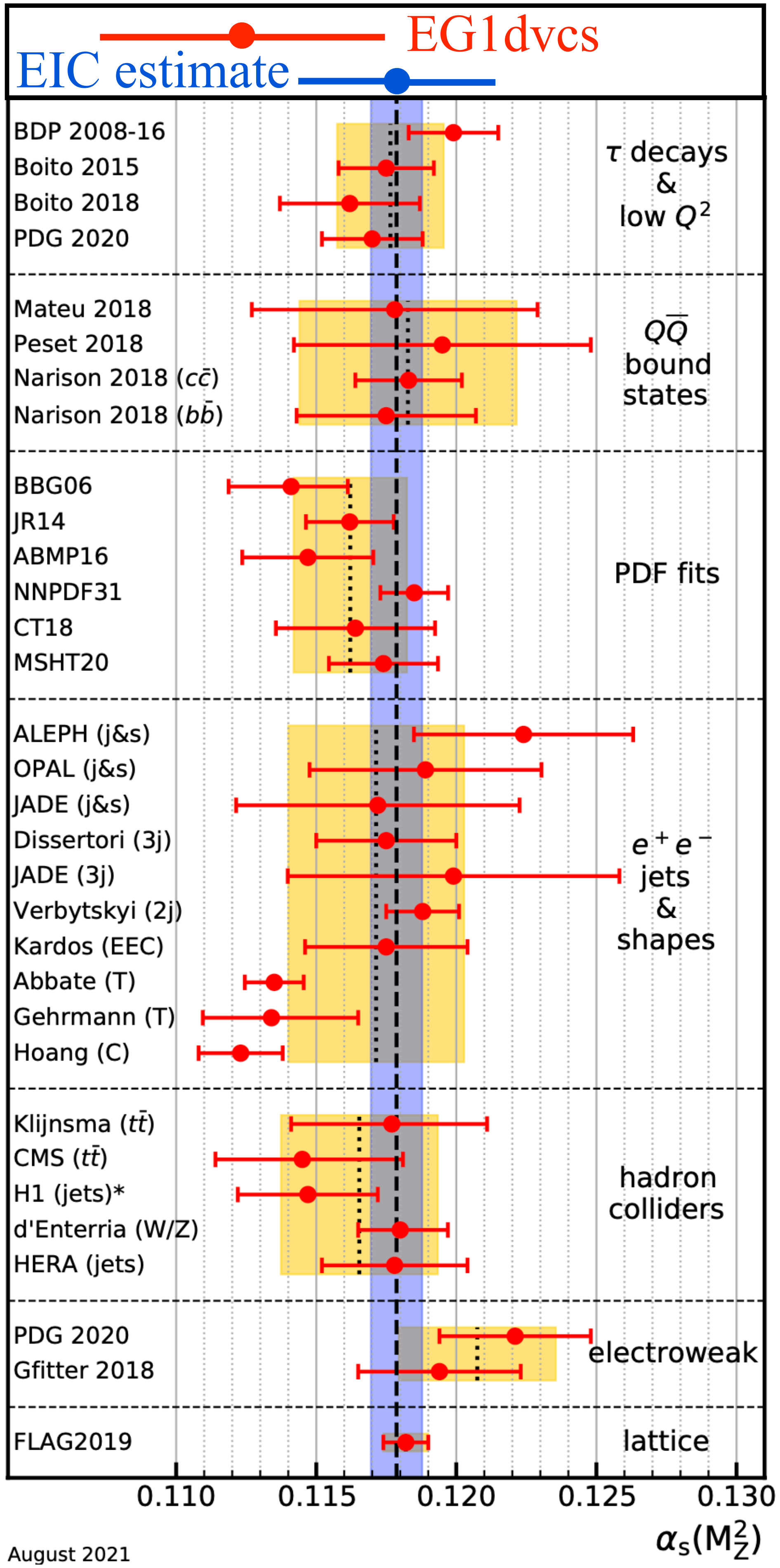



Fitting the data yields:
$\alpha_s(M_Z)=0.1175\pm0.0033\pm0.0005$

(Adding next data point yields:
$\alpha_s(M_Z)=0.1162\pm0.0030\pm0.0021$
$\Rightarrow$ Not worth including more point).

