

# DPS as MPI in ep/pA collisions

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**in collaboration with**

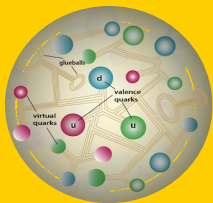
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Istituto Nazionale di Fisica Nucleare





# Outline

INTRODUCTION

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THE DATA

3

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NUCLEUS  
INTERACTIONS

5

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DISTRIBUTION  
FUNCTION

2

DPS VIA  
PHOTON-PROTON  
INTERACTIONS

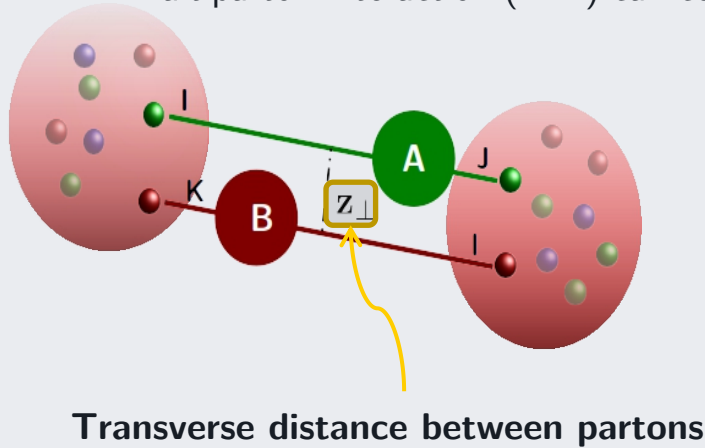
4

CONCLUSIONS

6

# 1 Double Parton Scattering

Multiparton interaction (MPI) can contribute to the, pp and pA, cross section @ the LHC:



The cross section for a double parton scattering (DPS) event can be written in the following way:

N. Paver, D. Treleani, *Nuovo Cimento* 70A, 215 (1982)  
 Mekhfi, *PRD* 32 (1985) 2371  
 M. Diehl et al, *JHEP* 03 (2012) 089

double PDF (dPDF)

$$d\sigma \propto \int d^2z_{\perp} \underbrace{F_{ik}(x_1, x_2, \vec{z}_{\perp}; \mu_A, \mu_B)}_{\text{double PDF (dPDF)}} \cdot \underbrace{F_{jl}(x_3, x_4, \vec{z}_{\perp}; \mu_A, \mu_B)}_{\text{double PDF (dPDF)}}$$

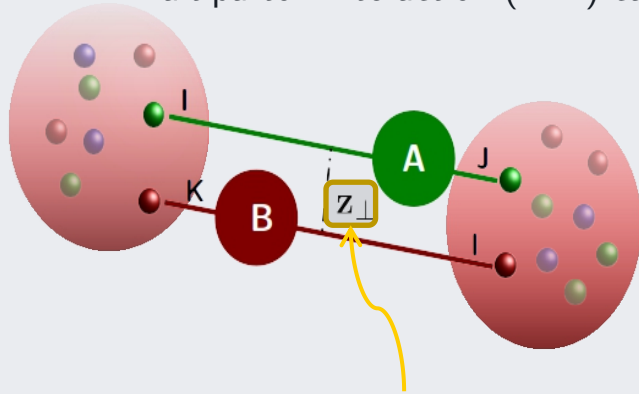
Momentum scales

Momentum fractions carried by the parton inside the proton

DPS processes are important for fundamental studies, e.g. the background for the research of new physics and to grasp information on the **3D PARTONIC STRUCTURE OF THE PROTON**

# 1 Double Parton Scattering

Multiparton interaction (MPI) can contribute to the, pp and pA, cross section @ the LHC:



Transverse distance between partons

The cross section for a double parton scattering (DPS) event can be written in the following way:

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double PDF (dPDF)

$$d\sigma \propto \int d^2z_{\perp} \overbrace{F_{ik}(x_1, x_2, \vec{z}_{\perp}; \mu_A, \mu_B) \cdot F_{jl}(x_3, x_4, \vec{z}_{\perp}; \mu_A, \mu_B)}$$

A **formal all-order proof** of the factorization formulae in perturbative QCD **has been achieved for DPS** in the case of a **colorless final state**, both for the TMD and the collinear case. Current status is at the **same level as for the SPS** counterpart.

Nagar's slides MPI 2021

Diehl et al. *JHEP* 03 (2012) 089, *JHEP* 01 (2016) 076

Vladimirov *JHEP* 04 (2018) 045

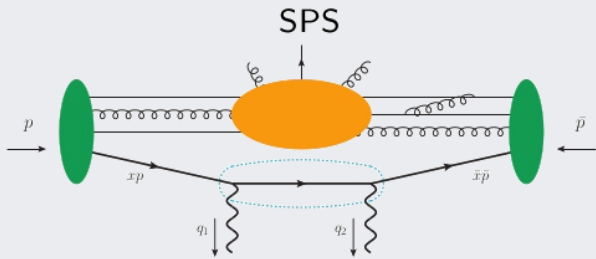
Buffing et al. *JHEP* 01 (2018) 044

Diehl, RN *JHEP* 04 (2019) 124

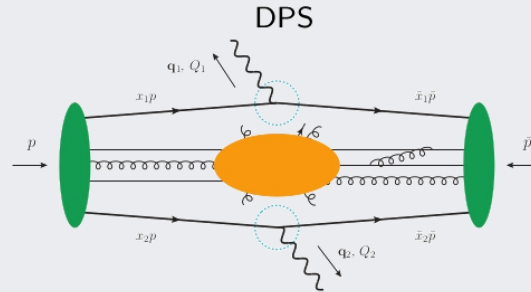


# 1 Double Parton Scattering

Scale analysis of SPS and DPS processes



$$|\mathbf{q}_1^\perp + \mathbf{q}_2^\perp| \sim \Lambda \ll Q$$



$$|\mathbf{q}_1^\perp| \sim \Lambda \ll Q$$

$$|\mathbf{q}_2^\perp| \sim \Lambda \ll Q$$

where:

- $Q = \min(Q_1, Q_2)$
- $\Lambda$  transverse momentum scale
- $\Lambda_{QCD} \ll \Lambda \ll Q$

Usually:

$$\frac{\sigma_{DPS}}{\sigma_{SPS}} \sim \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

$$\frac{d^2\sigma_{SPS}}{d^2q_1 d^2q_2} \sim \frac{d^2\sigma_{DPS}}{d^2q_1 d^2q_2}$$

First appearance in theory studies:

Politzer *Nucl. Phys.* B172 (1980) 349

Paver, Treleani *Nuovo Cim.* A70 (1982) 215

Mekhfi *Phys. Rev.* D32 (1985) 2371

Other ground-setting works:

Gaunt, Stirling *JHEP* 03 (2010) 005

Blok et al. *Eur. Phys. J.* C72 (2012) 1963

Diehl et al. *JHEP* 03 (2012) 089

Manohar, Waalewijn *Phys. Rev.* D85 (2012) 114009

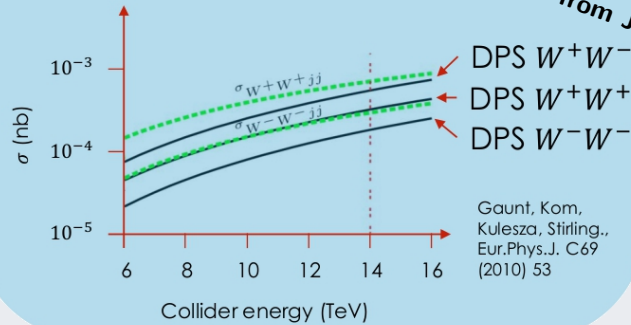
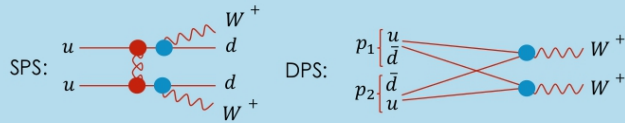
Ryskin, Snigirev *Phys. Rev.* D86 (2012) 014018

...

Nagar's slides MPI 2021

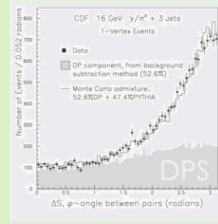
# 2 where and why DPS?

DPS can give a significant contribution to processes where SPS is suppressed by small/multiple coupling constants:



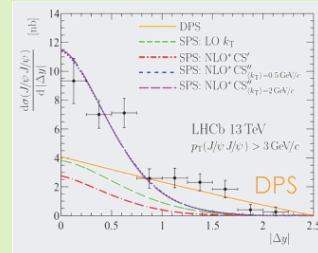
Intrinsically interesting: tells us about **correlations** between partons!

...or in certain phase space regions

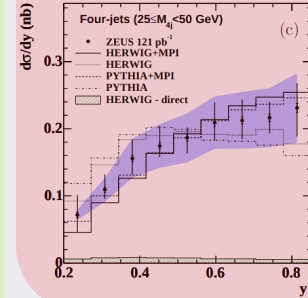


CDF,  $\gamma + 3j$ , Phys.Rev. D56 (1997) 3811-3832

LHCb, double  $J/\psi$ , JHEP 06, 047, (2017)



..or in ep colliders!



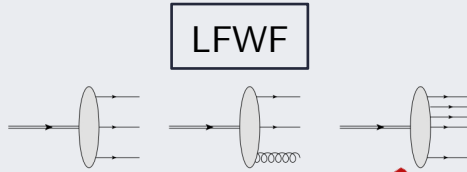
Access to:

- double parton correlations
- the transverse distance distribution of partons!!

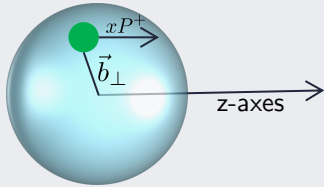
all **UNKNOWN**

# 3 Multidimensional Pictures of Hadron

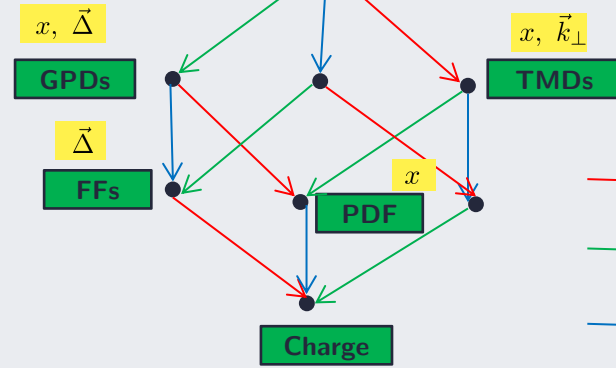
1-body



GPDs

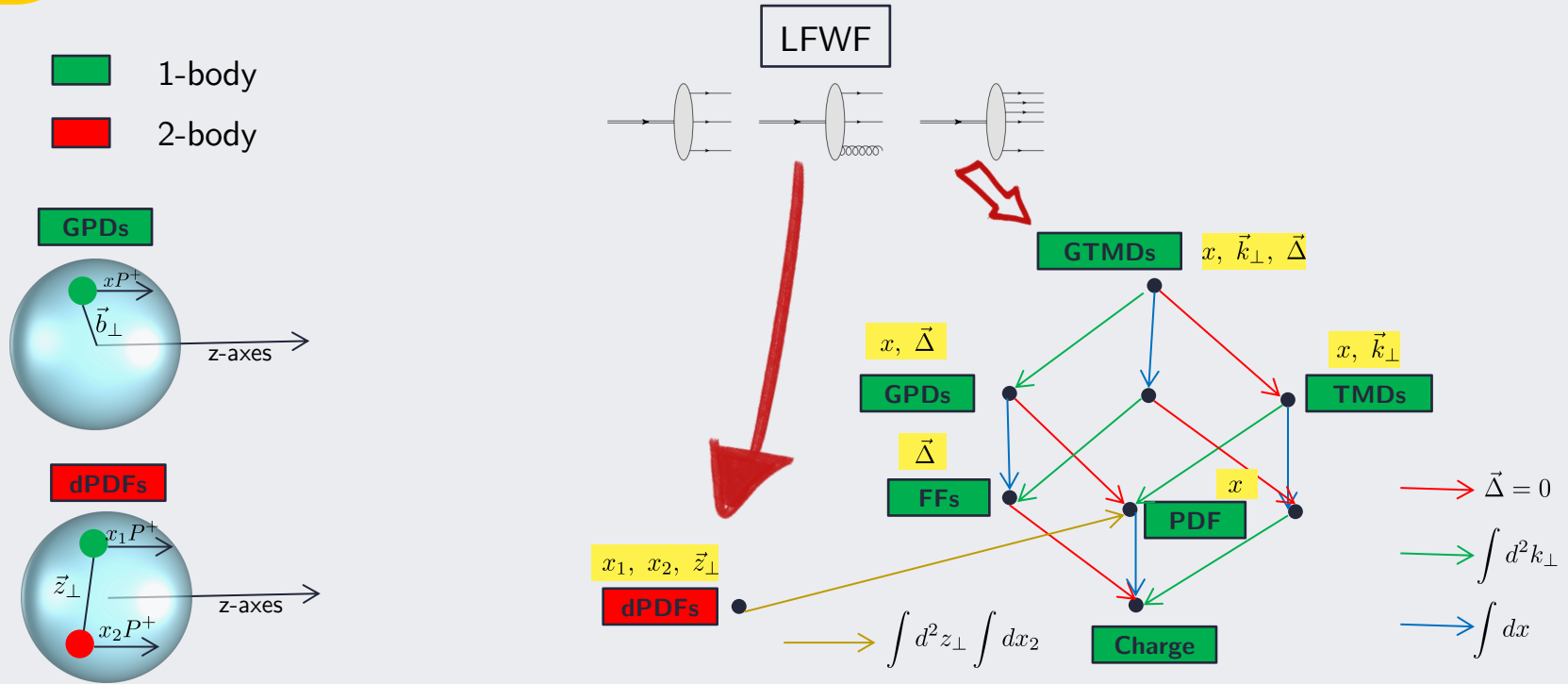


GTMDs  $x, \vec{k}_\perp, \vec{\Delta}$

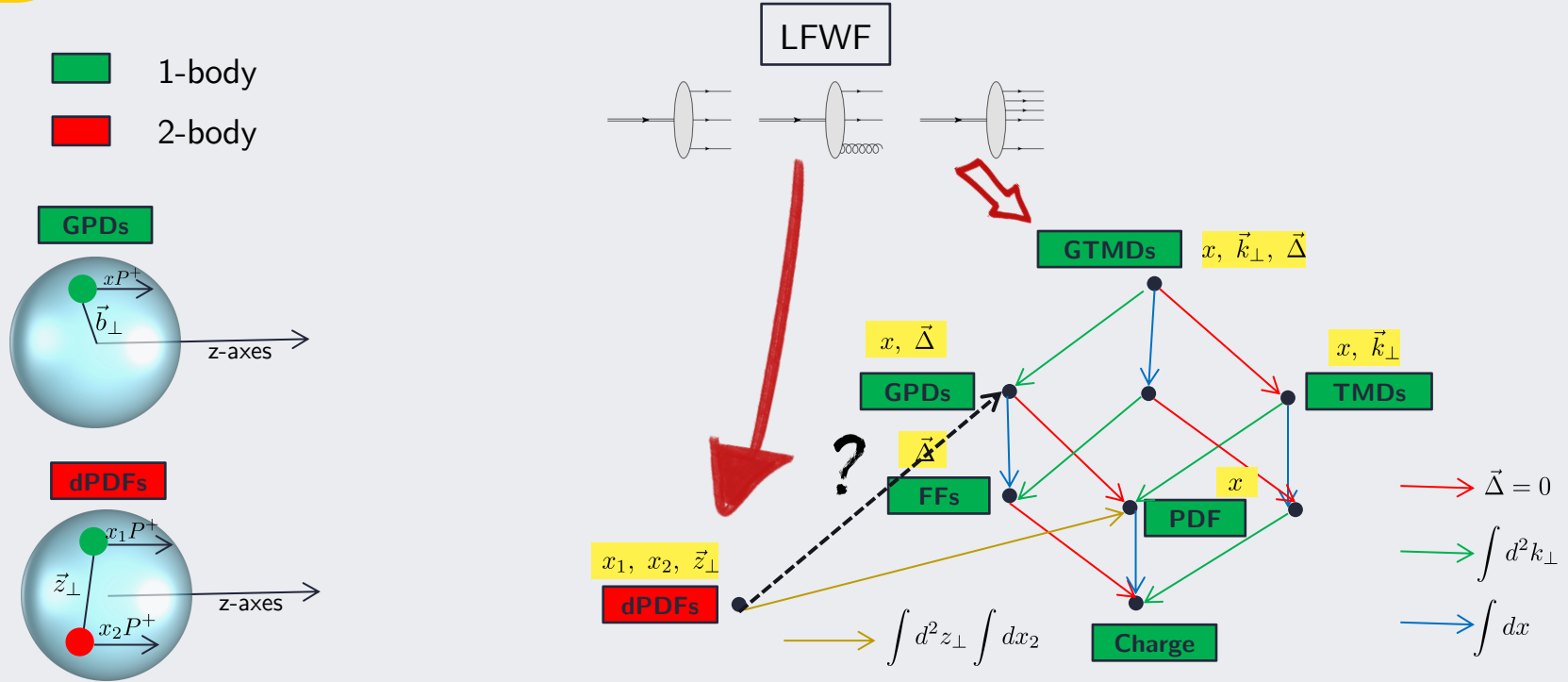


$\vec{\Delta} = 0$   
 $\int d^2 k_\perp$   
 $\int dx$

# 3 Multidimensional Pictures of Hadron



# 3 Multidimensional Pictures of Hadron



## 4 A link to GPDs?

The **dPDF** is formally defined through the Light-cone correlator:

$$F_{12}(x_1, x_2, \vec{z}_\perp) \propto \left( \sum_X \right) \int dz^- \left[ \prod_{i=1}^2 dl_i^- e^{ix_i l_i^- p^+} \right] \langle p | O(z, l_1) | X \rangle \langle X | O(0, l_2) | p \rangle \Big|_{\substack{\vec{l}_{1\perp} = \vec{l}_{2\perp} = 0 \\ l_1^+ = l_2^+ = z^+ = 0}}$$

Approximated by the proton state!

$$\int \frac{dp'^+ d\vec{p}'_\perp}{p'^+} |p'\rangle \langle p'|$$

$$F_{12}(x_1, x_2, \vec{k}_\perp) \sim f(x_1, 0, \vec{k}_\perp) f(x_2, 0, \vec{k}_\perp)$$

GPDs

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Approximated by the proton state!

$$\int \frac{dp'^+ d\vec{p}'_\perp}{p'^+} |p'\rangle \langle p'|$$

$$F_{qq}(x_1, x_2, \vec{k}_\perp, Q^2) \approx H^q(x_1, \xi = 0, -k_\perp^2, Q^2) H^q(x_2, \xi = 0, -k_\perp^2, Q^2) + \frac{k_\perp^2}{4M_p^2} E^q(x_1, \xi = 0, -k_\perp^2, Q^2) E^q(x_2, \xi = 0, -k_\perp^2, Q^2)$$

M. R., et al, JHEP 10, 063 (2016)

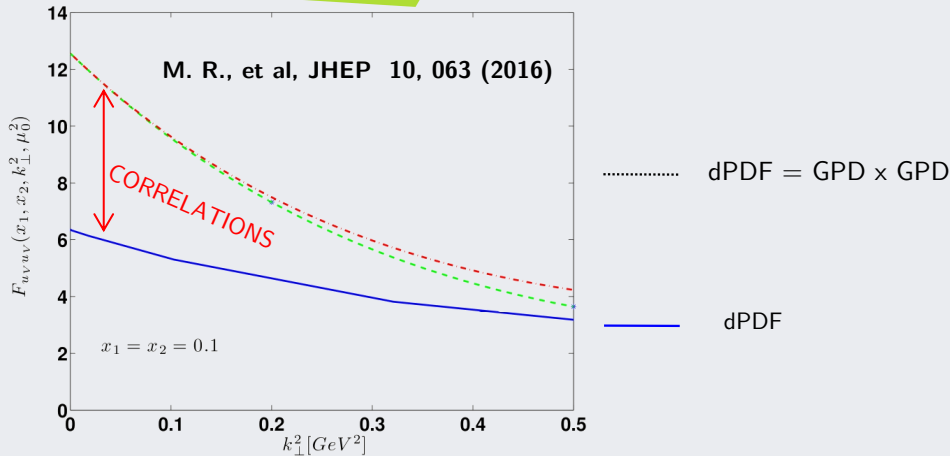
M. Diehl et al, JHEP 03, 089 (2012)

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M. R., et al, JHEP 10, 063 (2016)

M. Diehl et al, JHEP 03, 089 (2012)



Violation due to:

- ✓ Correlations between  $(x_1, x_2)$ 
  - M. R., et al, PRD 87, 114021 (2013)
  - M. R., et al, JHEP 12, 028 (2014)
  - M. R., et al, JHEP 10, 063 (2016)
- ✓ Correlations between  $(x_1, x_2) - k_\perp$ 
  - M. R., et al, PRD 87, 114021 (2013)
  - M. R., et al, JHEP 12, 028 (2014)
  - M. R., and F.A. Ceccoperi, JHEP 09, 097 (2019)
  - M. R., and F.A. Ceccoperi, PRD 95, 034040 (2017)

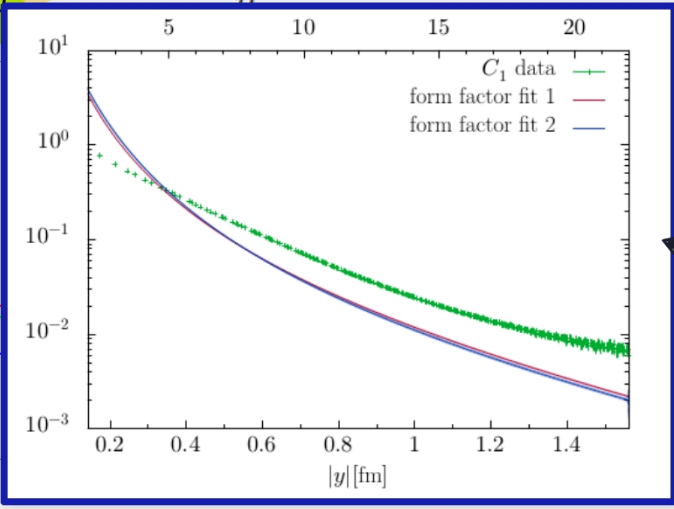
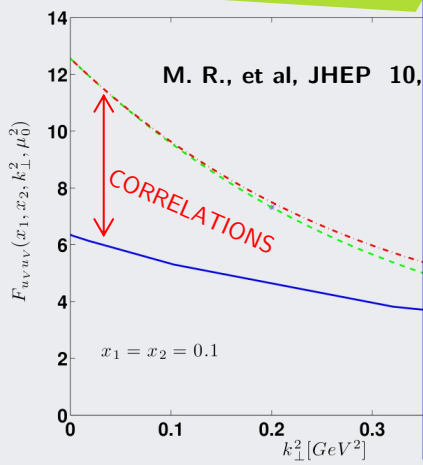


# 4 A link to GPDs?

$$F_{qq}(x_1, x_2, \vec{k}_\perp, Q^2) \approx H^q(x_1, \xi = 0, -k_\perp^2, Q^2)H^q(x_2, \xi = 0, -k_\perp^2, Q^2) + \frac{k_\perp^2}{4M_p^2} E^q(x_1, \xi = 0, -k_\perp^2, Q^2)E^q(x_2, \xi = 0, -k_\perp^2, Q^2)$$

