# Quarkonia's properties at finite temperature from lattice QCD

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## Quarkonia as Tools 2022

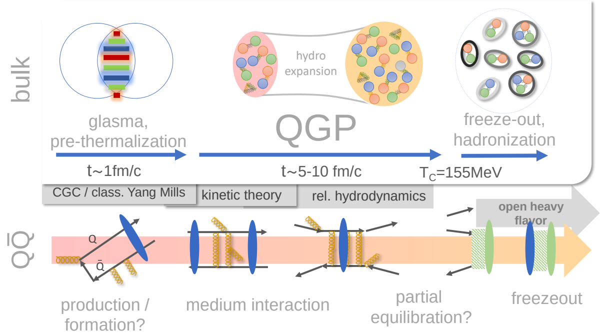
Centre Paul Langevin, Aussois, France, 01/14/2022

**Chromo-electric screening length in 2+1 flavor QCD**,
P. Petreczky, S. Steinbeißer, JHW, arXiv:2112.00788[hep-lat]
**Bottomonium melting from screening correlators at high temperature**,
P. Petreczky, S. Sharma, JHW, Phys.Rev.D 104 (2021) 5, 054511
*Bottomonia via lattice NRQCD*,
R. Larsen, S. Meinel, S. Mukherjee, P. Petreczky, Phys.Rev.D 100 (2019) 7, 074506; Phys.Lett.B 800 (2020) 135119; Phys.Rev.D 102 (2020) 114508
**Heavy Quark Potential in QGP: DNN meets LQCD**,
S. Shi, K. Zhou, J. Zhao, S. Mukherjee, P. Zhuang, arXiv:2105.07862[hep-ph]
**Static quark anti-quark interactions at non-zero temperature from lattice QCD**,
D. Bala, O. Kaczmarek, R. Larsen, S.Mukherjee, G. Parkar,
P. Petreczky, A. Rothkopf, JHW, arXiv:2110.11659[hep-lat]
**Static Potential At Non-zero Temperatures From Fine Lattices**,
+A. Bazavov, D. Hoying, arXiv:2110.00565[hep-lat]

# Outline

1. Hors d'œuvre: heavy quarkonia in heavy-ion phenomenology
   - Motivation why we study "Heavy quarkonia at $T > 0$"
   - Historical perspective on "Heavy quarkonia at $T > 0$"
2. Appetizer: modern, weak coupling picture of "Heavy quarkonia at $T > 0$"
3. Salad: not lettuce, but lattice field theory
4. Main course: lattice QCD results on "Heavy quarkonia at $T > 0$"
   - Relativistic bottomonia on the lattice
   - Nonrelativistic bottomonia on the lattice
   - In-medium static quarkonia
5. Dessert: bringing "Heavy quarkonia at $T > 0$" full circle

# Why focus on hard probes in heavy-ion collisions?

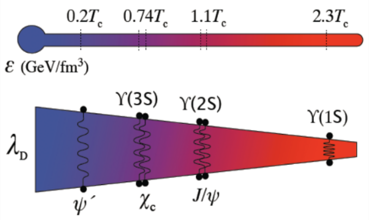



source: *Rothkopf, Phys.Rept. 858 (2020) 1-117*

- Hard probes are produced in a few **hard processes** in initial collision, neither created nor destroyed afterwards, but can alter their nature
- Most important probes: *jets*, open heavy flavor & **heavy quarkonia**
- **What happens to quarkonium if we increase the temperature?**

# Heavy quarkonia in the hot medium as a local thermometer

- Idea to look at **quarkonia** in the QGP is old and famous
  *Matsui, Satz, PLB 178 (1986)*
- **Debye screening** length $1/m_D$ of electric gluons ($A_0$) limits the radii of hadronic bound states
- Consequence: QGP formation $\Leftrightarrow$ **quarkonia suppression**

