

Quarkonia As Tools 2022
January 12th

Gluon TMDs and Quarkonia

Francesco Giovanni Celiberto

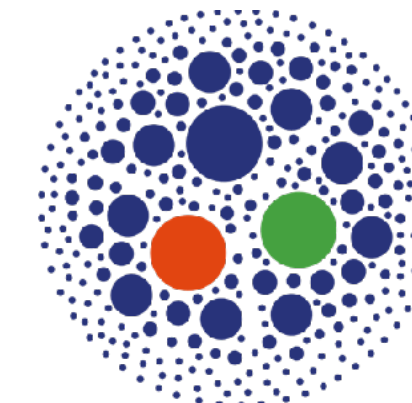
ECT*/FBK Trento & INFN-TIFPA

ECT*

EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS



Trento Institute for
Fundamental Physics
and Applications



HAS QCD

HADRONIC STRUCTURE AND
QUANTUM CHROMODYNAMICS

The background features a complex, multi-layered illustration of gluon Transverse Momentum Distributions (TMDs). It consists of several overlapping, semi-transparent circular regions. Each region contains a central point from which multiple wavy, yellow lines radiate outwards, representing gluon emissions. These lines are decorated with small, colorful spheres (red, blue, green) and arrows, indicating the direction and polarization of the gluons. The overall color palette is light and airy, with soft blues, greens, and yellows, creating a sense of depth and scientific complexity.

An overview on gluon TMDs

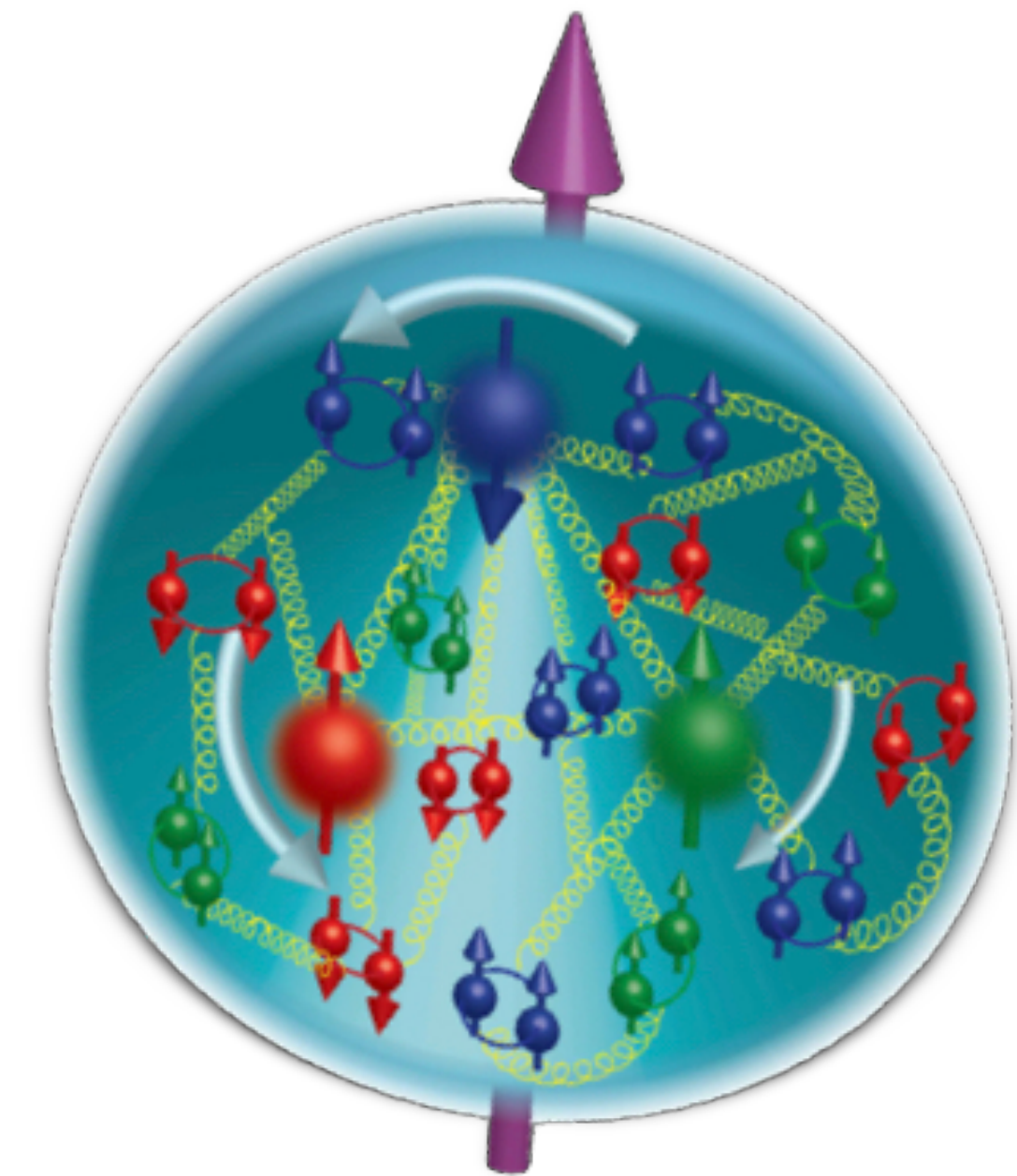
Parton densities: hors d'œuvre



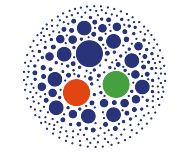
Parton densities → relevant for the search of **New Physics**...

→ ...crucial role in the understanding and exploration of **QCD**

- Describe the internal structure of the nucleon in terms of its elementary constituents (quarks and gluons)
- Nonperturbative** objects that enter the expression of cross sections
- Can be *extracted* from experiments via *global fits*



Parton densities: hors d'œuvre



Parton densities → relevant for the search of **New Physics**...

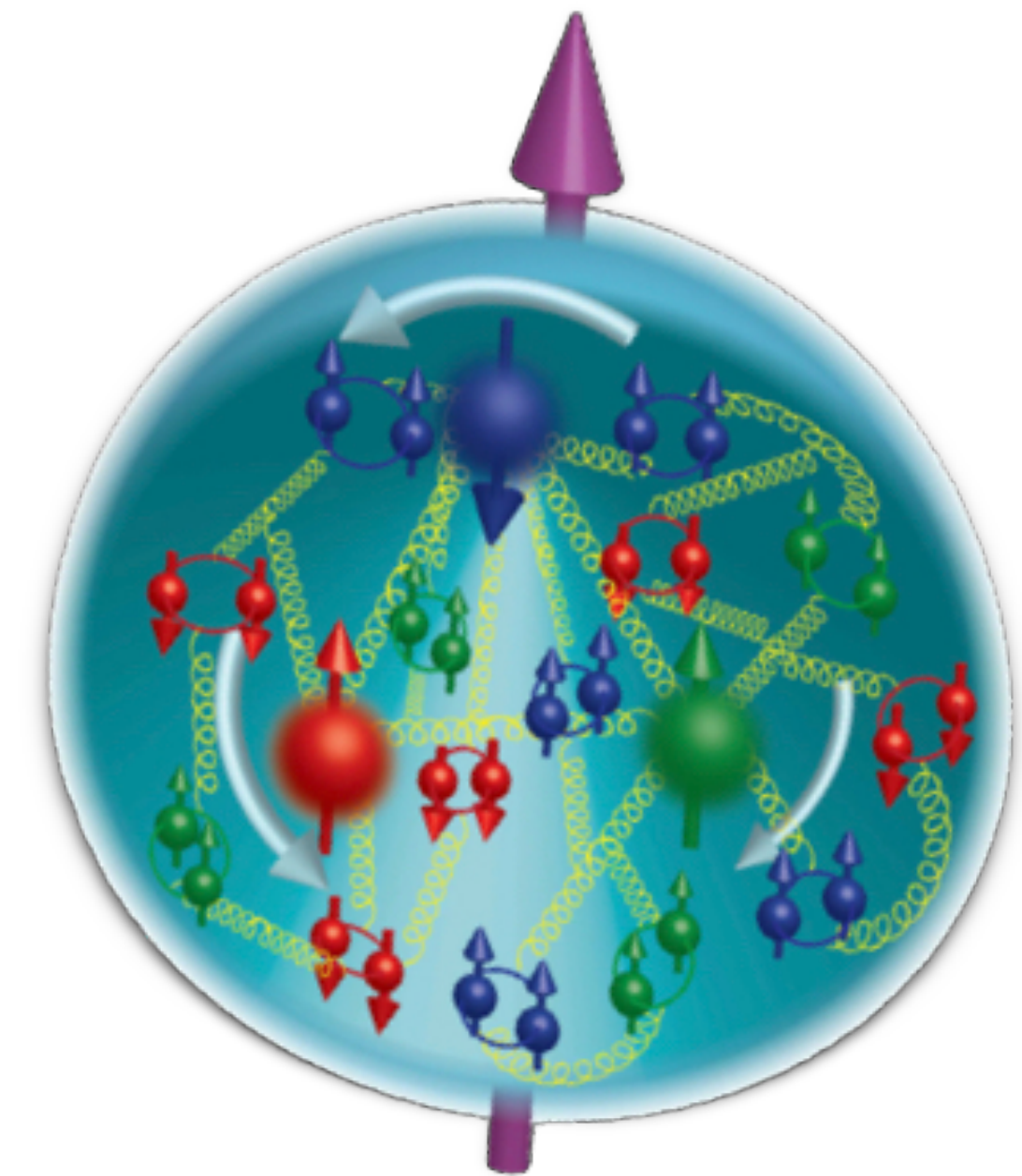
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- Describe the internal structure of the nucleon in terms of its elementary constituents (quarks and gluons)
- Nonperturbative** objects that enter the expression of cross sections
- Can be *extracted* from experiments via *global fits*



Several types of distributions (1D collinear, **3D TMD**, ...)

- Respect different **factorization theorems**
- Exhibit peculiar **universality properties**
- Obey distinct **evolution equations**



Gluon TMDs: gauge links and modified universality

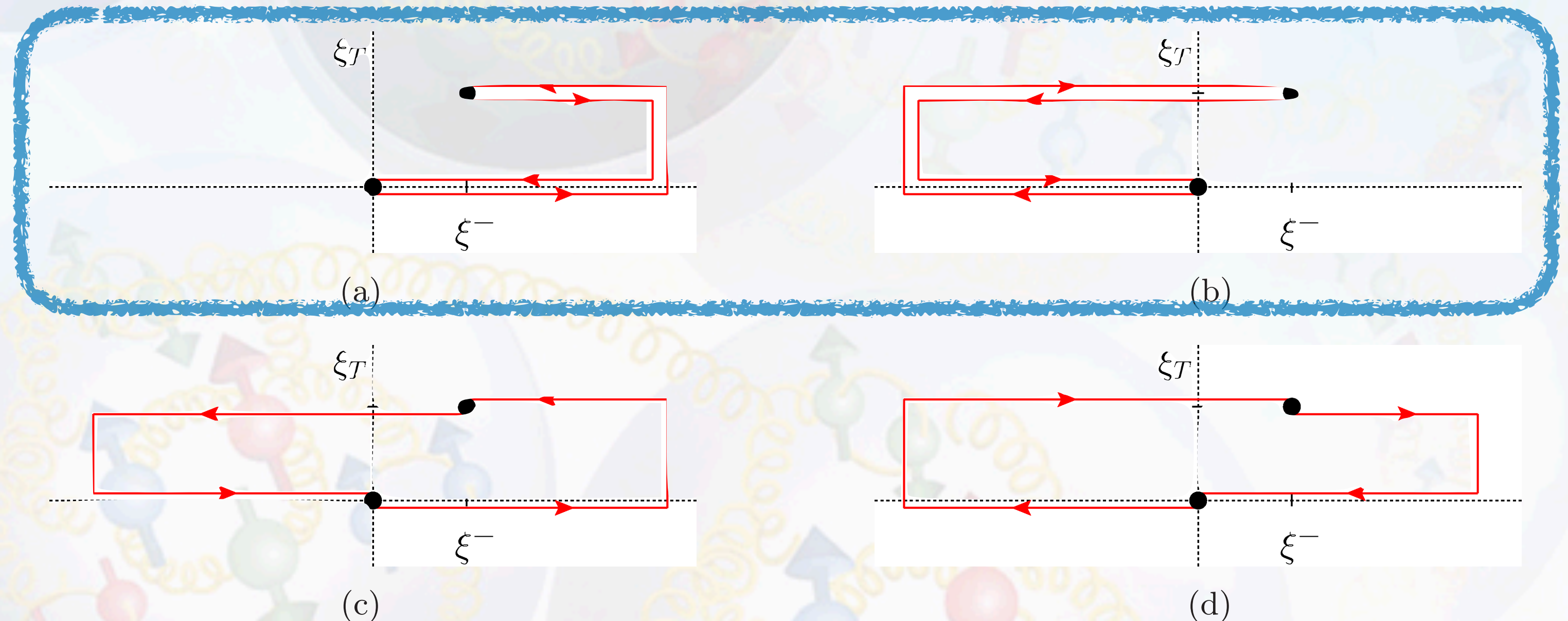
- * **Single-spin asymmetries** → process dependence of TMDs via **gauge links**
- * **Color flow** → integration paths of gauge links calculable
- * Gluon TMDs → more complicated structure with respect to quark **staple links**
- * **Factorization-preserving** processes → two main kinds of **modified universality**
- * Different classes of processes → distinct gluon TMDs, **not related** to each other

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f-type (WW)

(a) [+ , +] or (b) [- , -]



Gluon TMDs: gauge links and modified universality

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f-type (WW)

(a) [+ , +] or (b) [- , -]



d-type (dipole)

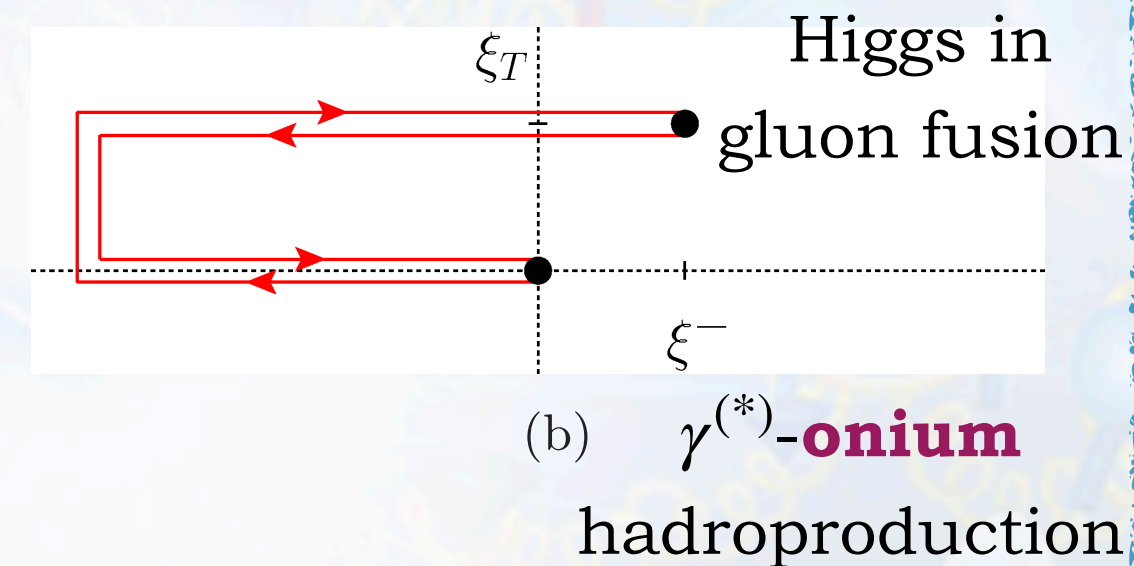
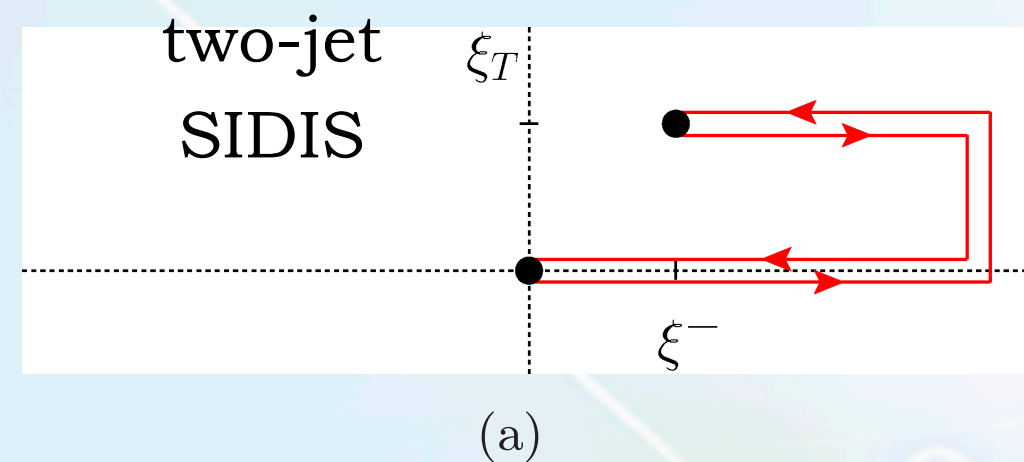
(c) [+ , -] or (d) [- , +]



Accessing WW and DP gluon TMDs

Weiszäcker-Williams (WW)

(a) [+ , +] or (b) [- , -]



* Color flow annihilated within final/initial state

* f -type gluon TMDs $\rightarrow f^{abc}$ color structure

* Modified universality:

$$f_1^{[+,+]} = f_1^{[-,-]},$$

$$f_{1T}^{\perp[+,+]} = -f_{1T}^{\perp[-,-]}$$

* Phenomenology: Higgs, **quarkonia** or $\gamma\gamma$ in pp ,
two-jet SIDIS, heavy-quark pair SIDIS

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Weizsäcker-Williams (WW)

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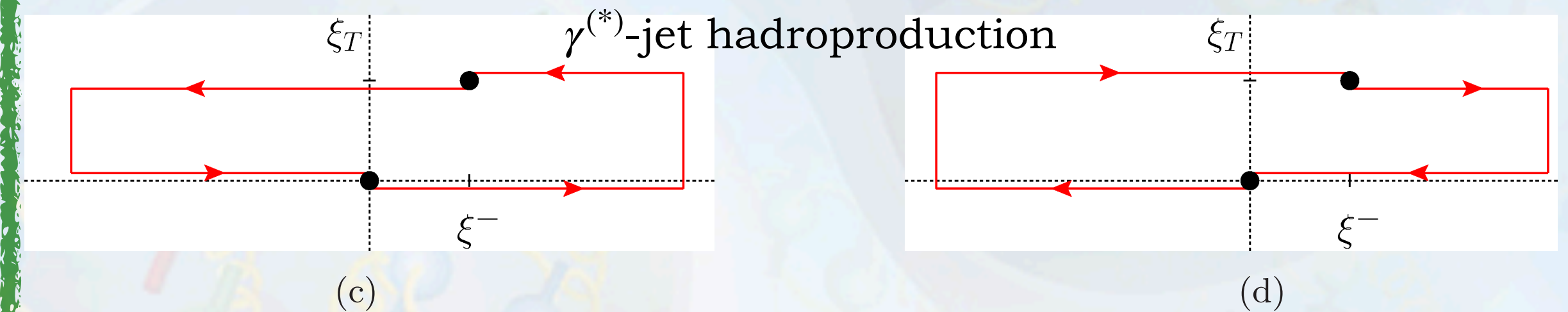
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- * Phenomenology: Higgs, **quarkonia** or $\gamma\gamma$ in pp , two-jet SIDIS, heavy-quark pair SIDIS

Dipole (DP)

(c) [+ , -] or (d) [- , +]



- * Color flow involving both initial and final states

- * d -type gluon TMDs $\rightarrow d^{abc}$ color structure

- * Modified universality:

$$f_1^{[+,-]} = f_1^{[-,+]},$$

$$f_{1T}^{\perp[+,-]} = -f_{1T}^{\perp[-,+]}$$

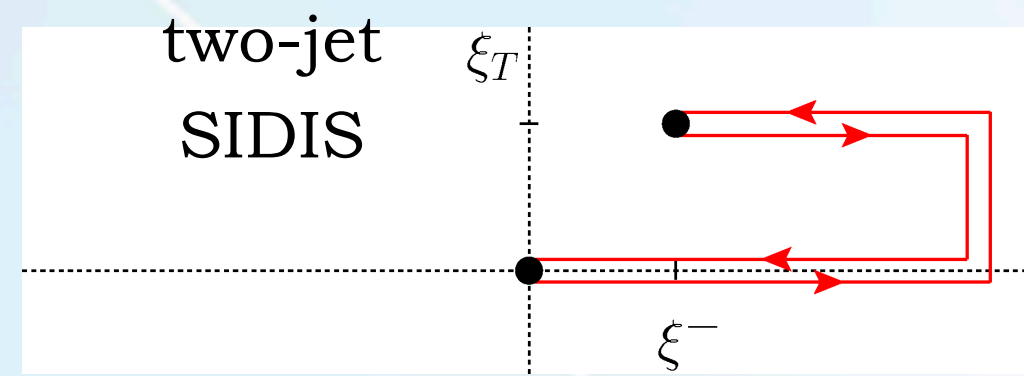
- * Phenomenology: single hadron or $\gamma^{(*)}$ -jet hadroproduction, SIDIS or Drell-Yan (subleading)

Gauge link \rightarrow two main independent sets of TMDs, **not related** to each other

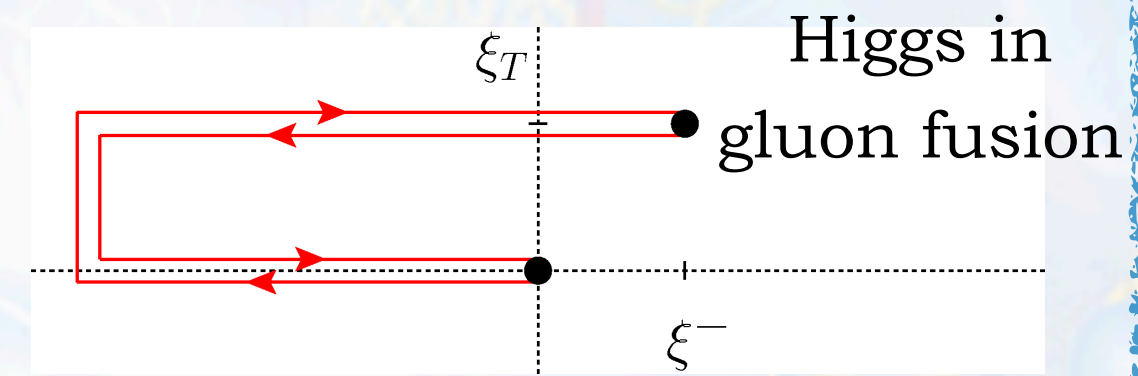
Accessing WW and DP gluon TMDs

Weizsäcker-Williams (WW)

(a) $[+, +]$ or (b) $[-, -]$



(a)

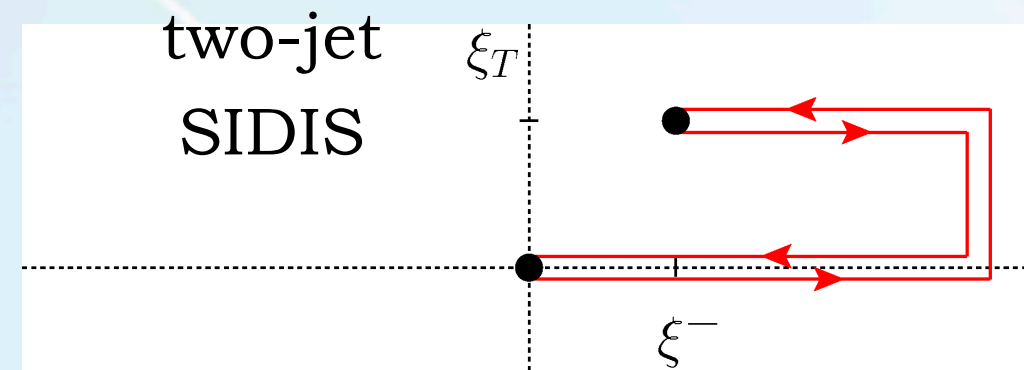


(b) $\gamma^{(*)}$ -onium
hadroproduction

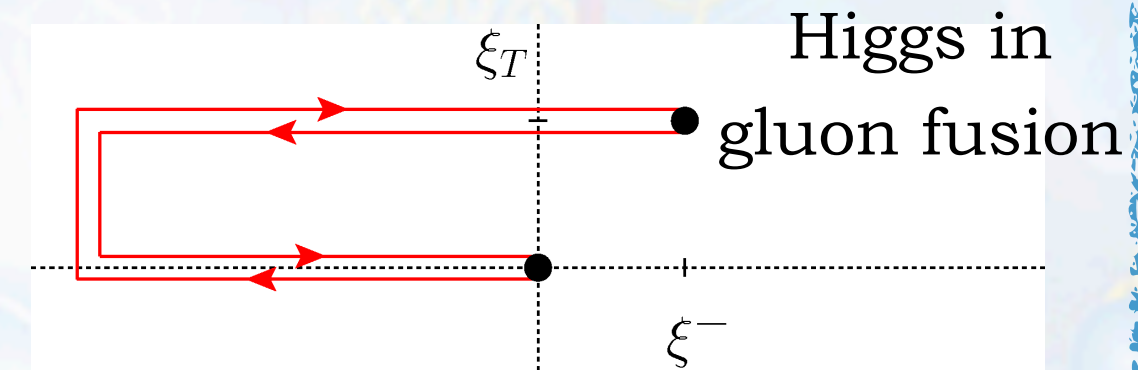
Accessing WW and DP gluon TMDs

Weiszäcker-Williams (WW)

(a) $[+, +]$ or (b) $[-, -]$



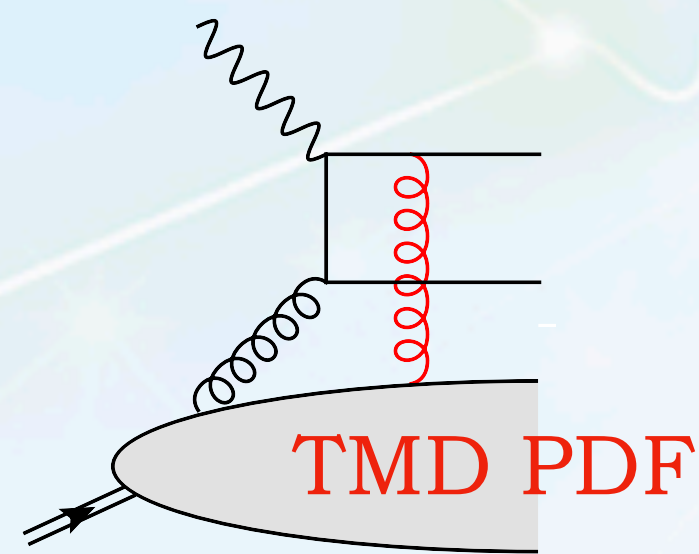
(a)



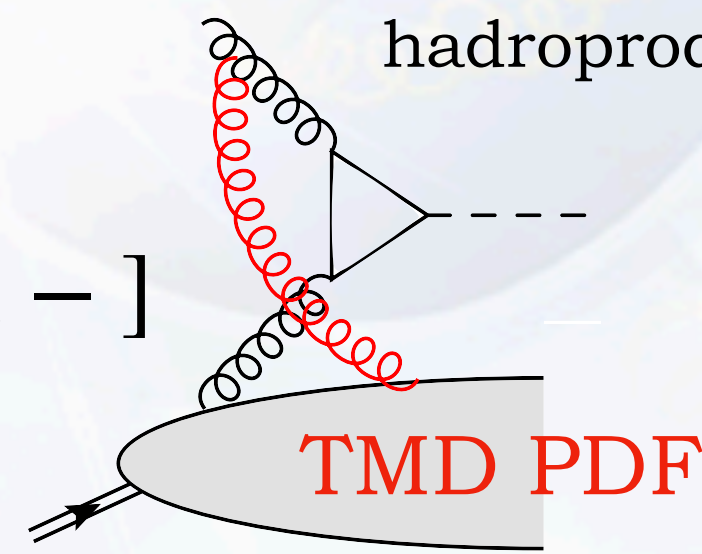
(b) $\gamma^{(*)}$ -onium

hadroproduction

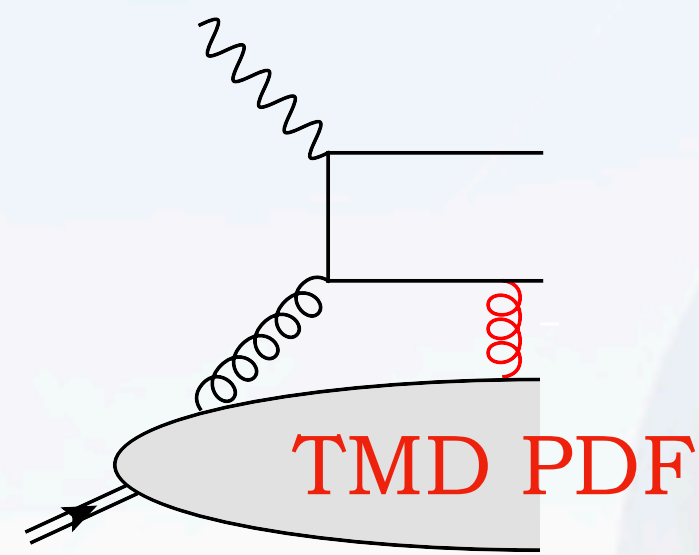
$[+, +]$



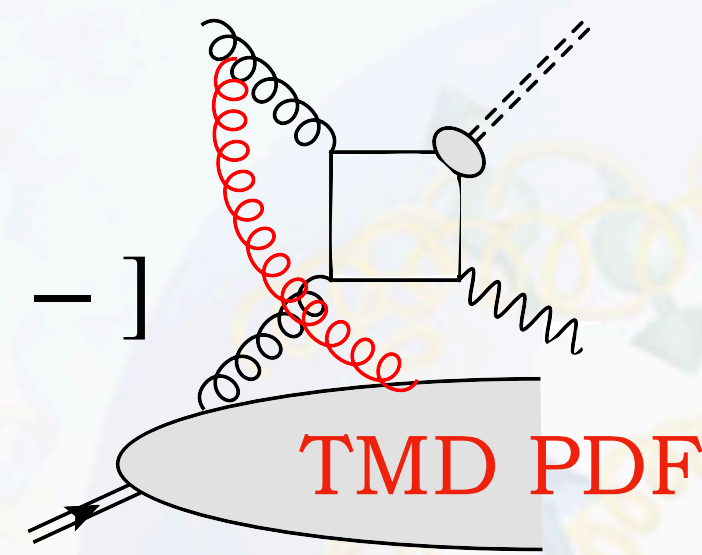
$[-, -]$



$[+, +]$



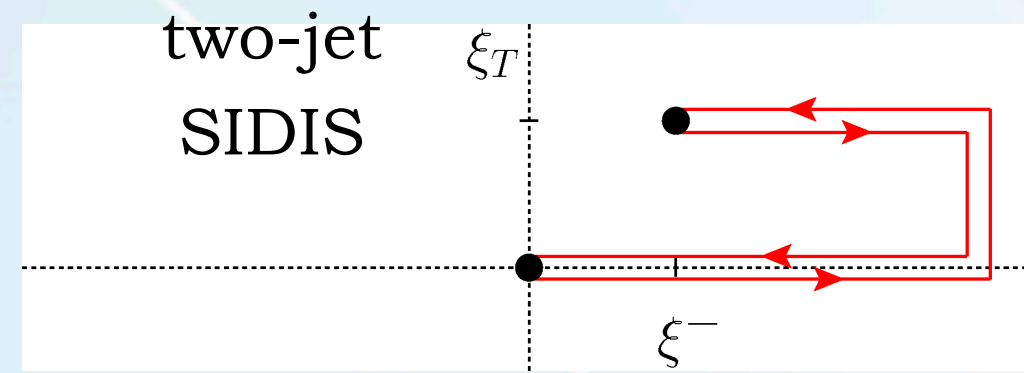
$[-, -]$



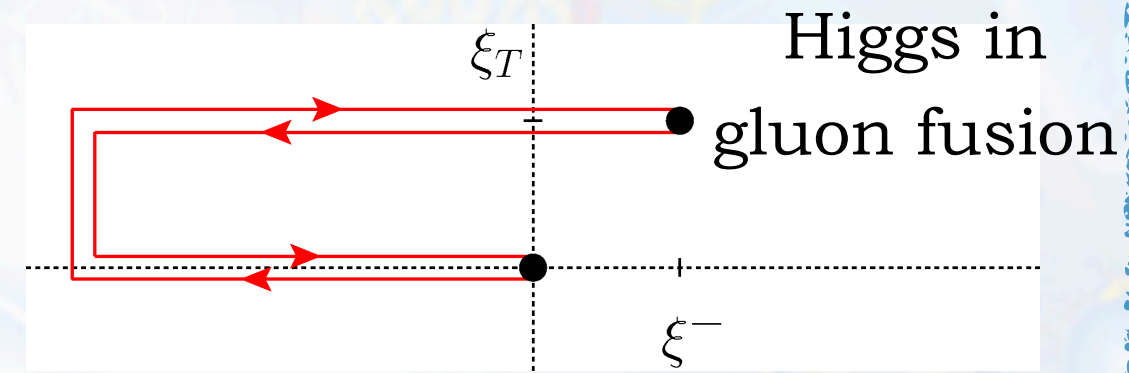
Accessing WW and DP gluon TMDs

Weizsäcker-Williams (WW)

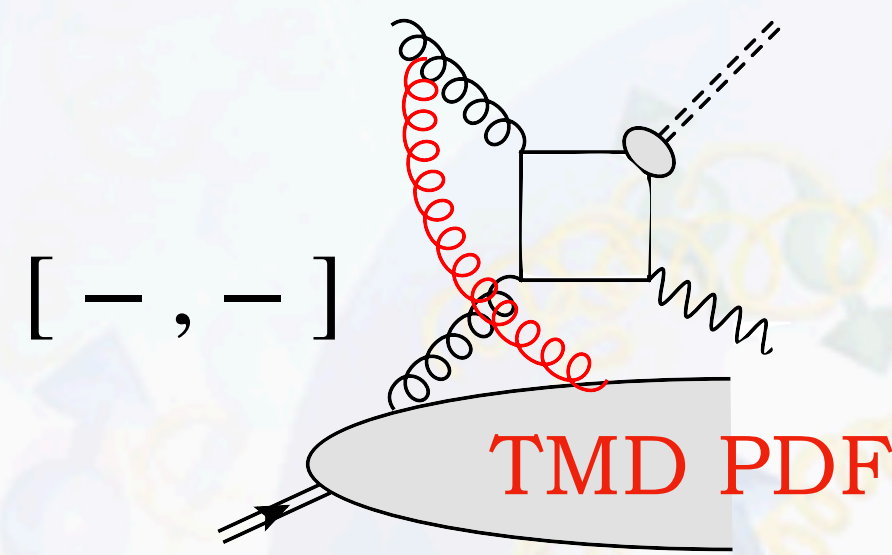
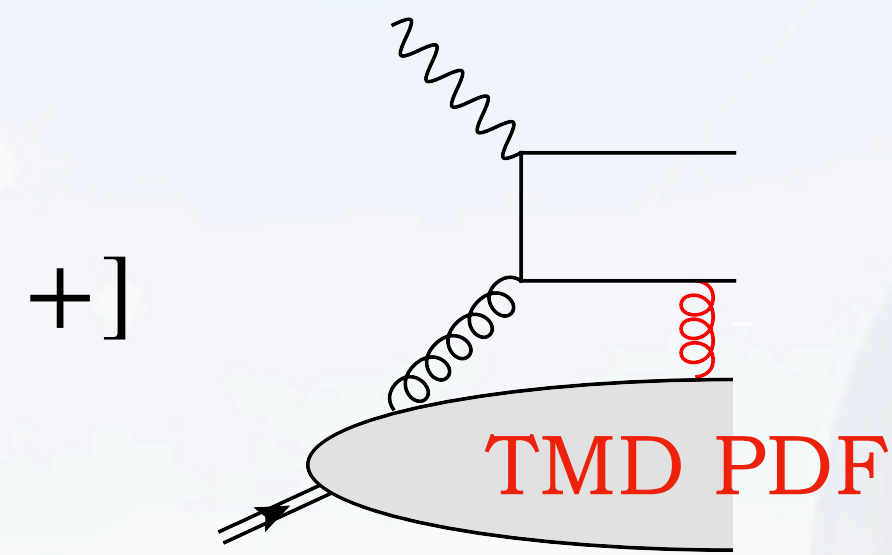
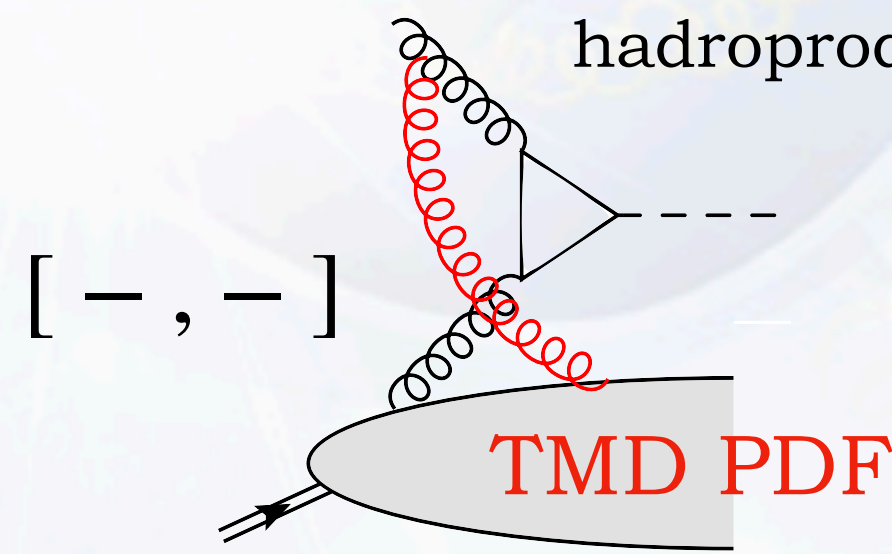
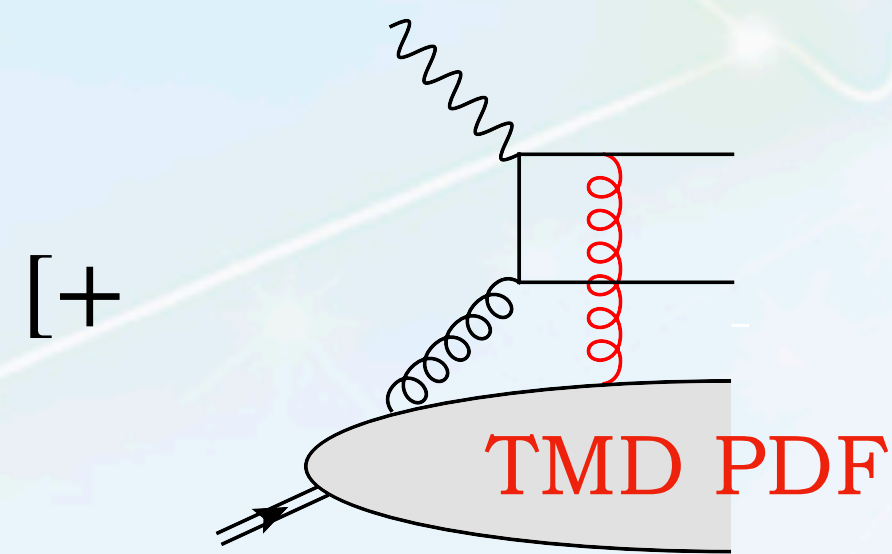
(a) $[+, +]$ or (b) $[-, -]$



(a)

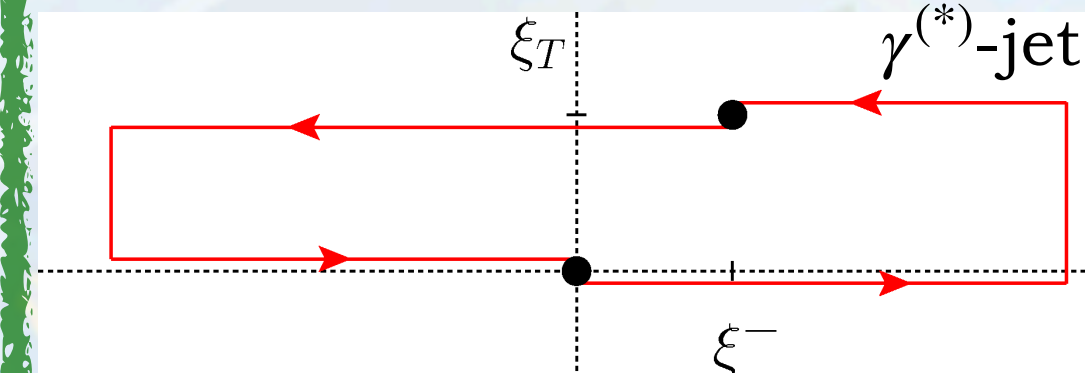


(b) $\gamma^{(*)}$ -onium hadroproduction

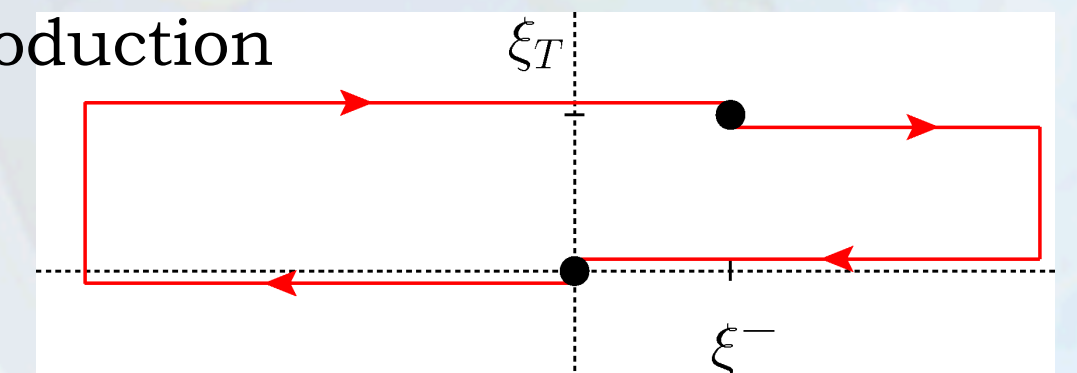


Dipole (DP)

(c) $[+, -]$ or (d) $[-, +]$



(c)

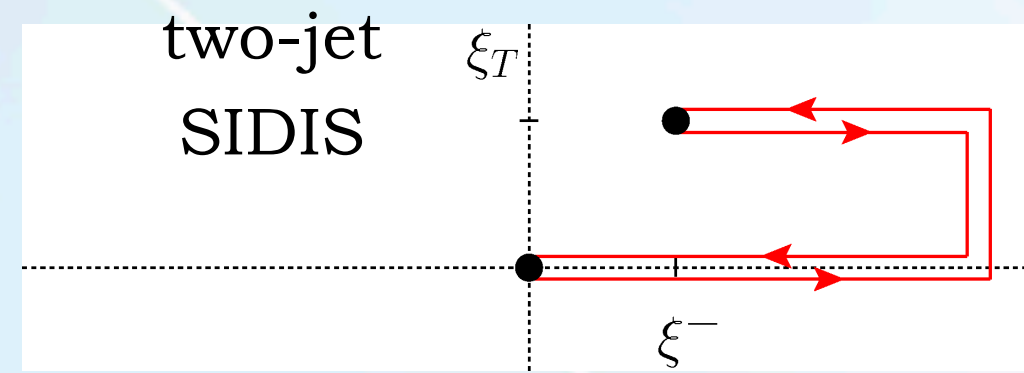


(d)

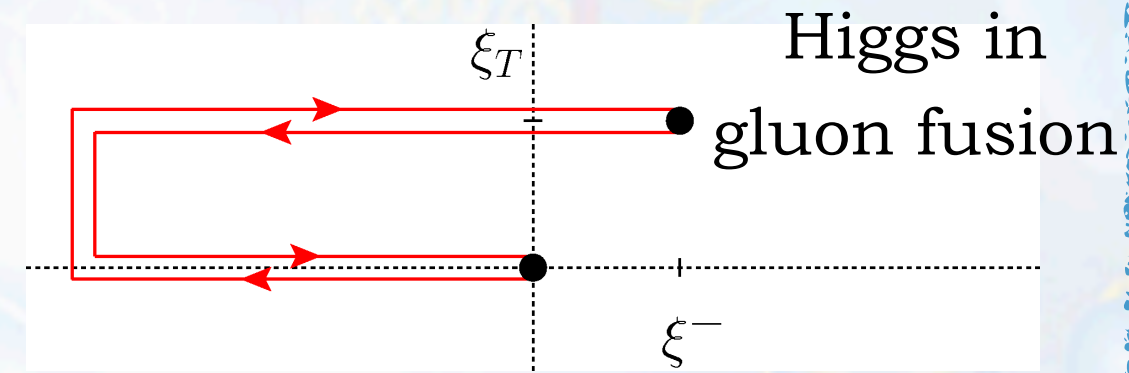
Accessing WW and DP gluon TMDs

Weizsäcker-Williams (WW)

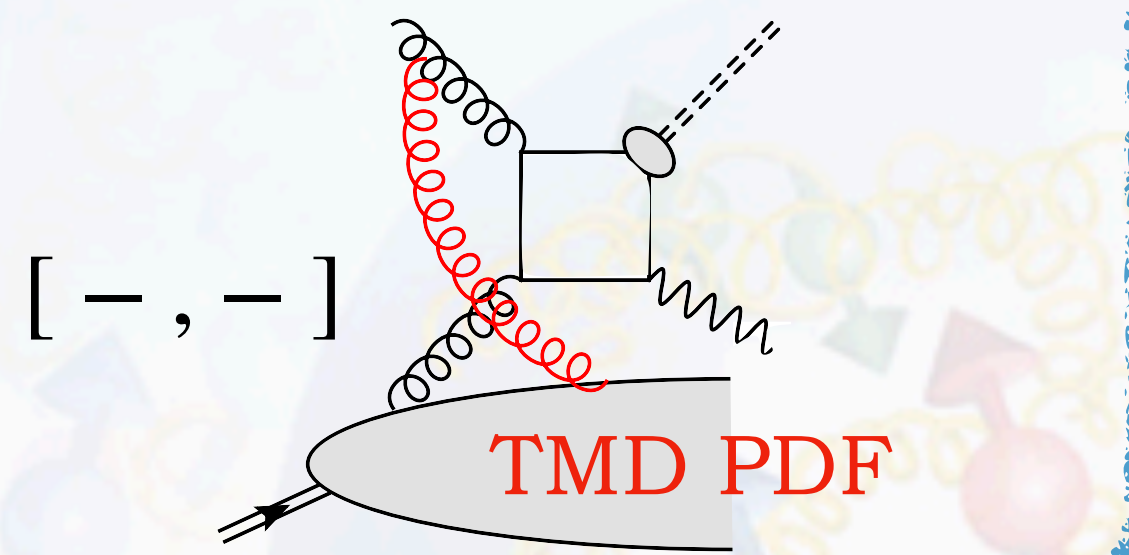
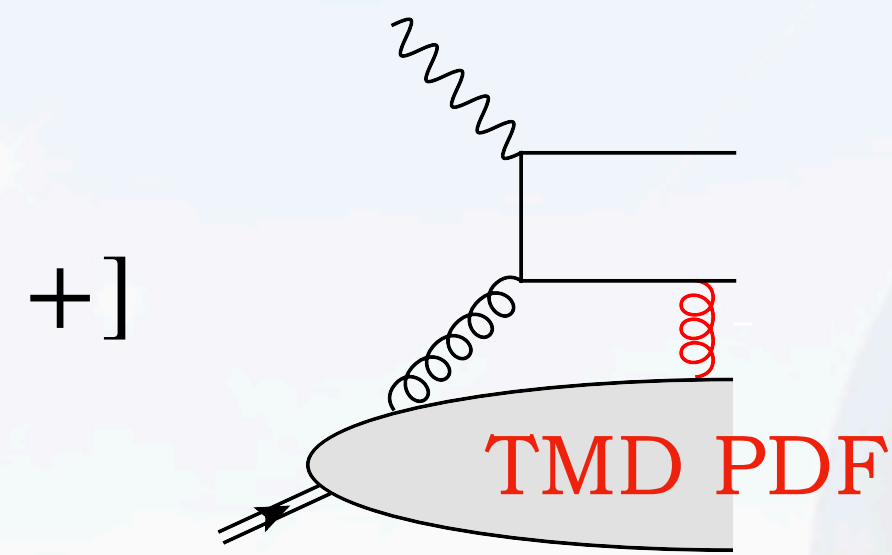
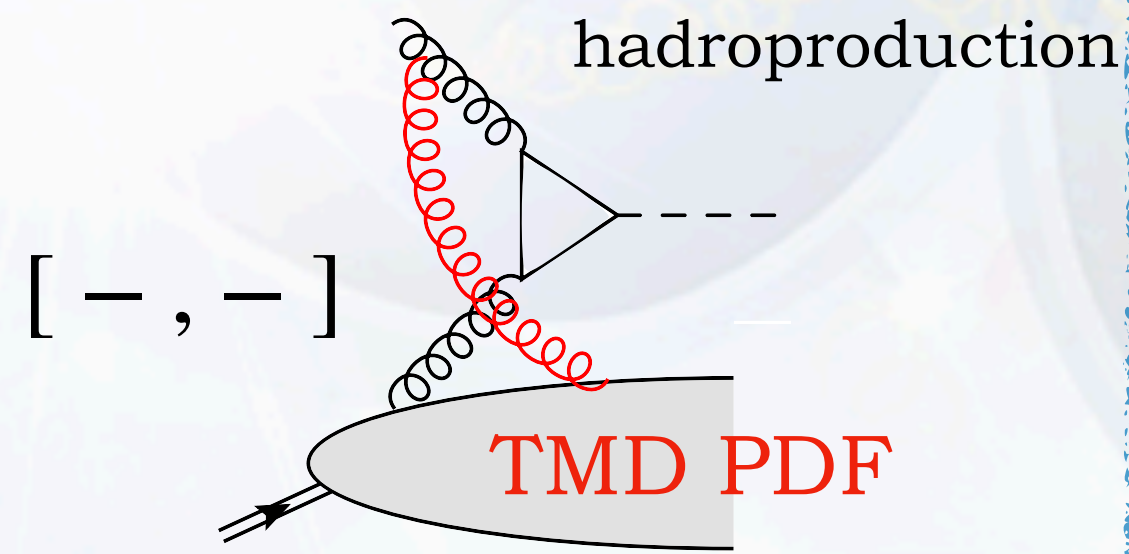
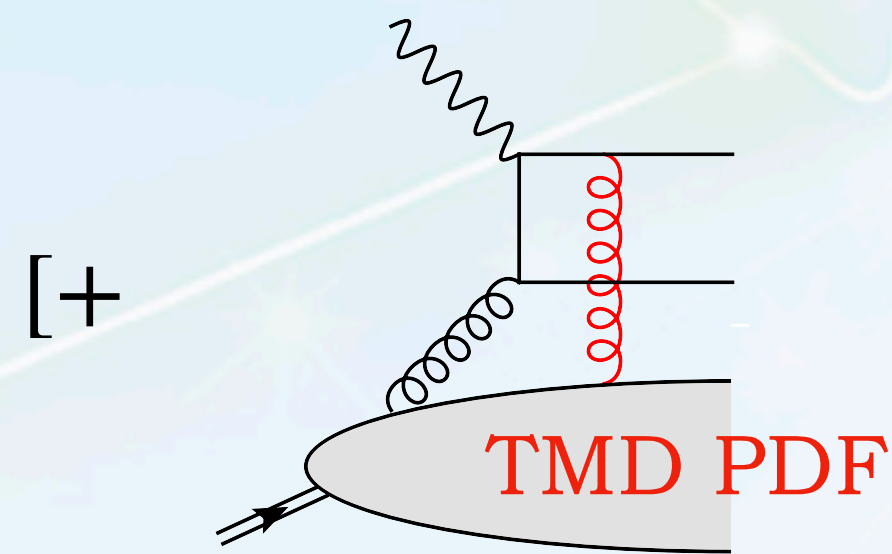
(a) $[+, +]$ or (b) $[-, -]$