M. R., et al, JHEP 10, 063 (2016)

M. Diehl et al, JHEP 03, 089 (2012)



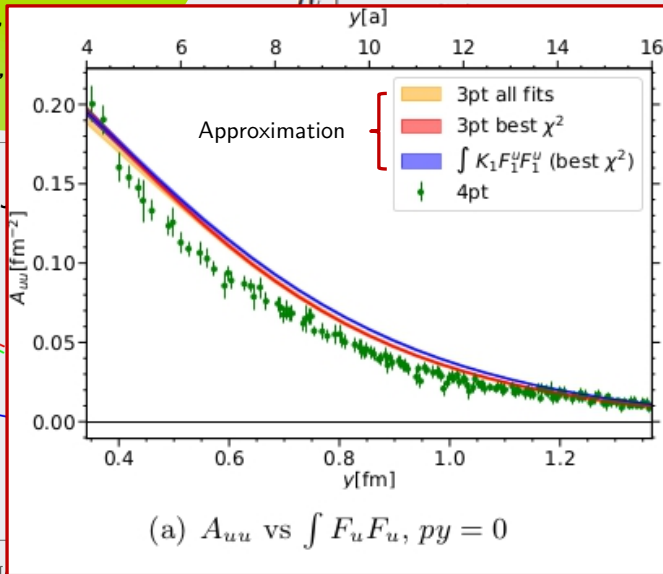
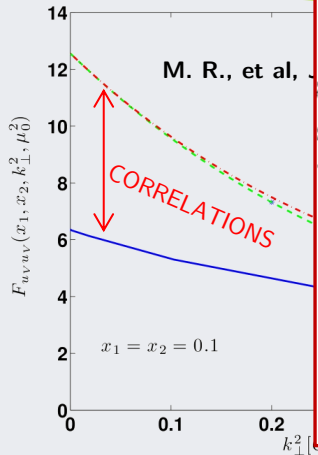
Similar violation observed in lattice analysis for the pion.  
G. S. Bali et al, JHEP 12 (2018) 061

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$$F_{qq}(x_1, x_2, \vec{k}_\perp, Q^2) \approx H^q(x_1, \xi = 0, -k_\perp^2, Q^2)H^q(x_2, \xi = 0, -k_\perp^2, Q^2) +$$

$$-k_\perp^2 E^q(x_2, \xi = 0, -k_\perp^2, Q^2)$$

M. R., et al, JHEP 10,  
M. Diehl et al, JHEP 03,



Similar violation observed in  
lattice analysis

G. S. Bali et al, JHEP 09 (2021) 121

## 4 A link to GPDs? What can we learn?

$$F_{qq}(x_1, x_2, \vec{k}_\perp, Q^2) \approx H^q(x_1, \xi = 0, -k_\perp^2, Q^2)H^q(x_2, \xi = 0, -k_\perp^2, Q^2) + \frac{k_\perp^2}{4M_p^2} E^q(x_1, \xi = 0, -k_\perp^2, Q^2)E^q(x_2, \xi = 0, -k_\perp^2, Q^2)$$

M. R., et al, JHEP 10, 063 (2016)

M. Diehl et al, JHEP 03, 089 (2012)

Since data on GPDs suggest that there is no factorization between  $(x_1, x_2)$  and  $k_\perp$  we can think that also in dPDFs such an approximation does not hold.

$$F_{12}(x_1, x_2, \vec{z}_\perp) \propto \sum_X \int dz^- \left[ \prod_{i=1}^2 dl_i^- e^{ix_i l_i^- p^+} \right] \langle p | O(z, l_1) | X \rangle \langle X | O(0, l_2) | p \rangle \Big|_{l_1^+ = l_2^+ = z^+ = 0}^{l_{1\perp} = l_{2\perp} = 0}$$

We can build the proton eff, for example:

$$T(k_\perp) = (1 + k_\perp^2/m_g^2)^{-4}$$

i.e., the square of the gluon form factor  
**B. Blok et al, EPJC 74, 2926 (2014)**

# 5 Relativistic effects: the breaking of factorization

Usually the intrinsic dPDF, at a given scale, are phenomenological assumed to be:

$$\underbrace{F_{ij}(x_1, x_2, k_\perp; Q^2)}_{\text{pheno}} = \underbrace{q_i(x_1; Q^2)q_j(x_2; Q^2)}_{\substack{\text{phenomenology from} \\ \text{PDFs}}} \overbrace{\theta(1 - x_1 - x_2)}^{\text{good support}} \underbrace{f(x_1, x_2, Q^2)}_{\text{sum rules}} \mathbf{F}(k_\perp) \left. \vphantom{F_{ij}} \right\} \begin{array}{l} \text{To be} \\ \text{modeled:} \\ \text{CQMs, GPDs...} \end{array}$$

Then pQCD can also be applied!

# 5 Relativistic effects: the breaking of factorization

- ⊙ Almost model independence
- ⊙ Almost scale independence

**SUGGEST:** *parameterize the impact of Melosh effects in dPDFs to encode **some** general correlations between  $x_i$  and  $k_\perp$*

$$\underbrace{F_{ij}(x_1, x_2, k_\perp; Q^2)}_{\text{pheno}} = \underbrace{q_i(x_1; Q^2)q_j(x_2; Q^2)}_{\substack{\text{phenomenology from} \\ \text{PDFs}}} \underbrace{\theta(1 - x_1 - x_2)}_{\text{good support}} \underbrace{f(x_1, x_2, Q^2)}_{\text{sum rules}} \underbrace{R(x_1, x_2, k_\perp)}_{\substack{\text{Melosh effects} \\ \text{correlations!}}} F(k_\perp) \left. \vphantom{R(x_1, x_2, k_\perp)} \right\} \text{To be modeled:} \\ \text{CQMs, GPDs...}$$

$$R(x_1, x_2, k_\perp) \equiv \frac{F_{[L]}^{\text{HO}}(x_1, x_2, k_\perp; Q^2)}{F_{[I]}^{\text{HO}}(x_1, x_2, k_\perp; Q^2)} = w(k_\perp) [x_1 x_2]^{t(k_\perp)} (1 - x_1 - x_2)^{|x_1 - x_2|} e(k_\perp) e^{-(1 - x_1 - x_2)h(k_\perp)}$$

The parameters  $w(k_\perp)$ ,  $e(k_\perp)$ ,  $t(k_\perp)$  and  $h(k_\perp)$  are fixed to reproduce and **READY TO BE USED!!!**

M. R. and F. A. Ceccopieri, JHEP 1909 (2019) 097

## 6 Double PDFs contributions:

From pQCD (double DGLAP + inhomogeneous term) analyses we can build the following decomposition:

$$F(z_{\perp}) = F_{\text{int}}(z_{\perp}) + F_{\text{sp}}(z_{\perp})$$

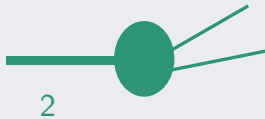
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The **first** term:

- it corresponds to:



- not divergent for  $z_{\perp} \rightarrow 0$

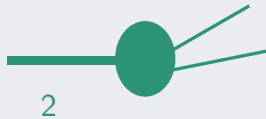
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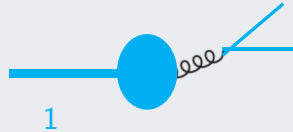
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The **second** term:

- it corresponds to:



- divergent  $1/z_{\perp}^2$

Diehl et al. JHEP 03 (2012) 089

Diehl et al. SciPost Phys. 7 (2019) 017

Diehl et al. JHEP 08 (2021) 040



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in principle we have the following terms:

The **first** term:

- it corresponds to:




2

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The **second** term:

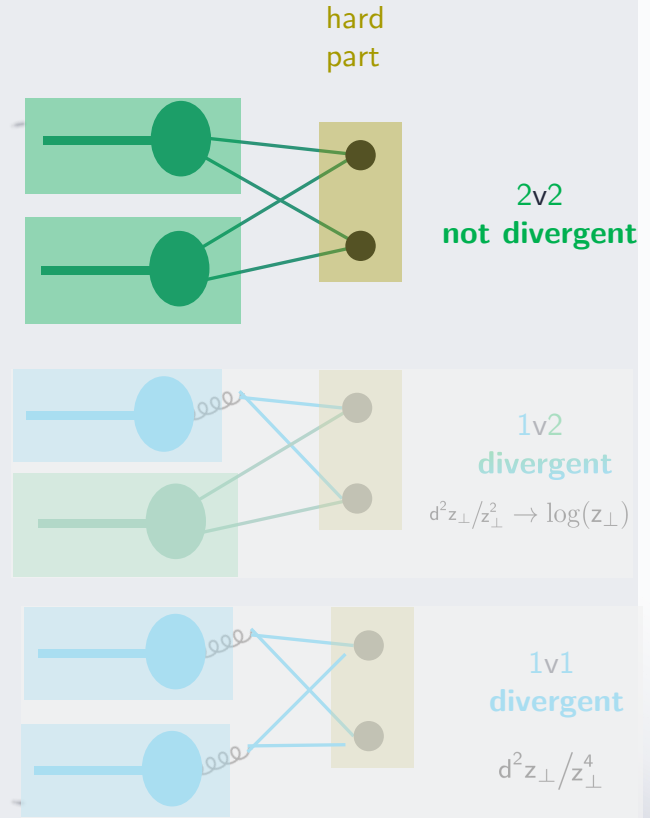
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1

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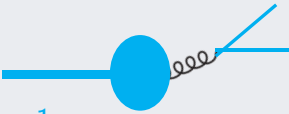


2

- not divergent for  $z_{\perp} \rightarrow 0$

The **second** term:

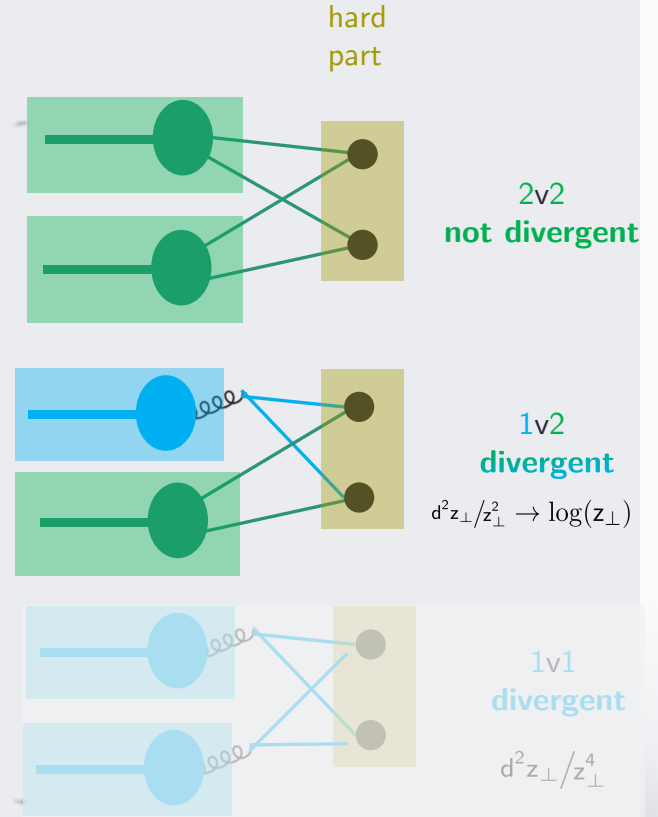
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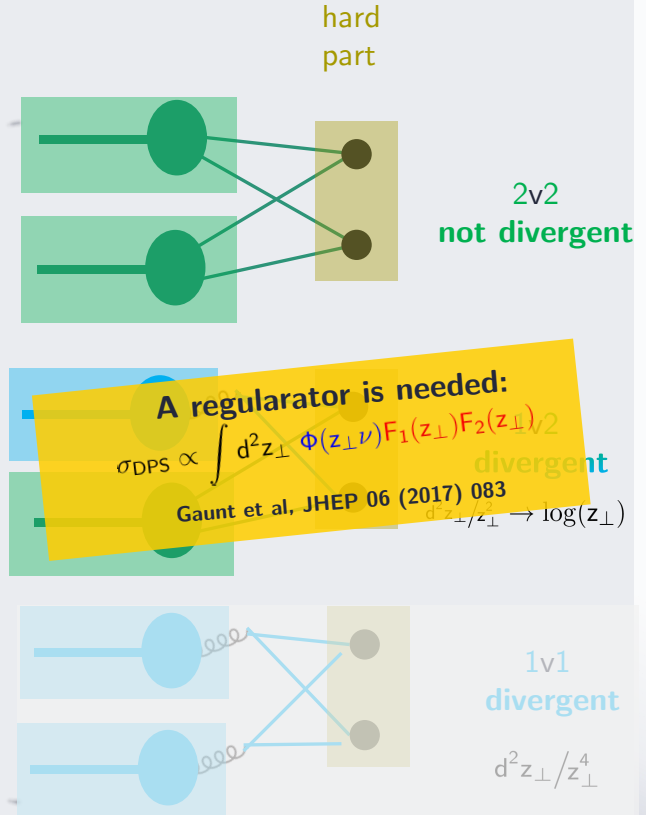
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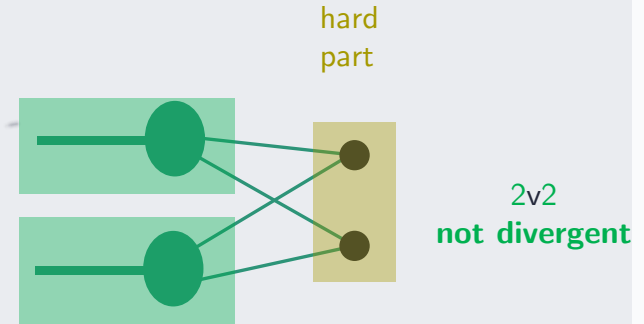
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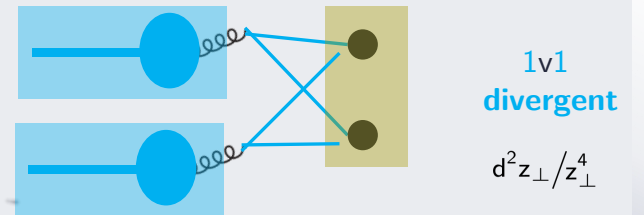
**A regulator is needed:**

$$\sigma_{\text{DPS}} \propto \int d^2z_{\perp} \phi(z_{\perp}) F_1(z_{\perp}) F_2(z_{\perp})$$

divergent

Gaunt et al. JHEP 06 (2017) 083

$d^2z_{\perp}/z_{\perp}^2 \rightarrow \log(z_{\perp})$




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2

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The **second** term:

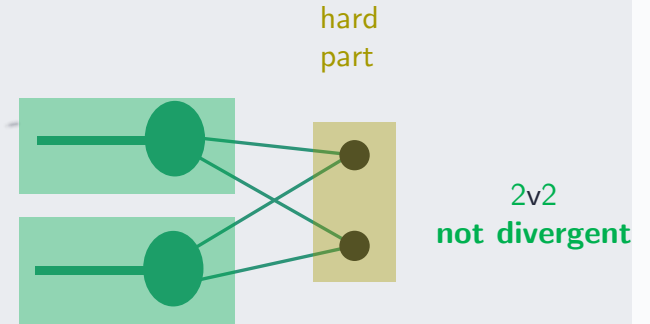
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1

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Diehl et al. JHEP 03 (2012) 089  
 Diehl et al. SciPost Phys. 7 (2019) 017  
 Diehl et al. JHEP 08 (2021) 040



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divergent

Gaunt et al, JHEP 06 (2017) 083

$d^2 z_{\perp}/z_{\perp} \rightarrow \log(z_{\perp})$

**This term must be subtracted!**



divergent

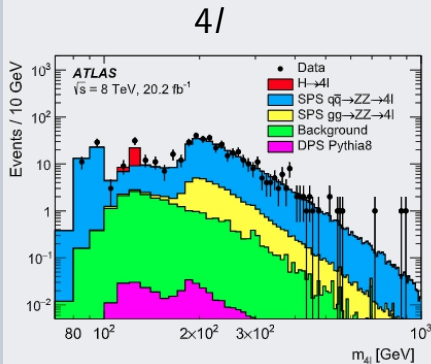
Diehl, Gaunt, JHEP 06 (2017) 083

$d^2 z_{\perp}/z_{\perp}^4$

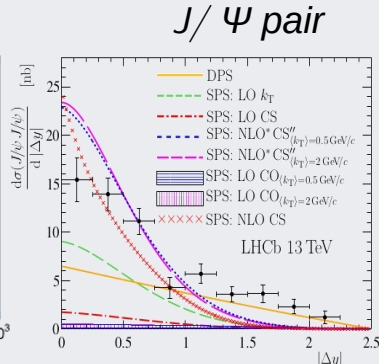
# 7 Some Data

Here some experimental and phenomenological analyses. Usually relevant final states are:

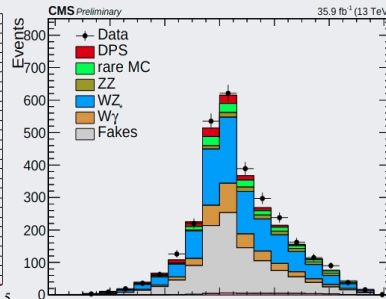
**WW (same sign are very promising), W+J/ $\Psi$ , J/ $\Psi$ +J/ $\Psi$ , W+jets, 4 jets,  $\Upsilon$ +3 jets, ZZ....**



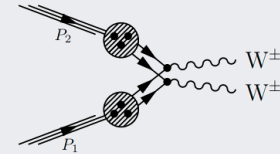
[ATLAS], PLB 790 (2019), 595-614



[LHCb] JHEP 06, 046 (2017)



CMS PAS FSQ-16-009



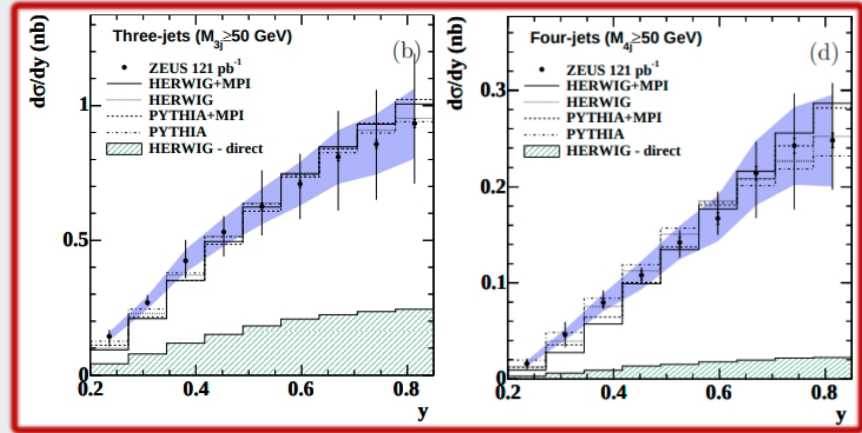
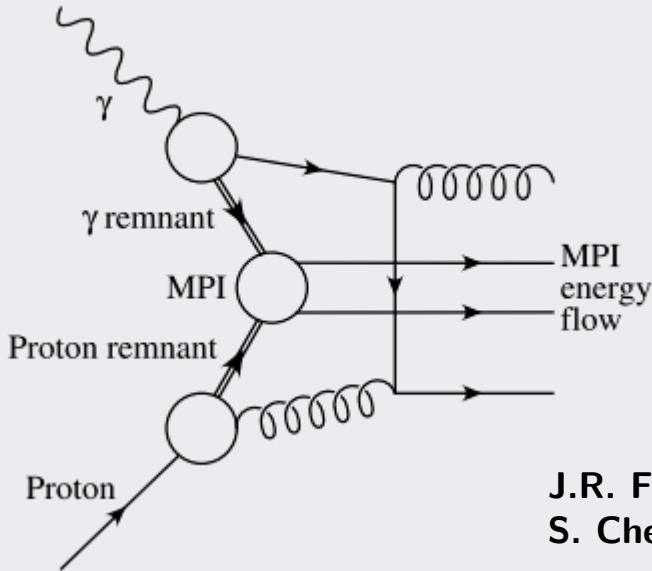
Gaunt et al, EPJC 69 (2010) 53  
first pheno. predictions

M.R. et al PRD 95 (2017) 3, 034040  
Same sign WW= golden process to  
access double parton correlations!

T. Kasemets et al, JHEP 10 (2020) 214  
Golden process to access spin  
correlations!

# 7 Some Data

We just mention here the importance of MPI for the **3,4 jets photo-production** at HERA:



J.R. Forshaw et al, Z phys. C 72, 637

S. Chekanov et al [ZEUS coll.], Nucl. Phys B 792,1 (2008)

# 8 Data and Effective Cross Section

A tool for the comprehension of the role of DPS in hadron-hadron collisions is the so called “effective X-section”.

$$\sigma_{\text{eff}}^{\text{pp}} = \frac{m}{2} \frac{\sigma_{\text{A}}^{\text{pp}} \sigma_{\text{B}}^{\text{pp}}}{\sigma_{\text{DPS}}^{\text{pp}}}$$

POCKET FORMULA

Differential cross section for the process:  
 $pp \rightarrow A(B) + X$

Differential cross section for a DPS event:  
 $pp \rightarrow A + B + X$



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Differential cross section for the process:  
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Differential cross section for a DPS event:  
pp  $\rightarrow$  A + B + X

$$\sigma_{\text{eff}}(x_1, x_2, x_3, x_4) = \frac{\sum_{i,j,k,l} \text{color factors } C_{ik} C_{jl} F_i(x_1) F_j(x_2) F_k(x_3) \text{ PDF } F_l(x_4)}{\sum_{i,j,k,l} C_{ik} C_{jl} \int d^2 z_{\perp} F_{ij}(x_1, x_3, z_{\perp}) F_{kl}(x_3, x_4, z_{\perp})}$$

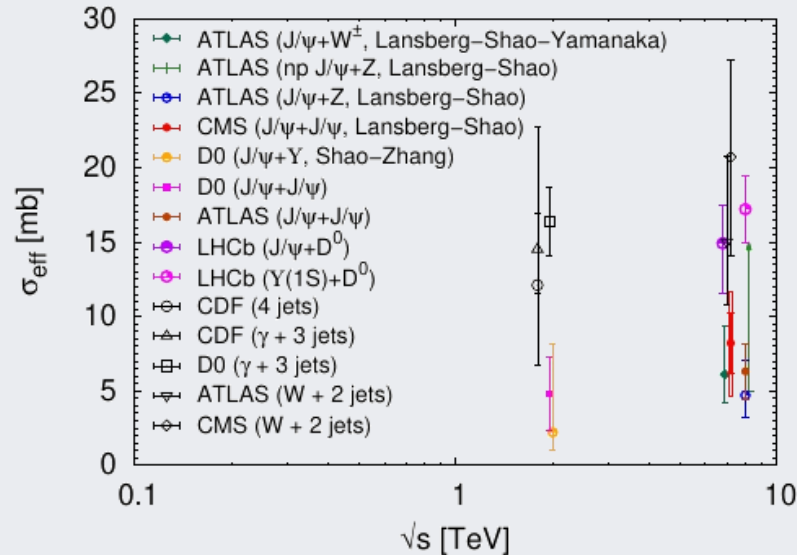
M.R., S. Scopetta et al, PLB 752

M. Traini, M.R., S. Scopetta and V. Vento, PLB 768 (2017)

# 8 Data and Effective Cross Section

J.P. Lansberg's slide  
MPI-2019 workshop

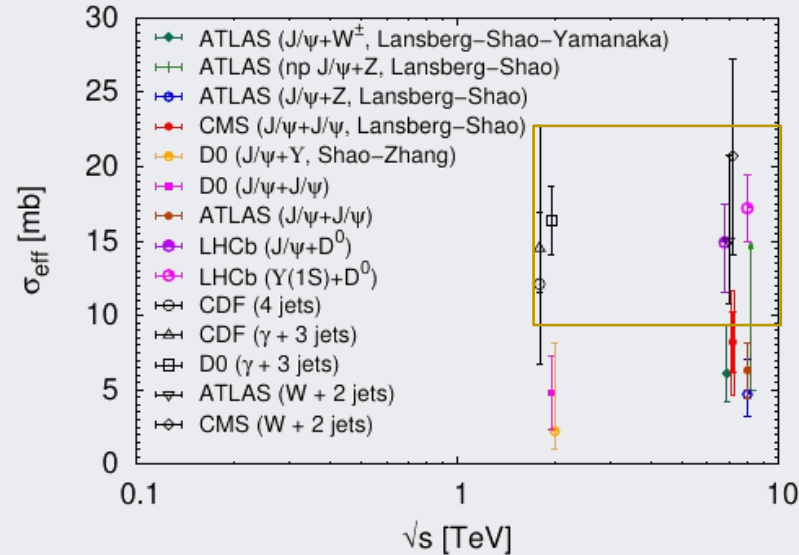
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J.P. Lansberg's slide  
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J.P. Lansberg's slide  
MPI-2019 workshop

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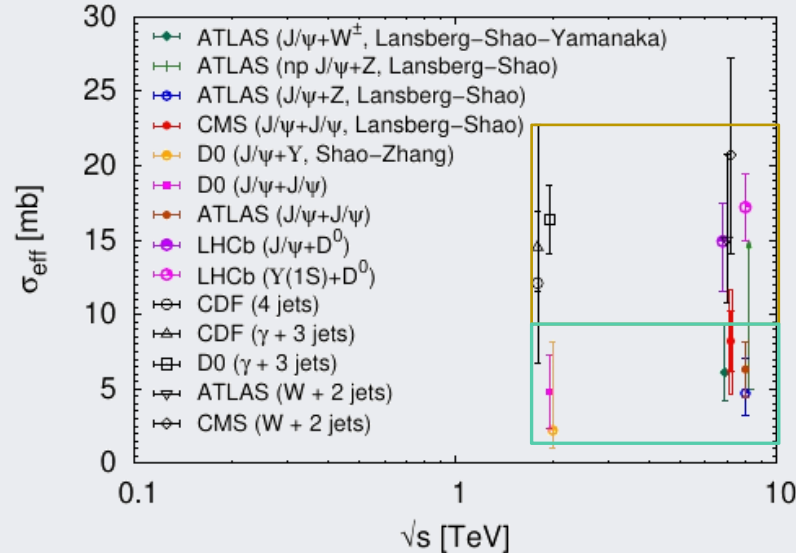
- SENSITIVE TO CORRELATIONS
- PROCESS DEPENDENT?
- SENSITIVE TO INFORMATION ON THE PROTON STRUCTURE?

