

Spin alignment in quarkonium production in SIDIS

In collaboration with: U. D'Alesio, F. Murgia, C. Pisano, R. Sangem



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QUARKONIUM PUZZLE

Quarkonium formation is described via different **models**
different ways to evaluate *short-* and *long-* distance scales

CSM

$$\sigma(\mathcal{Q}) = \hat{\sigma}(Q\bar{Q})|R(0)|^2$$

R. Baier & R. Rückl (1983)
E.L. Berger & D.L. Jones (1981)

CEM

$$\sigma(\mathcal{Q}) = P_{\mathcal{Q}} \int_{2m_Q}^{M_T} \frac{d\hat{\sigma}(m_{Q\bar{Q}})}{dm_{Q\bar{Q}}} dm_{Q\bar{Q}}$$

H. Fritzsch (1977)
F. Halzen (1977)

NRQCD

G.T. Bodwin, E. Braaten, G.P. Lepage (1997)
P. Cho & K. Leibovich (1996)

$$\sigma(\mathcal{Q}) = \sum_n \hat{\sigma}(Q\bar{Q}[n]) \langle \mathcal{O}[n] \rangle$$

FF

G. C. Nayak, J. W. Qiu, G. Sterman (2005)
Z. B. Kang, J. W. Qiu, G. Sterman (2014)

$$\begin{aligned} \sigma_{\mathcal{Q}}(p_T \gg m_{\mathcal{Q}}) &= d\hat{\sigma}_i(p_T/z) \otimes D_{i \rightarrow \mathcal{Q}}(z, m_{\mathcal{Q}}) \\ &+ d\sigma_{Q\bar{Q}[c]}(P_{Q\bar{Q}[c]} = p_T/z) \otimes D_{Q\bar{Q}[c] \rightarrow \mathcal{Q}}(z, m_{\mathcal{Q}}) \end{aligned}$$

POLARIZATION AS A KEY OBSERVABLE

Polarization is **less dependent** from theoretical uncertainties

(for spin-1)

(factorization scale, heavy quark masses, LDME, ...)

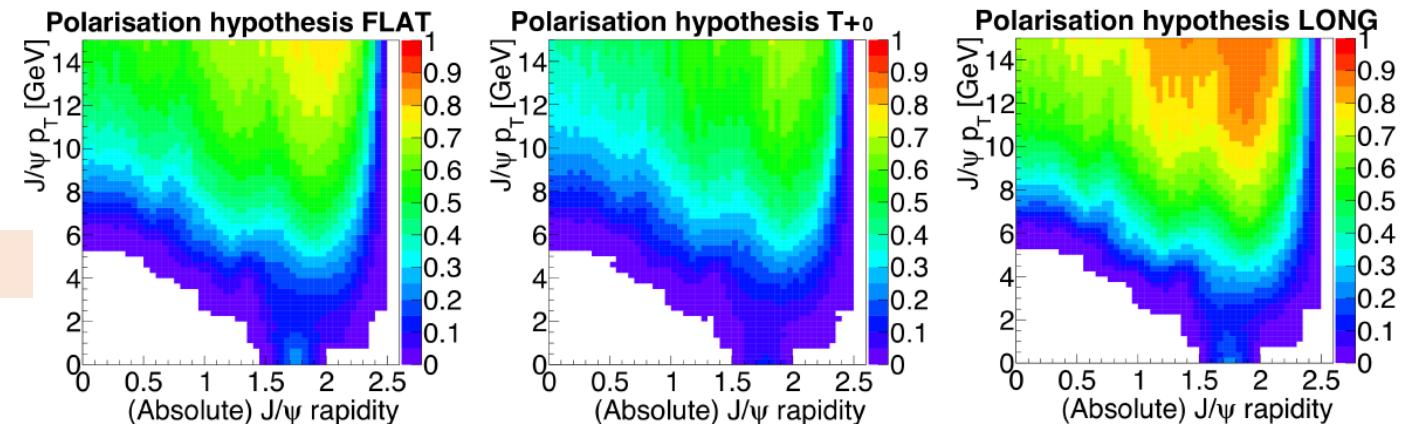
$$d\sigma \sim 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi$$

Angular parameters are defined through **ratio** of polarized cross sections

Complete analysis of quarkonium polarization requires
measurement of both **polar** and **azimuthal** angles

Impact on detector acceptance:

Aad et al. (ATLAS Collaboration), NuclPhysB 850 (2011)



OUTLINE

Matching procedure (in NRQCD)
Section A

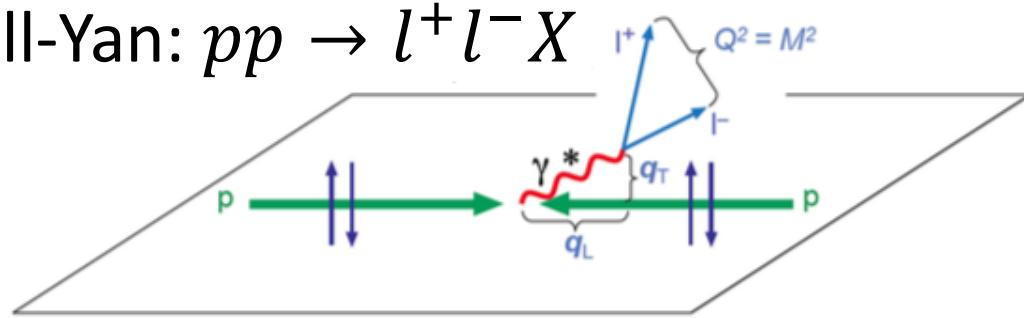
HERA recap
Section B

EIC preliminary results for the collinear region
Section C

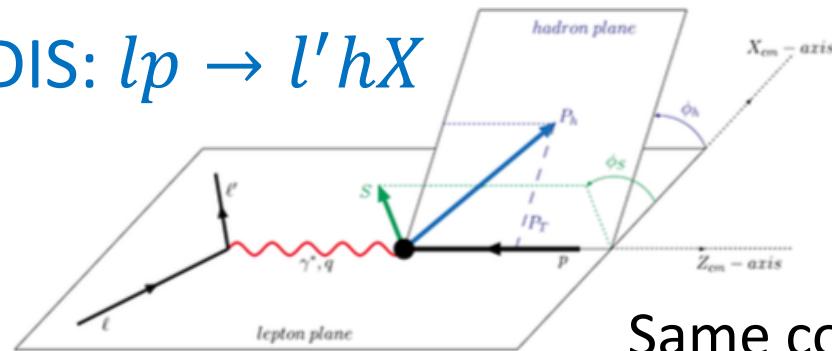
TMD FACTORIZATION

TMD factorization is formally proven only for few processes

Drell-Yan: $pp \rightarrow l^+ l^- X$

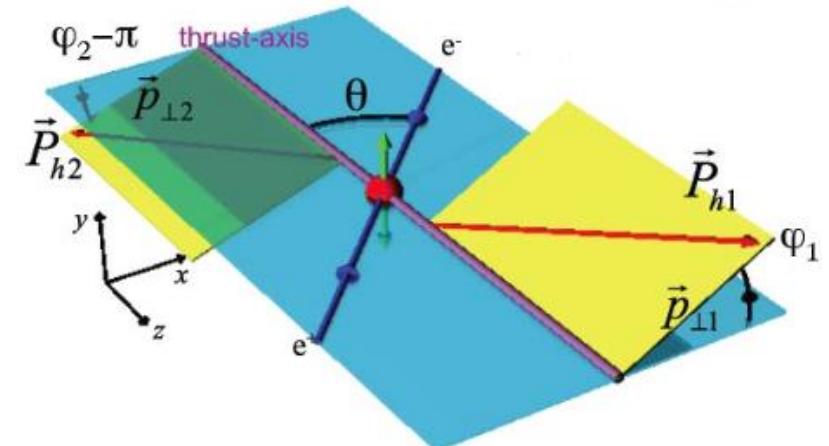


SIDIS: $lp \rightarrow l' h X$



Collins, Cambridge University Press (2011)

Echevarría Idibi Scimemi, JHEP 07 (2012)
lepton annihilation: $e^+ e^- \rightarrow \pi\pi X$



Same color flow for quarkonium production
→ No factorization breaking expected

ANGULAR STRUCTURE OF THE CROSS SECTION

J/ ψ polarization is accessed by the angular distribution of its decay products

$$J/\psi \rightarrow l^+ l^-$$

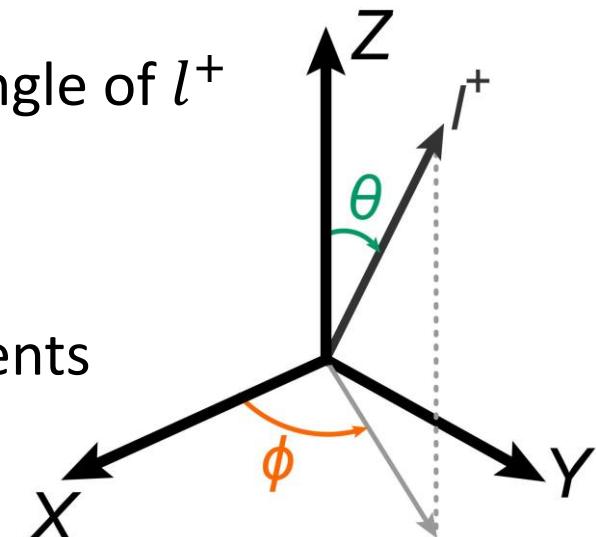
Faccioli, Lourenço, Seixas, Wöhri, EPJC 69 (2010)

SIDIS cross section is parameterized as

$$d\sigma \propto \mathcal{W}_T(1 + \cos^2 \theta) + \mathcal{W}_L(1 - \cos^2 \theta) + \mathcal{W}_\Delta \sin 2\theta \cos \phi + \mathcal{W}_{\Delta\Delta} \sin^2 \theta \cos 2\phi$$

with $\Omega(\theta, \phi)$ solid angle of l^+

Boer & Vogelsang, PRD 74 (2006)