(a)

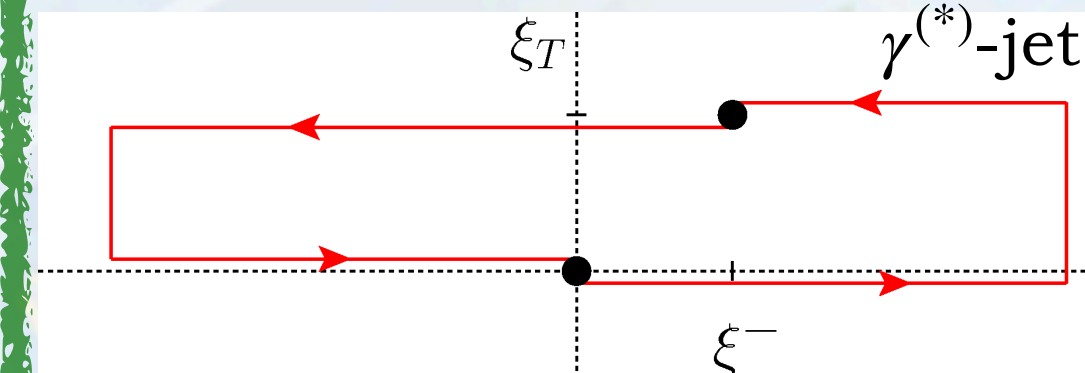


(b) $\gamma^{(*)}$ -onium hadroproduction

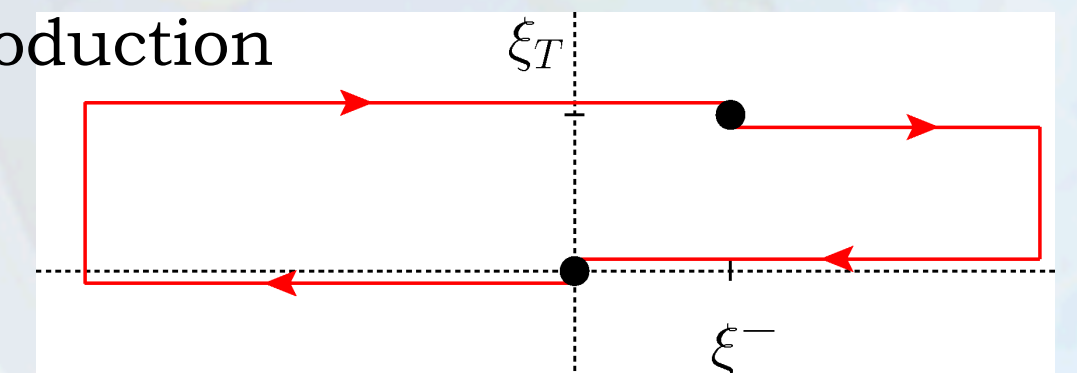


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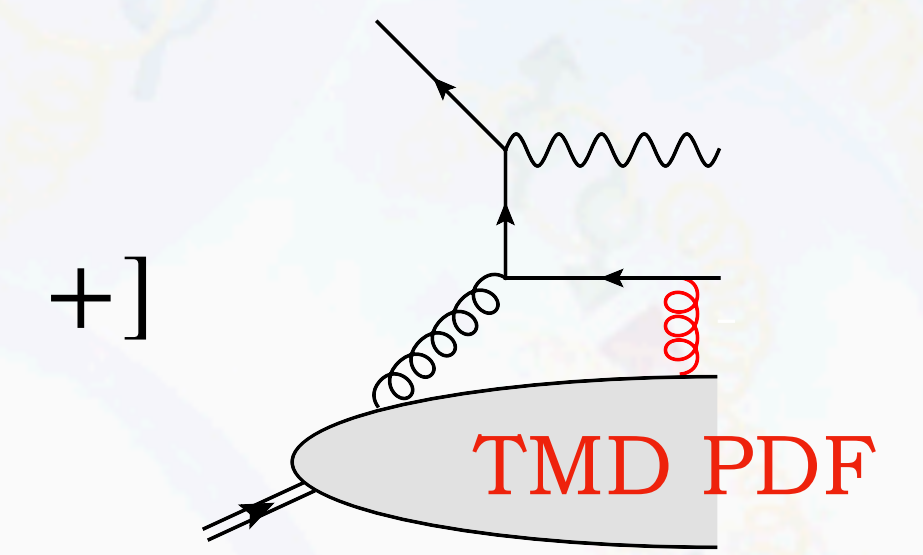
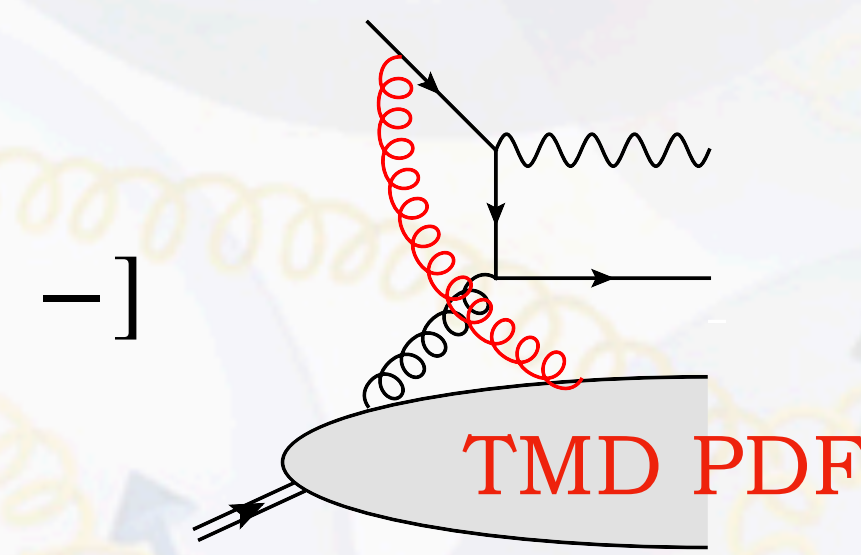
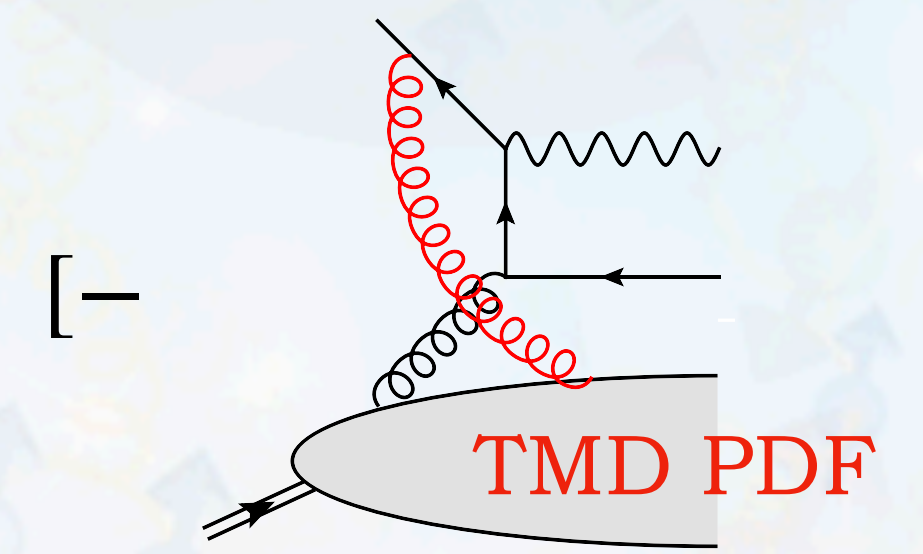
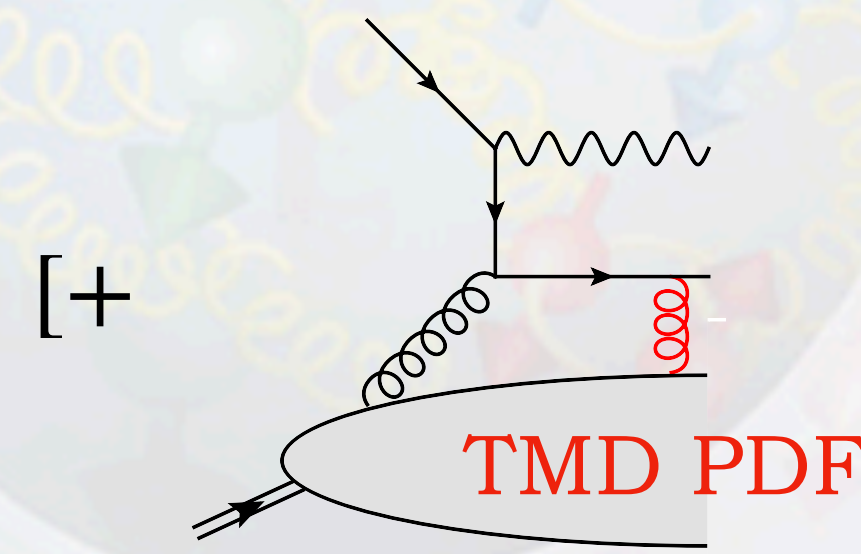
(c) $[+, -]$ or (d) $[-, +]$



(c)



(d)



Dihadron hadroproduction and factorization breaking

* Proof of factorization violation  [T. J. Rogers, P. J. Mulders (2010)]

* Assumed factorization in SCET and CGC

* Significance of low- x studies

* Size of factorization-breaking effects small?

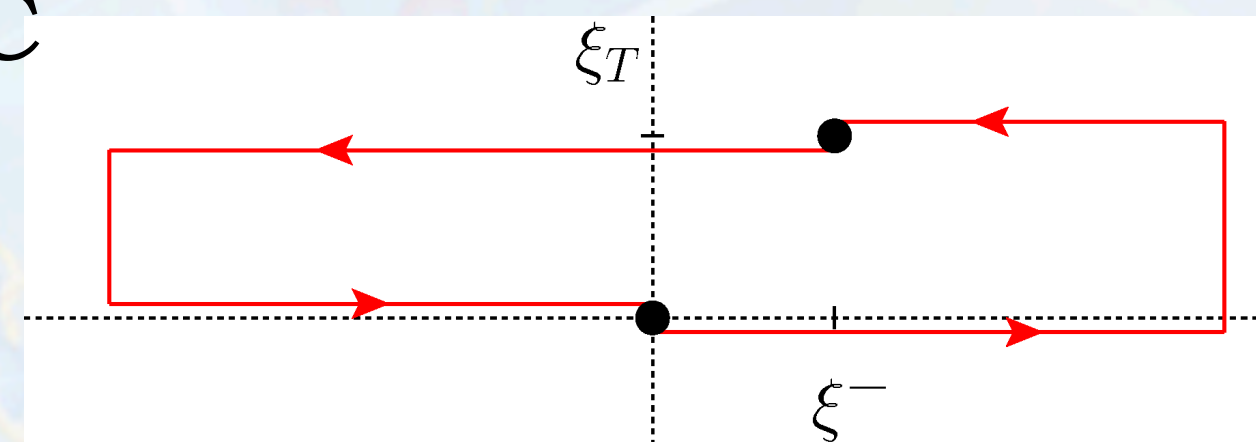
* DP TMDs:

(c) $[+, -]$ and (d) $[-, +]$

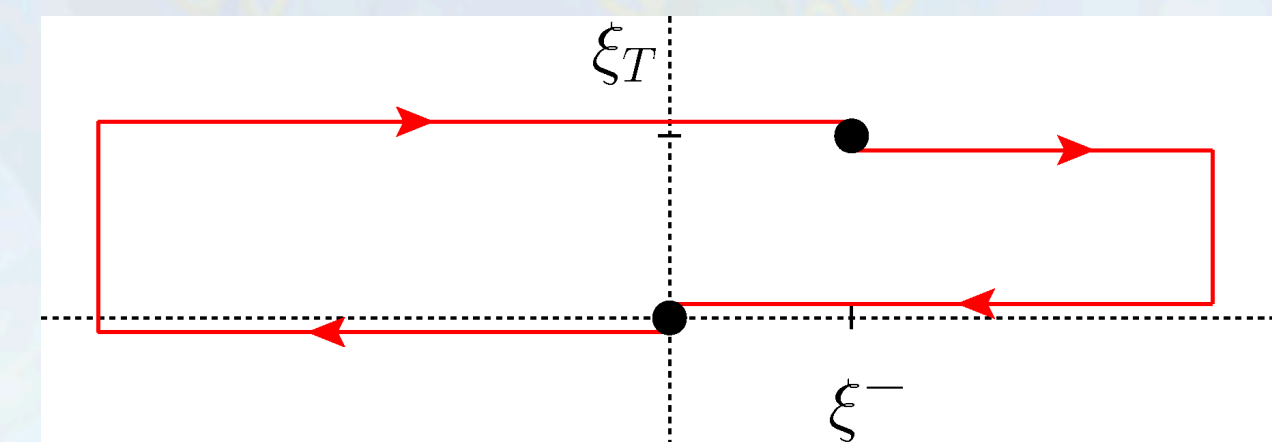
* Appearance of new gauge **loop links**:

(e) $[+\square, +\square]$, (f) $[+, +\square]$,

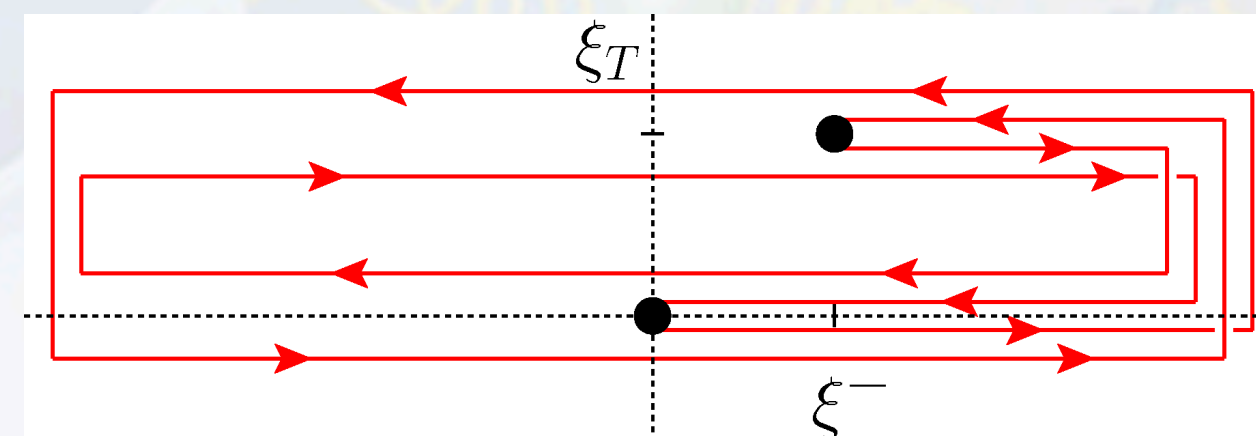
(g) $[\square, \square]$, and (h) $[\square, \square]$



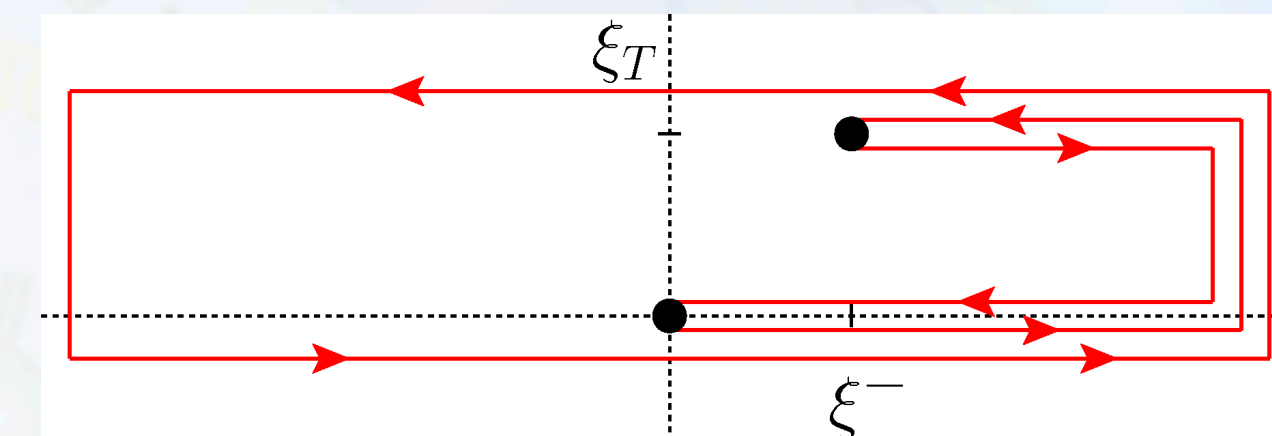
(c)



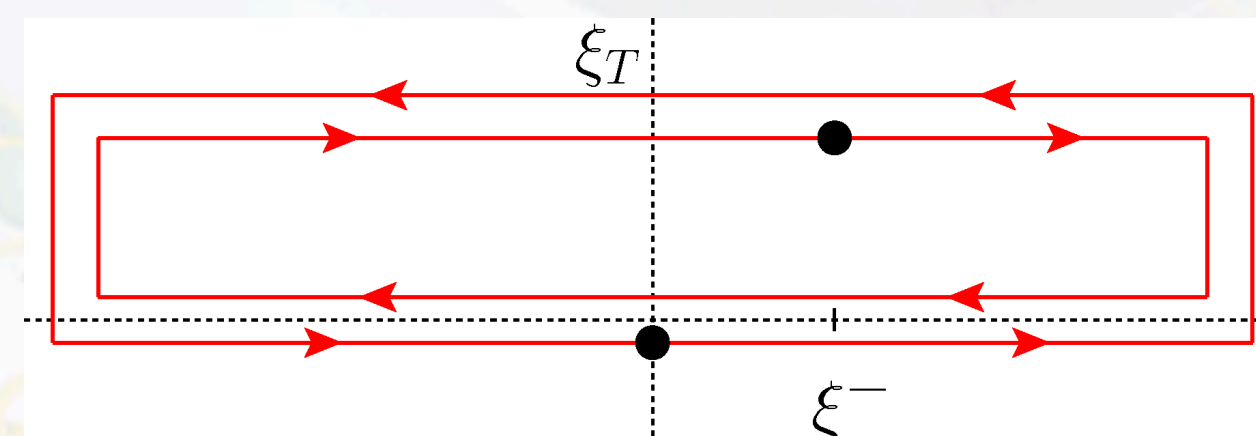
(d)



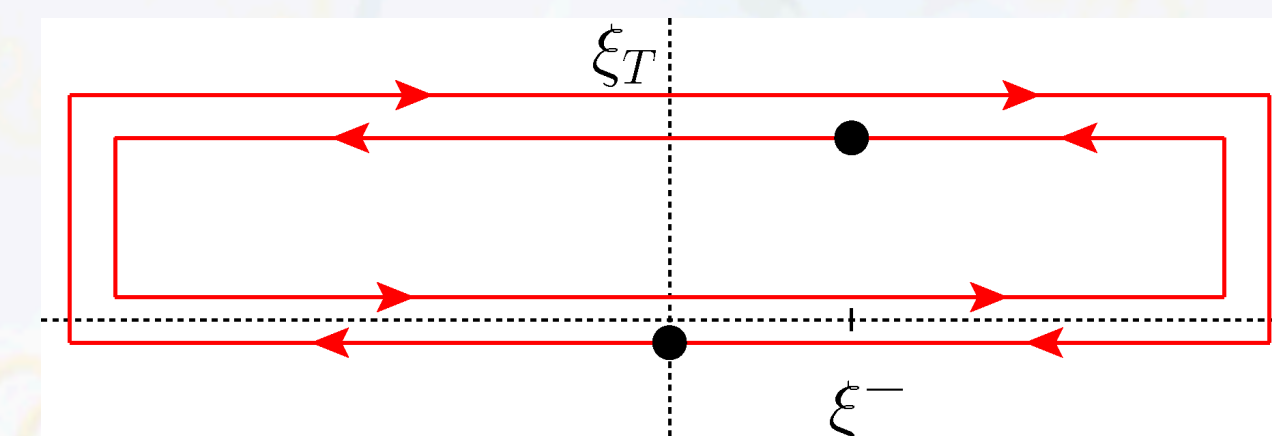
(e)



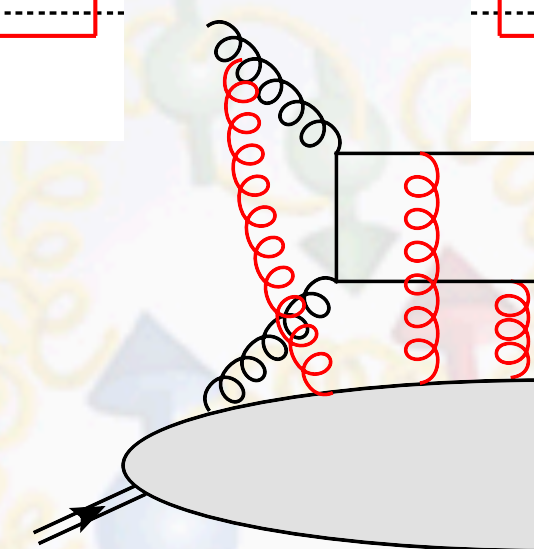
(f)



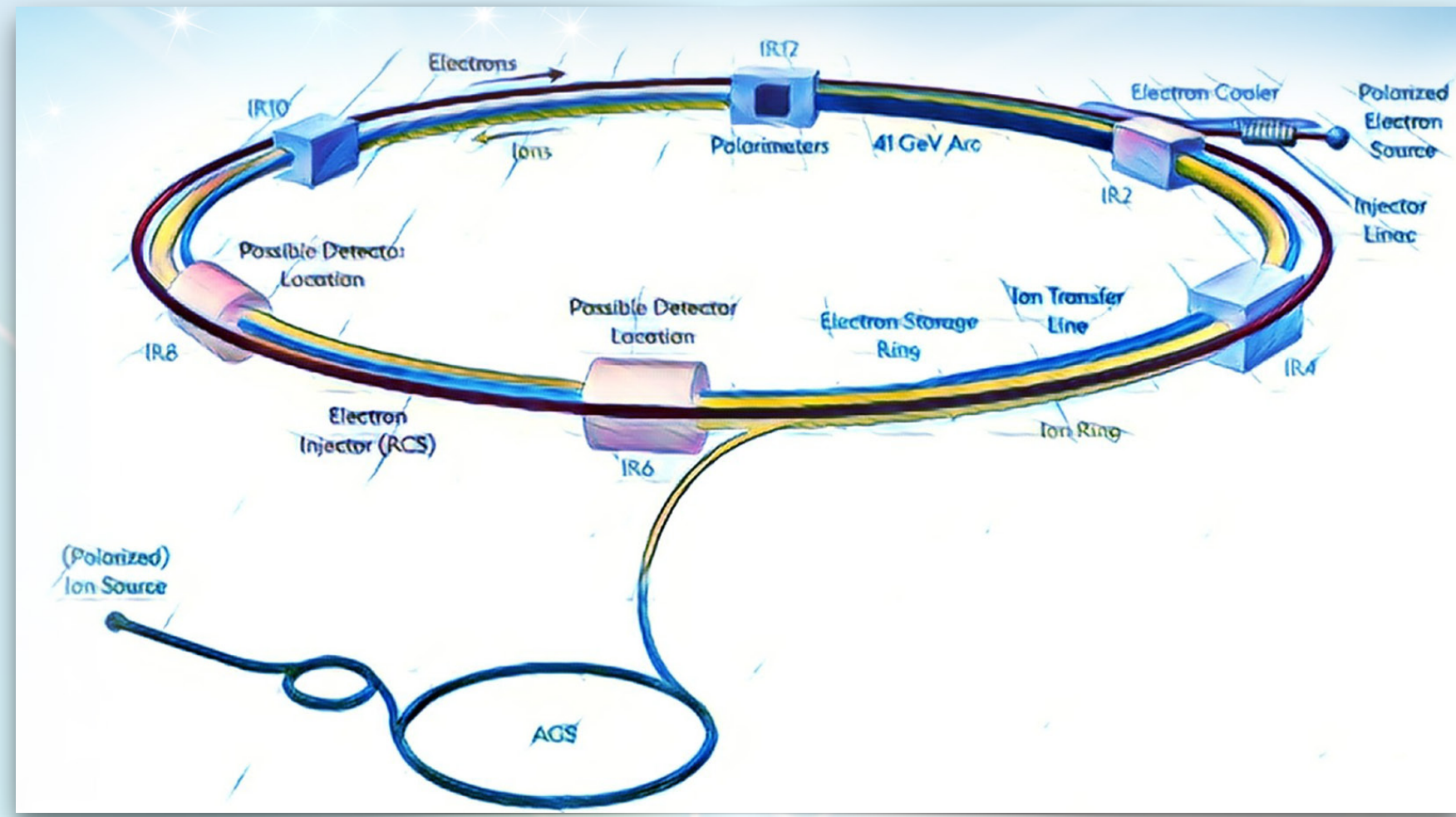
(g)



(h)



3D proton tomography at new-generation colliders

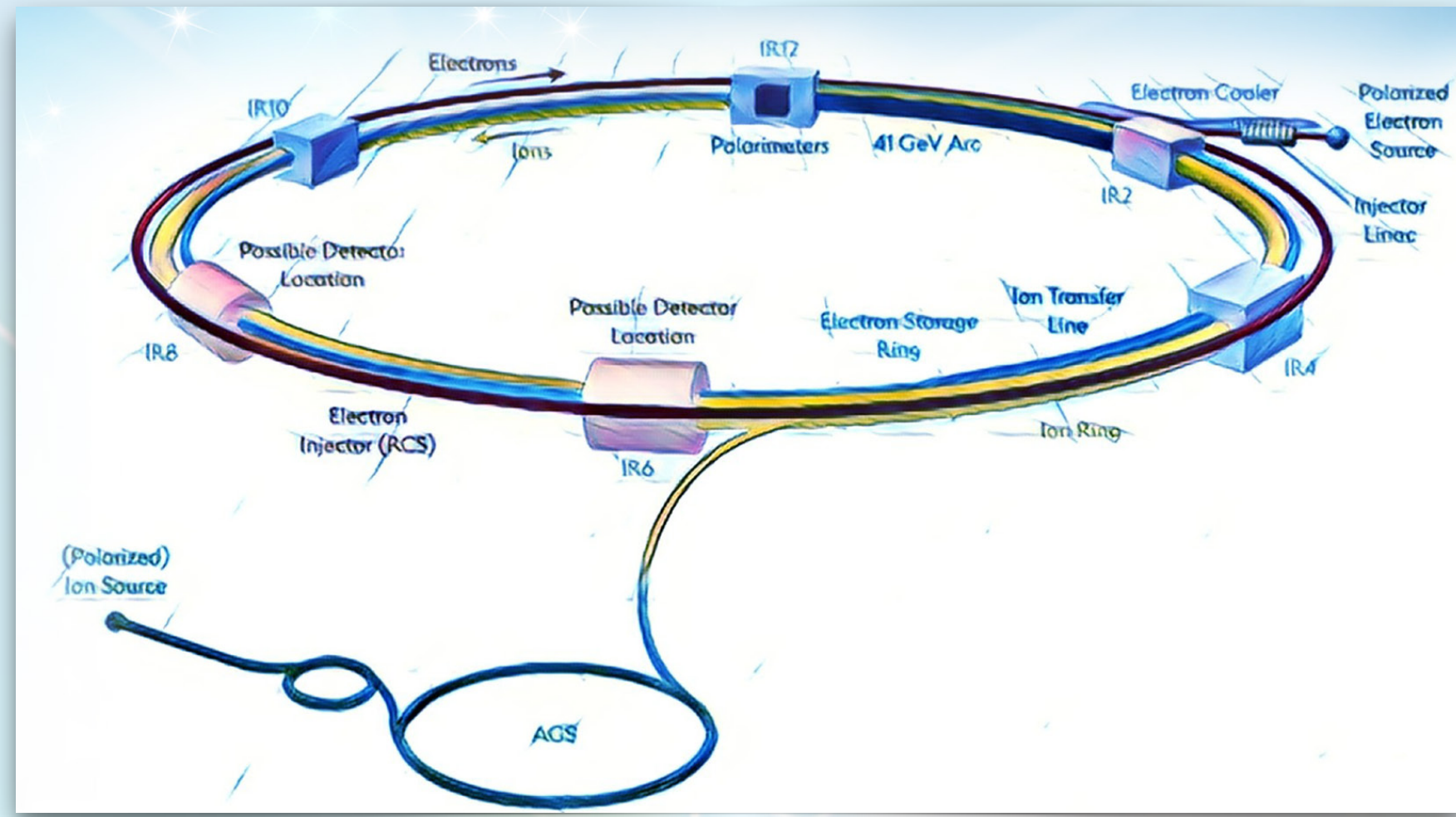


EIC

EIC Yellow Report [\[EICUG \[arXiv:2103.05419\]\]](#)

Accessing the proton content

3D proton tomography at new-generation colliders



EIC

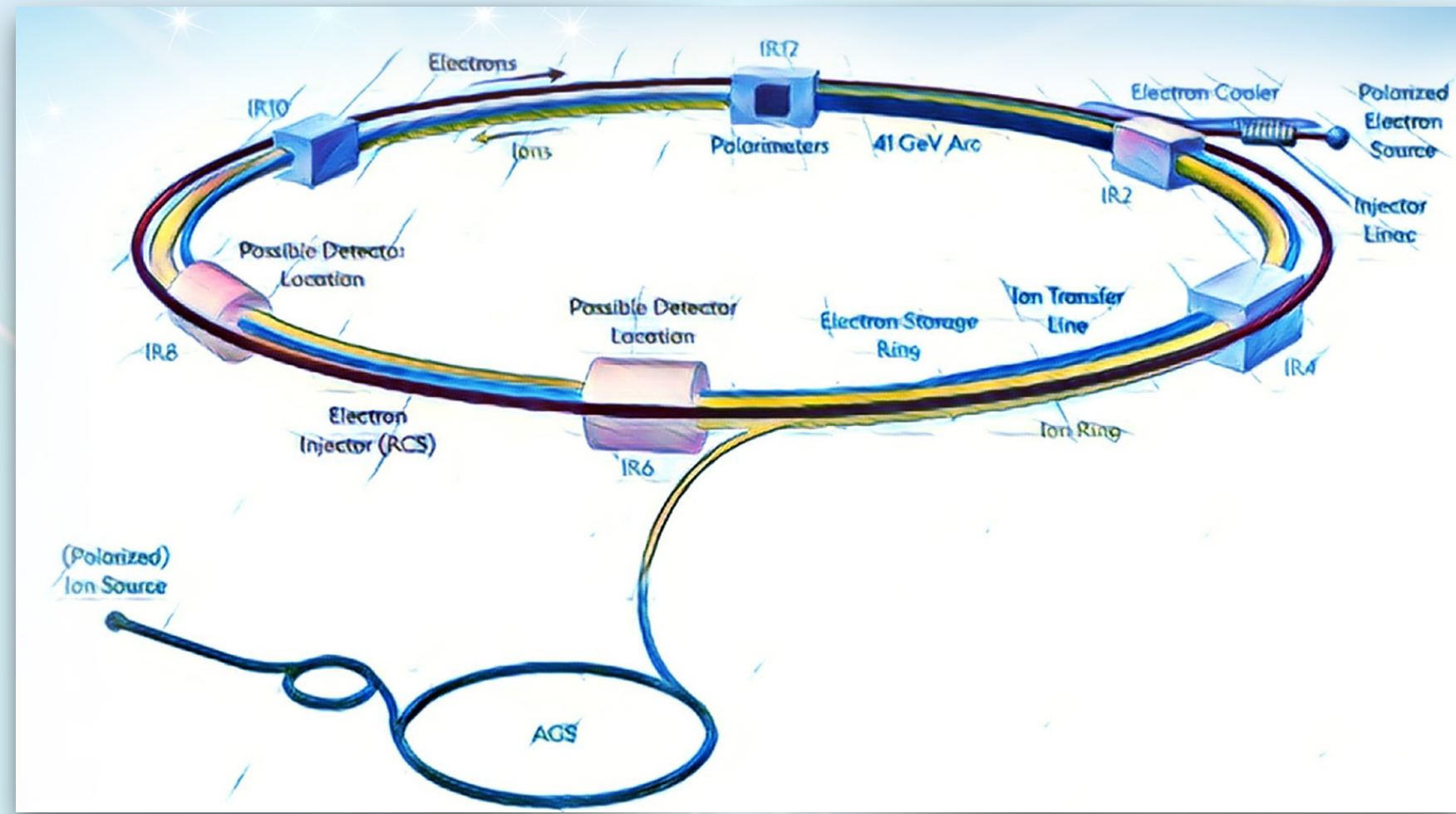
EIC Yellow Report  [EICUG [arXiv:2103.05419]]

Accessing the proton content



Core sector of **EIC** analyses

3D proton tomography at new-generation colliders



EIC

NICA-SPD

EIC Yellow Report [\[EICUG \[arXiv:2103.05419\]\]](#)

Gluon content at **NICA-SPD** [\[NICA \[arXiv:2011.15005\]\]](#)

Accessing the proton content

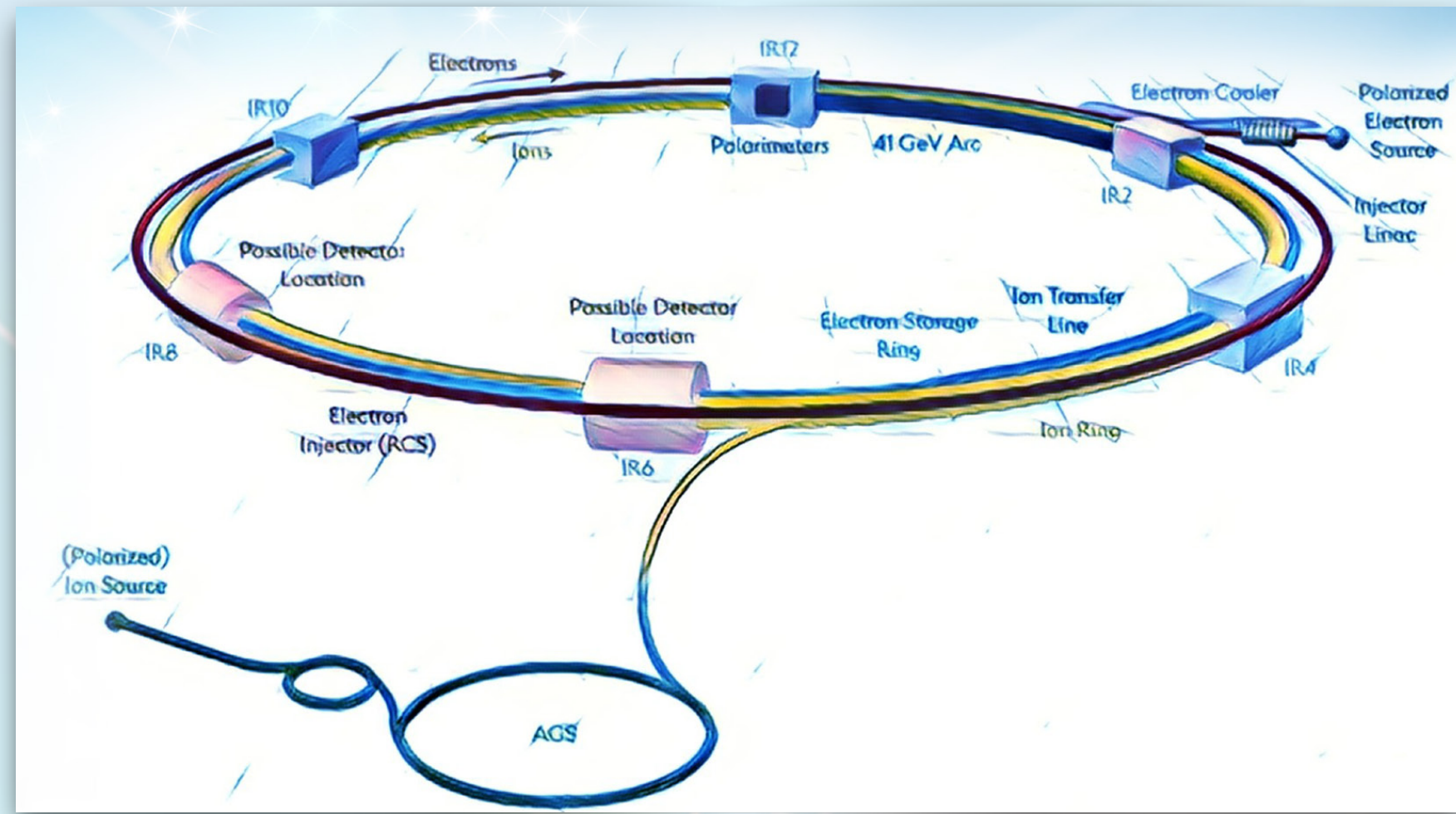


Core sector of **EIC** analyses



Significance of large- x studies at **NICA-SPD**

3D proton tomography at new-generation colliders



EIC

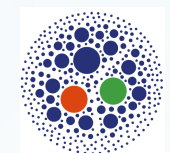
NICA-SPD

JLab12

EIC Yellow Report [\[EICUG \[arXiv:2103.05419\]\]](#)

Gluon content at NICA-SPD [\[NICA \[arXiv:2011.15005\]\]](#)

Accessing the proton content



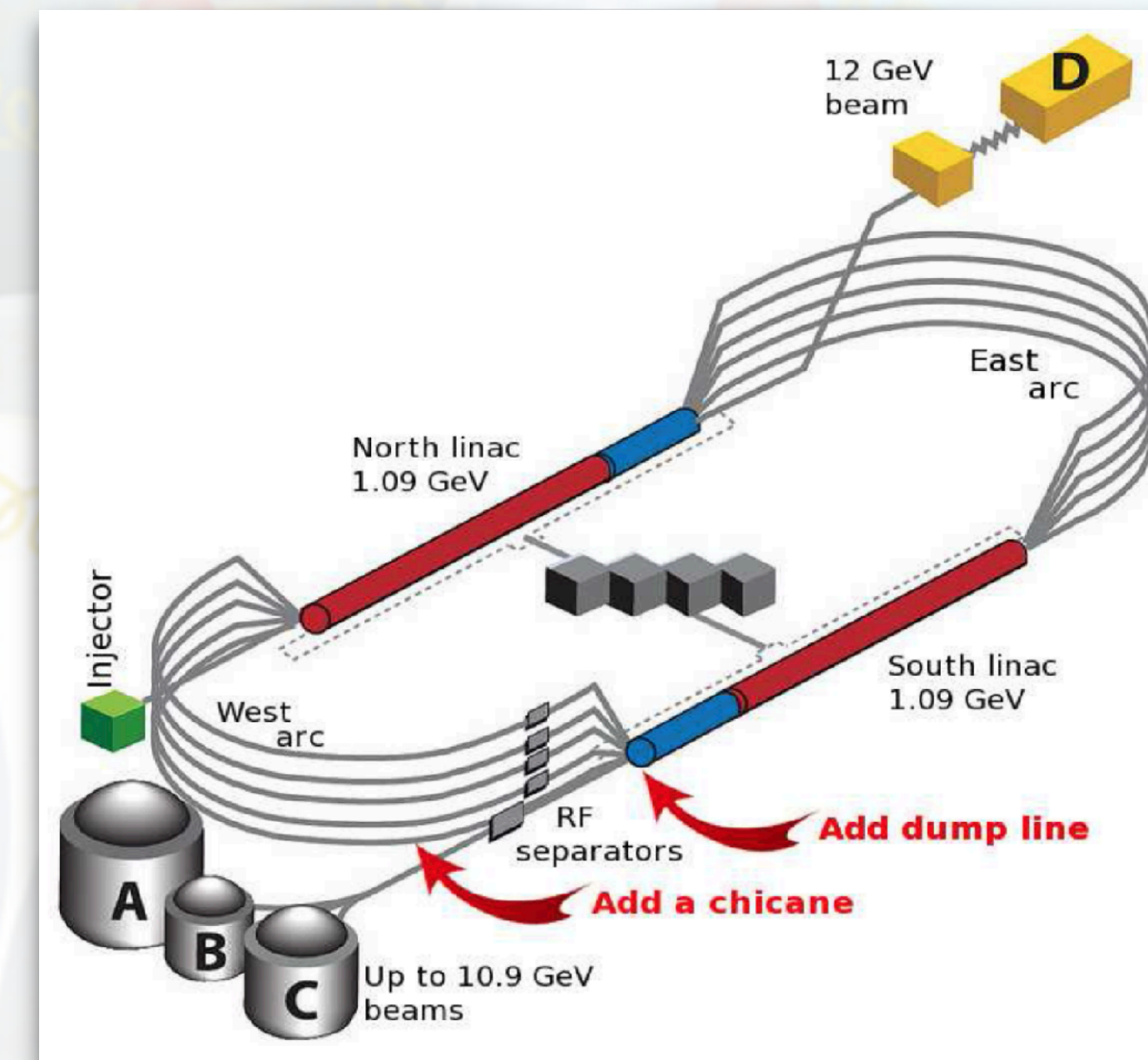
Core sector of EIC analyses



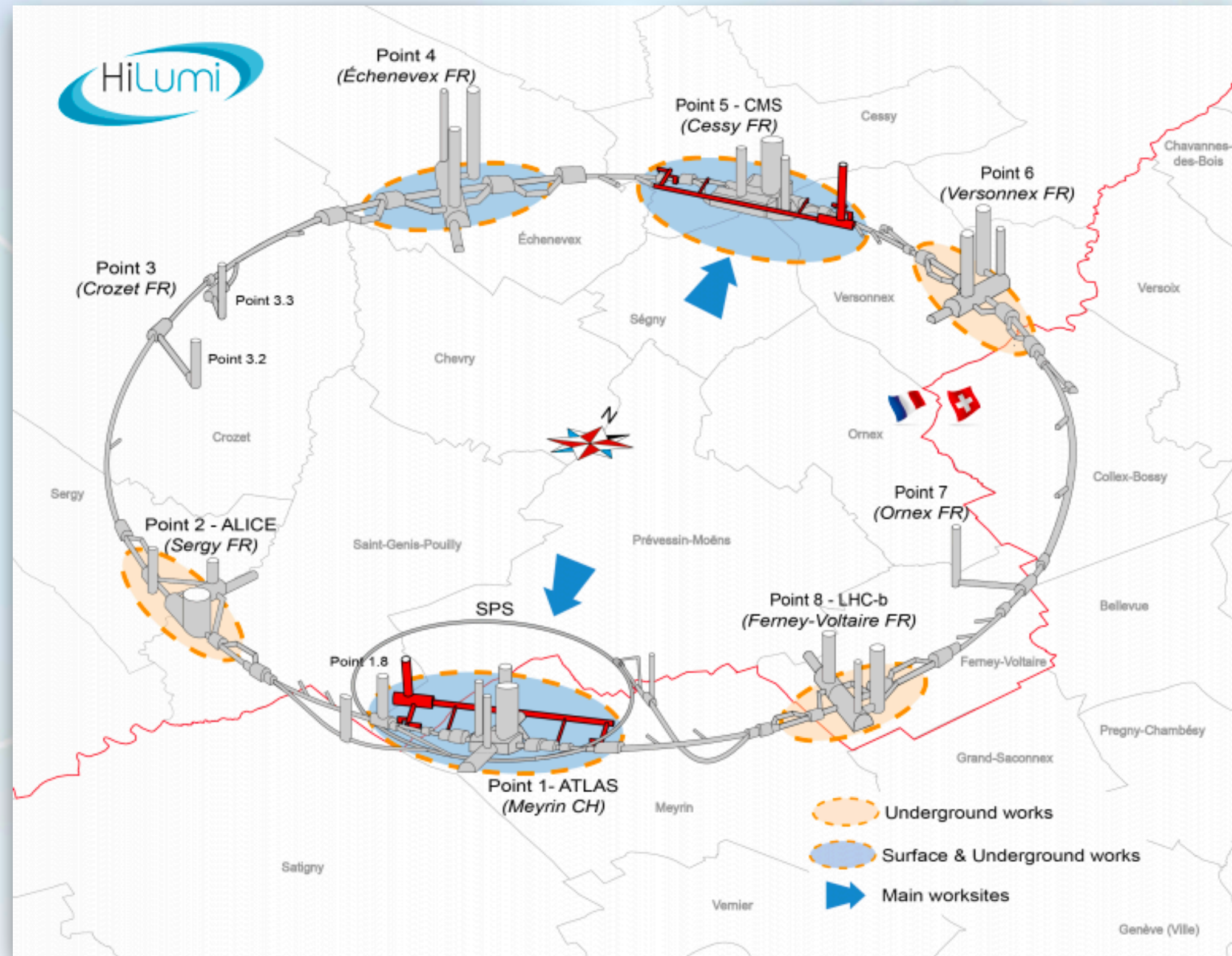
Significance of large- x studies at NICA-SPD



Gluon-TMD *dynamics* in JLab12 physics



Connections with high-energy physics

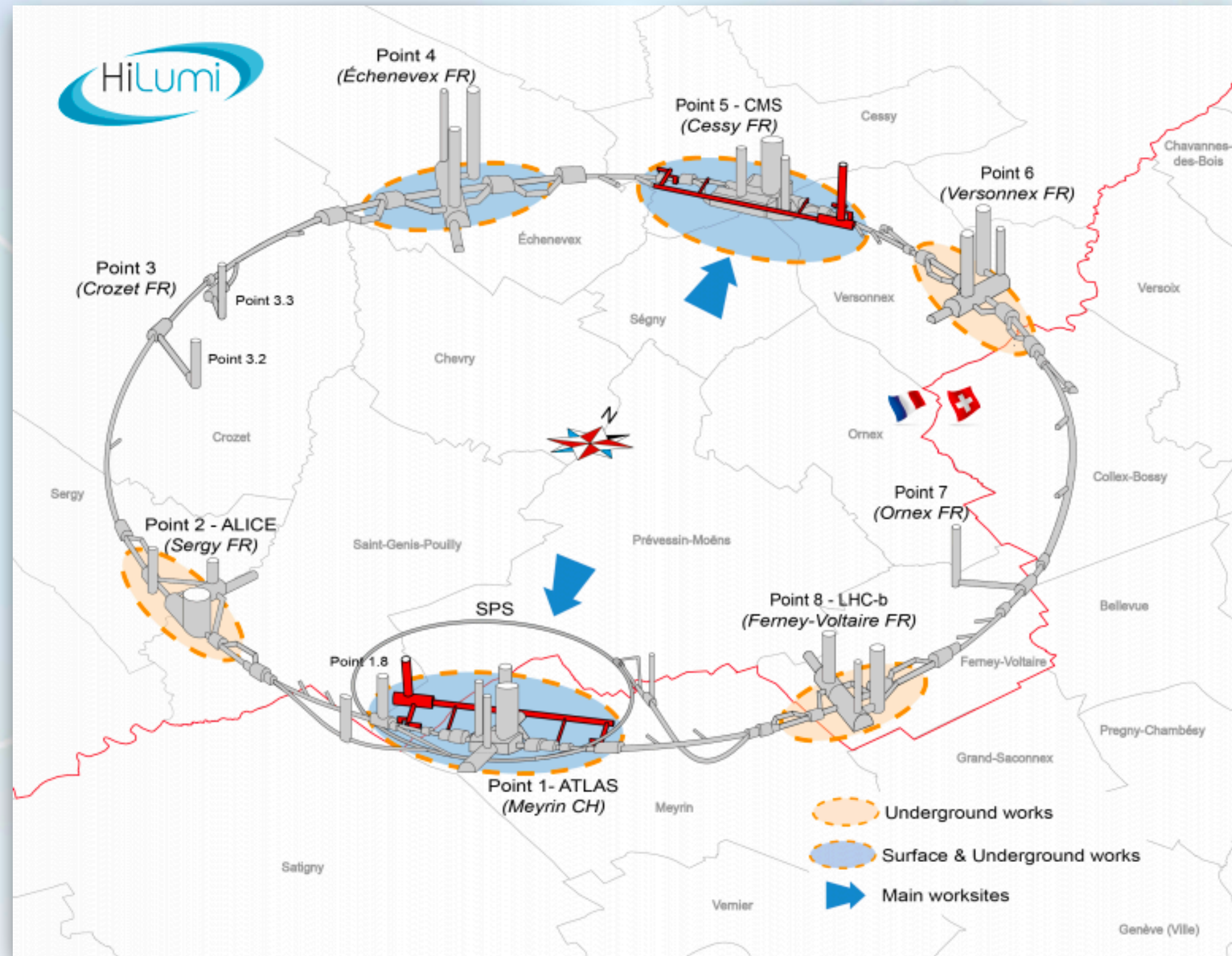


HL-LHC

Quarkonium studies at **HL-LHC**  [QAT [arXiv:2012.14161]]

Hadronic structure at high energies

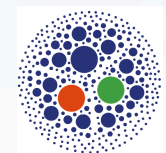
Connections with high-energy physics



HL-LHC

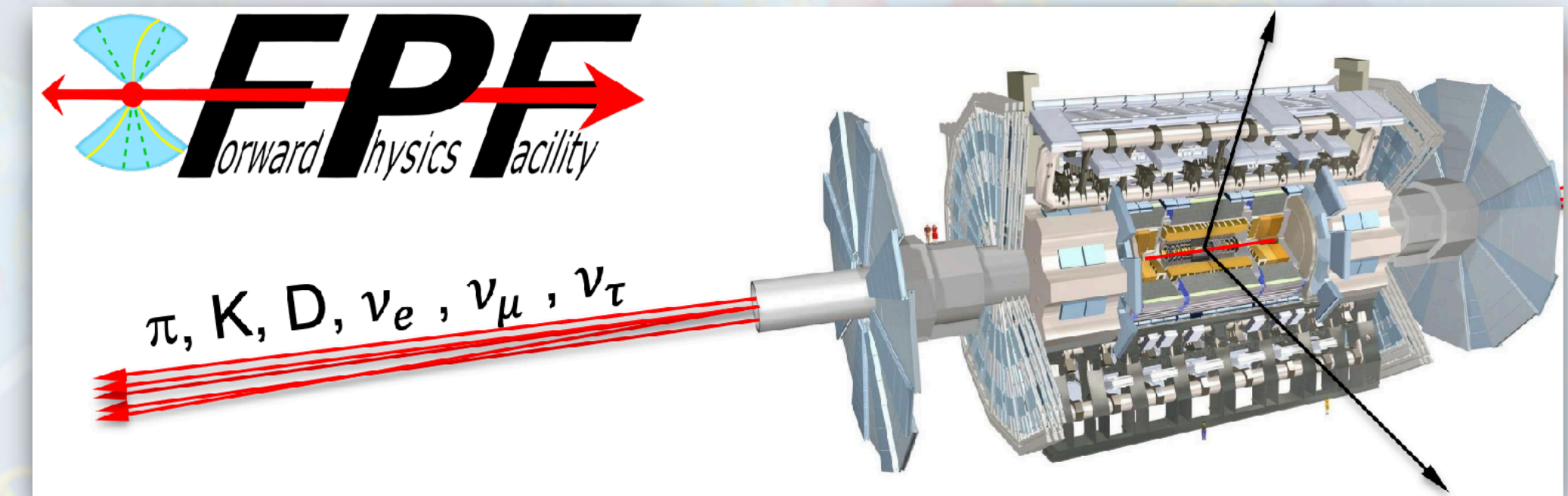
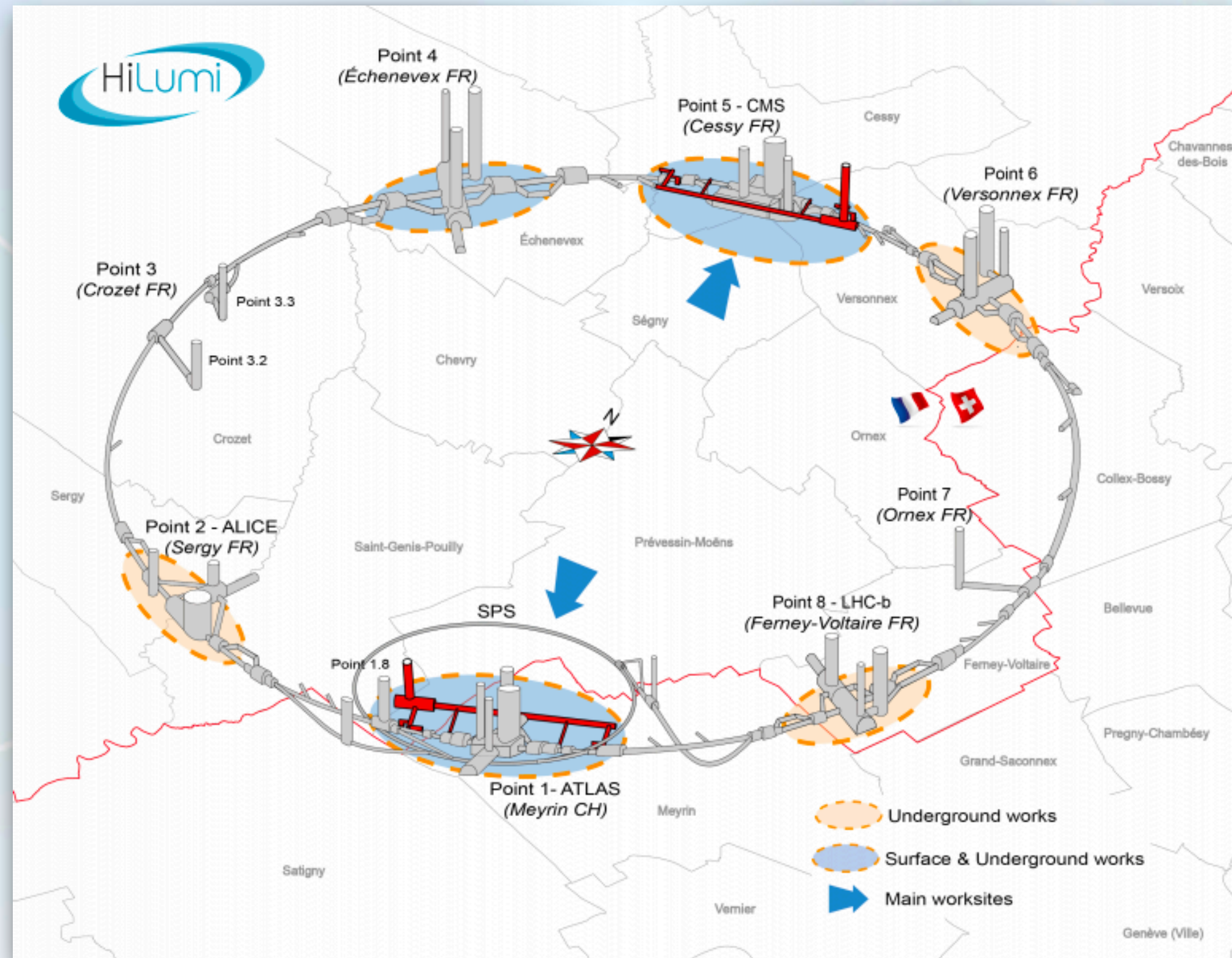
Quarkonium studies at HL-LHC [QAT [arXiv:2012.14161]]

Hadronic structure at high energies



Intrinsic effect of gluon polarization in **unpolarized** pp collisions

Connections with high-energy physics



HL-LHC

Forward Physics Facility

Quarkonium studies at **HL-LHC** [QAT [arXiv:2012.14161]]

The Forward Physics Facility (**FPF**) [FPF [arXiv:2109.10905]]

Hadronic structure at high energies

Intrinsic effect of gluon polarization in **unpolarized** pp collisions

Precision studies of proton structure via **natural stability** of high-energy resummation

The background features a repeating pattern of circular diagrams illustrating gluon Transverse Momentum Distributions (TMDs). Each diagram shows a central gluon (represented by a red sphere) interacting with a quark (represented by a blue sphere) and a gluon (represented by a green sphere). The gluon is shown as a wavy line with a red arrow indicating its spin. The quark and gluon are shown as spheres with blue arrows indicating their transverse momenta. The diagrams are arranged in a grid-like pattern, with some overlapping. The overall color scheme is light blue and green, with a subtle grid pattern.