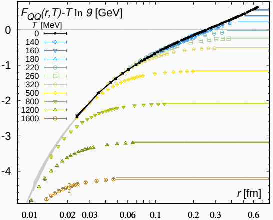



source: *USQCD whitepaper 2018, EPJ A 55 (2019)*

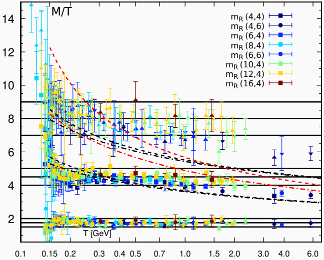
- But what is the **correct in-medium potential** for quarkonia?
- Two different phenomenological scenarios for the in-medium potential predict vastly different **melting points** for all quarkonia:
  1. **Weak-binding scenario**: Debye-screened potential goes flat as $V \sim F$
     $T(\Upsilon(1S)) \sim 2\, T_{\mathrm{pc}},\ T(J/\Psi) \sim \quad T_{\mathrm{pc}}$
  2. **Strong-binding scenario**: Remnant of confinement as $V \sim U = F + TS$
     $T(\Upsilon(1S)) \sim 3\, T_{\mathrm{pc}},\ T(J/\Psi) \sim 3/2\, T_{\mathrm{pc}}$

# Screening from Polyakov loop correlators

- **Color screening** usually studied via **Polyakov loop correlator**
  $$C_P(r,T) = \langle P(0) P^\dagger(\boldsymbol{r}) \rangle_T^{\mathrm{ren}} = e^{-F_{Q\bar{Q}}(r,T)/T}$$
- $rT \ll 1$: **singlet**/**octet** decomposition
  $$C_P(r,T) = {}^{1}/_{9} e^{-F_S(r,T)/T} + {}^{8}/_{9} e^{-F_O(r,T)/T}$$
- $rT \lesssim 0.4$: via $T=0$ potentials and adjoint Polyakov loop: no screening!
  $$C_P(r,T) = {}^{1}/_{9} e^{-V_s(r)/T} + L_A(T)\, {}^{8}/_{9} e^{-V_o(r)/T} + \mathcal{O}(\alpha_s^3)$$




source: *Brambilla, et al., PRD 98 (2018)*




⇒ *Petreczky, et al., PoS(LATTICE2021) 471*

- $rm_D \gtrsim 1$: screening regime; decompose
  $$C_P(r,T) = C_R(r,T) + C_I(r,T)$$
  $$\begin{matrix} C_R(r,T) \\ C_I(r,T) \end{matrix} = \begin{matrix} \langle \mathrm{Re}\, P(0) \mathrm{Re}\, P(\boldsymbol{r}) \rangle_T^{\mathrm{ren}} \\ \langle \mathrm{Im}\, P(0) \mathrm{Im}\, P(\boldsymbol{r}) \rangle_T^{\mathrm{ren}} \end{matrix} \to \mathcal{C} \begin{matrix} \text{even} \\ \text{odd} \end{matrix}$$
- Asymptotically 1PE: $C_{R,I}(r,T) \sim e^{-m_{R,I} r}/rT$
- EQCD: $m_R \sim 2m_D$, $m_I \sim 3m_D$ ($m_D$ per $A_0$)
  LQCD: $\frac{m_R}{T} \approx 4.5$, $\frac{m_I}{T} \approx 8$, $\frac{m_I}{m_R} \approx 1.75$

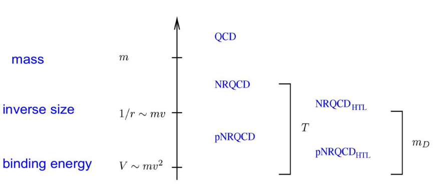
$$\Rightarrow \quad {}^{1}/_{m_D} \approx {}^{2}/_{m_R} \approx {}^{3}/_{m_I} = \{0.38 - 0.44\}/T$$
$r_{\Upsilon(1S)} \approx 0.21$ fm survives until $T \sim 380$ MeV
$r_{J/\Psi} \approx 0.43$ fm survives until $T \sim 190$ MeV

# Screening is not the whole story... (at weak coupling)

Matsui & Satz's idea of the **quarkonium suppression mechanism** was turned inside out by **weak-coupling EFT results** emerging 15 years ago

NR hierarchy:

$v \sim \alpha_s$




Thermal hierarchy:

