Recent Developments In Factorization Formalisms for Heavy Quarkonium Production

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Quarkonia As Tools 2022

Outline

- NRQCD matrix elements in potential NRQCD
- Soft Gluon Factorization
- Status of NRQCD Factorization
- QCD Factorization at large p_T

NRQCD Factorization

- NRQCD factorization formula for inclusive production of heavy quarkonium $\mathcal{Q}$

Bodwin, Braaten, Lepage, PRD51, 1125 (1995)

Nayak, Qiu, Sterman, PLB613, 45 (2005)

$$\sigma_{\mathcal{Q}+X} = \sum_N \sigma_{Q\bar{Q}(N)} \langle \Omega | \mathcal{O}^{\mathcal{Q}}(N) | \Omega \rangle$$

Lightlike Wilson lines

$$\mathcal{O}^{\mathcal{Q}}(N_{\text{color singlet}}) = \chi^\dagger \mathcal{K}_N \psi \mathcal{P}_{\mathcal{Q}(\boldsymbol{P}=\boldsymbol{0})} \psi^\dagger \mathcal{K}'_N \chi$$

$$\mathcal{O}^{\mathcal{Q}}(N_{\text{color octet}}) = \chi^\dagger \mathcal{K}_N T^a \psi \Phi_\ell^{\dagger ab}(0) \mathcal{P}_{\mathcal{Q}(\boldsymbol{P}=\boldsymbol{0})} \Phi_\ell^{bc}(0) \psi^\dagger \mathcal{K}'_N T^c \chi$$

$$\mathcal{P}_{\mathcal{Q}(\boldsymbol{P})} = a^\dagger_{\mathcal{Q}(\boldsymbol{P})} a_{\mathcal{Q}(\boldsymbol{P})}$$ projection onto states that include $\mathcal{Q}$

- $Q\bar{Q}$ can be color singlet or color octet.
  Singlet and octet can mix under evolution due to soft gluons (*P*-wave production/decay at leading order in v).

- Even when octet contributions are suppressed by powers of v, singlet contributions can be suppressed by powers of m/p_T, so that octet contributions can dominate cross section at large p_T (J/ψ, Υ production)

NRQCD matrix elements in pNRQCD

- One major difficulty in NRQCD description of quarkonium production is the unknown matrix elements.

- While singlet matrix elements can be related to decay matrix elements, so far it has been not known how to determine octet matrix elements.

- For quarkonium decays, potential NRQCD (pNRQCD) provides expressions for singlet ***and*** octet matrix elements in terms of wavefunctions at the origin, and universal gluonic correlators. This leads to more symmetries and more predictive power than standard NRQCD.

Brambilla, Eiras, Pineda, Soto, Vairo, PRL88, 012003 (2002)
Brambilla, Eiras, Pineda, Soto, Vairo, PRD67, 034018 (2003)
Brambilla, **HSC**, Müller, Vairo, JHEP04 (2020) 095

- Our recent work extends this to production matrix elements.

Brambilla, **HSC**, Vairo, PRL 126, 082003 (2021) and JHEP 09 (2021) 032

pNRQCD

- pNRQCD describes energy scales below mv. Quarkonium state is described as bound state of Q and $\bar{Q}$ in a potential.

Pineda, Soto, NPB Proc. Suppl. 64, 428 (1998)
Brambilla, Pineda, Soto, Vairo, NPB566, 275 (2000)
Brambilla, Pineda, Soto, Vairo, Rev. Mod. Phys. 77, 1423 (2005)

- We can match NRQCD matrix elements on pNRQCD by integrating out the scale mv. We do this under the assumption $\Lambda_{\rm QCD} \gg mv^2$ (strongly coupled pNRQCD).

- The matching coefficients are universal, as they only involve light degrees of freedom and are independent of heavy quark flavor.

- For decay matrix elements, it was sufficient to describe the heavy quarkonium state $|\mathcal{Q}\rangle$. To compute production matrix elements, we must describe the projection operator $\sum_X |\mathcal{Q}+X\rangle\langle\mathcal{Q}+X| = a^\dagger_{\mathcal{Q}} a_{\mathcal{Q}}$.
X includes an arbitrary number of soft gluons of scale mv.

P-wave production in pNRQCD

- χ_{QJ} cross section at leading order in v

$$\sigma_{\chi_{QJ}+X} = (2J+1)\sigma_{Q\bar{Q}({}^3P_J^{[1]})}\langle \mathcal{O}^{\chi_{Q0}}({}^3P_0^{[1]})\rangle + (2J+1)\sigma_{Q\bar{Q}({}^3S_1^{[8]})}\langle \mathcal{O}^{\chi_{Q0}}({}^3S_1^{[8]})\rangle$$

$$\mathcal{O}^{\chi_{Q0}}({}^3P_0^{[1]}) = \frac{1}{3}\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma}\right)\psi\mathcal{P}_{\chi_{Q0}}\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma}\right)\chi$$

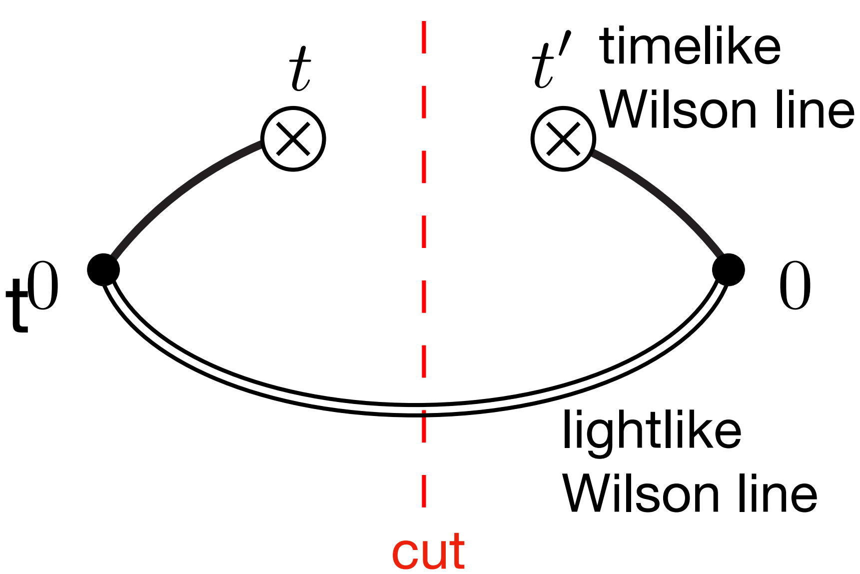
$$\mathcal{O}^{\chi_{Q0}}({}^3S_1^{[8]}) = \chi^\dagger\sigma^i T^a\psi\Phi_\ell^{\dagger ab}\mathcal{P}_{\chi_{Q0}}\Phi_\ell^{bc}\psi^\dagger\sigma^i T^c\chi$$

- pNRQCD gives Brambilla, **HSC**, Vairo, PRL 126, 082003 (2021) and JHEP 09 (2021) 032

$$\langle \mathcal{O}^{\chi_{Q0}}({}^3P_0^{[1]})\rangle = \frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2 \qquad \langle \mathcal{O}^{\chi_{Q0}}({}^3S_1^{[8]})\rangle = \frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2\frac{\mathcal{E}}{9N_c m^2}$$

$$\mathcal{E} = \frac{3}{N_c}\int_0^\infty t\,dt\int_0^\infty t'\,dt'\langle\Omega|\Phi_\ell^{\dagger ab}\Phi_0^{\dagger da}(0,t)gE^{d,i}(t)gE^{e,i}(t')\Phi_0^{ec}(t',0)\Phi_\ell^{bc}|\Omega\rangle$$

