Roundtable discussion: extraction of PDFs from CEP measurements

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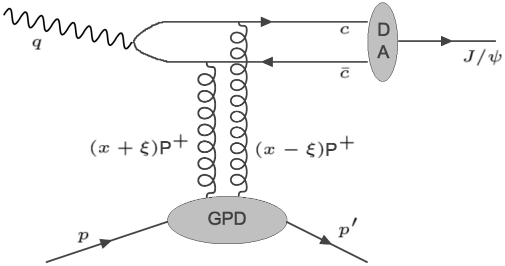
Theoretical description

We want to extract information on gluon PDFs at very low $x \sim 10^{-6} - 10^{-3}$ from exclusive J/ψ photoproduction events at HERA and LHCb.



LO depiction of J/ψ photoproduction. ξ is the **skewness** parameter measuring the transfer of plus-momentum to the proton. x is the average plus-momentum of the active parton.

The photon emanates from the colliding beam ($e^{\pm}$ at HERA, p or Pb at LHCb). In the framework of collinear factorization, the event can be understood in terms of **generalized parton distributions (GPDs)** and **non-relativistic QCD matrix element** for moderate or small photon virtuality $Q^2 = -q^2$ where the hard scale is provided by m_c^2, or alternatively **distribution amplitudes** (DAs) for very large Q^2 which provides the hard scale.

$$P = (p + p')/2, \quad \xi = \frac{p^+ - p'^+}{p^+ + p'^+}, \quad t = (p' - p)^2$$

Theoretical description

Collinear factorization allows to describe the vector meson production amplitude thanks to form factors of the following form at leading twist

- NR QCD formalism up to NLO [Ivanov *et al*, 2004]

$$\mathcal{F}(\xi, t) = \left(\frac{\langle O_1 \rangle_V}{m_c} \right)^{1/2} \sum_{a=q,g} \int_{-1}^{1} \mathrm{d}x \, T^a_{NR}(x, \xi) \, F^a(x, \xi, t) \tag{1}$$

  where $\langle O_1 \rangle_V^{1/2}$ is the NR QCD matrix element, T a hard-scattering kernel and $F(x, \xi, t)$ is the GPD.

- Distribution amplitude (DA) formalism at all orders [Brodsky *et al*, 1994, Collins *et al*, 1997]

$$\mathcal{F}(\xi, t) = \sum_{a=q,g} \int_{0}^{1} \mathrm{d}u \int_{-1}^{1} \mathrm{d}x \, \phi^a(u) \, T^a_{DA}(x, \xi, u) \, F^a(x, \xi, t) \tag{2}$$

  where $\phi(u)$ is the leading-twist DA.

At LO, the imaginary part of the hard-scattering kernel boils down to $\delta(x - \xi)$, so the **imaginary part of the amplitude is only sensitive to the diagonal** $F^a(\xi, \xi, t)$. Even at NLO, $x = \mathcal{O}(\xi)$ remains the dominant region in the computation of the integral.

Generalized parton distributions

Properties of GPDs [Müller *et al*, 1994], [Radyushkin, 1996] and [Ji, 1997]

- Depending on the helicity of the target and parton, several types of GPDs. Depending on the specific choice of observable, different sensitivity to various GPDs can be obtained. We focus on H in the following.
- The **forward limit** $t \to 0$ – and consequently $\xi \to 0$ – gives back the usual PDFs

$$H^q(x, \xi = 0, t = 0) = f^q(x) \tag{3}$$

$$H^g(x, \xi = 0, t = 0) = x f^g(x) \tag{4}$$

- **Polynomiality of the Mellin moments** Due to Lorentz covariance [Ji, 1998], [Radyushkin, 1999]

$$\int_{-1}^{1} \mathrm{d}x \, x^n H^q(x, \xi, t) = \sum_{k=0 \text{ even}}^{n+1} H^q_{n,k}(t) \xi^k \tag{5}$$

This has important consequences on the modellization of GPDs (double distribution formalism). Since $t \ll m_c^2$ and has little relevance in the following, we drop it from now on.

