Open quantum systems for quarkonium

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Quarkonia as Tools 2022

Collaborators:

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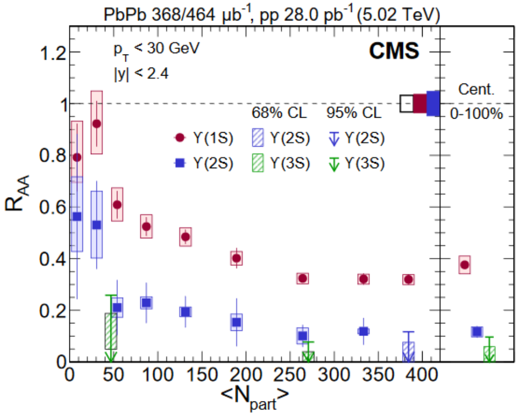
- Thierry Gousset
- Roland Katz
- Aoumeur Daddi Hammou

Quarkonium suppression

- Matsui & Satz (1986): Sequential suppression
- Quarkonium states have different binding energies $\Rightarrow$ Different dissociation temperatures
- Quarkonia viewed as thermometer

$$R_{AA} = \frac{N_{AA}}{\langle N_{\mathrm{coll}} \rangle N_{pp}}$$

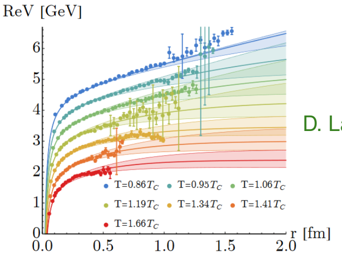


Quarkonium in heavy-ion collisions

Static screening

T $\neq$ 0 $\rightarrow$ Suppression of color attraction

Melting of pairs at high T
$\Rightarrow$ Suppression

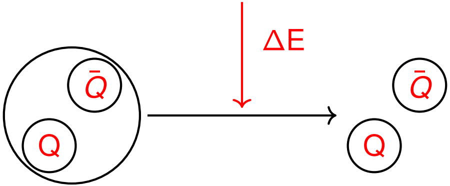


D. Lafferty, A. Rothkopf (2020)

Dynamical processes

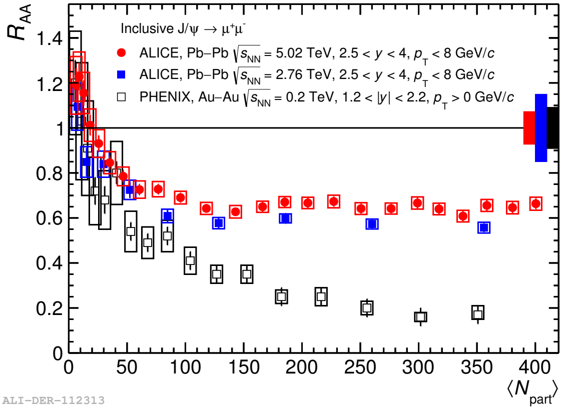
Collisions with medium partons
$\rightarrow$ Pair dissociation

$\Rightarrow$ Suppression



Often described by an imaginary potential

Quarkonium in heavy-ion collisions



Recombination

Initially uncorrelated heavy quarks form a quarkonium

Can happen below the dissociation temperature

Essential to have a formalism that can treat this effect

Theoretical models

- Statistical Recombination:
  - Quasi-stationnary medium
  - $Q\bar{Q}$ dissociated
  - Recombination at freeze-out
- Transport:
  - Semi-classical treatment of states
  - Recombination and dissociation included in reaction rates
- Open quantum systems:
  - Full quantum treatment of the dynamics
  - Treatment of multiple pairs still challenging

Lots of effort in the last decade using open quantum systems

Open quantum systems

System + environment

$$\rho(t) = \rho_{Q\bar{Q}} \otimes \rho_{QGP}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -i[H, \rho]$$