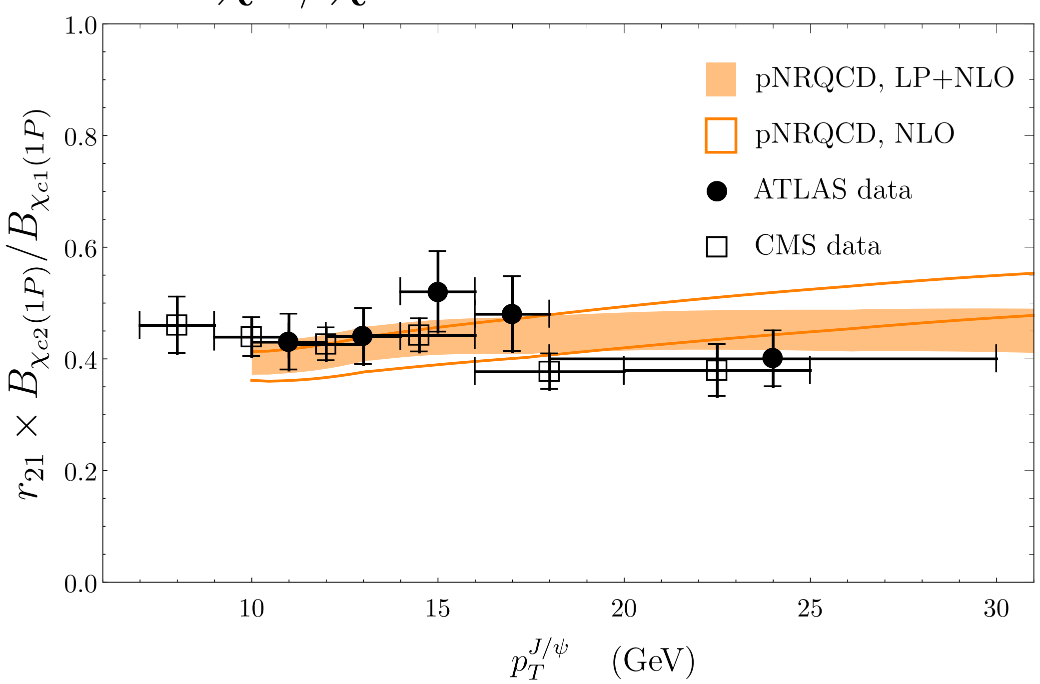
- A single universal quantity $\mathcal{E}$ determines all *P*-wave charmonium and bottomonium production rates. RG evolution of $\mathcal{E}$ consistent with NRQCD. $$\frac{d}{d\log\Lambda}\mathcal{E}(\Lambda) = 12C_F\frac{\alpha_s}{\pi} + O(\alpha_s^2)$$



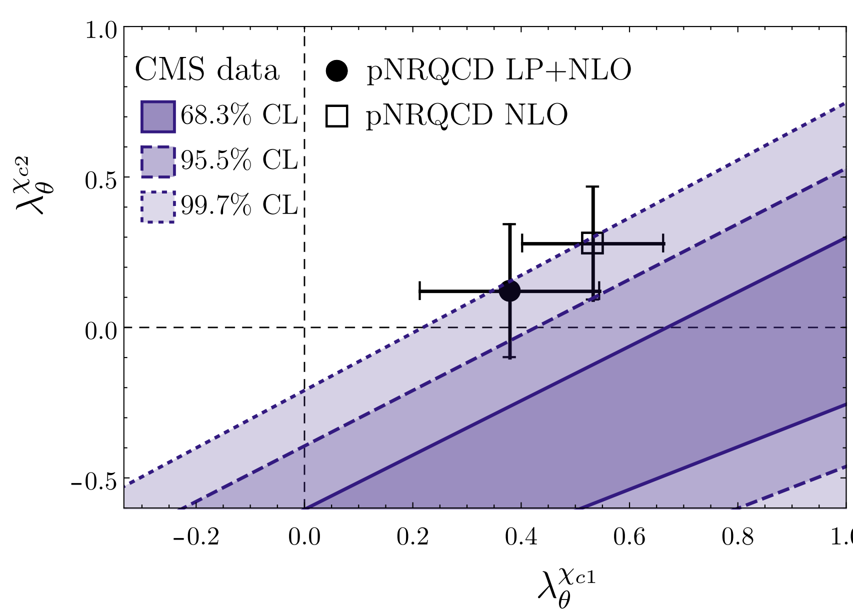
P-wave production in pNRQCD

- Determine $\mathcal{E}$ from charmonium production data, compute charmonium and bottomonium cross sections.

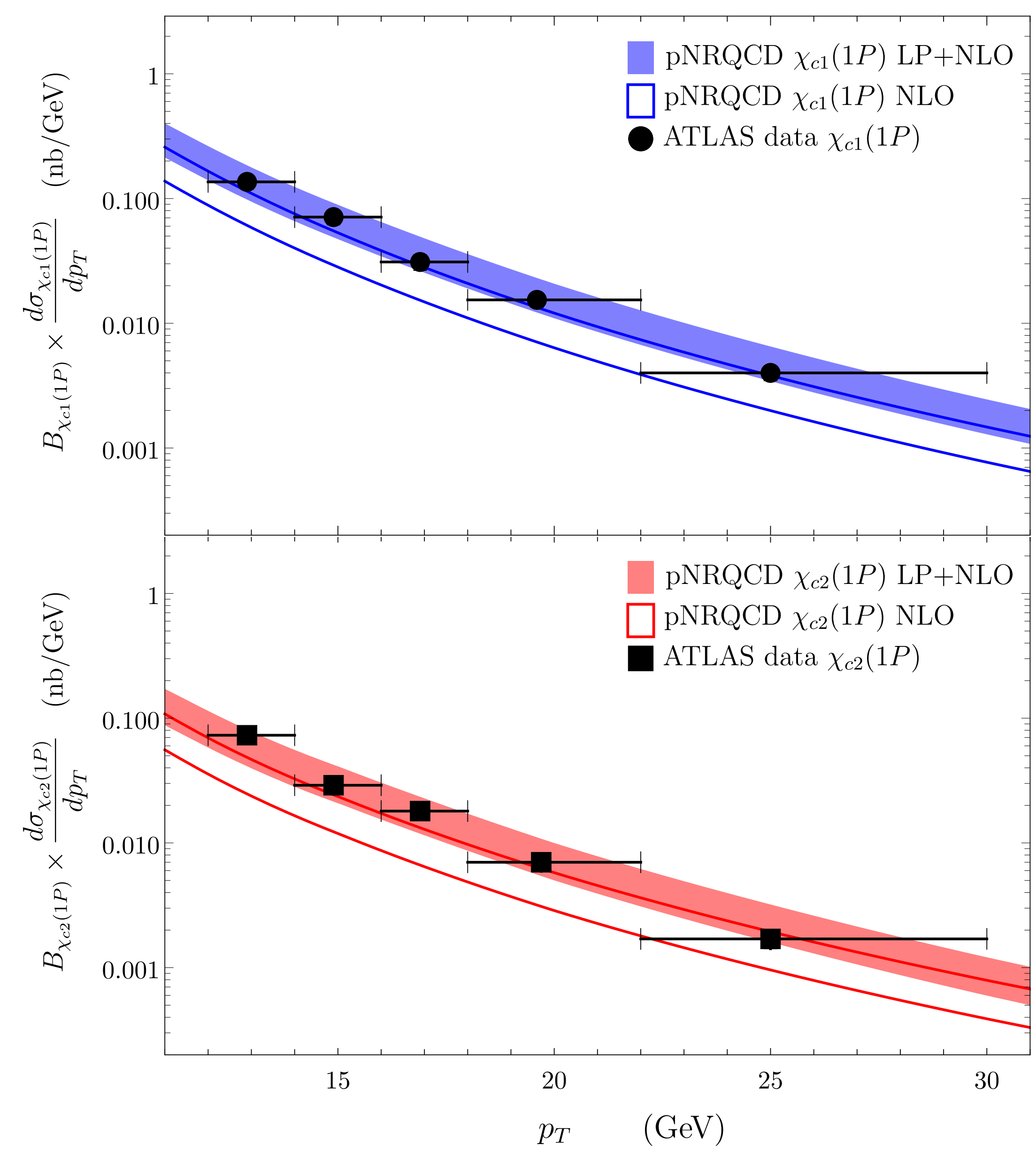
Use χ_{c2}/χ_{c1} ratio to determine $\mathcal{E}$