$m_D \sim gT$

- For $1/r \sim m_D \ll T$: $\mathrm{Re}\,[V_s] = F_S + \mathcal{O}(g^4)$ and $\mathrm{Im}\,[V_s] \sim \mathcal{O}(g^2 T)$

$$V_s(T,r) = -C_F\alpha_s \left\{ \frac{e^{-rm_D}}{r} + m_D + \mathrm{i}T\phi(rm_D) \right\}, \phi(x) = 2\int_0^\infty \frac{dz\; z}{(z^2+1)^2}\left\{1 - \frac{\sin(zx)}{zx}\right\}$$

*Laine, et al., JHEP 03 (2007)*

- For $\Delta V \ll 1/r \ll m_D \ll T$: $\mathrm{Re}\,[V_s] = V_s + \mathcal{O}(g^4)$ and $\mathrm{Im}\,[V_s] \sim \mathcal{O}(g^4 r^2 T^3, g^6 T)$

$$V_s(T,r) = \frac{-C_F\alpha_s}{r} + r^2T^3 \left\{ \mathcal{O}(g^4) + \mathrm{i}\mathcal{O}\left(g^4, \frac{g^6}{(rT)^2}\right) \right\}$$

*Brambilla, et al., PRD 78 (2008)*

But an imaginary part leads to **dissociation** – is **screening** even relevant?

# QCD regularized on a lattice

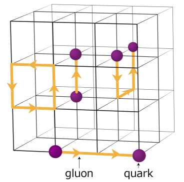
- QCD path integral needs **regularization**, e.g. dim. reg. or **lattice**
- Dim. reg. can only be applied perturbatively, i.e. order by order
- **Lattice** is a systematically improvable, non-perturbative regulator; breaks translation & rotation invariance, no space-time derivatives

$$U_\mu(x) = \exp[\mathrm{i} a g_0 A_\mu(x)] \qquad \text{gauge link}$$

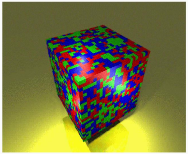
$$U_{\mu,\nu}(x) = U_\mu(x) U_\nu(x+a\hat{\mu}) U_\mu^\dagger(x+a\hat{\nu}) U_\nu^\dagger(x) \qquad \text{plaquette}$$

$$D_\mu[U_\mu(x)]\psi^f(x) = \frac{U_\mu(x)\psi^f(x+a\hat{\mu}) - U_\mu^\dagger(x-a\hat{\mu})\psi^f(x-a\hat{\mu})}{2a} + \mathcal{O}(a^2)$$

$$S_{QCD}[U,\bar{\psi},\psi] = a^4 \sum_x \sum_f \bar{\psi}^f(x) \left( \not{D}[U(x)] + m_f \right) \psi^f(x) - a^4 \sum_x \sum_{\mu<\nu} \frac{2}{g_0^2} \mathrm{Re}\,\mathrm{tr}\left\{ 1 - U_{\mu,\nu}(x) + \mathcal{O}(a^2) \right\}$$




$\overset{\text{HPC}}{\Longrightarrow}$



# Stochastic lattice QCD simulations in a box on a computer

Stochastically sample the (**Euclidean**) QCD path integral

$$\langle O\rangle_{\mathrm{QCD}} = \frac{1}{Z}\frac{1}{N}\sum_{\{U\}}^{N} O[U]\prod_{f}\det\left(\not{D}[U]+m_f\right)\exp\left(-S_g[U]\right)+\mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$$

using MCMC algorithm with **importance sampling**

QCD on a lattice with spacing $a$ in a box of $N_\sigma^3 \times N_\tau$ points

- **space**: periodic for gluons and quarks
  **always in a finite volume (must be large enough)**
- **imaginary time**: periodic for gluons, antiperiodic for quarks
  **always at finite temperature**: $aN_\tau = 1/T$
- **quark flavors**: usually $N_f = 2+1$ or $N_f = 2+1+1$, or $N_f = 0$
  **non-degenerate light quarks only in QCD+QED setup**
- **quark masses**: light quarks at the physical point are expensive
  **control the quark mass dependence through $\chi$PT**
- **scale setting**: spacing $a$ determined a posteriori from $(g_0, \{m_f\})$
  **line of constant physics: one observable per quark mass**
- **continuum limit**: $a \to 0$ realized **for each LCP** through $g_0 \to 0$
  **some matrix elements may require renormalization**