Evolution of GPDs

At LO, the imaginary part of the amplitude is probing the diagonal $H^g(\xi,\xi)$. Since $\xi \sim 10^{-5}$ at LHCb, one could be tempted to write

$$H^g(\xi, \xi = 10^{-5}) \approx H^g(\xi, 0) = \xi f^g(\xi) \tag{6}$$

or maybe

$$H^g(\xi, \xi = 10^{-5}) \approx H^g(2\xi, 0) = 2\xi f^g(2\xi) \tag{7}$$

But there is a problem...

GPDs depend on a factorization and renormalization scale whose RG equation generalizes the DGLAP equation and reads at LO for $x > \xi > 0$ [from Bertone - singlet quark GPD]

$$\frac{\mathrm{d}H^{q+}}{\mathrm{d}\mu}(x,\xi,\mu) = \frac{C_F \alpha_s(\mu)}{\pi\mu}\left\{2\int_x^1 \mathrm{d}y\, \frac{H^{q+}(y,\xi,\mu) - H^{q+}(x,\xi,\mu)}{y-x} + H^{q+}(x,\xi,\mu)\left[\frac{3}{2} + \log\left(\frac{(1-x)^2}{x^2-\xi^2}\right)\right] - \int_x^1 \mathrm{d}y\, H^{q+}(y,\xi,\mu)\left(\frac{x+y}{y^2-\xi^2}\right)\right\} \tag{8}$$

Evolution of GPDs

In the limit $\xi = 0$, we retrieve the usual DGLAP equation

$$\frac{\mathrm{d}f^{q+}}{\mathrm{d}\mu}(x,\mu) = \frac{C_F\alpha_s(\mu)}{\pi\mu}\Bigg\{\int_x^1 \mathrm{d}y\, \frac{f^{q+}(y,\mu) - f^{q+}(x,\mu)}{y-x}\left[1+\frac{x^2}{y^2}\right] + f^{q+}(x,\mu)\left[\frac{1}{2}+x+\log\left(\frac{(1-x)^2}{x}\right)\right]\Bigg\} \tag{9}$$

But in the limit $x = \xi$, we obtain

$$\frac{\mathrm{d}H^{q+}}{\mathrm{d}\mu}(x,x,\mu) = \frac{C_F\alpha_s(\mu)}{\pi\mu}\Bigg\{\int_x^1 \mathrm{d}y\, \frac{H^{q+}(y,x,\mu) - H^{q+}(x,x,\mu)}{y-x} + H^{q+}(x,x,\mu)\left[\frac{3}{2}+\log\left(\frac{1-x}{2x}\right)\right]\Bigg\} \tag{10}$$

Evolution of GPDs

To study how evolution generates a ξ dependence, let's consider an initial GPD with no ξ dependence at initial scale μ_0:

$$H^{q+}(x, \xi, \mu_0) = f^{q+}(x, \mu_0) \tag{11}$$

Then the **difference of derivatives between the diagonal and the forward limit** is

$$\frac{\mathrm{d}H^{q+}}{\mathrm{d}\mu}(x, x, \mu_0) - \frac{\mathrm{d}f^{q+}}{\mathrm{d}\mu}(x, \mu_0) = \frac{C_F \alpha_s(\mu_0)}{\pi \mu_0} \Bigg\{ - \int_x^1 \mathrm{d}y \, \frac{f^{q+}(y, \mu_0) - f^{q+}(x, \mu_0)}{y - x} \left[\frac{x^2}{y^2}\right] + f^{q+}(x, \mu_0) \left[1 - \log(2) - x - \log(1 - x)\right] \Bigg\} \tag{12}$$

In the limit $x \ll 1$, assuming a Regge-like behaviour

$$f^{q+}(x, \mu_0) \sim C x^{-\lambda} \tag{13}$$

$$\int_x^1 \mathrm{d}y \, \frac{f^{q+}(y, \mu_0) - f^{q+}(x, \mu_0)}{y - x} \left[\frac{x^2}{y^2}\right] \sim C \int_x^1 \mathrm{d}y \, \frac{y^{-\lambda} - x^{-\lambda}}{y - x} \left[\frac{x^2}{y^2}\right] \sim (1 - H_{\lambda+1}) \, C x^{-\lambda} \tag{14}$$