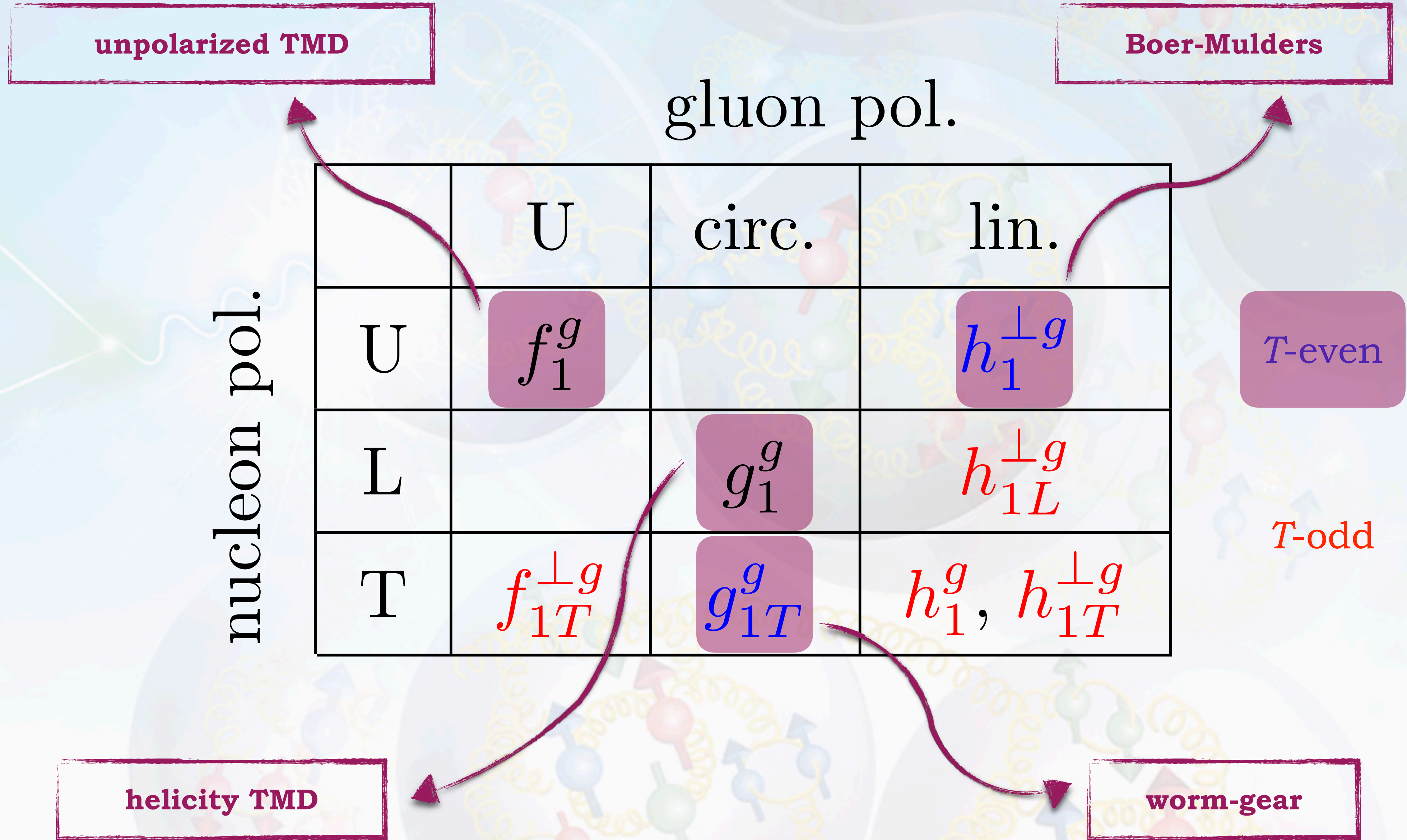
Modeling gluon TMDs

T -even and T -odd gluon TMD PDFs at leading-twist

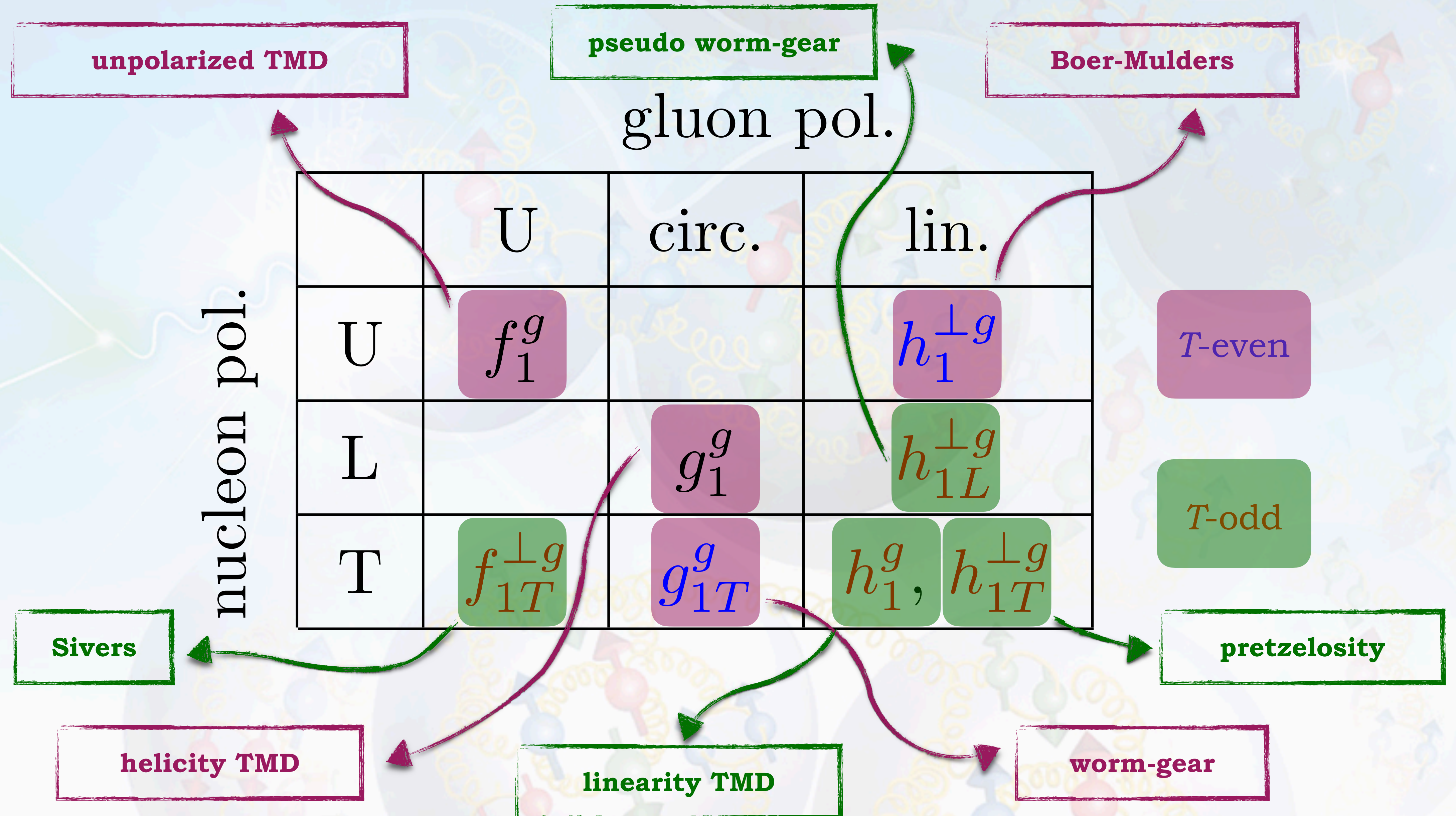
gluon pol.

nucleon pol.		U	circ.	lin.	
	U	f_1^g		$h_1^{\perp g}$	T -even
	L		g_1^g	$h_{1L}^{\perp g}$	T -odd
	T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g, h_{1T}^{\perp g}$	

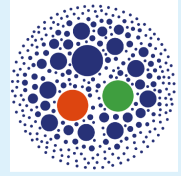
T-even and T-odd gluon TMD PDFs at leading-twist



T-even and T-odd gluon TMD PDFs at leading-twist

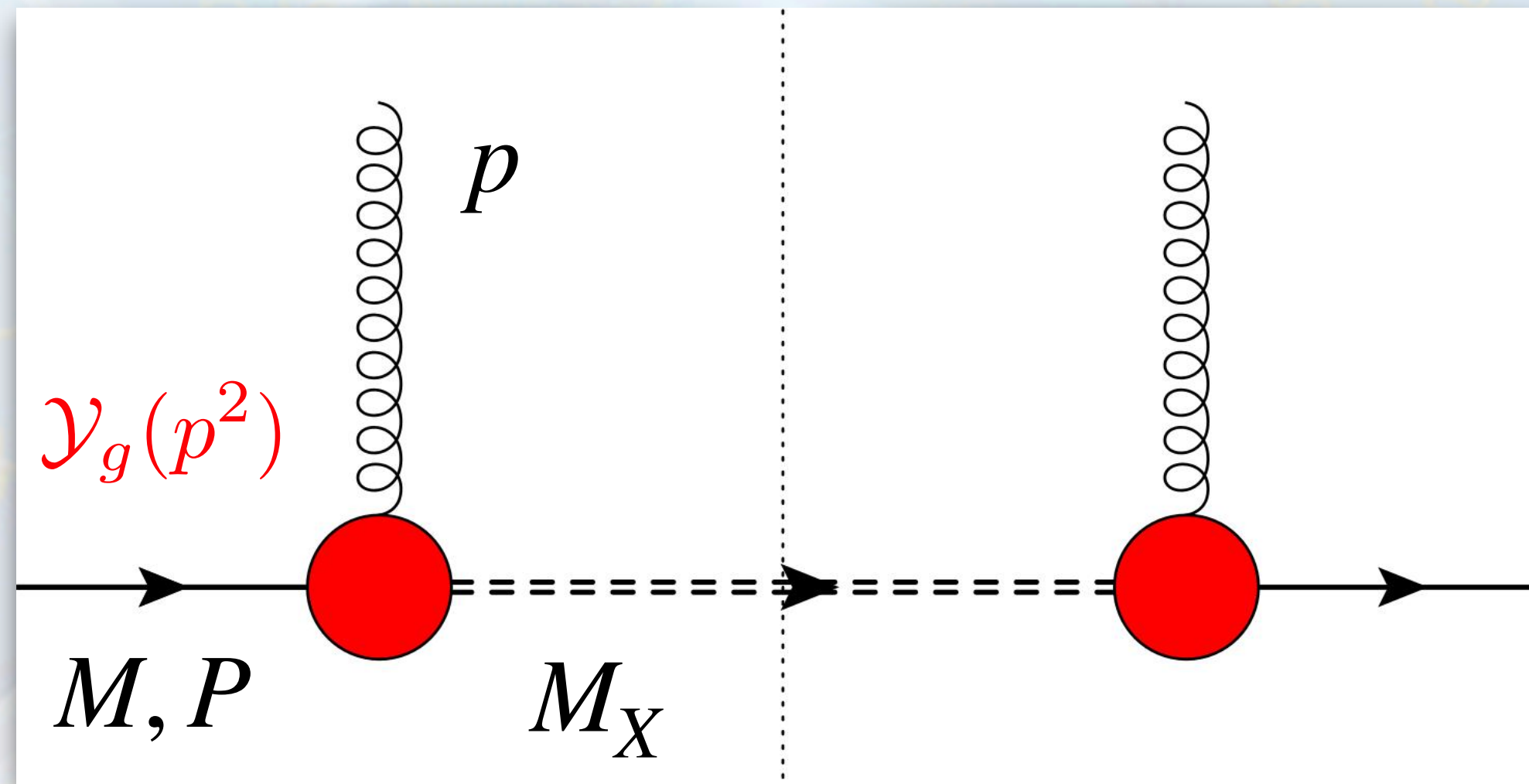


Assumptions of the model

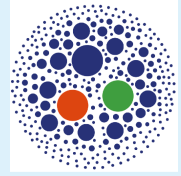


Spin-1/2 spectator

Lowest Fock state:
tri-quark spectator
on-shell and
with mass M_X

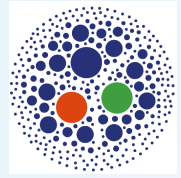
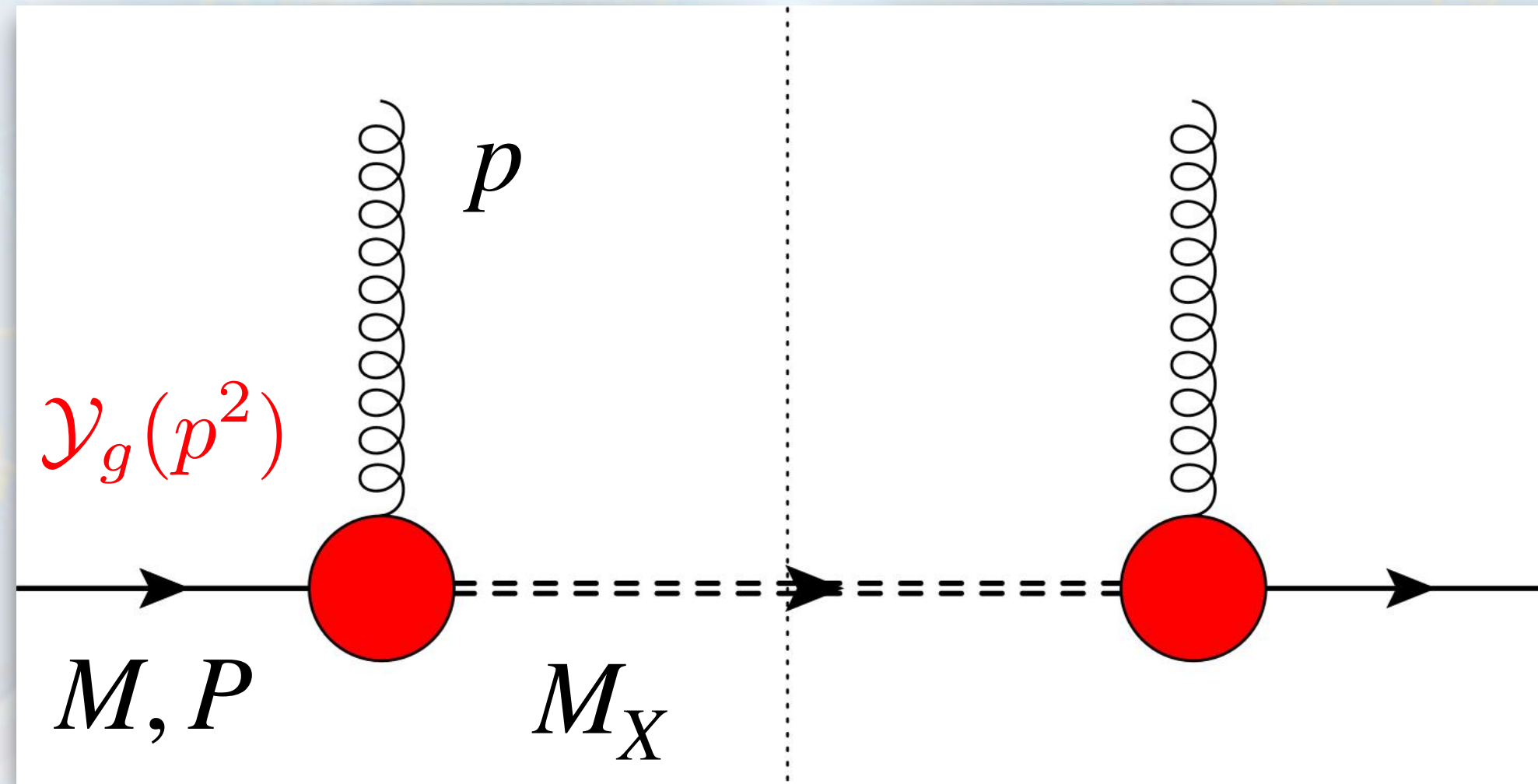


Assumptions of the model



Spin-1/2 spectator

Lowest Fock state:
tri-quark spectator
 on-shell and
 with mass M_X



Nucleon-gluon-spectator vertex

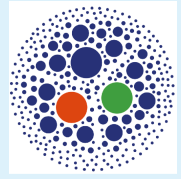
$$\Phi_g = \frac{1}{2(2\pi)^3(1-x)P^+} \text{Tr} \left[(\not{P} + M) \frac{1 + \gamma^5 \not{\not{P}}}{2} G_{\mu\rho}^*(p) G^{\nu\sigma}(p) \mathcal{Y}_g^{\rho*} \mathcal{Y}_{g\sigma} (\not{P} - \not{p} + M) \right]$$

$$\mathcal{Y}_g^\mu = g_1(p^2) \gamma^\mu + i \frac{g_2(p^2)}{2M} \sigma^{\mu\nu} p_\nu$$

mimics proton form factors
 (conserved EM current
 of a free nucleon)



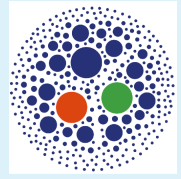
Assumptions of the model



Link with collinear factorization

1. p_T -integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution
2. TMDs and PDFs *decouple* due to evolution

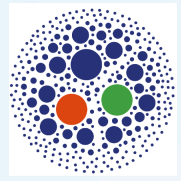
Assumptions of the model



Link with collinear factorization

1. p_T -integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution
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$$g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2}{|p^2 - \Lambda_X^2|^2}$$

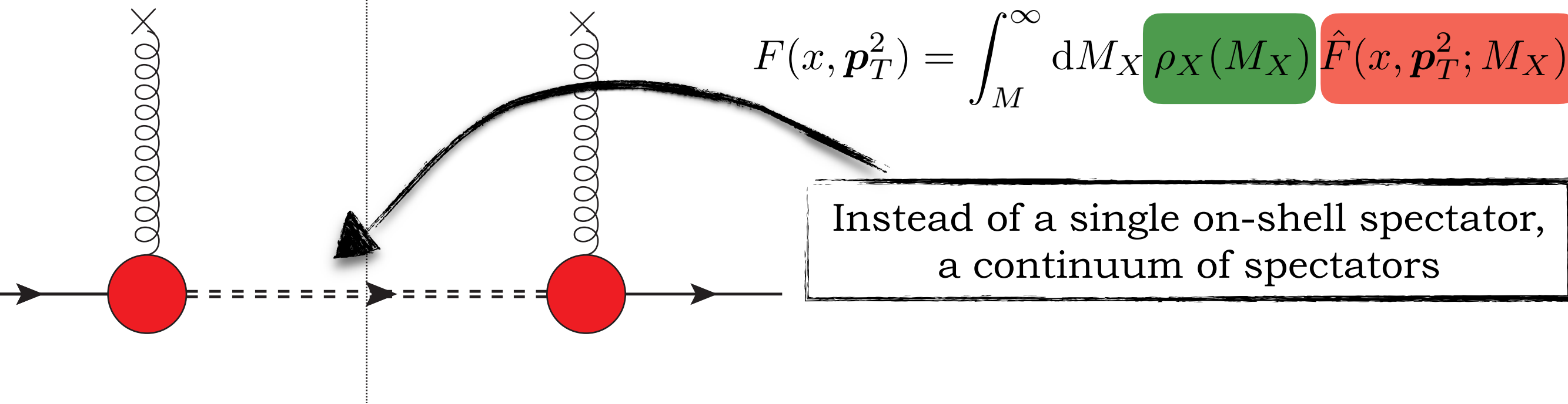


Dipolar form factor(s)

1. Cancels singularity of gluon propagator
2. Suppresses effects of high p_T
3. Compensates log divergences arising from p_T -integration
4. Adds three more parameters: $\kappa_{1,2}$ and Λ_X

Our model at a glance

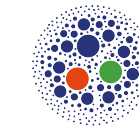
Spectator-system spectral-mass function



$$F(x, p_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, p_T^2; M_X)$$

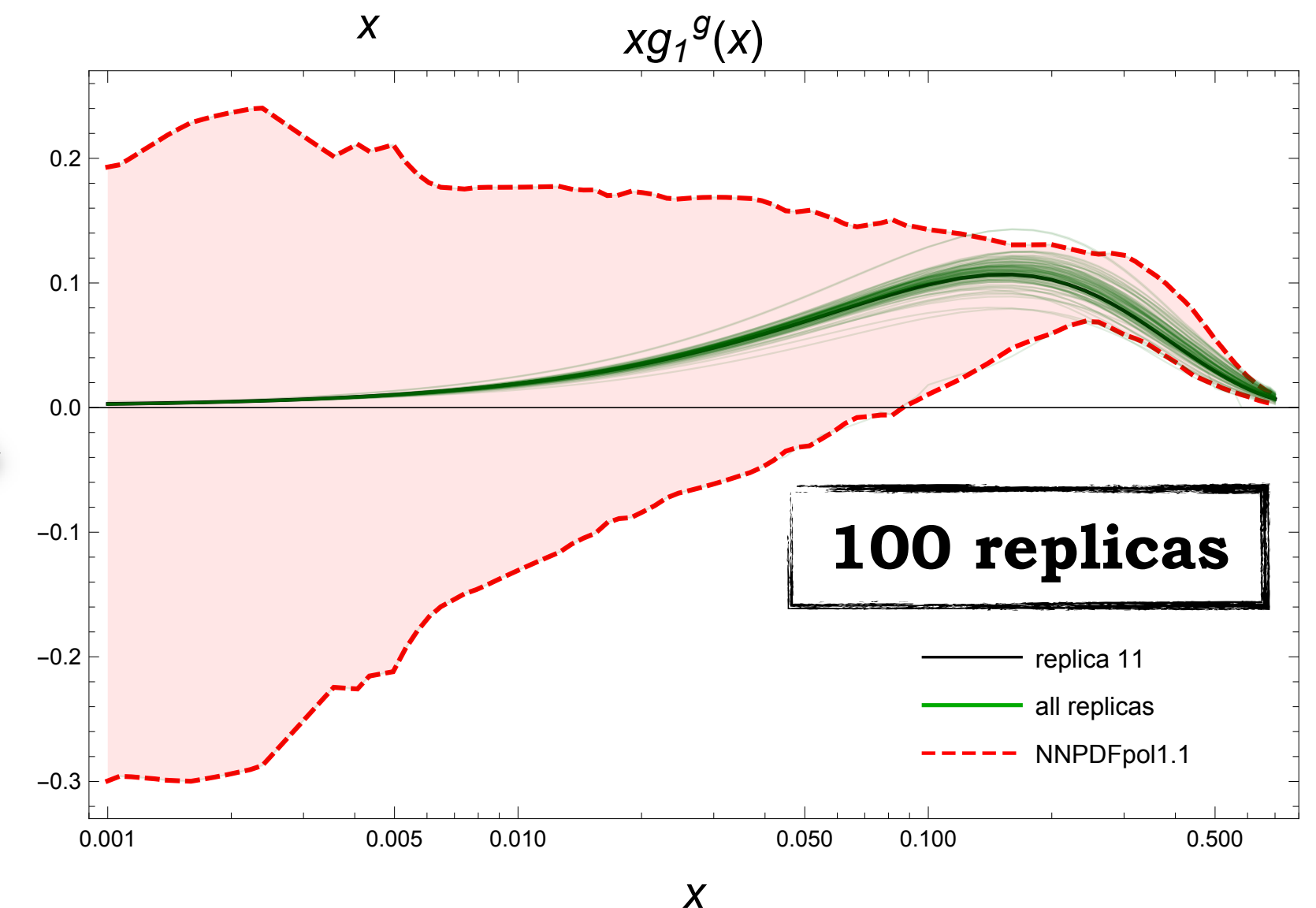
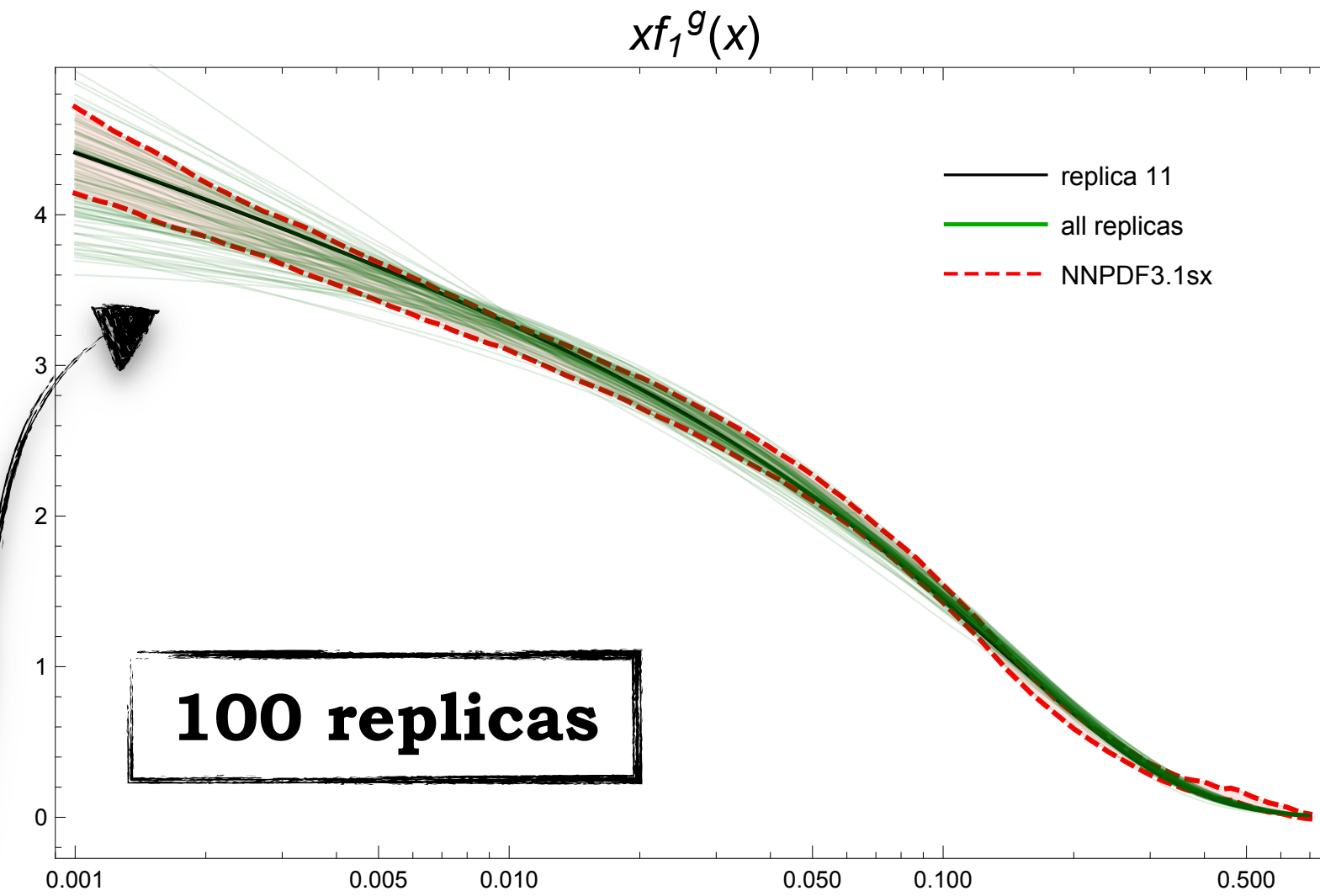
Spectral function **learns** small- and moderate- x info encoded in **NNPDF** collinear parametrizations (NNPDF3.1sx + NNPDFpol1.1)

- Simultaneous fit** of f_1 and g_1 PDFs
- Inclusion of small- x resummation effects (**BFKL**)
- Calculation of all leading-twist T -even gluon TMDs

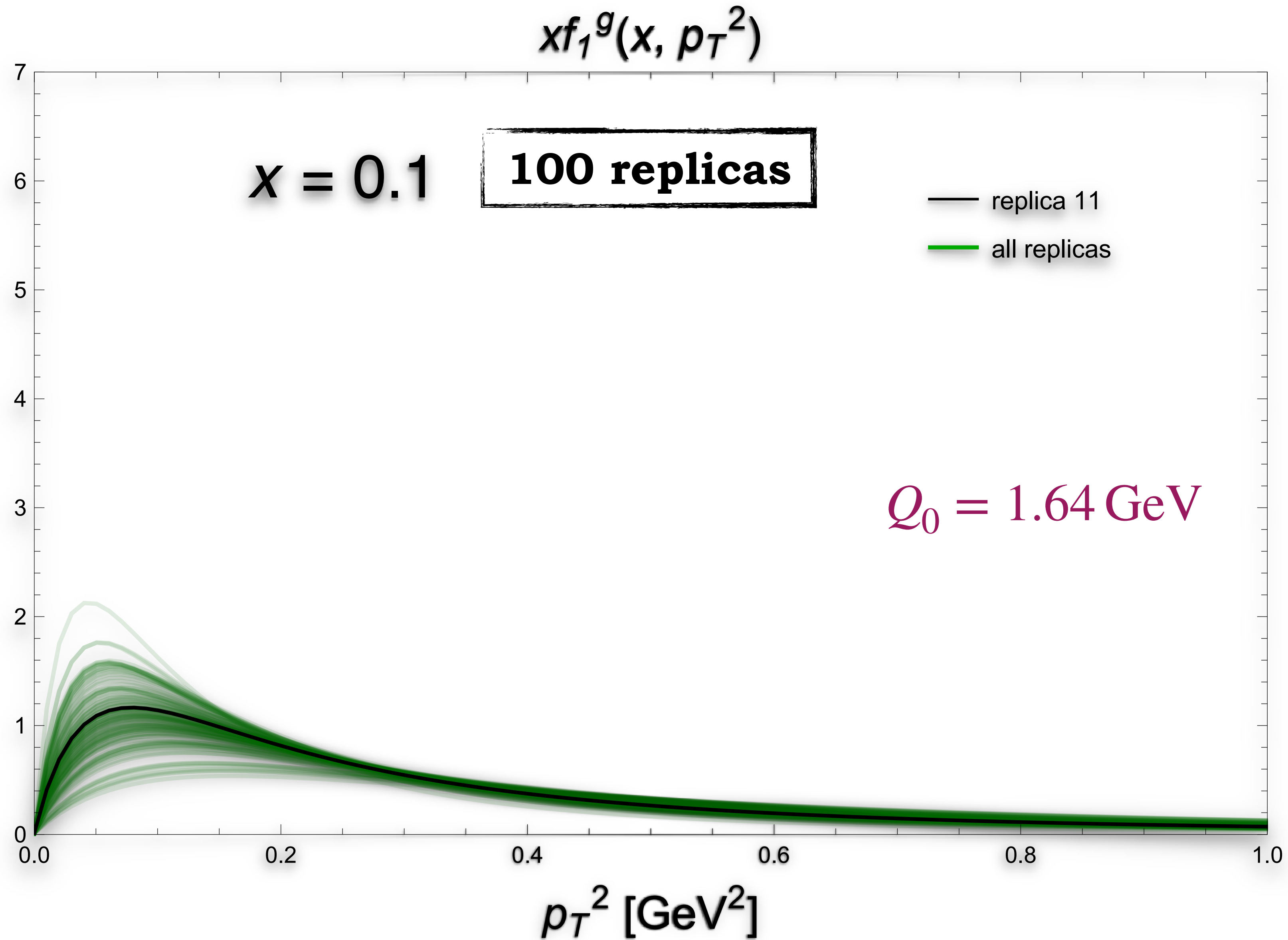
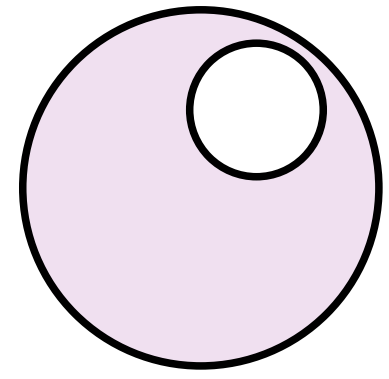


Link with collinear factorization

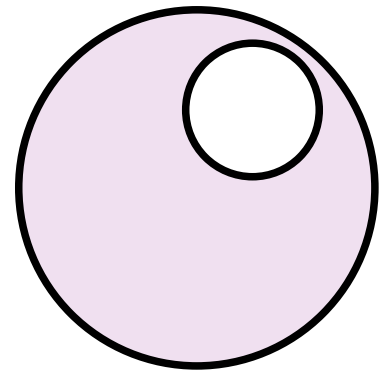
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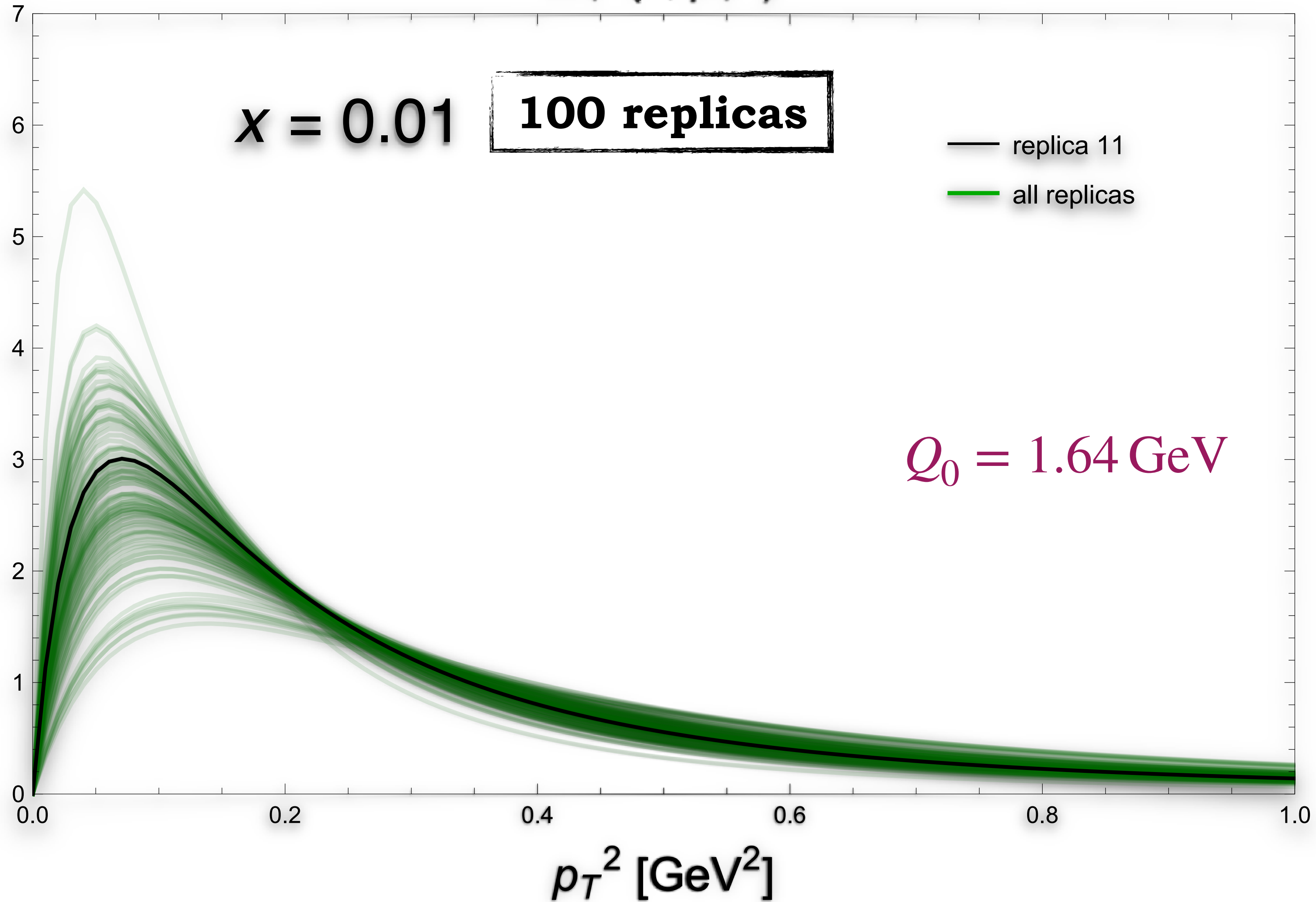
Unpolarized gluon TMD



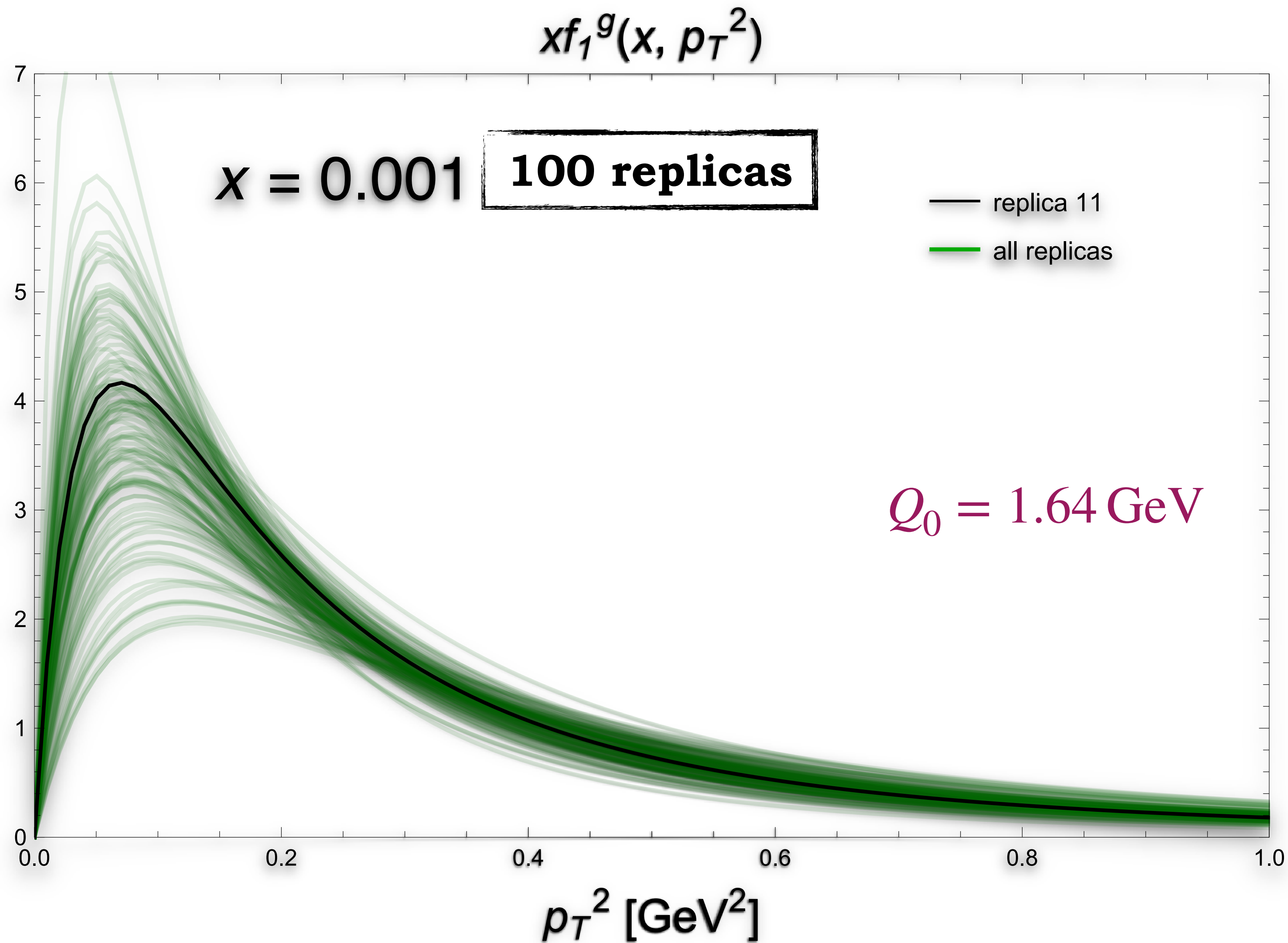
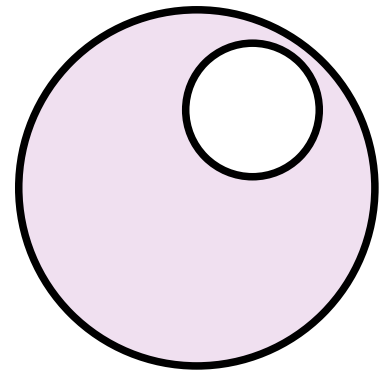
Unpolarized gluon TMD



$$xf_1^g(x, p_T^2)$$



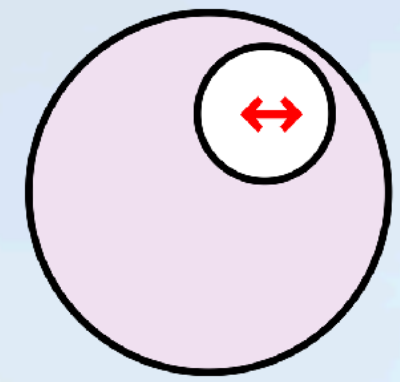
Unpolarized gluon TMD



The background features a complex, multi-layered illustration of a proton's internal structure. It shows various colored spheres (red, blue, green) representing quarks, connected by yellow wavy lines representing gluons. The entire scene is set against a light blue and green background with abstract, glowing patterns and lines, suggesting a high-energy physics environment.

**Glue TMDs
and quarkonia
in proton collisions**

Boer-Mulders effect in unpolarized pp collisions

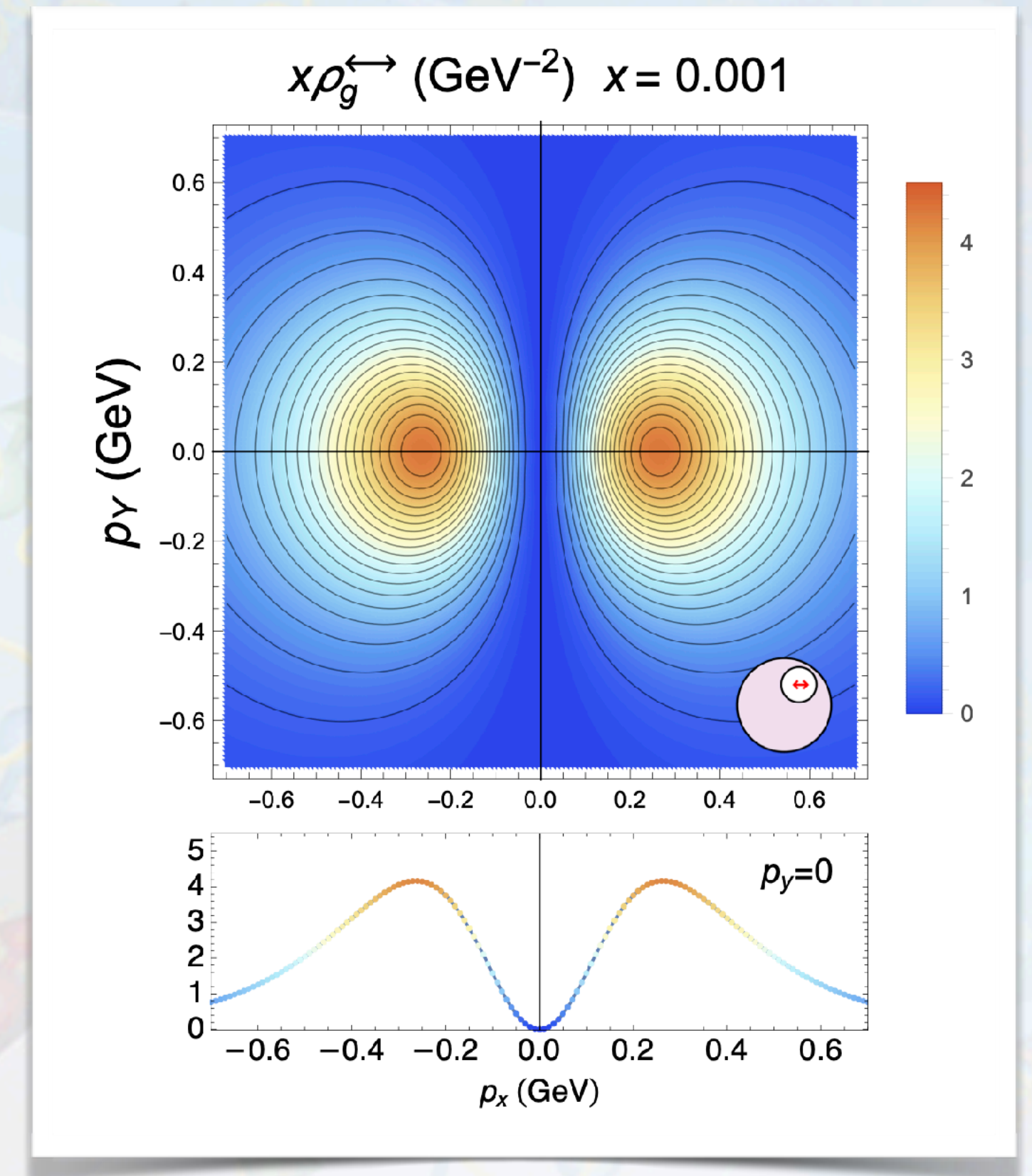


$[\leftrightarrow / \mathbf{u}]$

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

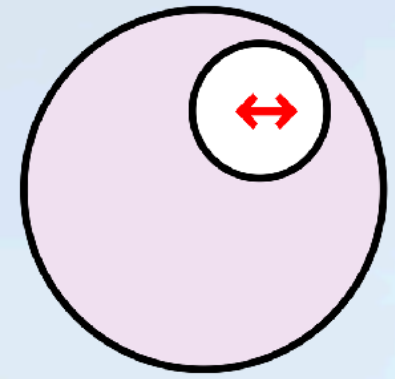
unpol.

Boer-Mulders



[A. Bacchetta, F.G. C., M. Radici, P. Taelis (2020)]

Boer-Mulders effect in unpolarized pp collisions

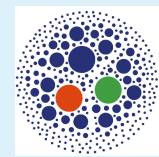


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unpol.