# Real-time dynamics from lattice QCD

- Importance sampling requires an imaginary-time formalism

⇒ **Dissociation** due to real-time dynamics not directly accessible

- Same **spectral functions** yield real- or imaginary-time correlators via different, analytically known integral kernels

$$G_T \begin{pmatrix} t \\ \tau \end{pmatrix} = \int d\omega \begin{pmatrix} K^M(T,\omega;t) \\ K^E(T,\omega;\tau) \end{pmatrix} \rho_T(\omega)$$

- **Spectral functions** encode the entire dynamics
  - Stable bound states ⇒ Delta functions
  - Unstable quasiparticles ⇒ regularized peaks, locally Breit-Wigner
  - On top of a UV continuum due to scattering or merged excited states
  - At $T > 0$ potentially a substantial IR tail below the "ground state"

⇒ Strategy for lattice QCD:

1. Compute imaginary-time correlators on the lattice
2. Reconstruct **spectral functions** by inverting spectral representation
3. Directly read off some state's properties from $\rho_T(\omega)$

- Spectral reconstruction is challenging: at best $N_\tau$ resp. $N_\tau/2$ data
- Usually, the best we can do at $T > 0$ is study **the lowest peak**

# At which $T$ are there either bound states or melted $q\bar{q}$ pairs?

- **Euclidean Correlators** are towers of exponential decays $G(\tau) = \sum_i A_i e^{-E_i \cdot \tau}$

  For **mesons**: same $E_i$ in temporal or spatial directions

- **Spatial $q\bar{q}$ pair correlators** are a **model-independent** analysis tool

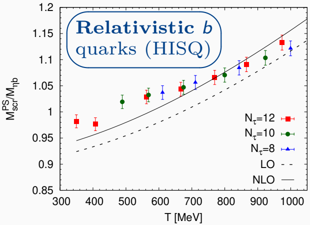
  *charm sector ⇒ Bazavov, et al., PRD 91 (2015)*

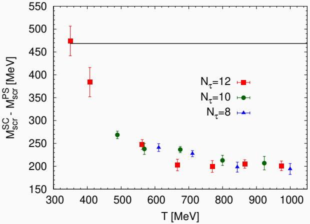
$$G_T(z) = \int_0^{1/T} d\tau \int d^2x_\perp \left\langle \mathcal{J}(\tau, \boldsymbol{x}_\perp, z) \mathcal{J}^\dagger(0) \right\rangle$$

$$= \int_0^{\infty} \frac{2d\omega}{\omega} \int_{-\infty}^{\infty} dp_z e^{ip_z z} \rho_T(\omega, p_z)$$

with **spectral function** $\rho_T(\omega, p_z)$

$$\sim \begin{cases} \delta\left[\omega^2 - p_z^2 - M_0^2\right] & \text{mesons} \\ \delta\left[\omega - \sum_{q_i} \sqrt{m_{q_i}^2 + [\pi T]^2}\right] & \text{free quarks} \end{cases}$$




source: *Petreczky, et al., PRD 104 (2021)*

- Survival of $\eta_b$ & $\Upsilon(1S)$ until $T \approx 400$ MeV; cf. $\eta_c$ & $J/\psi$ until $T \approx 200$ MeV
- Survival of $\chi_{b0}$ & $h_b$ until $T \approx 350$ MeV; cf. $\chi_{c0}$ & $\chi_{c1}$ until $T \sim T_{\rm pc}$
- How can we understand the **melting mechanism** at work?