As predicted by quark models

M.R. et al PLB 752,40 (2016)

M. Traini, M. R. et al, PLB 768, 270 (2017)

M. R. et al, Phys.Rev. D95 (2017) no.11, 114030



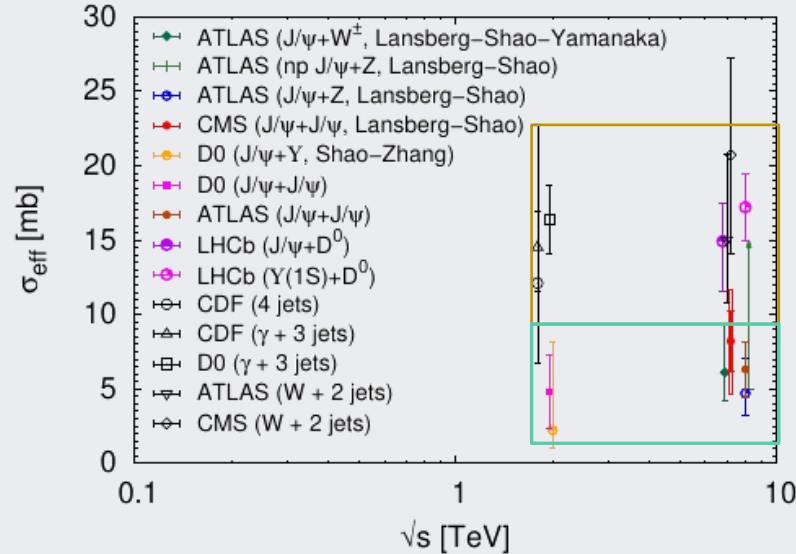
# 8 Data and Effective Cross Section

J.P. Lansberg's slide  
MPI-2019 workshop

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- SENSITIVE TO CORRELATIONS
- PROCESS DEPENDENT?
- SENSITIVE TO INFORMATION ON THE PROTON STRUCTURE?  
and phenomenological analyses  
T. Kasemets et al, JHEP 10 (2020) 214  
...



## 9 Clues from data?

If dPDFs factorize in terms of PDFs then

$$\sigma_{\text{eff}}^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi)^2} \boxed{T(k_{\perp})}^2$$

→ Effective form factor (EFF)

EFF can be formally defined as  
**FIRST MOMENT** of dPDF  
in momentum space

$$T(k_{\perp}) \propto \int dx_1 dx_2 \tilde{F}(x_1, x_2, k_{\perp})$$

## 9 Clues from data?

If dPDFs factorize in terms of PDFs then  $\sigma_{\text{eff}}^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi)^2} \boxed{T(k_{\perp})^2}$   $\rightarrow$  Effective form factor (EFF)

$k_{\perp}$  is the conjugate variable to  $z_{\perp}$ . In analogy with the charge form factor:

$$\langle z_{\perp}^2 \rangle \propto \frac{d}{k_{\perp} dk_{\perp}} T(k_{\perp}) \Big|_{k_{\perp}=0}$$

EFF can be formally defined as **FIRST MOMENT** of dPDF in momentum space

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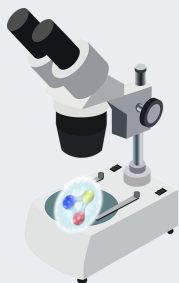
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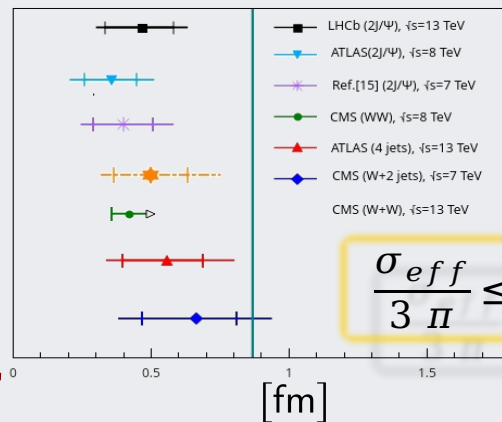
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DPS processes:

The vertical line stands for the transverse proton radius



$$\frac{\sigma_{\text{eff}}}{3\pi} \leq \langle z^2 \rangle \leq \frac{\sigma_{\text{eff}}}{\pi}$$



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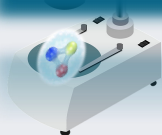
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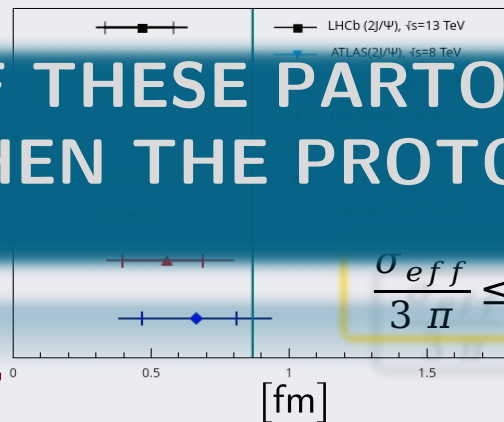
$k_{\perp}$  is the conjugate variable to  $z_{\perp}$  In analogy with the charge form factor:

$$\langle z_{\perp}^2 \rangle \propto \frac{d}{d^2 k_{\perp}} T(k_{\perp}) \Big|_{k_{\perp}=0}$$

**THE DISTANCE OF THESE PARTONS SEEMS TO BE SMALLER THEN THE PROTON RADIUS**



the transverse proton radius



$$\frac{\sigma_{\text{eff}}}{3\pi} \leq \langle z^2 \rangle \leq \frac{\sigma_{\text{eff}}}{\pi}$$

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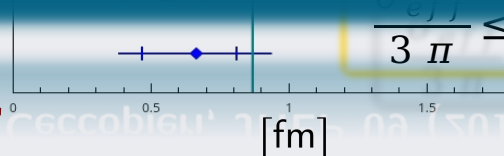
$$T(k_{\perp}) \propto \int dx_1 dx_2 \tilde{F}(x_1, x_2, k_{\perp})$$

$k_{\perp}$  is the conjugate variable to  $Z_{\perp}$  In analogy with the charge form factor:

$$\langle z_{\perp}^2 \rangle \propto \frac{d}{dk_{\perp}^2} T(k_{\perp}) \Big|_{k_{\perp}=0}$$

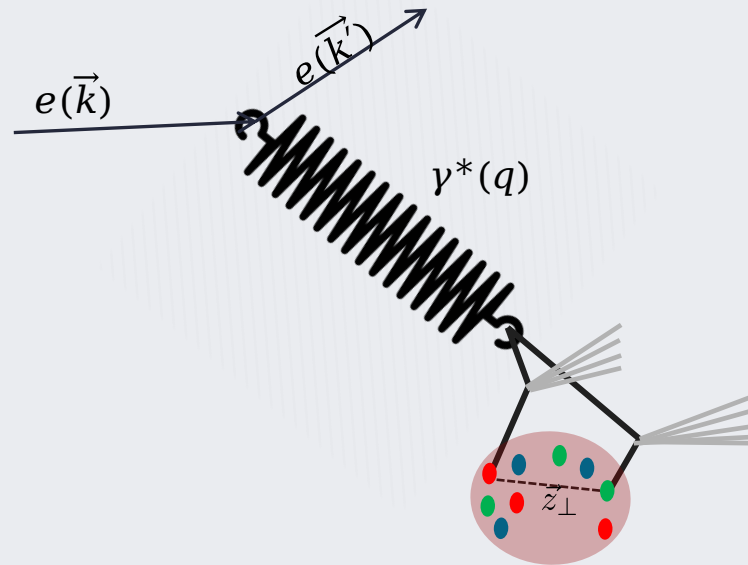
**HOWEVER FROM PROTON-PROTON COLLISIONS ONLY RANGES CAN BE ACCESSED**

M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097



# 10 New Idea: DPS via $\gamma$ -p interaction

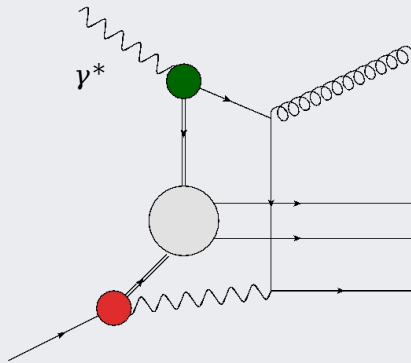
We consider the possibility offered by a DPS process involving a photon FLACTUATING in a quark-antiquark pair interacting with a proton:



M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

# 10 New Idea: DPS via $\gamma$ -p interaction

In order to study the impact of the DPS contribution to a process initiated via photon-proton interactions we evaluated the 4-JET photo-production at HERA (S. Chekanov et al. (ZEUS), Nucl. Phys B792, 1 (2008))



\*Single Parton Scattering (SPS)

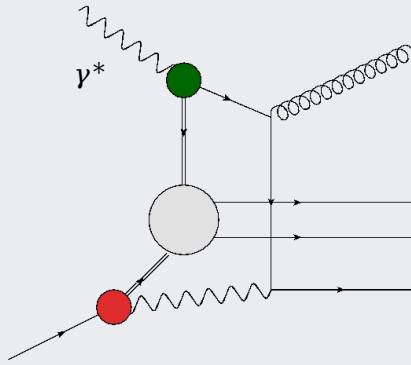
For this first investigation, we make use of the  
POCKET FORMULA:

$$\begin{aligned}
 d\sigma_{\text{DPS}}^{4j} = & \frac{1}{2} \sum_{ab,cd} \int dy dQ^2 \underbrace{f_{\gamma/e}(y, Q^2)}_{\text{Flux Factor}} \times \\
 & \times \int dx_{p_a} dx_{\gamma_b} f_{a/p}(x_{p_a}) f_{b/\gamma}(x_{\gamma_b}) d\hat{\sigma}_{ab}^{2j}(x_{p_a}, x_{\gamma_b}) \left. \vphantom{\int} \right\} \text{SPS}^* \\
 & \times \int dx_{p_c} dx_{\gamma_d} \underbrace{f_{c/p}(x_{p_c})}_{\text{p-PDF}} \underbrace{f_{d/\gamma}(x_{\gamma_d})}_{\text{\gamma-PDF (M. Gluck et al. PRD46, 1973 (1992))}} d\hat{\sigma}_{cd}^{2j}(x_{p_c}, x_{\gamma_d}) \left. \vphantom{\int} \right\} \text{SPS} \\
 & \text{(J. Pumplin et al. JHEP 07, 012 (2002) )}
 \end{aligned}$$

P. Nason et al, PLB319 339 (1993)

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For this first investigation, we make use of the  
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$$d\sigma_{DPS}^{4j} = \frac{1}{S} \sum_{i,j} \int dy dQ^2 \frac{f_{\gamma/e}(y, Q^2)}{\sigma_{eff}^{\gamma p}(Q^2)} \times$$

Flux Factor  
P. Nason et al, PLB319  
339 (1993)

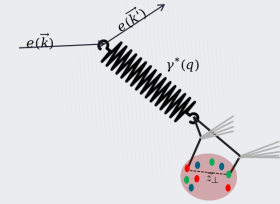
$$\times \left. \begin{aligned} & d\hat{\sigma}_{ab}^{2j}(x_{pa}, x_{\gamma b}) \quad \left. \vphantom{d\hat{\sigma}_{ab}^{2j}} \right\} \text{SPS}^* \\ & d\hat{\sigma}_{cd}^{2j}(x_{pc}, x_{\gamma d}) \quad \left. \vphantom{d\hat{\sigma}_{cd}^{2j}} \right\} \text{SPS} \end{aligned} \right\} \times$$

DF (M. Gluck et al. PRD46, 1973 (1992))  
(2002)

The main quantity we have to evaluate is:

$$\sigma_{eff}^{\gamma p}(Q^2)$$

# 10 The $\gamma$ -p effective cross section

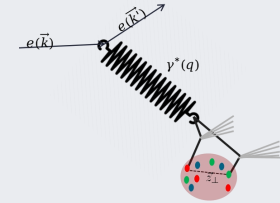


The expression of this quantity is very similar to the proton-proton collision case and can be formally derived by comparing the product of SPS cross sections and the DPS one obtained in **Gaunt, JHEP 01, 042 (2013)** and describing a DPS from a vector bosons splitting with given  $Q^2$  virtuality

$$[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} \overset{\text{Proton EFF}}{\boxed{T_p(k_{\perp})}} T_{\gamma}(k_{\perp}; Q^2)$$

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

# 10 The $\gamma$ -p effective cross section

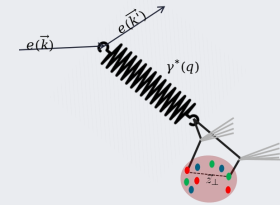


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M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

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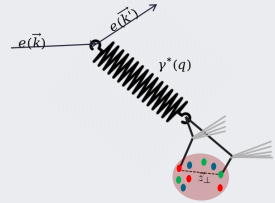
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The full DPS cross section depends on the amplitude of the splitting photon in a  $q \bar{q}$  pair. The latter can be formally described within a Light-Front (LF) approach in terms of LF wave functions.

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501



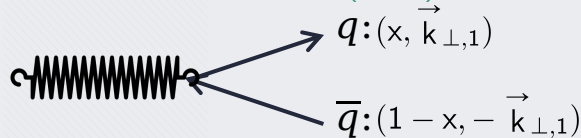
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$$[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$$

The full DPS cross section depends on the amplitude of the splitting photon in a  $q \bar{q}$  pair. The latter can be formally described within a Light-Front (LF) approach in terms of **LF wave functions (W.F.)**:

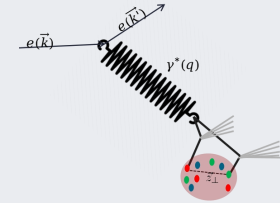


$$f_{q,\bar{q}}^{\gamma}(x, \vec{k}_{\perp}; Q^2) = \int d^2 k_{\perp,1} \psi_{q\bar{q}}^{\dagger\gamma}(x, \vec{k}_{\perp,1}; Q^2) \times \psi_{q\bar{q}}^{\gamma}(x, \vec{k}_{\perp,1} + \vec{k}_{\perp}; Q^2)$$

Similar definition of a meson dPDF

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

# 10 The $\gamma$ -p effective cross section

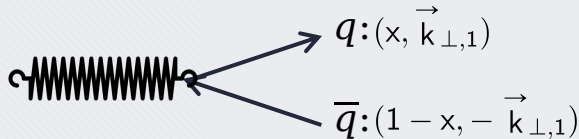


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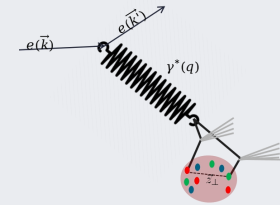
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$$T_{\gamma}(k_{\perp}; Q^2) = \frac{\sum_q \int dx f_{q,\bar{q}}^{\gamma}(x, k_{\perp}; Q^2)}{\sum_q \int dx f_{q,\bar{q}}^{\gamma}(x, k_{\perp} = 0; Q^2)}$$



M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

# 10 The $\gamma$ -p effective cross section

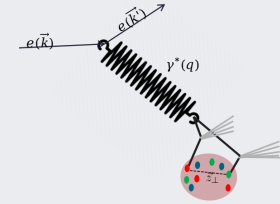


For the proton EFF use has been made of three choices:

- $[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$
- $T_p(k_{\perp})$  proton EFF
- $\psi/\gamma$  Photon WF

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

# 10 The $\gamma$ -p effective cross section



**1**  $[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$

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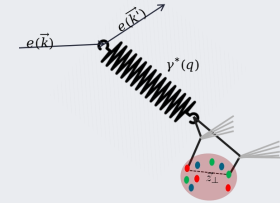
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For the proton EFF use has been made of three choices:

- 1) G1:  $e^{-\alpha_1 k_{\perp}^2}$        $\alpha_1 = 1.53 \text{ GeV}^{-2} \implies \sigma_{\text{eff}}^{\text{pp}} = 15 \text{ mb}$
- 2) G2:  $e^{-\alpha_2 k_{\perp}^2}$        $\alpha_2 = 2.56 \text{ GeV}^{-2} \implies \sigma_{\text{eff}}^{\text{pp}} = 25 \text{ mb}$

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

# 10 The $\gamma$ -p effective cross section



**1**  $[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi)^2} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$   
**2**  $T_p(k_{\perp})$  proton EFF  
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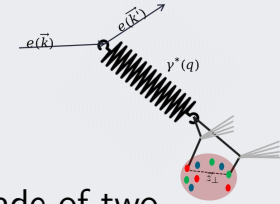
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- 2) G2:  $e^{-\alpha_2 k_{\perp}^2}$ ,  $\alpha_2 = 2.56 \text{ GeV}^{-2} \implies \sigma_{\text{eff}}^{\text{pp}} = 25 \text{ mb}$
- 3) S:  $\left(1 + \frac{k_{\perp}^2}{m_g^2}\right)^{-4}$ ,  $m_g^2 = 1.1 \text{ GeV}^2 \implies \sigma_{\text{eff}}^{\text{pp}} = 30 \text{ mb}$

B. Blok et al, EPJC74, 2926 (2014)

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

# 10 The $\gamma$ -p effective cross section



**1**  $[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$   
**2**  $T_p(k_{\perp})$  proton EFF  
**3**  $\psi_{\gamma}$  Photon WF

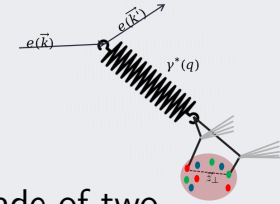
For the photon W.F. use has been made of two choices representing two extreme cases:

1) QED at LO (S.J. Brodsky et al. PRD50, 3134 (1994)):

$$\psi_{q,\bar{q}}^{\lambda=\pm}(x, k_{1\perp}; Q^2) = -e_f \frac{\bar{u}_q(k) \gamma \cdot \varepsilon^{\lambda} v_{\bar{q}}(q-k)}{\sqrt{x(1-x)} \left[ Q^2 + \frac{k_{1\perp}^2 + m^2}{x(1-x)} \right]}$$

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

# 10 The $\gamma$ -p effective cross section



**1**  $[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$   
**2**  $T_p(k_{\perp})$  proton EFF  
**3**  $\psi^{\gamma}$  Photon WF

For the photon W.F. use has been made of two choices representing two extreme cases:

1) QED at LO (S.J. Brodsky et al. PRD50, 3134 (1994)):

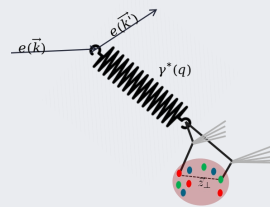
$$\psi_{q,\bar{q}}^{\lambda=\pm}(x, k_{1\perp}; Q^2) = -e_f \frac{\bar{u}_q(k) \gamma \cdot \varepsilon^{\lambda} v_{\bar{q}}(q-k)}{\sqrt{x(1-x)} \left[ Q^2 + \frac{k_{1\perp}^2 + m^2}{x(1-x)} \right]}$$

2) Non-Pertubative (NP) effects (E.R.Arriola et al, PRD74,054023 (2006))

$$\psi_A^{\gamma}(x, k_{1\perp}; Q^2) = \frac{6(1 + Q^2/m_{\rho}^2)}{m_{\rho}^2 \left( 1 + 4 \frac{k_{1\perp}^2 + Q^2 x(1-x)}{m_{\rho}^2} \right)^{5/2}}$$

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

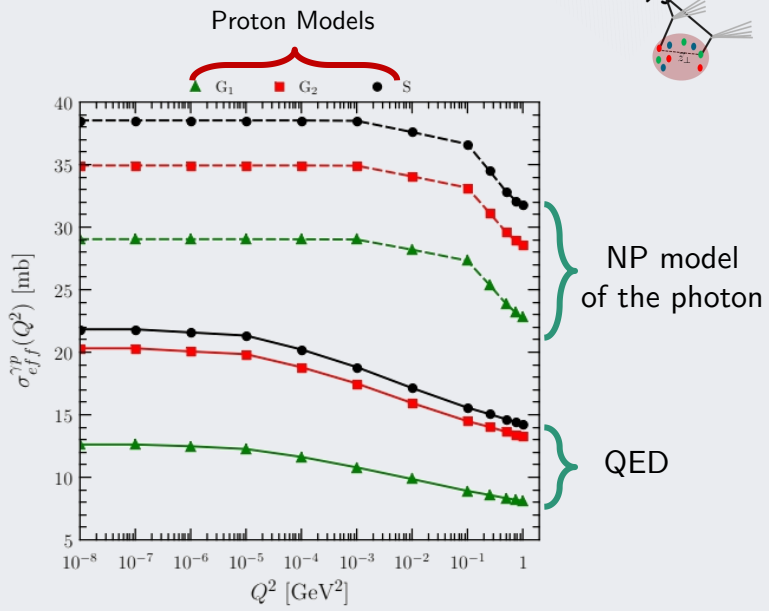
# 10 The $\gamma$ -p effective cross section



**1**  $[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$

**2**  $T_p(k_{\perp})$  proton EFF

**3**  $\psi/\gamma$  Photon WF



M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501



# 10 The 4 jet DPS cross section

The HERA KINEMATICS:

S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)

$$E_T^{\text{jet}} > 6 \text{ GeV}$$

Transverse energy of the jets

$$|\eta_{\text{jet}}| < 2.4$$

Pseudorapidity

$$Q^2 < 1 \text{ GeV}^2$$

Photon virtuality

$$0.2 \leq y \leq 0.85$$

Inelasticity

# 10 The 4 jet DPS cross section

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Inelasticity

The ZEUS collaboration quoted an integrated total 4-jet cross section of 136 pb

S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)

# 10 The 4 jet DPS cross section

KINEMATICS:

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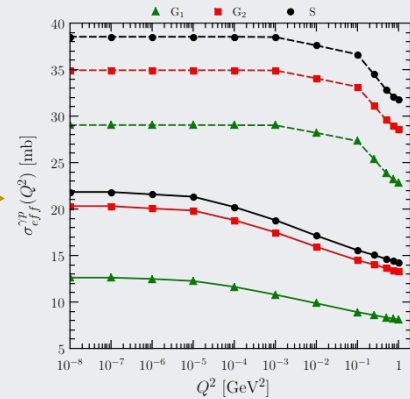
$$Q^2 < 1 \text{ GeV}^2$$

$$0.2 \leq y \leq 0.85$$

$$d\sigma_{\text{DPS}}^{4j} = \frac{1}{2} \sum_{ab,cd} \int dy dQ^2 \frac{f_{\gamma/e}(y, Q^2)}{\sigma_{\text{eff}}^{\gamma p}(Q^2)} \times$$

$$\times \int dx_{p_a} dx_{\gamma_b} f_{a/p}(x_{p_a}) f_{b/\gamma}(x_{\gamma_b}) d\hat{\sigma}_{ab}^{2j}(x_{p_a}, x_{\gamma_b})$$

$$\times \int dx_{p_c} dx_{\gamma_d} f_{c/p}(x_{p_c}) f_{d/\gamma}(x_{\gamma_d}) d\hat{\sigma}_{cd}^{2j}(x_{p_c}, x_{\gamma_d})$$