Evolution of the total system

$$\rho(t') = U(t', t)\left[\rho_{Q\bar{Q}} \otimes \rho_{QGP}\right] U(t', t)^{\dagger}$$

Trace out environment degrees of freedom

System ($Q\bar{Q}$ pair)

$$\rho_{Q\bar{Q}}(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}} = \mathcal{L}\rho_{Q\bar{Q}}$$

Evolution of the system

$$\rho_{Q\bar{Q}}(t') = Tr_{QGP}\left[U(t', t)\rho(t)U(t', t)^{\dagger}\right]$$

Lindblad equation

- Case of a Markovian time-evolution ⇒ Lindblad equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right] + \sum_i \gamma_i \left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2}\left\{L_i L_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

$H_{Q\bar{Q}}$: $Q\bar{Q}$ kinetics + screened potential V

L_i : Collapse operators (or dissipators), depend on the properties of the medium

$\langle n \vert \rho_{Q\bar{Q}} \vert n \rangle \geq 0 \; \forall \; n$	$\rho_{Q\bar{Q}}^\dagger = \rho_{Q\bar{Q}}$	$\mathrm{Tr}\left[\rho_{Q\bar{Q}}\right] = 1$
(Positivity)	(Hermiticity)	(Unitarity)

Two regimes

Quantum evolution of the system

$\tau_R \gg \tau_S$ → Quantum optical limit — X. Yao, T. Mehen...

$\tau_S \gg \tau_E$ → Quantum brownian motion — J.P. Blaizot, M.A. Escobedo; Y. Akamatsu, A. Rothkopf, M. Asakawa...; N. Brambilla, M.A. Escobedo, A. Vairo...*; S. D., T. Gousset, R. Katz, P.B. Gossiaux

τ_S System intrinsic timescale $\Delta E \times \tau_S \sim 1$

τ_E Environment correlation timescale $\langle O_E(t) O_E(0) \rangle \sim e^{-t/\tau_E}$

τ_R System relaxation timescale $\langle p \rangle \sim e^{-t/\tau_R}$

$\tau_R \gg \tau_E$: Markovian dynamics

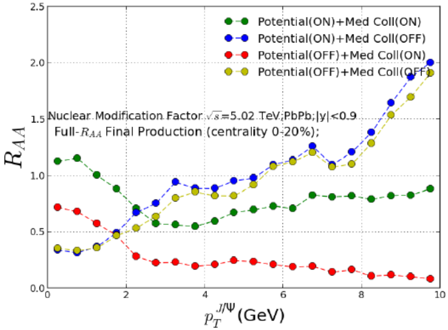
*See talk by Peter

Density operator model

- Based on Remler's model, describes the probability of being in a quarkonium state as function of time:
  $P^{\Phi}_{Q\bar{Q}}(t) = P^{\Phi}_{Q\bar{Q}}(t_0) + \int_{t_0}^{t} [\Gamma_{coll}(t') + \Gamma_{loc}(t')]\, dt'$
- Rates computed from Wigner distributions (obtained from the density operators)
- $Q\bar{Q}$ interaction potential taken from Rothkopf

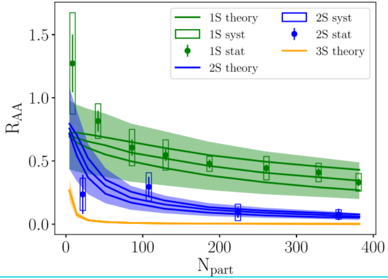
Denys Yen Arrebato Villar

Ph. D thesis

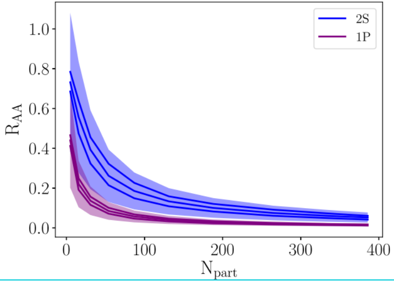


Duke approach

- pNRQCD derivation of the interaction Hamiltonian between the heavy quarks and the QGP particles X. Yao, T. Mehen (2019)
- Lindblad equation obtained in the quantum optical limit
- Wigner transform to obtain Boltzmann equations, including recombination and dissociation
  - Derivation of transport equations from first principles!



