Exclusive quarkonium production in CGC

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Quarkonia As Tools 2022

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Exclusive quarkonium production in deep inelastic scattering

- $\gamma^* + A \to V + A$, vector quarkonia $V = \mathrm{J}/\psi, \Upsilon \ldots$

Probes the gluon structure of the nucleus:

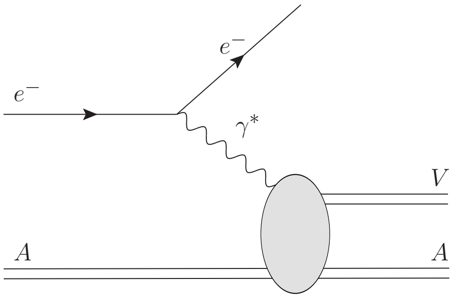
- Requires an exchange of at least two gluons

  $\Rightarrow$ Sensitive to the *squared* gluon density

- The momentum transfer $\boldsymbol{\Delta}$ can be measured

  - Conjugate of the impact parameter **b**

    $\Rightarrow$ Measures impact parameter dependent gluon distribution



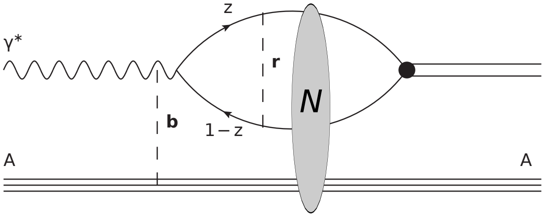
Quarkonium production at the leading order in the dipole picture

- Factorization at the high-energy limit:

Invariant amplitude for exclusive quarkonium production

$$\operatorname{Im} \mathcal{A}^\lambda = 2 \int \mathrm{d}^2\mathbf{b}\,\mathrm{d}^2\mathbf{r}\,\frac{\mathrm{d}z}{4\pi}\, e^{-i\left(\mathbf{b}+\left(\frac{1}{2}-z\right)\mathbf{r}\right)\cdot\boldsymbol{\Delta}}\, \Psi_{\gamma^*}^{q\bar{q}}(\mathbf{r}, z)\, N(\mathbf{r}, \mathbf{b}, Y)\, \Psi_V^{q\bar{q}*}(\mathbf{r}, z)$$

- $\Psi_{\gamma^*}^{q\bar{q}}$: Photon light-front wave function
- N: Dipole-target scattering amplitude
- $\Psi_V^{q\bar{q}}$: Quarkonium light-front wave function



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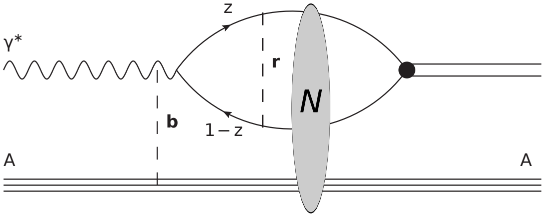
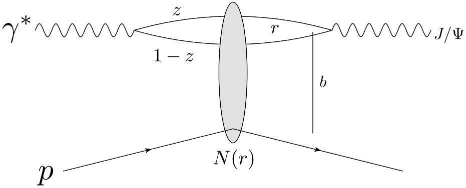
Mixed coordinate space:

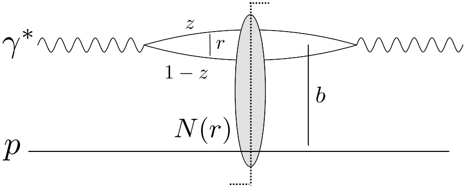
- Transverse separation **r**
- Longitudinal momentum fraction z
- Fixed in the dipole-target interaction