M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

# 10 The 4 jet DPS cross section

KINEMATICS

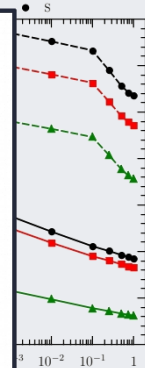
$$E_T^{\text{jet}} > 6 \text{ GeV}$$

$$|\eta_{\text{jet}}| < 2.4$$

$$Q^2 < 1 \text{ GeV}^2$$

$$0.2 \leq y \leq 0.8$$

		$\sigma_{DPS}$ [pb]			
		$Q^2 \leq 10^{-2}$ [GeV <sup>2</sup> ]	$10^{-2} \leq Q^2 \leq 1$ [GeV <sup>2</sup> ]	$Q^2 \leq 1$ [GeV <sup>2</sup> ]	$\frac{\sigma_{DPS}}{\sigma_{tot}}$ [%]
photon	G <sub>1</sub>	35.1	18.6	53.7	40
	G <sub>2</sub>	29.1	15.2	44.3	33
	S	26.4	13.7	40.1	30
QED	G <sub>1</sub>	87.8	54.3	142.1	101
	G <sub>2</sub>	54.3	33.4	87.7	65
	S	50.5	31.1	81.6	60



proton

# 10 The 4 jet DPS cross section

KINEMATICS

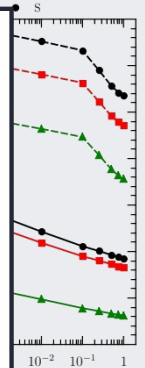
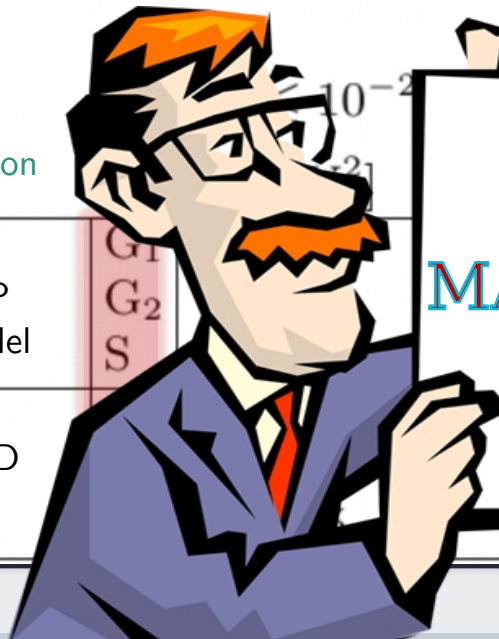
$$E_T^{\text{jet}} > 6 \text{ GeV}$$

$$|\eta_{\text{jet}}| < 2.4$$

$$Q^2 < 1 \text{ GeV}^2$$

$$0.2 \leq y \leq 0.8$$

		$\sigma_{DPS} \text{ [pb]}$	$Q^2 \leq 1 \text{ [GeV}^2\text{]}$	$\frac{\sigma_{DPS}}{\sigma_{tot}} \text{ [%]}$
photon				
NP model	G1		53.7	40
	G2		44.3	33
	S		40.1	30
QED			142.1	101
			87.7	65
			81.6	60



# 10 The 4 jet DPS cross section

KINEMATICS

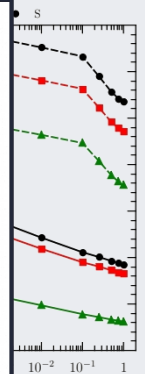
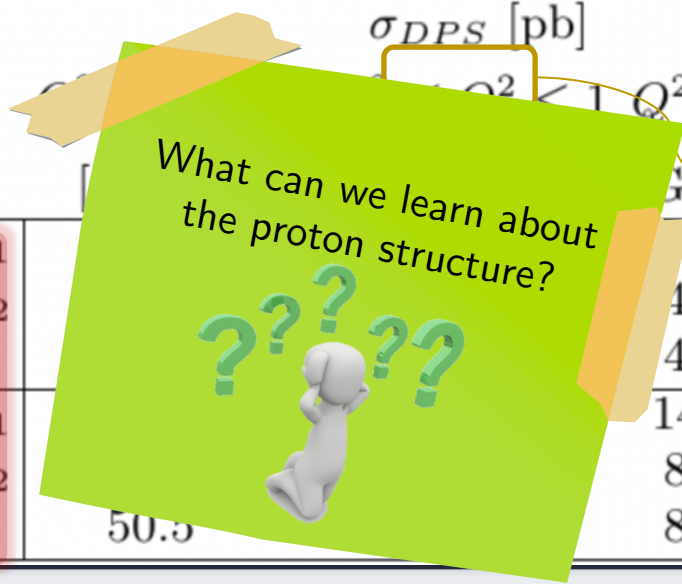
$E_T^{\text{jet}} > 6 \text{ GeV}$

$|\eta_{\text{jet}}| < 2.4$

$Q^2 < 1 \text{ GeV}^2$

$0.2 \leq y \leq 0.8$

		$Q^2 < 1 \text{ GeV}^2$	$Q^2 \leq 1 \text{ GeV}^2$	$\frac{\sigma_{DPS}}{\sigma_{tot}}$ [%]
photon				
NP model	G <sub>1</sub>		3.7	40
	G <sub>2</sub>		44.3	33
	S		40.1	30
QED	G <sub>1</sub>		142.1	101
	G <sub>2</sub>		87.7	65
	S	50.5	81.6	60



proton

# 10 The effective cross section: a key for the proton structure

The effective cross section can be also written in terms of Fourier Transform of the EFF:

$$\tilde{F}_2^\gamma(z_\perp; Q^2) = \sum_n C_n(Q^2) z_\perp^n$$

$$\left[ \sigma_{\text{eff}}^{\gamma p}(Q^2) \right]^{-1} = \int d^2 z_\perp \tilde{F}_2^p(z_\perp) \tilde{F}_2^\gamma(z_\perp; Q^2)$$

$$= \sum_n C_n(Q^2) \langle (z_\perp)^n \rangle_p$$

This coefficient can be determined from the structure of the photon described in a given approach

If we can measure the dependence of the effective-cross section on the photon VIRTUALITY

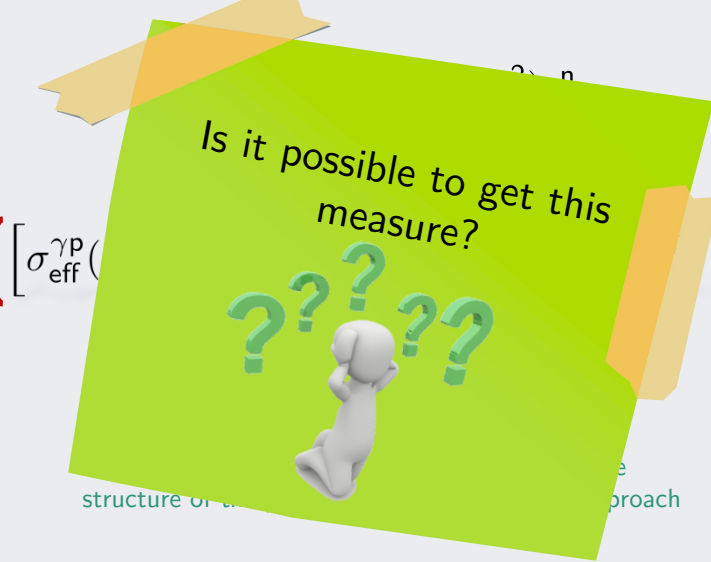
We could access for the first time the mean transverse distance between partons in the proton



M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

# 10 The effective cross section: a key for the proton structure

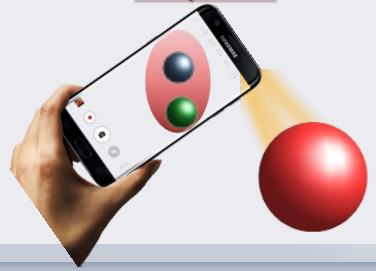
The effective cross section can be also written in terms of Fourier Transform of the EFF:



$$\left[ \sigma_{\text{eff}}^{\gamma p}(\dots) \right]$$

If we can measure the dependence of the effective-cross section on the photon VIRTUALITY

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M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

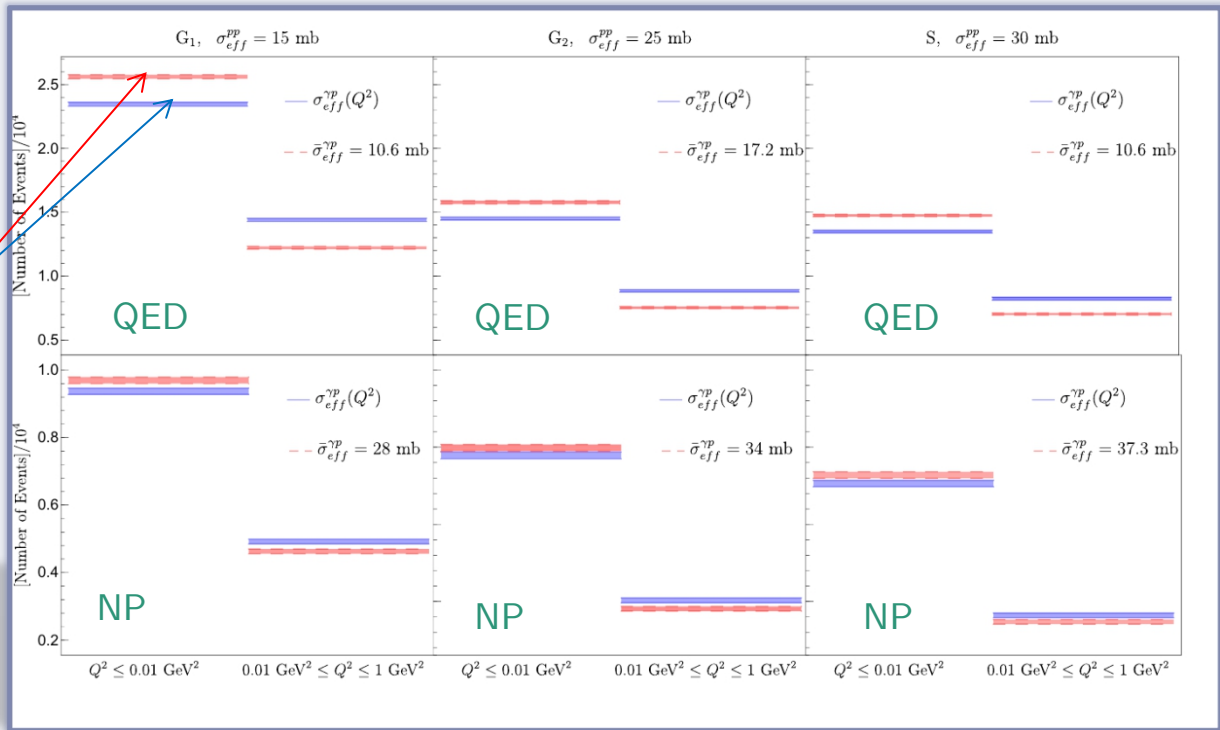


# 10 The effective cross section: a key for the proton structure

Proton →  
 h  
o  
t  
o  
n ↓

With an integrated luminosity of 200 pb<sup>-1</sup> we can separate:

- effective X-section constant
- Full effective X-section



# 11 DPS in pA collisions

A lot of effort (**slides from**):

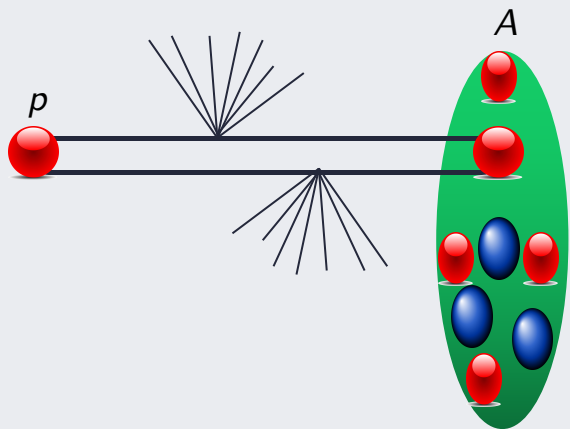
- **Boris Blok**
- **Federico Alberto Ceccopieri**
- **Mark Strikman**
- **Massimiliano Alvioli**
- **Daniele Treleani**

References:

- **B. Blok and F. A. Ceccopieri, EPJC 80 (2020) 8, 762**
- **B. Blok and F. A. Ceccopieri, PRD 101 (2020) 9, 094029**
- **M. Alvioli and M. Strikman, PRC 100 (2019), no. 2, 024912**
- **M. Strikman and D. Treleani, PRL 88 (2002), 031801**
- **M. Alvioli, M. Azarkin, B. Blok and M. Strikman, EPJC 79 (2019), 482**

# 11 DPS in pA collisions

In this case we have two mechanisms that contribute:



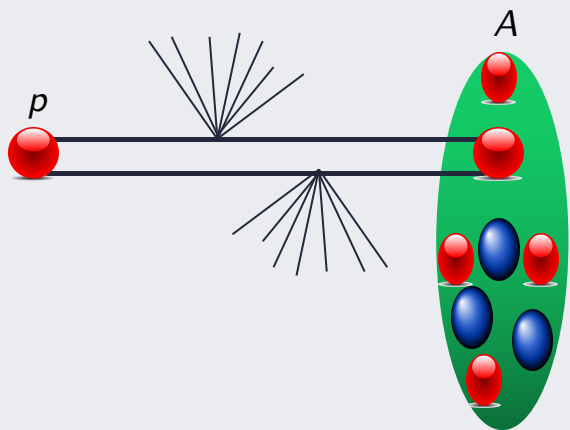
DPS 1

- A lot of effort (slides from):
- Boris Blok
  - Federico Alberto Ceccopieri
  - Mark Strikman
  - Massimiliano Alvioli
  - Daniele Treleani

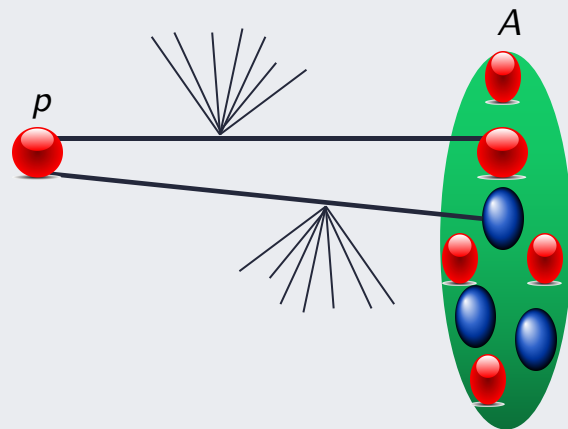
# 11 DPS in pA collisions

In this case we have two mechanisms that contribute:

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DPS 1

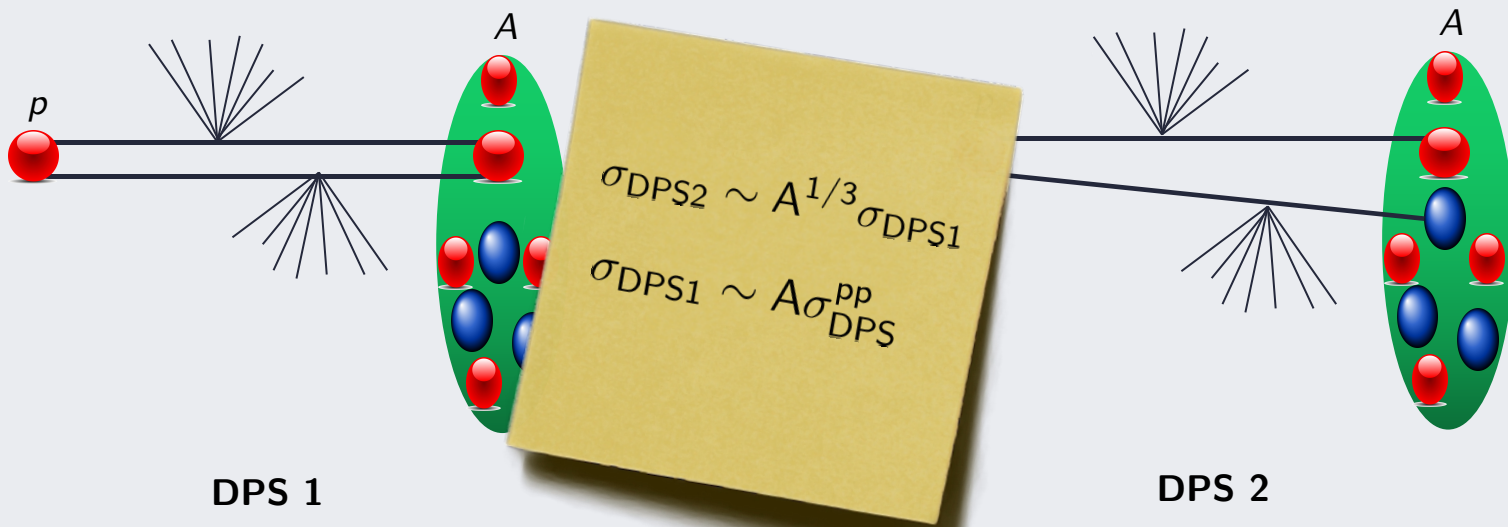


DPS 2

# 11 DPS in pA collisions

A lot of effort (slides from):  
- Boris Blok  
- Federico Alberto Ceccopieri  
- Mark Strikman  
- Massimiliano Alvioli  
- Daniele Treleani

In this case we have two mechanisms that contribute:



# 11 DPS in pA collisions

The DPS cross-section

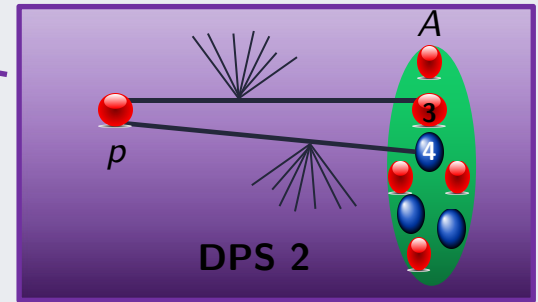
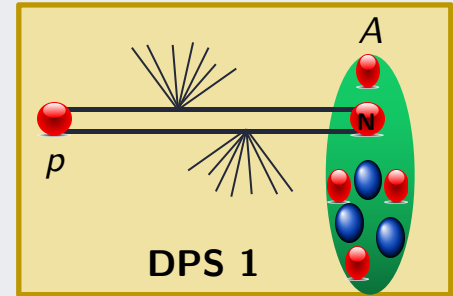
$$d\sigma_{\text{DPS}}^{\text{ML}} = \frac{m}{2} \sum_{i,j,k,l} d\hat{\sigma}_{ik}^{\text{M}} d\hat{\sigma}_{jl}^{\text{L}} \int d^2b_{\perp} F_p^{ij}(x_1, x_2, \vec{b}_{\perp}) \int d^2B \left\{ \right.$$

$$\sum_{N=p,n} F_N^{kl}(x_3, x_4, \vec{b}_{\perp}) \bar{T}_N(B)$$

+

$$\sum_{N_3, N_4=p,n} f_{N_3/A}^k(x_3) f_{N_4/A}^l(x_4) \bar{T}_{N_3}(B) \bar{T}_{N_4}(B)$$

}



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The ingredients:

- the nucleon dPDFs = PDF × PDF ×  $\tilde{T}(b_{\perp})$

$$\int d^2b_{\perp} \tilde{T}(b_{\perp}) = 1 \quad \int d^2b_{\perp} \tilde{T}(b_{\perp})^2 = 1/\sigma_{\text{eff}}^{\text{PP}}$$

$\sigma_{\text{eff}}^{\text{PP}} \sim 18 \pm 6 \text{ mb}$  (average ATLAS and CMS for W production)

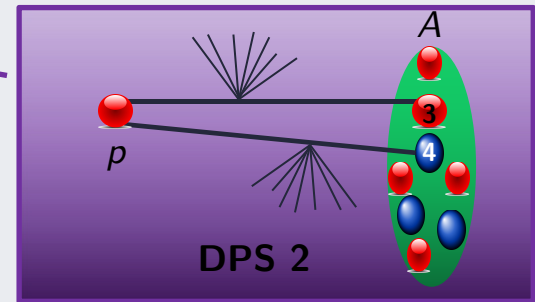
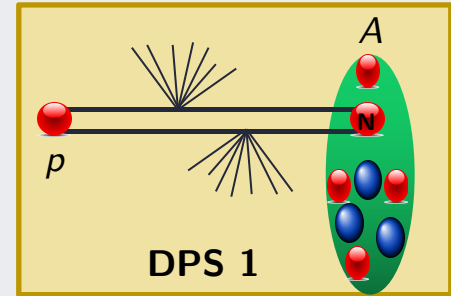
$$\sum_{N=p,n} \boxed{F_N^{kl}(x_3, x_4, \vec{b}_{\perp})} \tilde{T}_N(B)$$

+

$$\sum_{N_3, N_4=p,n} f_{N_3/A}^k(x_3) f_{N_4/A}^l(x_4) \tilde{T}_{N_3}(B) \tilde{T}_{N_4}(B)$$

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The DPS cross-section

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$$\sigma_{\text{eff}}^{\text{pp}} \sim 18 \pm 6 \text{ mb}$$

- the contribution of nucleon to the nuclear PDF

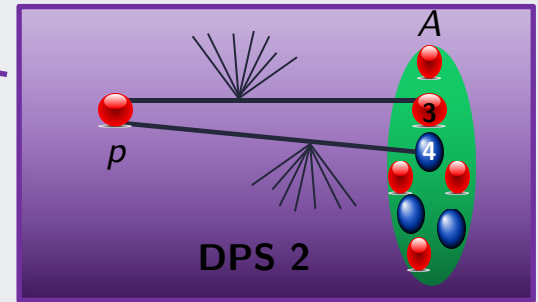
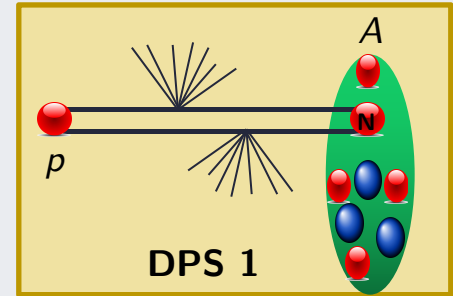
$$\sum_{N_3, N_4=p,n} \boxed{f_{N_3/A}^k(x_3)} f_{N_4/A}^l(x_4) \bar{T}_{N_3}(B) \bar{T}_{N_4}(B)$$

$$\sum_{N=p,n} F_N^{kl}(x_3, x_4, \vec{b}_{\perp}) \bar{T}_N(B)$$

+

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# 11 DPS in pA collisions

The DPS cross-section

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The ingredients:

- the nucleon dPDFs = PDF × PDF ×  $\tilde{T}(b_{\perp})$

$$\int d^2b_{\perp} \tilde{T}(b_{\perp}) = 1 \quad \int d^2b_{\perp} \tilde{T}(b_{\perp})^2 = 1/\sigma_{\text{eff}}^{\text{pp}}$$

$$\sigma_{\text{eff}}^{\text{pp}} \sim 18 \pm 6 \text{ mb}$$

- the contribution of nucleon to the nuclear PDF

- the thickness function as a function of the impact parameter  $B$ :

$$\tilde{T}(\vec{b}_{\perp} + \vec{B}) \sim \tilde{T}(\vec{B})$$

$$\tilde{T}_N(B) = \int dz \underbrace{\rho_N(\sqrt{B^2 + z^2})}_{\text{Wood-Saxon distribution for pb normalized to A}}$$