X. Yao et al. (2021)



Osaka/Stavanger approach

- Description of the interaction between the heavy quarks and the plasma particles (neglecting magnetic interactions) using NRQCD
- Lindblad equation for a single heavy quark-antiquark pair in the quantum brownian regime Y. Akamatsu (2015)
- 3 different strategies:
  - Turn the Lindblad equation into a Stochastic Schrödinger Equation (SSE) by using a stochastic potential Y. Akamatsu (2015)

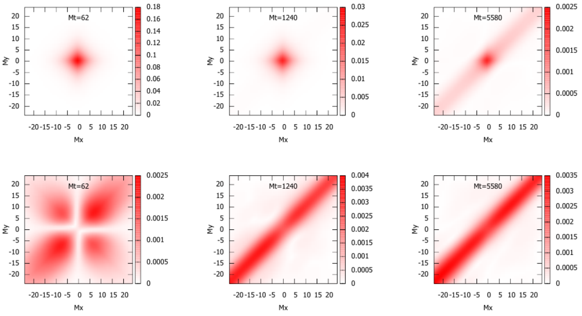
    1D, U(1): Y. Akamatsu, A. Rothkopf (2012); S. Kajimoto, Y. Akamatsu, M. Asakawa, A. Rothkopf (2018)

    3D, U(1): A. Rothkopf (2014) 1D, SU(3): Y. Akamatsu, M. Asakawa, S. Kajimoto (2021)
  - Use the Quantum State Diffusion method to obtain a SSE from the Lindblad equation

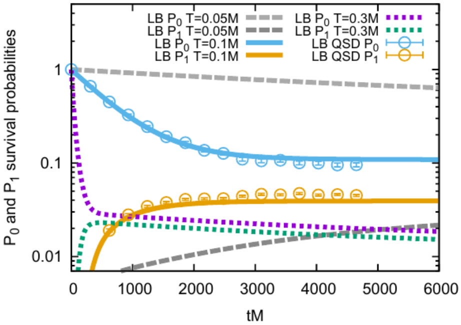
    1D, U(1): Y. Akamatsu, M. Asakawa, S. Kajimoto, A. Rothkopf (2018)

    T. Miura, Y. Akamatsu, M. Asakawa, A. Rothkopf (2020) 1D, SU(3): Y. Akamatsu, T. Miura (2021)
  - Directly resolve the Lindblad equation 1D, U(1): O. Ålund et al. (2021)