The dipole amplitude



$$\sigma^{\gamma^* p \to Vp} \sim |\text{dipole amplitude } N|^2$$



Optical theorem:

$$\sigma^{\gamma^* p} \sim \text{dipole amplitude } N$$

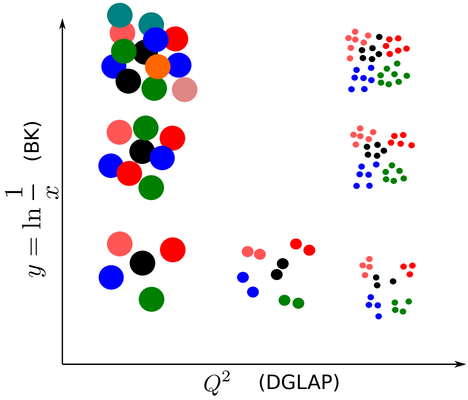
Universal dipole amplitude

The same dipole amplitude $N = 1 - \frac{1}{N_c} \operatorname{Tr}\left[V(\mathbf{x}) V^\dagger(\mathbf{y})\right]$ in appears different processes

- Convenient degrees of freedom at high energy:

  Wilson lines $V(\mathbf{x})$ and the dipole amplitude N

Rapidity evolution of the dipole amplitude



Perturbative evolution equation in rapidity $Y = \ln \frac{1}{x}$

- JIMWLK equation

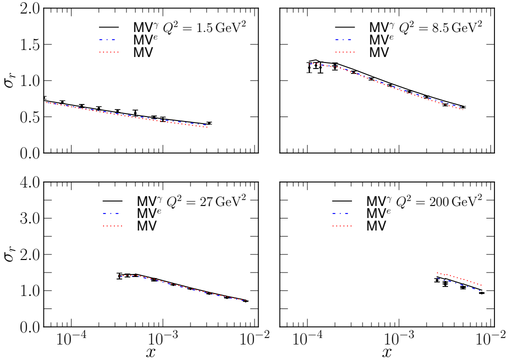
$\overset{\text{large } N_c}{\Rightarrow}$ Balitsky-Kovchegov (BK) equation:

$$\frac{\partial}{\partial Y} N(\mathbf{x}_{01}) = \frac{N_c \alpha_s}{2\pi^2} \int \mathrm{d}^2\mathbf{x}_2 \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2 \mathbf{x}_{21}^2}$$
$$\times \left[N(\mathbf{x}_{02}) + N(\mathbf{x}_{12}) - N(\mathbf{x}_{01}) - N(\mathbf{x}_{02}) N(\mathbf{x}_{12}) \right]$$

Needs a nonperturbative initial condition

Saturation at high energy (large Y): CGC

Initial condition for the dipole amplitude



Lappi, Mäntysaari, 1309.6963

- Common ansatz for the initial condition: the MV model and its generalizations

$$N_{\mathrm{MV}}(\mathbf{r}) = 1 - \exp\left[-\frac{1}{4}\mathbf{r}^2 Q_s^2 \ln\left(\frac{1}{\Lambda_{\mathrm{QCD}}^2 \mathbf{r}^2} + e\right)\right]$$

Q_s = saturation scale

- The initial condition can be fitted to HERA F_2 data
  - Gives a very good description of the data

Back to quarkonium production: Quarkonium wave function

Invariant amplitude for exclusive quarkonium production

$$\operatorname{Im} \mathcal{A}^\lambda = 2 \int \mathrm{d}^2\mathbf{b}\,\mathrm{d}^2\mathbf{r} \frac{\mathrm{d}z}{4\pi} e^{-i(\mathbf{b}+(\frac{1}{2}-z)\mathbf{r})\cdot\boldsymbol{\Delta}} \Psi_{\gamma^*}^{q\bar{q}}(\mathbf{r}, z) N(\mathbf{r}, \mathbf{b}, Y) \Psi_V^{q\bar{q}*}(\mathbf{r}, z)$$