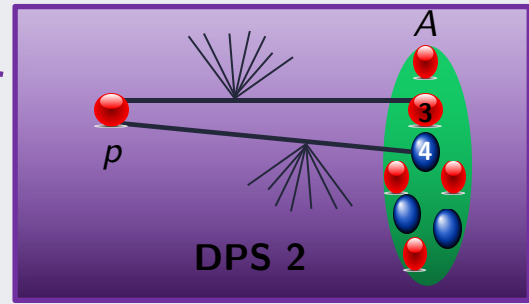
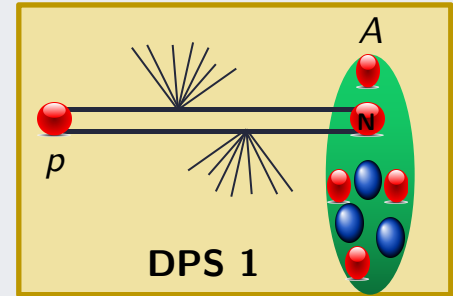
Wood-Saxon distribution for pb normalized to A

$$\sum_{N=p,n} F_N^{kl}(x_3, x_4, \vec{b}_{\perp}) \tilde{T}_N(B)$$

+

$$\sum_{N_3, N_4=p,n} f_{N_3/A}^k(x_3) f_{N_4/A}^l(x_4) \tilde{T}_{N_3}(B) \tilde{T}_{N_4}(B)$$

}



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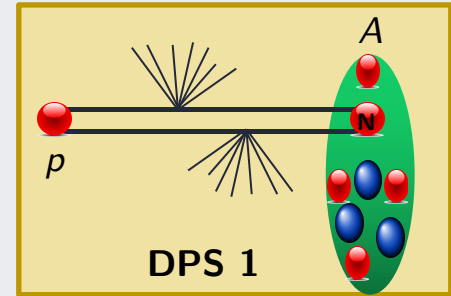
# 11 DPS in pA collisions

The DPS cross-section

$$d\sigma_{\text{DPS}}^{\text{ML}} = \frac{m}{2} \sum_{i,j,k,l} d\hat{\sigma}_{ik}^{\text{M}} d\hat{\sigma}_{jl}^{\text{L}} \int d^2b_{\perp} F_{\text{p}}^{ij}(x_1, x_2, \vec{b}_{\perp}) \int d^2B \left\{ \right.$$

DPS1 LINEAR IN  $\bar{T}(B)$

$$\sum_{N=p,n} F_{\text{N}}^{kl}(x_3, x_4, \vec{b}_{\perp}) \bar{T}_{\text{N}}(B)$$

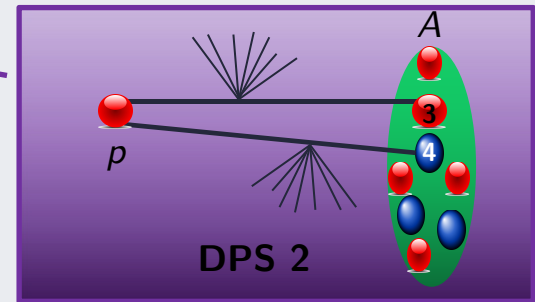


+

$$\sum_{N_3, N_4=p,n} f_{N_3/A}^k(x_3) f_{N_4/A}^l(x_4) \bar{T}_{N_3}(B) \bar{T}_{N_4}(B)$$

DPS2 QUADRATIC IN  $\bar{T}(B)$

}



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# 11 DPS in pA collisions

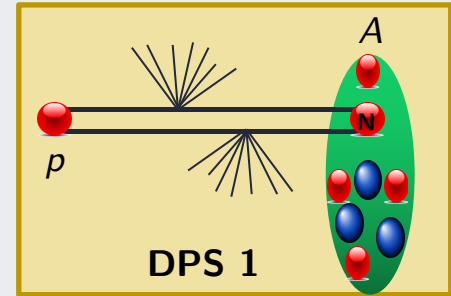
The DPS cross-section

$$d\sigma_{\text{DPS}}^{\text{ML}} = \frac{m}{2} \sum_{i,j,k,l} d\hat{\sigma}_{ik}^{\text{M}} d\hat{\sigma}_{jl}^{\text{L}} \int d^2b_{\perp} F_p^{ij}(x_1, x_2, \vec{b}_{\perp}) \int d^2B \left\{ \right.$$

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  - Daniele Treleani

DPS1 LINEAR IN  $\bar{T}(B)$

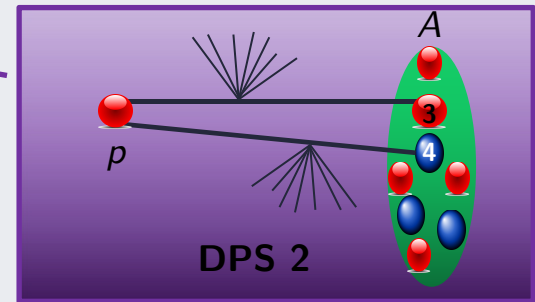
Experimental techniques have been developed to estimate the centrality (B) of the pA or AA collisions. In principle it is possible to separate these contributions!



$$\sum_{N_3, N_4=p,n} f_{N_3}^k$$

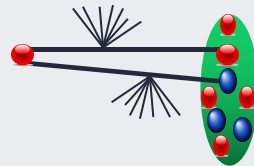
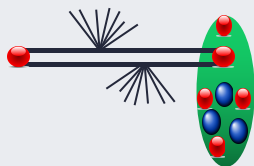
LHCb, PRL 125 (2020), 21, 212001

DPS2 QUADRATIC IN  $\bar{T}(B)$



# 11 DPS in pA collisions

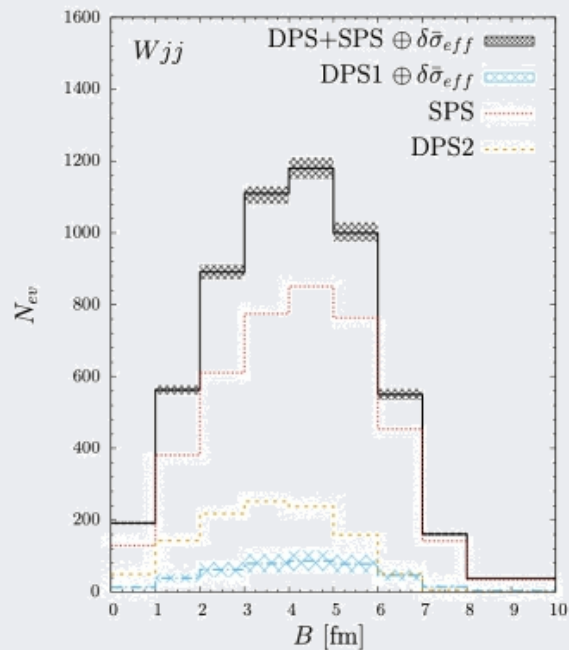
W+di-jets



- A lot of effort (slides from):
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  - Mark Strikman
  - Massimiliano Alvioli
  - Daniele Treleani

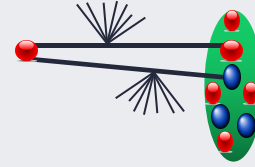
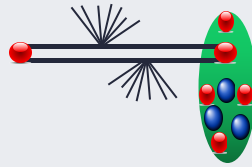
$\sigma^{Wjj}$	$p_T^j > 20 \text{ GeV}$	$p_T^j > 25 \text{ GeV}$	$p_T^j > 30 \text{ GeV}$
	[nb]	[nb]	[nb]
DPS1	$19 \pm 6$	$8 \pm 3$	$4 \pm 2$
DPS2	49	22	11
SPS	81	57	41
Tot	$149 \pm 6$	$87 \pm 3$	$56 \pm 2$

- SPS dominant
- DPS2 bigger than DPS1 has expected



# 11 DPS in pA collisions

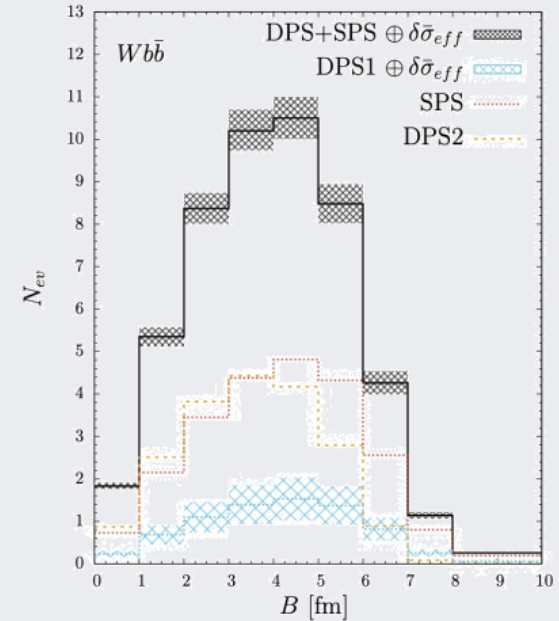
W+2 b-jets



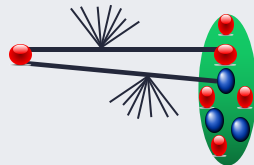
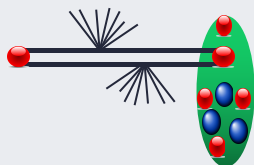
- A lot of effort (slides from):
- Boris Blok
  - Federico Alberto Ceccopieri
  - Mark Strikman
  - Massimiliano Alvioli
  - Daniele Treleani

$\sigma^{Wb\bar{b}}$	$p_T^b > 20$ GeV [pb]	$p_T^b > 25$ GeV [pb]	$p_T^b > 30$ GeV [pb]
DPS1	$74 \pm 25$	$35 \pm 12$	$18 \pm 6$
DPS2	196	92	48
SPS	234	158	114
Tot	$504 \pm 25$	$285 \pm 12$	$180 \pm 6$

- for  $20 < p_T < 25$  GeV the DPS is bigger than SPS
- also for  $B < 4$  fm



# 11 DPS in pA collisions



A lot of effort (slides from):

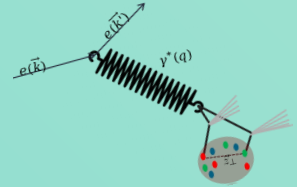
- Boris Blok
- Federico Alberto Ceccopieri
- Mark Strikman
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- Daniele Treleani

$Zjj$	DPS1 (pb)	DPS2 (pb)	SPS (pb)
$p_T^{j_1, j_2} > 20, 20 \text{ GeV}$	2971	7814	15,940
$p_T^{j_1, j_2} > 25, 25 \text{ GeV}$	1270	3341	11,024
$p_T^{j_1, j_2} > 30, 30 \text{ GeV}$	621	1632	8030

	DPS1	DPS2	SPS	Sum
	( $\mu\text{b}$ )	( $\mu\text{b}$ )	( $\mu\text{b}$ )	( $\mu\text{b}$ )
$2b2j$				
$p_T^{b, j} > 20 \text{ GeV}$	2.2	6.2	13.0	21.4
$p_T^{b, j} > 25 \text{ GeV}$	0.4	1.2	4.7	6.4
$p_T^{b, j} > 30 \text{ GeV}$	0.1	0.3	1.9	2.3

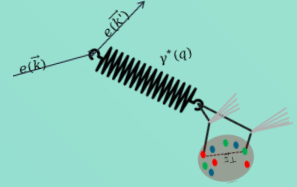
	DPS1	DPS2	SPS	Sum
	( $\mu\text{b}$ )	( $\mu\text{b}$ )	( $\mu\text{b}$ )	( $\mu\text{b}$ )
$4j$				
$p_T^{j_3, j_4} > 20 \text{ GeV}$	26.0	72.2	170.9	269.2
$p_T^{j_3, j_4} > 25 \text{ GeV}$	10.8	30.2	92.9	133.9
$p_T^{j_3, j_4} > 30 \text{ GeV}$	5.1	14.3	51.4	70.9

# CONCLUSIONS (?)



- 1) Properties of dPDF are better understood BUT could we go forward (relation to GPDs, TMDs or mechanical properties)?
- 2) Can we try to add non perturbative correlations in MC?
- 3) Can we observe DPS at EIC, also using quarkonium production?
- 4) Also DPS with nuclear targets at the EIC? We have the technology for light nuclei for which realistic calculations can be performed.
- 5) Moreover, DPS in ultra-peripheral AA collisions ?
- 6) many others.....

# CONCLUSIONS (?)



We are collecting ideas, feelings and impressions for a possible dedicated **workshop** on DPS where all these topics can be discussed merging these communities!

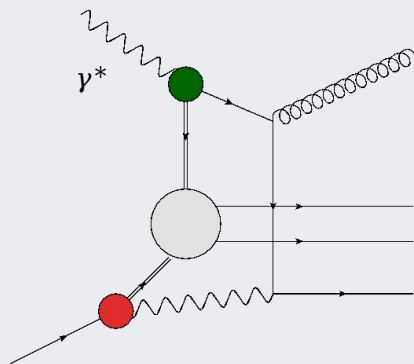
If you are interested any suggestion is welcome!





## 6 New Idea: DPS via $\gamma$ -p interaction

In order to study the impact of the DPS contribution to a process initiated via photon-proton interactions we evaluated the 4-JET photoproduction at HERA (S. Chekanov et al. (ZEUS), Nucl. Phys B792, 1 (2008))



In

- 1) **G. Abbiend et al, Phys. Commun 67, 465 (1992)**
- 2) **J.R. Forshaw et al, Z. Phys. C 72, 637 (1992)**

It has been shown that the agreement with data improves if MPI are included in the Monte Carlo



**WE EVALUATE THE DPS CONTRIBUTION TO THIS PROCESS**

## 6 The effective cross section: a key for the proton structure

The effective cross section can be also written in terms of Fourier Transform of the EFF:

$$\tilde{F}_2^\gamma(z_\perp; Q^2) = \sum_n C_n(Q^2) z_\perp^n$$

$$\left[ \sigma_{\text{eff}}^{\gamma p}(Q^2) \right]^{-1} = \int d^2 z_\perp \tilde{F}_2^p(z_\perp) \tilde{F}_2^\gamma(z_\perp; Q^2)$$
$$= \sum_n C_n(Q^2) \langle (z_\perp)^n \rangle_p$$

If we can measure the dependence of the effective-cross section on the photon VIRTUALITY

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

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$$= \sum_n \underbrace{C_n(Q^2)} \langle (z_\perp)^n \rangle_p$$

This coefficient can be determined from the structure of the photon described in a given approach

If we can measure the dependence of the effective-cross section on the photon VIRTUALITY

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

# 6

## The effective cross section: a key for the proton structure

The effective cross section can be also written in terms of  
Fourier Transform of the EFF:

$$\tilde{F}(z_{\perp})$$

The probability of finding a parton pair at distance

$$z_{\perp}$$

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

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M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

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## The effective cross section: a key for the proton structure

To test if in future a dependence of the effective cross section on the photon virtuality could be observed, we considered again the 4 JET photoproduction:

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501



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To test if in future a dependence of the effective cross section on the photon virtuality could be observed, we considered again the 4 JET photoproduction:

1) We divided the integral of the cross section on  $Q^2$  in two intervals:

$$Q^2 \leq 10^{-2} \quad \text{and} \quad 10^{-2} \leq Q^2 \leq 1 \quad \text{GeV}^2$$

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2) We have estimated for each photon and proton models a constant effective cross section  $\bar{\sigma}_{\text{eff}}^{\gamma\text{P}}$  (with respect to  $Q^2$ ) such that the total integral of the cross section on  $Q^2$  reproduce the full calculation obtained by means of  $\sigma_{\text{eff}}^{\gamma\text{P}}(Q^2)$

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

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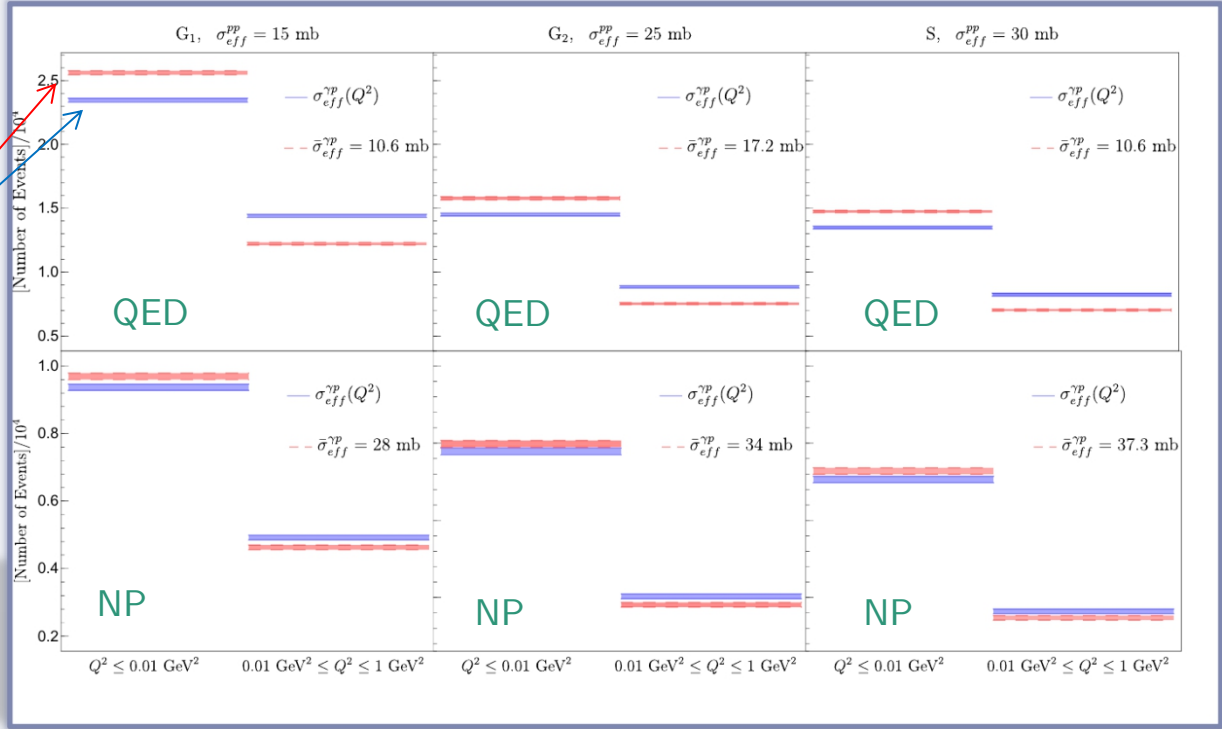
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3) We estimate the minimum luminosity to distinguish the two cases

M. R. and F. A. Ceccopieri, PRD 105 (2022) L011501

# 6 The effective cross section: a key for the proton structure

With an integrated luminosity of  $200 \text{ pb}^{-1}$  we can separate:



# 6 Relativistic effects

- ⊙ Almost model independence
- ⊙ Almost scale independence



**SUGGEST:** *parametrize the impact of Melosh effects in dPDFs to encode **some** general correlations between  $x_i$  and  $k_{\perp}$*

$$\underbrace{F_{ij}(x_1, x_2, k_{\perp}; Q^2)}_{\text{pheno}} = \underbrace{q_i(x_1; Q^2)q_j(x_2; Q^2)}_{\substack{\text{phenomenology from} \\ \text{PDFs}}} \underbrace{\theta(1 - x_1 - x_2)}_{\text{good support}} \underbrace{f(x_1, x_2, Q^2)}_{\text{sum rules}} \underbrace{R(x_1, x_2, k_{\perp})}_{\substack{\text{Melosh effects} \\ \text{correlations!}}} F(k_{\perp}) \} \text{To be modeled:} \\ \text{CQMs, GPDs...}$$

$$R(x_1, x_2, k_{\perp}) \equiv \frac{F_{[L]}^{\text{HO}}(x_1, x_2, k_{\perp}; Q^2)}{F_{[I]}^{\text{HO}}(x_1, x_2, k_{\perp}; Q^2)} = w(k_{\perp}) [x_1 x_2]^{t(k_{\perp})} (1 - x_1 - x_2)^{|x_1 - x_2|} e^{(k_{\perp})} e^{-(1 - x_1 - x_2)h(k_{\perp})}$$

# 6 Relativistic effects

Let us consider the LF expression of the dPDF with its non relativistic (NR) limit:

$$F_{[I]}(x_1, x_2, k_\perp) = \int d\vec{k}_1 d\vec{k}_2 f(\vec{k}_1, \vec{k}_2, k_\perp) \delta\left(x_1 - \frac{k_1^+}{M_P}\right) \delta\left(x_2 - \frac{k_2^+}{M_P}\right) \quad \text{NR}$$

$$F_{[L]}(x_1, x_2, k_\perp) = \int d\vec{k}_1 d\vec{k}_2 f(\vec{k}_1, \vec{k}_2, k_\perp) \langle SPIN | O_1(\vec{k}_1, \vec{k}_2, k_\perp) O_2(\vec{k}_1, \vec{k}_2, k_\perp) | SPIN \rangle \times \delta\left(x_1 - \frac{k_1^+}{M_0}\right) \delta\left(x_2 - \frac{k_2^+}{M_0}\right) \quad \text{LF}$$

$f(\vec{k}_1, \vec{k}_2, k_\perp)$  = product of canonical w.f.  
(momentum space)

Melosh Operators!

*They rotate canonical spin into LF ones*

In the small x region, the main difference between  $F_{[I]}$  and  $F_{[L]}$  is given by the Melosh operators.

## More on the LO QED photon EFF and effective x-section

- 1) Since the photon starts to be a **small** system, the effective-form factor must be similar to a constant (to be properly related to the FT of the probability distribution)
- 2) as a consequence, the effective cross section should be of the same order of that for pp collisions.
- 3) why this two effective x-section are similar if the system are different?
- 4) a possible explanation can be obtained by considering:

$$\frac{\sigma_{eff}}{3\pi} \leq \langle b^2 \rangle \leq \frac{\sigma_{eff}}{\pi}$$

(proven for pp collisions)



Inverting this inequality one gets:

$$\pi \langle b^2 \rangle \leq \sigma_{eff}^{pp} \leq 3\pi \langle b^2 \rangle$$

## More on the LO QED photon EFF and effective x-section

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$$\pi \langle \mathbf{b}^2 \rangle \leq \sigma_{\text{eff}}^{\text{pp}} \leq 3\pi \langle \mathbf{b}^2 \rangle \quad \longrightarrow$$

(proven for pp collisions)

therefore, similar effective x-sections can be related to different **distances**, i.e. **different geometrical structures!**



## (Proton) Model Independent conclusions

- 1) in arXiv:2103.1340 we show that high virtual behavior of the effective cross sections correctly follows the result in **J.R. Gaunt JHEP 01, 042 (2013)**, i.e.:

$$\sigma_{eff}^{\gamma p}(Q^2 \rightarrow \infty) = \sigma_{1v2}^{pp} = \left[ \int \frac{d^2 k_{\perp}}{(2\pi)^2} T_p(k_{\perp}) \right]^{-1}$$

- 2) In Ref. **M.Rinaldi and F.A: Ceccopieri JHEP 09, 097 (2019)**, we prove, in a general framework:

$$\frac{\sigma_{\text{eff},2v1}}{2\pi} \leq \langle b^2 \rangle \leq \frac{2 \sigma_{\text{eff},2v1}}{\pi}$$

therefore, by inverting this relation one gets:

$$\frac{\pi}{2} \langle b^2 \rangle \leq \sigma_{eff}^{\gamma p}(Q^2 \rightarrow \infty) \leq 2\pi \langle b^2 \rangle$$

## (Proton) Model Independent conclusions

$$\frac{\pi}{2} \langle b^2 \rangle \leq \sigma_{eff}^{\gamma p} (Q^2 \rightarrow \infty) \leq 2\pi \langle b^2 \rangle$$

3) in arXiv:2103.1340, for the moment being we considered proton model producing a (2v2) effective cross section of 15-30 mb (**in new analysis we can relax this condition**).