Compute χ_{c2} and χ_{c1} polarization



Compute χ_{c2} and χ_{c1} cross sections

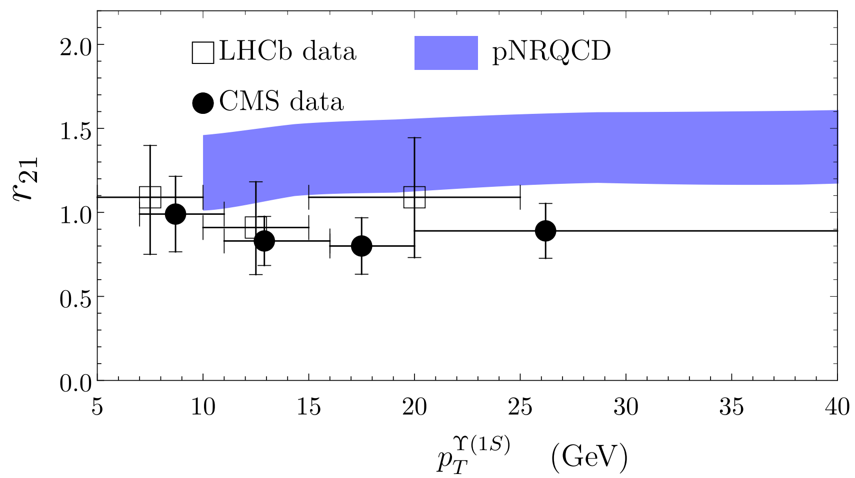


Brambilla, **HSC**, Vairo, JHEP 09 (2021) 032

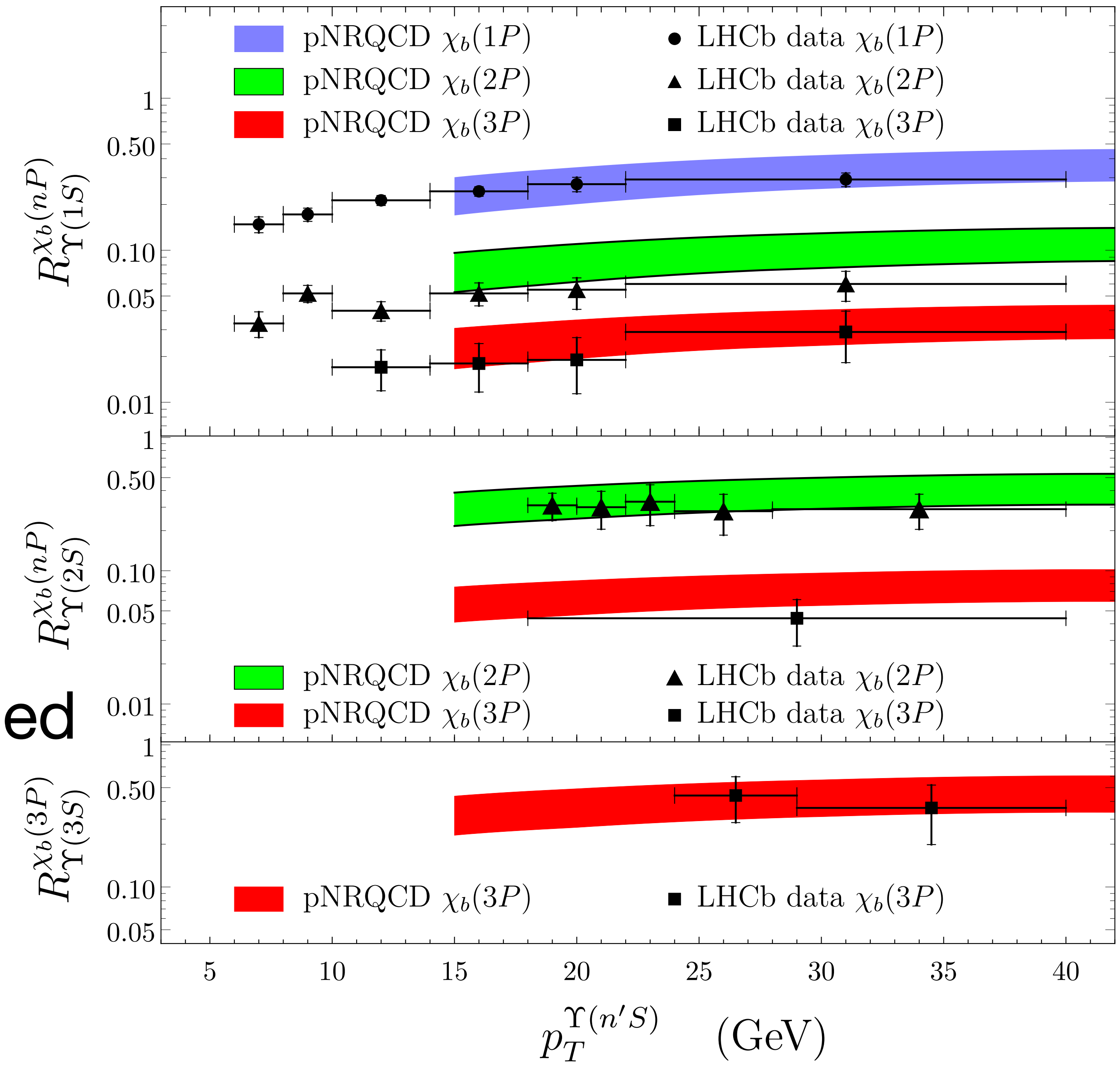
P-wave production in pNRQCD

- Determine $\mathcal{E}$ from charmonium production data, compute charmonium and bottomonium cross sections.