- Now we have:
  - Photon wave function Ψ_{γ^*} (perturbative, calculate using light-cone perturbation theory)
  - Dipole amplitude N (nonperturbative initial condition + perturbative evolution)
- Final ingredient: quarkonium light-front wave function
- The quarkonium wave function is nonperturbative – a major source of uncertainty
- Nonrelativistic limit: $q\bar{q}$ at rest $\Psi_V^{q\bar{q}}(\vec{k}) \sim \delta^{(3)}(\vec{k}) \Leftrightarrow \Psi_V^{q\bar{q}}(\mathbf{r}, z) \sim \delta(z - \frac{1}{2})$

Light-front wave function from rest-frame wave function

- One way to constrain the light-front wave function is to start from the rest frame
- The rest frame wave function is described in terms of spin and 3-position $\vec{r} = (x^1, x^2, x^3)$
  - J^{PC} conservation $\Rightarrow$ a combination of S and D waves
- From NRQCD: D wave velocity-suppressed $\Rightarrow$ usually only S wave is considered:

$$\phi^{q\bar{q}}_{s\bar{s}}(\lambda, \vec{r}) = \frac{1}{\sqrt{2}} \xi_s^\dagger \epsilon^\lambda \cdot \sigma \chi_{\bar{s}} \, \phi(r)$$

  - ϵ^λ = the polarization vector of the quarkonium
  - ξ_s and $\chi_{\bar{s}}$ = quark and antiquark spinors with spins s and $\bar{s}$
  - ϕ = "scalar" part of the wave function – depends only on $r = |\vec{r}|$
- To get the light-front wave function in terms of helicity:

  Need to change the coordinate system and the spinor basis (Melosh rotation)

NRQCD-motivated wave function

- Relativistic effects can be introduced order by order using NRQCD
- Expand the wave function in the *rest frame* Lappi, Mäntysaari, J.P, 2006.02830

$$\phi(r) = \underbrace{\phi(0)}_{\mathcal{O}(v^0)} + \frac{1}{6}\underbrace{\nabla^2\phi(0)r^2}_{\mathcal{O}(v^2)} + \mathcal{O}(v^4)$$

- Two unknown constants $\phi(0), \nabla^2\phi(0)$
  - Related to wave function and its derivative at the origin $\vec{r} = 0$
  - Can be written in terms of NRQCD *long-distance matrix elements* (LDMEs)
  - For J/ψ: LDMEs determined from charmonium decay widths Bodwin et al., 0710.0994

$$\Rightarrow \phi(0) = 0.213\ \mathrm{GeV}^{3/2}, \quad \frac{1}{6}\nabla^2\phi(0) = -0.0157\ \mathrm{GeV}^{7/2}$$

Other approaches to the quarkonium light-front wave function

Basis Light-Front Quantization (*BLFQ*)

Li, Maris, Vary, 1704.06968

- A light-cone Hamiltonian approach
- Solve the eigenstates of the light-cone Hamiltonian $\Rightarrow$ quarkonium states
- Fitted to the quarkonium masses

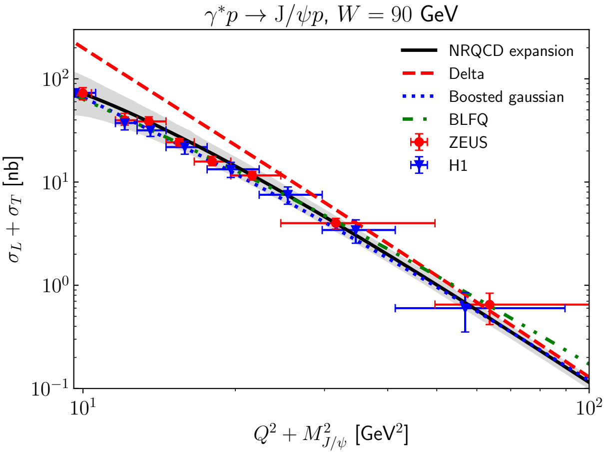
Boosted Gaussian

Kowalski, Motyka, Watt, hep-ph/0606272

- Assume a Gaussian form for the wave function
- The same spin structure as photon (a mixture of S and D waves)
- Fitted to the leptonic width