Evolution of GPDs

So for $x \ll 1$,

$$\frac{\mathrm{d}H^{q+}}{\mathrm{d}\mu}(x, x, \mu_0) \approx \frac{\mathrm{d}f^{q+}}{\mathrm{d}\mu}(x, \mu_0) + \frac{C_F \alpha_s(\mu_0)}{\pi \mu_0} \left(H_{\lambda+1} - \log(2)\right) f^{q+}(x, \mu_0) \tag{15}$$

The diagonal, which at initial scale is equal to the forward limit by our simplifying assumption, **evolves at a different rate** than the forward limit. From Eq. (15) results

Proportionality of the diagonal and forward limit at $x \ll 1$ - Skewness function

$$\frac{H^{q+}(x, x, \mu_1)}{f^{q+}(x, \mu_1)} = 1 + (\mu_1 - \mu_0) \frac{C_F \alpha_s(\mu_0)}{\pi \mu_0} \left(H_{\lambda+1} - \log(2)\right) + \mathcal{O}(\mu_1 - \mu_0)^2 \tag{16}$$

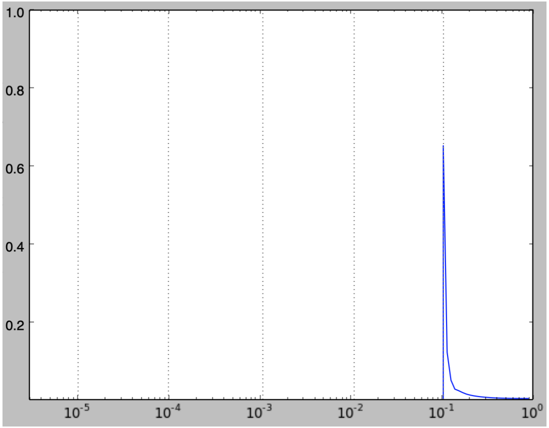
Any assumption $H^{q+}(x, x, \mu_0) \approx f^{q+}(x, \mu_0)$ at $x \ll 1$ is therefore immediately violated by evolution. In general, we can even demonstrate

$$\frac{H^{q+}(x, \xi, \mu_1)}{f^{q+}(x, \mu_1)} = 1 + (\mu_1 - \mu_0) \frac{C_F \alpha_s(\mu_0)}{\pi \mu_0} R_\lambda\left(\frac{\xi}{x}\right) + \mathcal{O}(\mu_1 - \mu_0)^2 \tag{17}$$

An alternative procedure

Thanks to Valerio's APFEL++, we can access numerically

$$\xi H^{q+}(\xi,\xi,\mu_1) = \int_{\xi}^{1} \mathrm{d}x_\beta \, \Gamma^{qq}_{\mu_0\to\mu_1}(x_\beta,\xi) x_\beta H^{q+}(x_\beta,\xi,\mu_0) \tag{18}$$



$\Gamma^{qq}_{1\to 10\ \mathrm{GeV}}(x_\beta, \xi = 1e-1)$

The evolved GPD depends mostly on the value of the GPD at initial scale at the same value of x, and to a marginal extend on higher values of x.

An alternative procedure

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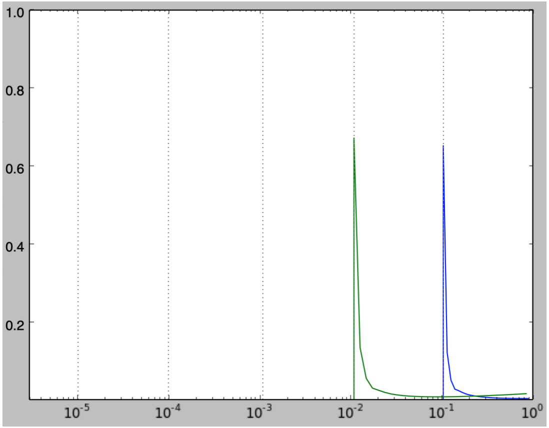