Now in **M. Rinaldi and F. A. Ceccopieri PRD 97 (2018) 7, 071501**, we prove:

$$\frac{\sigma_{eff}^{pp}}{3\pi} \leq \langle b^2 \rangle \leq \frac{\sigma_{eff}^{pp}}{\pi}$$

combining everything:

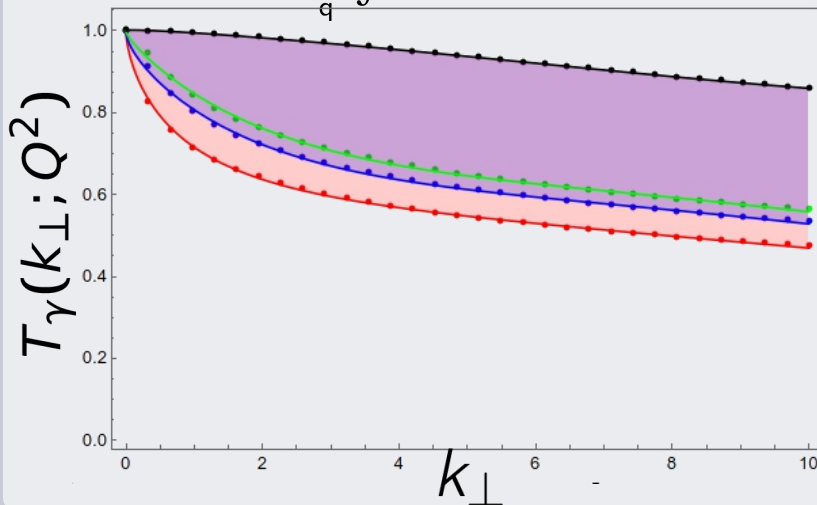
**VERIFIED!!**

$$\frac{\sigma_{eff}^{pp}}{6} \leq \sigma_{eff}^{\gamma p} (Q^2 \rightarrow \infty) \leq 2\sigma_{eff}^{pp}$$

## More on the LO QED photon EFF and effective x-section

$$T_\gamma(k_\perp; Q^2) = \frac{\sum_q \int dx f_{q,\bar{q}}^\gamma(x, k_\perp; Q^2)}{\sum_q \int dx f_{q,\bar{q}}^\gamma(x, k_\perp = 0; Q^2)}$$

$$f_{q,\bar{q}}^\gamma(x, \tilde{k}_\perp; Q^2) = \int d^2k_{\perp,1} \psi_{q\bar{q}}^{\dagger\gamma}(x, \vec{k}_{\perp,1}; Q^2) \times \psi_{q\bar{q}}^\gamma(x, \vec{k}_{\perp,1} + \vec{k}_\perp; Q^2)$$



$$Q^2 = 10 \text{ GeV}^2 \quad \langle z_\perp^2 \rangle_\gamma \propto \frac{1}{Q^2}$$

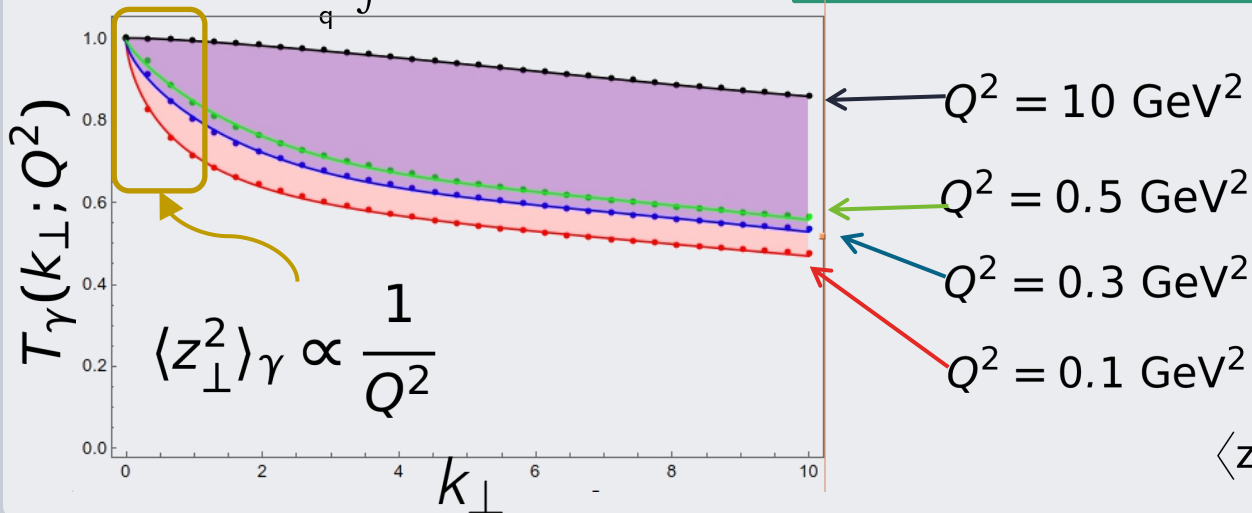
Small system  $\rightarrow$  EFF **SLOWLY** decreasing:

$$T_\gamma(k_\perp; Q^2 \gg 1 \text{ GeV}^2) \sim 1$$

## More on the LO QED photon EFF and effective x-section

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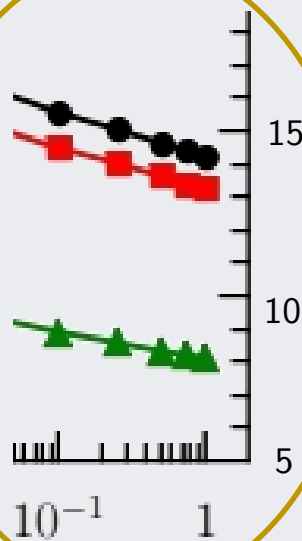
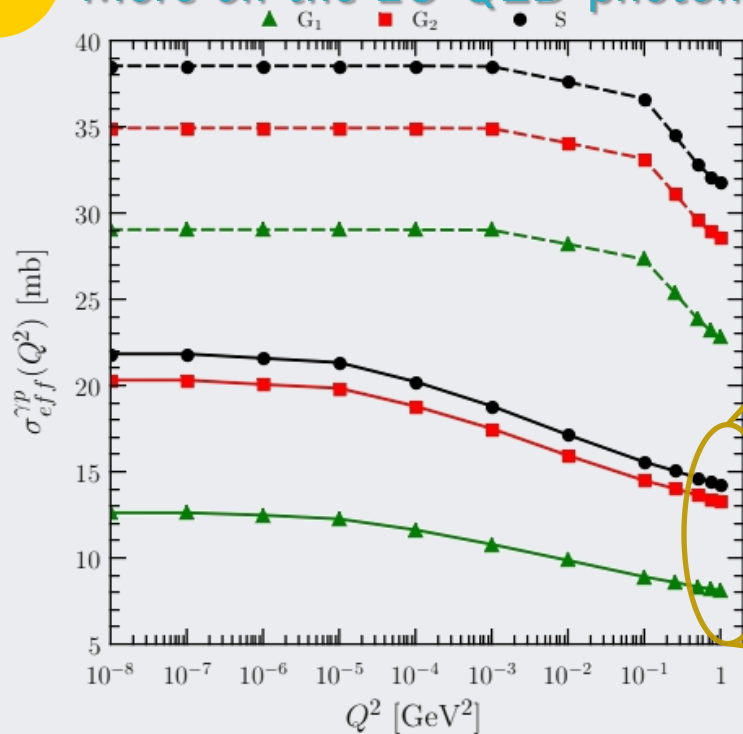
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$$\langle z_\perp^2 \rangle_\gamma \propto \frac{1}{Q^2}$$

$$\langle z_\perp^2 \rangle \propto \left. \frac{d}{dk_\perp} T(k_\perp) \right|_{k_\perp=0}$$

## More on the LO QED photon EFF and effective x-section

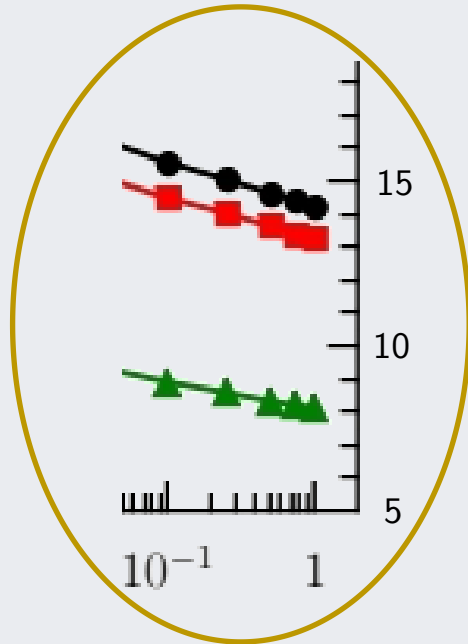


$\sim 30/2$  mb

$\sim 25/2$  mb

$\sim 15/2$  mb

## More on the LO QED photon EFF and effective x-section



~ 30/2 mb  
~ 25/2 mb

~ 15/2 mb

$$[\sigma_{eff}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi)^2} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$$

$$[\sigma_{eff}^{\gamma p}(Q^2)]^{-1} \underset{Q^2 \gg 1}{\sim} \int \frac{d^2 k_{\perp}}{(2\pi)^2} T_p(k_{\perp}) \times 1$$

For the proton models we have used:


$$\int \frac{d^2 k_{\perp}}{(2\pi)^2} T_p(k_{\perp}) \sim 2 \int \frac{d^2 k_{\perp}}{(2\pi)^2} T_p(k_{\perp})^2$$



$$\sigma_{eff}^{\gamma p}(Q^2 \gg 1 \text{ GeV}^2) \sim \sigma_{eff}^{pp}/2$$

# 1 Double PDFs (intrinsic) of the proton

$F_{ik}(x_1, x_2, \vec{z}_\perp)$  is unknown. However @LHC kinematics (small  $x$  and many partons produced)

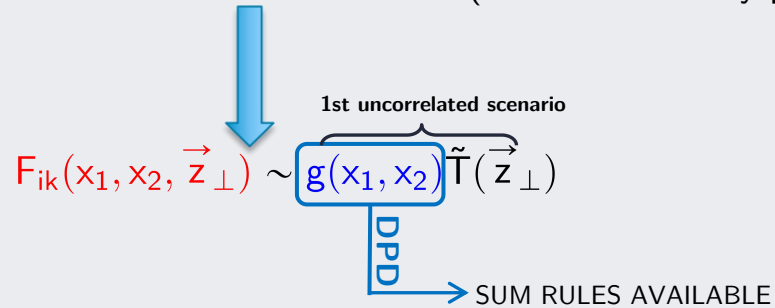


1st uncorrelated scenario

$$F_{ik}(x_1, x_2, \vec{z}_\perp) \sim g(x_1, x_2) \tilde{T}(\vec{z}_\perp)$$

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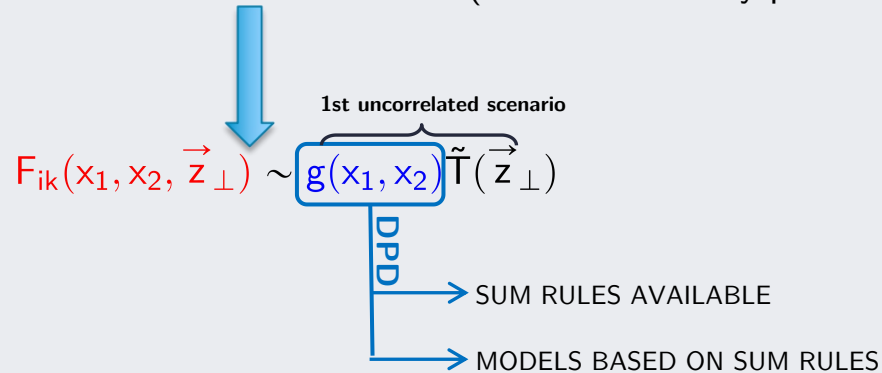
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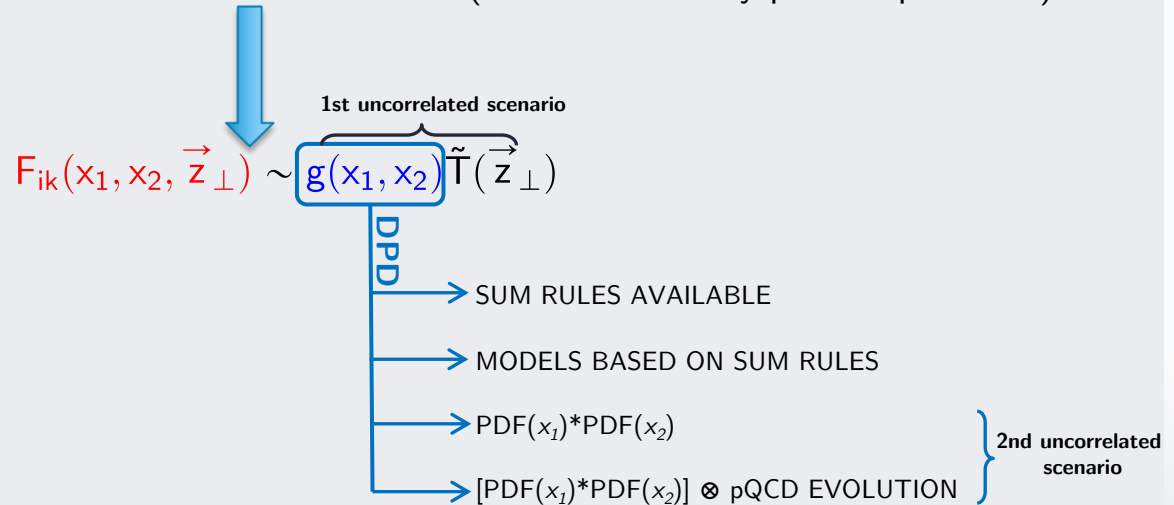
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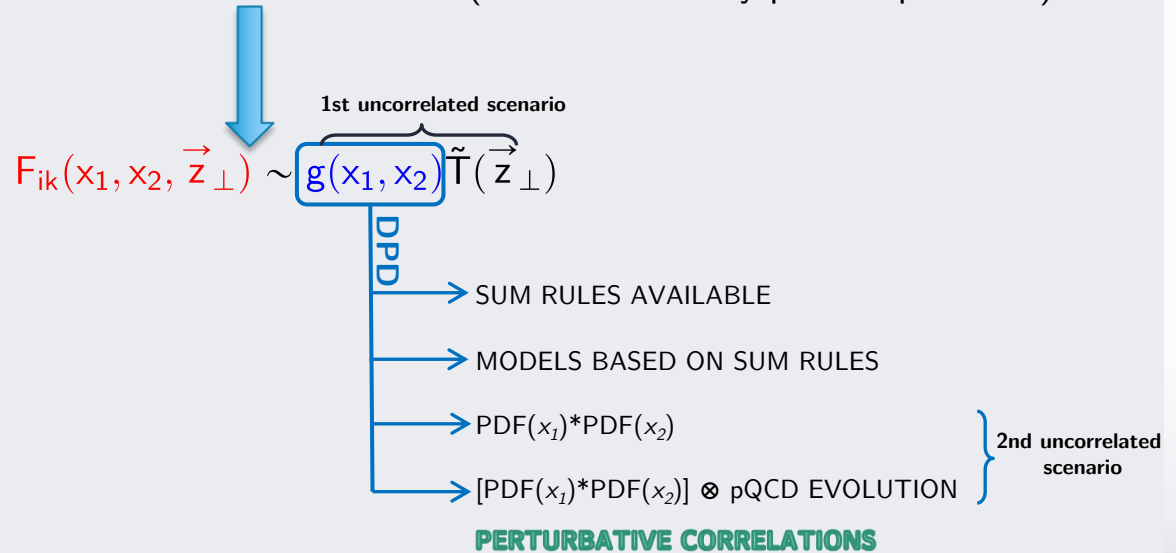
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J. R. Gaunt and W. J. Stirling, JHEP 03, 005 (2010)

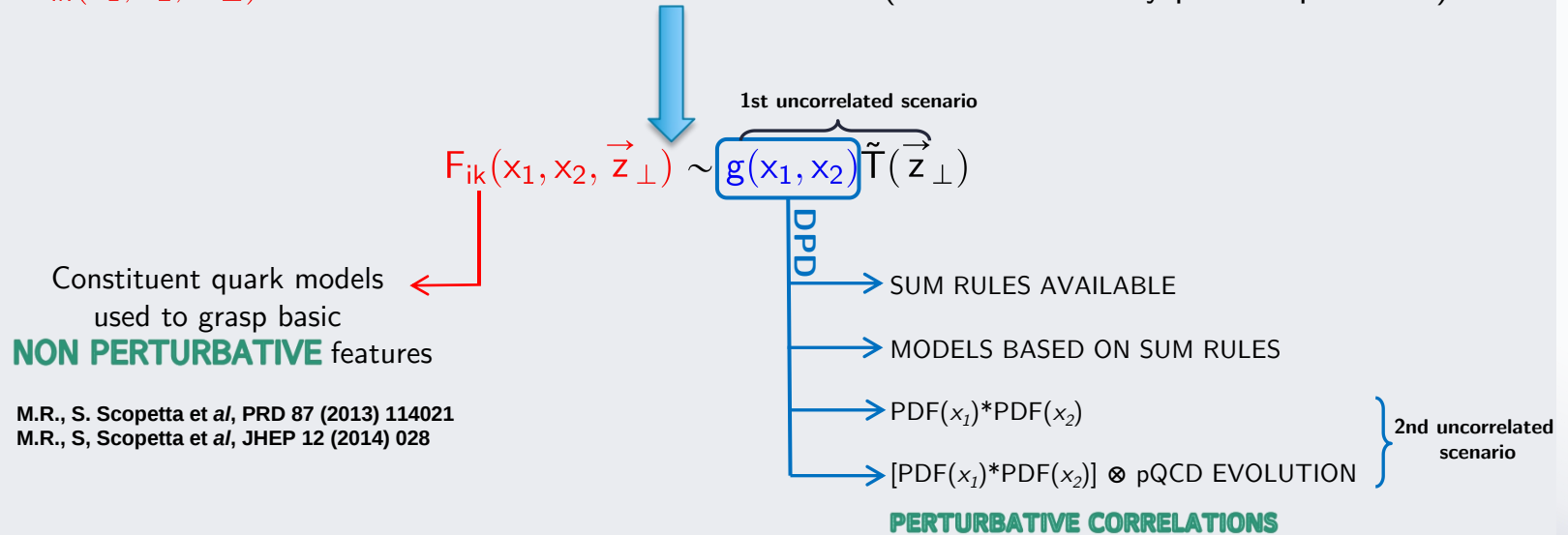
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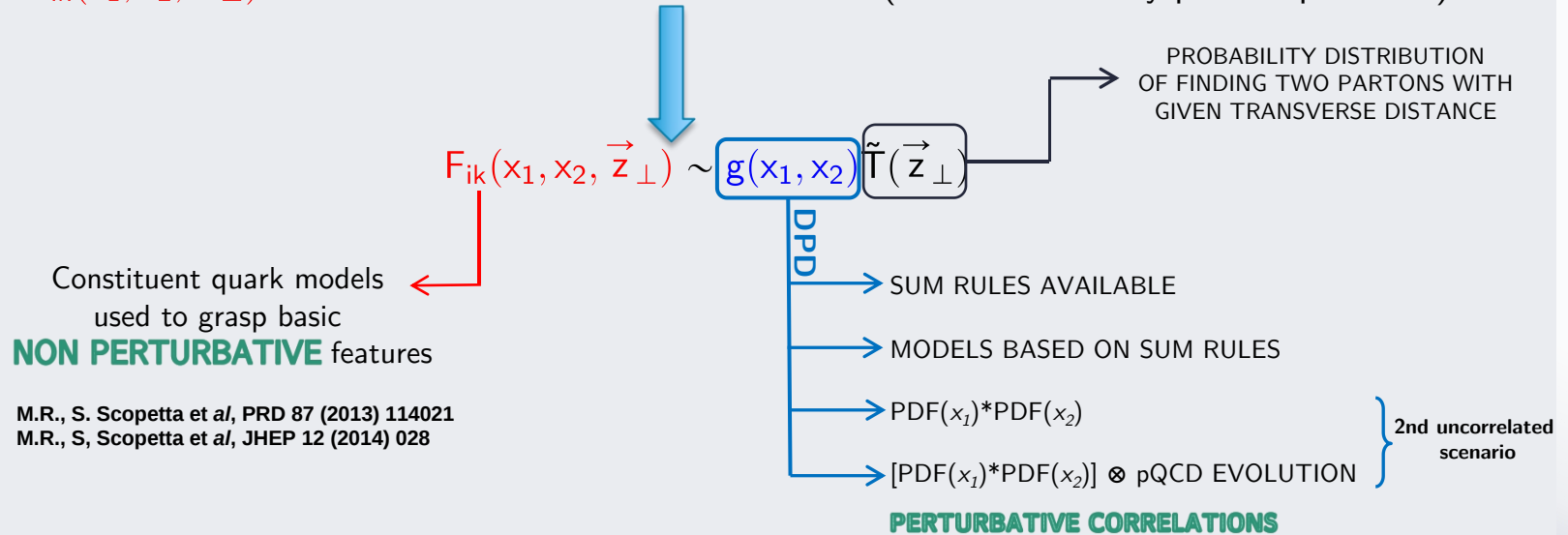
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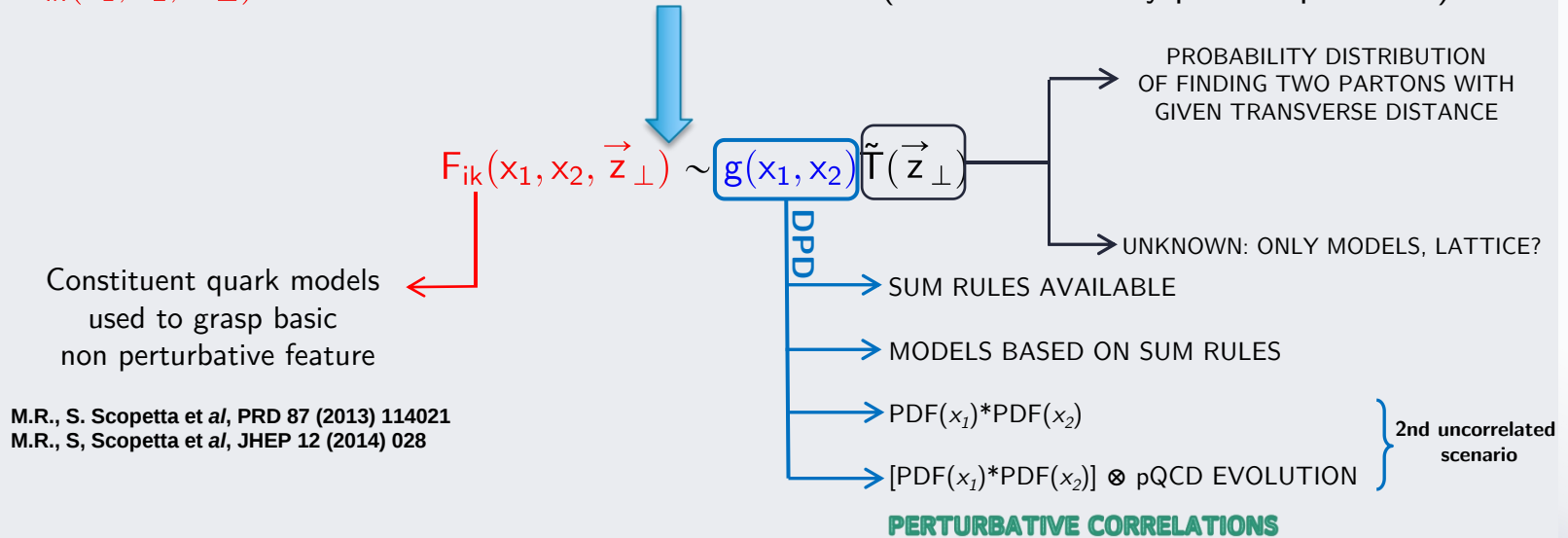
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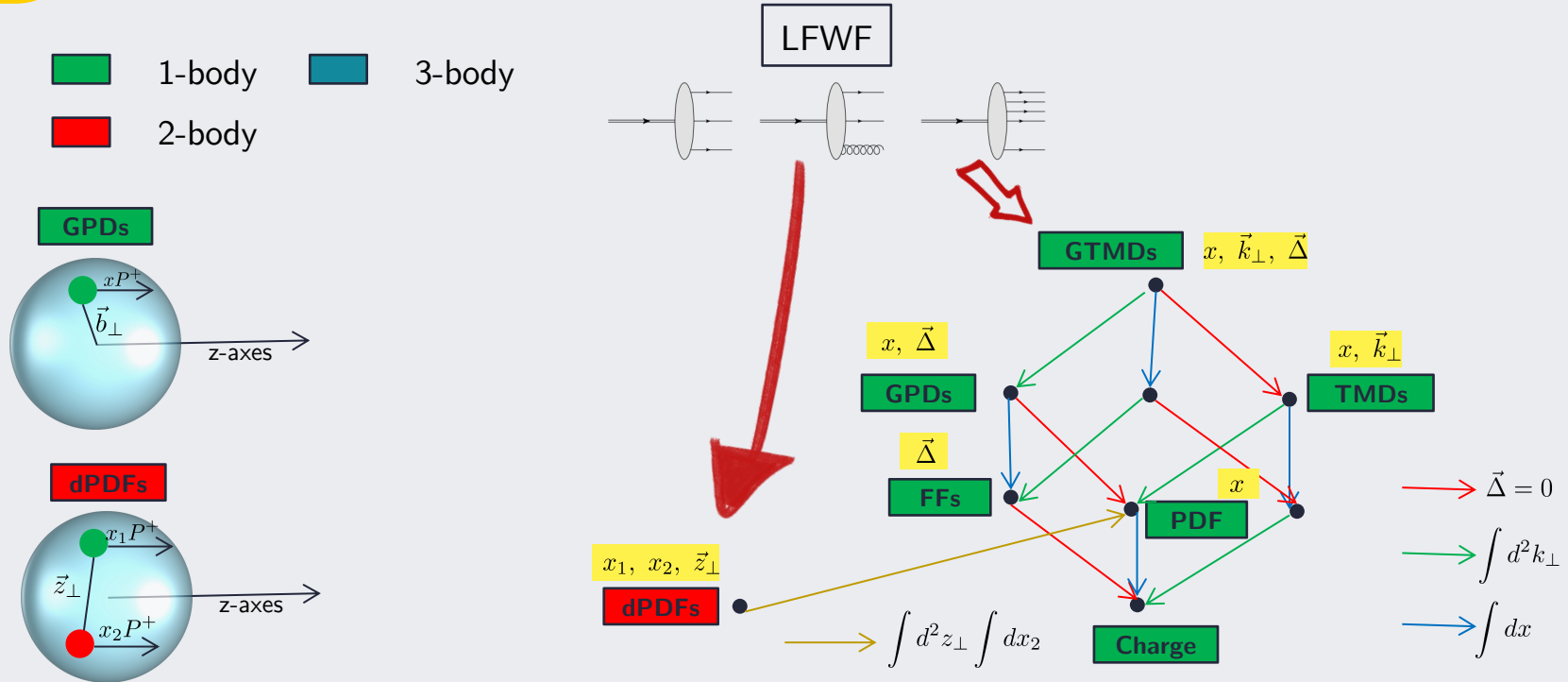