Also other approaches, such as deducing the rest-frame wave function from potential models (see e.g. Cepila et al., 1901.02664)

Quarkonium production at LO as a function of the photon virtuality Q^2



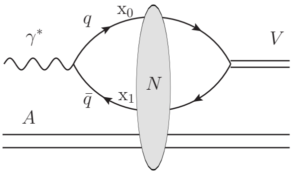
Lappi, Mäntysaari, J.P, 2006.02830

HERA data from hep-ex/0510016 and hep-ex/0404008

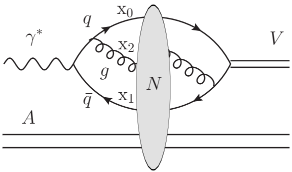
- Delta = nonrelativistic limit
- Probes the wave function at the distance $\mathbf{r} \sim 1/(Q^2 + M^2_{\mathrm{J}/\psi})$ $\Rightarrow$ relativistic effects more important at small Q^2
- Q^2 dependence of Delta in disagreement with the data
- Relativistic effects needed: Other wave functions describe the data well

Exclusive quarkonium production at NLO

LO:



NLO:

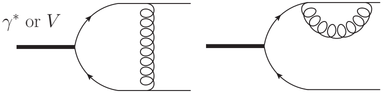


- At higher orders: perturbative corrections from gluon emission
  - Can be incorporated into the photon and quarkonium light-front wave functions
- Calculate α_s corrections at the nonrelativistic limit for the quarkonium wave function
  - Relativistic effects higher order in NRQCD power counting
  - Nonrelativistic limit can be written in terms of the leptonic width

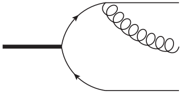
    – no unknown nonperturbative constants coming from the wave function

Corrections to the wave functions

$q\bar{q}$ (virtual corrections):



$q\bar{q}g$ (real corrections):



- Corrections from virtual gluon loops and real gluon emission
  - Nonperturbative elements of the $V \to q\bar{q}g$ wave function suppressed by velocity v
- Calculations of the NLO corrections to the light-front wave function are very recent developments
  - Quarkonium Escobedo, Lappi, 1911.01136
  - Virtual photon with massive quarks Beuf, Lappi, Paatelainen, 2103.14549, 2112.03158

Cancellation of divergences

- UV divergences between the $q\bar{q}$ and $q\bar{q}g$ parts of the calculation cancel
- IR divergences cancel when one takes into account:
  - Renormalization of the leading-order wave function $\phi^{q\bar{q}}(\vec{r}=0)$
    - Can be written in terms of the leptonic width

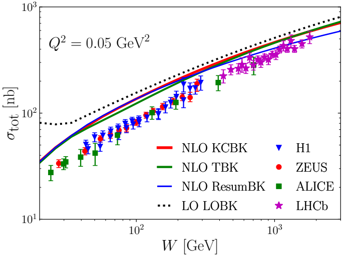
$$\Gamma(V \to e^- e^+) \sim \left|\phi^{q\bar{q}}(0)\right|^2 \left[1 + \frac{2\alpha_s C_F}{\pi}\left(\frac{1}{2\alpha} - 2\right)\right], \alpha = \text{gluon IR cutoff}$$

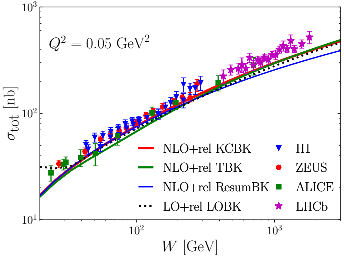
  - The rapidity dependence of the dipole amplitude which can be described in terms of the Balitsky-Kovchegov equation

⇒ The total production amplitude is finite and can be numerically evaluated

- Longitudinal NLO production: Mäntysaari, J.P, 2104.02349
- Transverse NLO production: Mäntysaari, J.P, 22XX.XXXXX