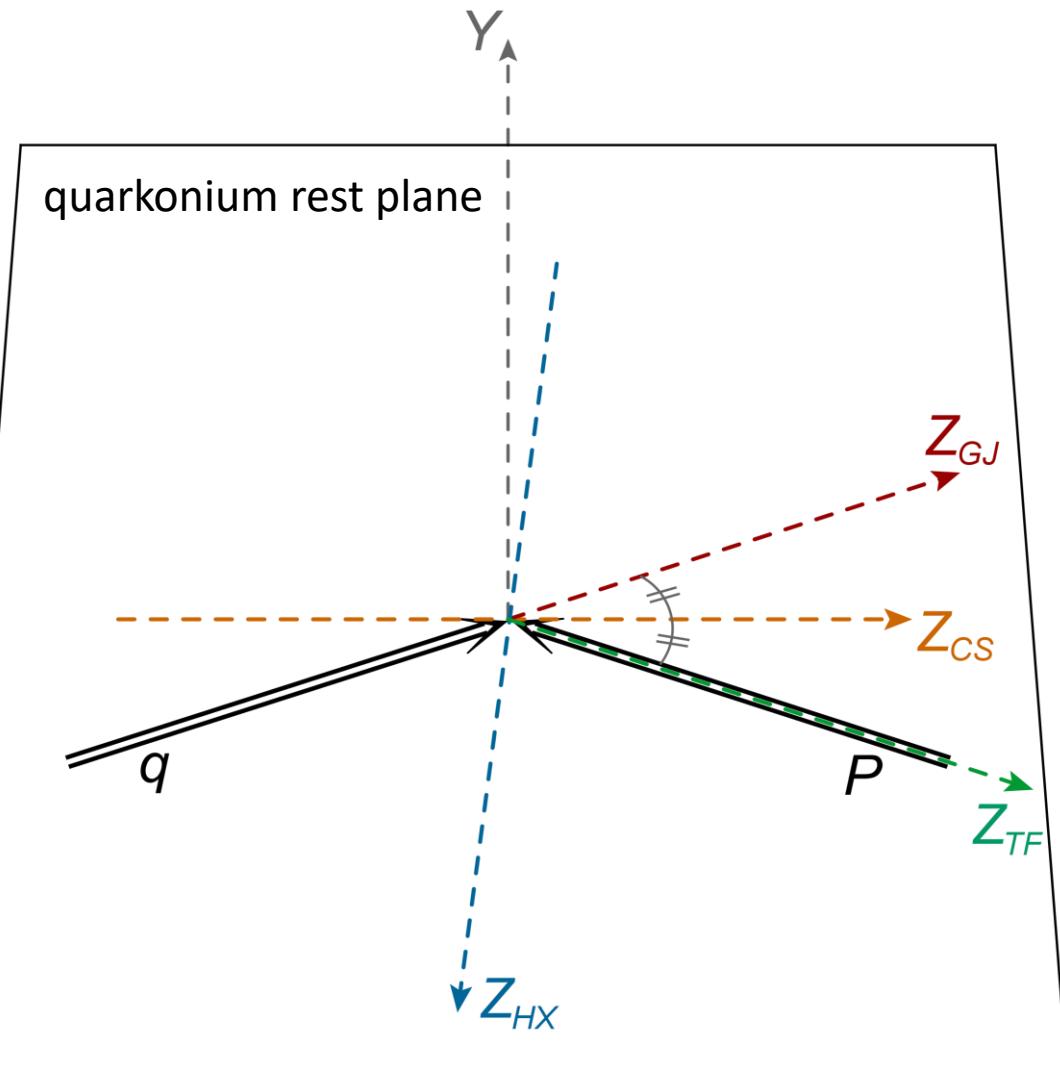
The parameterization could be obtained from model independent arguments

Hermiticity

Parity conservation

Gauge invariance

SPIN-QUANTIZATION FRAME



J/ψ polarization is studied in the
quarkonium rest frame

$$\gamma^*(q) + p(P) \rightarrow J/\psi(P_\psi) + X$$

Different choices for the reference frame

GJ *Gottfried-Jackson frame*

CS *Collins-Soper frame*

HX *Helicity frame*

TF *Target frame*

Frames are related by a rotation around Y -axis

J/ψ POLARIZATION IN NRQCD

In the NRQCD approach there is a double expansion: α_s and v

up to v^4 order

$^3S_1^{[1]}$,

$^1S_0^{[8]}$,

$^3S_1^{[8]}$,

$^3P_J^{[8]}$

unpolarized

$J = 0, 1, 2$

NRQCD symmetries allow **interference** among states with same **L** and **S**

$$\mathcal{W}_\Lambda^\mathcal{P} = \mathcal{W}_\Lambda^\mathcal{P} \left[{}^3S_1^{(1)} \right] + \mathcal{W}_\Lambda^\mathcal{P} \left[{}^1S_0^{(8)} \right] + \mathcal{W}_\Lambda^\mathcal{P} \left[{}^3S_1^{(8)} \right] + \mathcal{W}_\Lambda^\mathcal{P} \left[\{L = 1, S = 1\}^{(8)} \right]$$

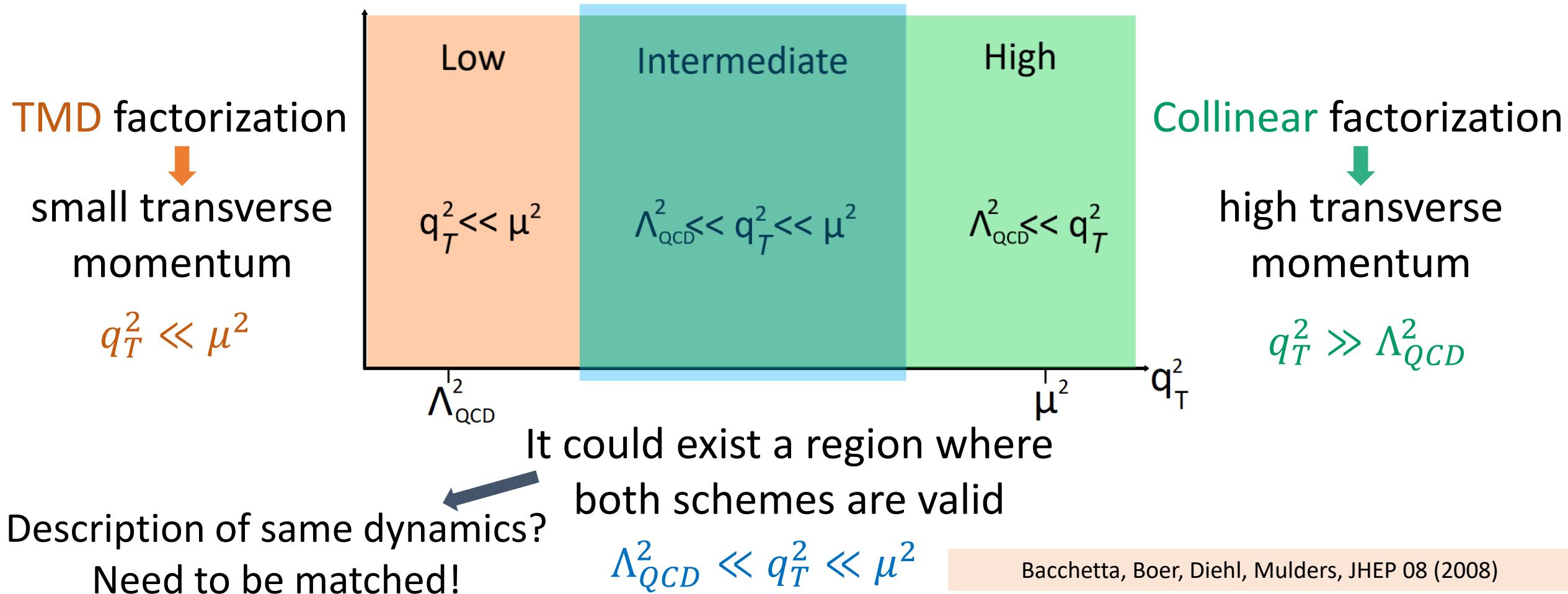
→ $\Lambda = T, L, \Delta, \Delta\Delta$ J/ψ helicity

$\mathcal{P} = T, L$ γ^* polarization

Beneke, Krämer, Vänttinen, PRD 57 (1998)

FACTORIZATION SCHEME

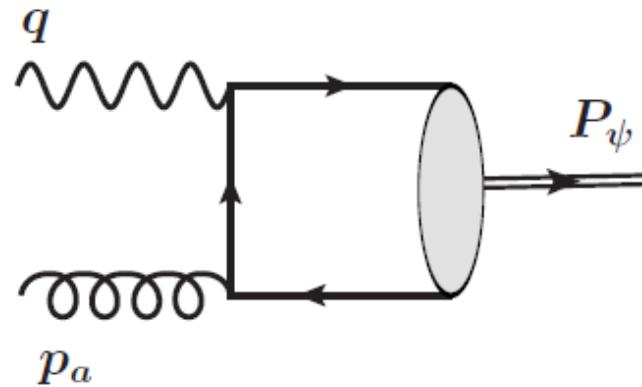
In the J/ψ rest frame the virtual photon has a transverse momentum (TM) q_T



J/ψ POLARIZATION AT SMALL q_T

Partonic subprocesses at order $\alpha\alpha_s$

$$\gamma^*(q) + g(p_a) \rightarrow c\bar{c}[n](P_\psi)$$



described by
polarized partonic cross section \tilde{w}_Λ^P

4 frame independent \mathcal{W} helicity structure functions survive

$$\mathcal{W}_T^\perp = \tilde{w}_T^\perp f_1(x, \mathbf{q}_T^2)$$

$$\mathcal{W}_L^\perp = \tilde{w}_L^\perp f_1(x, \mathbf{q}_T^2)$$

$$\mathcal{W}_L^{\parallel} = \tilde{w}_L^{\parallel} f_1(x, \mathbf{q}_T^2)$$

$$\mathcal{W}_{\Delta\Delta}^\perp = \tilde{w}_{\Delta\Delta}^\perp h_1^\perp(x, \mathbf{q}_T^2)$$

Neglecting smearing effects:

D'Alesio, LM, Murgia, Pisano, Sangem, under review

proportional to $\langle \mathcal{O}_8[{}^3P_0] \rangle$

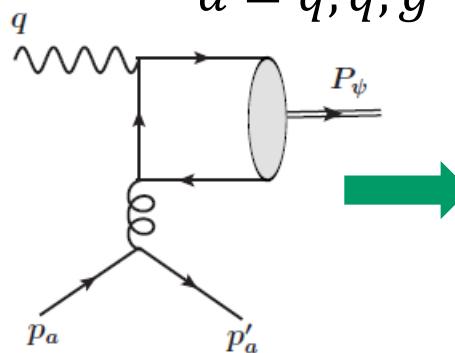
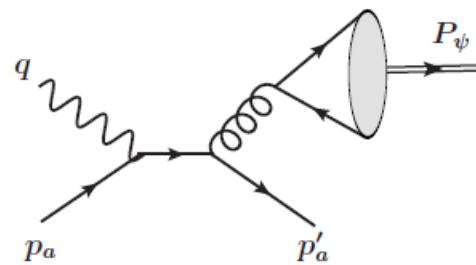
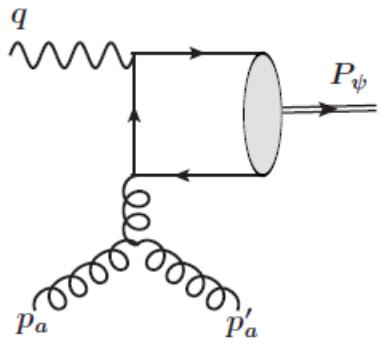
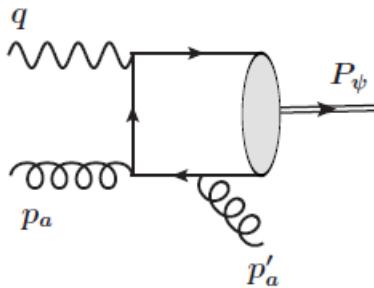
access to