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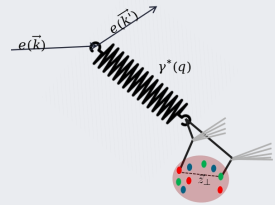
$F_{ik}(x_1, x_2, \vec{z}_\perp)$  is unknown. However @LHC kinematics (small  $x$  and many partons produced)



# 1 Multidimensional Pictures of Hadron



# 6 The $\gamma$ -p effective cross section



The expression of this quantity is very similar to the one for the collision case and can be formally derived from the effective cross sections and the DPS on a proton. The DPS from a

## 1 INGREDIENTS OF THE CALCULATIONS

$$[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2k_{\perp}}{(2\pi)^2}$$

$$T_p(k_{\perp}) \quad \text{proton EFF}$$

$$\int dx f_{q,\bar{q}}^{\gamma}(x, k_{\perp}; Q^2)$$

## 2

$$f_{q,\bar{q}}^{\gamma}(x, \tilde{k}_{\perp}; Q^2) = \int d^2k_{\perp}$$

$$\psi/\gamma \quad \text{Photon WF}$$

$$\int dx f_{q,\bar{q}}^{\gamma}(x, k_{\perp} = 0; Q^2)$$

## 3

$$\int d^2k_{\perp} d^2k'_{\perp} (x' k'_{\perp} = 0; Q^2)$$

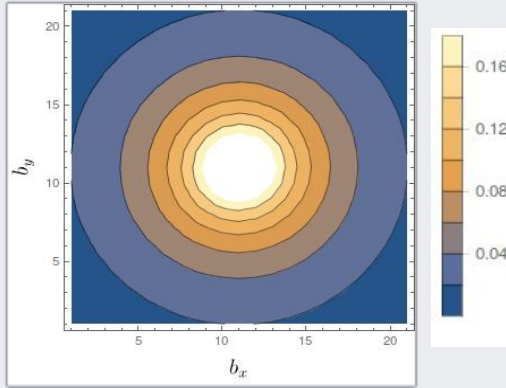


$$q: (1-x, k_{\perp,1})$$

M. R. and F. A. Ceccopieri, arXiv:2103.13480

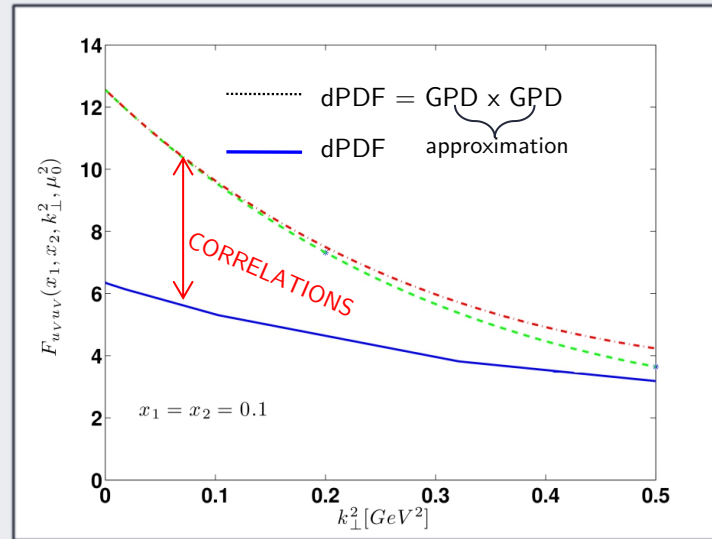


## 2 Information from Quark Models



M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

1) e.g. the distance distribution of two gluons in the proton



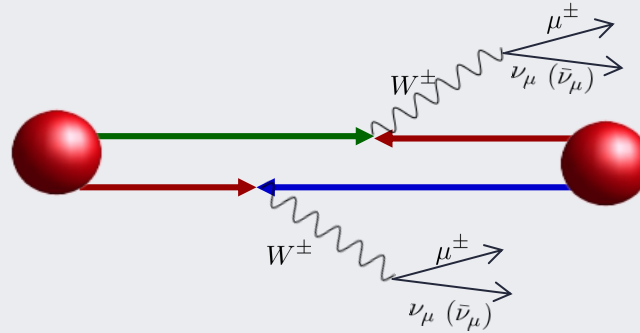
2) Correlations are important

M.R., S. Scopetta et al,  
JHEP 10 (2016) 063

M.R. and F. A. Ceccopieri  
PRD 95 (2017) 034040

## 4 Same sign W's production at the LHC

M. R. et al, Phys.Rev.  
D95 (2017) no.11,  
114030



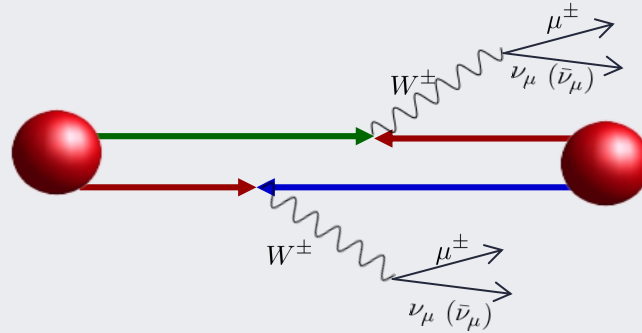
In this channel, the single parton scattering (usually dominant w.r.t to the double one) starts to contribute to higher order in strong coupling constant.



*“Same-sign W boson pairs production is a promising channel to look for signature of double Parton interactions at the LHC.”*

## 4 Same sign $W$ 's production at the LHC

M. R. et al, Phys.Rev.  
D95 (2017) no.11,  
114030



In this channel, the single parton scattering (usually dominant w.r.t to the double one) starts to contribute to higher order in strong coupling constant.

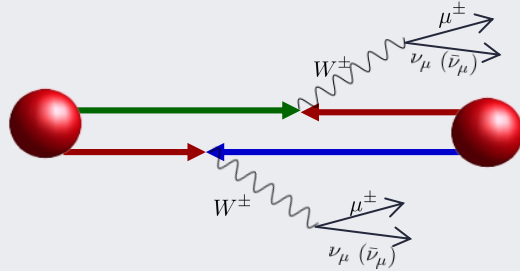


**Can double parton correlations be observed for the first time in the next LHC run ?**

# 4 Same sign W's production at the LHC

M. R. et al, Phys.Rev.  
D95 (2017) no.11,  
114030

Kinematical cuts



$$\begin{aligned}
 &pp, \sqrt{s} = 13 \text{ TeV} \\
 &p_{T,\mu}^{\text{leading}} > 20 \text{ GeV}, \quad p_{T,\mu}^{\text{subleading}} > 10 \text{ GeV} \\
 &|p_{T,\mu}^{\text{leading}}| + |p_{T,\mu}^{\text{subleading}}| > 45 \text{ GeV} \\
 &|\eta_\mu| < 2.4 \\
 &20 \text{ GeV} < M_{\text{inv}} < 75 \text{ GeV} \text{ or } M_{\text{inv}} > 105 \text{ GeV}
 \end{aligned}$$

**DPS cross section:**

$$\frac{d^4 \sigma_{pp \rightarrow \mu^\pm \mu^\pm X}}{d\eta_1 dp_{T,1} d\eta_2 dp_{T,2}} = \sum_{i,k,j,l} \frac{1}{2} \int d^2 \vec{b}_\perp F_{ij}(x_1, x_2, \vec{b}_\perp, M_W) F_{kl}(x_3, x_4, \vec{b}_\perp, M_W) \frac{d^2 \sigma_{ik}^{pp \rightarrow \mu^\pm X}}{d\eta_1 dp_{T,1}} \frac{d^2 \sigma_{jl}^{pp \rightarrow \mu^\pm X}}{d\eta_2 dp_{T,2}} \mathcal{I}(\eta_i, p_{T,i})$$

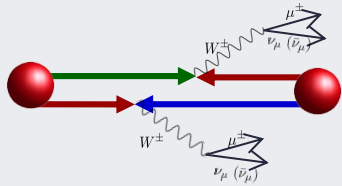
In order to estimate the role of double parton correlations we have used as input of dPDFs:

1) Longitudinal and transverse correlations arise from the relativistic CQM model describing three valence quarks

2) These correlations propagate to sea quarks and gluons through pQCD evolution

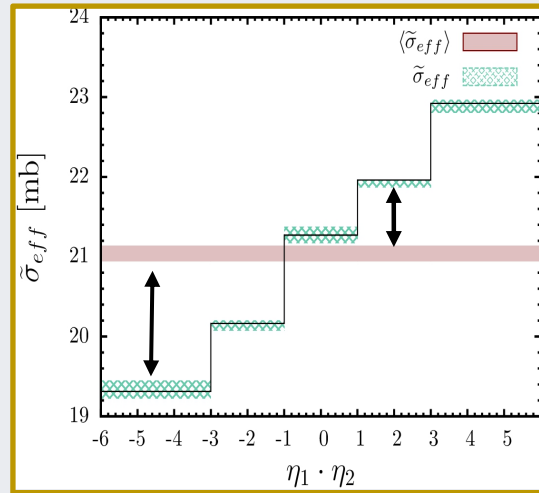
# 4 Same sign W's production at the LHC

M. R. et al, Phys.Rev.  
D95 (2017) no.11,  
114030



$$\eta_1 \cdot \eta_2 \simeq \frac{1}{4} \ln \frac{x_1}{x_3} \ln \frac{x_2}{x_4}$$

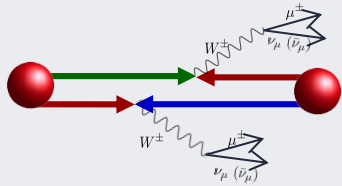
$$\langle \tilde{\sigma}_{eff} \rangle = 21.04^{+0.07}_{-0.07} (\delta Q_0)^{+0.06}_{-0.07} (\delta \mu_F) \text{ mb} .$$



Difference  $\left[ \updownarrow \right]$  between  
green and red line is due  
to correlations effects

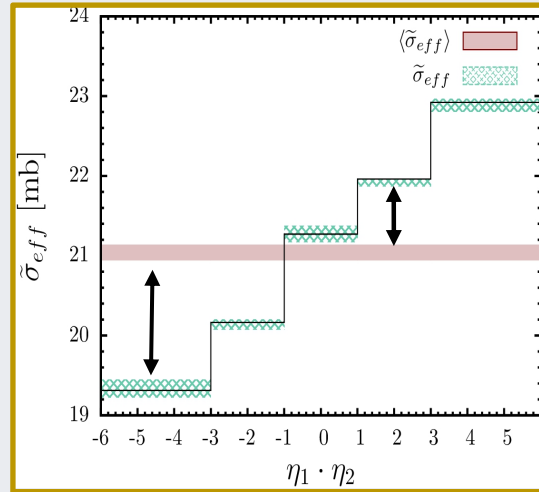
# 4 Same sign W's production at the LHC

M. R. et al, Phys.Rev. D95 (2017) no.11, 114030



$$\eta_1 \cdot \eta_2 \simeq \frac{1}{4} \ln \frac{x_1}{x_3} \ln \frac{x_2}{x_4}$$

$$\langle \tilde{\sigma}_{eff} \rangle = 21.04^{+0.07}_{-0.07} (\delta Q_0)^{+0.06}_{-0.07} (\delta \mu_F) \text{ mb} .$$



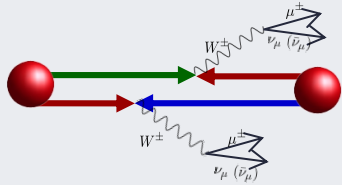
x- dependence of effective x-section  
M.Rinaldi et al PLB 752,40 (2016)  
M. Traini, M. R. et al, PLB 768, 270 (2017)

Assuming that the results of the first and the last bins can be distinguished if they differ by 1 sigma, we estimated that:

$$\mathcal{L} = 1000 \text{ fb}^{-1}$$

is necessary to observe correlations  
\* to be updated to new CMS cuts

## 4 Same sign $W$ 's production at the LHC

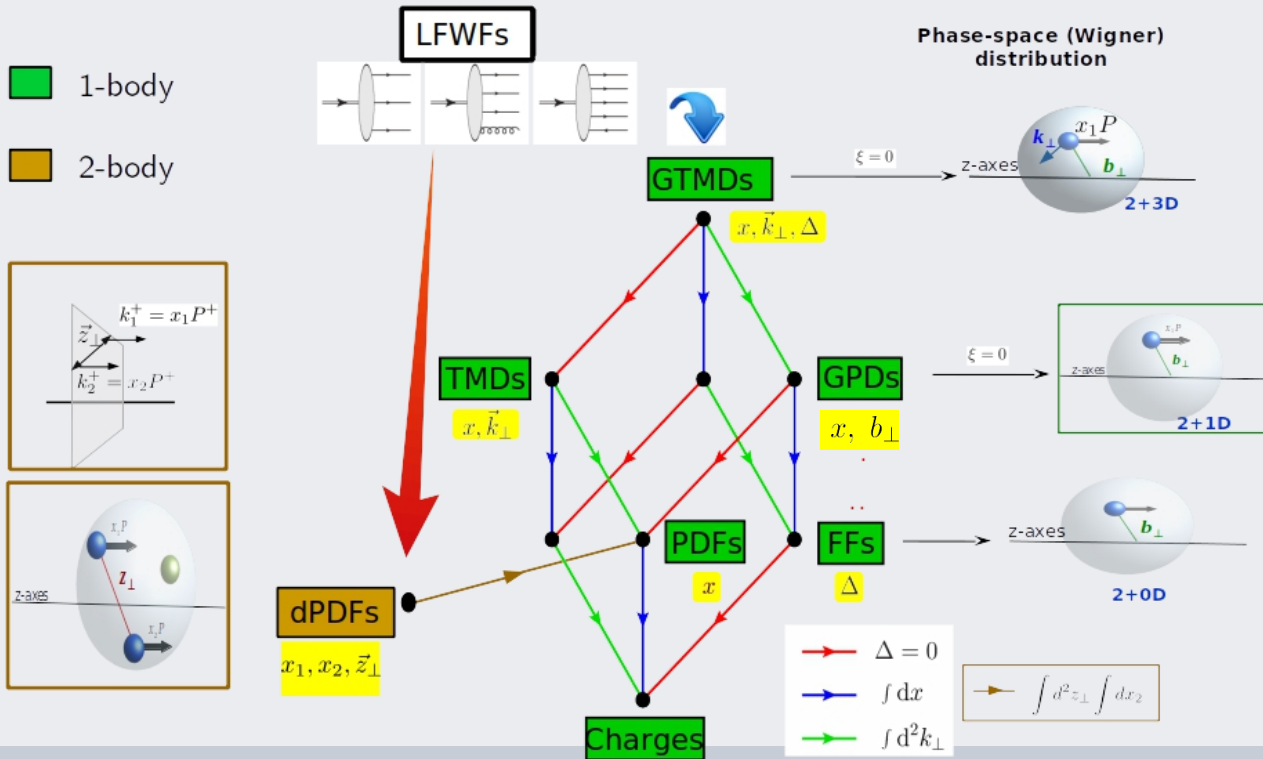


In Ref. **S. Cotogno et al, JHEP 10 (2020) 214**, it has been shown that several experimental observable are sensitive to **double spin correlations**.

The LHC has the potential to access these new information!

*IN THIS CHANNEL, WE ESTABLISHED THE POSSIBILITY TO OBSERVE, FOR THE FIRST TIME, TWO-PARTON CORRELATIONS IN THE NEXT LHC RUN!*

# 2 Multidimensional Pictures of Hadron

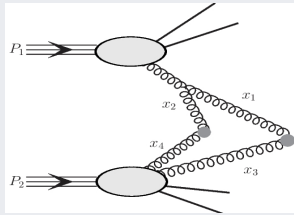




# 4 Further implementations

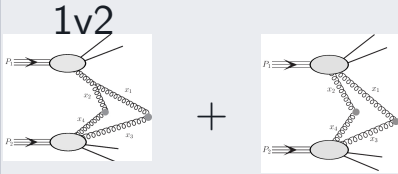
Relevant for processes involving heavy particles is the splitting term (1v2 mechanism) i.e.:

$$*D_{j_1 j_2}(x_1, x_2) = \int d^2 b_{\perp} \tilde{F}_{j_1 j_2}(x_1, x_2, b_{\perp})$$



In pQCD evolution:  $\frac{dD_{j_1 j_2}(x_1, x_2; t)}{dt} = \left\{ \begin{array}{l} \text{Homogeneous term (double DGLAP)} \\ + \\ \sum_{j'} F_{j'}(x_1 + x_2; t) \underbrace{\frac{1}{x_1 + x_2} P_{j' \rightarrow j_1 j_2}}_{\text{SPLITTING TERM}} \left( \frac{x_1}{x_1 + x_2} \right) \end{array} \right.$

Gaunt J.R. and Stirling W. J., JHEP 03 (2010)



J.R. Gaunt, R. Maciula and A. Szczurek, PRD 90 (2014) 054017

2v2

$$\frac{\sigma_{eff}}{3\pi} \left( 1 + \frac{3}{2} r_v \right) \leq \langle b^2 \rangle \leq \frac{\sigma_{eff}}{\pi} \left( 1 + 2 r_v \right)$$

SPLITTING TERM

$$r_v \sim \frac{F_{j_1 j_2}^{splitting}(x_1, x_2, k_{\perp} = 0; t)}{F_{j_1 j_2}(x_1, x_2, k_{\perp} = 0; t)}$$

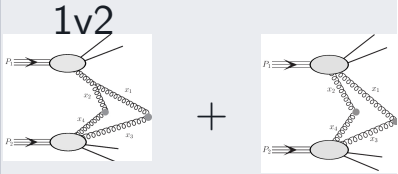
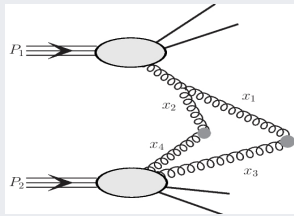
Due to the difficulty in the estimate of the 2 contributions:

with:  
 $0 \leq r_v \leq 1$

Absolute minimum  $r_v = 0$   $\frac{\sigma_{eff}}{3\pi} \leq \langle b^2 \rangle \leq \frac{3 \sigma_{eff}}{\pi}$  Absolute maximum  $r_v = 1$

# 4 Further implementations

Relevant for processes involving heavy particles is the splitting term (1v2 mechanism) i.e.:

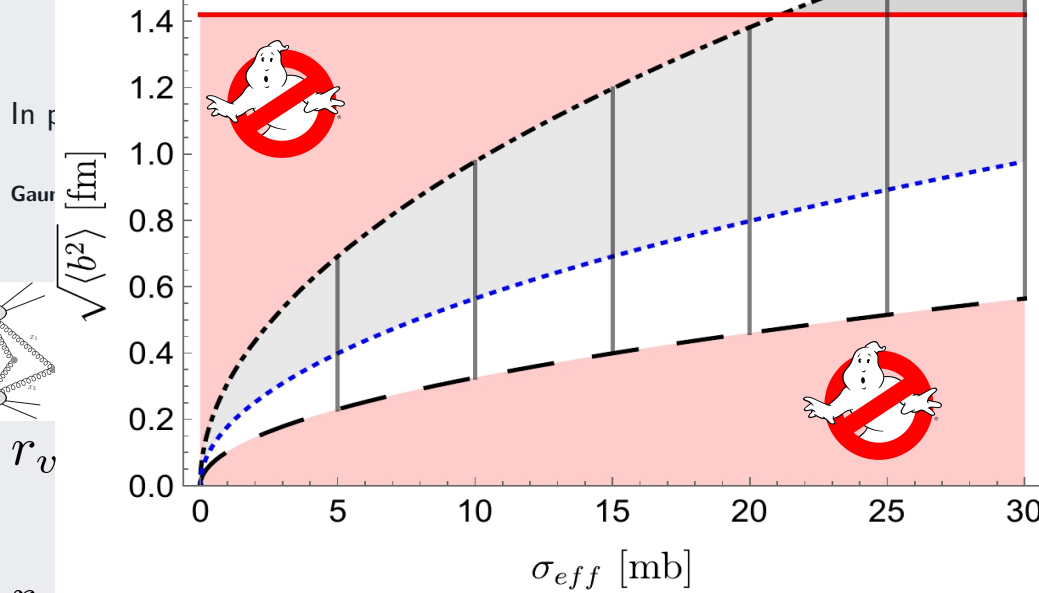


1) Minimum as function of  $r_v$

$$m(r_v)$$

2) Maximum as function of  $r_v$

$$M(r_v)$$



Absolute minimum  $\frac{\sigma_{eff}}{3\pi} \leq \langle b^2 \rangle \leq \frac{3\sigma_{eff}}{\pi}$  Absolute maximum

$r_v = 0$   $r_v = 1$

$$F_{j_1 j_2}(x_1, x_2, k_{\perp} = 0; t) = \int d^2 b_{\perp} \tilde{F}_{j_1 j_2}(x_1, x_2, b_{\perp})$$

(DGLAP)

$$F_{j_1 j_2} \left( \frac{x_1}{x_1 + x_2} \right)$$

$$\frac{F_{j_1 j_2}^{splitting}(x_1, x_2, k_{\perp} = 0; t)}{F_{j_1 j_2}(x_1, x_2, k_{\perp} = 0; t)}$$