# Compared to EG1dvcs and best world data (PDG 2021):



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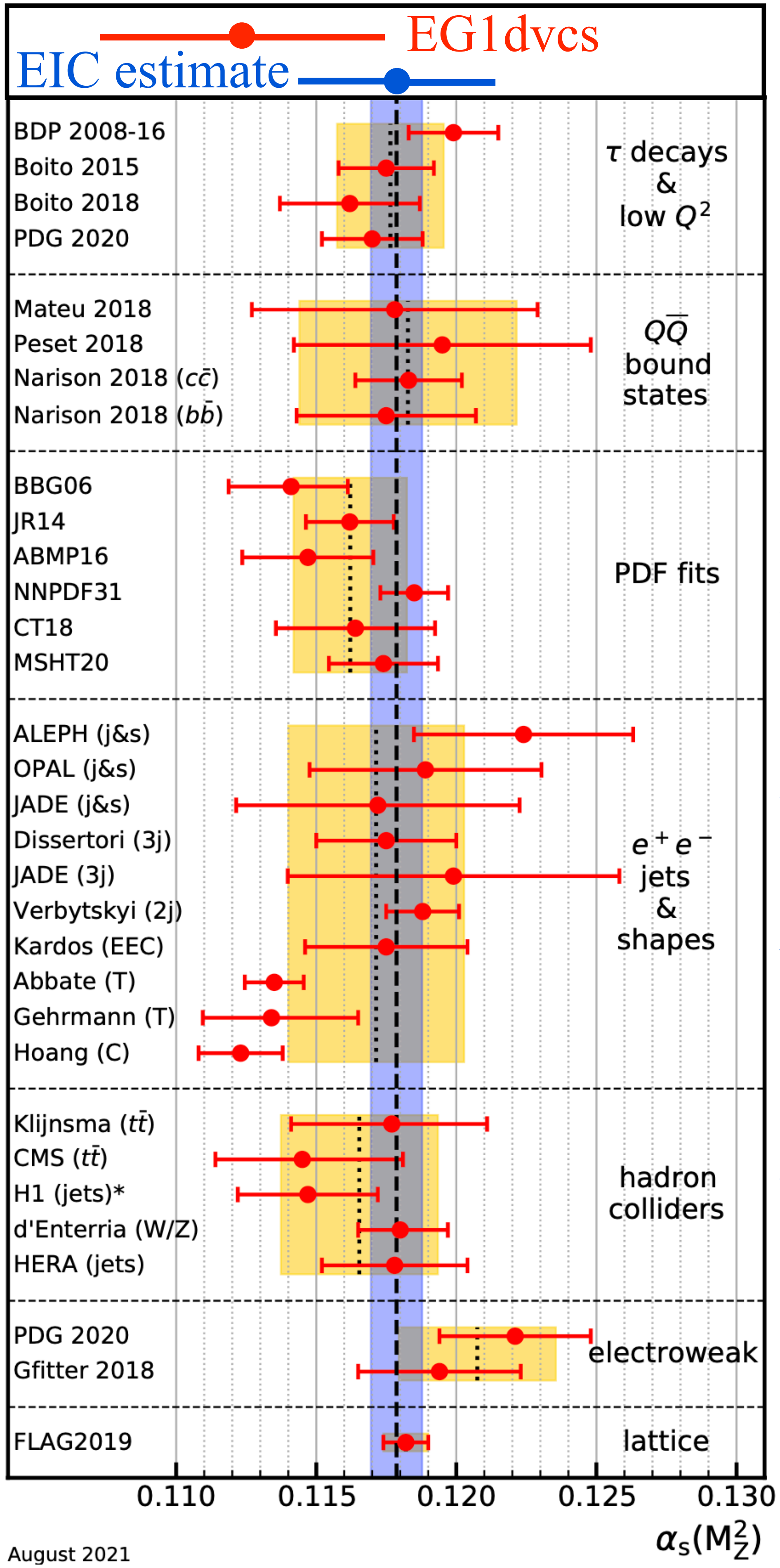


## Conclusion:

- Under reasonable assumptions, EIC can yield an accurate measurement of moderate precision.
- Assumed statistics similar to a typical CLAS experiment aiming at measuring inclusive spin structure functions.
- Increasing statistics by factor 10 would yield: $\Delta\alpha_s(M_Z)$=±0.0021±0.0003.
- If proton tagging is added with D data taking: $\Delta\alpha_s(M_Z)$=±0.0016±0.0003. Competitive measurement.