Total cross section at NLO as a function of the center-of-mass energy W





- Total J/ψ production at NLO with different dipole amplitudes
- NLO calculated in the nonrelativistic limit
- Lower plot: v^2 relativistic corrections at LO included
  - NRQCD: $1 > \alpha_s > v^2 > \ldots$
- Including relativistic corrections results in a better agreement with the data

  $\Rightarrow$ Both NLO and v^2 corrections numerically important
- Caveat: quark masses missing from the dipole amplitude fit

Summary

- In the CGC framework, exclusive quarkonium production can be divided into three parts
  - Photon splitting into a $q\bar{q}$ dipole
  - Dipole amplitude describing the interaction of the dipole with the nucleus
  - Formation of the quarkonium
- This framework has been successful in describing exclusive quarkonium production at LO
  - Main uncertainty coming from the quarkonium wave function
  - Relativistic effects important at small photon virtualities
- NLO calculations starting to become available in the nonrelativistic limit

Backup

Backup - Relativistic corrections to the wave function

We use the wave function from Lappi, Mäntysaari, JP, 2006.02830 which includes the relativistic corrections at the order v^2:

$$\Psi^{\lambda=0}_{+-}(\mathbf{r},z) = \Psi^{\lambda=0}_{-+}(\mathbf{r},z) = \frac{\pi\sqrt{2}}{\sqrt{m_c}}\left[A\delta(z-1/2) + \frac{B}{m_c^2}\left(\left(\frac{5}{2}+\mathbf{r}^2 m_c^2\right)\delta(z-1/2) - \frac{1}{4}\partial_z^2\delta(z-1/2)\right)\right]$$

$$\Psi^{\lambda=1}_{++}(\mathbf{r},z) = \Psi^{\lambda=-1}_{--}(\mathbf{r},z) = \frac{2\pi}{\sqrt{m_c}}\left[A\delta(z-1/2) + \frac{B}{m_c^2}\left(\left(\frac{7}{2}+\mathbf{r}^2 m_c^2\right)\delta(z-1/2) - \frac{1}{4}\partial_z^2\delta(z-1/2)\right)\right]$$

$$\Psi^{\lambda=1}_{+-}(\mathbf{r},z) = -\Psi^{\lambda=1}_{-+}(\mathbf{r},z) = \left(\Psi^{\lambda=-1}_{-+}(\mathbf{r},z)\right)^* = \left(-\Psi^{\lambda=-1}_{+-}(\mathbf{r},z)\right)^* = -\frac{2\pi i}{m_c^{3/2}}B\delta(z-1/2)(r_1+ir_2)$$

$$\Psi^{\lambda=1}_{--}(\mathbf{r},z) = \Psi^{\lambda=-1}_{++}(\mathbf{r},z) = \Psi^{\lambda=0}_{++}(\mathbf{r},z) = \Psi^{\lambda=0}_{--}(\mathbf{r},z) = 0$$

$$A = \phi(0) = 0.213\ \mathrm{GeV}^{3/2}, \quad B = \frac{1}{6}\nabla^2\phi(0) = -0.0157\ \mathrm{GeV}^{7/2}$$

Backup - Final expression (longitudinal production)

$$-i\mathcal{A}^L = -Q\sqrt{\Gamma(V\to e^-e^+)\frac{3M_V}{16\pi^2\alpha_{\mathrm{em}}}}\int \mathrm{d}^2\mathbf{x}_{01}\int \mathrm{d}^2\mathbf{b}\left\{\mathcal{K}^{\mathrm{LO}}_{q\bar q}(Y_0)+\frac{\alpha_s C_F}{2\pi}\mathcal{K}^{\mathrm{NLO}}_{q\bar q}(Y_{\mathrm{dip}})+\frac{\alpha_s C_F}{2\pi}\int \mathrm{d}^2\mathbf{x}_{20}\int_{z_{\min}}^{1/2}\mathrm{d}z_2\,\mathcal{K}_{q\bar q g}(Y_{\mathrm{qqg}})\right\}$$