Boer-Mulders



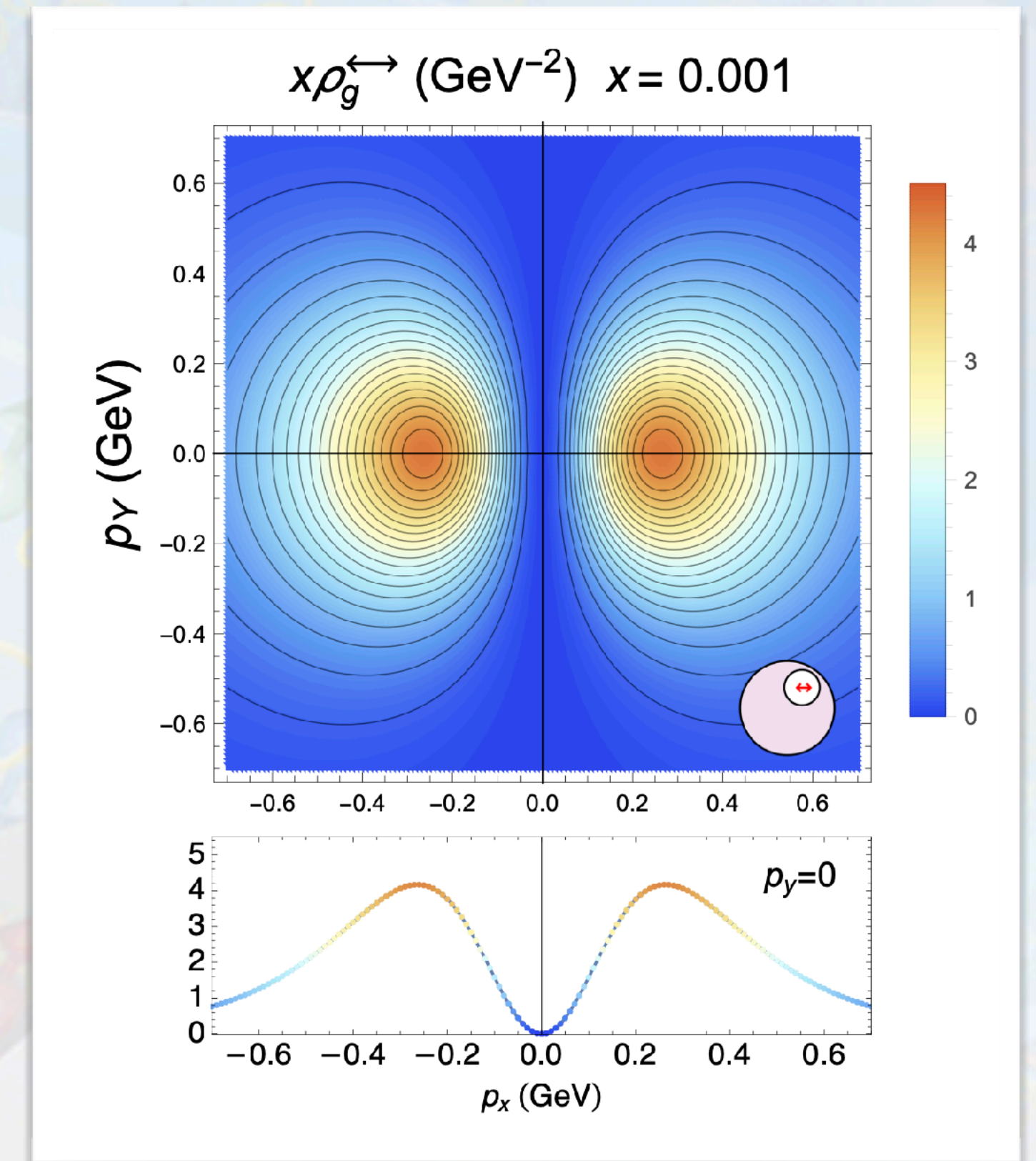
(Pseudo)scalar Higgs p_T -distribution

$$\frac{E d\sigma^{H(A)}}{d^3\vec{q}} \Big|_{q_T \ll m_H} = \frac{\pi\sqrt{2}G_F}{128m_H^2 S} \left(\frac{\alpha_s}{4\pi}\right)^2 |\mathcal{A}_{H(A)}(\tau)|^2 \times \left(C [f_1^g f_1^g] \pm C [w_H h_1^{\perp g} h_1^{\perp g}] \right) + \mathcal{O}\left(\frac{q_T}{m_H}\right)$$

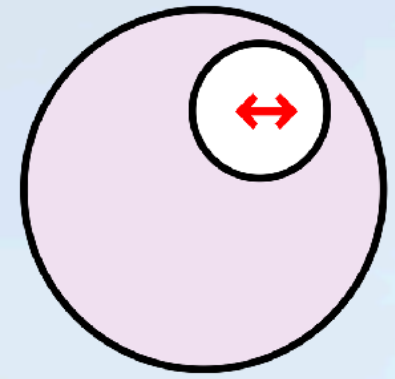
[D. Boer, W.J. den Dunnen, C. Pisano, M. Schlegel, W. Vogelsang (2012)]
(Higgs+jet angular distributions)

[D. Boer, C. Pisano (2015)]

[A. Bacchetta, F.G. C., M. Radici, P. Taelis (2020)]



Boer-Mulders effect in unpolarized pp collisions

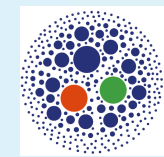


$[\leftrightarrow / \mathbf{u}]$

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

unpol.

Boer-Mulders



(Pseudo)scalar Higgs p_T -distribution

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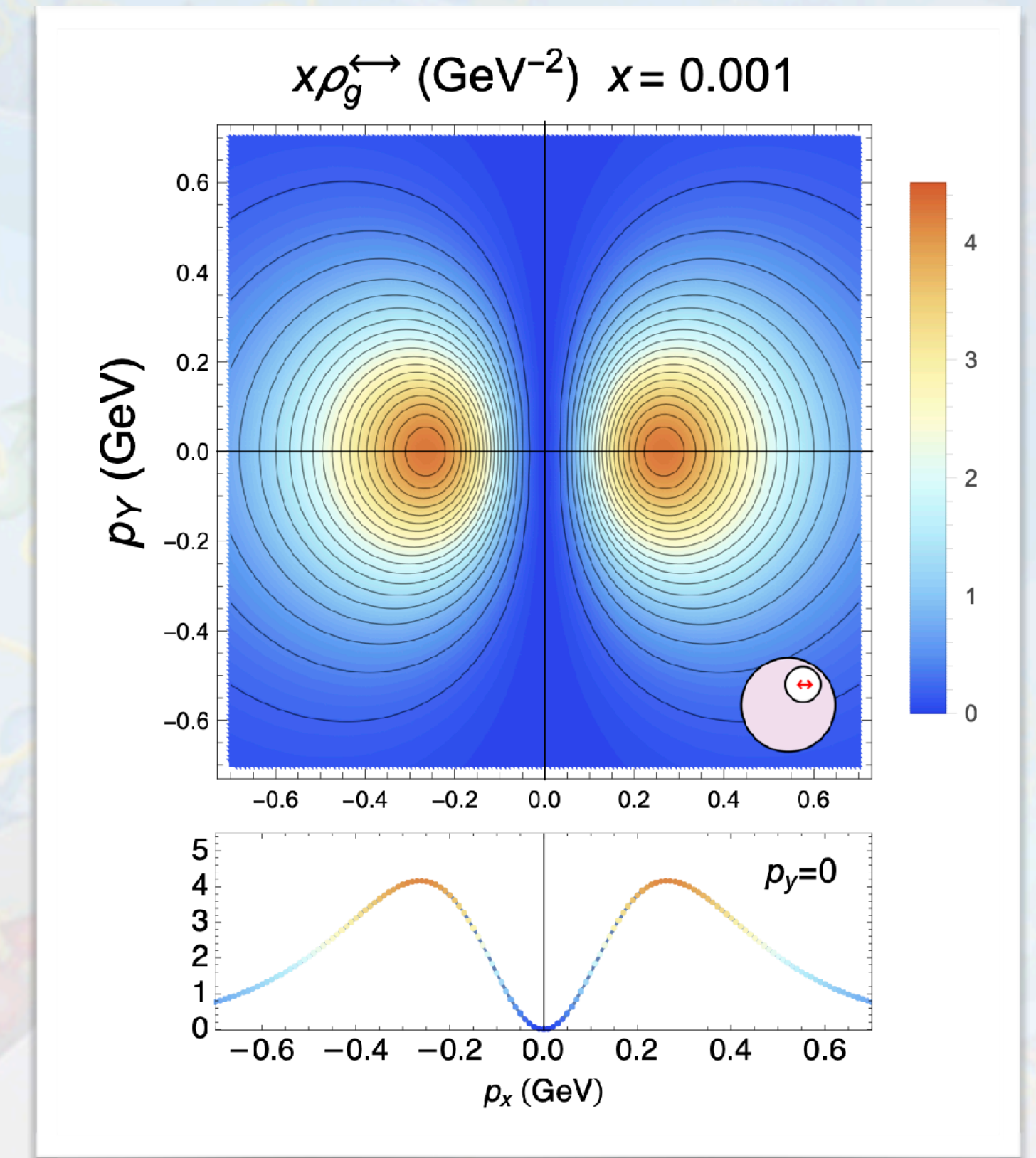
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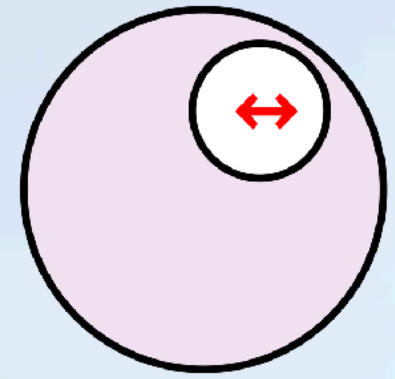


Model prediction at low- x

$$\frac{f_1^g(x, p_T^2)}{h_1^{\perp g}(x, p_T^2)} \underset{x \rightarrow 0^+}{\sim} \text{constant}$$



Boer-Mulders effect in unpolarized pp collisions

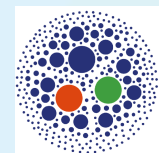


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unpol.

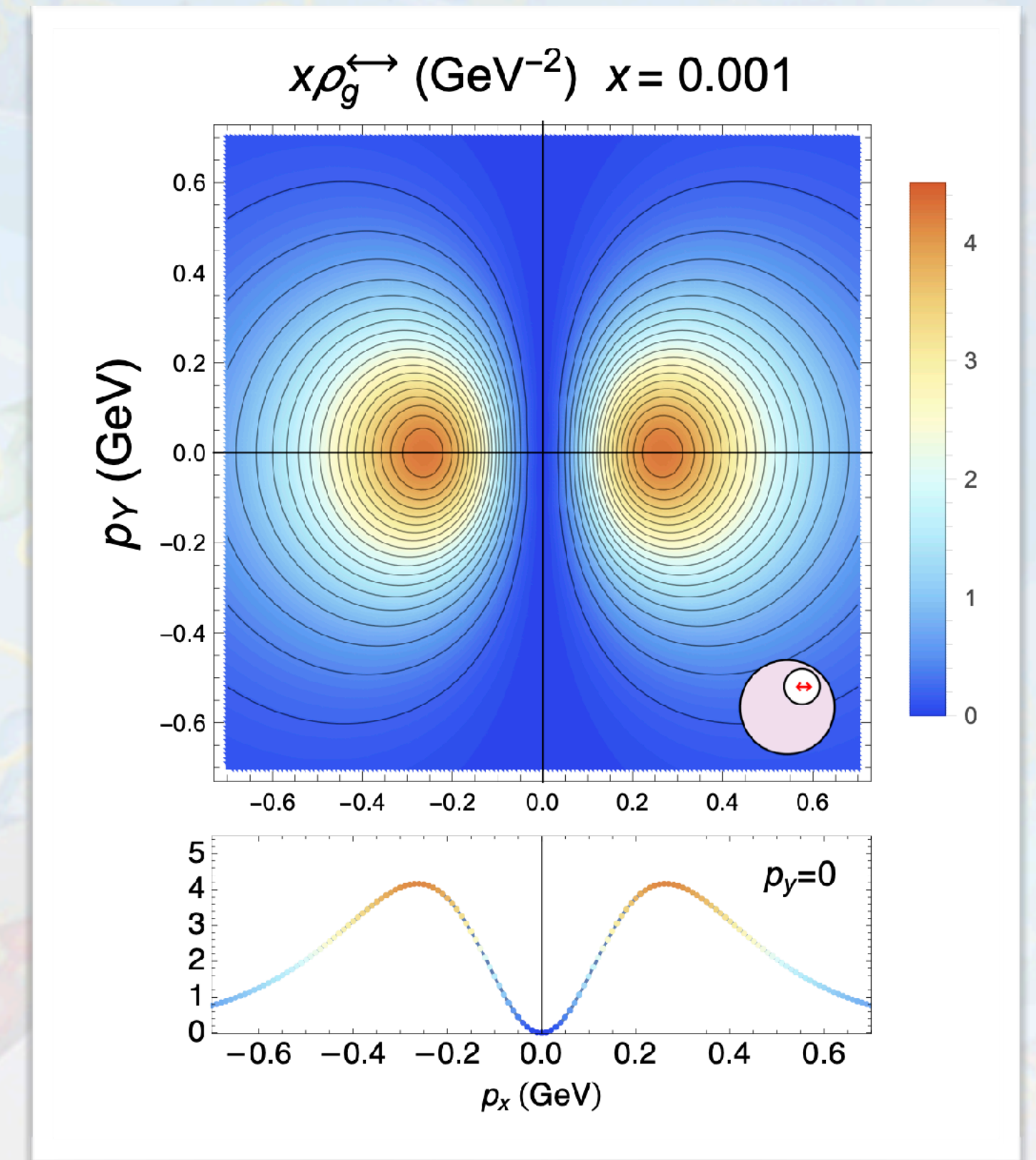
Boer-Mulders



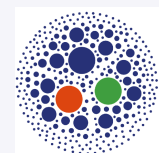
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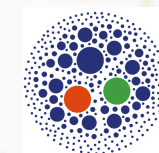


[A. Bacchetta, F.G. C., M. Radici, P. Taelis (2020)]



Model prediction at low- x

$$\frac{f_1^g(x, p_T^2)}{h_1^{\perp g}(x, p_T^2)} \underset{x \rightarrow 0^+}{\sim} \text{constant}$$



HEF regime (linear low- x evolution)

$$f_1^g(x, p_T^2) = h_1^{\perp g}(x, p_T^2) + \text{higher twist}$$



Anatomy of gluon TMDs

$$F(x, \mathbf{b}; \mu, \zeta) = \sum_j \left(C_j^{(F)} \otimes F^j \right) (x, b_*; \mu_b) e^{S(b_*; \mu_b, \mu, \zeta)} e^{S_{\text{NP}}(b)} F_{\text{NP}}(x, b)$$

matching coefficients collinear PDF nonperturbative Sudakov nonperturbative TMD function

perturbative expansion in $\alpha_s(\mu)$

perturbative Sudakov

resummation of

$$L = \ln \frac{Q^2}{\mu_b^2}$$

define logarithmic ordering

Anatomy of gluon TMDs

$$F(x, \mathbf{b}; \mu, \zeta) = \sum_j \left(C_j^{(F)} \otimes F^j \right) (x, b_*; \mu_b) e^{S(b_*; \mu_b, \mu, \zeta)} e^{S_{\text{NP}}(b)} F_{\text{NP}}(x, b)$$

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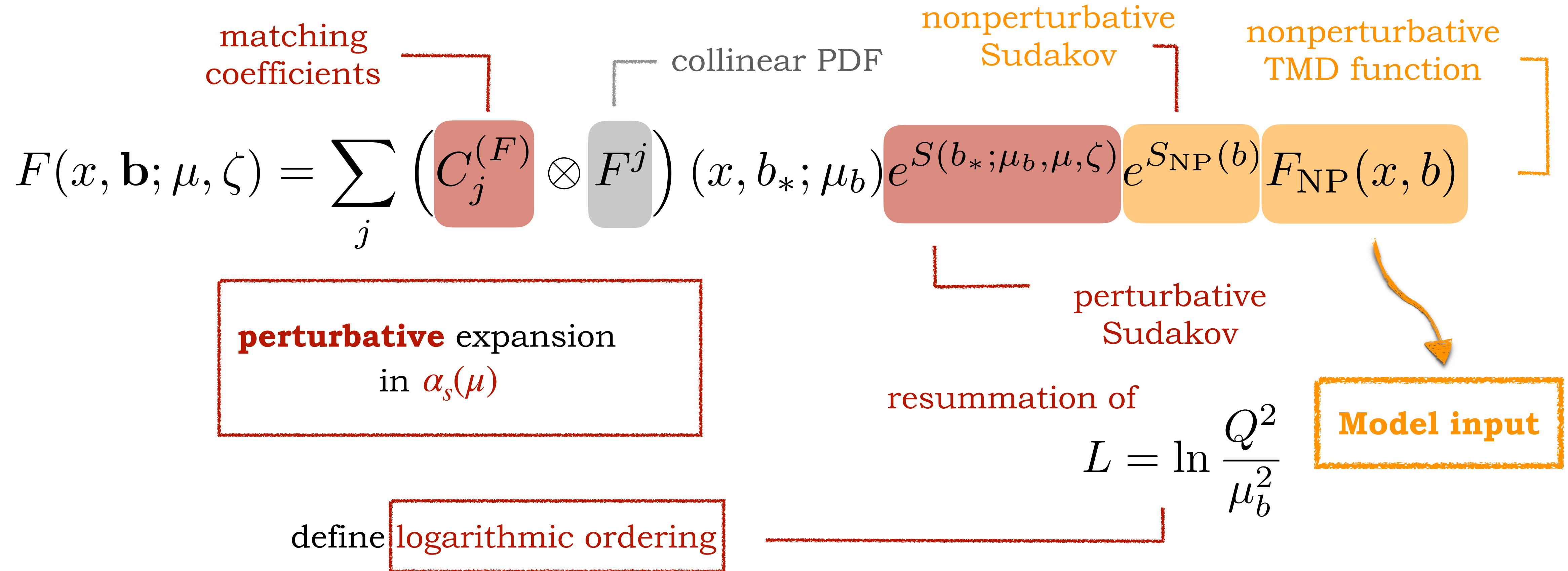
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Model input

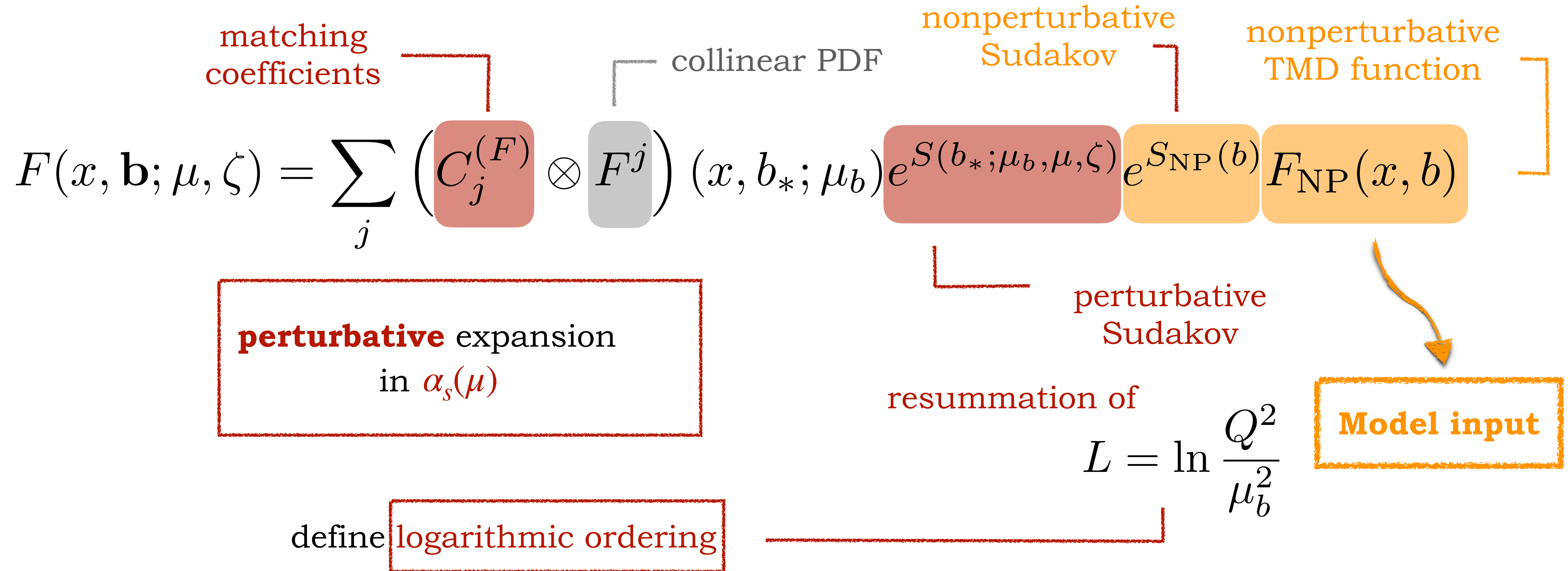
define logarithmic ordering

Anatomy of gluon TMDs



$$f_1(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(f_1)} \otimes f_1^j = \left[1 + \mathcal{O}(\alpha_s) \right]_j \otimes f_1^j$$

Anatomy of gluon TMDs



$$f_1(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(f_1)} \otimes f_1^j = [1 + \mathcal{O}(\alpha_s)]_j \otimes f_1^j$$

$$h_1^\perp(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(h_1^\perp)} \otimes f_1^j = [\mathcal{O}(\alpha_s)]_j \otimes f_1^j$$

Anatomy of gluon TMDs

$$F(x, \mathbf{b}; \mu, \zeta) = \sum_j \left(C_j^{(F)} \otimes F^j \right) (x, b_*; \mu_b) e^{S(b_*; \mu_b, \mu, \zeta)} e^{S_{\text{NP}}(b)} F_{\text{NP}}(x, b)$$

matching coefficients $C_j^{(F)}$ (red box)
 collinear PDF F^j (grey box)
 nonperturbative Sudakov $e^{S(b_*; \mu_b, \mu, \zeta)}$ (red box)
 nonperturbative TMD function $F_{\text{NP}}(x, b)$ (orange box)

perturbative expansion in $\alpha_s(\mu)$ (red box)
 resummation of $L = \ln \frac{Q^2}{\mu_b^2}$ (red text)
 perturbative Sudakov (red text)
 Model input (orange box)

define logarithmic ordering (red box)

$$f_1(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(f_1)} \otimes f_1^j = [1 + \mathcal{O}(\alpha_s)]_j \otimes f_1^j$$

$$h_1^\perp(x, \mathbf{b}, \mu, \zeta) \rightarrow C_j^{(h_1^\perp)} \otimes f_1^j = [\mathcal{O}(\alpha_s)]_j \otimes f_1^j$$

Suppression of genuine NP effects! ←

$\eta_{b,c}$ production in unpolarized pp collisions

TMD phenomenology: from JLab to the LHC

Andrea Signori

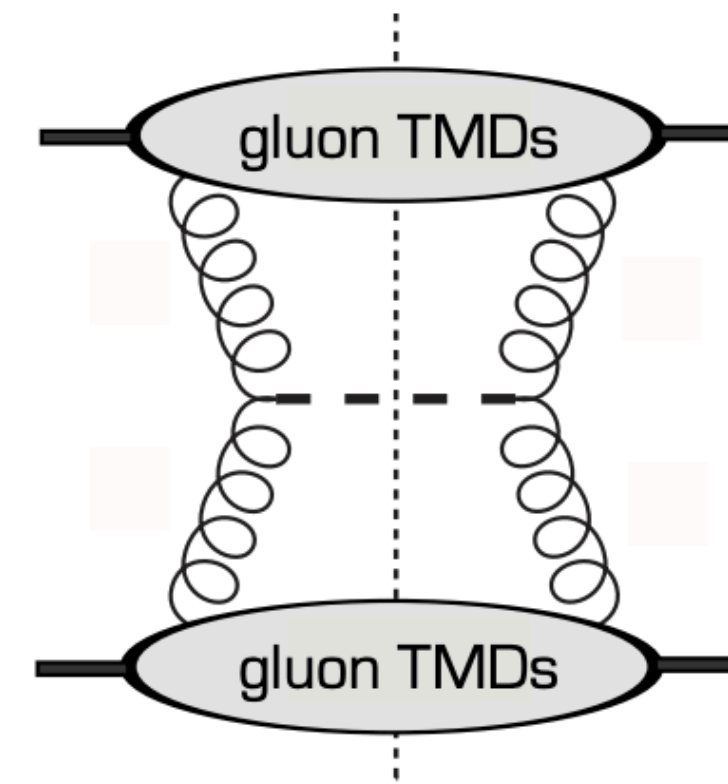
Spatial and momentum
tomography
of hadrons and nuclei

INT 17-3
Sept 25 2017

NRQCD

$$\frac{\text{CS}}{\text{CO}} \sim \frac{1}{v^4}$$

gluon TMD PDFs



pseudoscalar quarkonium production:

$$p p \rightarrow \eta_b X \quad M = 9.39 \text{ GeV}$$

$$p p \rightarrow \eta_c X \quad M = 2.98 \text{ GeV}$$

(see also talk by C. Pisano week 4)

$$\frac{d\sigma}{dq_T} \sim \Phi_A^U \Phi_B^U |\mathcal{M}|^2$$

$$\sim \underbrace{\mathcal{C} \left[f_1^{g/A} \quad f_1^{g/B} \right]}_{\text{unpolarized gluons}} \pm \underbrace{\mathcal{C} \left[h_1^{\perp g/A} \quad h_1^{\perp g/B} \right]}_{\text{lin. polarized gluons}}$$

unpolarized cross section
at low transverse momentum
for (pseudo)scalar state

C_{ff}

C_{hh}

Jefferson Lab

22

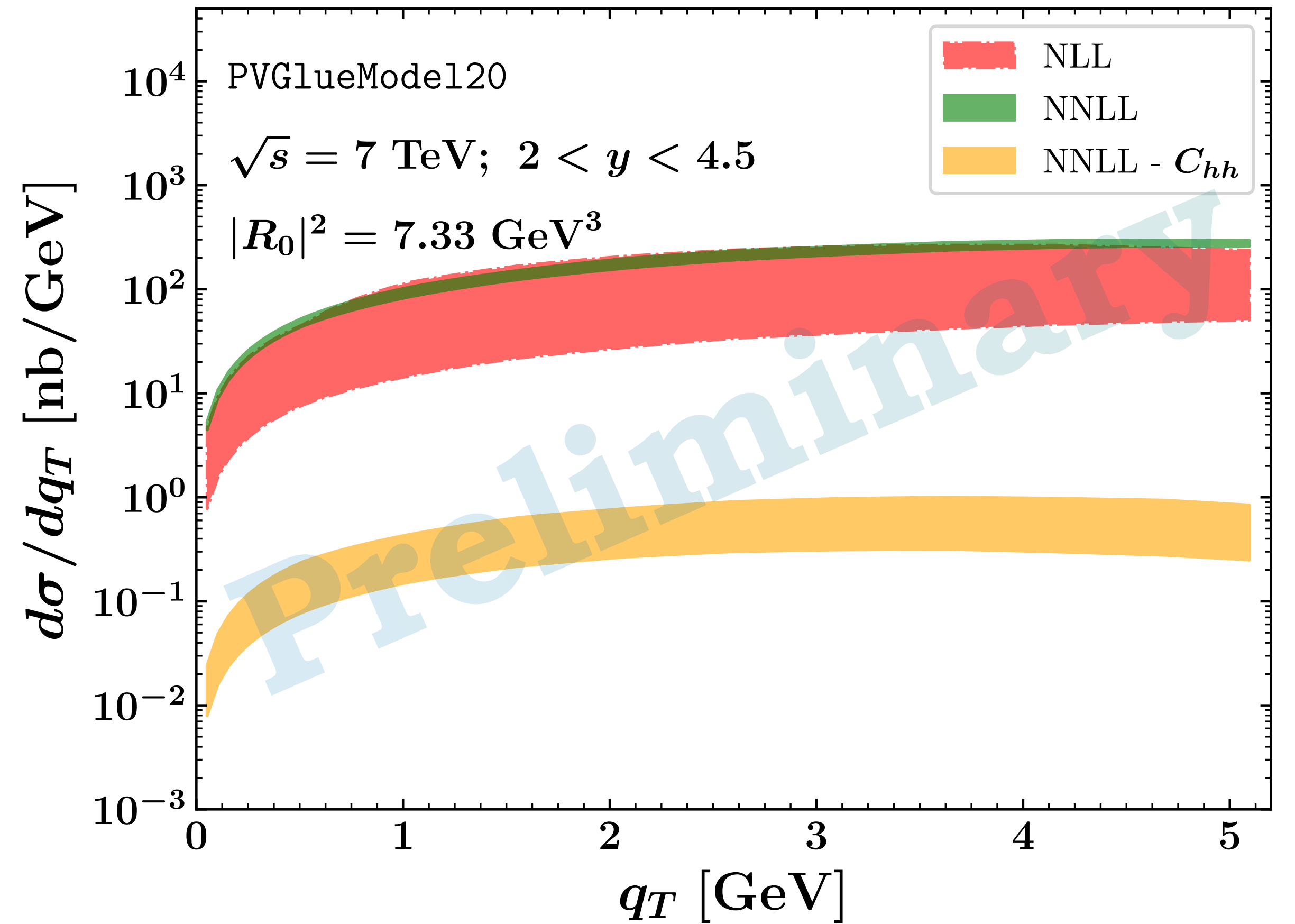
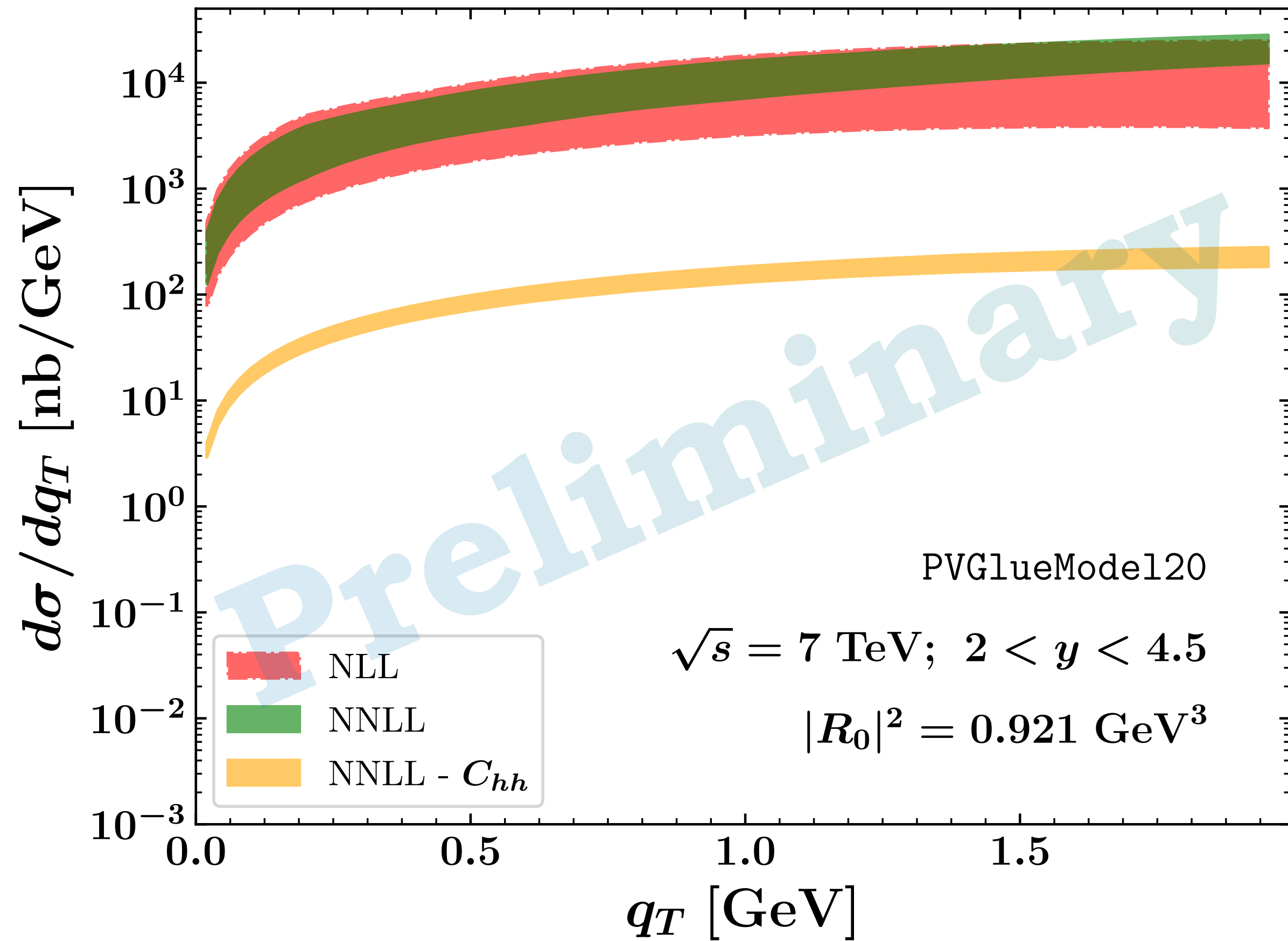
$\eta_{b,c}$ production @ 7TeV LHCb



Perturbative-scale variation, NP-evolution parameters fixed, TMD central replica

$$p(P_1) + p(P_2) \rightarrow \eta_c(q_T)$$

$$p(P_1) + p(P_2) \rightarrow \eta_b(q_T)$$



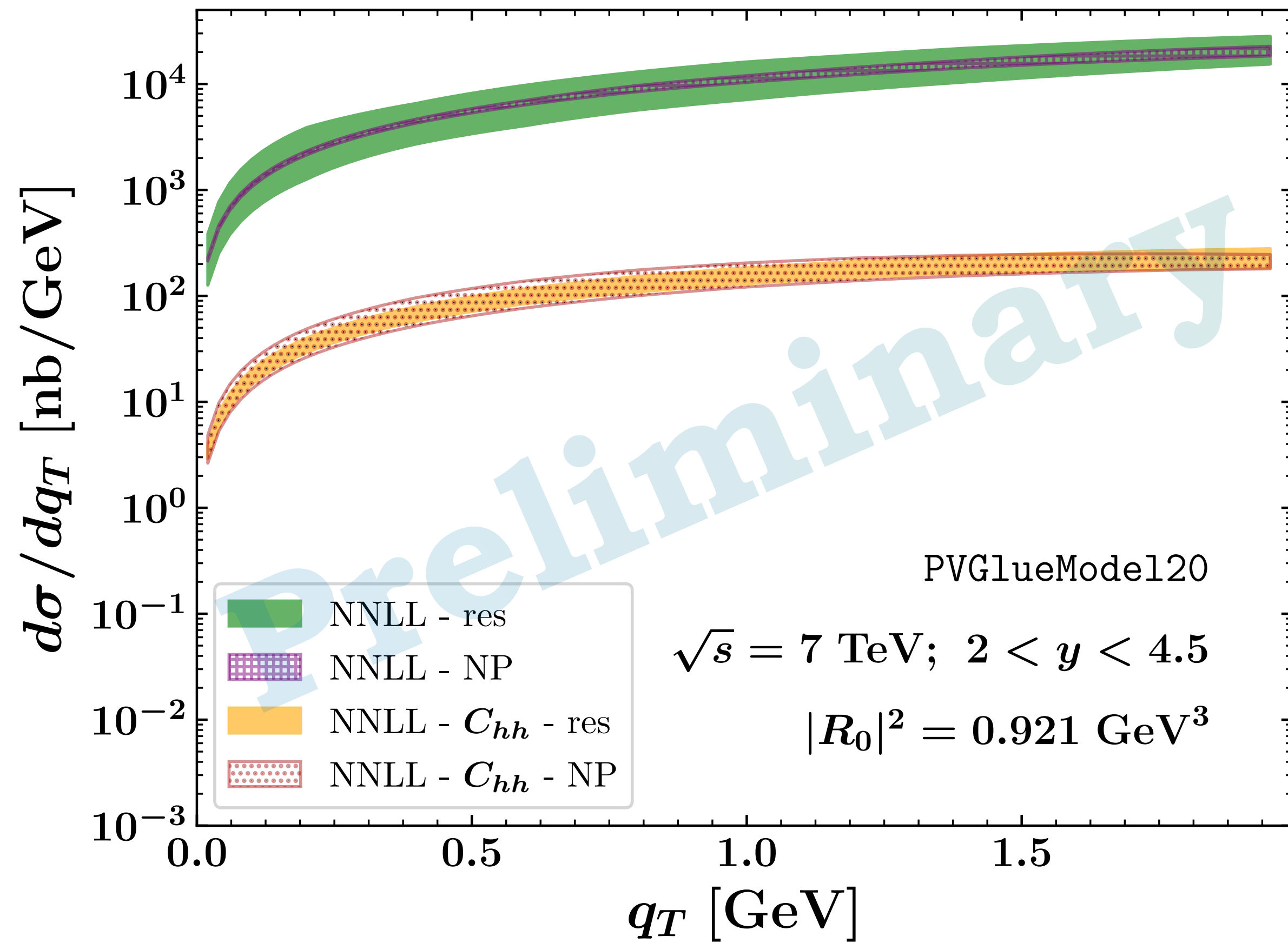
[A. Bacchetta, F.G. C., J.-P. Lansberg, A. Signori (in preparation)]

$\eta_{b,c}$ production @ 7TeV LHCb

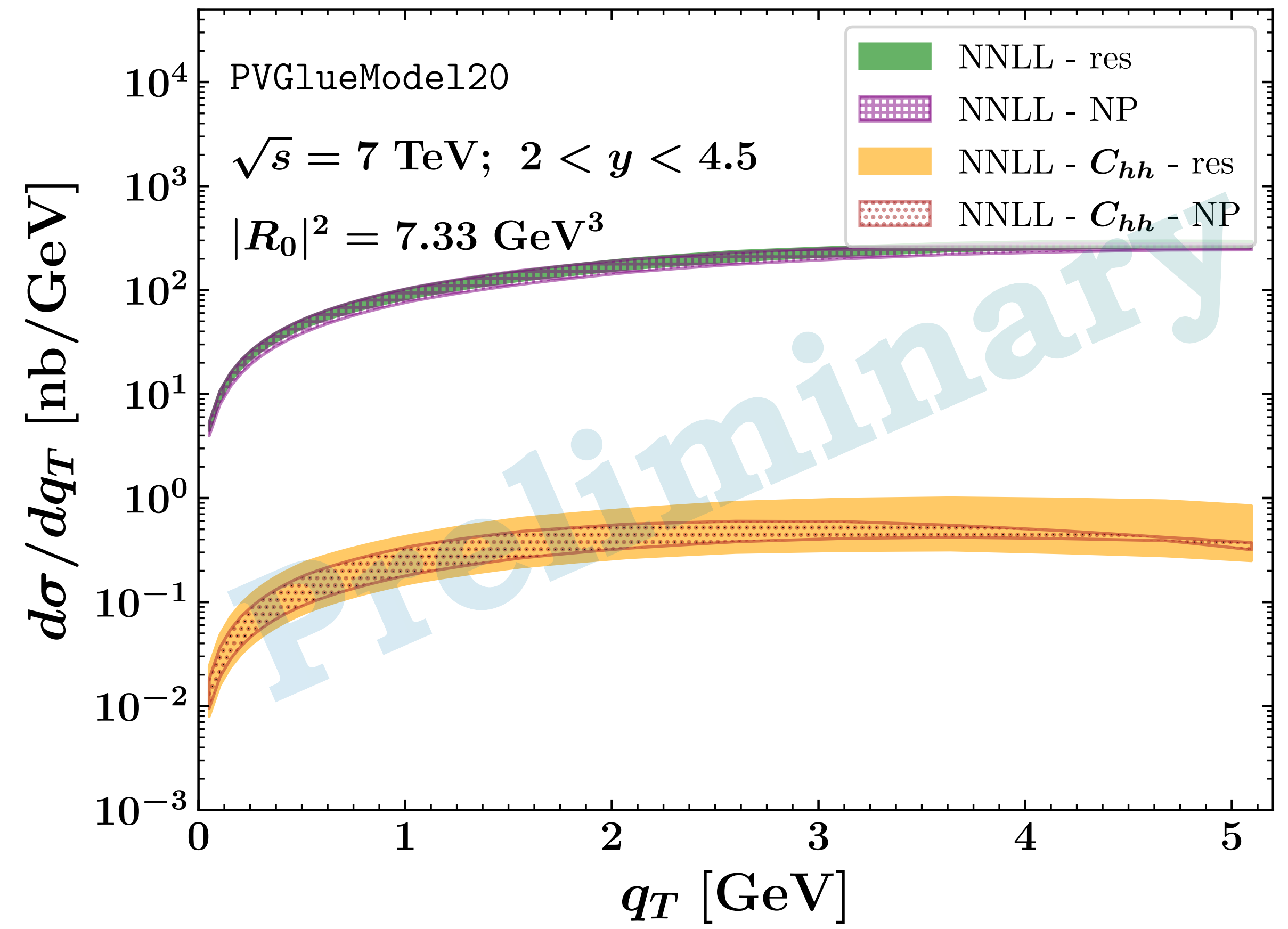


Perturbative scales fixed, **NP-evolution parameter** variation, TMD central replica

$$p(P_1) + p(P_2) \rightarrow \eta_c(q_T)$$



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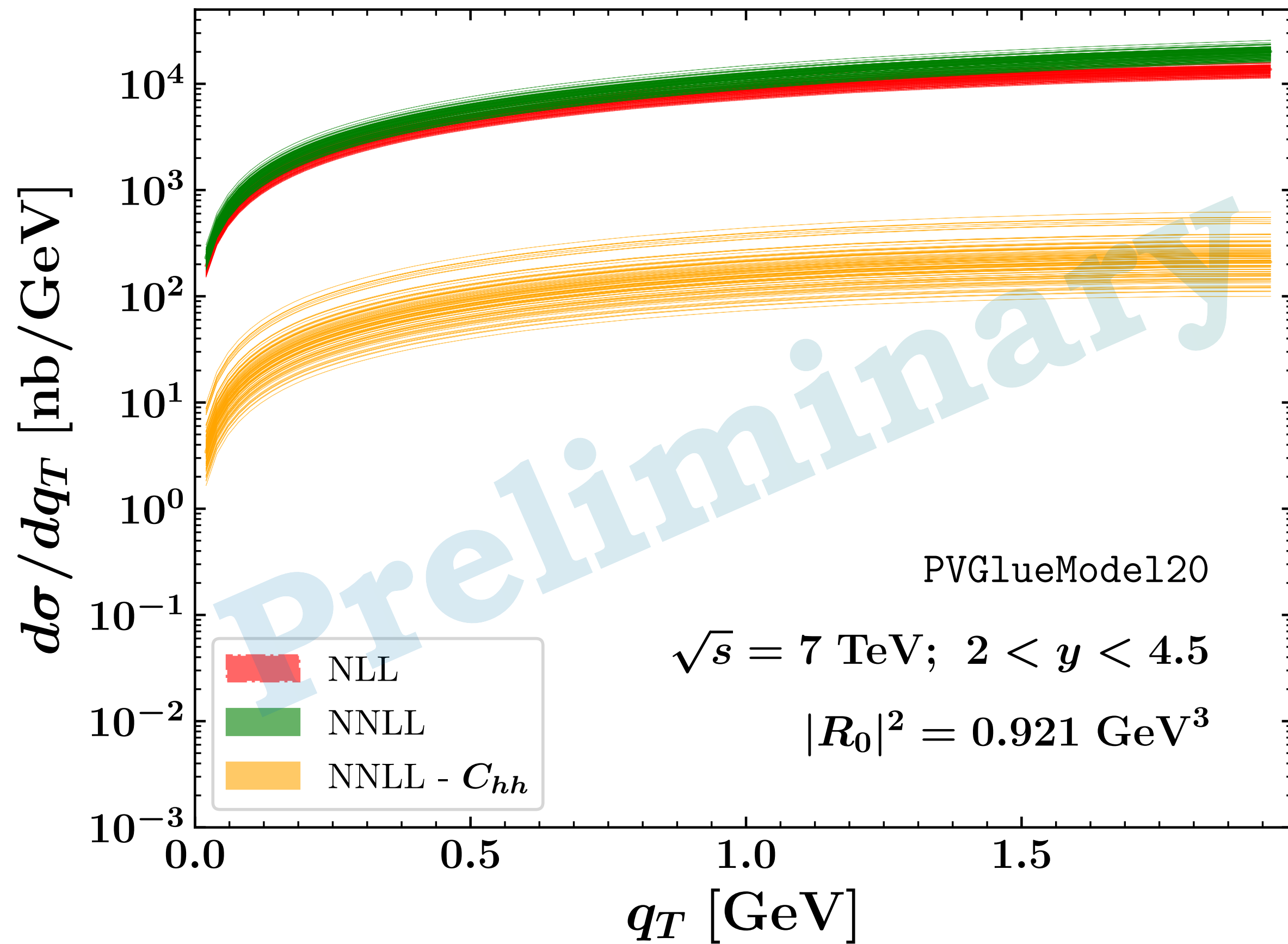
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$\eta_{b,c}$ production @ 7TeV LHCb

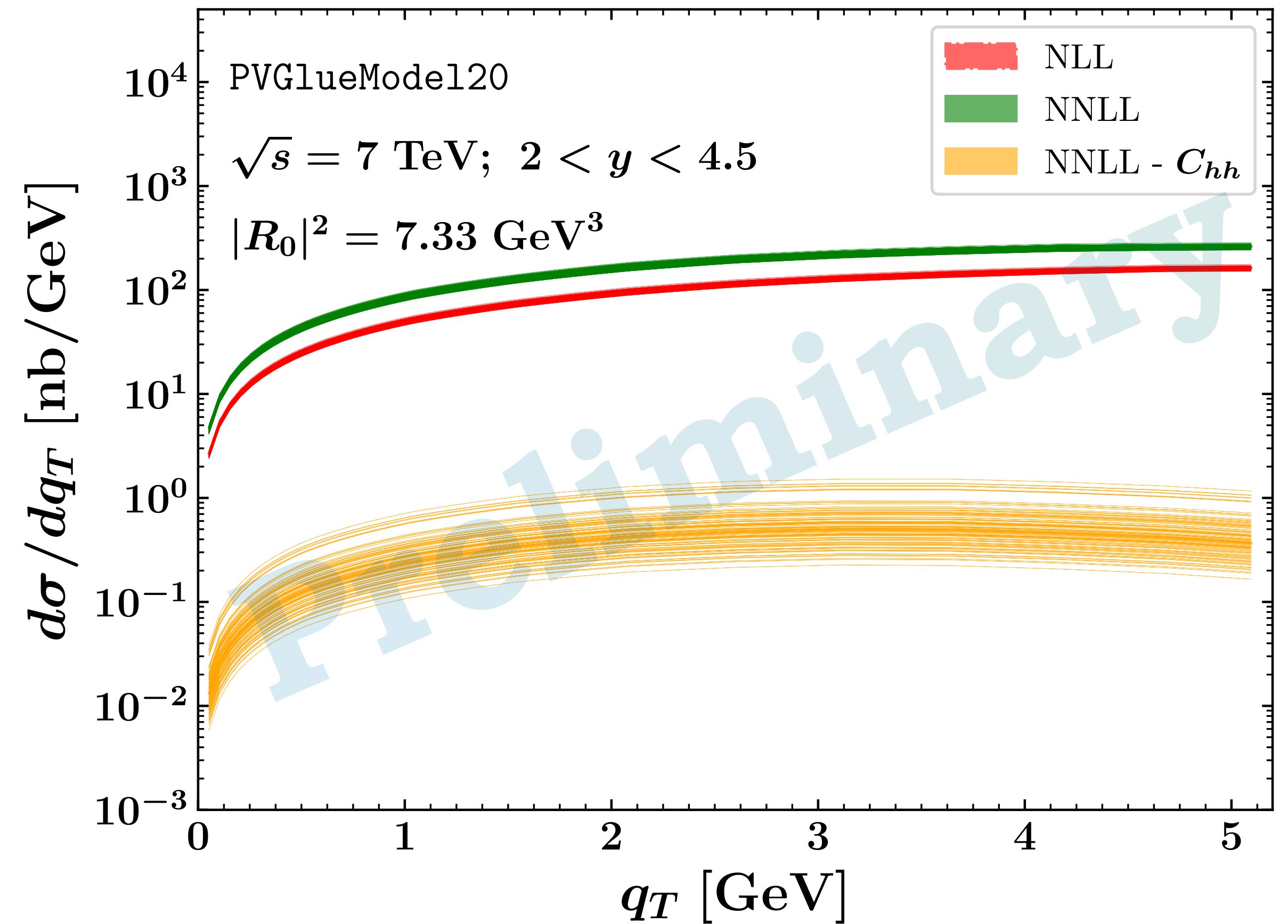


Perturbative scales fixed, NP-evolution parameters fixed, **TMD 100-replica analysis**

$$p(P_1) + p(P_2) \rightarrow \eta_c(q_T)$$



$$p(P_1) + p(P_2) \rightarrow \eta_b(q_T)$$



[A. Bacchetta, F.G. C., J.-P. Lansberg, A. Signori (in preparation)]