# Nonrelativistic bottomonium with extended sources (HotQCD)

- Lattice NRQCD: no continuum limit $\Rightarrow$ upper limit on $T$ due to $N_\tau = {}^1/_{aT}$
- NRQCD correlator study on $T = 0$ or $N_\tau = 12$ lattices: **extended sources**

  *Larsen, et al., ...*

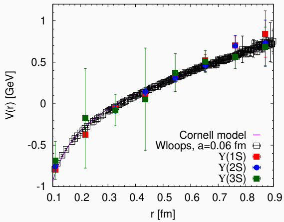
  1. Point source vs Gaussian smearing, new scheme for removing UV part: **thermal width**, small mass shift *PRD 100 (2019)*
  2. Cornell pot. eigenstates $\rightarrow$ GEVP $\Rightarrow$ lattice **NBS amplitudes** $\phi_\alpha(r)$
  3. Recover potential from NBS amp.

$$\left(\frac{-\Delta}{m_b} + V(r)\right)\phi_\alpha(r) = E_\alpha \phi_\alpha(r)$$

*PLB 800 (2020) + PRD 102 (2020)*

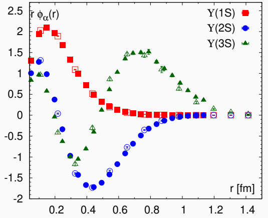
  4. **Small** $\tau$: NBS amplitudes at $T = 0$ and $T > 0$ almost **$T$-independent**
  5. **Large** $\tau$: $3S$ NBS amp. at $T > 0$ visibly **modified** $\Leftrightarrow$ thermal width

$T = 0$, $a = 0.06$ fm: NRQCD vs static $q\bar{q}$




source: *Larsen, et al., PRD 102, (2020)*

$N_\tau = 12$, $\tau \sim 0.4$ fm
$T = 334$ MeV vs $151$ MeV

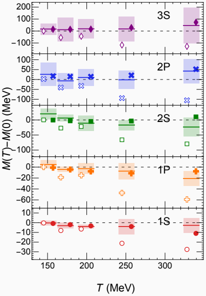


# Machine learning the potential from NRQCD amplitudes

Is there room for another interpretation? Let an algorithm figure it out...

Machine learning (DNN) applied to lattice NBS amp.: $T > 0$ potential

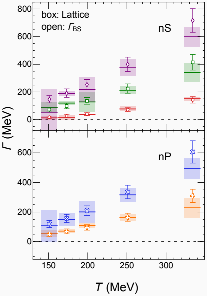
*Shi, et al., arXiv:2105.07862*




Lattice NBS amplitudes fed into DNN ⇒ can reconstruct a nonperturbative potential $V^{\mathrm{ML}}$

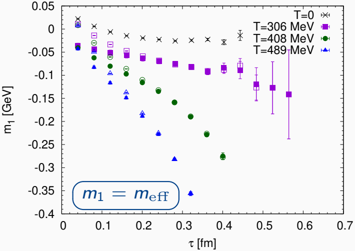
Clearly smaller thermal mass shift and larger width than in **Hard Thermal Loop** (HTL) perturbation theory $(\mathrm{Re}\,(V_s^{\mathrm{HTL}}) \sim F_S)$:

$\mathrm{Re}\,[V^{\mathrm{ML}}] \sim V_s(T=0)$
$\mathrm{Im}\,[V^{\mathrm{ML}}] \gg \mathrm{Im}\,[V_s^{\mathrm{HTL}}]$



# Static $q\bar{q}$ pair at $T > 0$ on the lattice

$a \approx 0.04$ fm, $r/a = 6$, $N_\tau = 64, 16, 12, 10$




source: *Bala, et al., arXiv:2110.11659*

- **Static $q\bar{q}$** interaction is encoded in (real-time) **Wilson loops**[a]

$$W_{[r,T]}(t) = \left\langle e^{ig \oint_{r\times t} dz^\mu A_\mu} \right\rangle_{\mathrm{QCD},T}$$

- Stable (ground) state $\Omega_r$ exists if

$$\Omega_{[r,T]} \equiv -\mathrm{i} \lim_{t\to\infty} \partial_t W_{[r,T]}(t)$$

[a] We use Wilson line correlators in Coulomb gauge.