J/ψ POLARIZATION AT HIGH q_T

Partonic subprocesses at order $\alpha\alpha_s^2$

$$\gamma^*(q) + a(p_a) \rightarrow c\bar{c}[n](P_\psi) + a(p'_a)$$

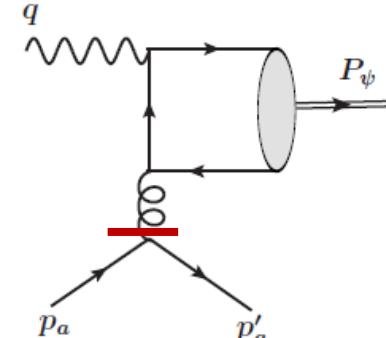
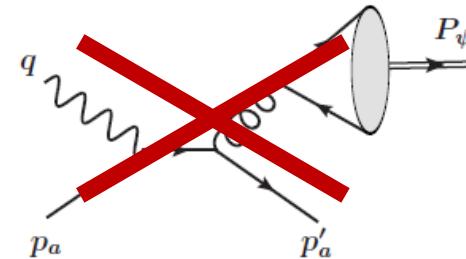
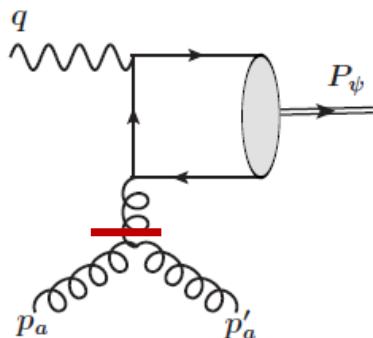
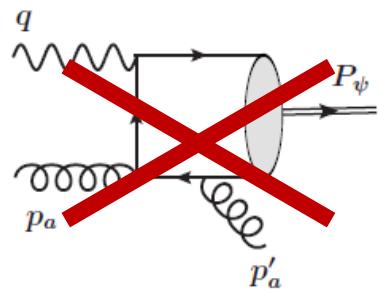
$a = q, \bar{q}, g$



in total:
12 diagrams

Full \mathcal{W}_Λ^P evaluated for all $q_T \gg \Lambda_{\text{QCD}}$

For small q_T limit
divergences are
related to $\alpha\alpha_s$
 γ^*g subprocesses



MATCHING AND SMEARING EFFECT

In the $\Lambda_{QCD} \ll q_T \ll Q$ region

$$(\Lambda, \mathcal{P}) = (L, \perp), (T, \perp), (L, \parallel)$$

$$\mathcal{W}_{\Delta\Delta}^\perp|_{\text{coll}} - \mathcal{W}_{\Delta\Delta}^\perp|_{\text{TMD}} = 0 \quad \mathcal{W}_\Lambda^{\mathcal{P}}|_{\text{coll}} - \mathcal{W}_\Lambda^{\mathcal{P}}|_{\text{TMD}} \neq 0$$

(P. Taels talk)

→ matching requires *shape functions*

Echevarria, JHEP 10 (2019)

Fleming, Makris, Mehen, JHEP 04 (2020)

Shape function $\Delta^{[n]}$ is a TMD generalization of NRQCD LDME

$$f_1^g \longrightarrow \mathcal{C} [f_1^g \Delta^{[n]}] \longrightarrow \Delta^{[n]}(\mathbf{k}_T; \mu) = C_A \frac{\alpha_s}{2\pi^2 \mathbf{k}_T^2} <\mathcal{O}[n]> \ln \frac{\mu^2}{\mathbf{k}_T^2} k_T^2 \gg m_p^2$$

$$h_1^{\perp g} \longrightarrow \mathcal{C} [w h_1^{\perp g} \Delta^{[n]}] \longrightarrow \Delta^{[n]}(\mathbf{k}_T, \mu^2) \text{ not observable at this } \alpha_s \text{ order}$$

Boer, D'Alesio, Murgia, Pisano, Taels, JHEP 09 (2020)

D'Alesio, LM, Murgia, Pisano, Sangem, under review

COLLINEAR PHENOMENOLOGY

Experimentally a different parameterization is usually adopted for $d\sigma \equiv \frac{d\sigma}{dx_B dy dz d^4 P_\psi d\Omega}$

$$d\sigma \propto 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi$$

$$\lambda = \frac{\mathcal{W}_T - \mathcal{W}_L}{\mathcal{W}_T + \mathcal{W}_L} \quad \mu = \frac{\mathcal{W}_\Delta}{\mathcal{W}_T + \mathcal{W}_L} \quad \nu = \frac{2\mathcal{W}_{\Delta\Delta}}{\mathcal{W}_T + \mathcal{W}_L} \quad \text{where} \quad \begin{array}{ll} \lambda = +1 & \xrightarrow{\hspace{1cm}} \text{transverse} \\ \lambda = -1 & \xrightarrow{\hspace{1cm}} \text{longitudinal} \end{array}$$

Next: focus on λ in CSM and NRQCD at scale $\mu_0/2 < \mu < 2\mu_0$ $\mu_0 = \sqrt{M_\psi^2 + Q^2}$

NRQCD with different LDME choices

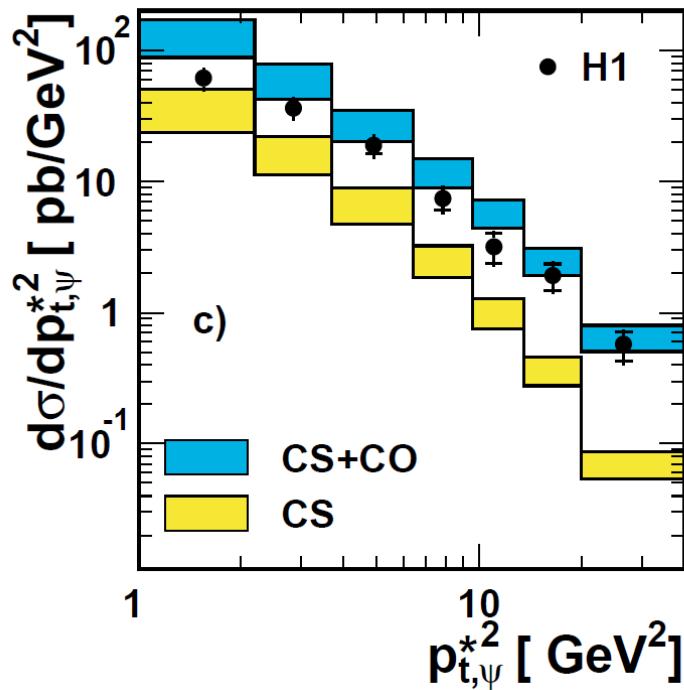
- | | | | |
|------|---|-------------------|---|
| C12 | Chao, Ma, Shao, Wang, Zhang, PRL 108 (2012) | \longrightarrow | include polarization data |
| G13 | Gong, Wan, Wang, Zhang, PRL 110 (2013) | \longrightarrow | tested on polarization data |
| BK11 | Butenschoen & Kniehl, PRD 84 (2011) | \longrightarrow | include low- P_T and photoproduction unpolarized data |

HERA UNPOLARIZED DATA

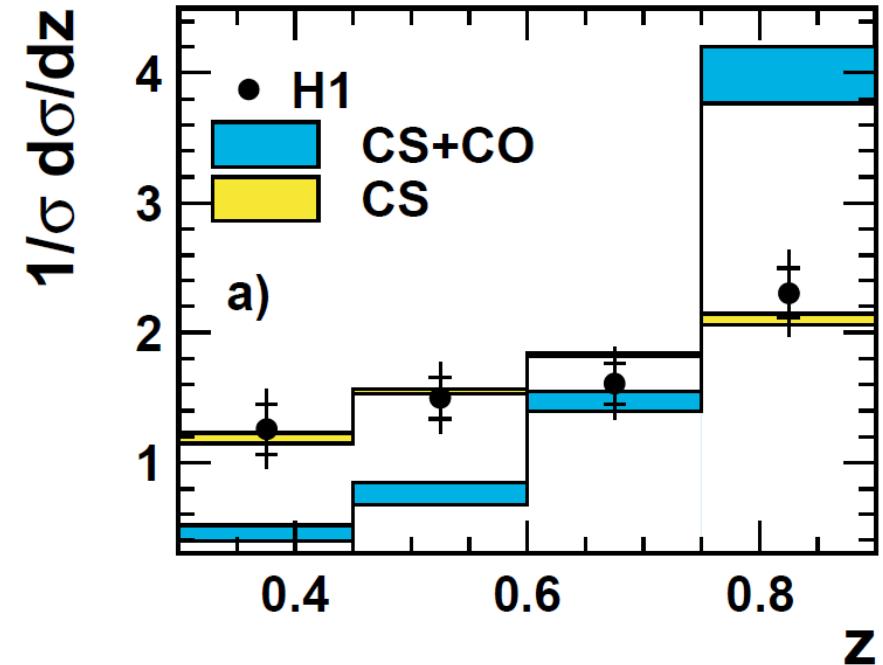
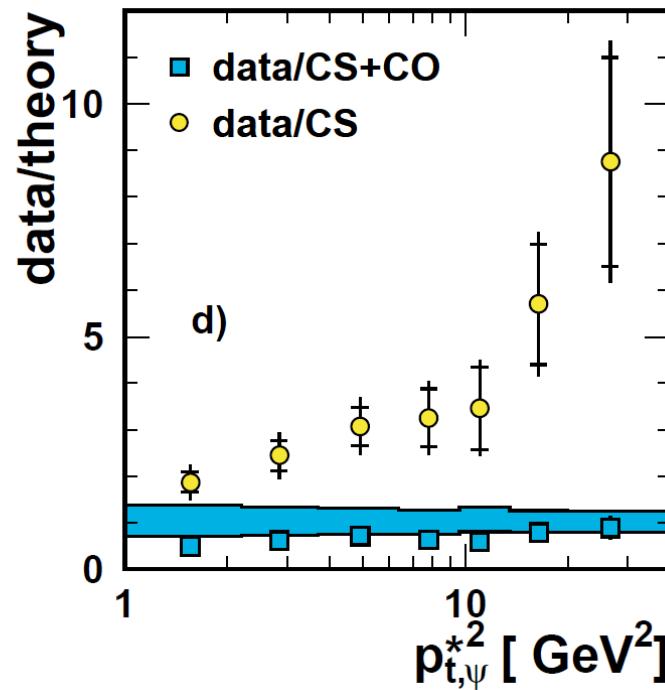
Adloff et al. (H1 Collaboration), EPJ C 25 (2002)

Kniehl & Zwirner, NPB 621 (2002)