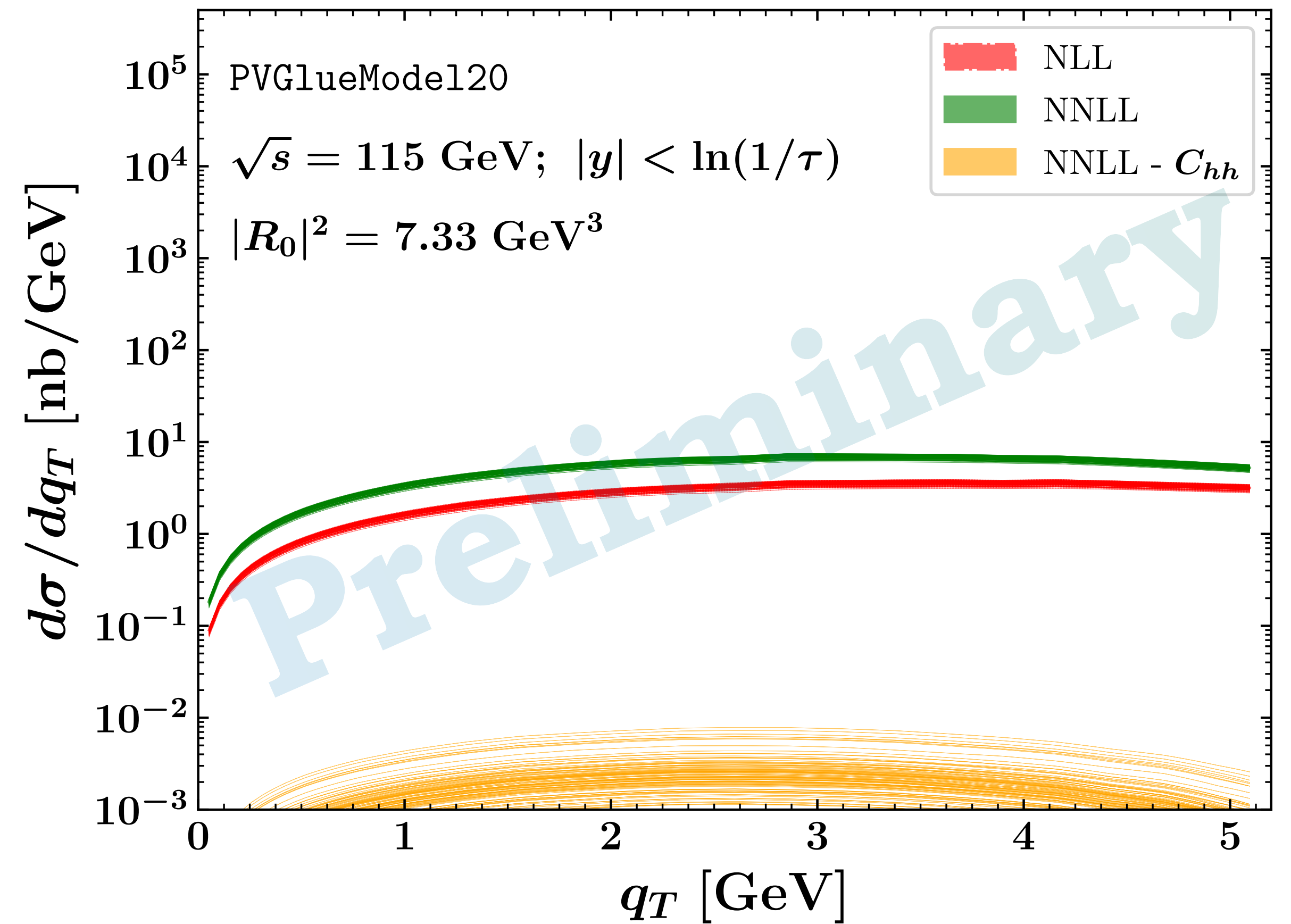
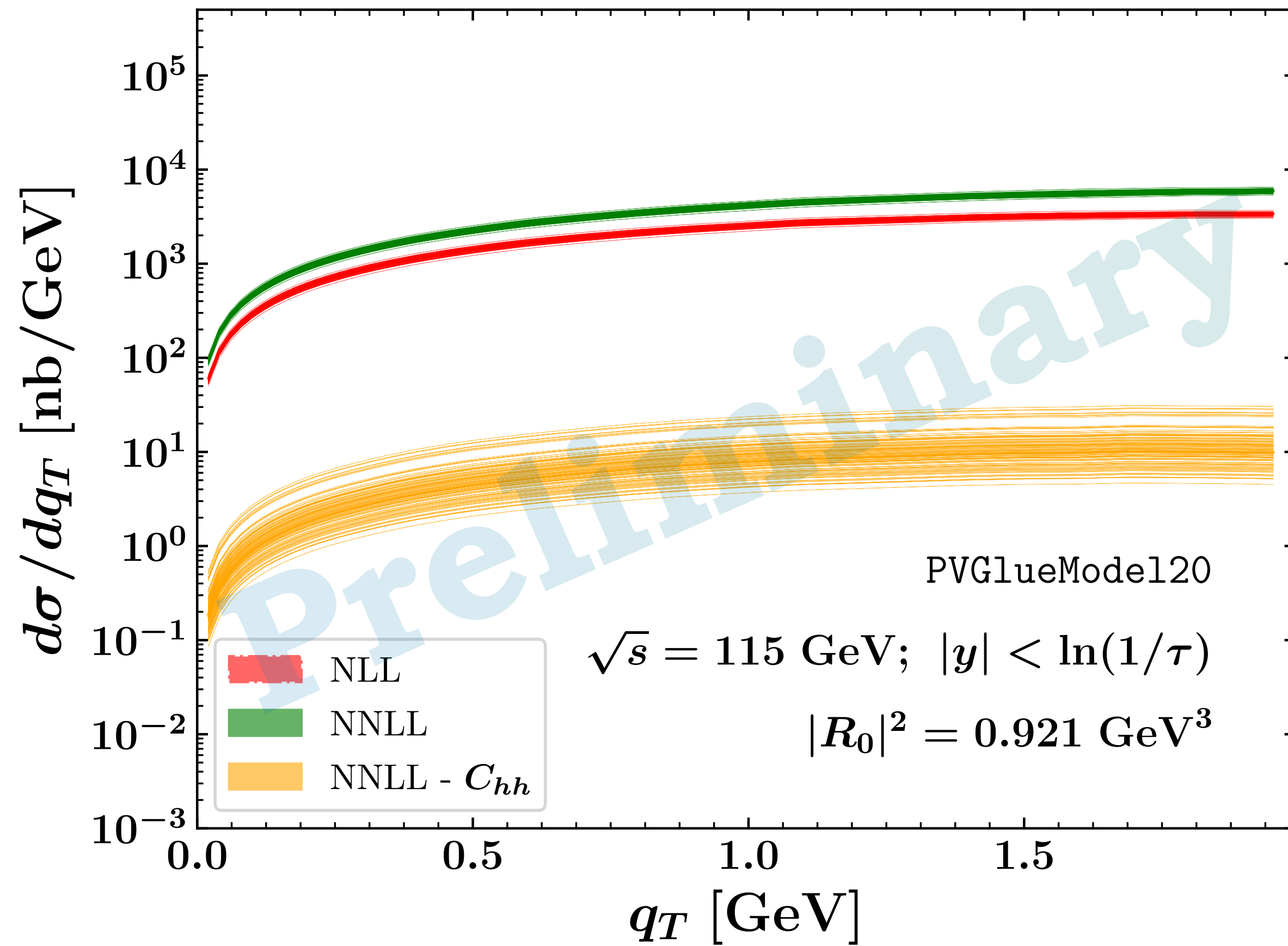
$\eta_{b,c}$ production @ 115 GeV After@LHC



Perturbative scales fixed, nonperturbative parameters fixed, **TMD 100-replica analysis**

$$p(P_1) + p(P_2) \rightarrow \eta_c(q_T)$$

$$p(P_1) + p(P_2) \rightarrow \eta_b(q_T)$$

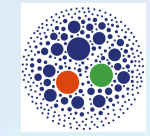


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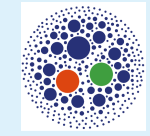


**...towards twist-2
T-odd gluon TMDs**

T-odd gluon TMDs in a spectator model

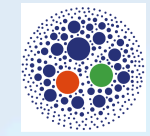


No residual gluon-spectator interaction at tree level

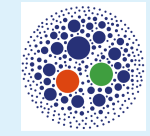


Interference with one-gluon exchange (*eikonal*)

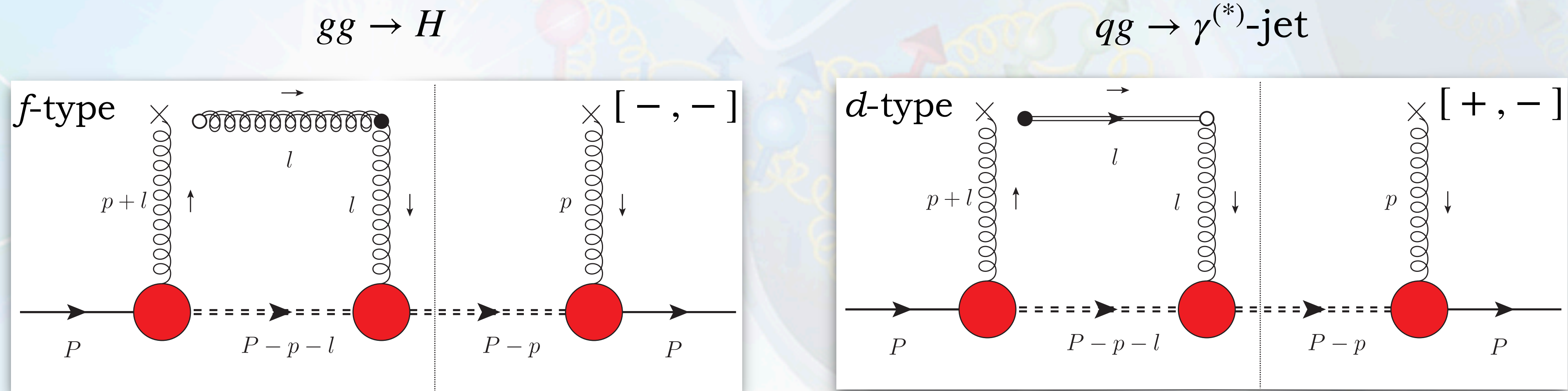
T -odd gluon TMDs in a spectator model



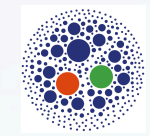
No residual gluon-spectator interaction at tree level



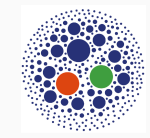
Interference with one-gluon exchange (*eikonal*)



Leading-twist one-gluon-exchange of the gauge-link operator



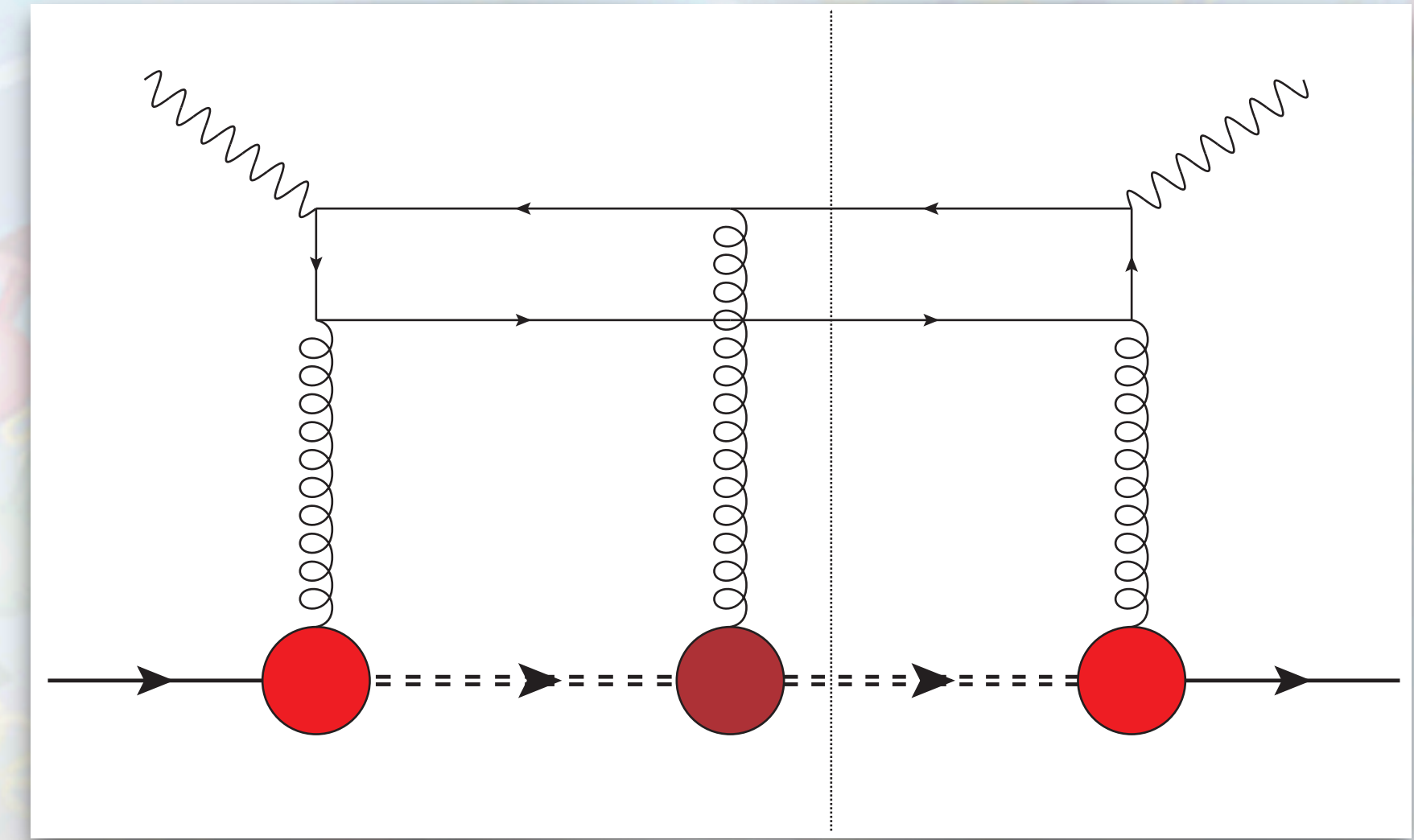
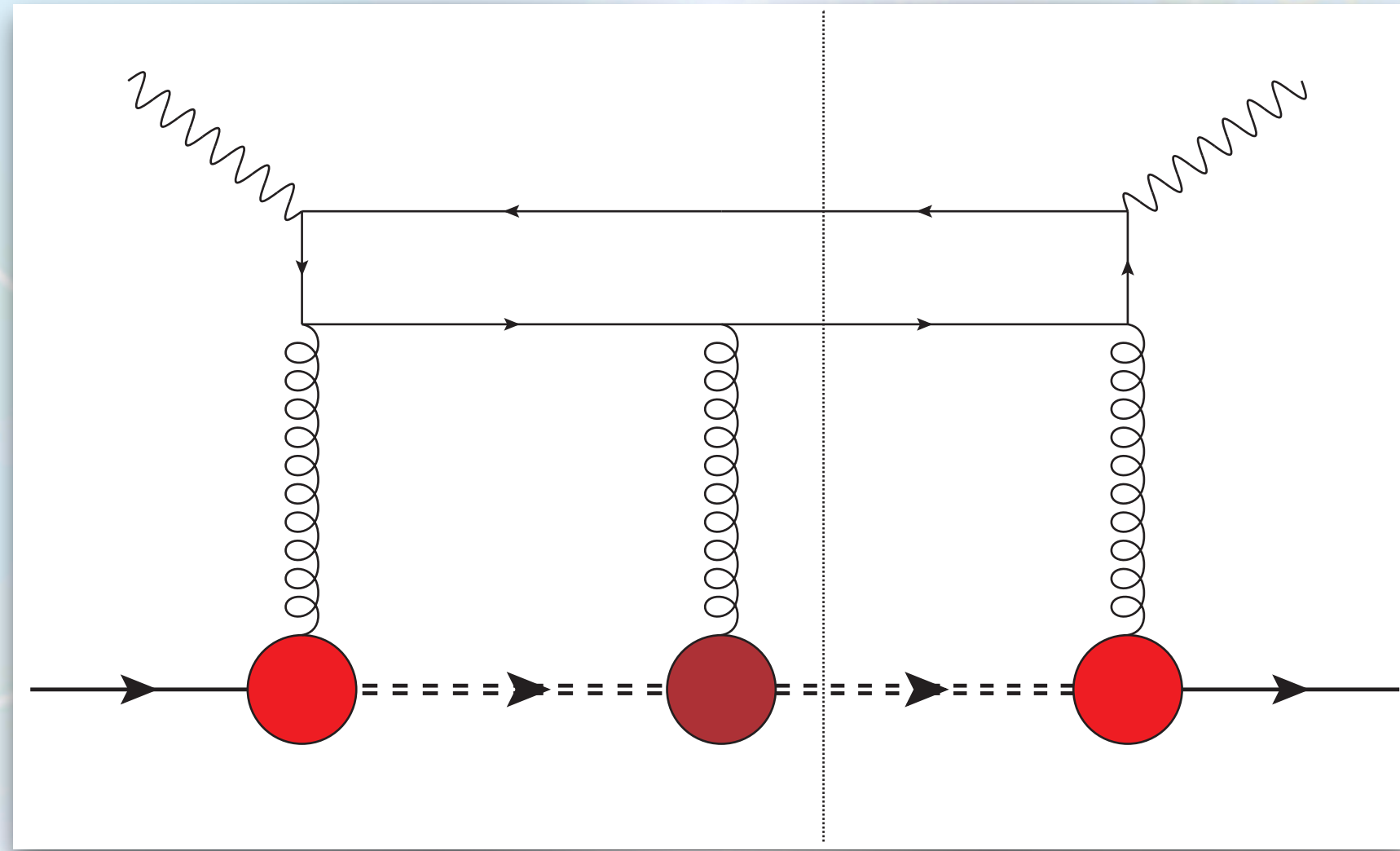
Sensitivity to *f*- and *d*-type structures



Preliminary results for **Sivers** and **linearity** functions

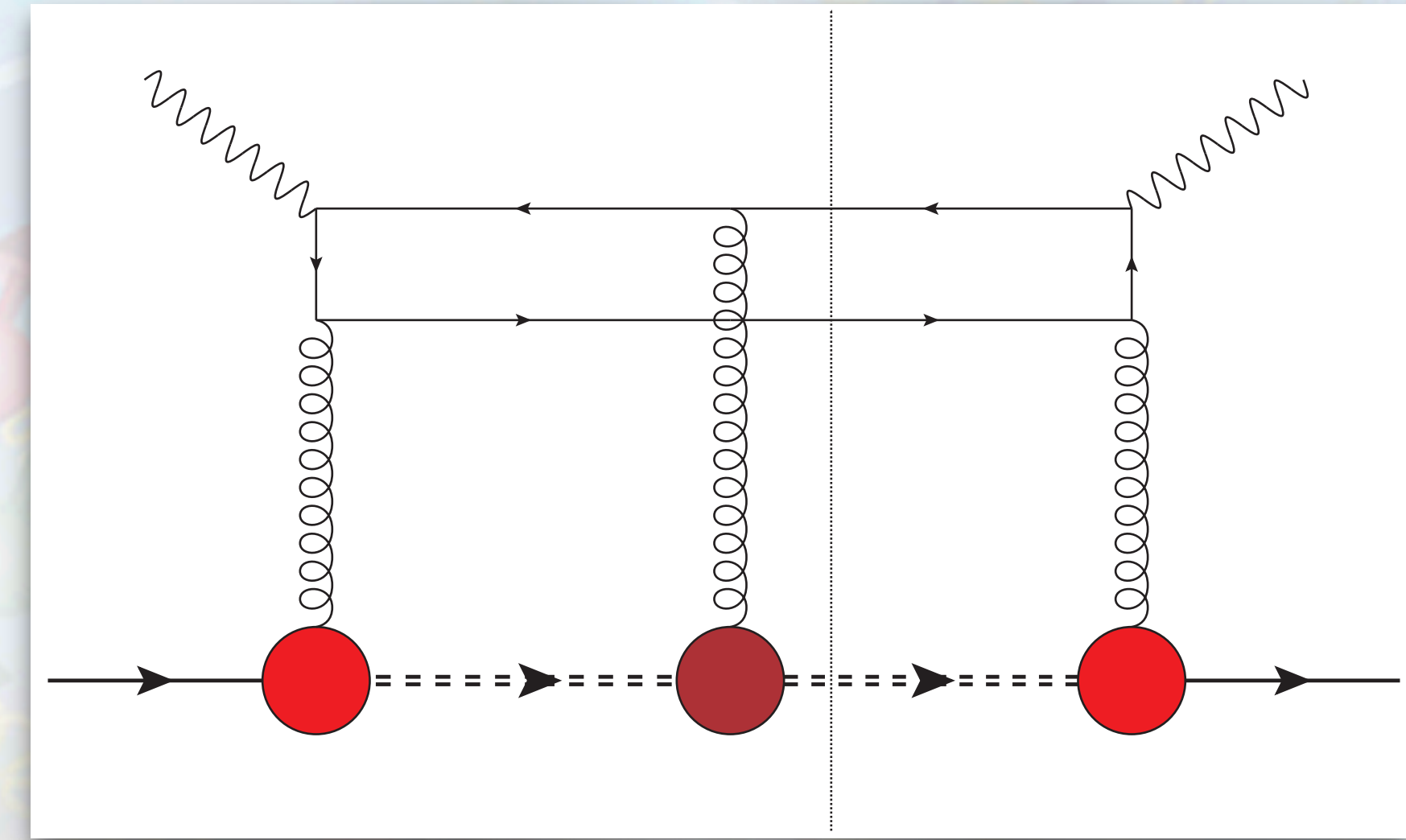
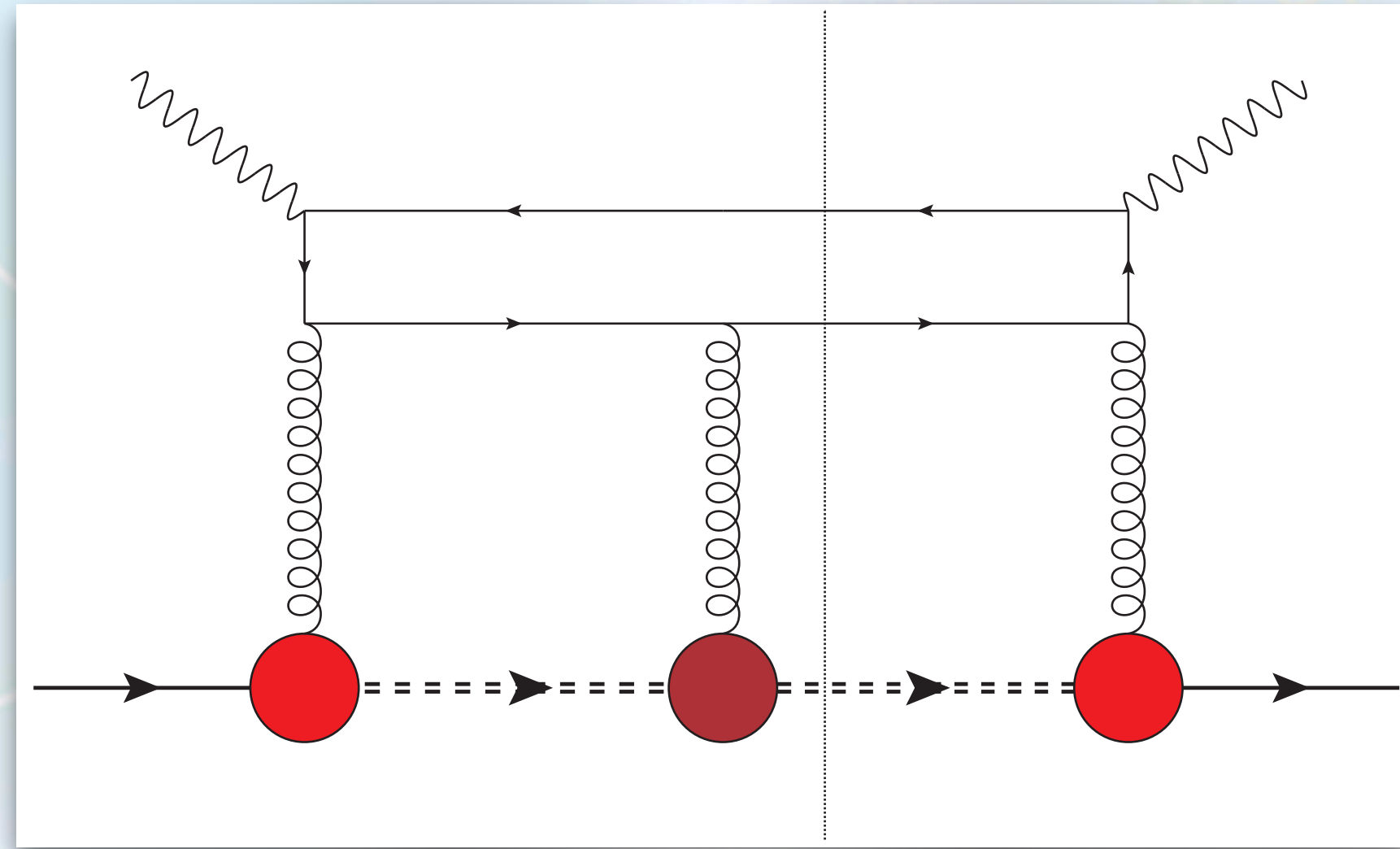
Analytic structure of T -odd gluon TMDs

Two-jet SIDIS $\Rightarrow f$ -type $[+, +]$



Analytic structure of T -odd gluon TMDs

Two-jet SIDIS $\Rightarrow f$ -type [+ , +]

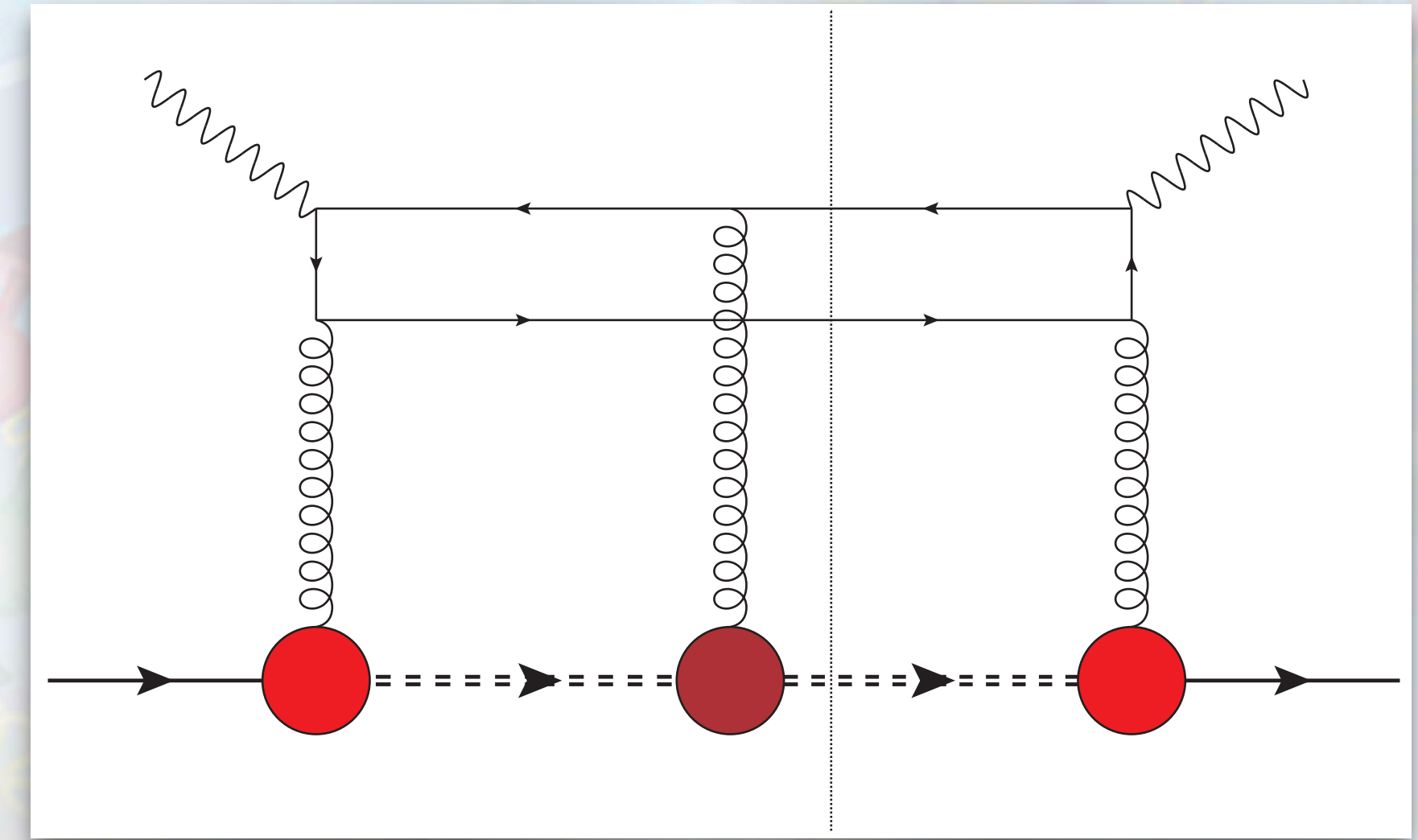
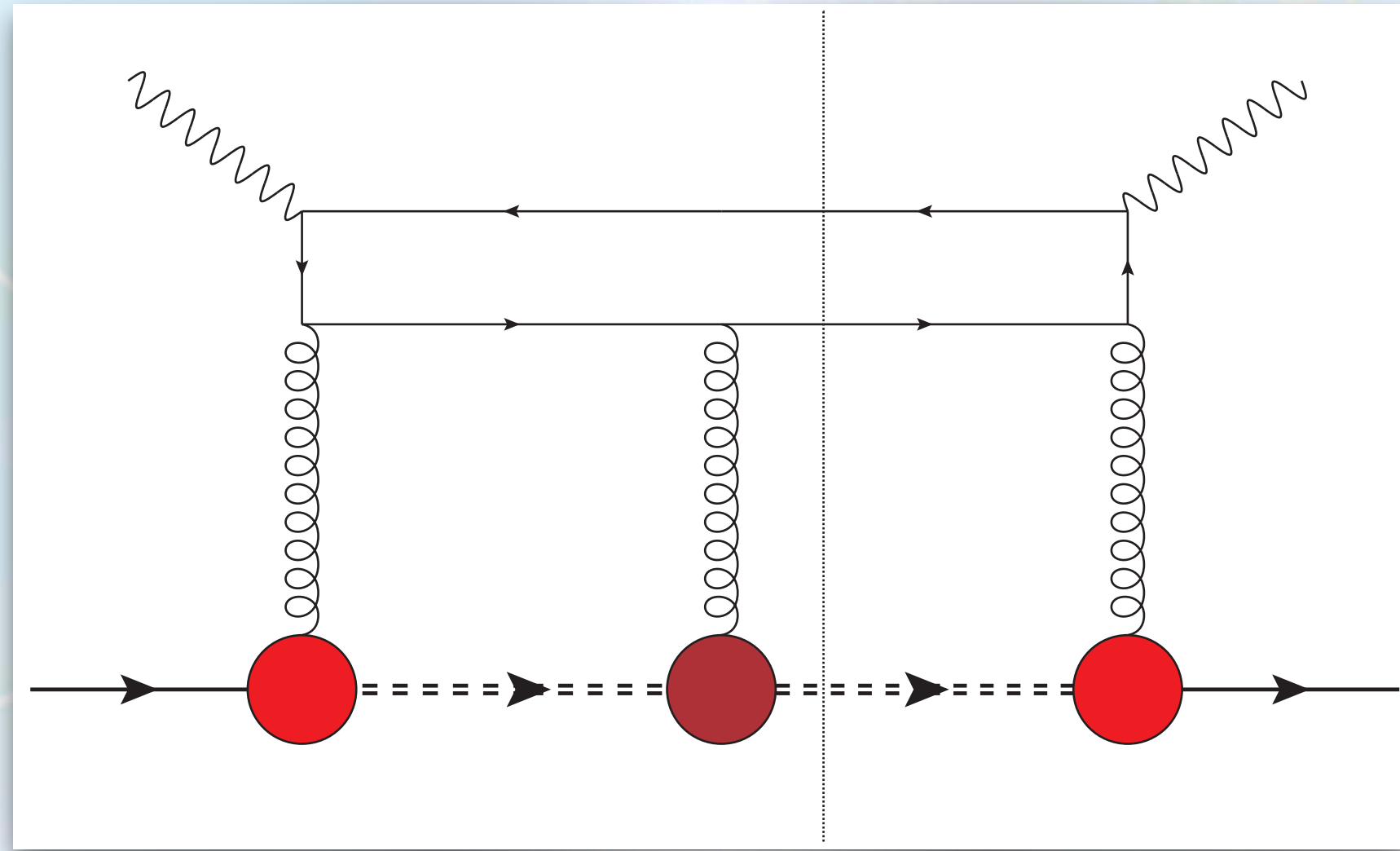


- nucleon-gluon-spectator
- spectator-gluon-spectator

$$\mathcal{Y}_g^\mu = g_1(p^2) \gamma^\mu + i \frac{g_2(p^2)}{2M} \sigma^{\mu\nu} p_\nu$$

Analytic structure of T -odd gluon TMDs

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8 \times 7 \times 4

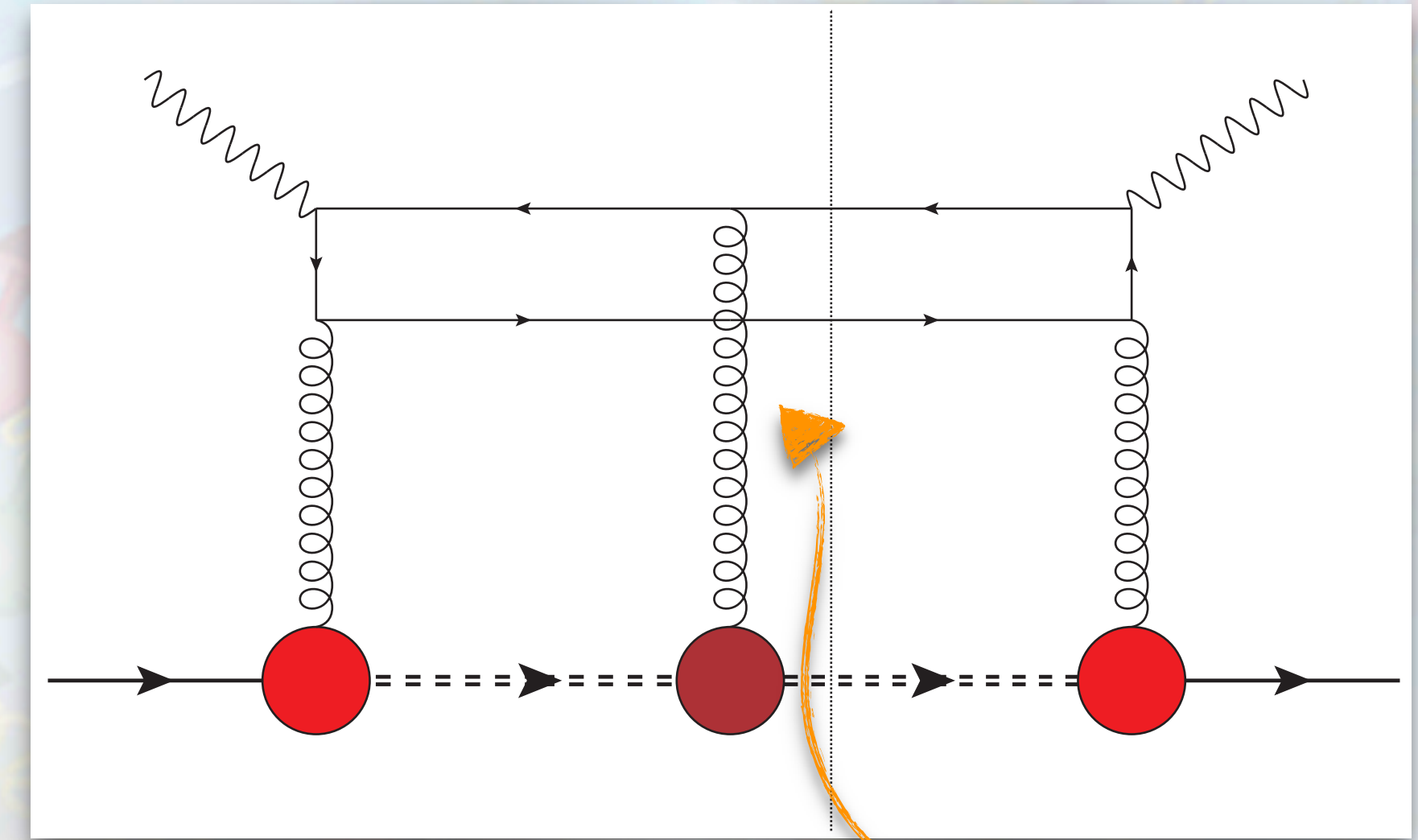
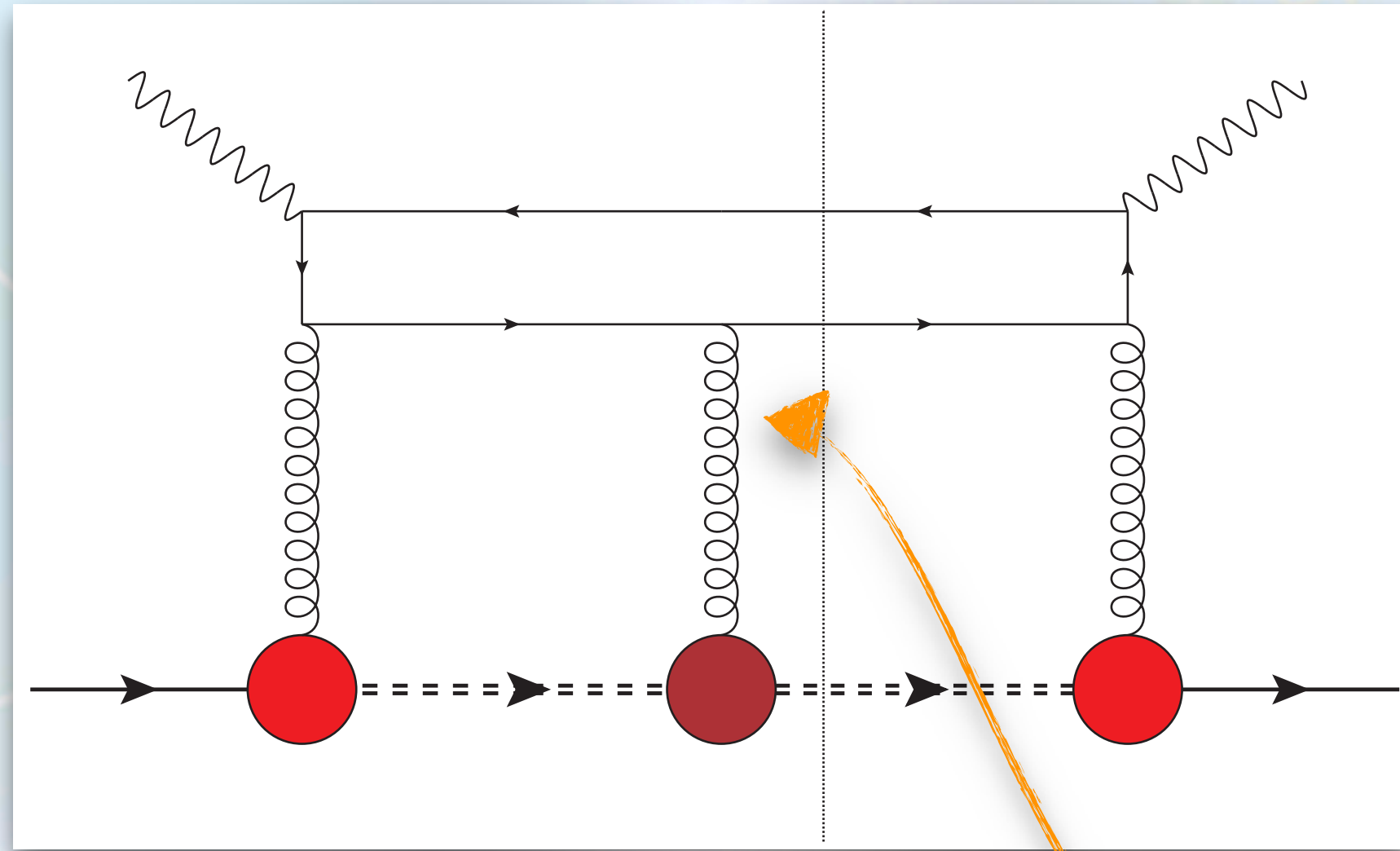
$$F(x, \mathbf{p}_T^2) = \sum_{i,j,k}^{1,2} C_{ijk}^{(F)}(x, \mathbf{p}_T^2) g_i(\mathbf{p}_T^2) g_j(\mathbf{p}_T^2) g_k(\mathbf{p}_T^2)$$

$$C_{ijk}^{(F)}(x, \mathbf{p}_T^2) = \sum_{l=1}^7 C_{ijk}^{(F),l}(x, \mathbf{p}_T^2) \mathcal{D}_l(x, \mathbf{p}_T^2)$$

[A. Bacchetta, F.G. C., M. Radici, *in preparation*]

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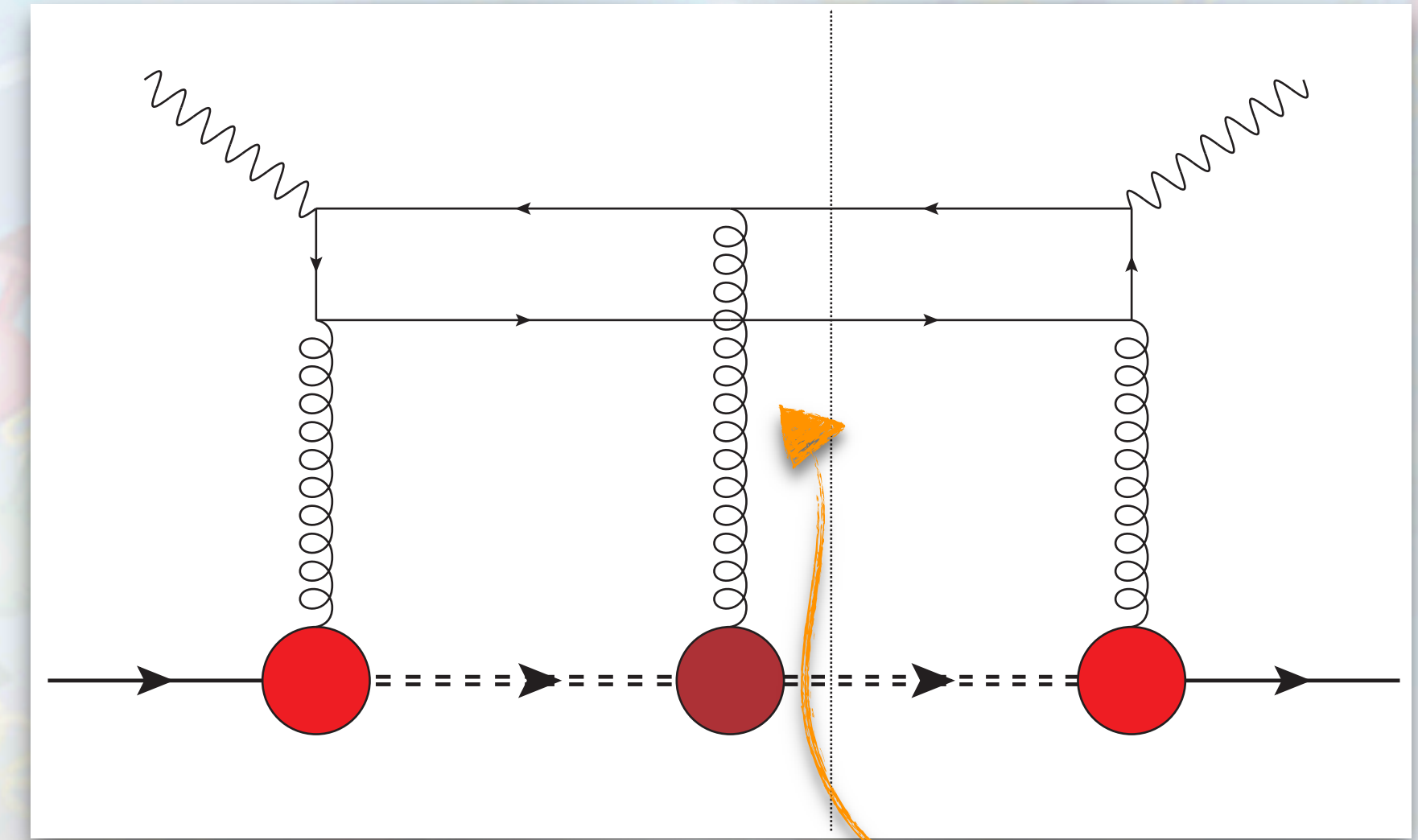
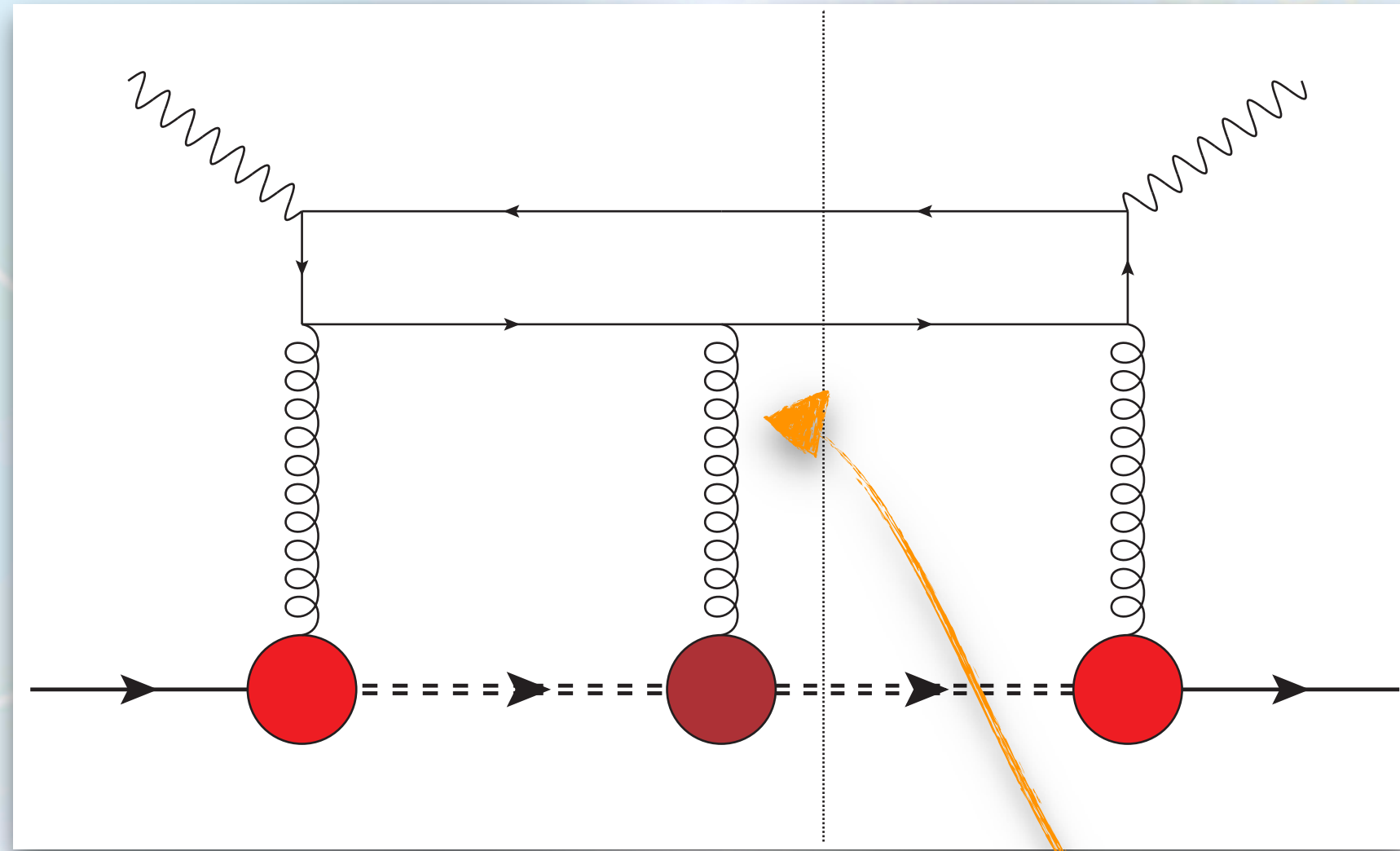
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➔

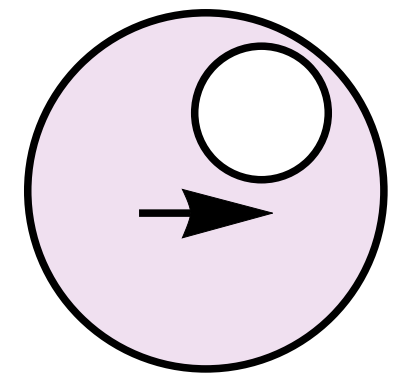
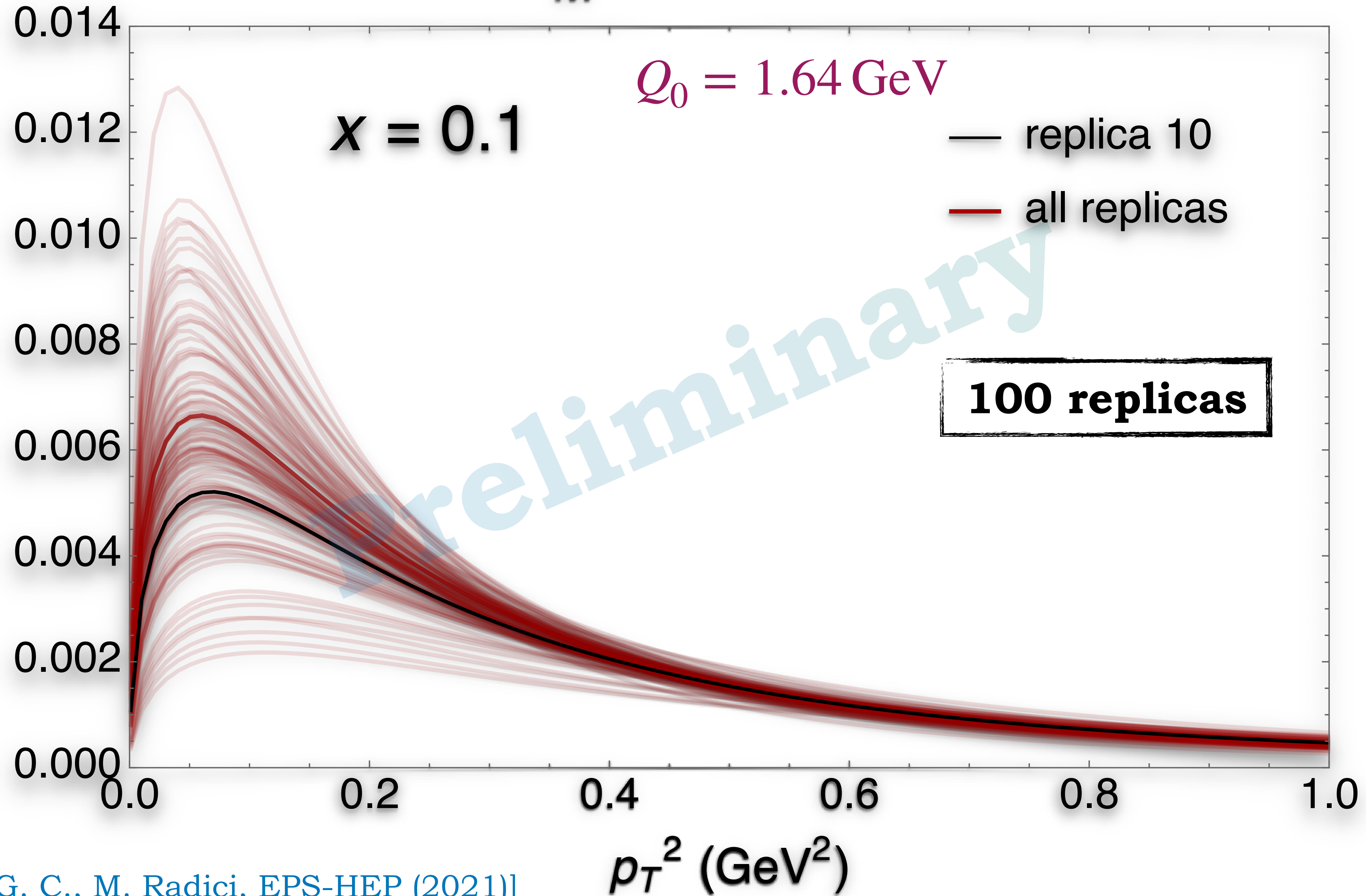
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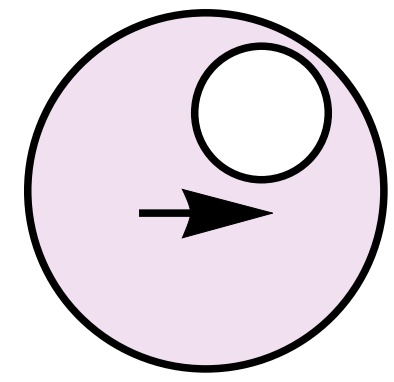
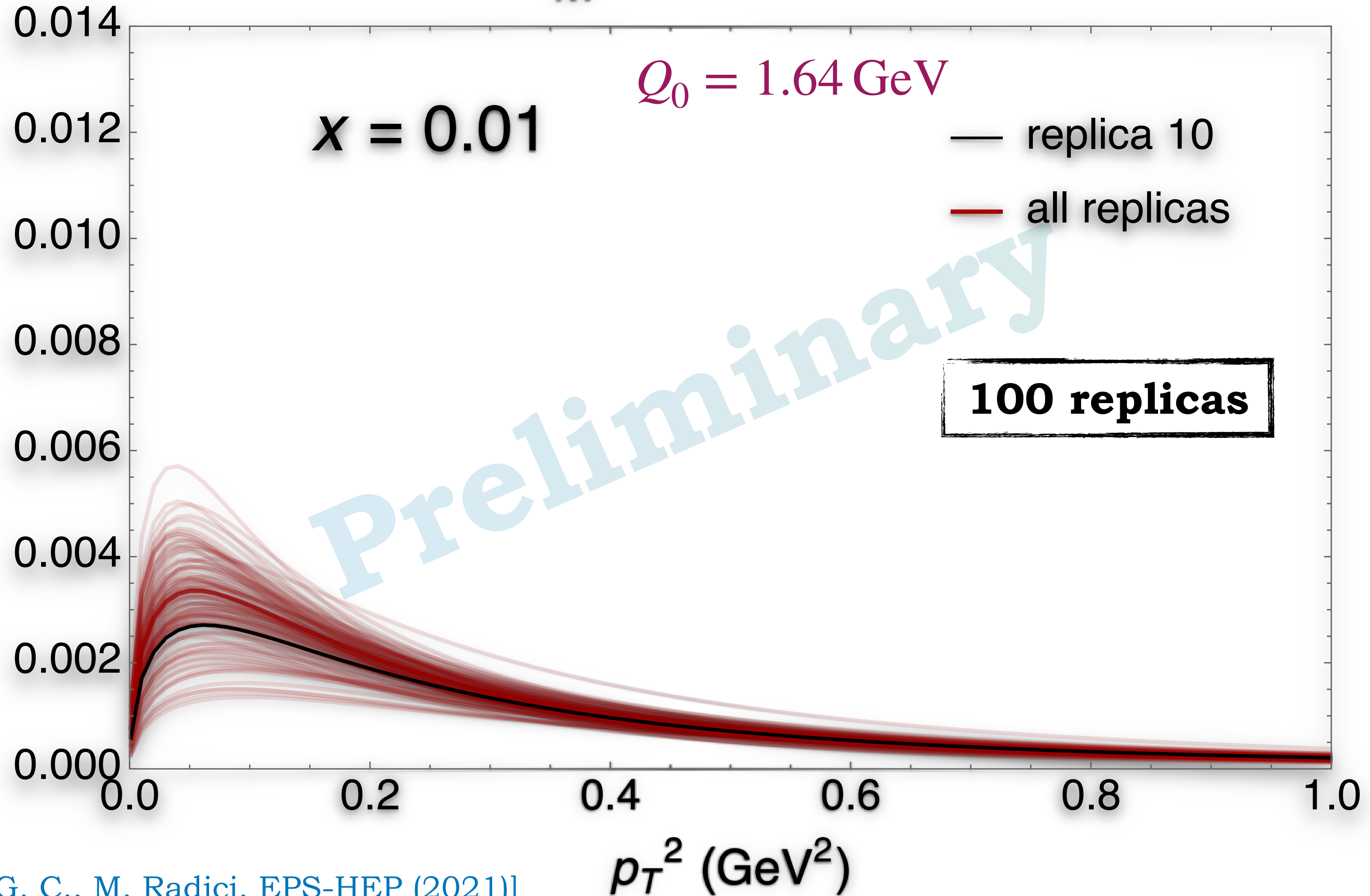
f -type Sivers gluon TMD