- Same **spectral functions** yield real- or imaginary-time correlators

$$W_{[r,T]}\begin{pmatrix} t \\ \tau \end{pmatrix} = \int d\omega \begin{pmatrix} e^{+\mathrm{i}\omega t} \\ e^{-\omega\tau} \end{pmatrix} \rho_{[r,T]}(\omega)$$

- Motivates **generic decomposition**

$$\rho_{[r,T]}(\omega) = \rho^{\{\Omega;\mathcal{O}(T)\}}_{[r,T]}(\omega) + \rho^{\mathrm{tail}}_{[r,T]}(\omega) + \rho^{\mathrm{UV}}_{[r,T]}(\omega)$$

- UV continuum $\rho^{\mathrm{UV}}_{[r,T]}(\omega)$ is far above **lowest feature $\Omega$ + effects of $\mathcal{O}(T)$**

$\Rightarrow$ Guess $\rho^{\mathrm{UV}}_{[r,T]}(\omega)$ via $\rho^{\mathrm{UV}}_{[r,0]}(\omega) \Rightarrow$ subtract

**Note:** "tail" due to backward propagating UV physics (vacuum excited states) at $\tau \lesssim 1/T$.

# Cumulants of spectral functions – what can we expect?

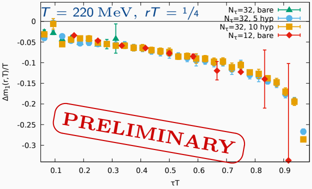
- Access **cumulants of** $\rho_{[r,T]}(\omega)e^{-\omega\tau}$ via $\tau$ **(log) derivatives** of $W_{[r,T]}(\tau)$

$$m_1^{[r,T]}(\tau) = -\partial_\tau \ln W_{[r,T]}(\tau) \quad [\equiv m_{\text{eff}}^{[r,T]}(\tau)],$$

$$m_n^{[r,T]}(\tau) = -\partial_\tau m_{n-1}^{[r,T]}(\tau), \quad n > 1$$

- For $N_\tau \leq 16$ obtain up to $m_3^{[r,T]}(\tau)$: supports $\leq 5$ parameters for $\rho_{[r,T]}(\omega)$
- Higher cumulants at small $\tau$ need at least $N_\tau > 16$: bad signal-to-noise

Fully vacuum-subtracted result




see: *Hoying, et al., arXiv:2110.00565 [hep-lat]*

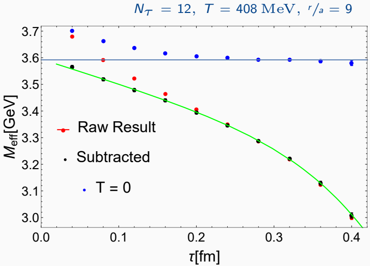
Feasibility study with $N_\tau = 32$: $m_n^{[r,T]}$, $n > 2$ ?

- Fine lattices: $a^{-1} \approx 7\,\text{GeV}$ $m_\pi \approx 0.3\,\text{GeV}$
- UV filtering (HYP) for **noise reduction**

→ distortions cancel in vacuum subtraction

- Definitely still work in progress

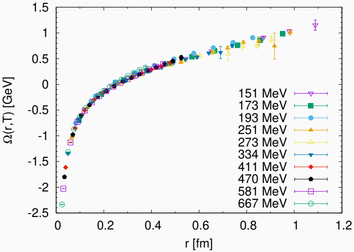
# Lowest spectral feature from fits using Gaussian approximation

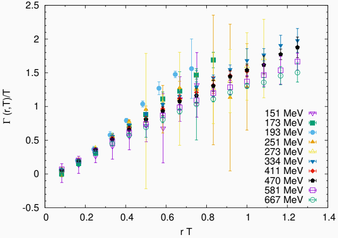



- **Quasiparticles** are represented as **Breit-Wigner** in $\rho_{[r,T]}(\omega)$
- Ansatz: **approximate BW of** $\rho_r^{\{\Omega;\mathcal{O}(T)\}}(\omega)$ locally as **Gaussian**, include delta function for $\rho_r^{\rm tail}(\omega)$

$$W_{[r,T]}(\tau) = A_{[r,T]}^{\{\Omega;\mathcal{O}(T)\}} e^{-\Omega_{[r,T]}\tau + (\Gamma^G_{[r,T]})^2\tau^2/2} + A^{\rm tail}_{[r,T]} e^{-\omega^{\rm tail}_{[r,T]}\tau}, \quad \omega^{\rm tail}_{[r,T]} \ll \Omega_{[r,T]}$$