# Summary and perspective

- We discussed the extraction of $\alpha_s(M_Z)$ from the Bjorken sum $\Gamma_1^{p-n}(Q^2)=\int g_1^{p-n}(x,Q^2)dx$.
- The JLab 6 GeV experiment EG1dvcs yielded $\alpha_s(M_Z)=0.1123\pm0.0061$ (PRD, 2014). On the low side, but agreeing with the PDG-2021 world average.
- EG12: JLab CLAS12 with 11 GeV beam. Expected precision similar to EG1dvcs, but better accuracy and with $Q^2$ up to 6 GeV$^2$.
- EIC: exploratory estimate suggests an accurate measurement of acceptable precision, even without dedicated experiment $\Rightarrow$ it seems worth to seriously investigate this possibility.
- Not discussed here:
  - Improving extraction from $\Gamma_1^{p-n}(Q^2)$ with lower $Q^2$ points, using either
    - Higher-twist corrections.
    - Non-perturbative *model* of $\alpha_s(Q^2)$. Matching between AdS/QCD and pQCD in their overlap domain has yield competitive estimate.
  - $\alpha_s(M_Z)$ from $Q^2$-evolution of
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    - Proton's $\int g_1^p(x,Q^2)dx$.

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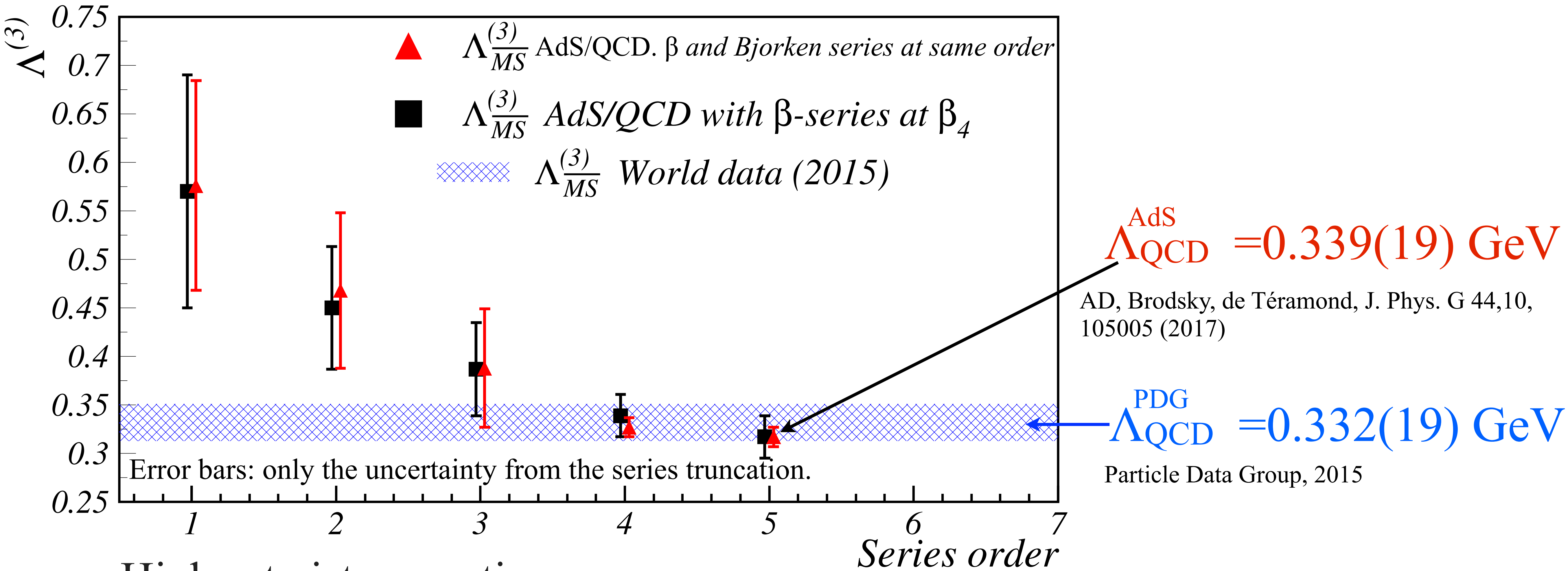


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