Osaka/Stavanger approach



Y. Akamatsu, T. Miura (2021)



O. Ålund et al. (2021)

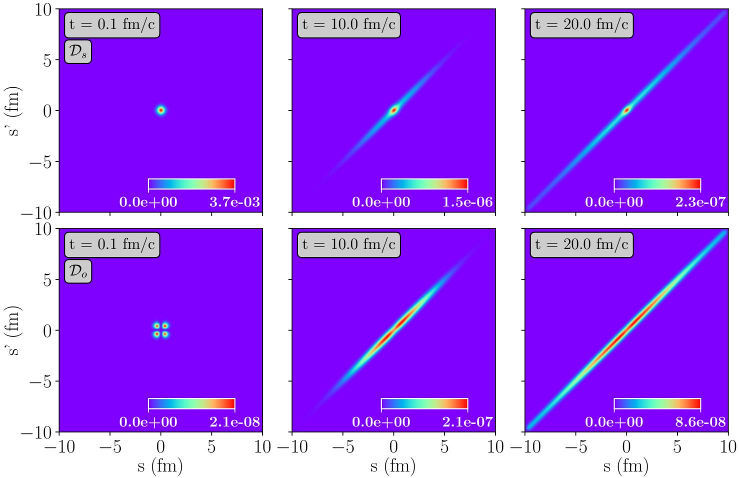
Saclay approach

- Heavy quarks-Plasma interaction also described using NRQCD
- Derivation of quantum master equations to describe the evolution of the density operator (not Lindblad equations)
- Semi-classical approximation to obtain Langevin equations
  $\Rightarrow$ Treatment of multiple $Q\bar{Q}$ pairs! U(1): J-P. Blaizot, D. De Boni, P. Faccioli, G. Garberoglio (2016)
  SU(3): J-P. Blaizot, M. Escobedo (2018)
- Extension in the Abelian case to satisfy properties of a Lindblad equation (direct resolution) D. De Boni (2017)

Nantes approach (Delorme, Gossiaux, Gousset, Katz)

- Extension of Saclay approach in the non-Abelian case (preservation of positivity)
- Direct resolution in 1D and application to charmonium system
- Study of validity of a semi-classical treatment
- New potential developped specifically for 1D studies
  R. Katz, S.D., P-B. Gossiaux (in preparation)

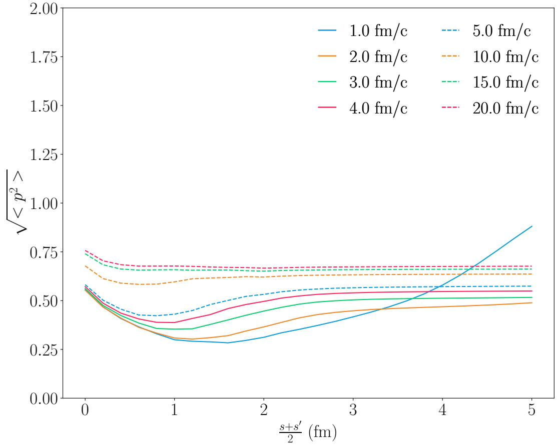
Singlet initial state



- Initial 1S-like singlet state
- QGP at T = 300 MeV (fixed)
- Octet populated as a dipole
- Delocalization of initial state along $s = s'$ axis

Compatible with a semi-classical treatment?

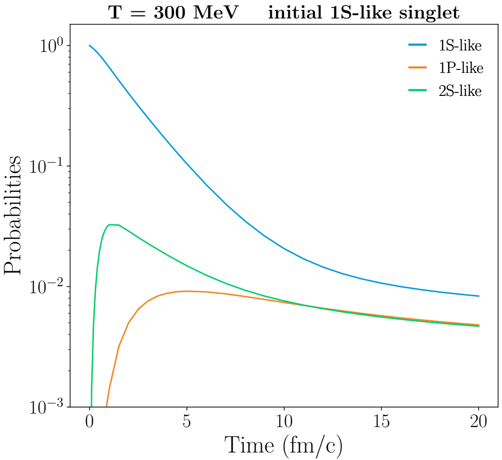
Singlet initial state



- Compute discretized Wigner transform of singlet density operator $f(\frac{s+s'}{2}, p)$
- For fixed values of $\frac{s+s'}{2}$, compute $\sqrt{\langle p^2 \rangle}$
- Convergence towards an asymptotic value for most of the range in $\frac{s+s'}{2}$
- Still a range where it is not the case: effect of the binding potential