$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$



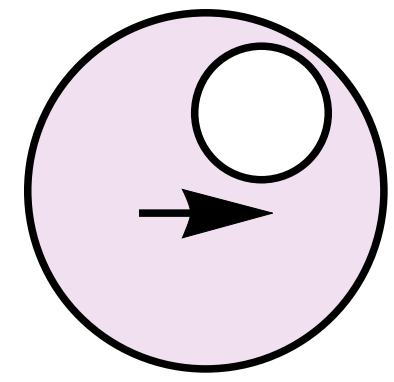
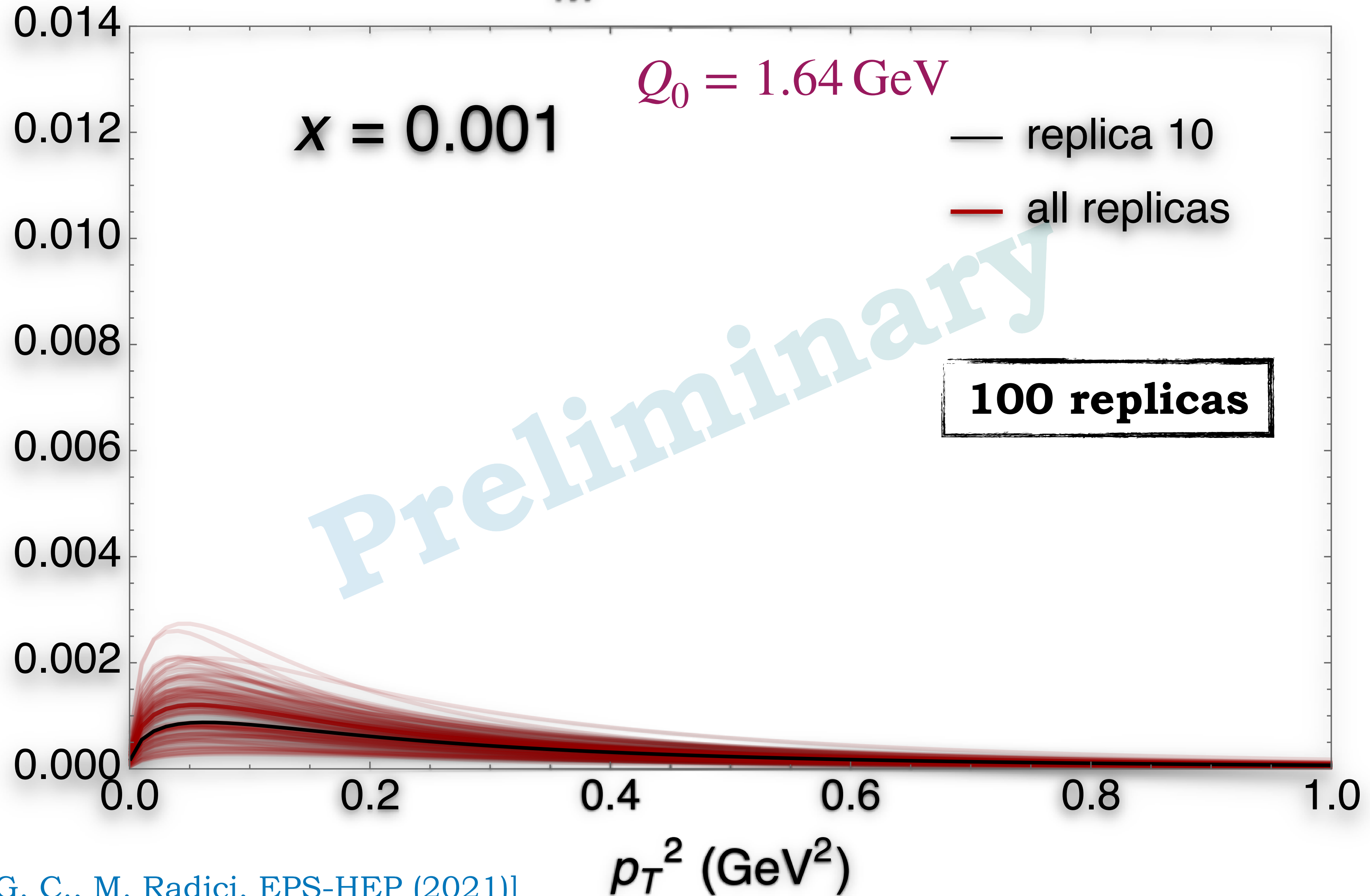
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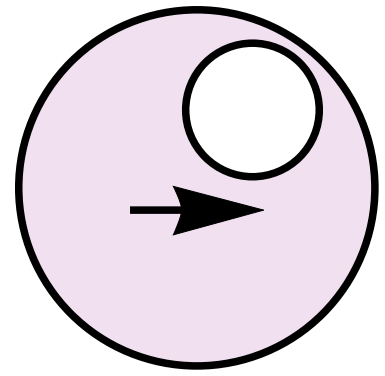


f-type Sivers gluon TMD

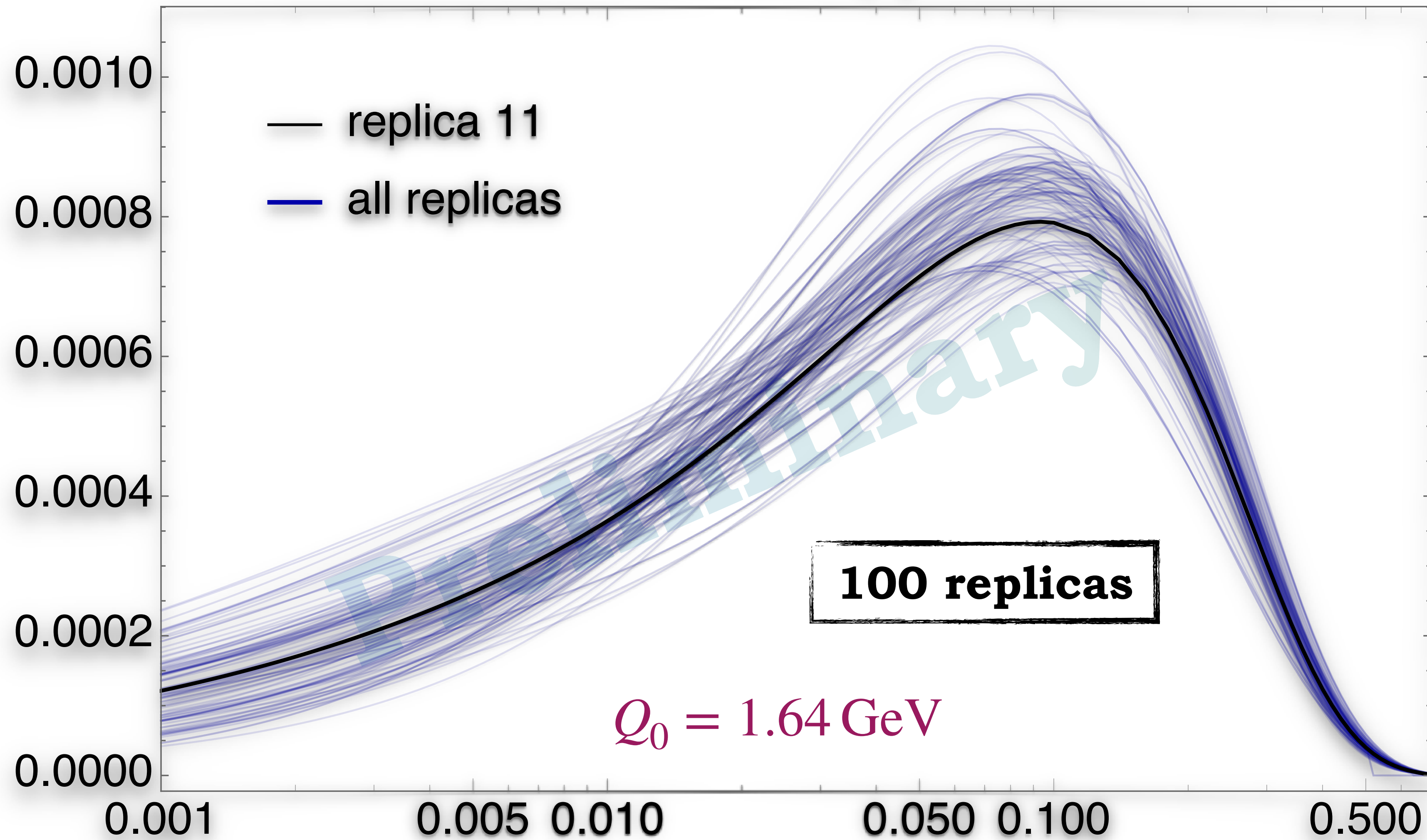
$$x \frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)$$



f -type Qiu-Sterman twist-3 gluon PDF



$$xf_{1T}^{\perp(f)}(x)$$




$$f_{1T}^{\perp(f)}(x) = \int d^2p_T \frac{p_T^2}{2M^2} f_{1T}^{\perp[+,+]}(x, p_T^2)$$

Checkpoints and further steps

- ☑ Systematic calculation of all twist-2 T -even gluon TMDs with CSS evolution
- ☑ Spectral mass to catch small- and large- x effects
- ☑ **Simultaneous fit** of f_1 and g_1 PDFs via **replica method**
- ☑ Inclusion of standard CSS evolution (investigation on *perturbative tail*)

Checkpoints and further steps

- Systematic calculation of all twist-2 T -even gluon TMDs with CSS evolution
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- Quarkonia** As Tools \rightarrow clean channels for f -type gluon TMDs
- Pheno: **spin asymmetries**, **pseudodata** and **impact studies**
- Twist-2 T -odd gluon TMDs (**Sivers**, etc.) almost done!
- Explorative studies on gauge-link sensitivity and factorization

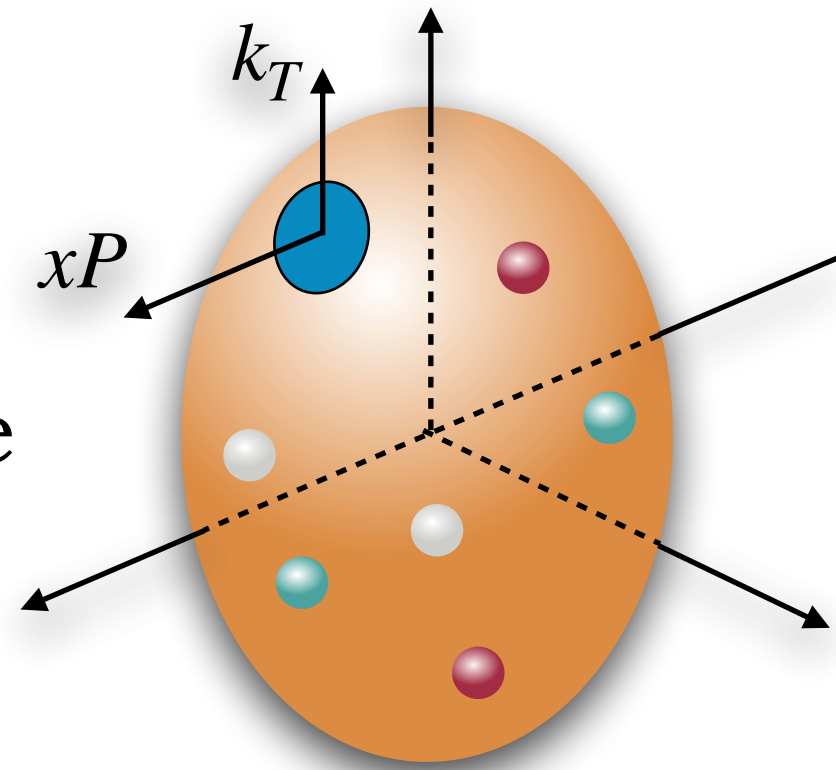
The background features a repeating pattern of DNA double helix structures. The helices are rendered in a light, semi-transparent style, with yellow and blue strands. Various colored spheres (red, blue, green) and arrows are attached to the strands, representing molecular components or data points. The overall color palette is soft and pastel, dominated by light blues, greens, and yellows, with a subtle gradient and some light flare effects.

**Backup
slides**

Parton densities: an incomplete family tree

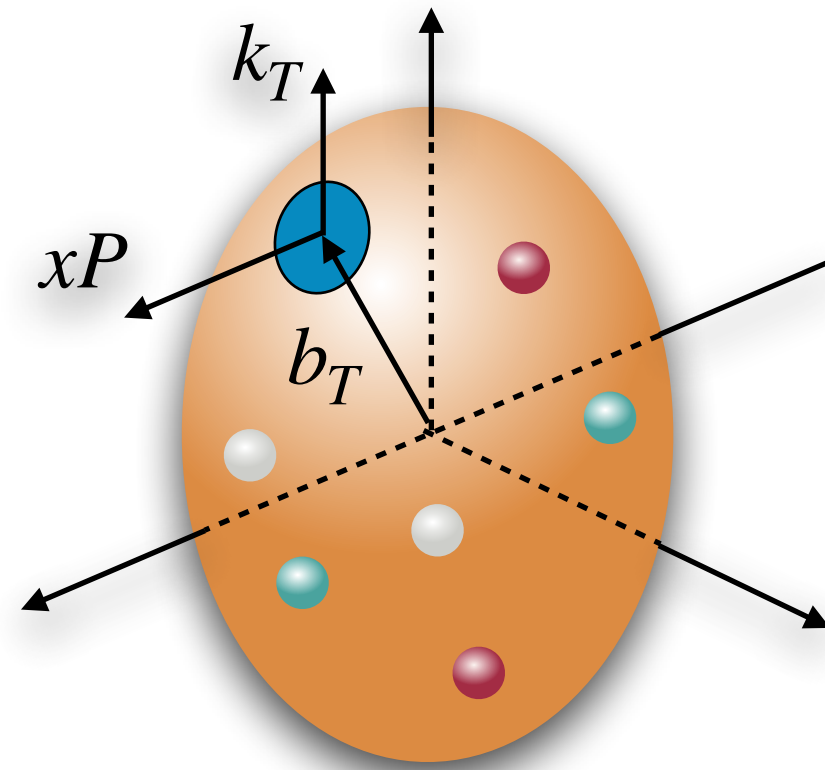
3D

TMDs
(semi-)inclusive



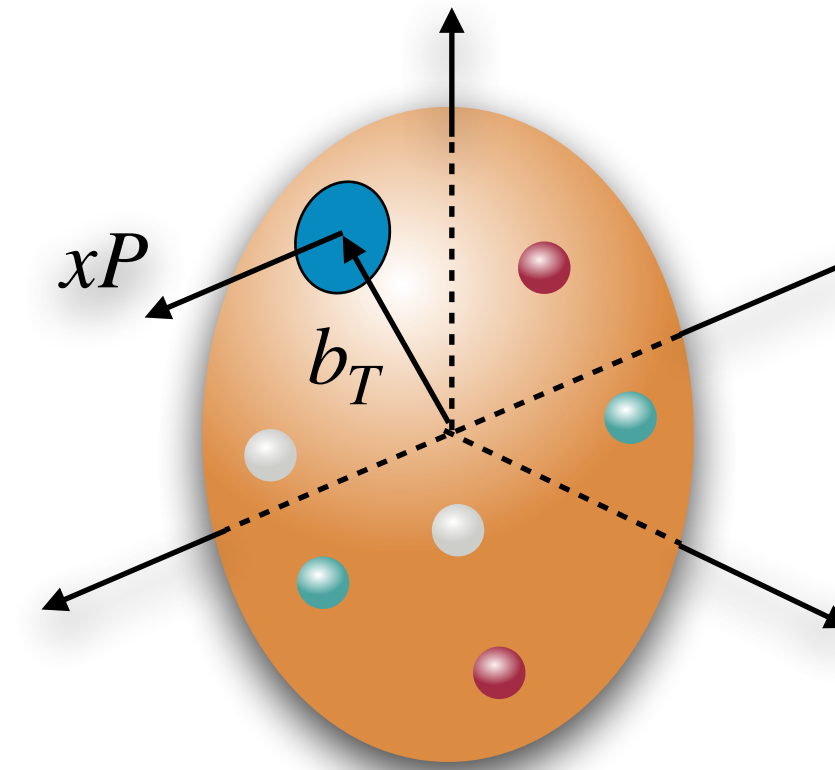
5D

Wigner distributions

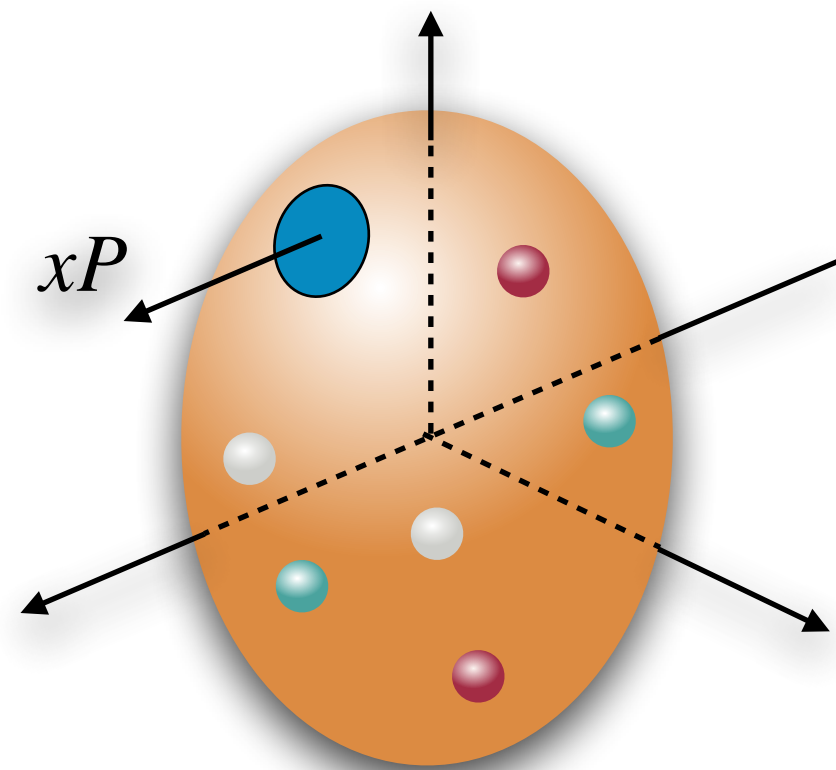


3D

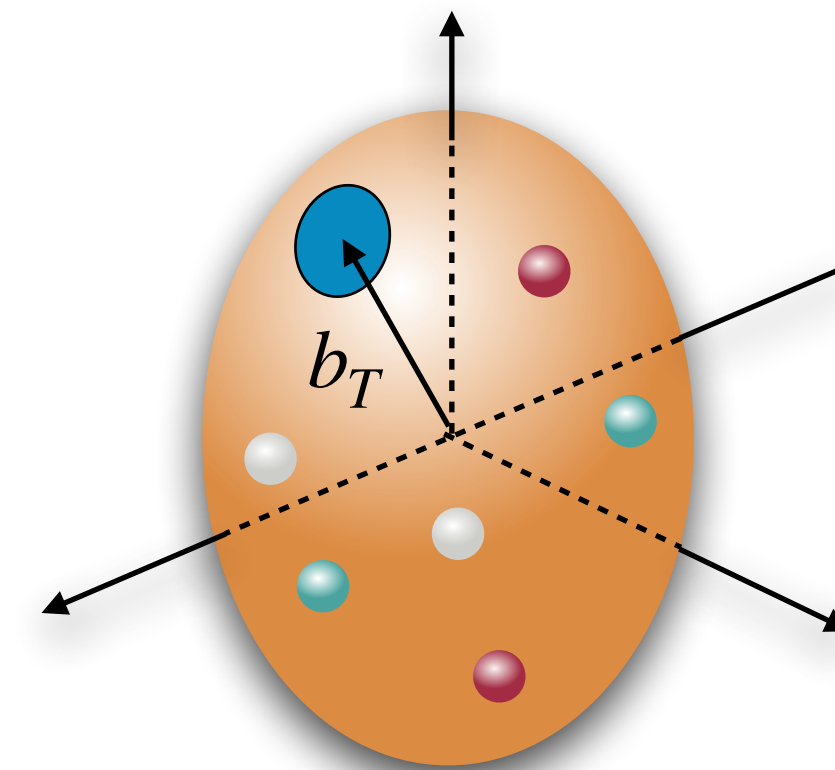
FT of GPDs
exclusive



PDFs
(semi-)inclusive



FT of Form Factors



1D

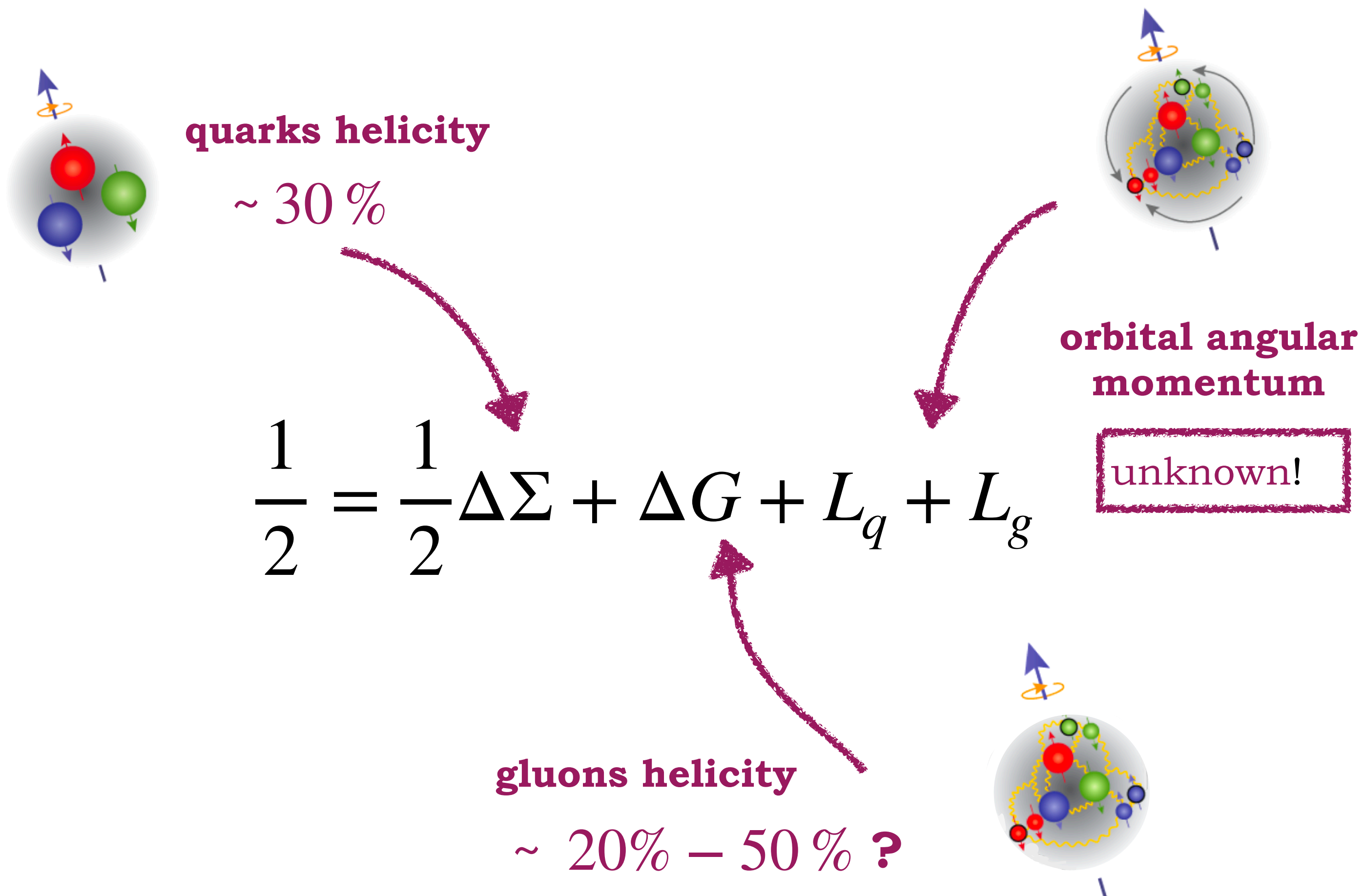
→ \vec{b}_\perp dependence
→ \vec{k}_\perp dependence



these two variables are NOT Fourier conjugate

2D

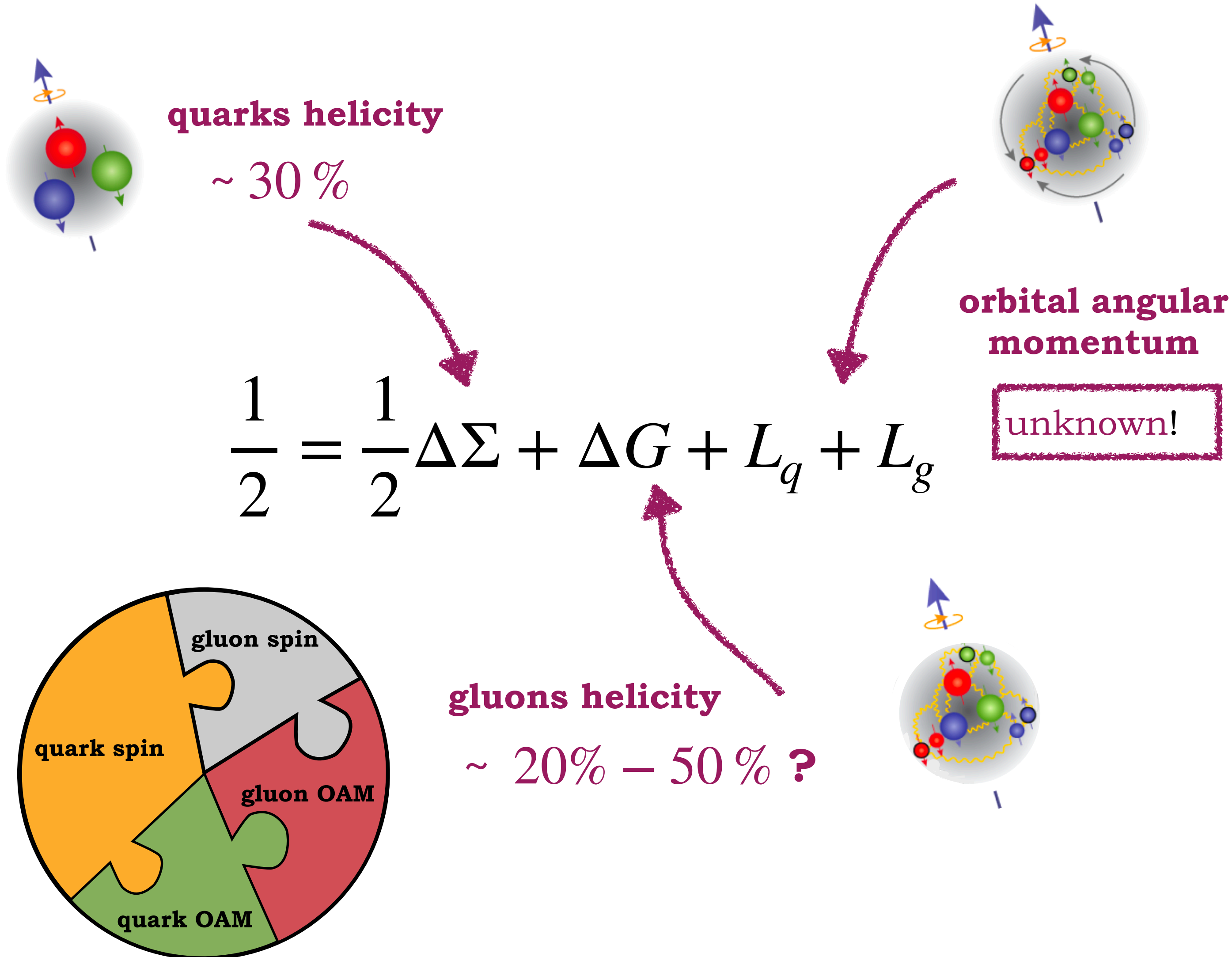
The proton spin crisis



Total spin carried by quarks and gluons does not amount to 1/2, one needs orbital angular momentum, then a 3D description...

(proton spin crisis) [EMC Collaboration, CERN (1987)]

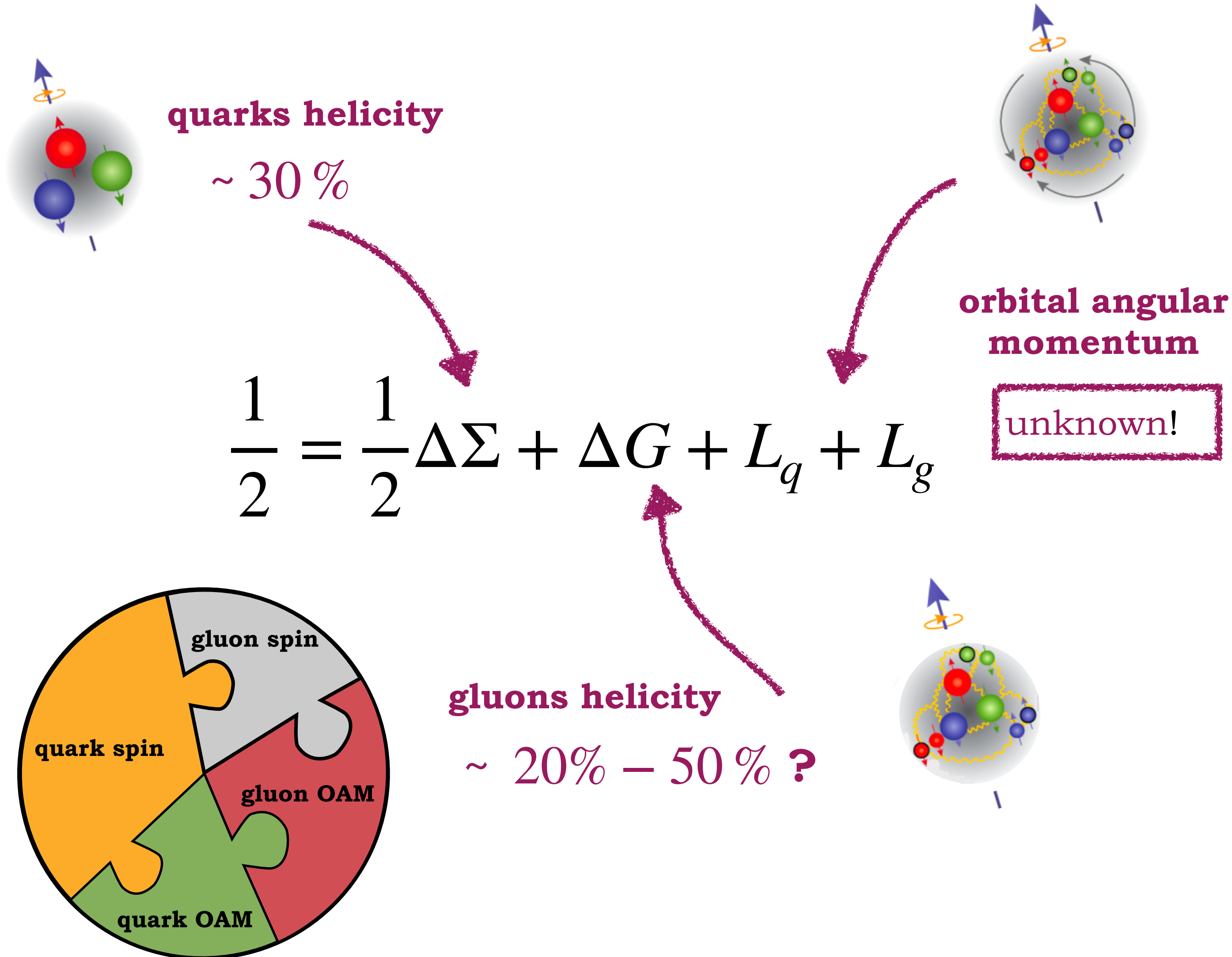
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The proton spin crisis



...many other effects in hadronic interactions cannot be understood in the purely collinear approach

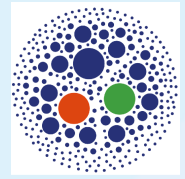
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slide adapted from C. Bissolotti

Backup

Assumptions of the model



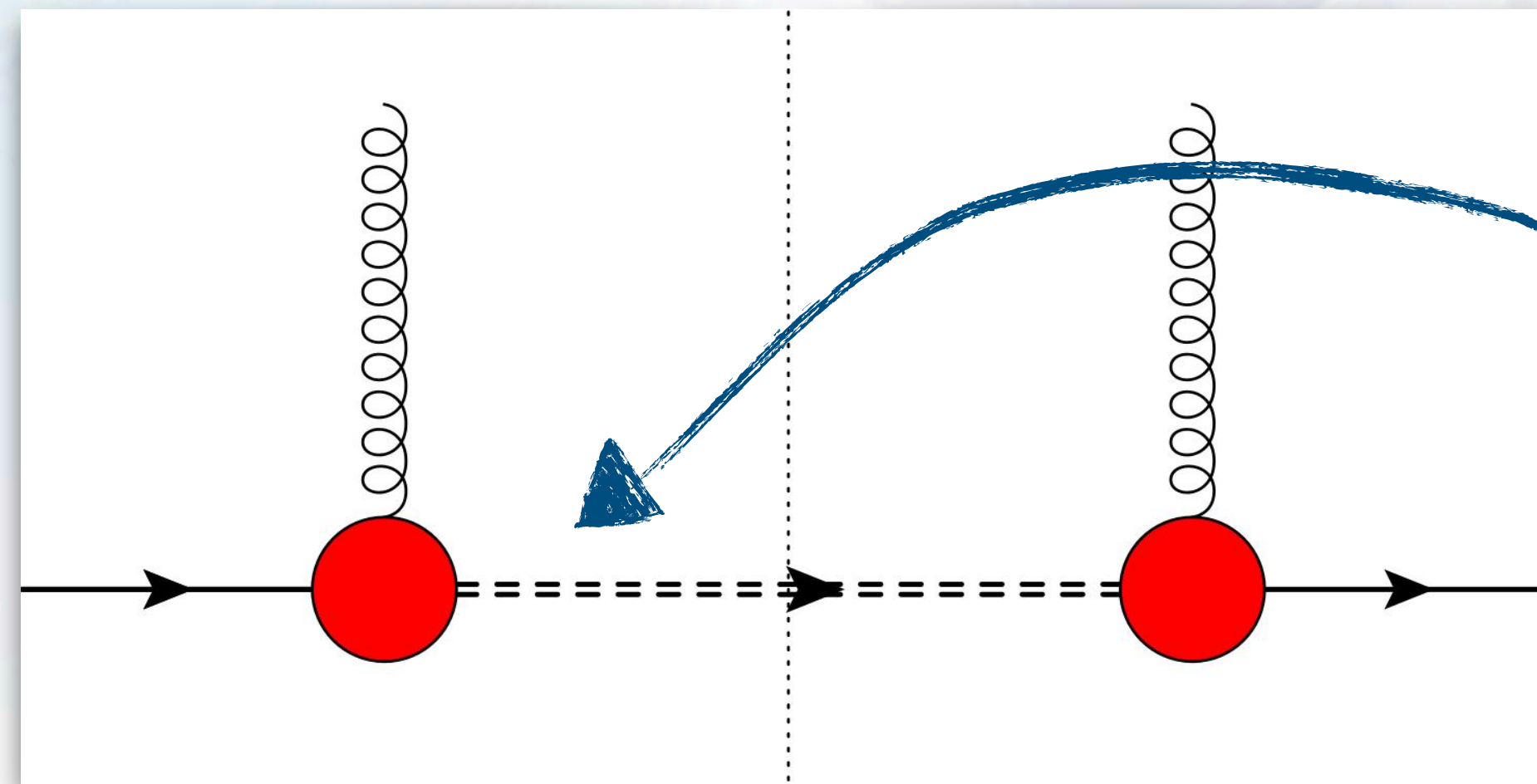
Spectator-system spectral-mass function

$$F(x, \mathbf{p}_T^2) = \int_M^\infty dM_X \rho_X(M_X) \hat{F}(x, \mathbf{p}_T^2; M_X)$$

spectral-mass function

spectator-model TMD

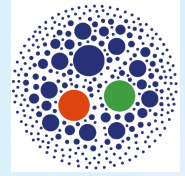
[Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]



Instead of a single on-shell spectator, a continuum of spectators

$\mathcal{Y}_g(p^2)$

Assumptions of the model



Spectator-system spectral-mass function

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spectral-mass function

spectator-model TMD

ⓘ [Inspired by G.R. Goldstein, J.O.G. Hernandez, S. Liuti (2011)]

$$\rho_X \left(M_X; \{X^{(\text{pars})}\} \equiv \{A, B, a, b, C, D, \sigma\} \right) = \mu^{2a} \left[\frac{A}{B + \mu^{2b}} + \frac{C}{\pi\sigma} e^{-\frac{(M_X - D)^2}{\sigma^2}} \right]$$

low- x (high- μ^2) tail $\propto (a - b)$

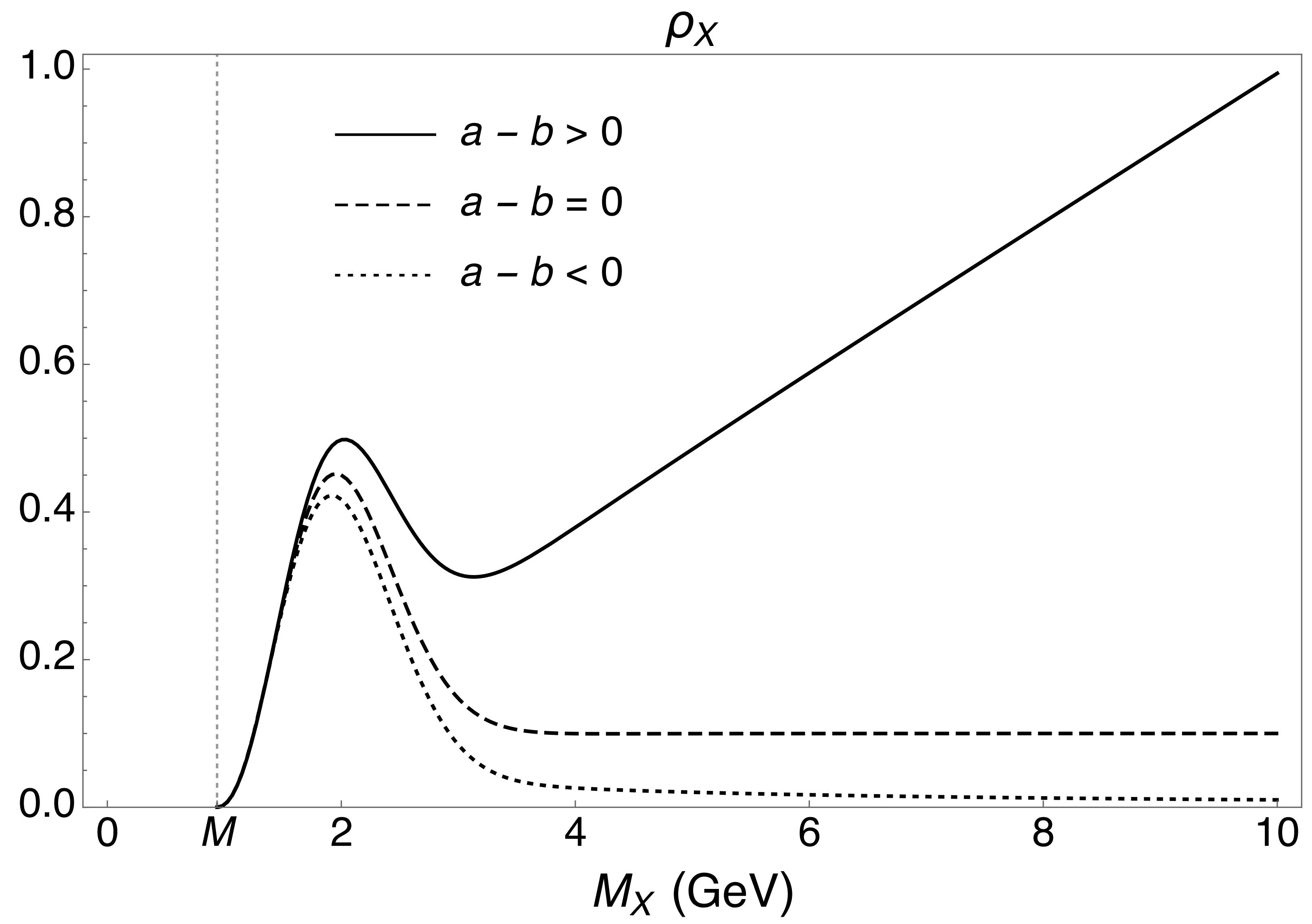
$q\bar{q}$ contributions energetically available at large M_X

$$\mu^2 = M_X^2 - M^2$$

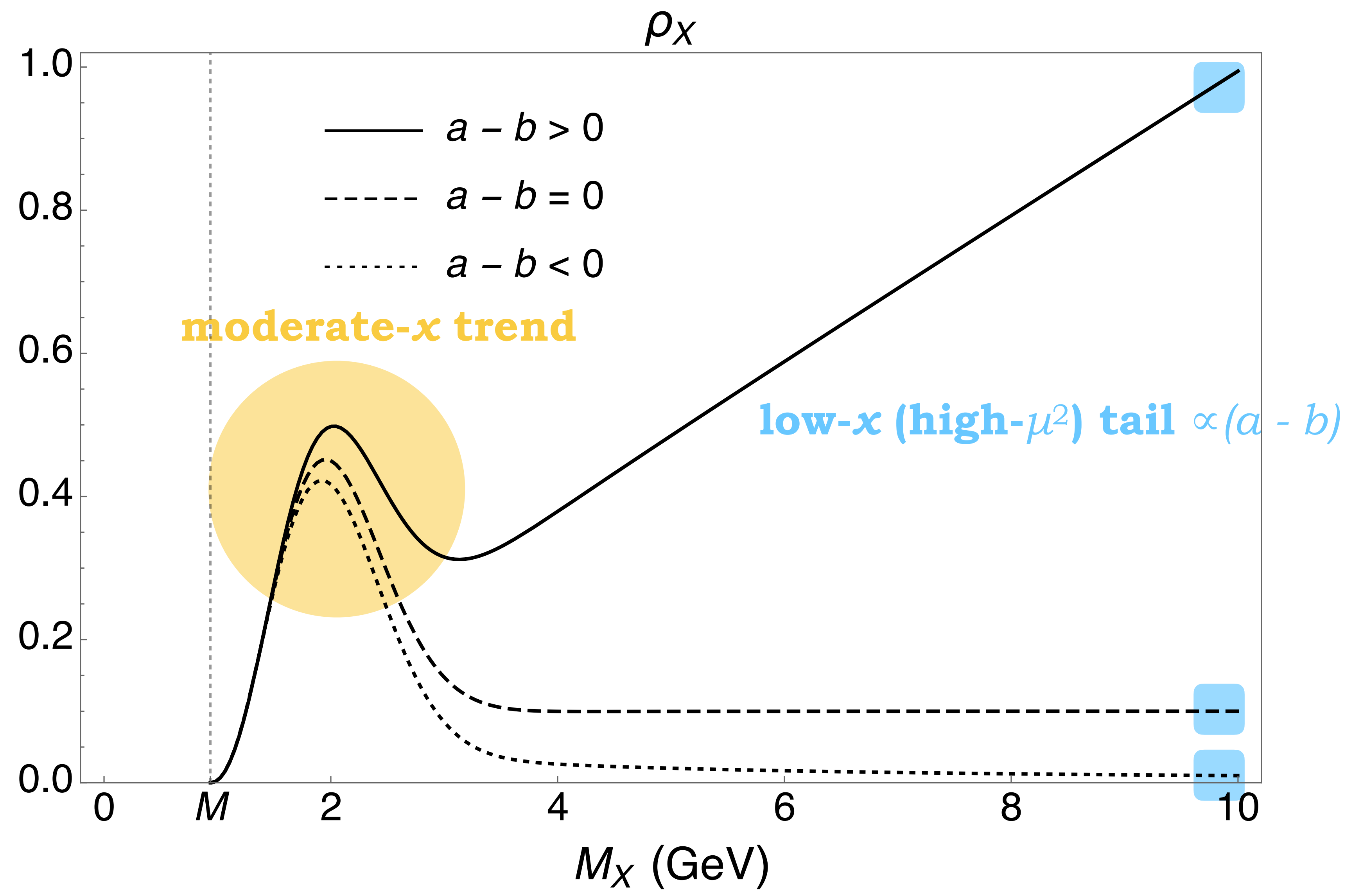
moderate- x trend

pure tri-quark contribution at low M_X

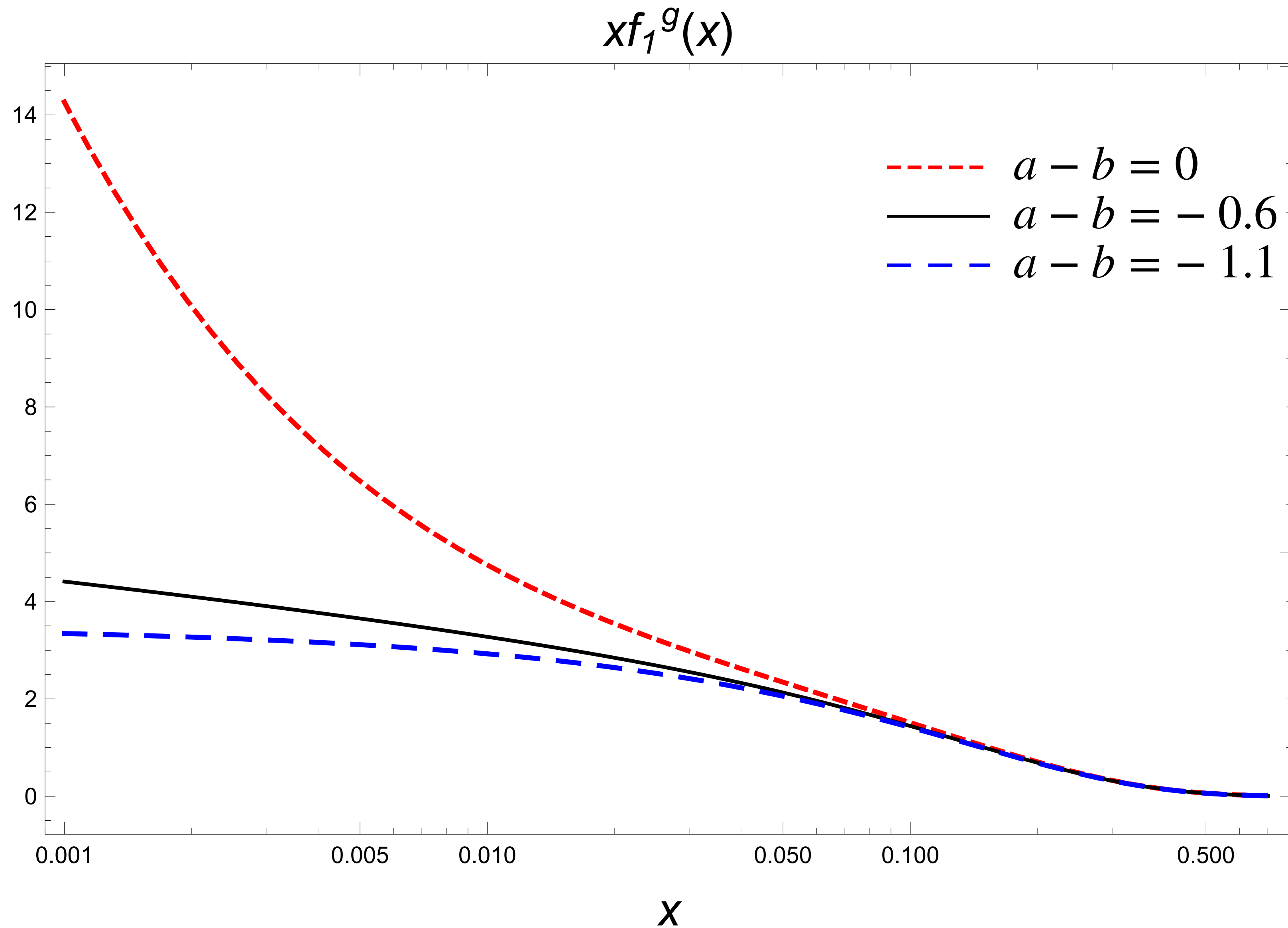
Spectral function vs $(a - b)$



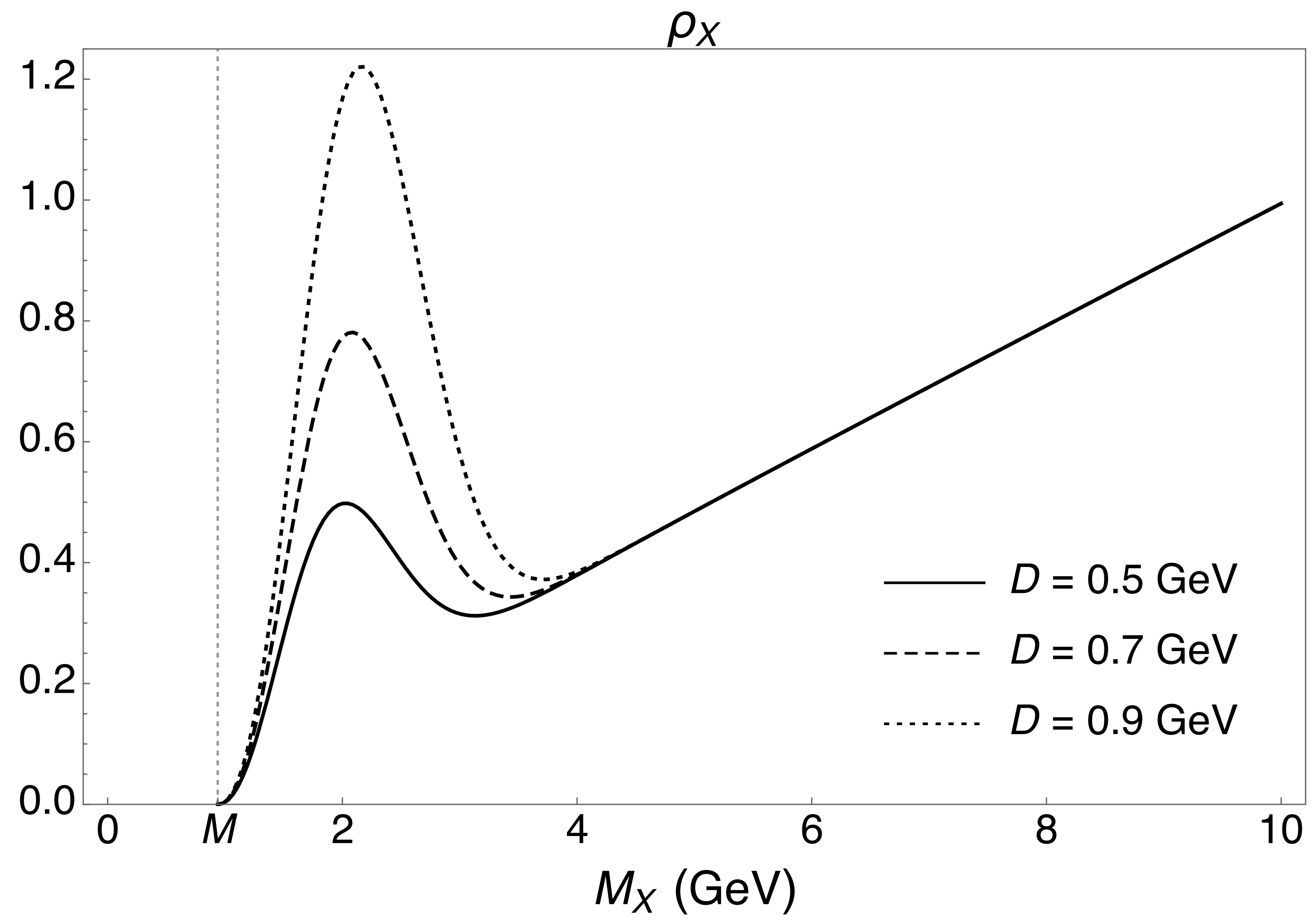
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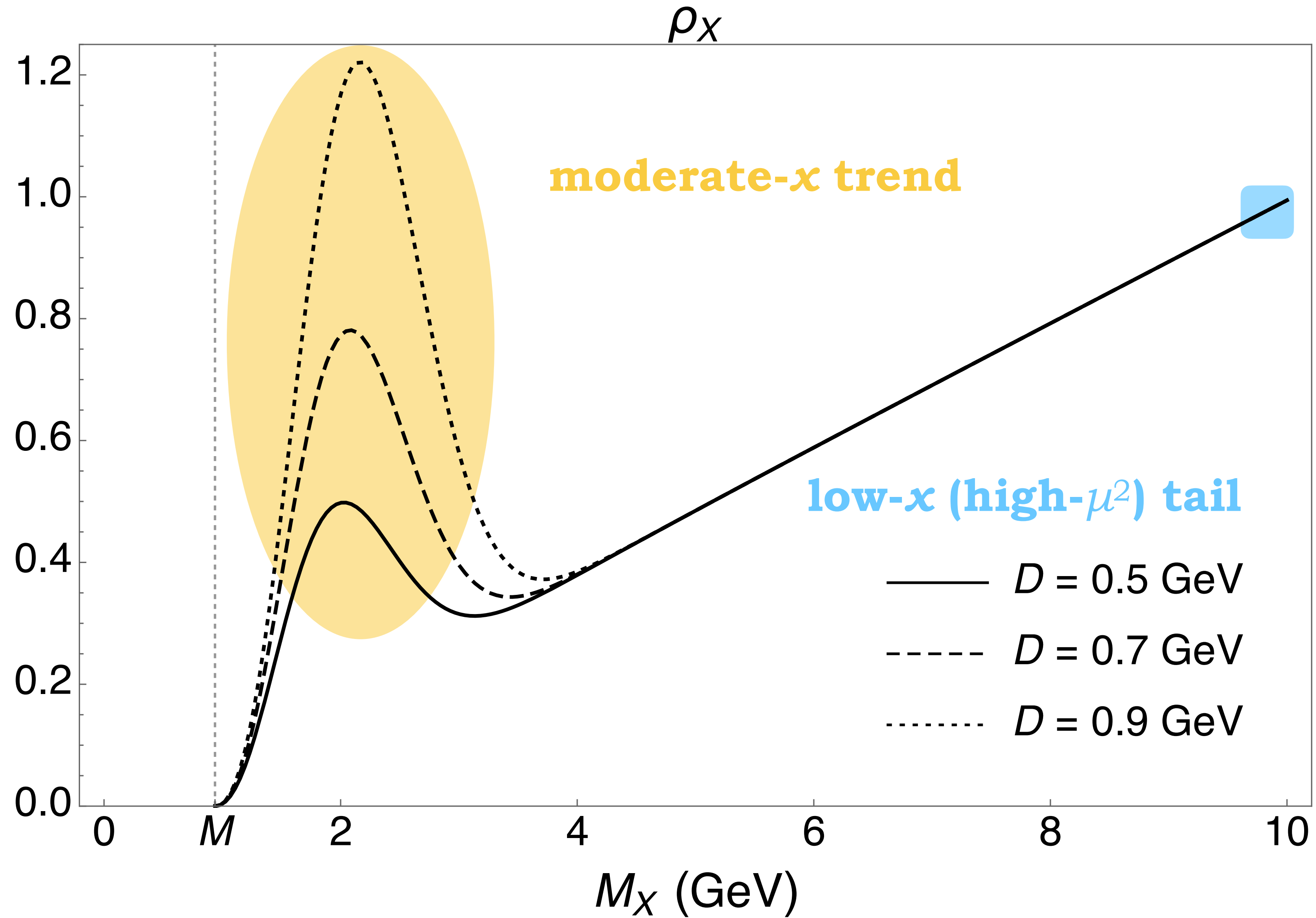
xf_1 collinear PDF vs $(a - b)$