$N_\tau = 12$, $\Omega(r,T) \equiv \Omega_{[r,T]}$, $\Gamma(r,T) \equiv \sqrt{2\ln 2}\,\Gamma^G_{[r,T]}$, subtracted correlators



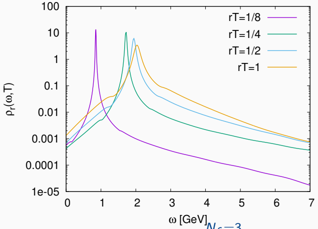




source: *Bala, et al., arXiv:2110.11659*

- Almost no $T$ dependence in $\Omega_{[r,T]}$ (naive correspondence: $\mathrm{Re}\, V_s(r,T)$)
- Naively expected scaling of $\Gamma(r,T)/T \approx \Gamma(rT)/T$ down to $T \approx T_{pc}$

# Comparison: lattice QCD vs HTL

**HTL spectral function** for $T = 667$ MeV




[NLO, 2-loop $\alpha_s(2\pi T)$, $\Lambda_{\overline{\rm MS}}^{N_f=3} = 332$ MeV]

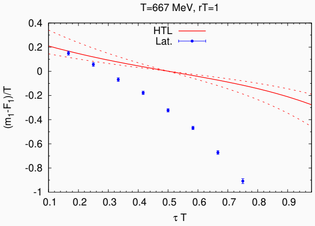
source: *Bala, et al., arXiv:2110.11659*

- **HTL** is an attractive proposition: **motivated & regularized BW**
- **HTL** result is **antisymmetric** around the midpoint $\tau = 1/2T$:

$$\log W_{[r,T]}(\tau) = -\mathrm{Re}\, V_s(r,T) \times \tau + \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left\{ e^{-\omega\tau} + e^{-\omega(1/T-\tau)} \right\} \times \{1 + n_B(\omega)\}\, \sigma_{[r,T]}(\omega)$$

- Leading **singularity** of $\sigma_{[r,T]}(\omega)$ (transv. gluon spec. fun.) fixes $\mathrm{Im}\, V_s(r,T)$

$N_\tau = 12$, $r/a = 12$, subtracted correlator



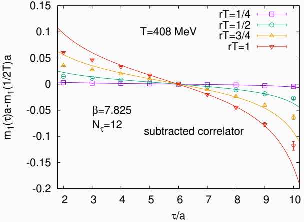

source: *Bala, et al., arXiv:2110.11659*

- **HTL** should work at $r \sim 1/m_D$
- *Subtleties* due to renormalons and regulators: consider $(m_1 - F_S)/T$

  Reminder: $\mathrm{Re}\,[V_s] = F_S + \mathcal{O}(g^4)$ in **HTL**
- No large UV component in HTL, compare UV-subtracted result
- $m_1$ at midpoint lower than **HTL**, and $m_2$ is much more negative

# Lowest spectral feature from fits using HTL-motivated Ansatz

$N_\tau = 12,\ T = 408\ \text{MeV},\ r/a = 9$



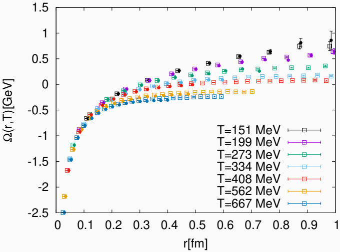

- Fit via HTL-motivated Ansatz

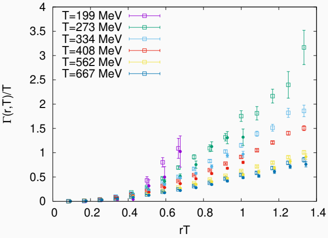
*Bala, Datta, PRD 101 (2020)*

$$W_{[r,T]}(\tau) = A^{BD}_{[r,T]} e^{-\Omega^{BD}_{[r,T]}\tau - \mathrm{i}\frac{\Gamma^{BD}_{[r,T]}}{\pi}\log\sin(\pi\tau T)}$$