Compute χ_{b2}/χ_{b1} ratio



Compute $\chi_b(nP)$ cross sections



- P-wave bottomonium matrix elements completely determined from charmonium data

Brambilla, **HSC**, Vairo, JHEP 09 (2021) 032

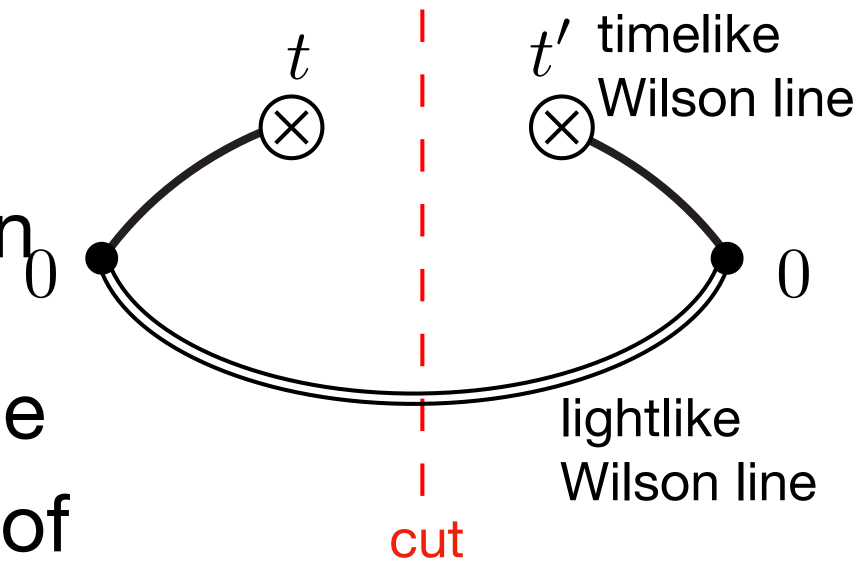
S-wave production in pNRQCD

- Similar calculation is being done for production of S-wave heavy quarkonia (J/ψ, $\psi(2S)$, η_c, Υ). Brambilla, **HSC**, Vairo, X.-P. Wang, in preparation

- General form of octet matrix element for S-wave quarkonium is

$$\langle \mathcal{O}(N) \rangle = \text{color and normalization factor} \times |R(0)|^2 \times \mathcal{E}_N$$

- $\mathcal{E}_N$ are given by gluon field-strength insertions onto temporal Wilson lines in this configuration



- Large p_T hadroproduction data underdetermine J/ψ, $\psi(2S)$, Υ octet matrix elements because of degeneracy in p_T shape : only certain linear combinations could be determined. See e.g. JHEP05 (2015) 103, PRD94, 014028 (2016)
  It is expected that the pNRQCD result will be able to resolve this degeneracy when both $c\bar{c}$ and $b\bar{b}$ data are considered.

Soft Gluon Factorization

- NRQCD is an expansion in powers of v. The convergence of the velocity expansion has long been a concern.

- The velocity expansion of kinematical effects can lead to bad convergence. For example, decay rates of S-wave quarkonia can involve $1/M^2$ (M : quarkonium mass), which can be rewritten in terms of the quark mass m by $1/(2\ m+E)^2$ and expanded in powers of v. For $\psi(2S)$ with $m = 1.4\ \mathrm{GeV}$, we have

$$\boxed{1/M^2 = 0.074\ \mathrm{GeV}^{-2}}$$

$$1/(2\ m+E)^2 = 1/m^2 - E/(4m^3) + \mathrm{O}(v^4)$$
$$\boxed{= 0.047\ \mathrm{GeV}^{-2} + \mathrm{O}(v^4)}$$

- For decays and exclusive production, some of these effects can be resummed.

Bodwin, Kang, Lee, PRD74, 014014 (2006)
Bodwin, **HSC**, Kang, Lee, Yu, PRD77, 094017 (2008)
Bodwin, Lee, Yu, PRD77, 094018 (2008)
Bodwin, **HSC**, Lee, Yu, PRD79, 014007 (2009)
HSC, Lee, Yu, PLB697, 48 (2011)
Fan, Lee, Yu, PRD87, 094032 (2013)
Bodwin, **HSC**, Ee, Lee, Petriello, PRD90, 113010 (2014)

- The situation is less clear in inclusive production, because of soft gluons.

Soft Gluon Factorization

- Soft gluon factorization aims to resum a subset of relativistic corrections in both quarkonium production and decay.
  Ma, Chao, PRD100, 094007 (2019) Li, Feng, Ma, JHEP 05 (2020) 009
  Chen, Ma, Chin.Phys.C45, 013118 (2021)
- For quarkonium production, cross section is described by (SGF-4d)

Intermediate $Q\bar{Q}$ momentum

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} \approx \sum_n \int \frac{d^4P}{(2\pi)^4} \mathcal{H}_n(P) F_{n\to H}(P, P_H).$$

quarkonium momentum

soft gluon distribution functions (SGD)

$$F_{n\to H}(P,P_H) = \int d^4b e^{-iP\cdot b} \langle 0|[\overline{\Psi}\mathcal{K}_n\Psi]^\dagger(0)\left(a_H^\dagger a_H\right)[\overline{\Psi}\mathcal{K}_n\Psi](b)|0\rangle_{\mathrm{S}}$$

- SGD is constructed in terms of QCD fields. $\boldsymbol{\Psi}$: Dirac spinor field
- Projectors $\mathcal{K}$ include relativistic corrections.

color projection, Wilson line

$$\mathcal{K}_n(rb) = \frac{\sqrt{M_H}}{M_H+2m} \frac{M_H+\not{P}_H}{2M_H} \Gamma_n \frac{M_H-\not{P}_H}{2M_H} \mathcal{C}^{[c]}$$

- SGDs are bilocal and depend on the 4-dimensional vector P. P can be different from quarkonium momentum P_H, and P dependence of SGD is in general nonperturbative.

Collinear approximation : SGF-1d

- Using $P \approx (M, \mathbf{0})$ in the rest frame, spatial components of P can be integrated out :

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} \approx \sum_n \int dz\, \mathcal{H}_n(P_H/z) F_{n\to H}(z)$$

$$F_{n\to H}(z) = \int \frac{d^4P}{(2\pi)^4} \delta(z - \sqrt{P_H^2/P^2}) F_{n\to H}(P, P_H)$$

Statical approximation : SGF-1d

- If soft gluon momenta can be neglected, we obtain

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} \approx \sum_n \mathcal{H}_n(P_H/z_n) \langle \widetilde{\mathcal{O}}_n^H \rangle$$

$$\langle \widetilde{\mathcal{O}}_n^H \rangle = \int \frac{d^4P}{(2\pi)^4} F_{n\to H}(P, P_H)$$