Data from HERA collaboration



Theoretical prediction obtained by Kniehl-Zwirner



P_T data show a general better agreement with NRQCD predictions

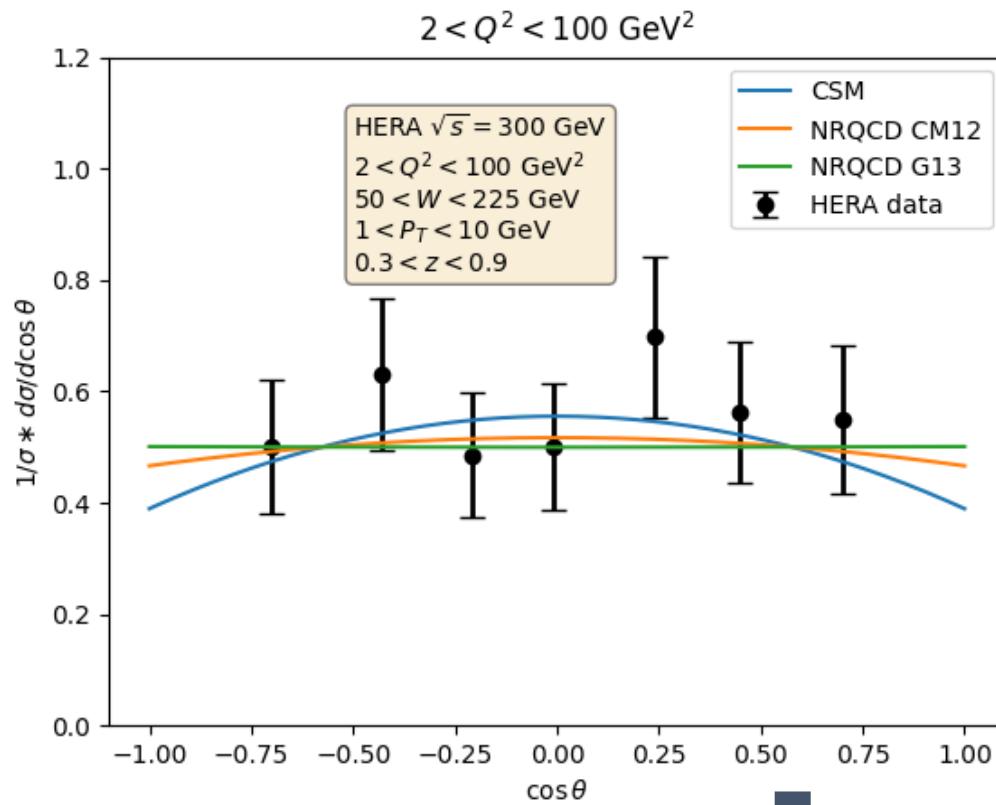
z (multiplicity) data show a general better agreement with CSM predictions

HERA POLARIZED DATA

Adloff et al. (H1 Collaboration), EPJ C 25 (2002)

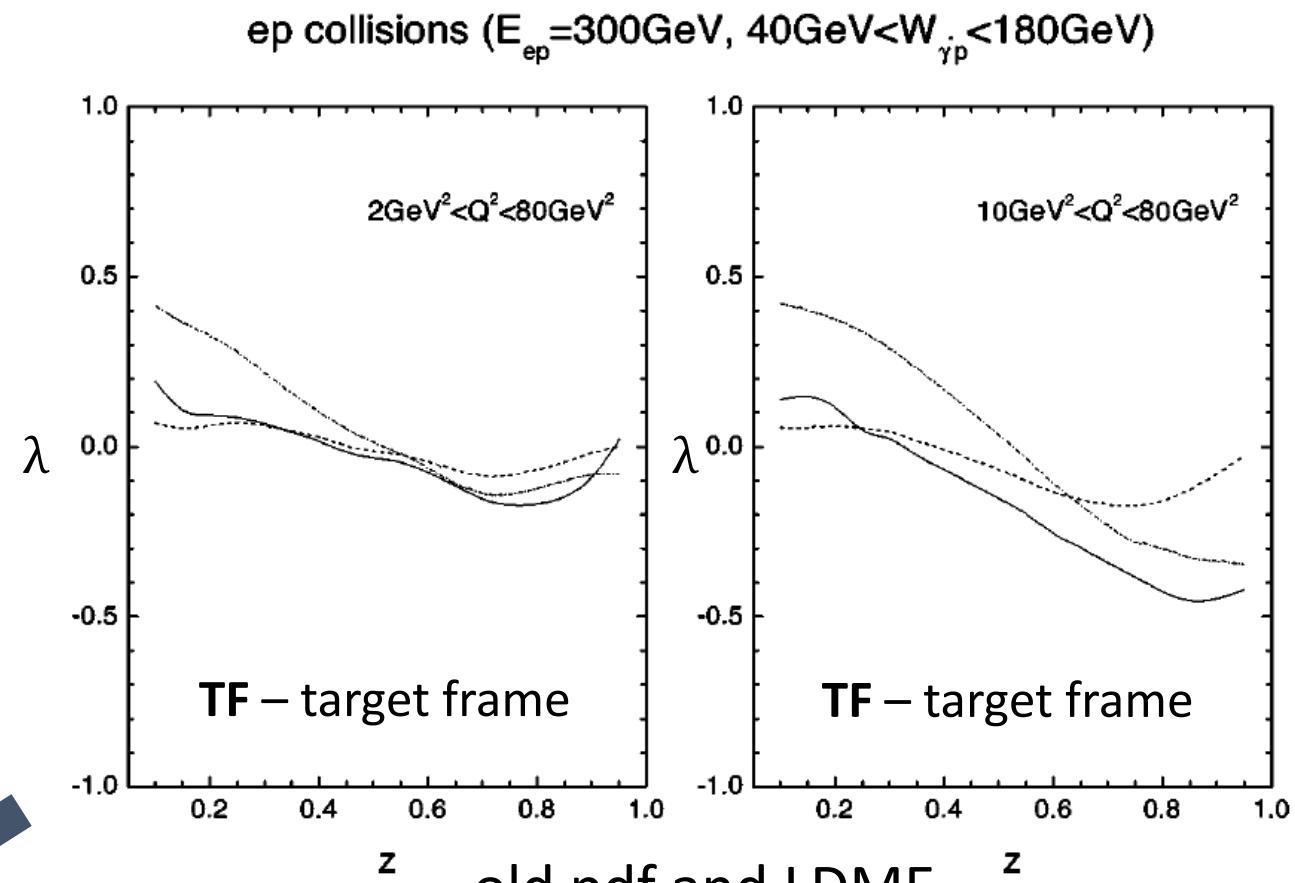
Yuan & Chao, PRD 63 (2001)

Data from HERA collaboration



Hard to extract information

From Yuan-Chao paper



old pdf and LDME
includes resolved photon contribution

CEM at HERA

No result yet regarding polarization in SIDIS within CEM

CEM predictions respect to photoproduction are promising for future SIDIS polarization result

Data from HERA collaboration

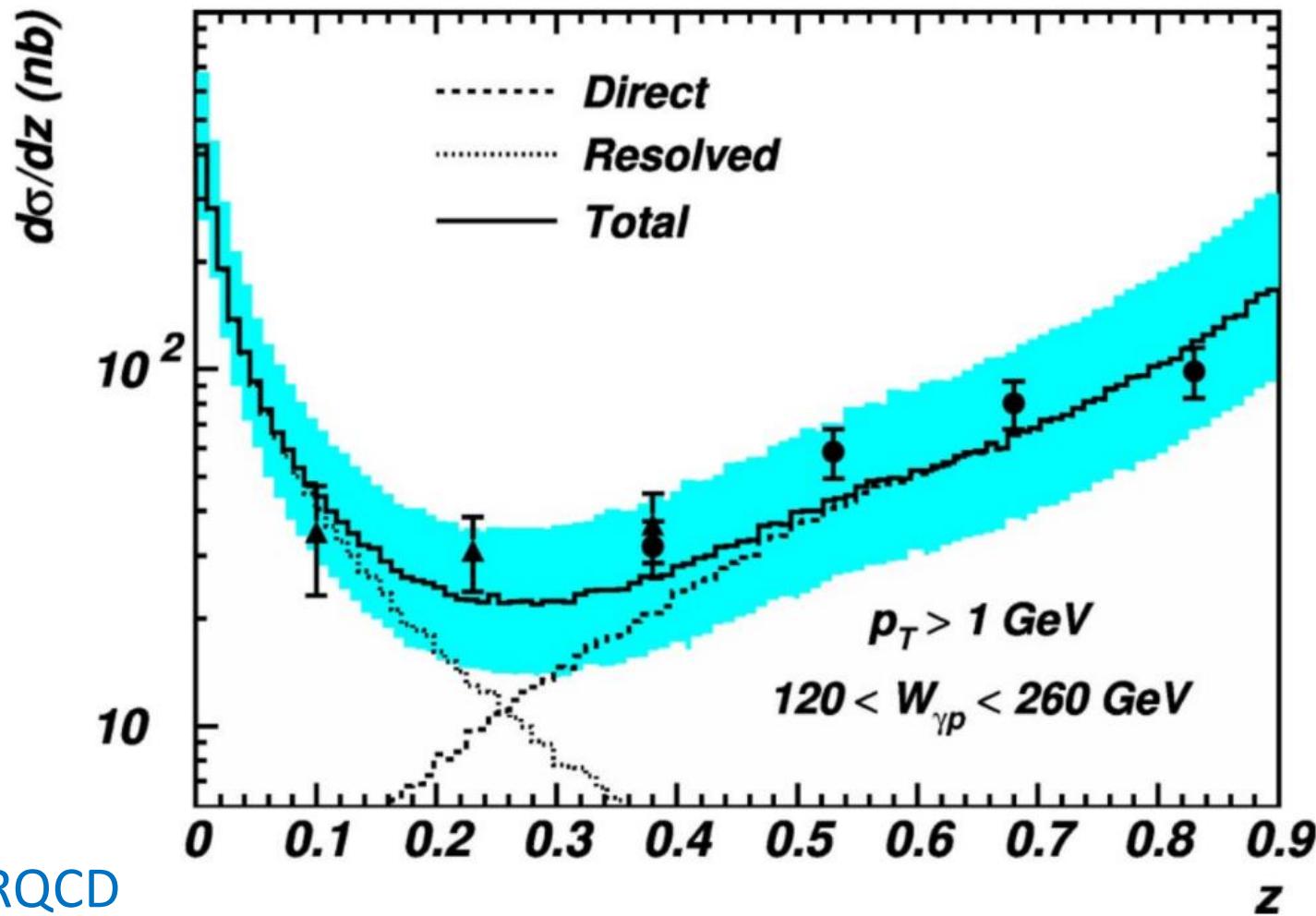
Adloff et al. (H1 Collaboration), EPJ C 25 (2002)