$$\Gamma^{qq}_{1\to 10\ \mathrm{GeV}}(x_\beta, \xi = 1e-2)$$

An alternative procedure

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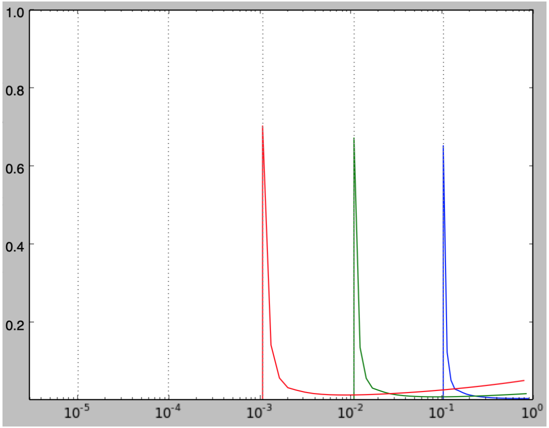
$$\xi H^{q+}(\xi,\xi,\mu_1) = \int_\xi^1 \mathrm{d}x_\beta\, \Gamma^{qq}_{\mu_0\to\mu_1}(x_\beta,\xi) x_\beta H^{q+}(x_\beta,\xi,\mu_0) \quad (18)$$

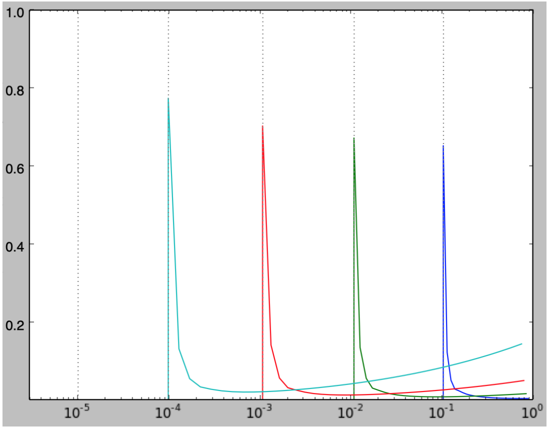


$\Gamma^{qq}_{1\to 10\ \mathrm{GeV}}(x_\beta,\xi = 1e-3)$

An alternative procedure

Thanks to Valerio's APFEL++, we can access numerically

$$\xi H^{q+}(\xi,\xi,\mu_1) = \int_\xi^1 \mathrm{d}x_\beta \, \Gamma^{qq}_{\mu_0\to\mu_1}(x_\beta,\xi) x_\beta H^{q+}(x_\beta,\xi,\mu_0) \quad (18)$$

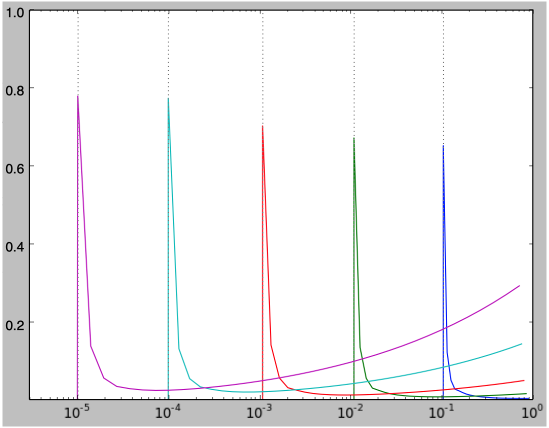


$\Gamma^{qq}_{1\to 10\ \mathrm{GeV}}(x_\beta, \xi = 1e-4)$

An alternative procedure

Thanks to Valerio's APFEL++, we can access numerically

$$\xi H^{q+}(\xi,\xi,\mu_1) = \int_\xi^1 \mathrm{d}x_\beta\, \Gamma^{qq}_{\mu_0\to\mu_1}(x_\beta,\xi) x_\beta H^{q+}(x_\beta,\xi,\mu_0) \quad (18)$$