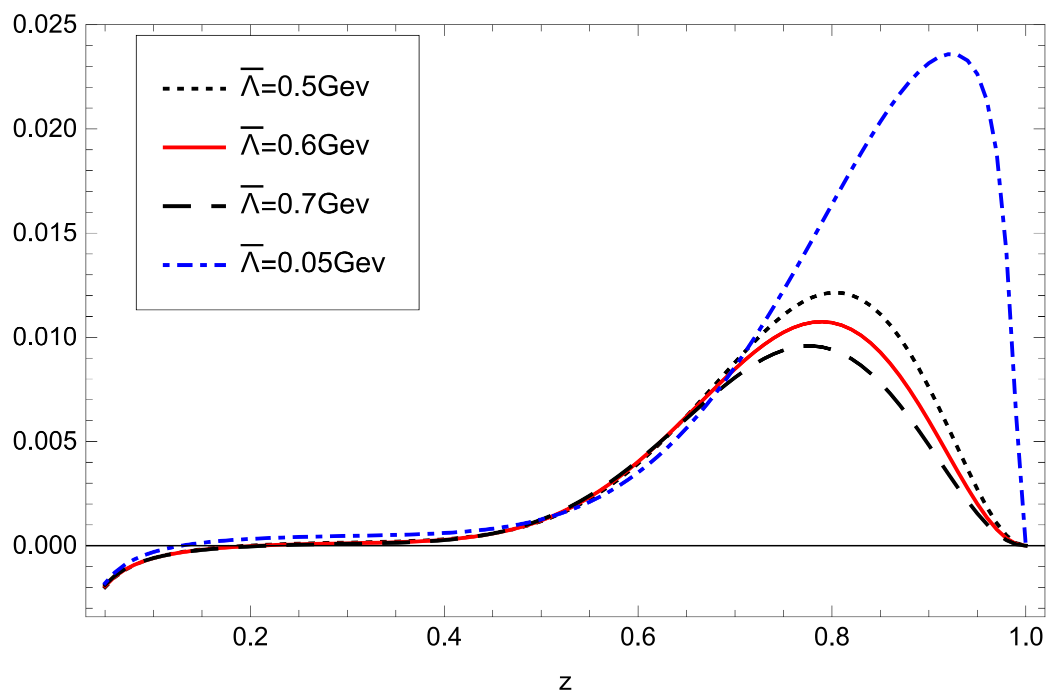
- While this looks like NRQCD factorization, $\langle \widetilde{\mathcal{O}}_n^H \rangle$ include resummed relativistic corrections, while NRQCD matrix elements do not.

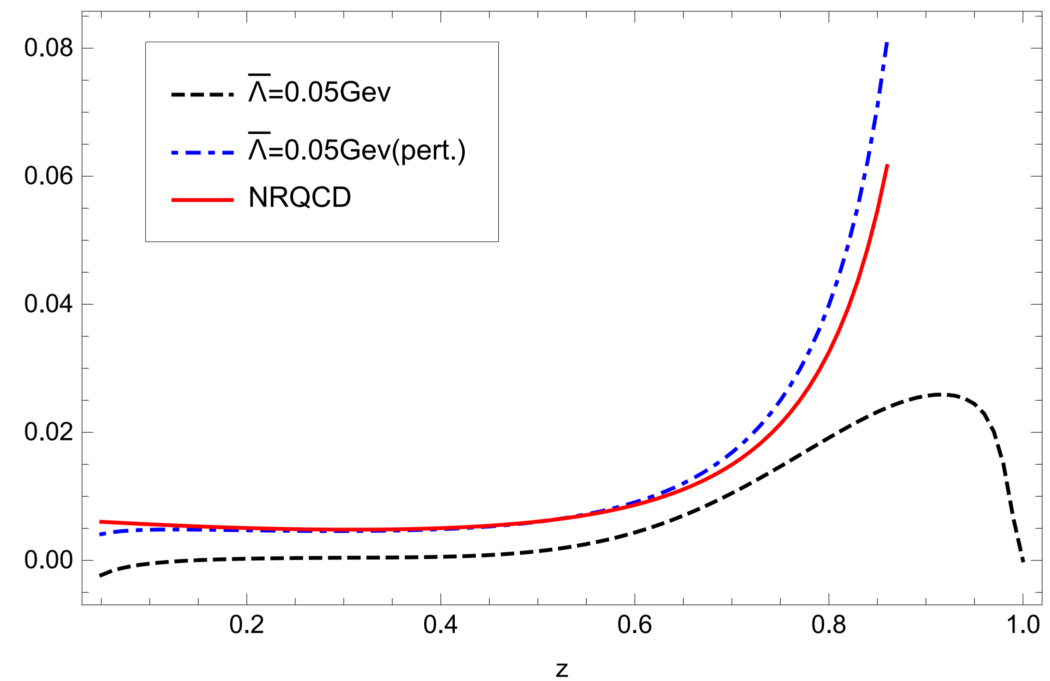
Comparison of SGF and NRQCD

- Gluon fragmentation $g \to Q\bar{Q}(^3S_1^{[8]})$ is an important contribution to large p_T quarkonium production.

- Fragmentation function in NRQCD and SGF :

Chen, Jin, Ma, Meng, JHEP 06 (2021) 046





- SGF result depends on model that describes nonperturbative shape ($\bar{\Lambda}$: model parameter).

- NRQCD shows large threshold logarithm near z=1, while SGF does not. NRQCD can lead to cross sections that are much larger than SGF by factor of ≈ 6, depending on model parameter.

NRQCD Factorization

- NRQCD factorization formula for inclusive production of heavy quarkonium $\mathcal{Q}$

Bodwin, Braaten, Lepage, PRD51, 1125 (1995)

Nayak, Qiu, Sterman, PLB613, 45 (2005)

$$\sigma_{\mathcal{Q}+X} = \sum_N \sigma_{Q\bar{Q}(N)} \langle \Omega | \mathcal{O}^{\mathcal{Q}}(N) | \Omega \rangle$$

Lightlike Wilson lines

$$\mathcal{O}^{\mathcal{Q}}(N_{\text{color singlet}}) = \chi^\dagger \mathcal{K}_N \psi \mathcal{P}_{\mathcal{Q}(\boldsymbol{P}=\boldsymbol{0})} \psi^\dagger \mathcal{K}'_N \chi$$

$$\mathcal{O}^{\mathcal{Q}}(N_{\text{color octet}}) = \chi^\dagger \mathcal{K}_N T^a \psi \Phi_\ell^{\dagger ab}(0) \mathcal{P}_{\mathcal{Q}(\boldsymbol{P}=\boldsymbol{0})} \Phi_\ell^{bc}(0) \psi^\dagger \mathcal{K}'_N T^c \chi$$

$$\mathcal{P}_{\mathcal{Q}(\boldsymbol{P})} = a^\dagger_{\mathcal{Q}(\boldsymbol{P})} a_{\mathcal{Q}(\boldsymbol{P})}$$ projection onto states that include $\mathcal{Q}$

- NRQCD matrix elements are local products of heavy quark fields, so short-distance coefficients do not distinguish Q and $\bar{Q}$ positions.

- Validity of NRQCD factorization is yet to be proven. In perturbative factorization, for factorization to be valid,
  - short-distance coefficients must be IR finite to all orders in α_s
  - must be consistent with parton cross sections (all relevant QCD dynamics must be included).

When/Why would NRQCD Work?

What does it take to prove NRQCD factorization?

- **NRQCD factorization of inclusive decay** can arise from four-fermion operators in the NRQCD Lagrangian, by integrating out the hard modes of scale m. This leaves us with products of local operators and Wilson coefficients, as is usual done in EFTs.

- In perturbative factorization of *inclusive decay*, ***KLN theorem*** ensures cancellation of infrared singularities in inclusive decays. As a result, short-distance coefficients are insensitive to low-energy nature of the quarkonium, and NRQCD matrix elements are given by local operator products. Bodwin, Braaten, Lepage, PRD51, 1125 (1995)

- ***The situation is different in inclusive production.*** Soft gluons ***can and do*** interact with other parts of Feynman diagrams and the $Q\bar{Q}$. Those interactions must be decoupled/ absorbed into matrix elements for us to obtain local operators. Otherwise, "short-distance" coefficients will be able probe the soft interactions between the Q and $\bar{Q}$.