where $\mathcal{K}^{\mathrm{LO}}_{q\bar q}(Y_0)=K_0(\zeta)N_{01}(Y_0)$, $\zeta=|\mathbf{x}_{01}|\sqrt{\frac{1}{4}Q^2+m_q^2}$,

$$\mathcal{K}^{\mathrm{NLO}}_{q\bar q}(Y_{\mathrm{dip}}) = \left[\mathcal{K}+\tilde{\mathcal{I}}_{\mathcal{V}}\left(z=\frac{1}{2},\mathbf{x}_{01}\right)+K_0(\zeta)\left(6-\frac{\pi^2}{3}+\Omega_{\mathcal{V}}\left(\gamma;z=\frac{1}{2}\right)+L\left(\gamma;z=\frac{1}{2}\right)-3\log\left(\frac{|\mathbf{x}_{10}|m}{2}\right)-3\gamma_E\right)\right]N_{01}(Y_{\mathrm{dip}})$$

and

$$\mathcal{K}_{q\bar q g}(Y_{\mathrm{q\bar qg}}) = -32\pi m_q\left\{\frac{i\mathbf{x}_{20}^i}{|\mathbf{x}_{20}|}K_1(2m_q z_2|\mathbf{x}_{20}|)\left[\left((1-z_2)^2+z_2^2\right)\mathcal{I}^i_{(f)}+(2z_2^2-1)(1-2z_2)\mathcal{I}^i_{(g)}\right]N_{012}(Y_{\mathrm{q\bar qg}})\right.$$
$$\left.+4m_q z_2^3K_1(2m_q z_2|\mathbf{x}_{20}|)\left[\mathcal{I}_{(f)}-\frac{1-2z_2}{1+2z_2}\mathcal{I}_{(g)}\right]N_{012}(Y_{\mathrm{q\bar qg}})+\frac{1}{8\pi^2}\left((1-z_2)^2+z_2^2\right)\frac{1}{m_q z_2|\mathbf{x}_{20}|^2}K_0(\zeta)e^{-\mathbf{x}_{20}^2/(\mathbf{x}_{10}^2e^{\gamma_E})}N_{01}(Y_{\mathrm{q\bar qg}})\right\}.$$

Equation for transverse production similar but more complicated.

Backup - Balitsky-Kovchegov equation

- The $q\bar{q}g$ part is singular at $\alpha \to 0$
- This is related to the rapidity evolution of the dipole amplitude, described by the Balitsky-Kovchegov (BK) equation:

$$\frac{\partial}{\partial Y} N_{01} = \frac{N_c \alpha_s}{2\pi^2} \int \mathrm{d}^2 \mathbf{x}_2 \, \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2 \mathbf{x}_{21}^2} \left[N_{02} + N_{12} - N_{01} - N_{02} N_{12} \right]$$

- In fact, we can write:

$$\begin{aligned} \frac{\alpha_s}{2\pi} \int \mathrm{d}^2 \mathbf{x}_2 \int_\alpha^{1/2} \mathrm{d} z_2 \, \mathcal{K}_{q\bar{q}g} &= K_0(\zeta) \int \mathrm{d}^2 \mathbf{x}_2 \int_\alpha^{1/2} \mathrm{d} z_2 \, \frac{N_c \alpha_s}{2\pi^2 z_2} \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2 \mathbf{x}_{21}^2} \left[N_{02} + N_{12} - N_{01} - N_{02} N_{12} \right] + \text{nonsingular part} \\ &= K_0(\zeta) \left[N_{01}\Big(Y(z_2 = 1/2) \Big) - N_{01}\Big(Y(z_2 = \alpha) \Big) \right] + \text{nonsingular part} \end{aligned}$$

- Combining this with the LO result, we get $Y(z_2 = \alpha) \to Y(z_2 = 1/2)$ for the evolution rapidity in the LO term