Prediction from

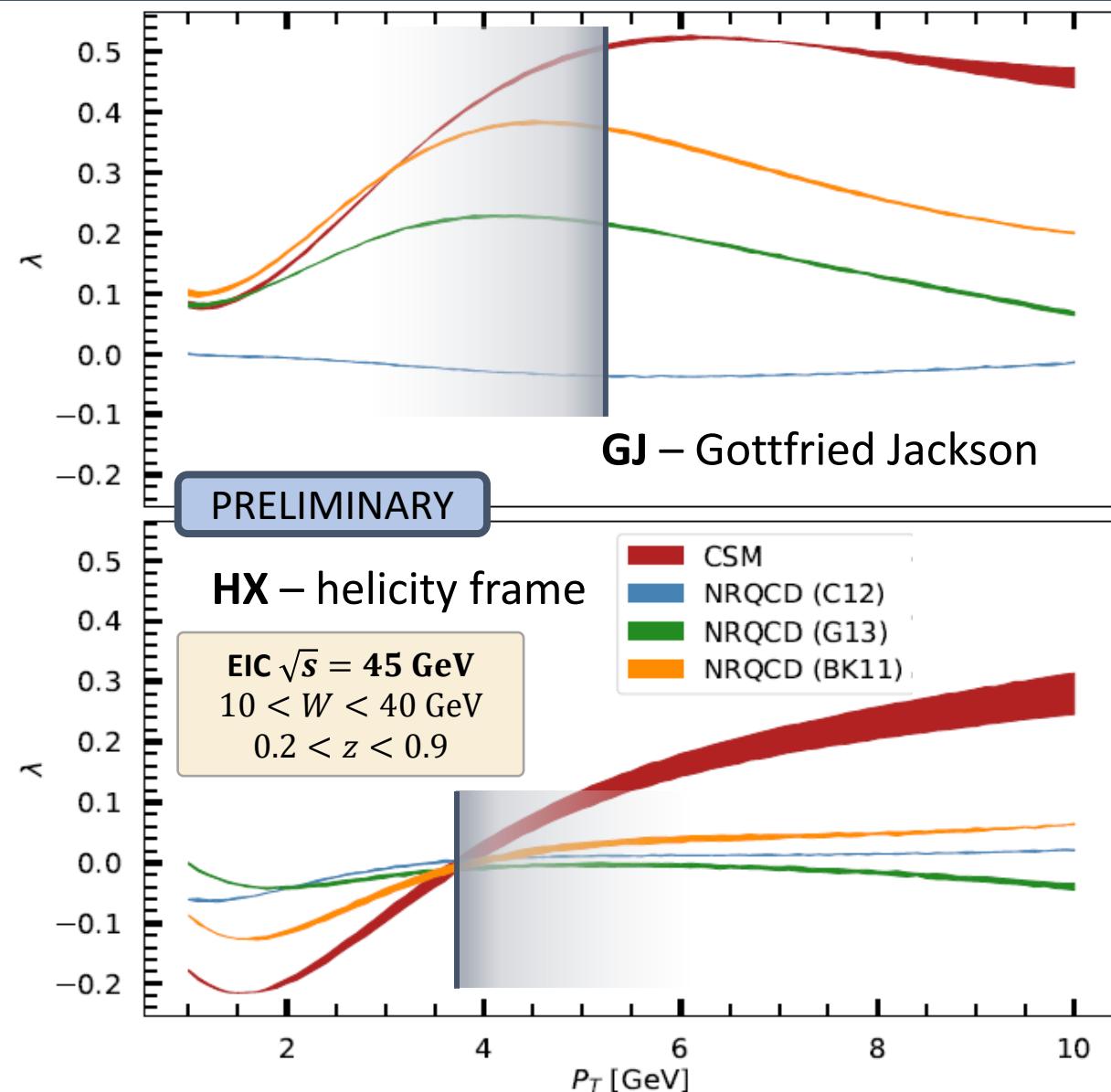
Eboli, Gregores, Halzen, PRD 67 (2003)

Corresponding prediction for CSM and NRQCD in photoproduction in

Butenschoen, Kniehl, PRL 107 (2011)

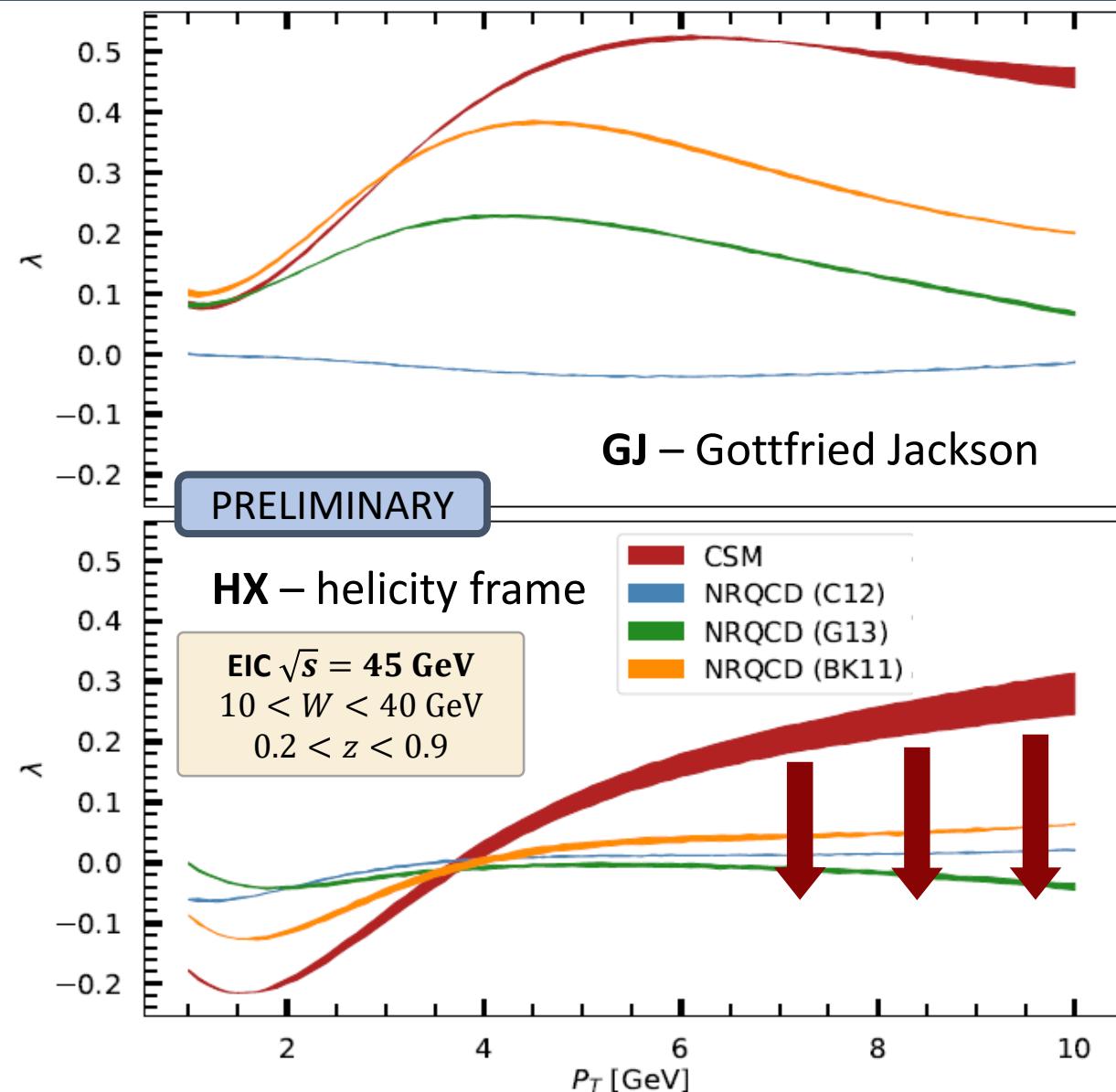


POLARIZATION AT EIC

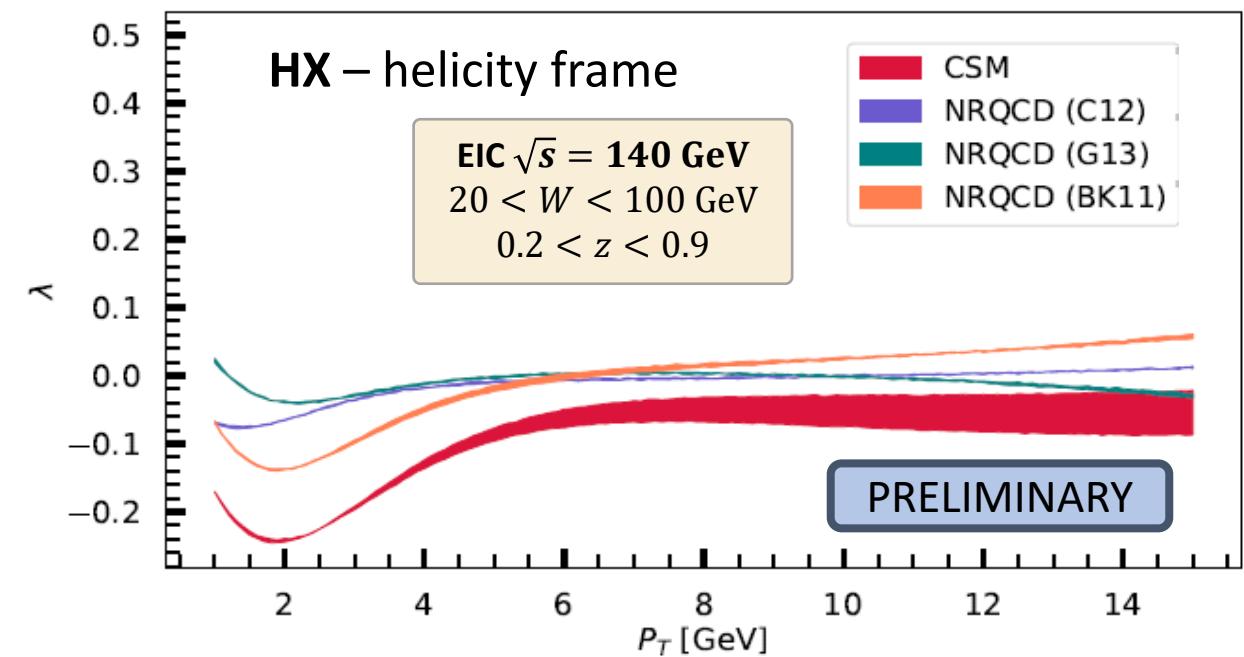


EIC collinear predictions for λ vs P_T
Analysis for different frames and energies
allows for a more complete picture
(resolved photon not included)

POLARIZATION AT EIC



EIC collinear predictions for λ vs P_T
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EIC INVARIANTS