Preliminary study, analysis under way

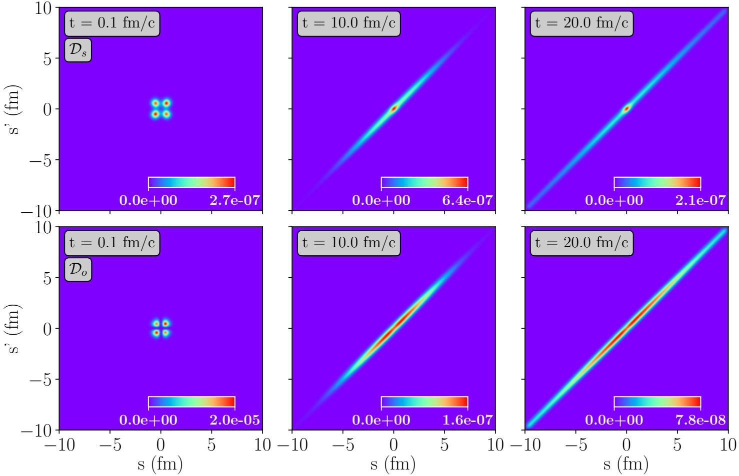
Singlet initial state



- Instantaneous projections on vacuum eigenstates
- 2S-like state first populated from 1S-like then population of 1P-like (different types of transitions)
- Decay phase afterwards, with same decay rate for all states

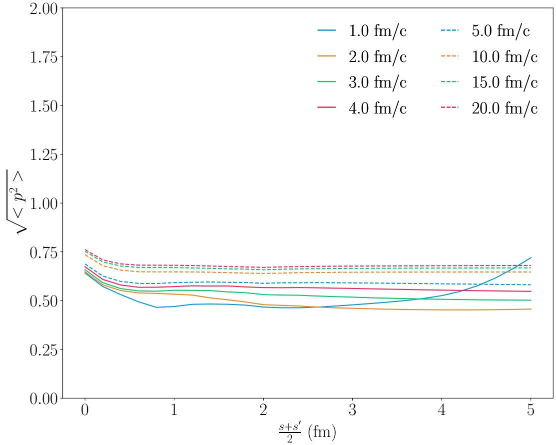
What happens if we start from a more realistic initial state?

Octet initial state



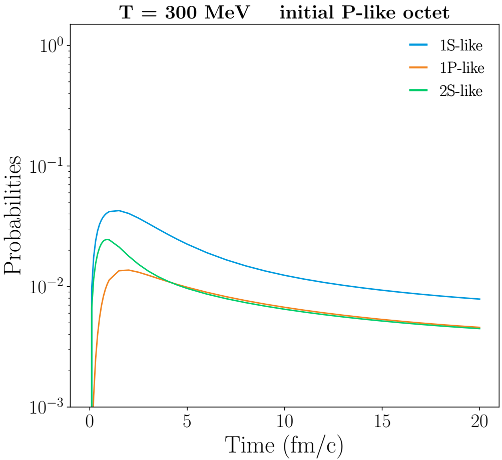
- Initial P-like octet state
- QGP at T = 300 MeV (fixed)
- Singlet populated via dipolar transitions
- Delocalization of initial state along $s = s'$ axis
- System seems to reach the same limit

Octet initial state



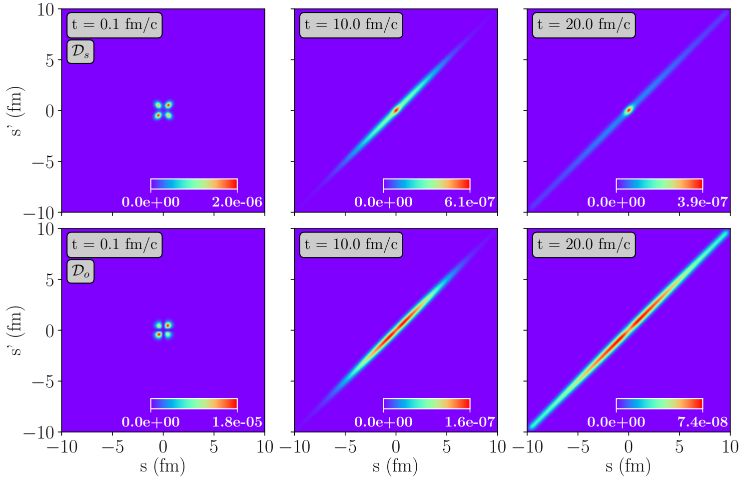
- Again convergence for most of the range
- Smaller range of effect of the potential but still plays a role