Spectral function vs D

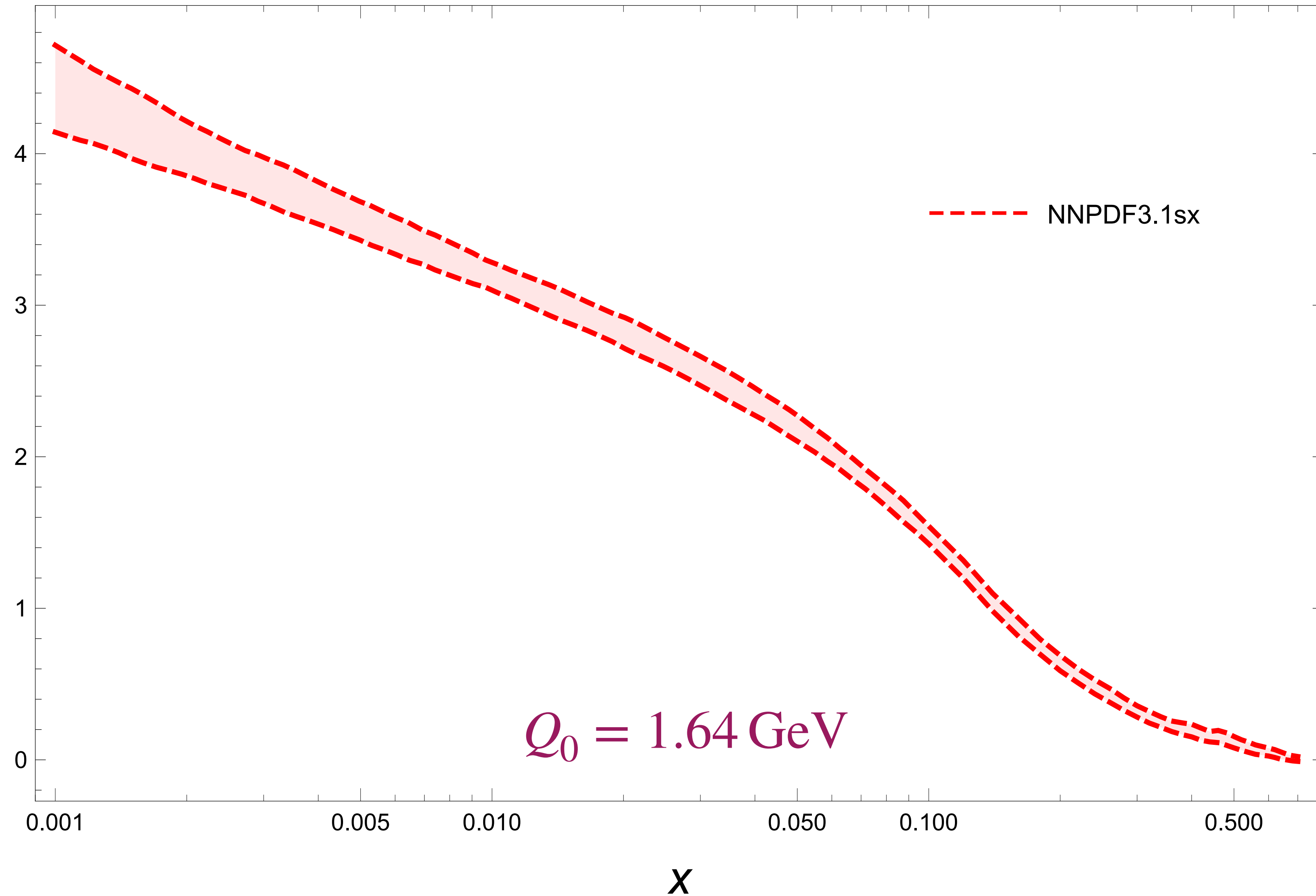


Spectral function vs D



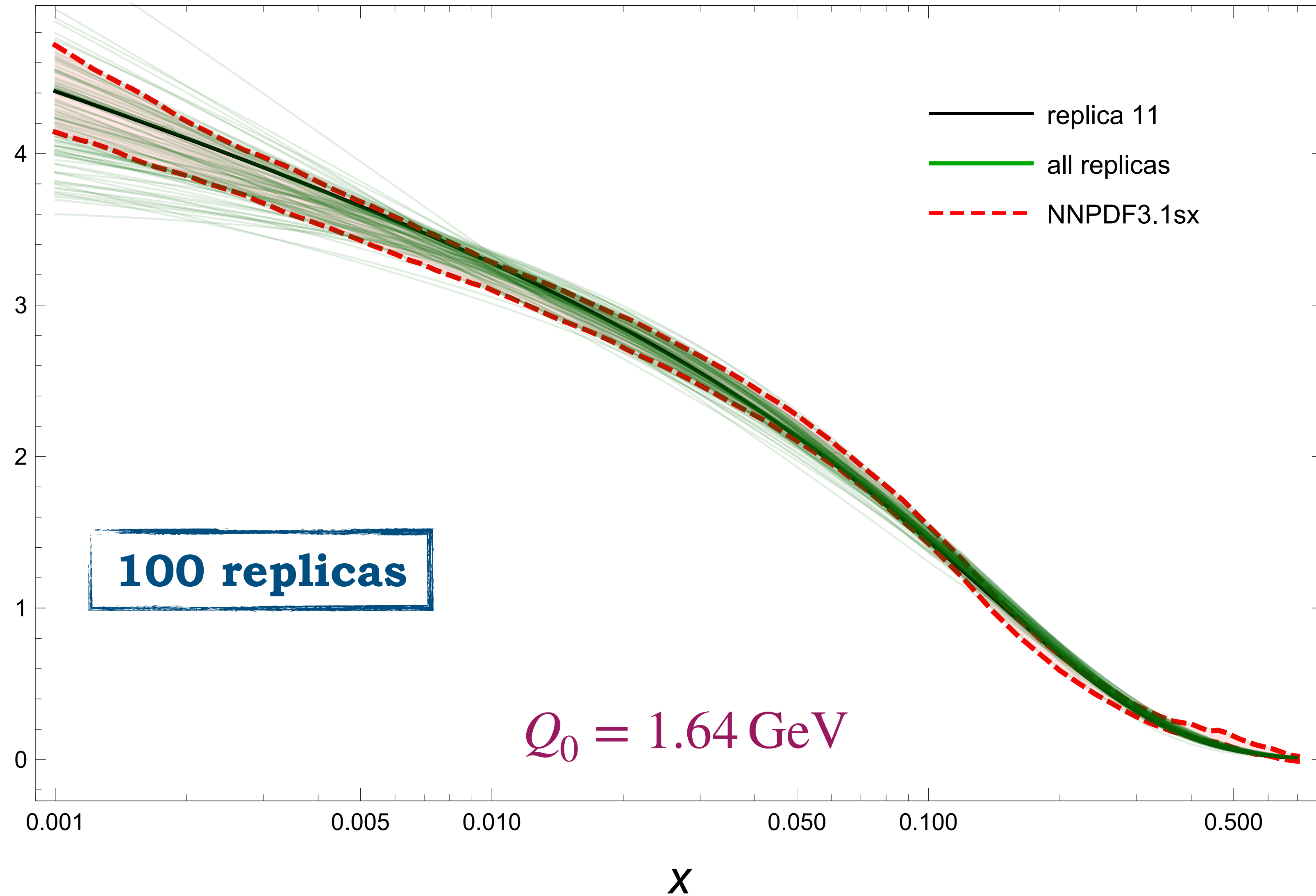
Unpolarized gluon PDF

$$xf_1^g(x)$$



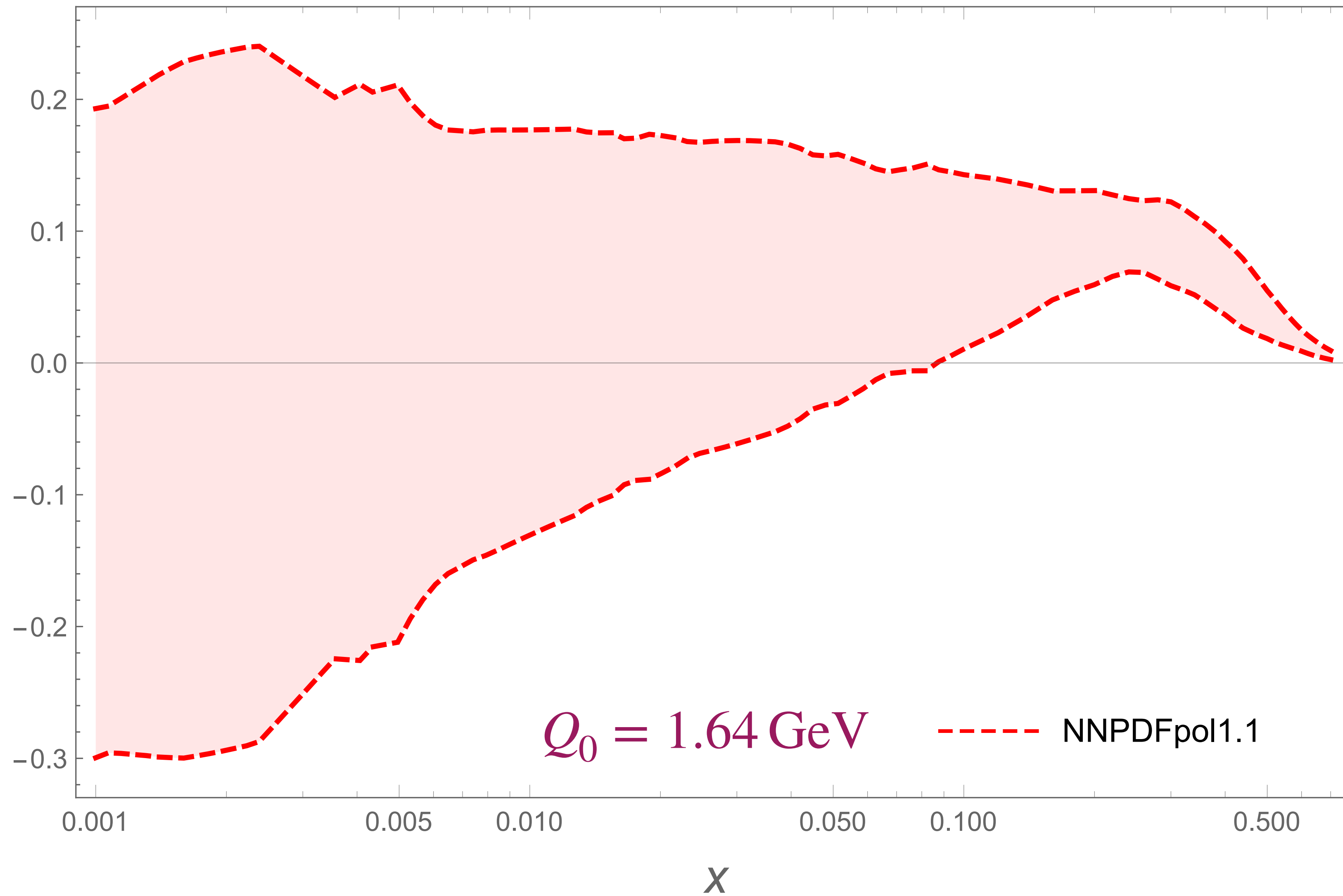
Unpolarized gluon PDF

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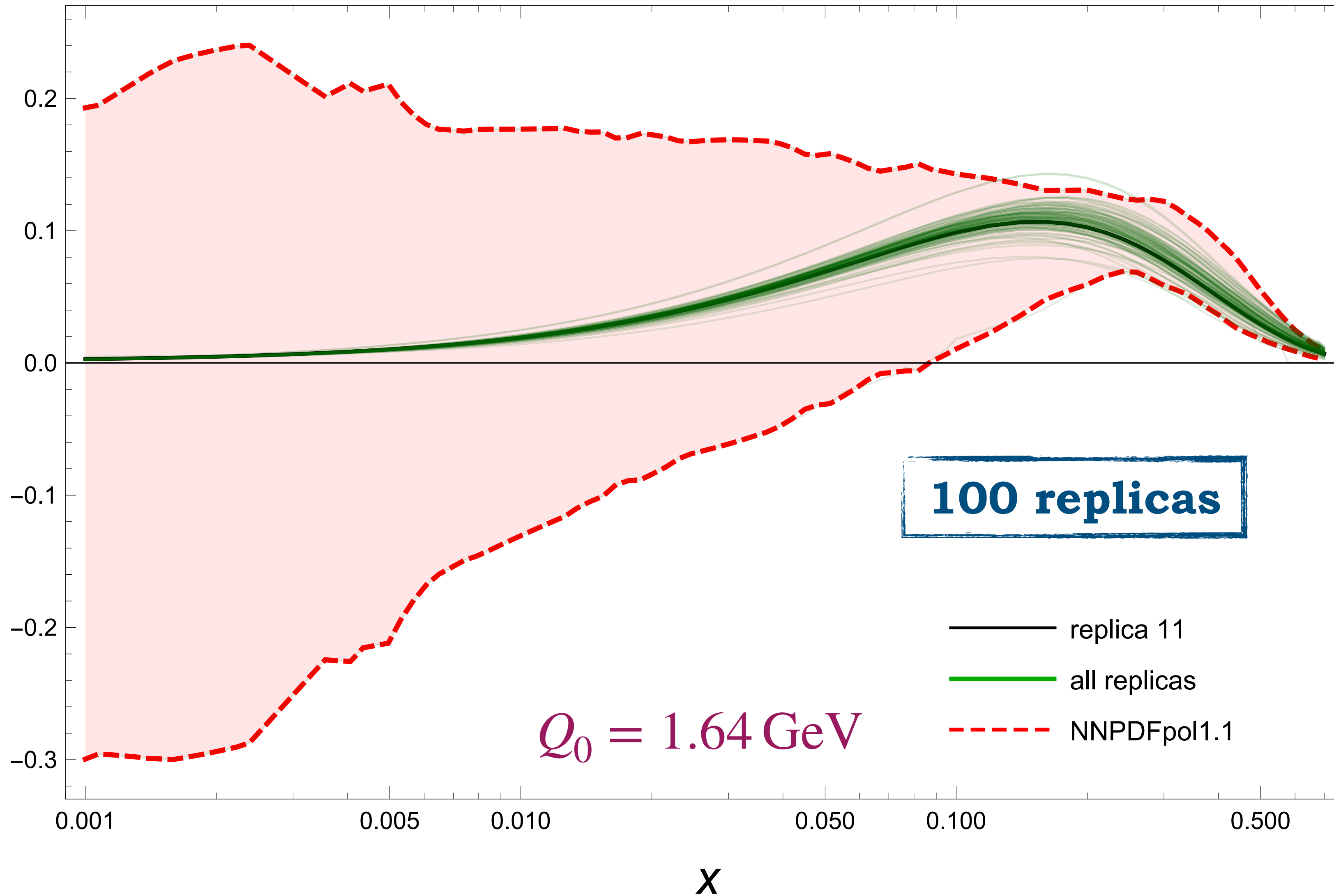
Helicity gluon PDF

$$xg_1^g(x)$$



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Fit specifics

$$\chi^2/\text{d.o.f.} = 0.54 \pm 0.38$$

no **overlearning**, just large errors for g_1

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Our model @ $Q_0 = 1.64$ GeV

Lattice @ $Q_0 = 2$ GeV

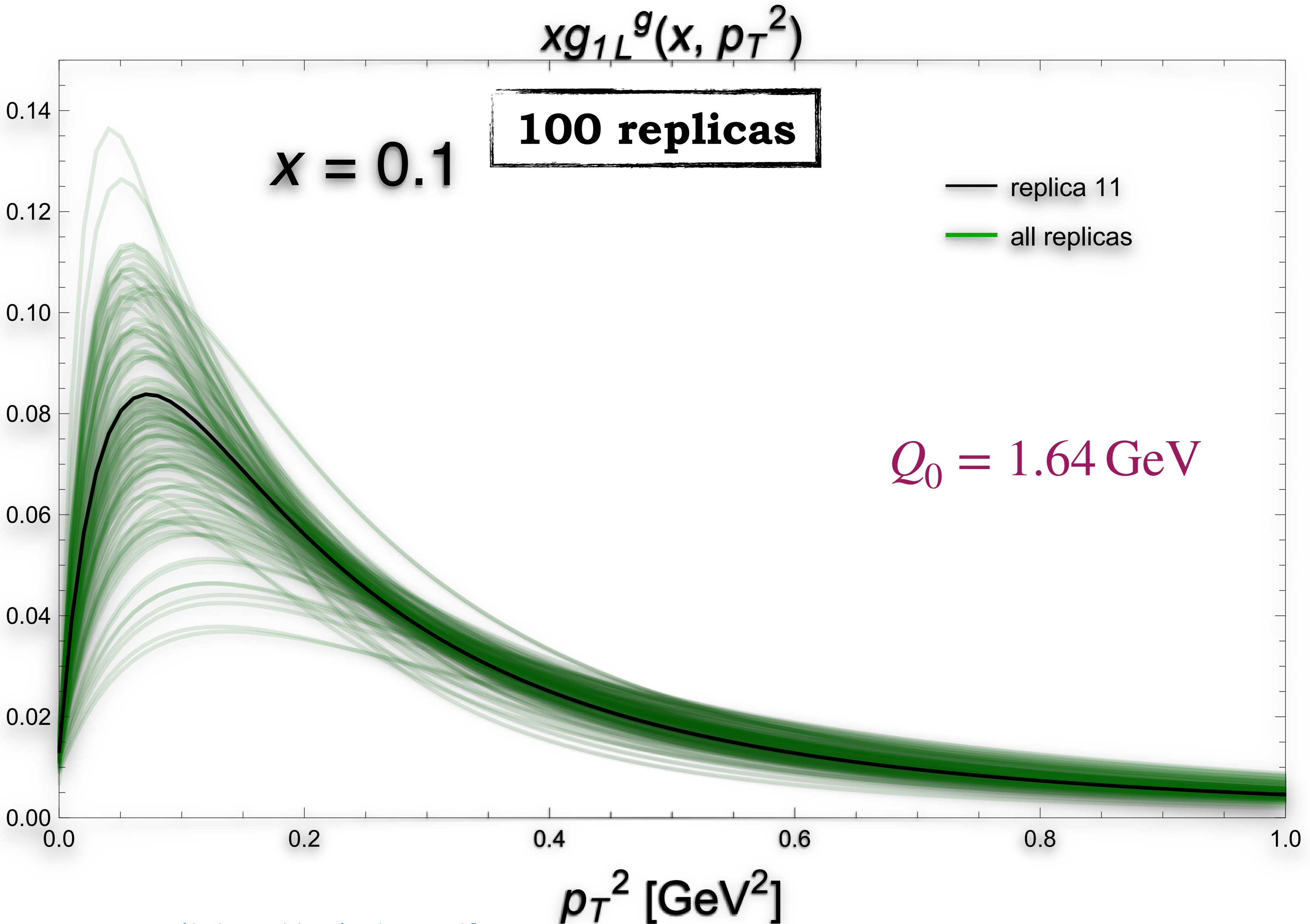
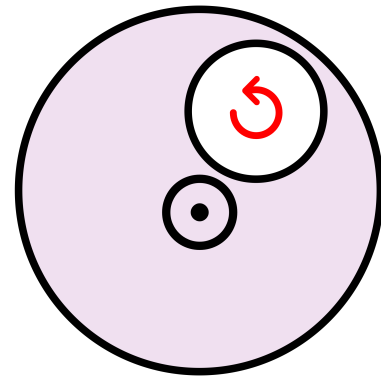
$$\langle x \rangle_g = 0.424(9)$$

$$\langle S \rangle_g = 0.159(11)$$

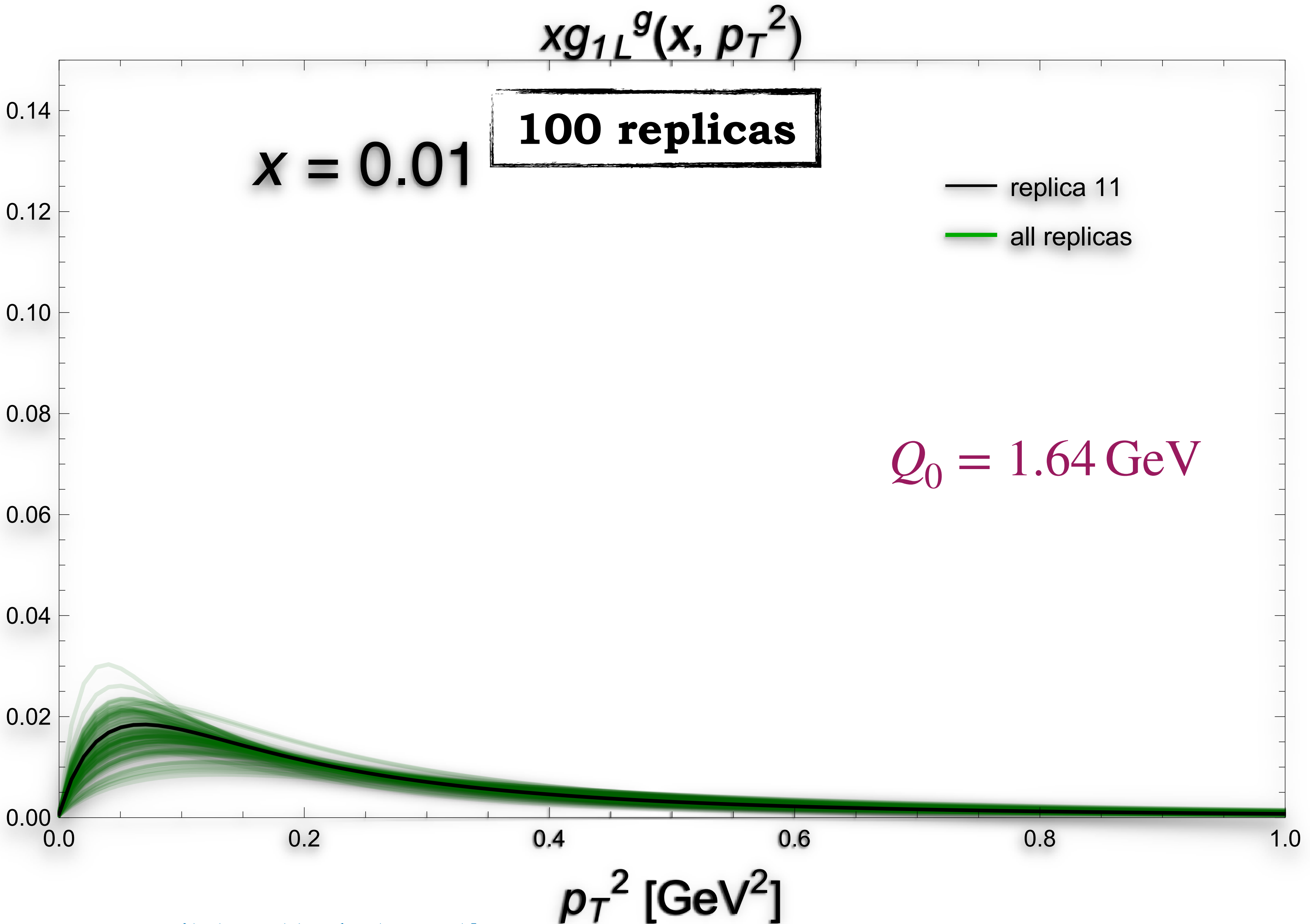
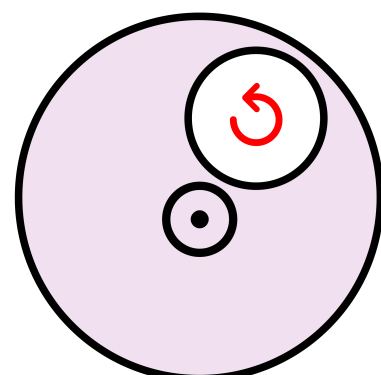
$$\langle x \rangle_g = 0.427(92)$$

$$\langle J \rangle_g = 0.187(46)$$

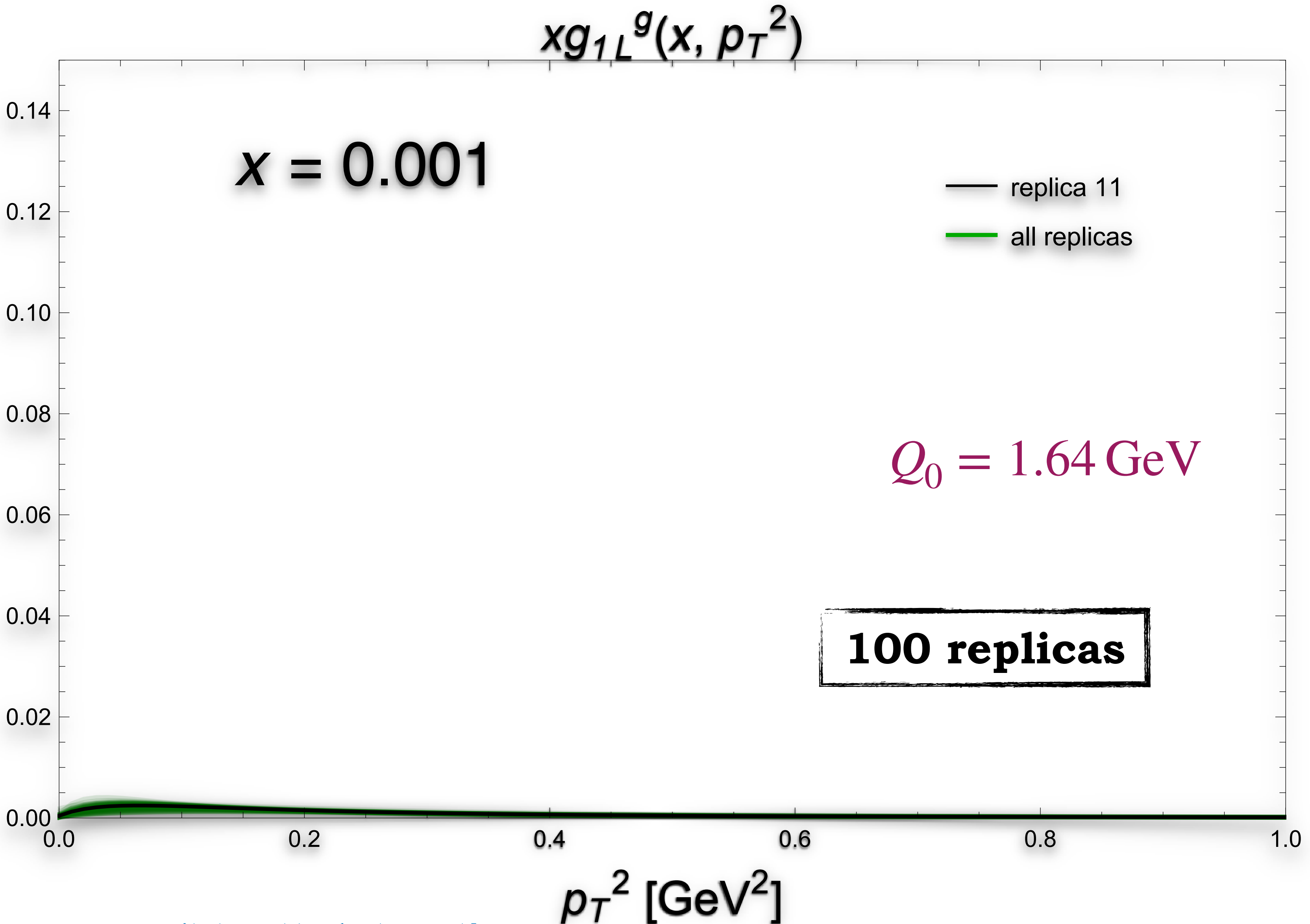
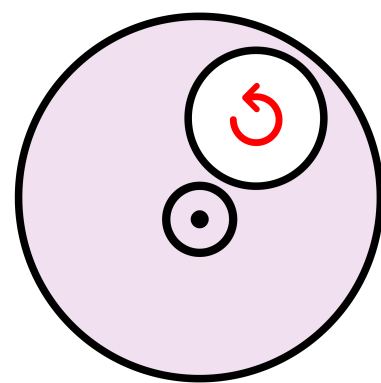
Helicity gluon TMD



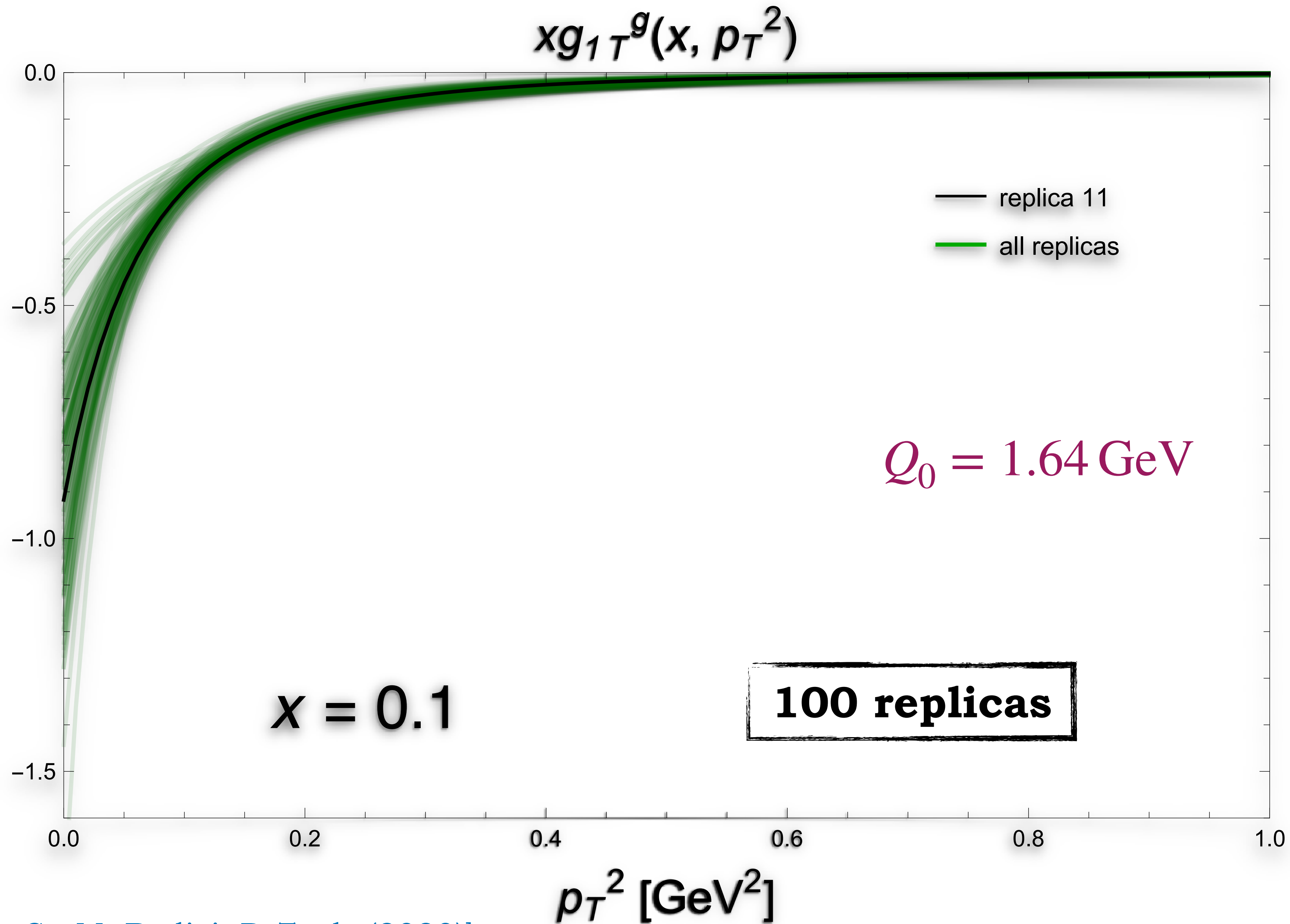
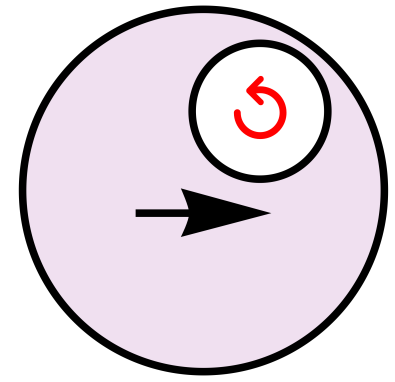
Helicity gluon TMD



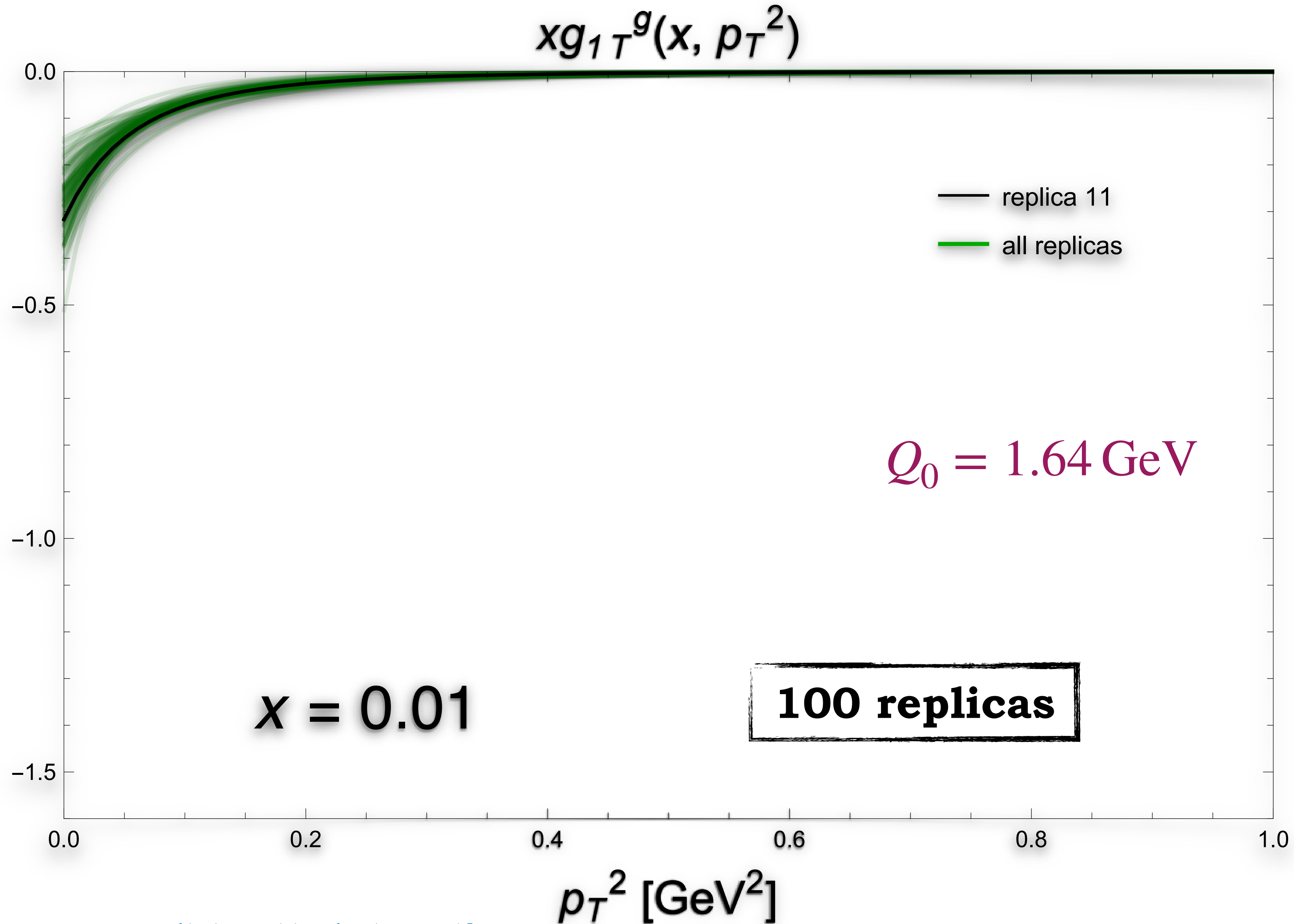
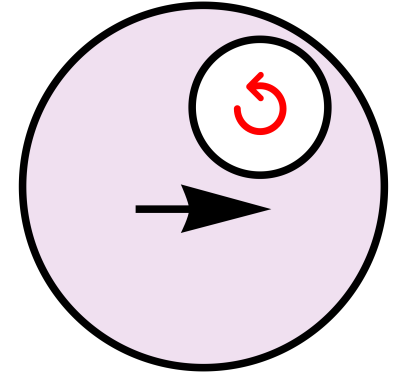
Helicity gluon TMD



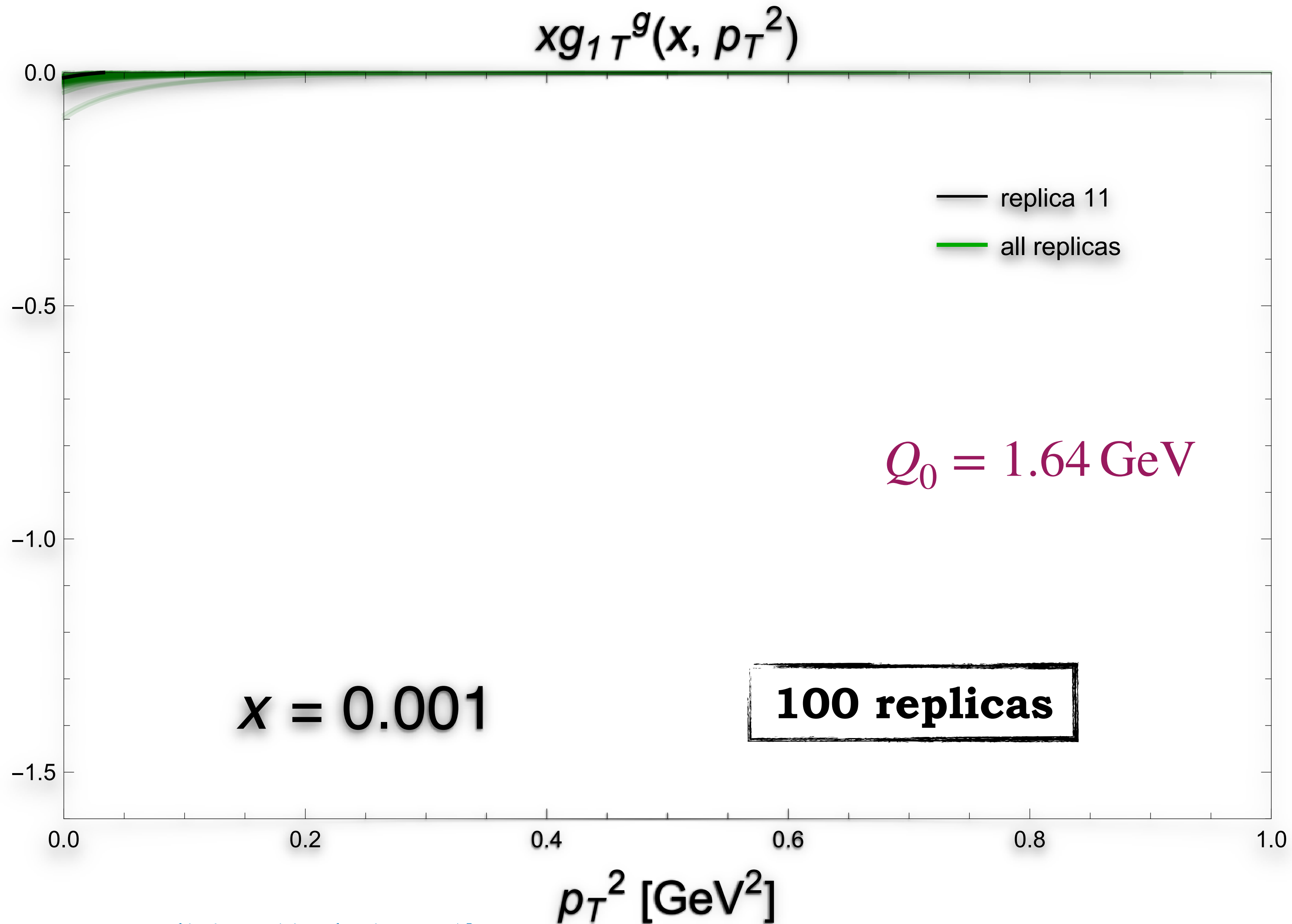
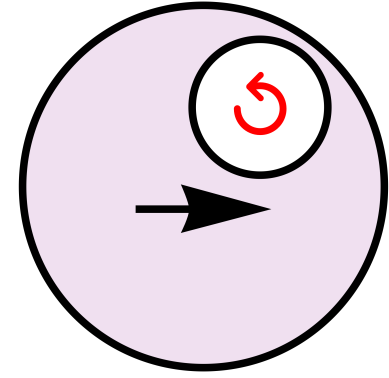
Worm-gear gluon TMD



Worm-gear gluon TMD

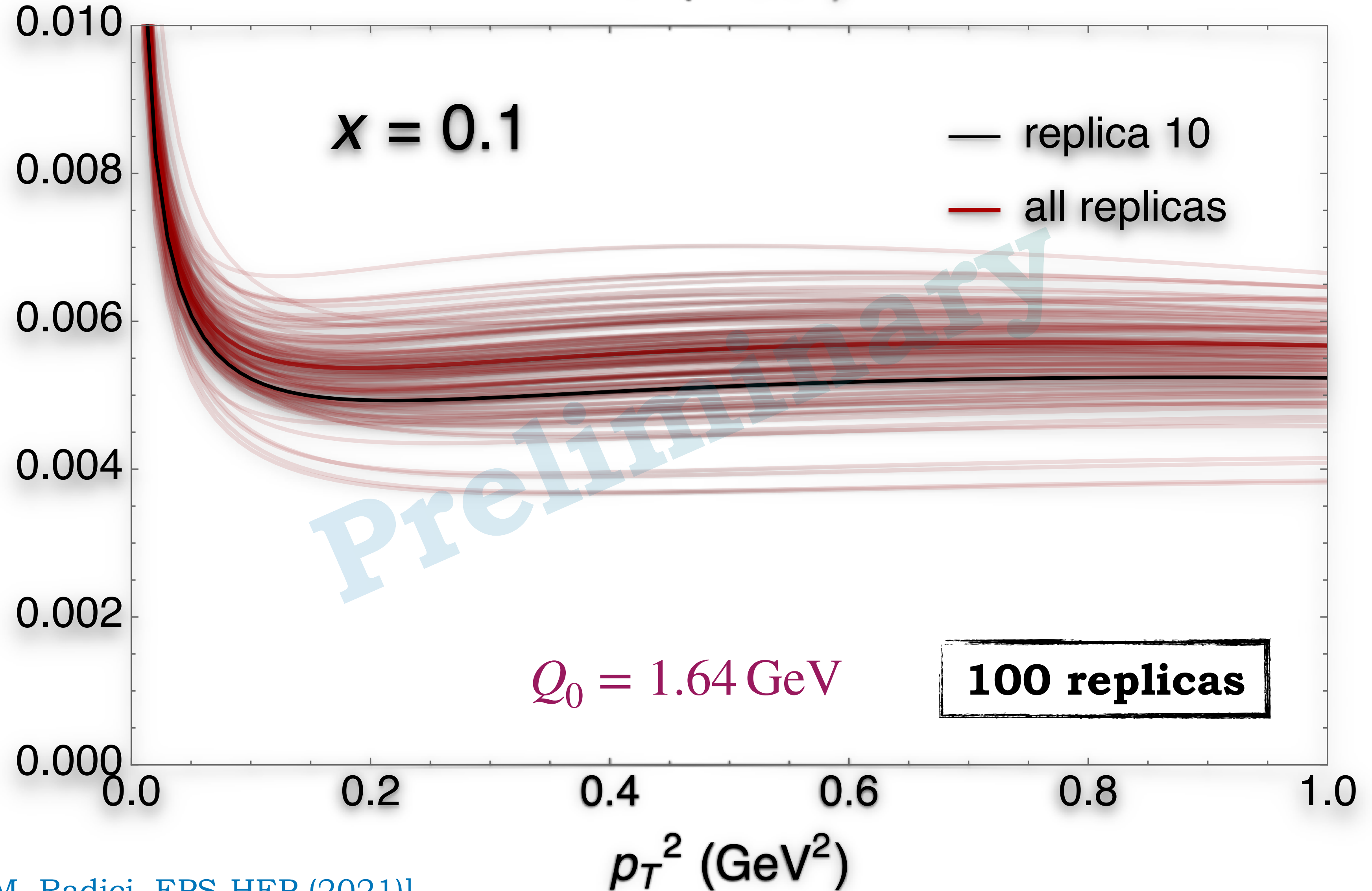
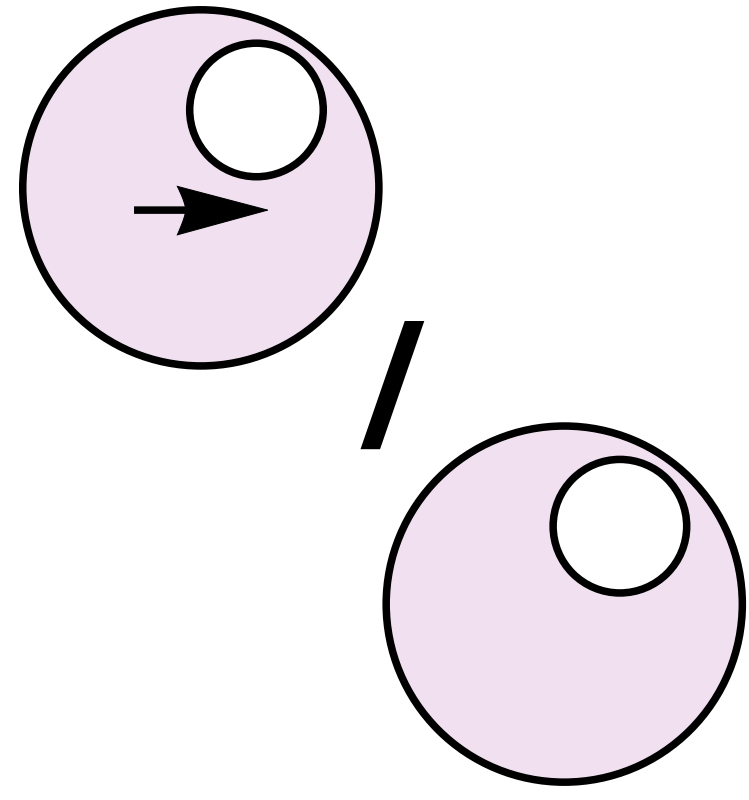