Upon rotation around Y-axis from a to b λ, μ, ν mix up

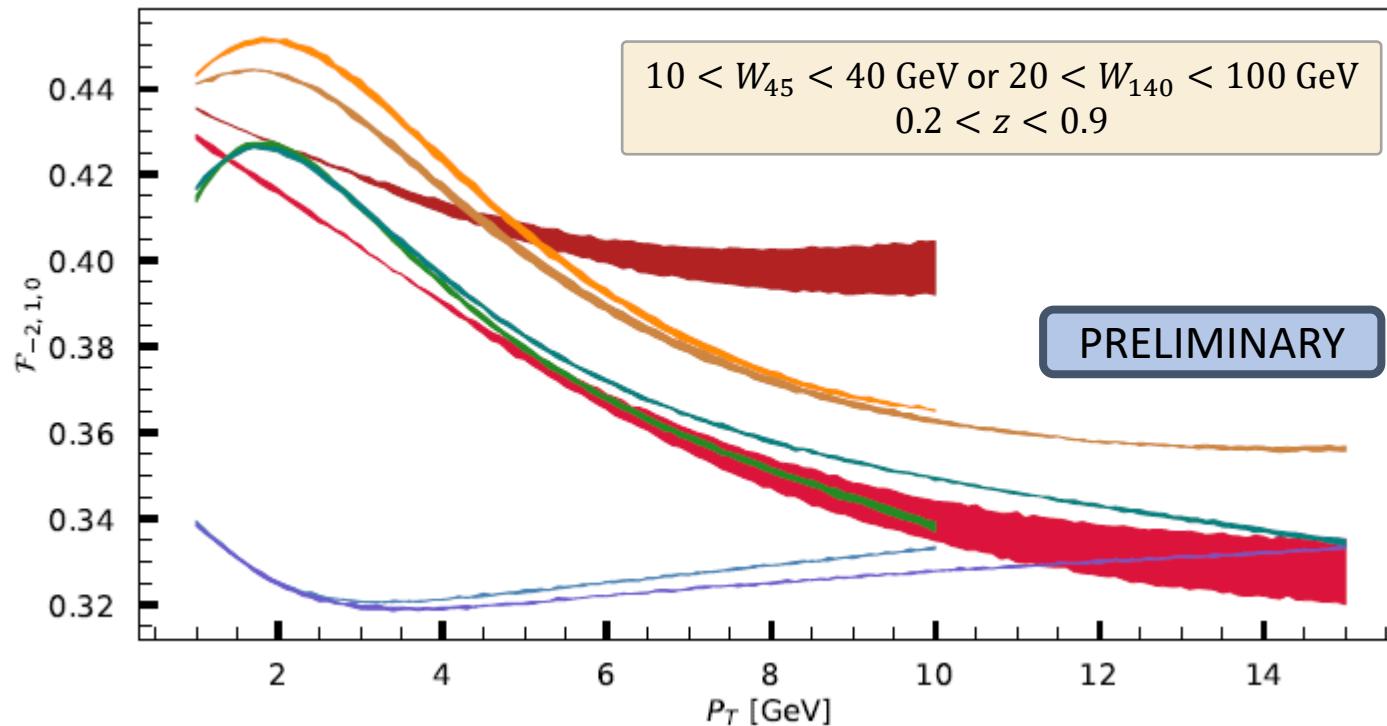
$$\begin{pmatrix} \lambda^b \\ \mu^b \\ \nu^b \end{pmatrix} = \frac{1}{1 + \Delta} \begin{pmatrix} 1 - \frac{3}{2} \sin^2 \theta_{ab} & -\frac{3}{2} \sin 2\theta_{ab} & \frac{3}{4} \sin^2 \theta_{ab} \\ \frac{1}{2} \sin 2\theta_{ab} & \cos 2\theta_{ab} & -\frac{1}{4} \sin 2\theta_{ab} \\ \sin^2 \theta_{ab} & \sin 2\theta_{ab} & 1 - \frac{1}{2} \sin^2 \theta_{ab} \end{pmatrix} \begin{pmatrix} \lambda^a \\ \mu^a \\ \nu^a \end{pmatrix}$$

with $\Delta = \frac{\sin^2 \theta_{ab}}{2} \left(\lambda - \frac{\nu}{2} \right) + \frac{\sin 2\theta_{ab}}{2} \mu$

One can identify a family of functions which are invariant under this rotation

$$\mathcal{F}_{c_1, c_2, c_3} = \frac{(3 + \lambda) + c_1(1 - \nu/2)}{c_2(3 + \lambda) + c_3(1 - \nu/2)}$$

EIC INVARIANTS



One can define variables invariant upon **rotation around Y-axis**

$$\mathcal{F}_{-2,1,0} = \frac{1 + \lambda + \nu}{3 + \lambda}$$

$$\sqrt{s} = 45 \text{ GeV}$$

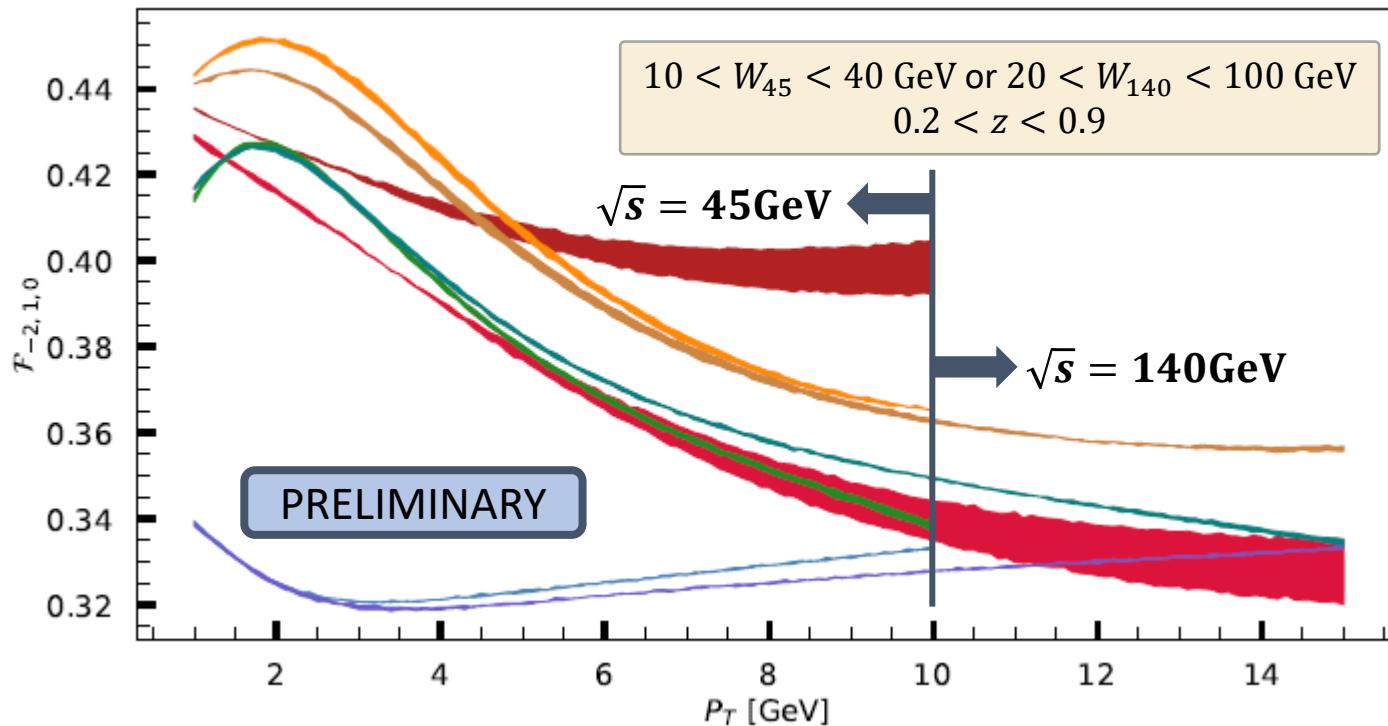
Color legend reminder:

$$\sqrt{s} = 140 \text{ GeV}$$

CSM
NRQCD (C12)
NRQCD (G13)
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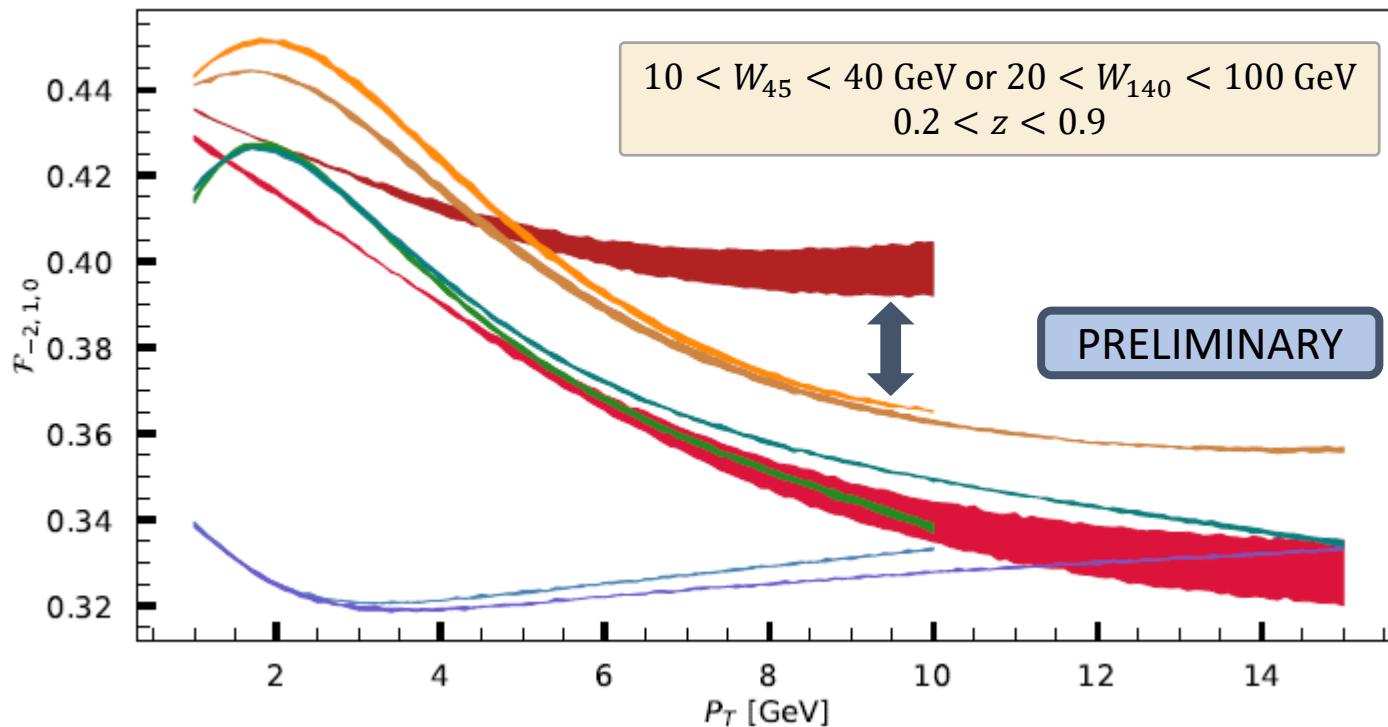
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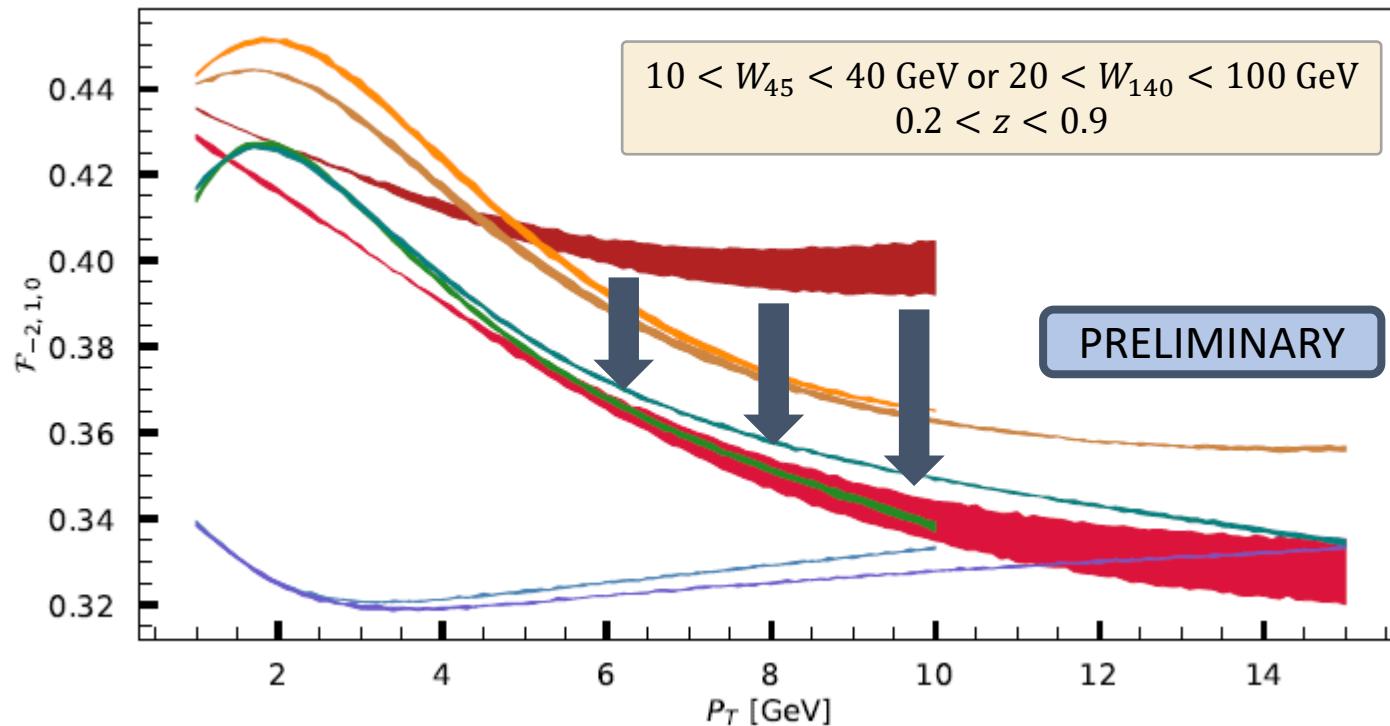
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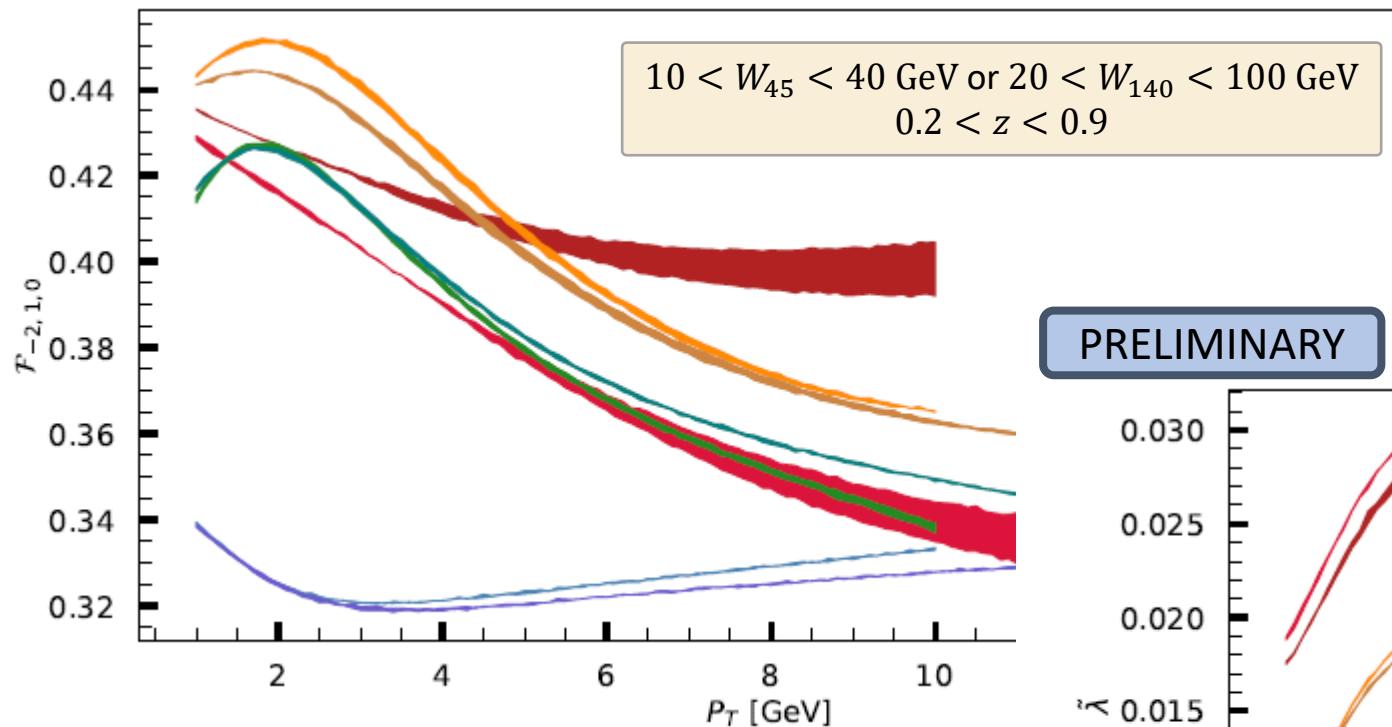
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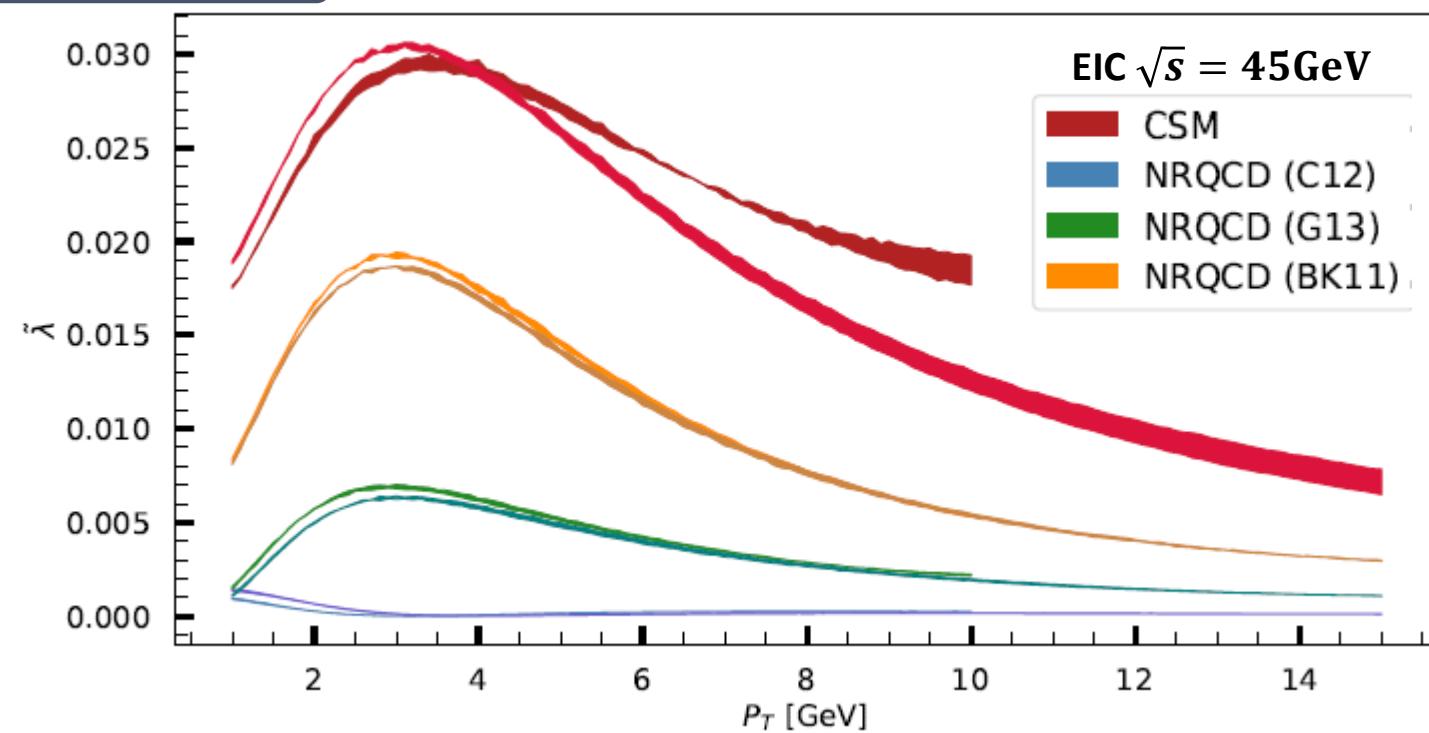
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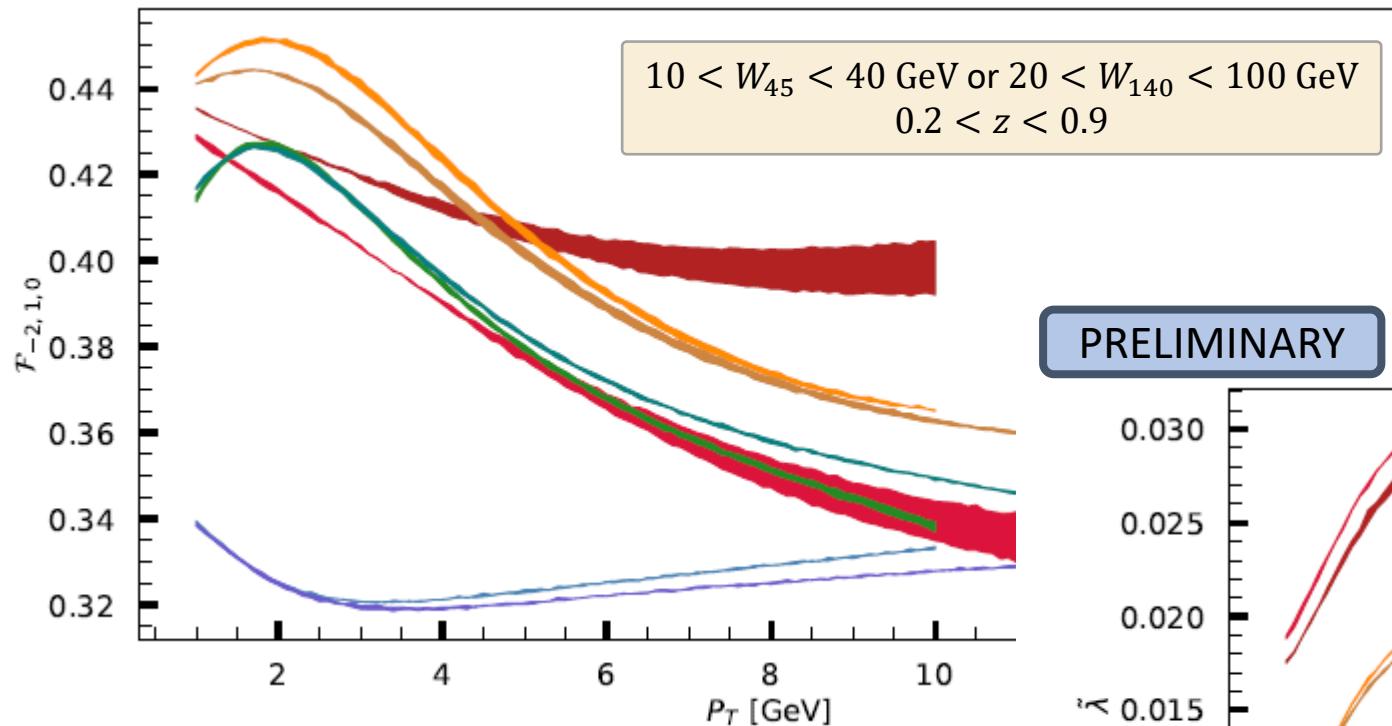
$$\tilde{\lambda} = \frac{(\lambda - \nu/2)^2 + 4\mu^2}{(3 + \lambda)^2}$$