- Note: similar result via Gaussian around midpoint $\tau = 1/2T$

$N_\tau = 12,\quad \Omega(r,T) \equiv \Omega^{BD}_{[r,T]},\quad \Gamma^{BD}(r,T) \equiv \Gamma^{BD}_{[r,T]},\quad$ (un-)subtracted correlators

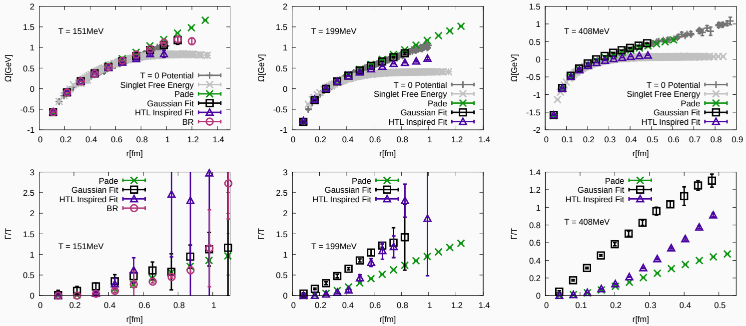



source: *Bala, et al., arXiv:2110.11659*

- Significant $T$ dependence in $\Omega_{[r,T]}$ (naive correspondence: Re $V_s(r,T)$)
- Weaker than naive scaling of $\Gamma(r,T)/T \approx \Gamma(rT)/T$

# Comparison: lowest spectral feature from four different methods




source: *Bala, et al., arXiv:2110.11659*

- Applied two further, independent methods (Padé rational approximation, Bayesian reconstruction) not discussed in detail
- $T \approx 150\,\mathrm{MeV}$ conclusive: $\Omega_{[r,T]} \approx F_S(r,T) \approx V_s(r)$ for $r \lesssim 0.8\,\mathrm{fm}$
- $T \lesssim 250\,\mathrm{MeV}$: all three methods yield $\Omega_{[r,T]} \gg F_S(r,T)$
- $T \approx 400\,\mathrm{MeV}$ inconclusive: $\Omega^{BD}_{[r,T]} \approx F_S(r,T)$ vs $\Omega^{G}_{[r,T]} \approx \Omega^{P}_{[r,T]} \approx V_s(r)$
- All methods find for all $T$ nontrivial $\Gamma_{[r,T]}$ that increases with $r$ or $T$

# Quarkonia's properties at finite temperature from lattice QCD

- **Spatial** correlation functions using **relativistic heavy quarks**
  - Model-independent study of quarkonia melting/survival in LQCD
  - $\eta_b$ or $\Upsilon(1S)$ until $T \approx 400$ MeV; $\chi_{b0}$ or $h_b$ until $T \approx 350$ MeV
  - $\eta_c$ or $J/\Psi$ until $T \approx 200$ MeV; $\chi_{c0}$ or $\chi_{c1}$ until $T \approx 150$ MeV
- **Polyakov loop correlators** (static picture → screening length)
  - $\eta_b$ or $\Upsilon(1S)$ until $T \approx 380$ MeV
  - $\eta_c$ or $J/\Psi$ until $T \approx 190$ MeV
- **Nonrelativistic bottomonia** in lattice NRQCD
  - Extended sources or BS wave functions boost resolving power of LQCD
  - Indicate **finite thermal widths**, but no significant thermal mass shifts
- **Static quarkonia ($q\bar{q}$ pair)**
  - Lowest spectral feature $\{\Omega; \mathcal{O}(T)\}$ + tail + UV continuum
  - Model-independent cumulant analysis → robust evidence for significant **thermal width being important for quarkonia melting**
  - Consistent with minor (Gaussian fit, Padé) or major (HTL-inspired fit) medium effects in real part → $N_\tau \leq 16$ has insufficient resolution

- So far, no conclusive statement about weak vs strong binding.
- Lattice + EFT are in good shape to deliver more robust and more realistic results needed for HIC phenomenology in the near future.

Merci bien!