Worm-gear gluon TMD



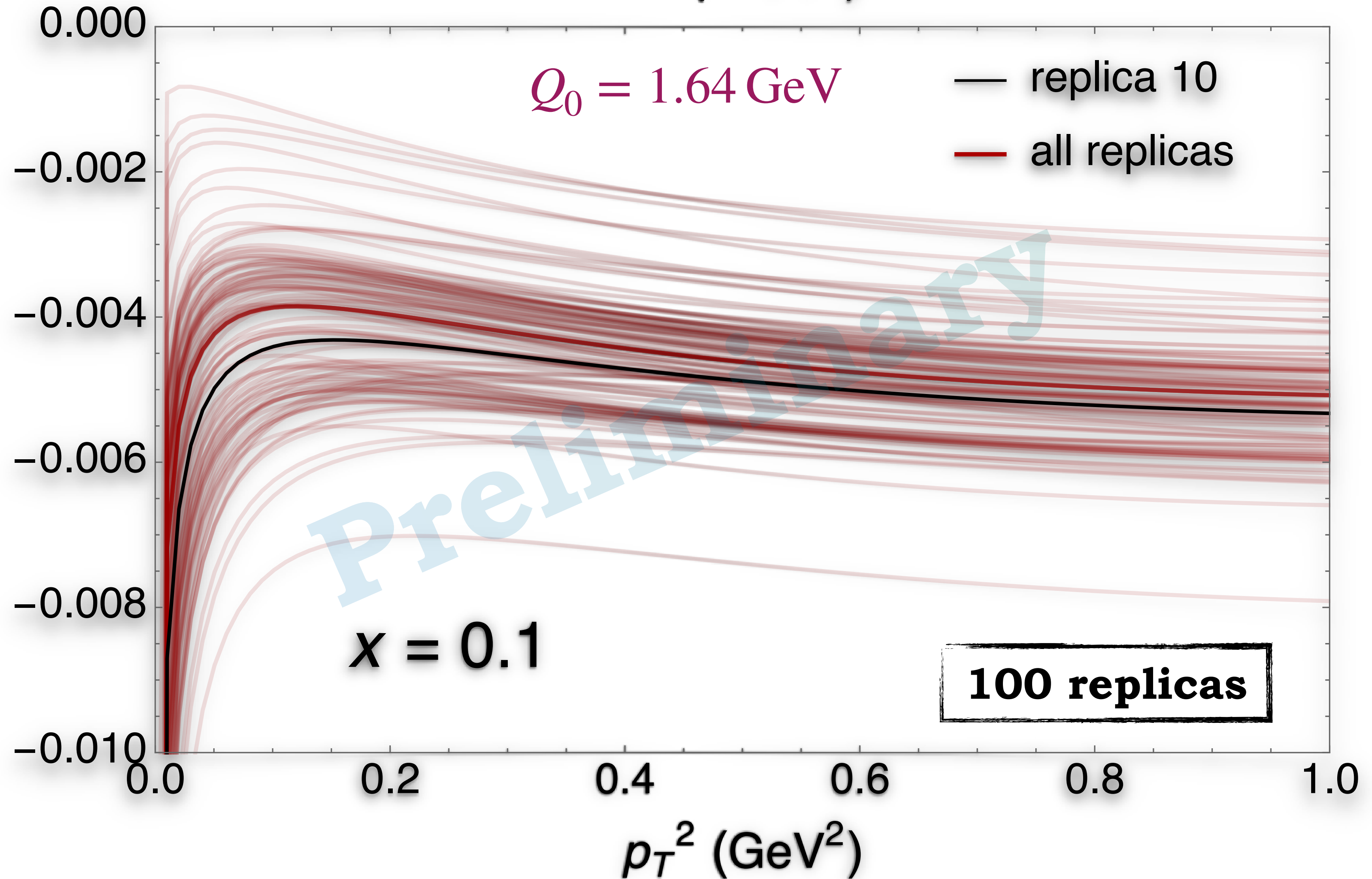
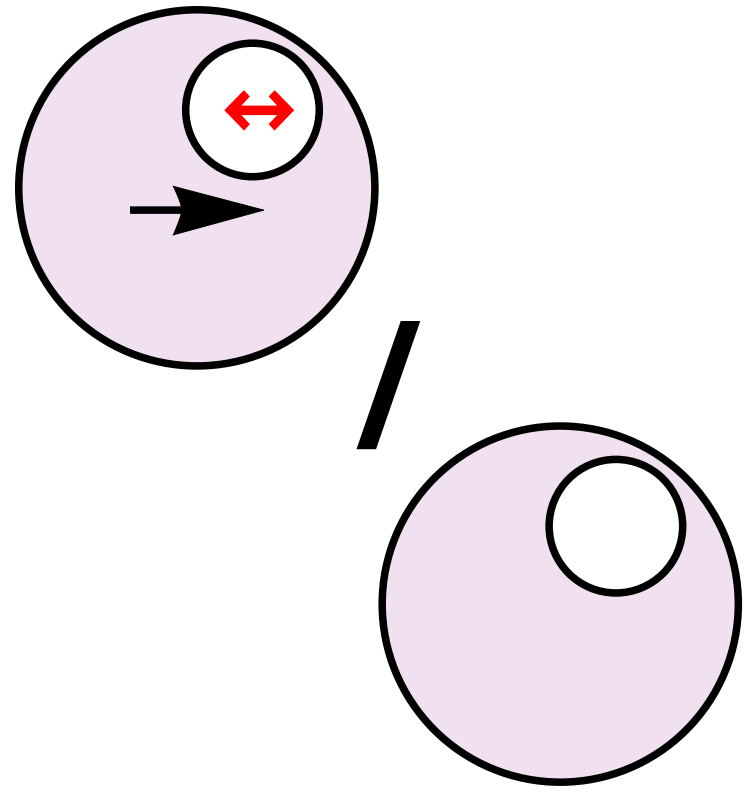
f-type Sivers / unpol.

$$\frac{\frac{p_T}{M} f_{1T}^{\perp[+,+]}(x, p_T^2)}{f_1^g(x, p_T^2)}$$



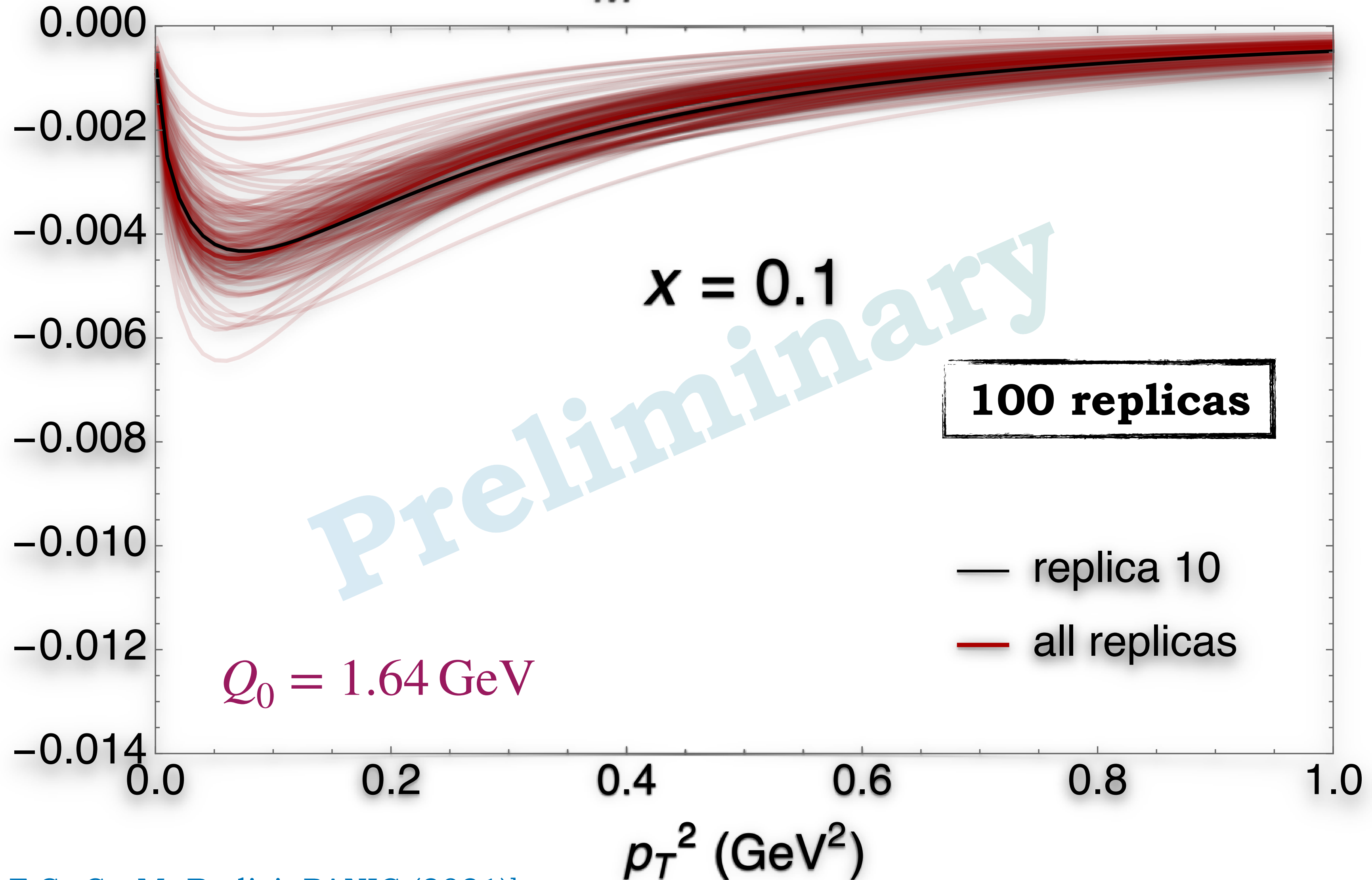
f -type linearity / unpol.

$$\frac{\frac{p_T}{M} h_1^{[+,+]}(x, p_T^2)}{f_1^g(x, p_T^2)}$$



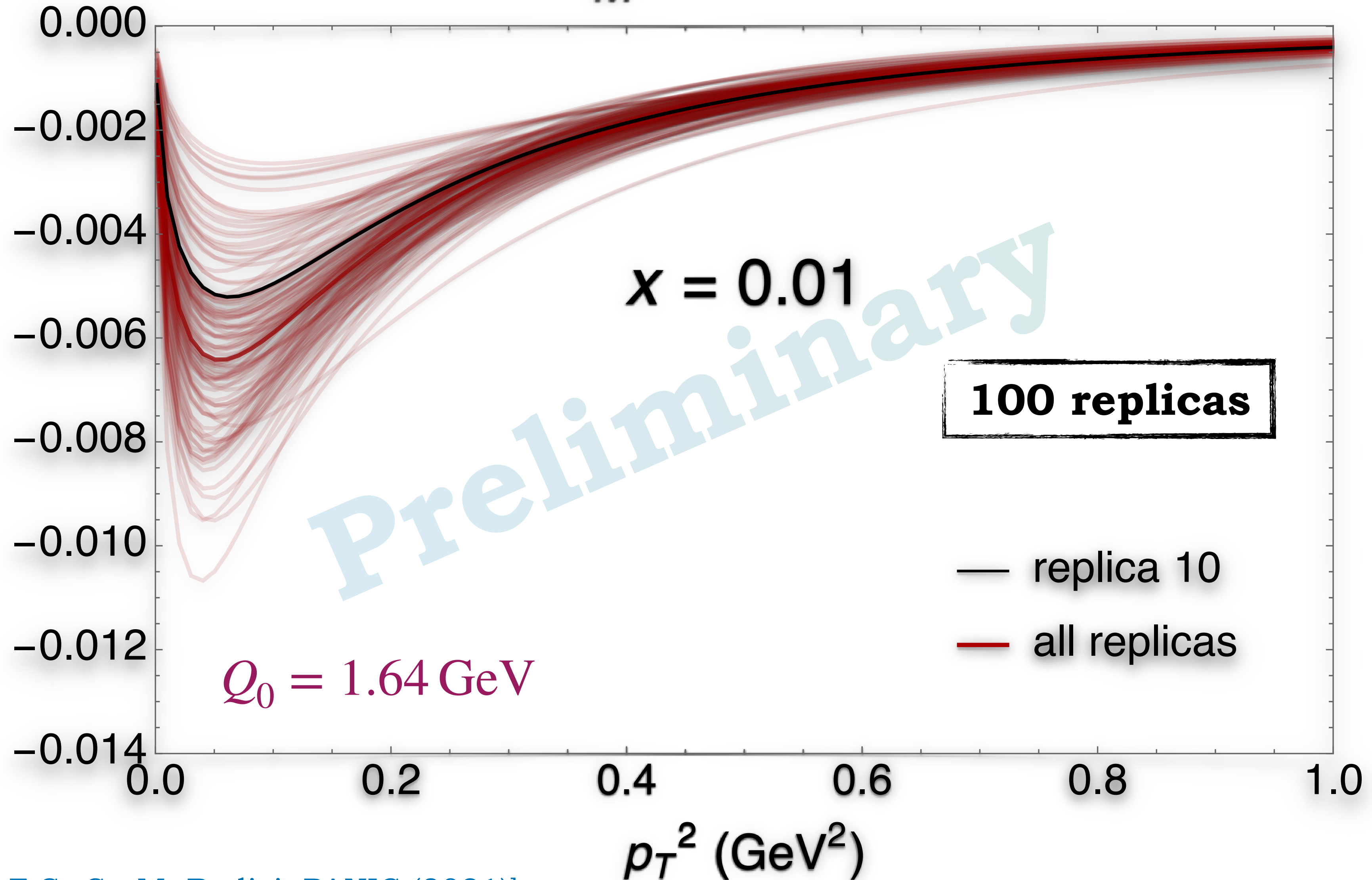
f -type linearity gluon TMD

$$x \frac{p_T}{M} h_1^{[+,+]}(x, p_T^2)$$



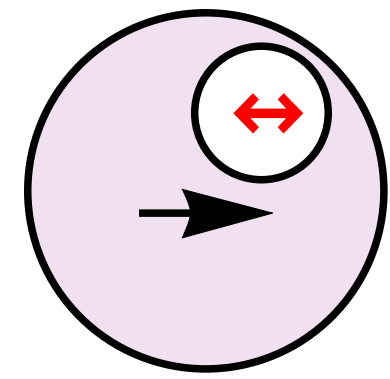
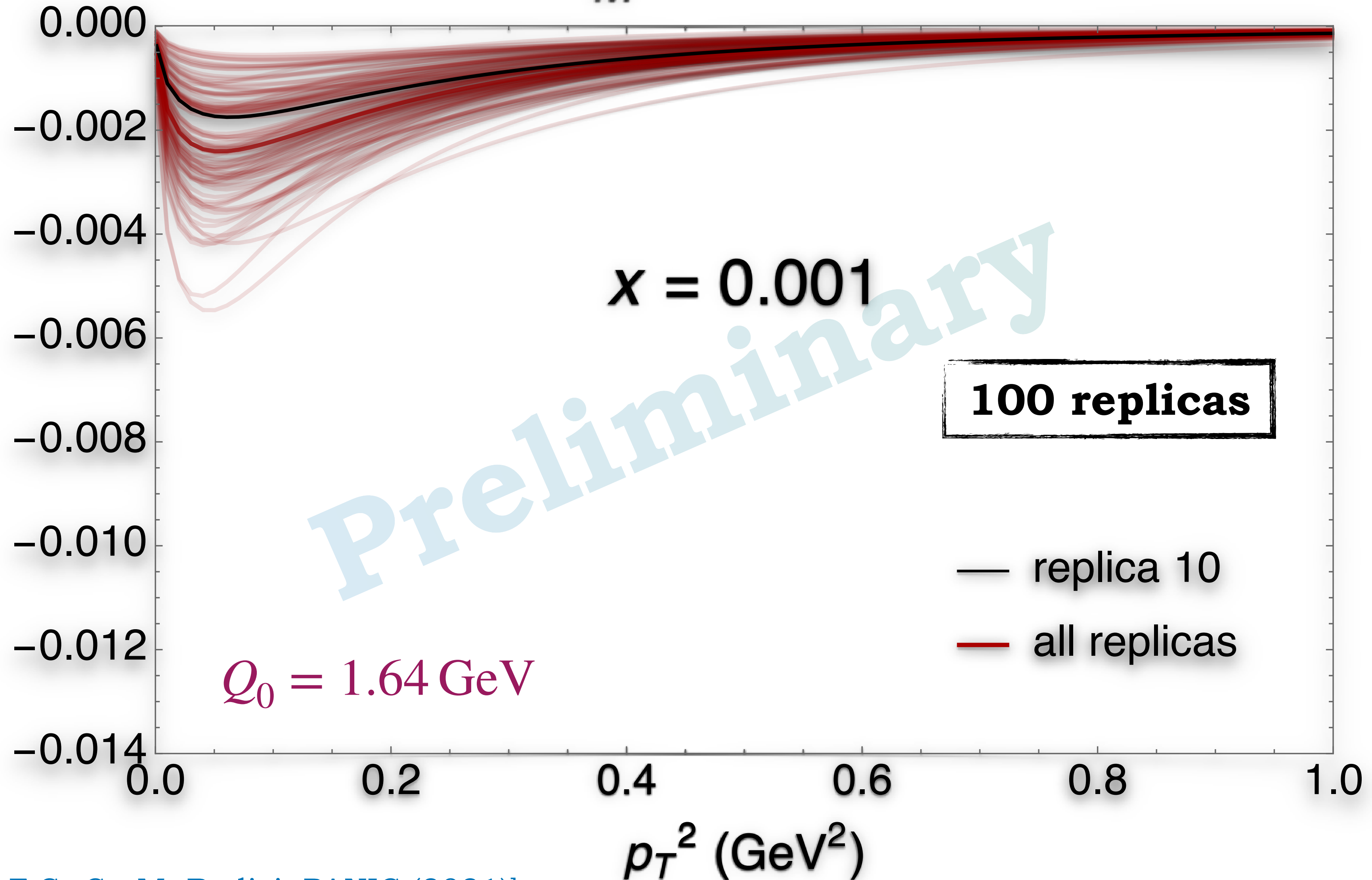
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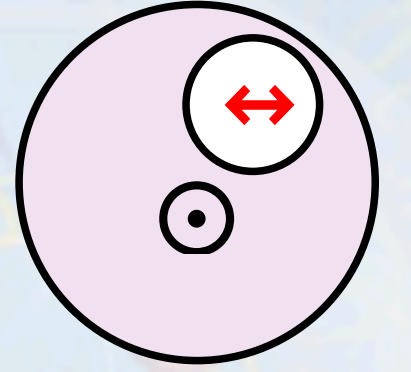
Gluon TMD correlator and T -odd gluon densities

$$\Gamma_U^{ij}(x, \mathbf{k}) = x \left[\delta_T^{ij} f_1(x, \mathbf{k}^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, \mathbf{k}^2) \right]$$

$$\Gamma_L^{ij}(x, \mathbf{k}) = x \left[i\epsilon_T^{ij} S_L g_1(x, \mathbf{k}^2) + \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^\perp(x, \mathbf{k}^2) \right]$$

$$\Gamma_T^{ij}(x, \mathbf{k}) = x \left[\frac{\delta_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}^2) + \frac{i\epsilon_T^{ij} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}^2) \right. \\ \left. - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, \mathbf{k}^2) - \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_T}{2M^3} h_{1T}^\perp(x, \mathbf{k}^2) \right]$$

Gluon TMD correlator and T -odd gluon densities



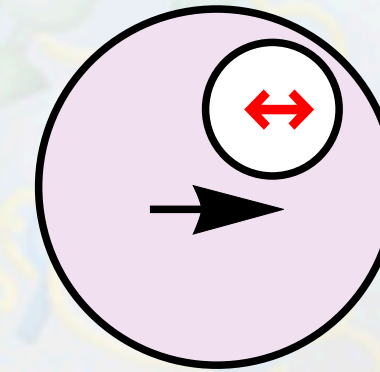
$$\Gamma_U^{ij}(x, \mathbf{k}) = x \left[\delta_T^{ij} f_1(x, \mathbf{k}^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, \mathbf{k}^2) \right]$$

$$\Gamma_L^{ij}(x, \mathbf{k}) = x \left[i\epsilon_T^{ij} S_L g_1(x, \mathbf{k}^2) + \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^\perp(x, \mathbf{k}^2) \right]$$

$$\Gamma_T^{ij}(x, \mathbf{k}) = x \left[\frac{\delta_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}^2) + \frac{i\epsilon_T^{ij} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}^2) \right.$$

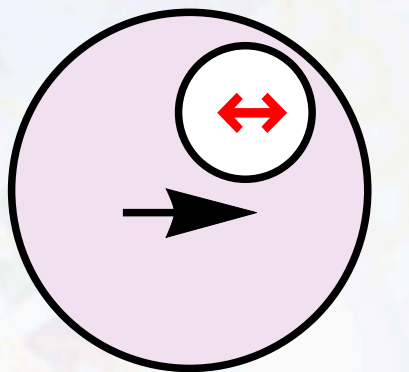
$$\left. - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, \mathbf{k}^2) - \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_T}{2M^3} h_{1T}^\perp(x, \mathbf{k}^2) \right]$$

pseudo worm-gear

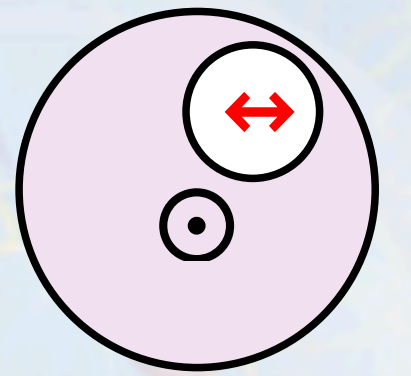


linearity TMD

pretzelosity



Gluon TMD correlator and T -odd gluon densities



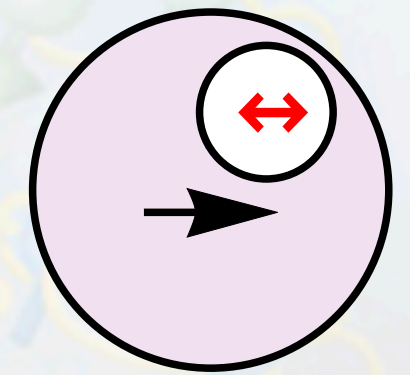
pseudo worm-gear

$$\Gamma_U^{ij}(x, k) = x \left[\delta_T^{ij} f_1(x, k^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, k^2) \right]$$

$$\Gamma_L^{ij}(x, k) = x \left[i\epsilon_T^{ij} S_L g_1(x, k^2) + \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_L}{2M^2} h_{1L}^\perp(x, k^2) \right]$$

$$\Gamma_T^{ij}(x, k) = x \left[\frac{\delta_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, k^2) + \frac{i\epsilon_T^{ij} \mathbf{k} \cdot \mathbf{S}_T}{M} g_{1T}(x, k^2) \right.$$

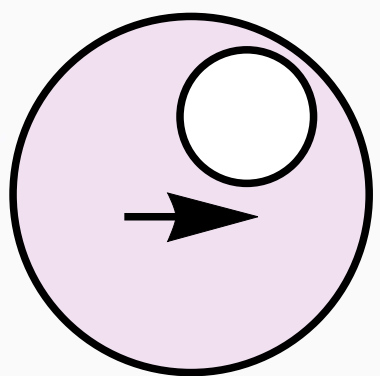
$$\left. - \frac{\epsilon_T^{k_T \{i} S_T^{j\}} + \epsilon_T^{S_T \{i} k_T^{j\}}}{4M} h_1(x, k^2) - \frac{\epsilon_T^{\{i} k_T^{j\}\alpha} S_T}{2M^3} h_{1T}^\perp(x, k^2) \right]$$



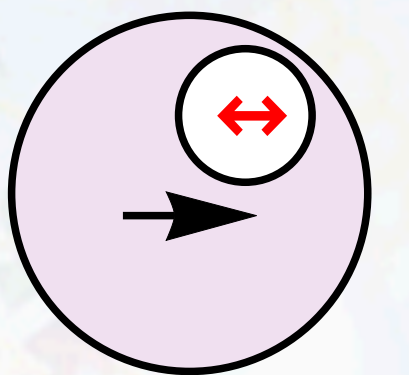
linearity TMD

pretzelosity

Sivers

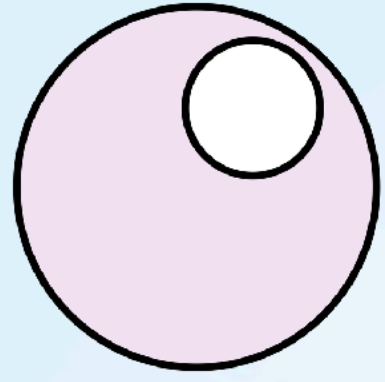


$$\frac{\epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, k^2) = \frac{1}{2} \delta_{Tij} \Gamma_T^{ij}(x, k)$$



Backup

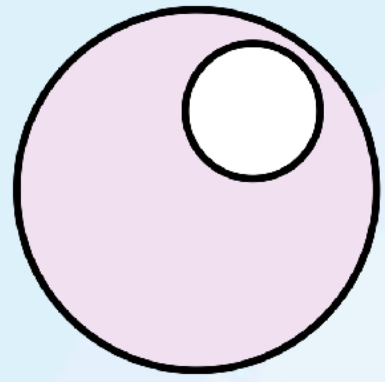
ρ -densities



Unpolarized [u/u]

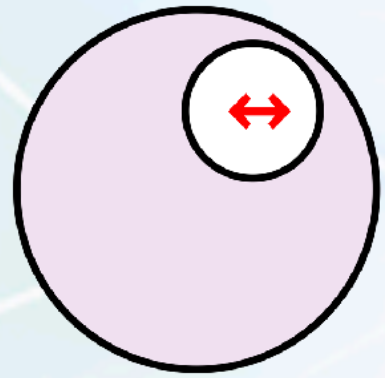
$$f_1(x, p_x, p_y)$$

ρ -densities



Unpolarized [u/u]

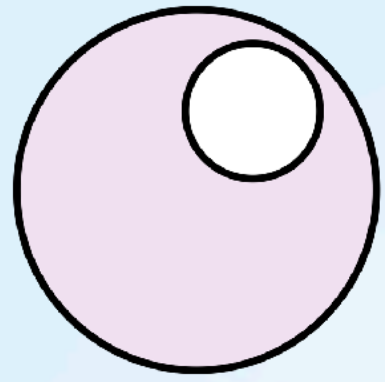
$$f_1(x, p_x, p_y)$$



Boer-Mulders [\leftrightarrow /u]

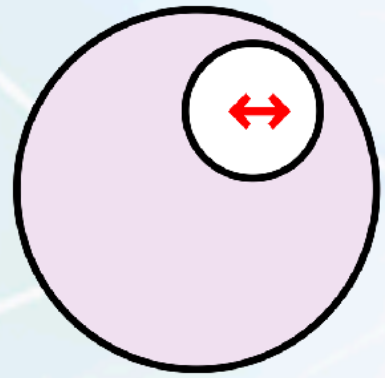
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$

ρ -densities



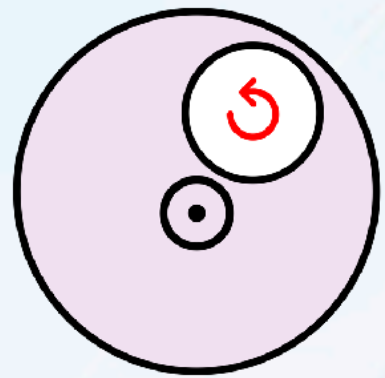
Unpolarized [u/u]

$$f_1(x, p_x, p_y)$$



Boer-Mulders [\leftrightarrow /u]

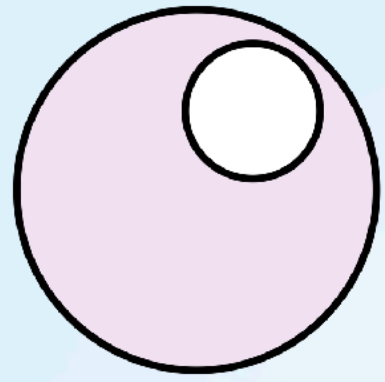
$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$



Helicity [\cup /+]

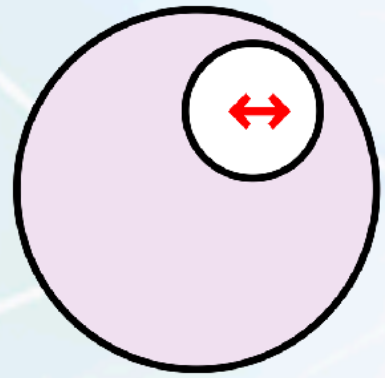
$$\frac{1}{2} \left[f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y) \right]$$

ρ -densities



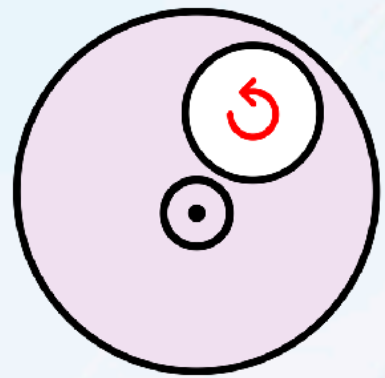
Unpolarized [u/u]

$$f_1(x, p_x, p_y)$$



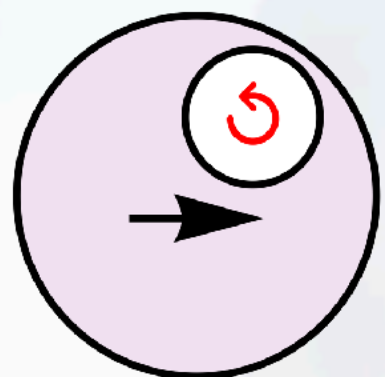
Boer-Mulders [\leftrightarrow /u]

$$f_1(x, p_x, p_y) + \frac{p_x^2 - p_y^2}{2M^2} h_1^\perp(x, p_x, p_y)$$



Helicity [\cup /+]

$$\frac{1}{2} \left[f_1(x, p_x, p_y) + g_{1L}(x, p_x, p_y) \right]$$



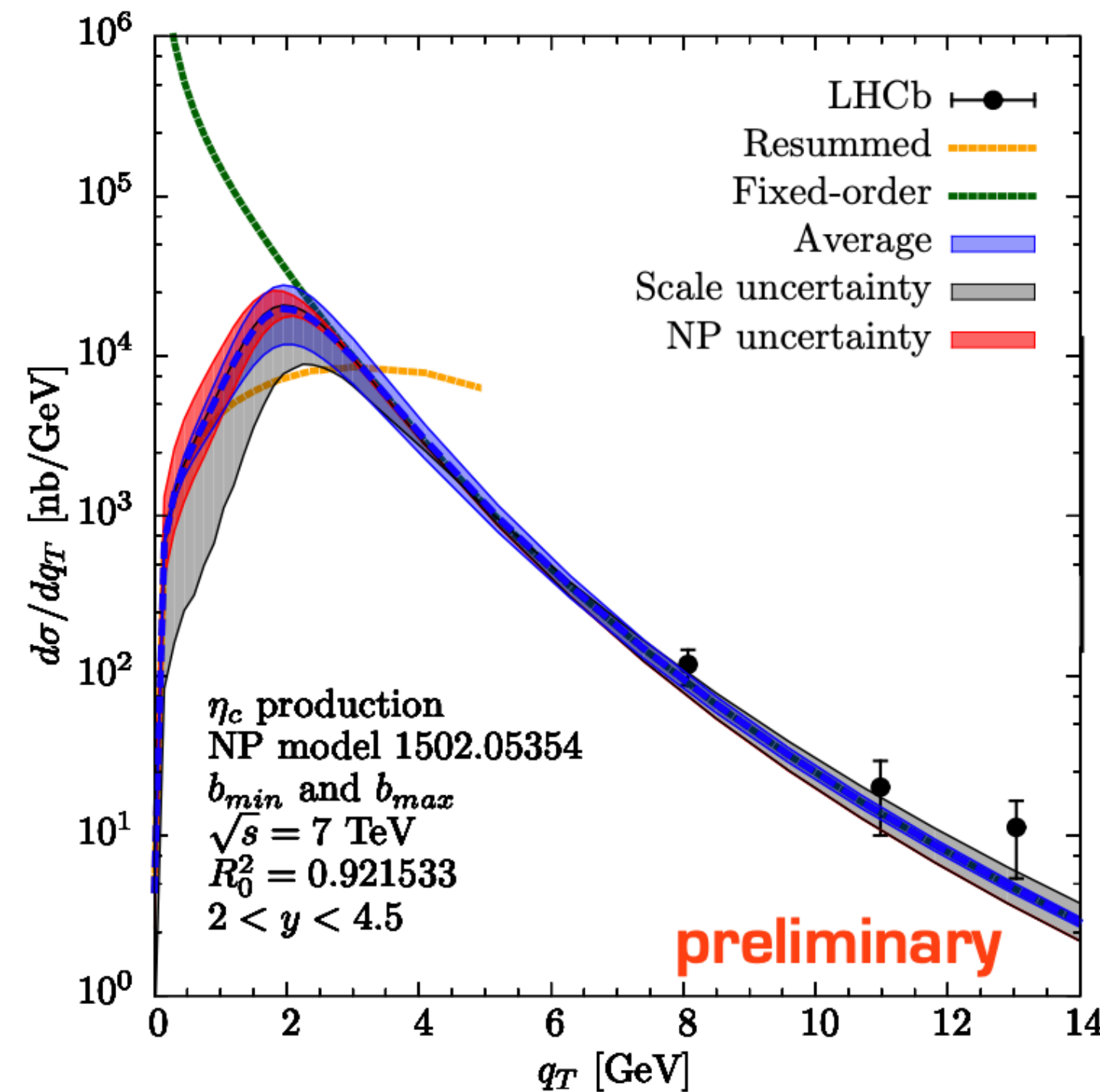
Worm-gear [\cup / \rightarrow]

$$f_1(x, p_x, p_y) - \frac{p_x}{M} g_{1T}(x, p_x, p_y)$$

η_c production @ 7TeV LHC

η_c production at LHC

full transverse momentum spectrum:
low q_T matched with high q_T region



blue band: uncertainty from matching

grey band: scale uncertainty

red band: nonpert. uncertainty

$$S_{NP}(\bar{b}_T) = - \left[\frac{a_1}{2} + \frac{a_2}{2} \ln Q^2 \right] \bar{b}_T^2$$

$a_i = 0.5 \text{ GeV}^2$, var. 50%, envelope

both for unpolarized and
linearly polarized distributions

the formalism is in good shape!
we need the data at low q_T

Jefferson Lab

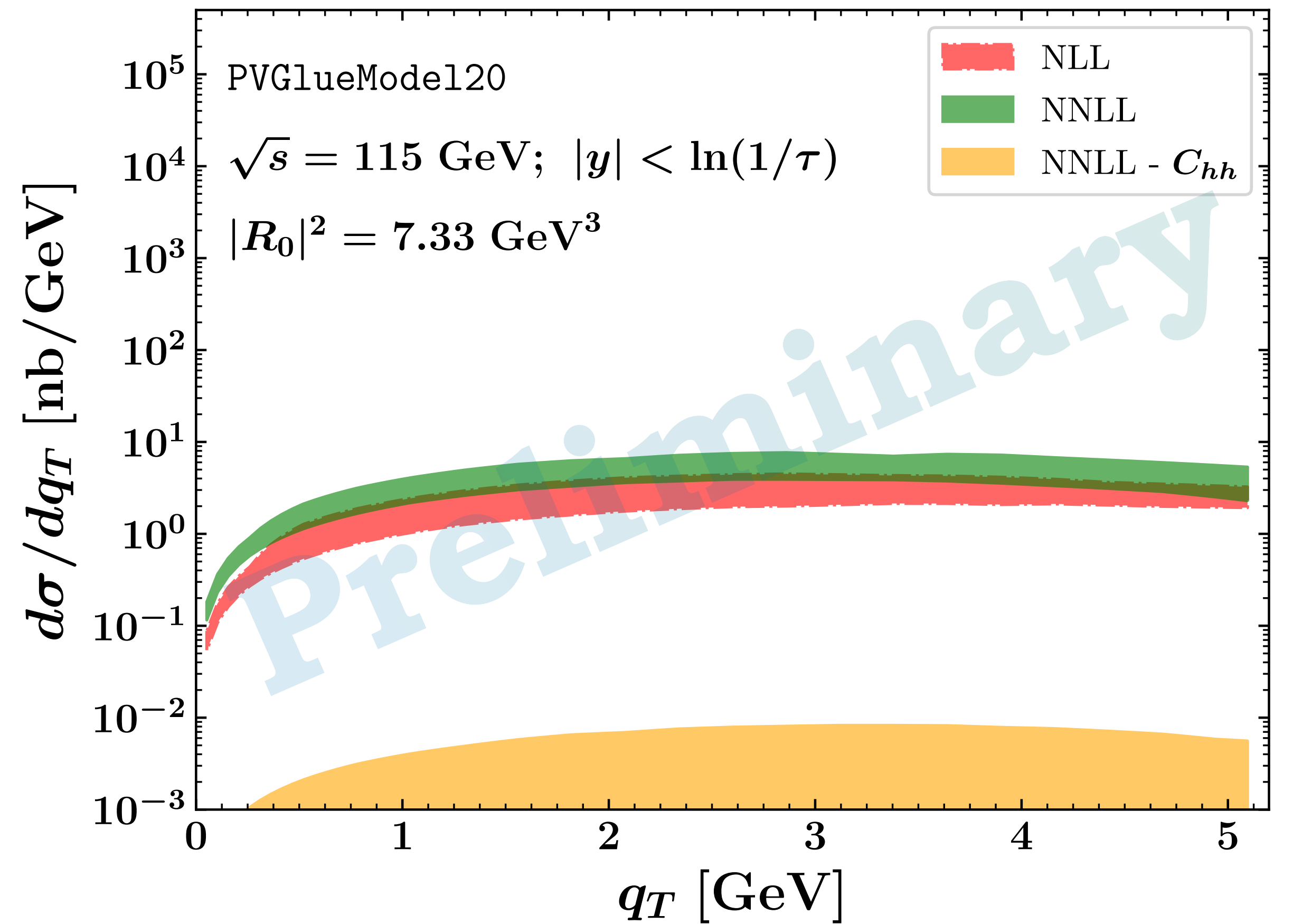
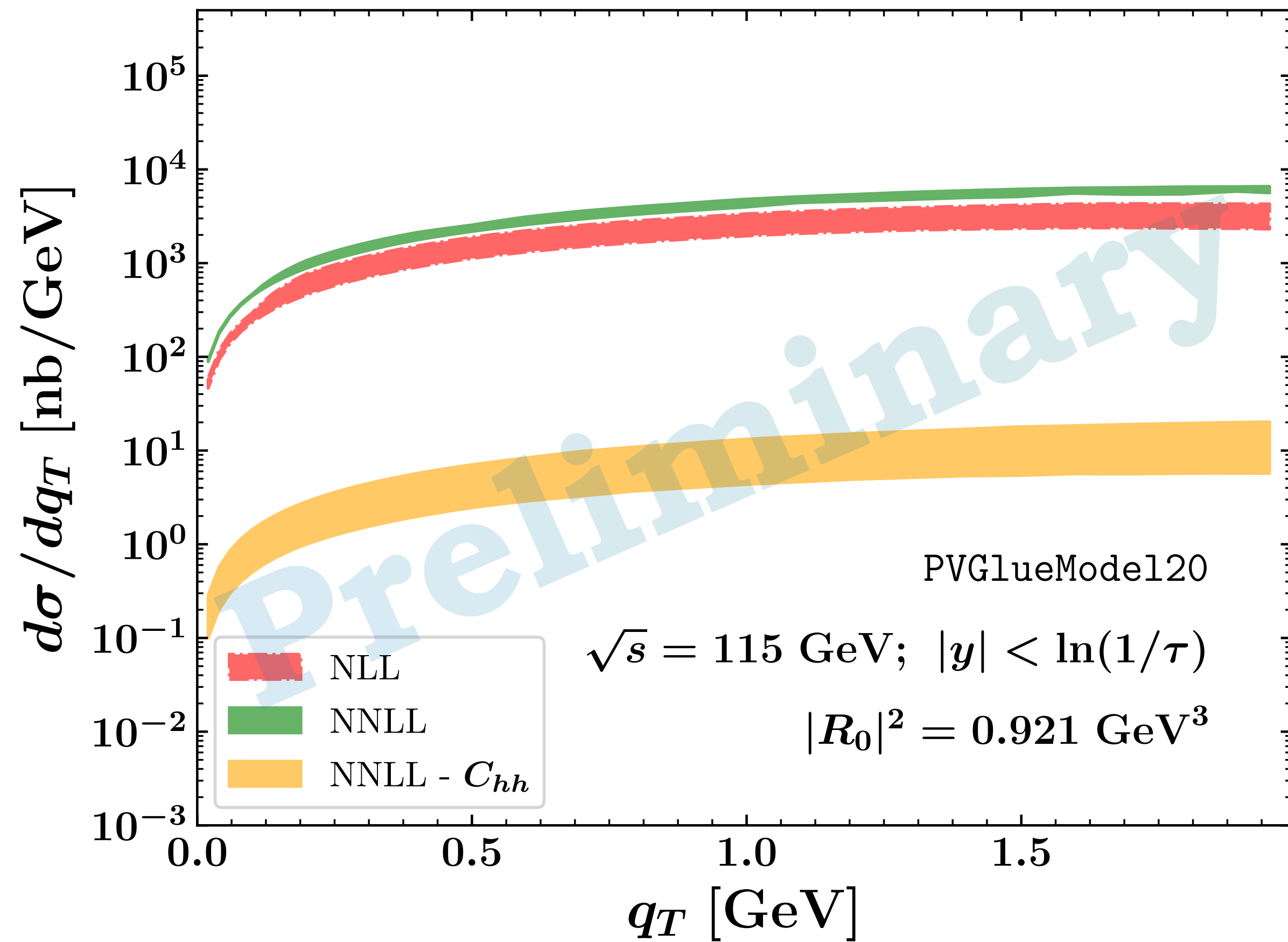
$\eta_{b,c}$ production @ 115 GeV After@LHC



Perturbative-scale variation, NP-evolution parameters fixed, TMD central replica

$$p(P_1) + p(P_2) \rightarrow \eta_c(q_T)$$

$$p(P_1) + p(P_2) \rightarrow \eta_b(q_T)$$



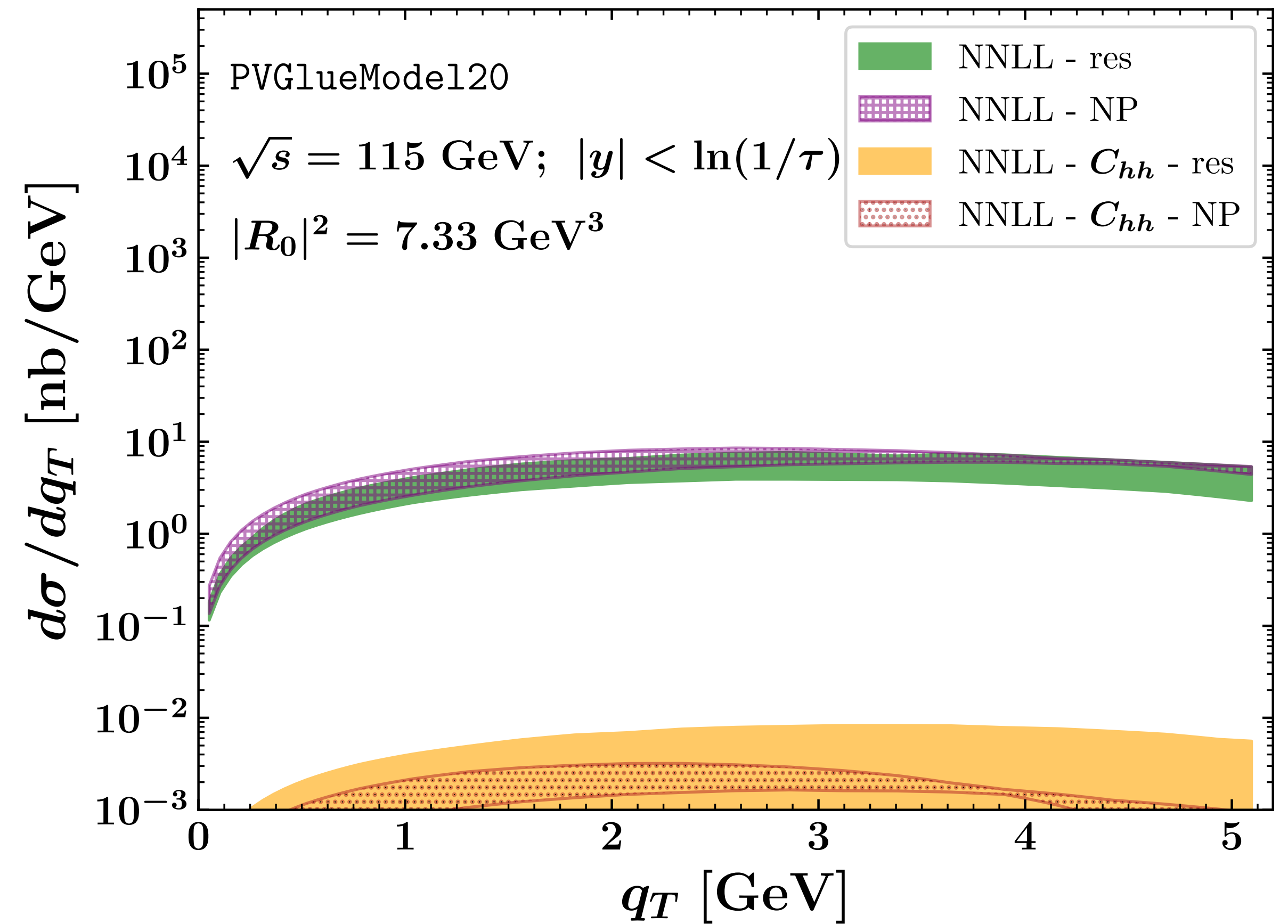
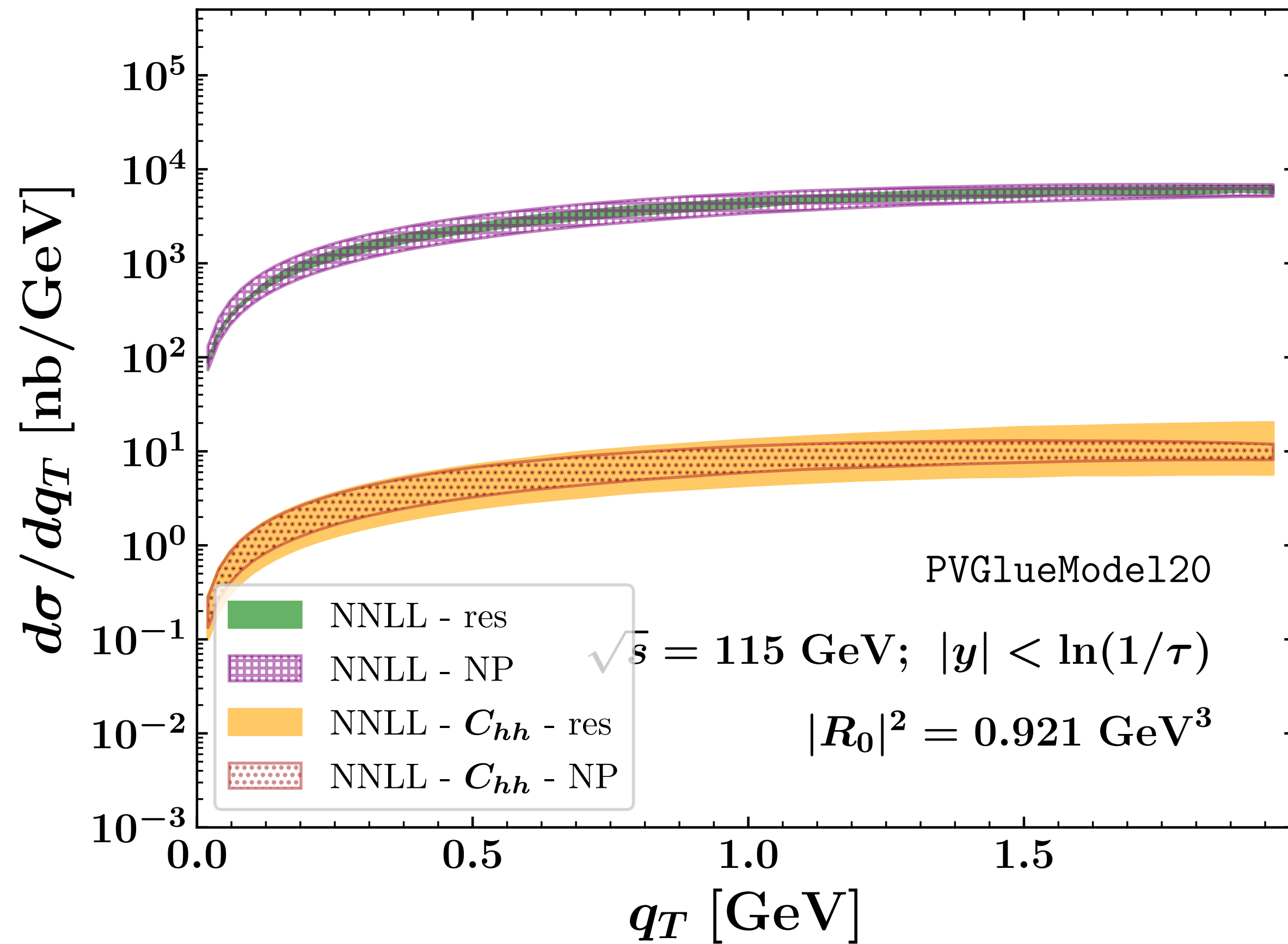
$\eta_{b,c}$ production @ 115 GeV After@LHC



Perturbative scales fixed, **NP-evolution parameter** variation, TMD central replica

$$p(P_1) + p(P_2) \rightarrow \eta_c(q_T)$$

$$p(P_1) + p(P_2) \rightarrow \eta_b(q_T)$$



[A. Bacchetta, F.G. C., J.-P. Lansberg, A. Signori (in preparation)]