When/Why would NRQCD Work?

What does it take to prove NRQCD factorization?

- The analysis is simpler in collinear factorization, which is given by power expansion in m/p_T.

  Nayak, Qiu, Sterman, PLB613, 45 (2005)

- Effects of IR singularities can be resummed in the form of lightlike Wilson lines

$$\Phi_\ell = P \exp\left[ig \int_0^\infty dz\, \ell \cdot A(\ell z) \right]$$

- In inclusive production, we must consider all possible lightlike directions ℓ, because it depends on the direction of other collinear particles, which are integrated over in the inclusive process.

- To absorb the effect of IR singularities into the matrix element, the Wilson line must be included in the definition.
  This is only possible with one lightlike direction ℓ.

$$\mathcal{O}^{\mathcal{Q}}(N_{\text{color octet}}) = \chi^\dagger \mathcal{K}_N T^a \psi \Phi_\ell^{\dagger ab}(0) \mathcal{P}_{\mathcal{Q}(\boldsymbol{P}=\boldsymbol{0})} \Phi_\ell^{bc}(0) \psi^\dagger \mathcal{K}'_N T^c \chi$$

- Hence, a necessary condition for NRQCD factorization to hold is that the IR singularities are independent of ℓ.

Checks of NRQCD Factorization

- Nayak, Qiu, Sterman, PLB613, 45 (2005) : the first work to introduce Wilson lines in octet matrix elements. Computed for the first time ℓ -dependent diagrams for gluon fragmentation to two loops in eikonal approximation at leading nontrivial order in v (order v^2), found IR poles independent of ℓ direction. Results and arguments presented in more detail in PRD72, 114012 (2005).
- Nayak, Qiu, Sterman, PRD74, 074007 (2006) : generalized previous result to all orders in v.
- Bodwin, **HSC**, Ee, Kim, Lee, PRD101, 096011 (2020) : computed ℓ -dependent diagrams for ${}^3S_1$ color octet matrix element using a covariant method. Reproduced previous results, but failed to generalize beyond two loops.
- Zhang, Meng, Ma, Chao, JHEP 08 (2021) 111 : byproduct of NLO correction to ${}^3P_J$ singlet/octet gluon fragmentation functions. Also showed that IR poles are independent of ℓ direction at two loop level at leading order in v.

QCD Factorization at large p_T

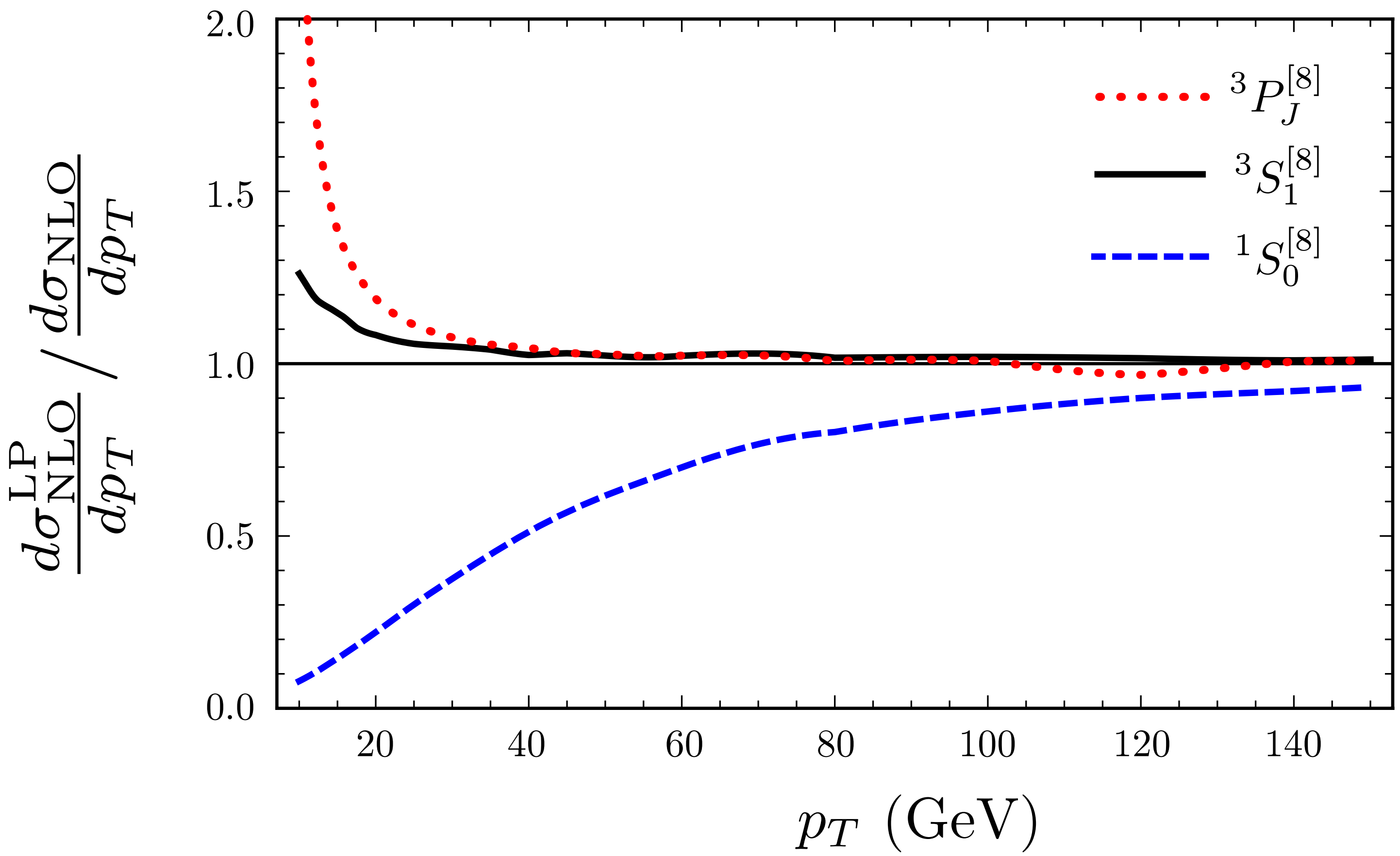
- It is likely that NRQCD factorization describes cross sections asymptotically as $p_T \to \infty$, if it works at all.
- If NRQCD factorization is valid at large p_T, it makes sense to compute short-distance coefficients after expansion in m/p_T.
- QCD factorization :
  Leading-power in $1/p_T$ is given by single parton fragmentation.
  Next-to-leading power is given by double parton fragmentation.

Kang, Qiu, Sterman, PRL 108, 102002 (2012)
Kang, Ma, Qiu, Sterman, PRD90, 034006 (2014)
Ma, Qiu, Sterman, Zhang, PRL 113, 142002 (2014)

- Combine QCD factorization with NRQCD :
  compute short-distance coefficients as parton cross sections convolved with $Q\bar{Q}$ fragmentation functions.
- This also allows resummation of logarithms in p_T/m using RG evolution.

NRQCD and LP fragmentation

- Ratio of LP fragmentation and standard NRQCD calculations of short-distance coefficients.