Octet initial state



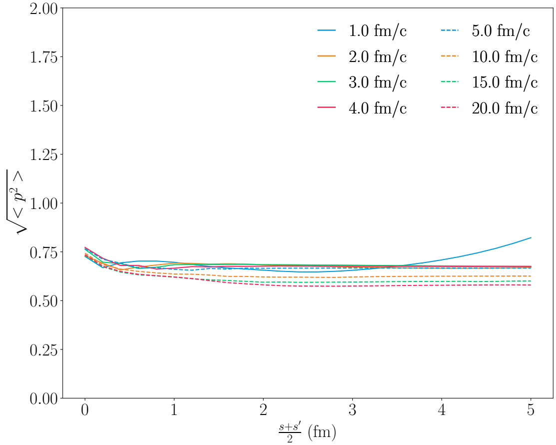
- Formation of bound states at early times
- 2S-like state dominant very early then 1S-like
- 1P-like state populated earlier (easier from P-like octet)
- Decay phase afterwards, with same decay rate for all states
- Same kind of late-time limit

Cooling medium



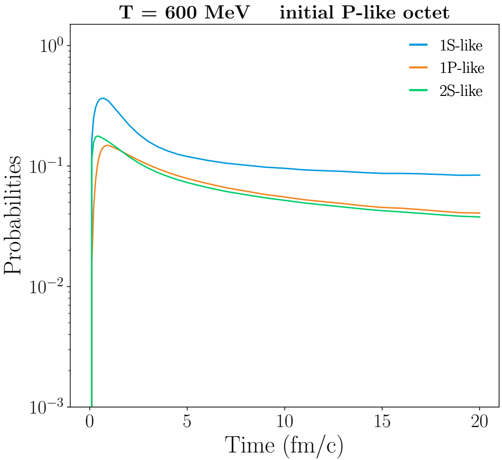
- Cooling medium
- $T(t) = T_0 \left(\frac{1}{1+t}\right)^{1/3}$
- $T_0 = 600$ MeV
- Delocalization of initial state along $s = s'$ axis
- Slightly different limit, closer to 1S-like state

Cooling medium



- Decreasing asymptotic value (cooling effect)
- Still a region where the potential plays a role, even at late times

Cooling medium



- Global evolution similar to the fixed temperature case for excited states
- Repopulation of ground state at large times

Overwiew of theoretical approaches

- Lindblad based approaches:

Approach	1D/3D	Color	Dissipation	Method	System
Munich	3D	yes	no	Quantum Jump	$b\bar{b}$
	1D	yes	no	Direct resolution	$b\bar{b}$
Osaka	both	yes	yes	Stochastic potential	$b\bar{b}$
	1D	both	yes	Quantum State Diffusion	$b\bar{b}$
	1D	no	yes	Direct resolution	$b\bar{b}$
Saclay	3D	yes	no	Semi-classical treatment	$c\bar{c}/b\bar{b}$
Nantes	1D	yes	yes	Direct resolution	$c\bar{c}$

- Other approaches:
  - Yao et al.: pNRQCD based Boltzmann equations (quantum optical regime)
  - Arrebato et al.: Density operator approach based on Remler's formalism

Conclusions, perspectives and challenges

- Many approaches using open quantum systems
- Each with their own strategies and focus
- Results presented compatible with other approaches
- Preliminary analysis of a semi-classical treatment

- Semi-classical analysis to complete
- Application to bottomonium system
- Modify initial conditions (quarks initially separated)

- Treatment of multiple $Q\bar{Q}$ pairs
- More realistic treatment of the medium
- How to treat the transition between quantum brownian regime (high temperature) and quantum optical regime (low temperature)?