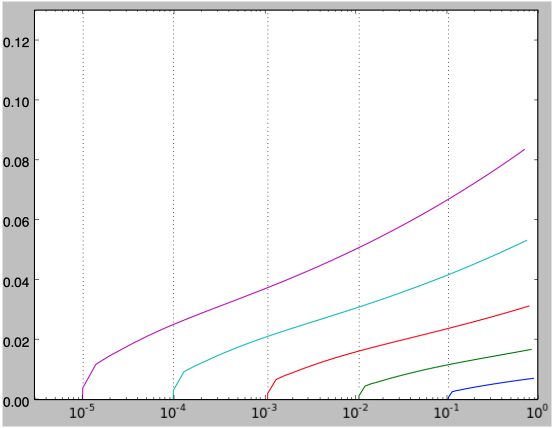
$\Gamma^{qq}_{1\to 10\ \mathrm{GeV}}(x_\beta, \xi = 1e-5)$
Crudely, a peak of width $\sim \xi$ for $x_\beta \approx \xi$, and a large x_β domain inflated by $\log(\xi)$, so a rule of thumb

$$\xi H^{q+}(\xi,\xi,\mu_1) \approx \xi^2 H^{q+}(\xi,\xi,\mu_0) + K \log(\xi) \int H^{q+}(x_\beta \gg \xi,\xi,\mu_0) \quad (19)$$

An alternative procedure

For gluons generated radiatively from a quark distribution on the range $1 \to 1.5$ GeV

$$\xi H^{g+}(\xi,\xi,\mu_1) = \int_\xi^1 \mathrm{d}x_\beta\, \Gamma^{gq}_{\mu_0\to\mu_1}(x_\beta,\xi) x_\beta H^{q+}(x_\beta,\xi,\mu_0) \tag{20}$$



$\Gamma^{gq}_{1\to 1.5\ \mathrm{GeV}}(x_\beta, \xi = 1e-5)$

An alternative procedure

Unless $H^{q+}(\xi,\xi)$ diverges stronger than ξ^{-2}, the region $x_\beta \gg \xi$ controls the behavior of evolution if ξ is small enough. But if $x_\beta \gg \xi$ and $\xi \ll 1$, then the approximation

$$H^{q+}(x_\beta,\xi) \approx f^{q+}(x_\beta) \tag{21}$$

makes sense this time! Hence

Alternative procedure [article in preparation]

- At a low scale Q_0, define the GPD as a simple PDF

$$H^{q+}(x,\xi,Q_0) = f^{q+}(x,Q_0)$$

- Evolve the GPD to the relevant scale Q_f
- The value of the diagonal of the GPD at Q_f is controlled by the large x region of the GPD at initial scale Q_0, where our assumption of GPD = PDF is valid.
- The value of Q_0 is irrelevant as long as $Q_f - Q_0$ is enough to ensure the domination of the large x region.

The Shuvaev transform

- Conformal moments of GPDs are defined as

$$O_n^q(\xi,\mu) = \xi^n \int_{-1}^{1} \mathrm{d}x\, C_n^{3/2}\left(\frac{x}{\xi}\right) H^q(x,\xi,\mu) \tag{22}$$

  where $C_n^{3/2}$ are Gegenbauer polynomials of degree n. Conformal moments have a particularly suitable behavior under evolution since

$$O_n^q(\xi,\mu) = O_n^q(\xi,\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_n/2\beta_0} \tag{23}$$

  where γ_n is the same anomalous dimension as the one involved in the evolution of Mellin moments of the PDF.
- The limit $\xi \to 0$ of GPD conformal moments give back PDF Mellin moments.

The Shuvaev transform

- Shuvaev (arXiv:9902410) has described how to reconstruct a GPD from the knowledge of its conformal moments:

$$H^q(x,\xi,\mu) = -\frac{1}{\pi}\,\mathrm{Im}\int_{-1}^{1}\frac{\mathrm{d}y}{y}\mathfrak{f}^q(y,\xi,\mu)\int_0^1 \mathrm{d}s\left(1-\frac{4ys(1-s)}{x+\xi(1-2s)}\right)^{-3/2} \quad (24)$$

$$\mathfrak{f}^q(y,\xi,\mu) = \int\frac{\mathrm{d}n}{2\pi i}\frac{4^{n+1}}{\sqrt{\pi}}\frac{\Gamma(n+3/2)}{\Gamma(n+2)}y^{-n}O_n^q(\xi,\mu) \quad (25)$$

where $\mathfrak{f}^q(y,\xi,\mu)$ is a so-called *effective forward distribution* (EFD), whose Mellin moments are essentially the Gegenbauer moments of the GPD. In particular, $\mathfrak{f}^q(y,\xi=0,\mu)$ gives back the usual PDF.