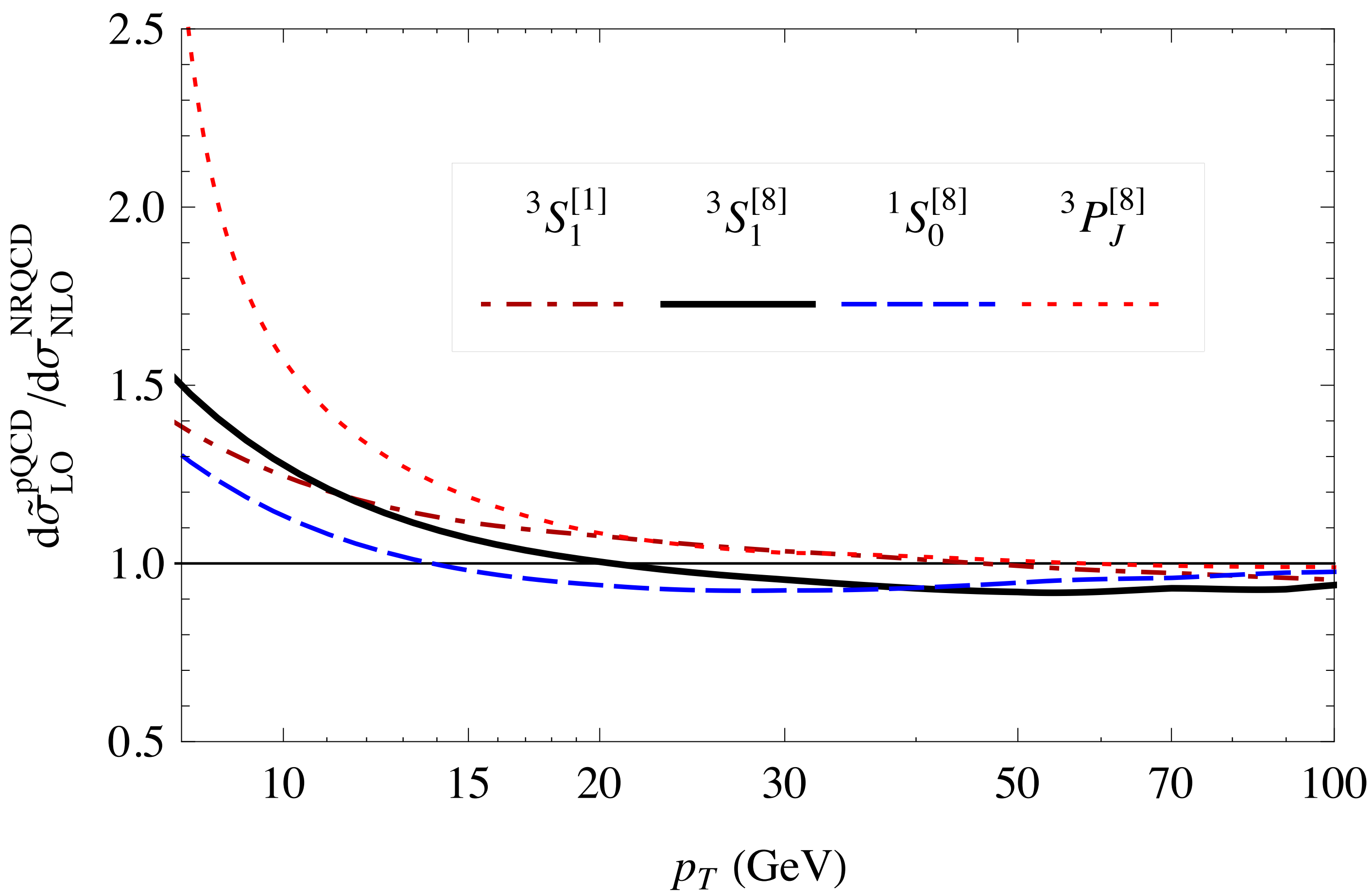


Bodwin, **HSC**, Kim, Lee, PRL113, 022001 (2014)
Bodwin, Chao, **HSC**, Kim, Lee, Ma, PRD93, 034041 (2016)

- LP fragmentation reproduces standard NRQCD calculation at large p_T, as expected.

NRQCD and LP+NLP

- Inclusion of NLP improves agreement at intermediate p_T (but still $p_T \gg m$)

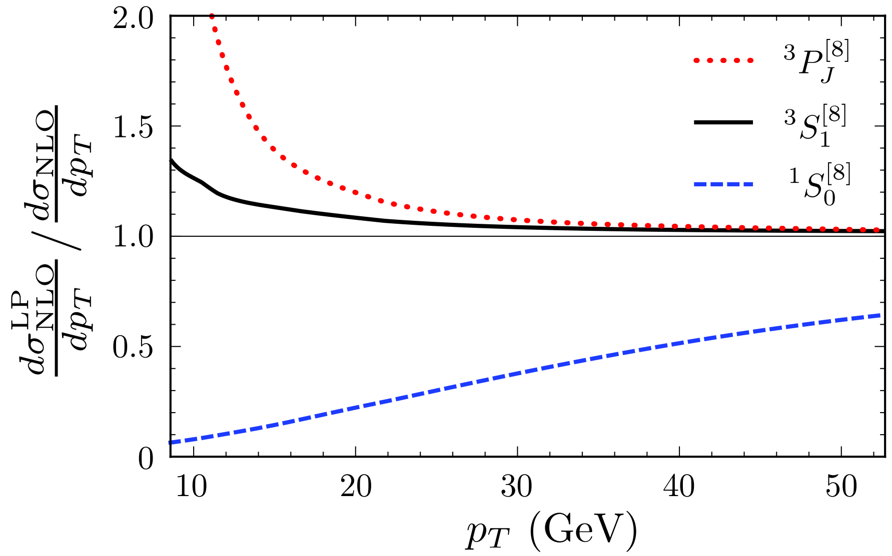


Ma, Qiu, Sterman, Zhang, PRL113, 142002 (2014)

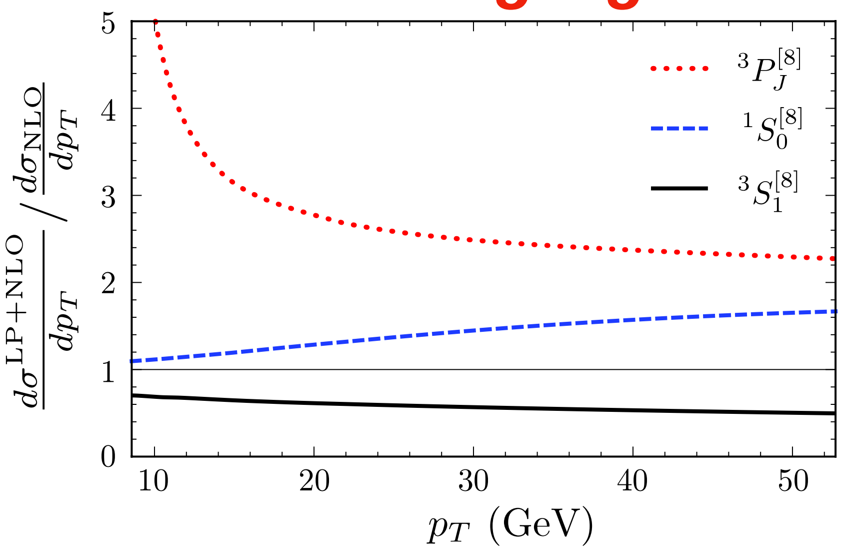
NRQCD and LP with RG

- The use of fragmentation functions allow resummation of logarithms of p_T/m.
- At LP, resummation is done by DGLAP evolution