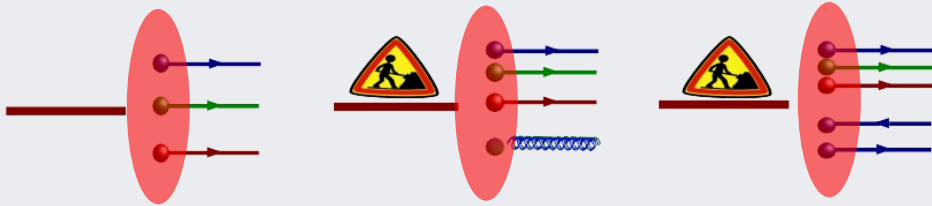
with  $0 \leq r_v \leq 1$

## 2 Double PDFs within the Light-Front

Extending the procedure developed in **S. Boffi, B. Pasquini and M. Traini, Nucl. Phys. B 649, 243 (2003)** for GPDs, we obtained the following expression of the **dPDF** in momentum space, often called  **$_2$ GPDs**:

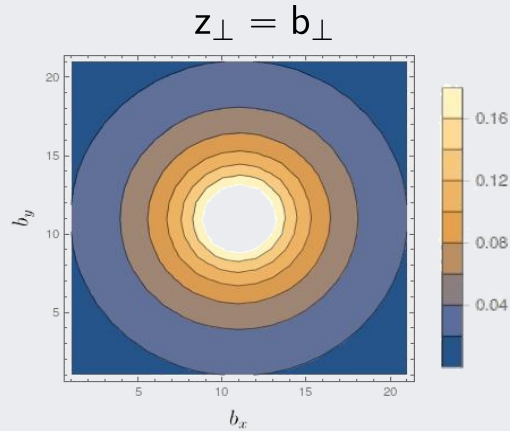
$$F_{ij}(x_1, x_2, k_\perp) = 3(\sqrt{3})^3 \int \prod_{i=1}^3 d\vec{k}_i \delta\left(\sum_{i=1}^3 \vec{k}_i\right) \underbrace{\Phi^*(\{\vec{k}_i\}, k_\perp) \Phi(\{\vec{k}_i\}, -k_\perp)}_{\text{LF wave-function}}$$

Conjugate to  $z_\perp$   $\times \delta\left(x_1 - \frac{k_1^+}{P_+}\right) \delta\left(x_2 - \frac{k_2^+}{P_+}\right)$



$$\Phi(\{\vec{k}_i\}, \pm k_\perp) = \Phi\left(\vec{k}_1 \pm \frac{\vec{k}_\perp}{2}, \vec{k}_2 \mp \frac{\vec{k}_\perp}{2}, \vec{k}_3\right)$$

## 2 Information from Quark Models

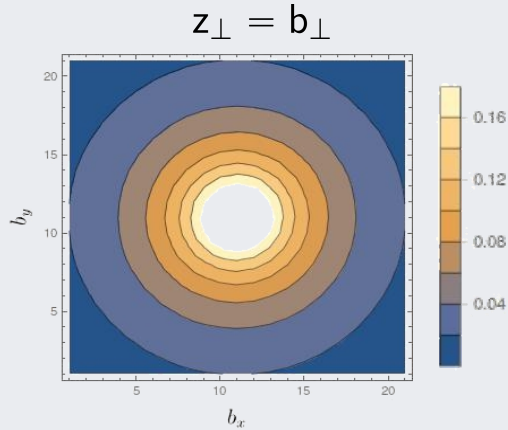


M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

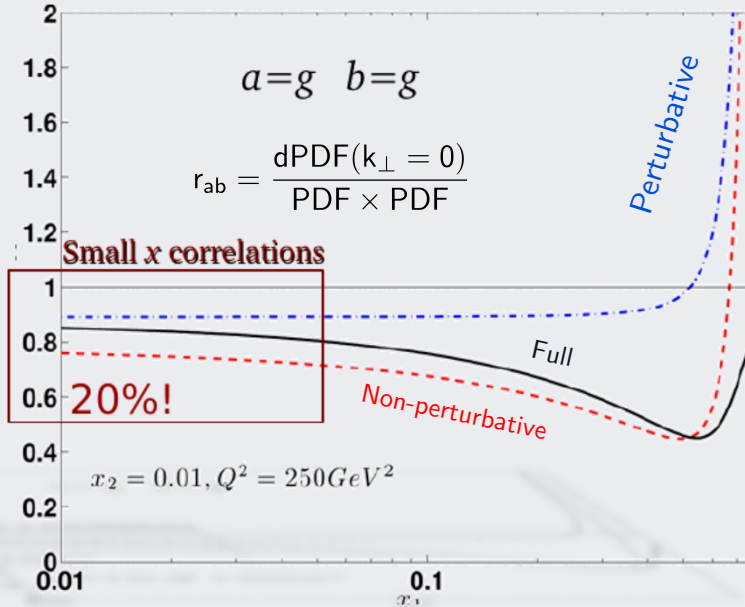
1) e.g. the distance distribution of **two gluons** in the proton

$$\langle z_{\perp}^2 \rangle_{x_1, x_2}^{ij} = \frac{\int d^2 z_{\perp} z_{\perp}^2 F_{ij}(x_1, x_2, z_{\perp})}{\int d^2 z_{\perp} F_{ij}(x_1, x_2, z_{\perp})}$$

## 2 Information from Quark Models



M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097



2) Correlations are important


M.R., S. Scopetta et al, JHEP 10 (2016) 063


M.R. and F. A. Ceccopieri PRD 95 (2017) 034040

## 4 Further implementations

IF WE DO NOT CONSIDER ANY FACTORIZATION ANSATZ IN DOUBLE PDFs:

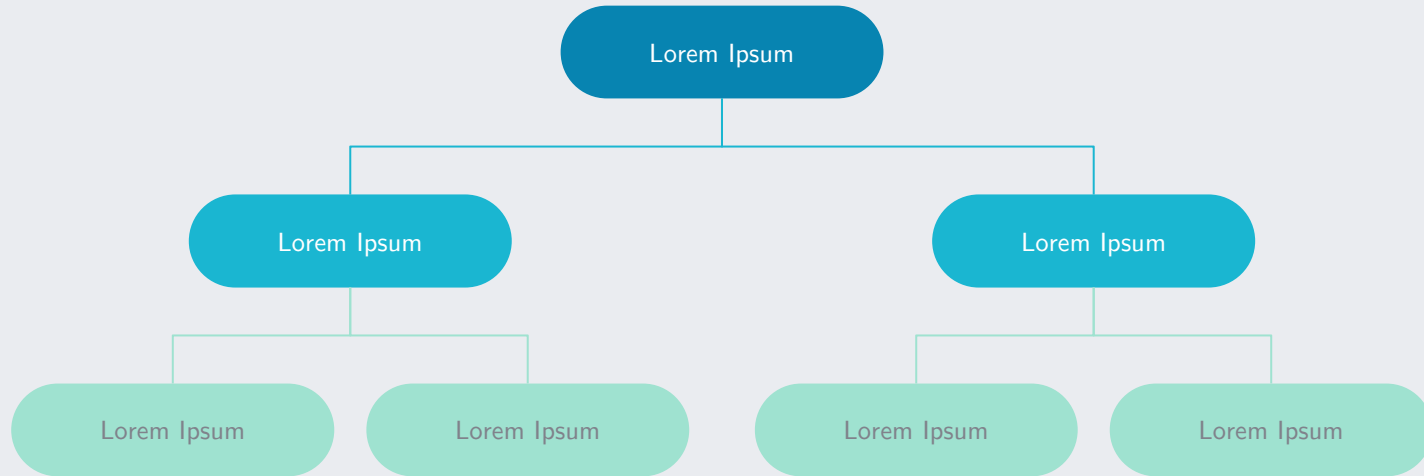
$$\frac{\sigma_{eff}(x_1, x_2)}{3\pi} \left[ r^{2v_2}(x_1, x_2)^2 + \frac{3}{2} r^{2v_1}(x_1, x_2)^2 r_v \right] \leq \langle b^2 \rangle_{x_1, x_2} \leq \frac{\sigma_{eff}(x_1, x_2)}{\pi} \left[ r^{2v_2}(x_1, x_2)^2 + 2r^{2v_1}(x_1, x_2)^2 r_v \right]$$


$$r^{2v_2}(x_1, x_2) = \frac{F(x_1, x_2, k_{\perp} = 0; t)}{F(x_1; t)F(x_2; t)}$$


$$r^{2v_1}(x_1, x_2) = \frac{F^{splitting}(x_1, x_2, k_{\perp} = 0; t)}{F(x_1; t)F(x_2; t)}$$



# Use diagrams to explain your ideas

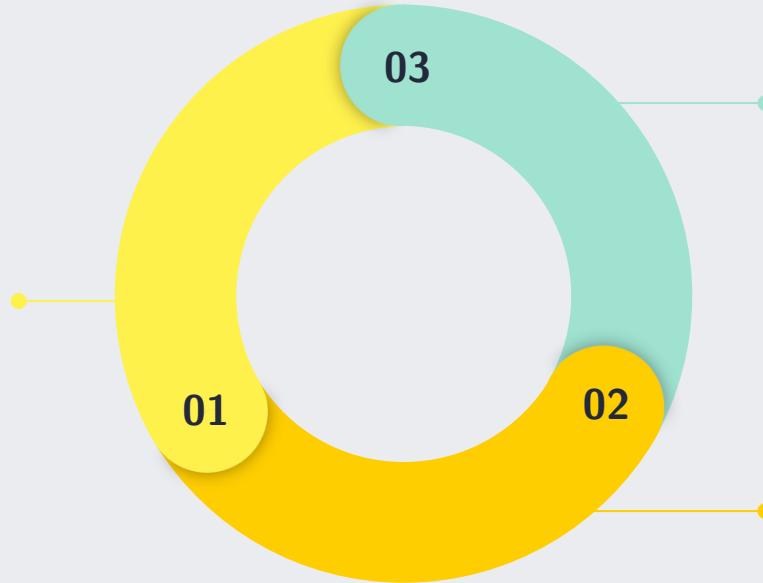




# Our process is easy

## Vestibulum congue tempus

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor. Donec facilisis lacus eget mauris.



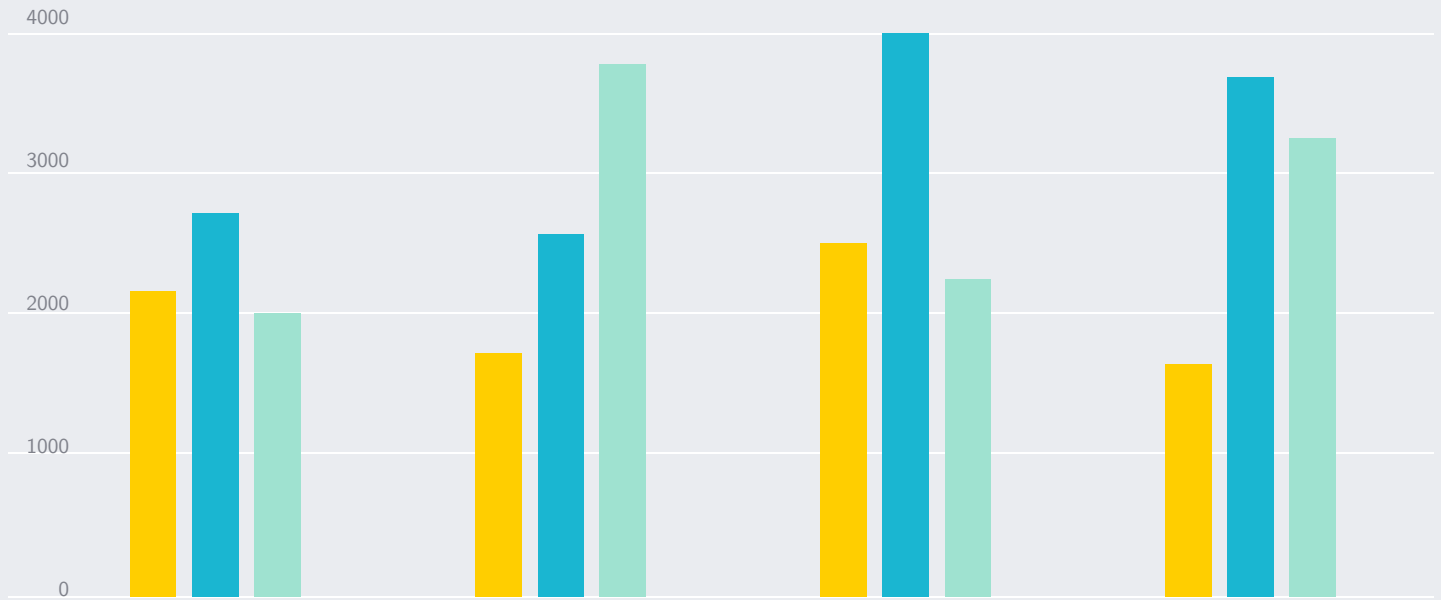
## Vestibulum congue tempus

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor. Donec facilisis lacus eget mauris.

## Vestibulum congue tempus

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor. Donec facilisis lacus eget mauris.





You can insert graphs from Excel or Google Sheets



## Desktop project

Show and explain  
your web, app or  
software projects  
using these gadget  
templates.





# Timeline

Blue is the colour of the clear sky and the deep sea

Red is the colour of danger and courage

Black is the color of ebony and of outer space

Yellow is the color of gold, butter and ripe lemons

White is the color of milk and fresh snow

Blue is the colour of the clear sky and the deep sea

JAN

FEB

MAR

APR

MAY

JUN

JUL

AUG

SEP

OCT

NOV

DEC

Yellow is the color of gold, butter and ripe lemons

White is the color of milk and fresh snow

Blue is the colour of the clear sky and the deep sea

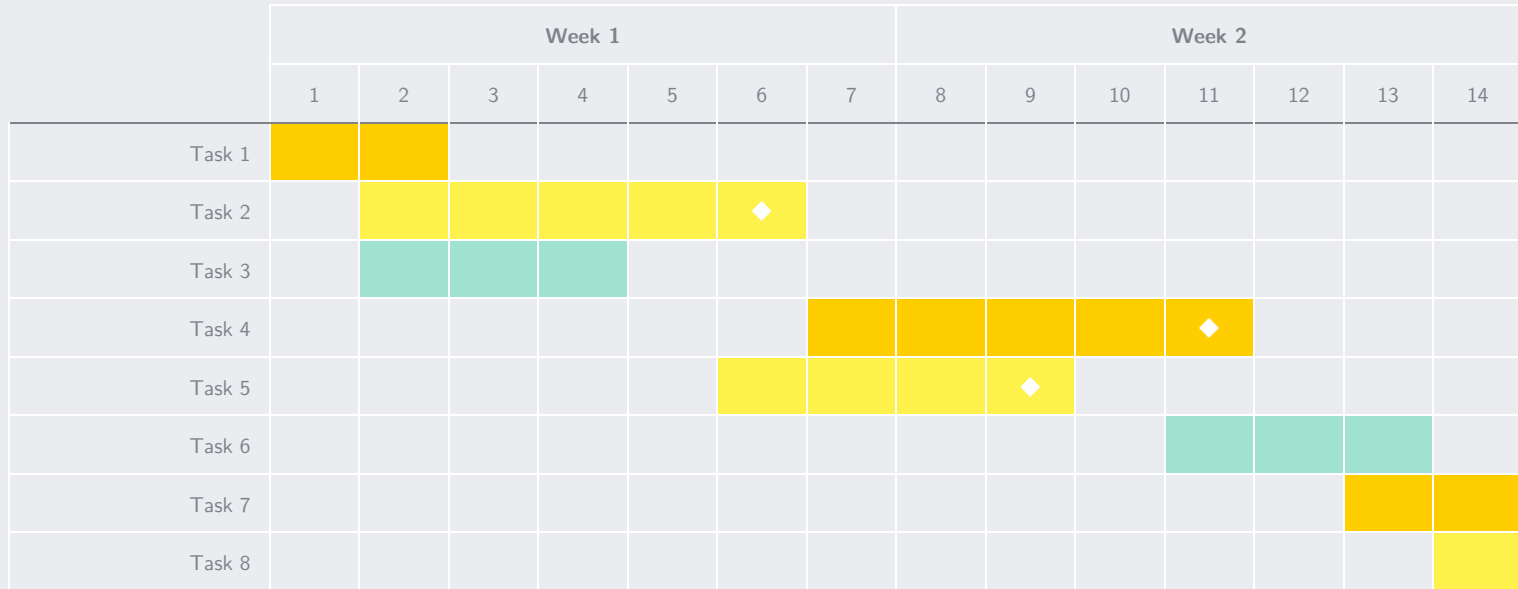
Red is the colour of danger and courage

Black is the color of ebony and of outer space

Yellow is the color of gold, butter and ripe lemons



# Gantt chart





# SWOT Analysis

## STRENGTHS

Blue is the colour of the clear sky and the deep sea

S

W

## WEAKNESSES

Yellow is the color of gold, butter and ripe lemons

O

T

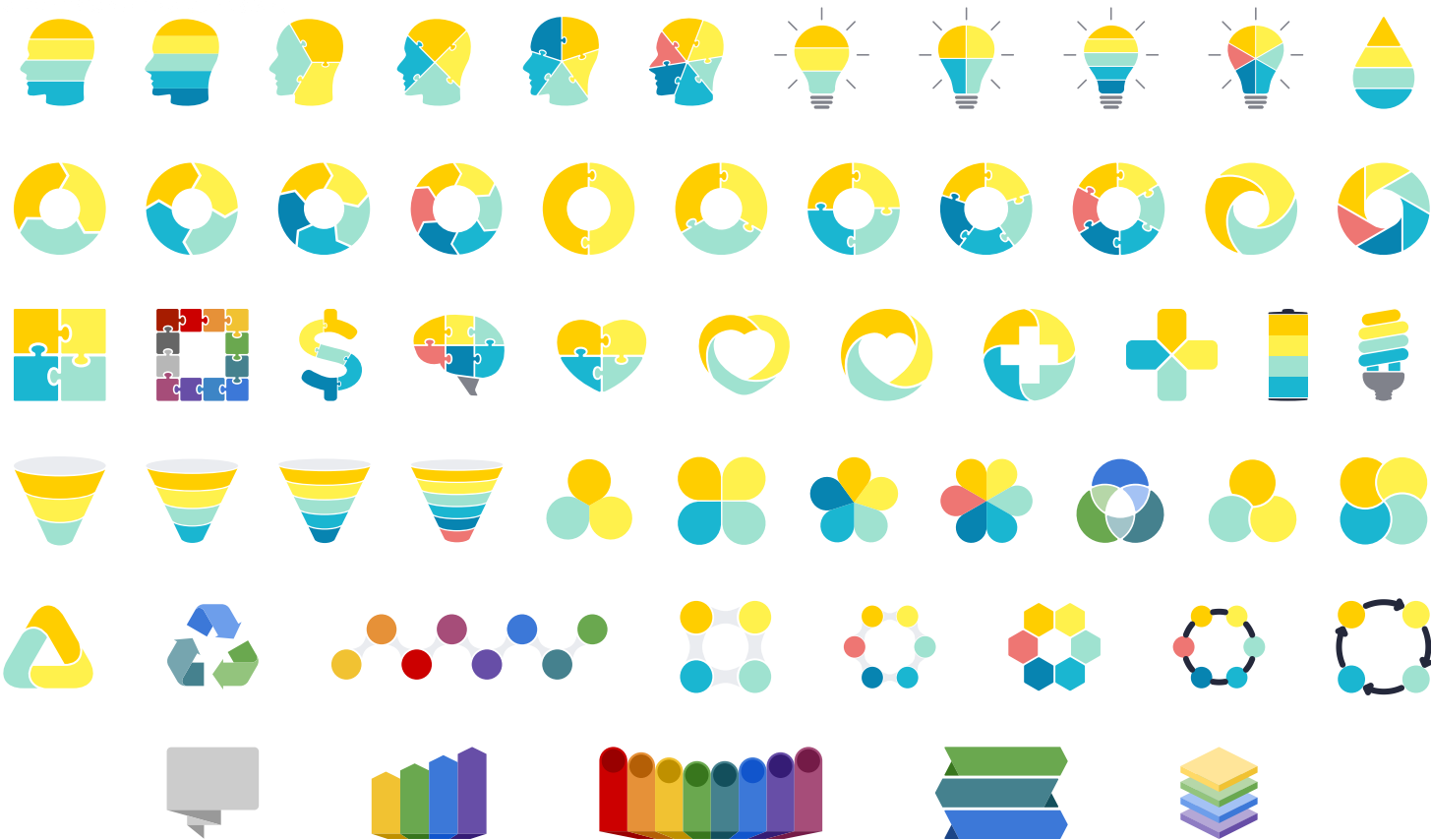
Black is the color of ebony and of outer space

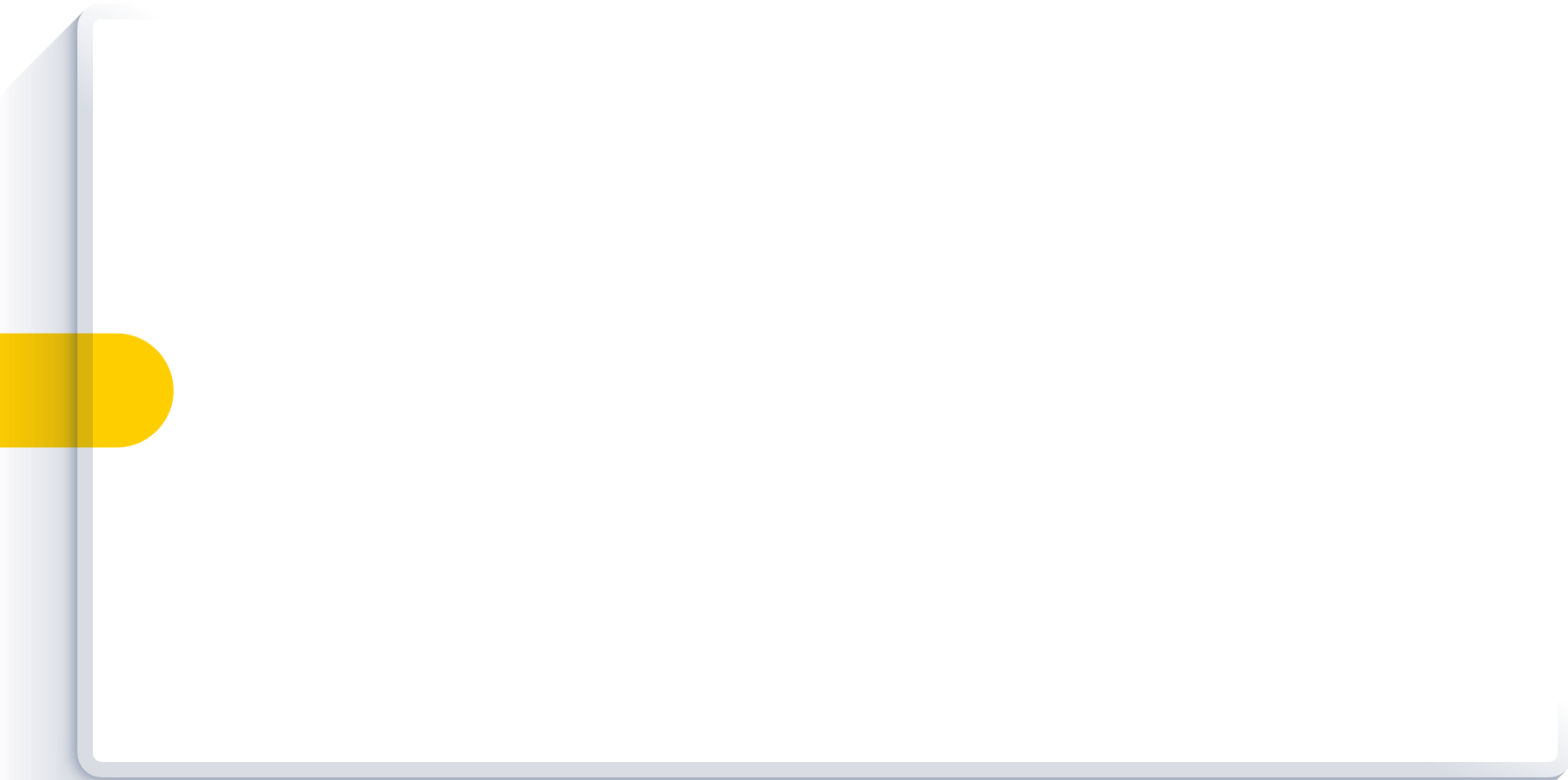
## OPPORTUNITIES

White is the color of milk and fresh snow

## THREATS

# Diagrams and infographics





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