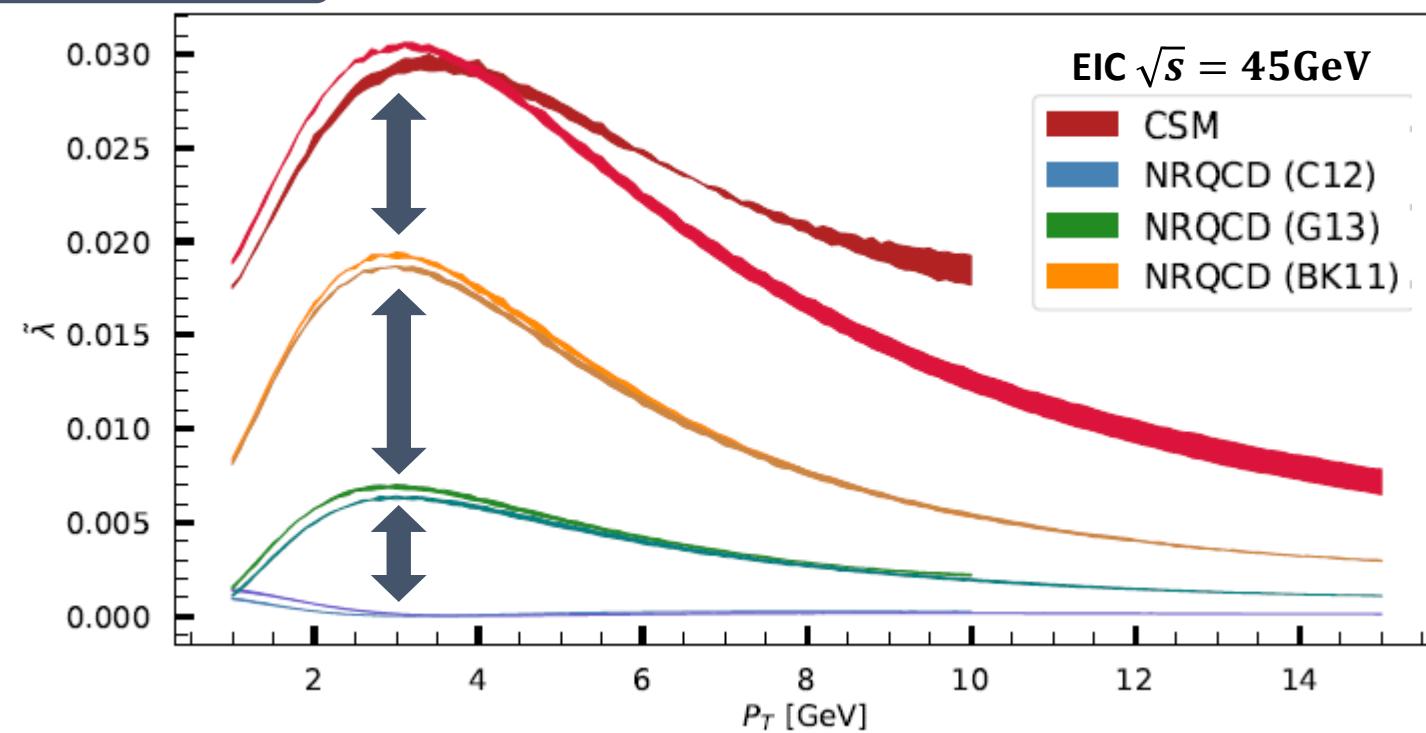
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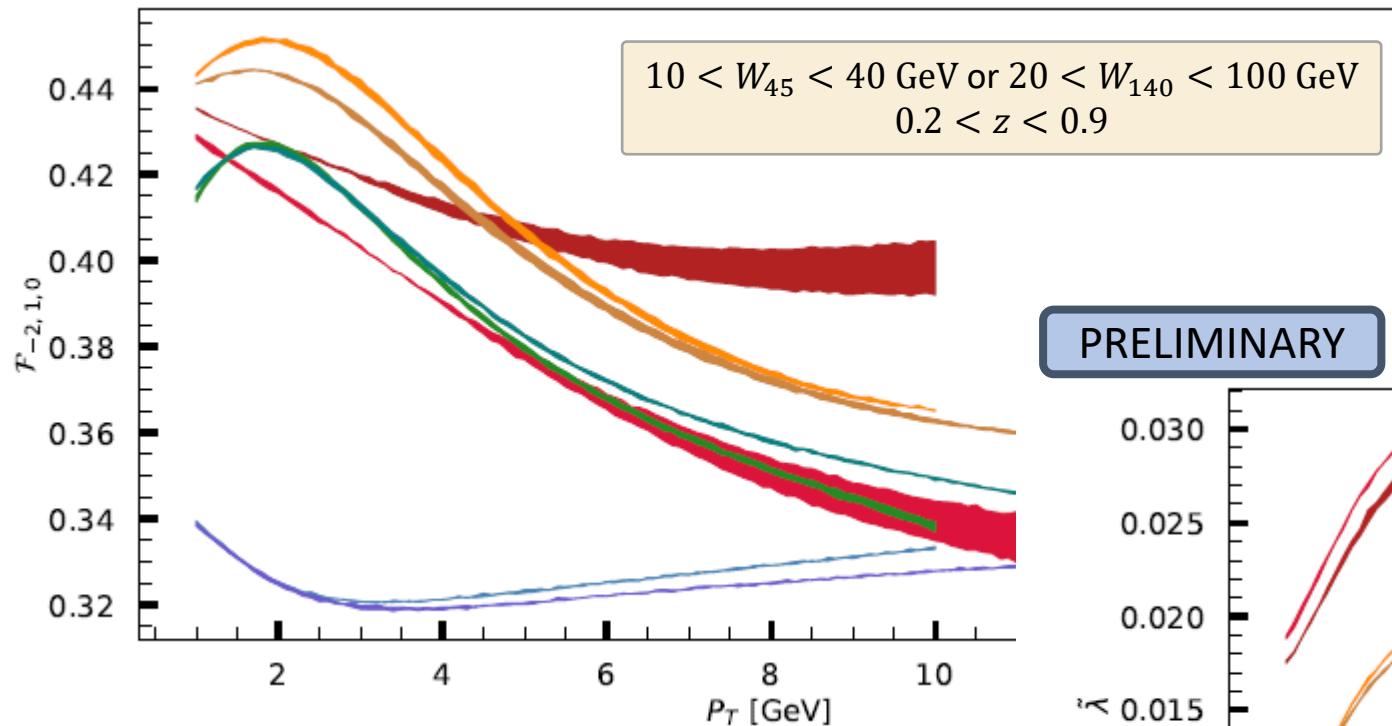
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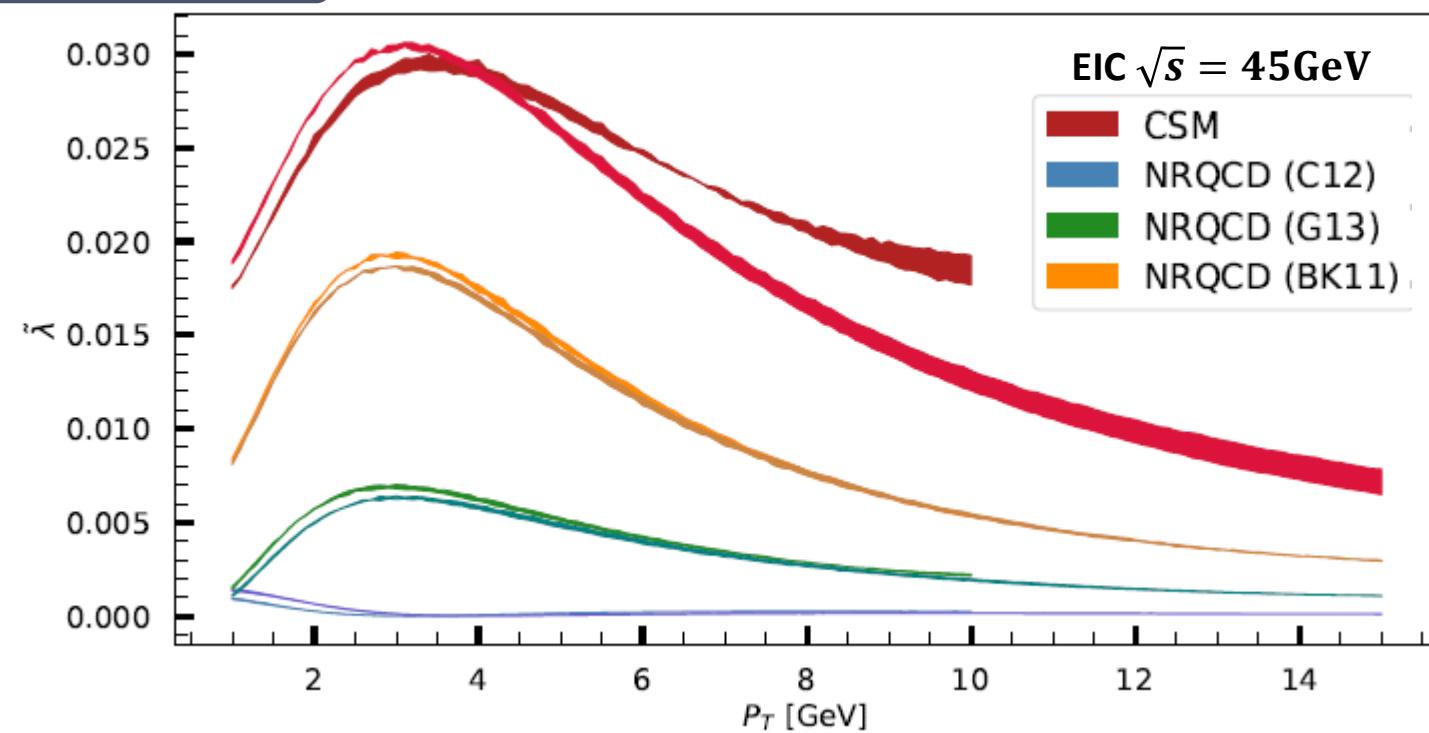


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More information encoded!

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CONCLUSIONS

Importance of polarization states analysis

Access to gluon TMD PDFs

In TMD region $\mathcal{W}_{\Delta\Delta}^\perp$ is related to the linearly polarized gluon distribution h_1^\perp

Proper **shape functions** are necessary to provide correct expressions in the intermediate q_T region

Comparison between HERA and EIC predictions shows the importance of **full** polarization measurement to achieve a complete picture

Thanks for the attention