No resum



Resummed leading logarithms at LP



Bodwin, **HSC**, Kim, Lee, PRL113, 022001 (2014)

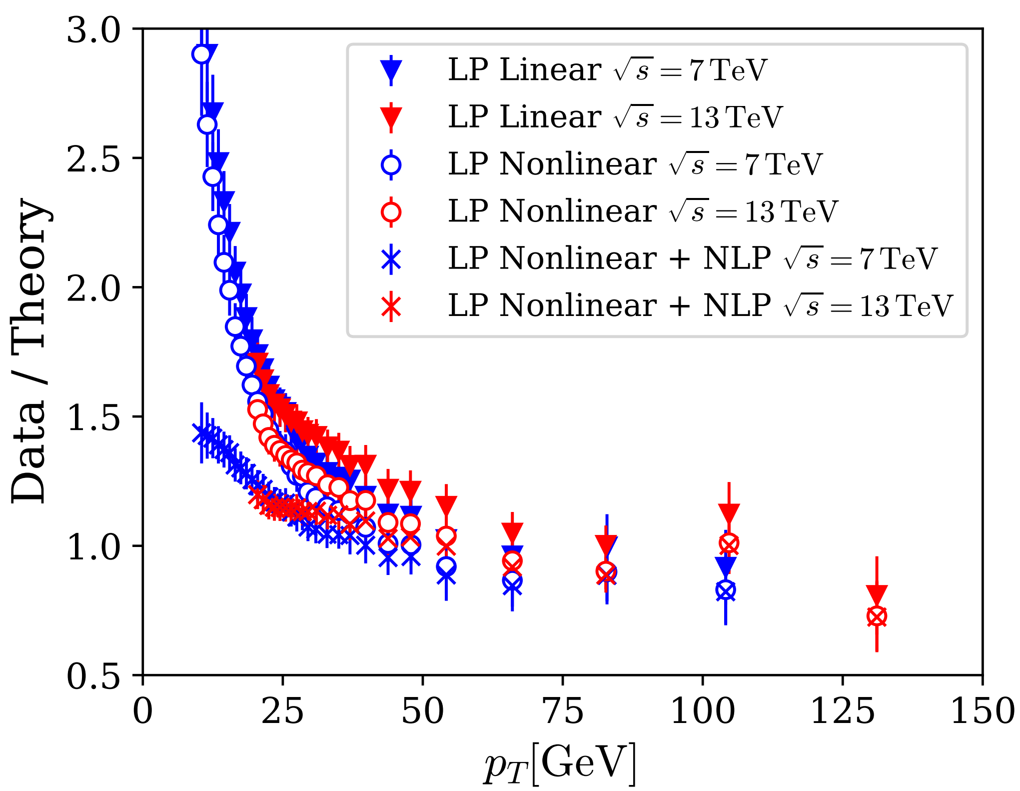
- Resummation can have significant effects in p_T shapes.

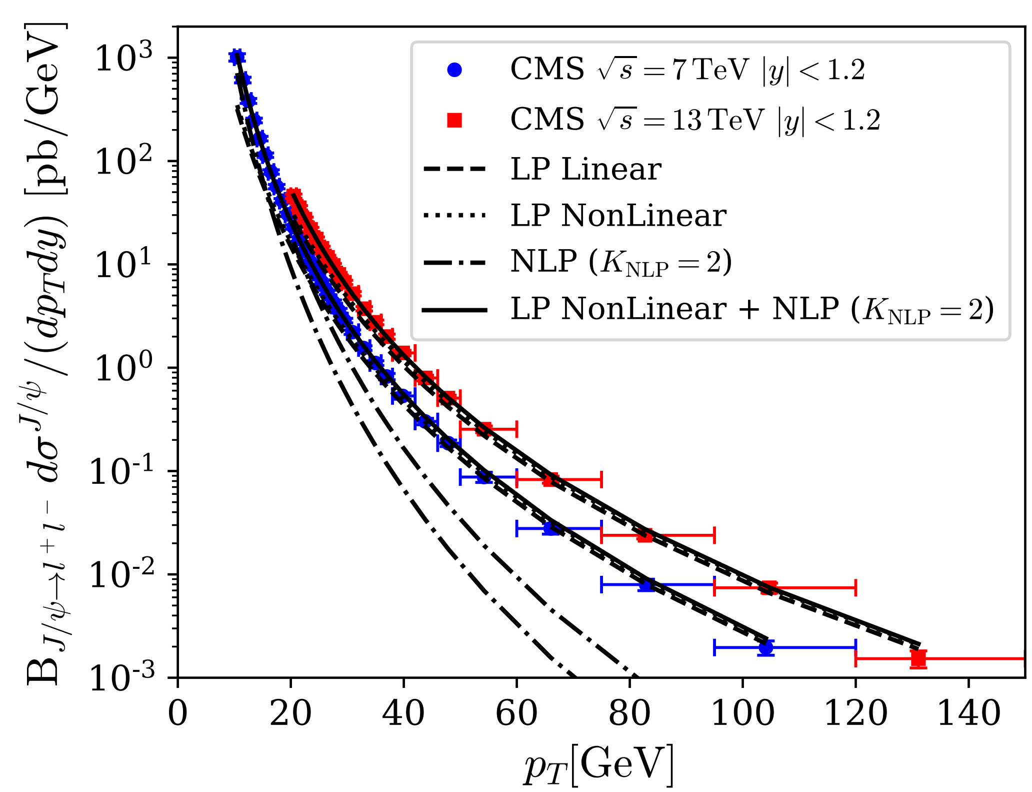
NRQCD and LP+NLP with RG

- Inclusion of NLP effect in resummation is complicated, due to mixing of LP and NLP contributions in evolution of LP fragmentation. This has been achieved recently.

$$\frac{\partial}{\partial \ln \mu^2} D_{f\to H}(z,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} \sum_{f'} \int_z^1 \frac{dz'}{z'} P_{f\to f'}\left(\frac{z}{z'}\right) D_{f'\to H}(z',\mu^2) \qquad \textbf{DGLAP}$$

$$+ \frac{\alpha_s^2(\mu)}{\mu^2} \sum_{\kappa} \int_z^1 \frac{dz'}{z'} P_{f\to[Q\bar{Q}(\kappa)]}\left(\frac{z}{z'}\right) D_{[Q\bar{Q}(\kappa)]\to H}\left(z',\mu^2\right) \qquad \textbf{NLP contribution}$$





Lee, Qiu, Sterman, Watanabe, 2108.00305

Summary

- Production matrix elements in potential NRQCD :
  pNRQCD provides expressions of singlet and octet matrix elements in terms of wavefunctions at the origin and universal gluonic correlators. More symmetries and more predictive power.
- Soft gluon factorization :
  SGF resums relativistic corrections coming from kinematical effects. Reproduces and generalizes previous resummation methods in decays and exclusive production. SGF may have better convergence than standard NRQCD.
- Status of NRQCD factorization :
  No violation of NRQCD factorization found up to two loops in specific processes.
- QCD Factorization at large p_T :
  Fragmentation formalism at LP and NLP can reproduce standard NRQCD calculations, and can resum logarithms. Inclusion of NLP effects in RG improvement computed recently.