The Shuvaev transform

- Following the earlier procedure, we could set the GPD as a simple PDF at a low scale Q_0, compute the conformal moments

$$O_n^q(\xi, Q_0) = \xi^n \int_{-1}^{1} \mathrm{d}x \, C_n^{3/2}\left(\frac{x}{\xi}\right) f^{q+}(x, Q_0) \tag{26}$$

  which still have a ξ dependence although the GPD has none, evolve the conformal moments to Q_f, compute the EFD and apply Shuvaev's transform to retrieve the diagonal of the GPD.
- However, the following simplification has been proposed. Writing directly

$$\mathfrak{f}^q(y, \xi, Q_0) = f^{q+}(y, Q_0) \tag{27}$$

  or in other words, **setting the conformal moments to be equal to the PDF Mellin moments, with no ξ dependence, at some scale Q_0. Since the anomalous dimensions of the conformal and Mellin moments are the same, it actually means the conformal moments and PDF Mellin moments are always equal and we could as well select $Q_0 = Q_f$.** The ξ dependence is purely generated by Shuvaev's reconstruction kernel.

Questions regarding the transform

- Noritzsch (arXiv:0004012) underlines that the support region of the EFD has to satisfy

$$2|x| \leq 1 + \sqrt{1 - \xi^2} \tag{28}$$

  which in particular forbids that the EFD has no ξ dependence, although it is a good approximation at low ξ.
- Kumericki and Müller (arXiv:0907.1207) while acknowledging the validity of the procedure at low x, consider it as a model among others and raise arguments about the non-commutativity of truncating the ξ expansion of conformal moments and analytically continuating them.
- With the latter simplification, the role of evolution is far less clear to me: it seems we have switched from the heuristic idea that evolution will favor the large x region where the assumption that GPD = PDF is sound to the model assumption that conformal moments are independent of ξ. How to compute the **systematic uncertainty** associated to this assumption?

Model uncertainty

- Going back to the original procedure in x-space, we want to assess the systematic uncertainty introduced. For that, we evaluate in accordance with the skewness ratio evoked earlier that the absolute uncertainty created by replacing a GPD by a PDF at a given scale is estimated by

$$\Delta H^{q+}(x,\xi,Q_0) = \frac{\xi}{x} f^{q+}(x,Q_0) \tag{29}$$

Using our very crude previous estimate that

$$H^{q+}(\xi,\xi,\mu_1) \approx \xi H^{q+}(\xi,\xi,\mu_0) + K\frac{\log(\xi)}{\xi}\int \mathrm{d}x_\beta \, H^{q+}(x_\beta \gg \xi, Q_0) \tag{30}$$

then

$$\Delta H(\xi,\xi,\mu_1) \approx \xi f^{q+}(\xi,Q_0) + K\log(\xi)\int \mathrm{d}x_\beta \, \frac{H^{q+}(x_\beta \gg \xi)}{x_\beta} \tag{31}$$

Unsurprisingly, the first term corresponding to the peak creates most of the uncertainty, so this would incline towards a relative uncertainty scaling as $O(\xi)$.

Conclusion

- We are in position to determine, depending on the value of ξ and the available evolution range, the regime where the large x region of the GPD at initial scale Q_0 controls the value of the evolved GPD at scale Q_f at small x and ξ, and to associate a systematic uncertainty to this procedure.
- This allows to determine, given a large x PDF at scale Q_0, the precise relation uniting $H(\xi, \xi, Q_f)$ to $H(\xi, 0, Q_f)$.
- Considering the subtlety of switching to conformal moment space, using x-space evolution solvers seems more practical.
- Extending the study at higher order is necessary, starting by understanding the operator weights at NLO.