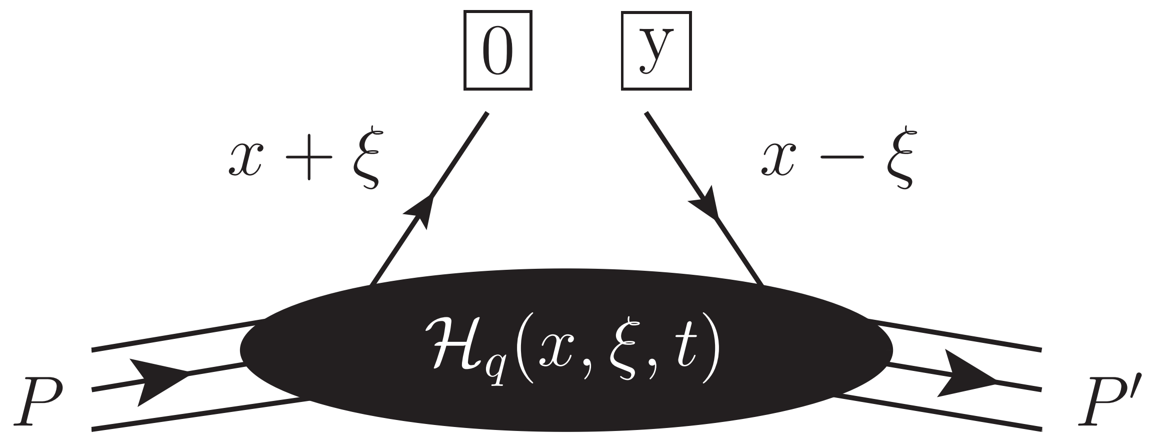
GPDs and the Shuvaev transform

GPDs generalise PDFs: outgoing/incoming partons carry different momentum fractions

Müller 94; Radyushkin 97; Ji 97



$$\langle P'|\,\overline{\psi}_q(y)\mathcal{P}\{\}\psi_q(0)\,|P\rangle$$

Shuvaev: Relates GPDs to PDFs at small x under physically motivated assumptions c.f analyticity

Shuvaev 99   Martin et al. 09

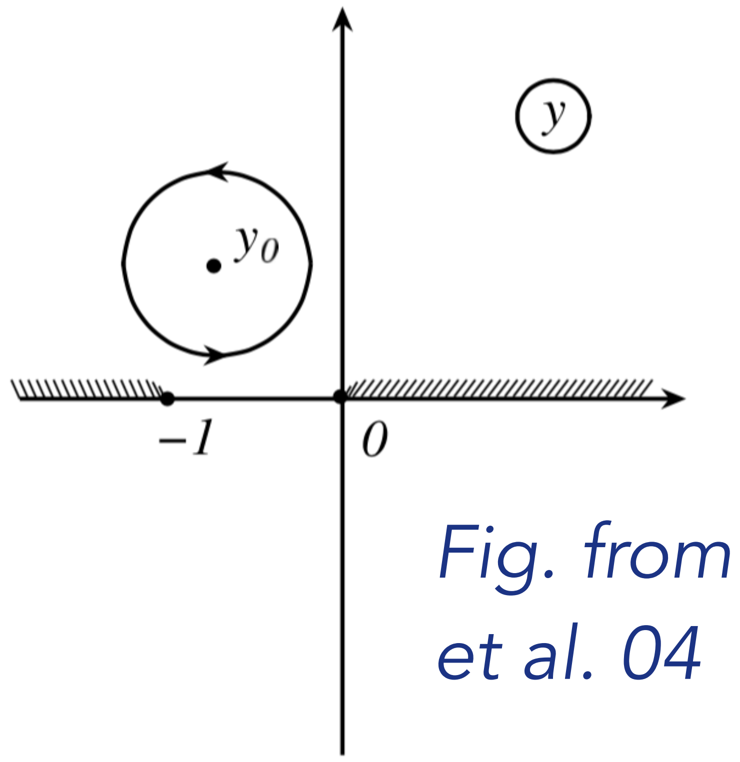


Fig. from Ivanov et al. 04

Idea: Conformal moments of GPDs = Mellin moments of PDFs

(up to corrections of order xi^2)

- Construct GPD grids in multidimensional parameter space x,xi/x,qsq with forward PDFs from LHAPDF
- Costly computationally due to slowly converging double integral transform
- Regge theory considerations => Shuvaev transform valid in space like (DGLAP) region only. In time like (ERBL) region imaginary part of coefficient is zero

Shuvaev Transform

Full Transform:

$$\mathcal{H}_q(x,\xi) = \int_{-1}^{1} \mathrm{d}x' \left[\frac{2}{\pi} \mathrm{Im} \int_0^1 \frac{\mathrm{d}s}{y(s)\sqrt{1-y(s)x'}} \right] \frac{\mathrm{d}}{\mathrm{d}x'} \left(\frac{q(x')}{|x'|} \right),$$

$$\mathcal{H}_g(x,\xi) = \int_{-1}^{1} \mathrm{d}x' \left[\frac{2}{\pi} \mathrm{Im} \int_0^1 \frac{\mathrm{d}s(x+\xi(1-2s))}{y(s)\sqrt{1-y(s)x'}} \right] \frac{\mathrm{d}}{\mathrm{d}x'} \left(\frac{g(x')}{|x'|} \right),$$

$$y(s) = \frac{4s(1-s)}{x+\xi(1-2s)}.$$

[Shuvaev et. al 1999]

Shuvaev Transform cont.

The conformal moments H_i^N of the GPDs are given by

$$H_i^N \equiv \int_{-1}^{1} \mathrm{d}x R_{N,i}(x_1, x_2) H_i(x, \xi), \qquad i = q, g,$$

Ohrndorf, 82

The conformal moments are polynomials in even powers of ξ,

$$H_i^N = \sum_{k=0}^{\lfloor (N+1)/2 \rfloor} c_{k,i}^N \xi^{2k} = c_{0,i}^N + c_{1,i}^N \xi^2 + c_{2,i}^N \xi^4 + \ldots, \qquad , \; c_{0,i}^N = f_i^N$$

Leading term is Mellin moment of PDF

- Provided inverse exists then can relate GPDs to PDFs with suppression of order xi (i.e. good low x approx)

Shuvaev Transform cont.

Widely debated, certain conditions needing upheld, e.g lack of singularities in Re N > 1 plane

e.g Diehl, Kugler, 08

Regge theory considerations => condition met Martin, Nockles, Ryskin, Teubner, 09

- Can check in physically motivated ansatz, e.g MSTW2008 global partons input parametrisation

Martin, Stirling,Thorne, Watt, 09

$$xg(x, Q_0^2) = A_g x^{\delta_g} (1-x)^{\eta_g} (1 + \epsilon_g \sqrt{x} + \gamma_g x) + A_{g'} x^{\delta_{g'}} (1-x)^{\eta_{g'}}.$$

Expand about x ~ 0

$$xg(x, Q_0^2) = A_g x^{\delta_g} + A_{g'} x^{\delta_{g'}} + \ldots,$$

Mellin transform:

$$xg^N(Q_0^2) = \int_0^1 \mathrm{d}x x^{N-1} (A_g x^{\delta_g} + A_{g'} x^{\delta_{g'}}) + \ldots$$
$$= \frac{A_g}{N + \delta_g} + \frac{A_{g'}}{N + \delta_{g'}} + \ldots,$$

Fits to data (including 1sig. errors) suggest $\delta_g > -1$ and $\delta_{g'} > -1$

- Shuvaev transform describes HVM and GDVCS data well

Kumericki, Muller, 10