FCC-ee Parameter Optimization at Different Energies

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Many thanks to all FCC-ee/FCCIS colleagues



Introduction

Parameter optimization has been reported on numerous occasions at previous conferences, and little has changed since then.

The main purpose of this talk is to brush up on the basic dependencies and constraints for luminosity in FCC-ee.

Summary of further presentation:

- Key factors to consider when optimizing parameters for maximum luminosity.
- Selection of parameters at different energies: Z, WW, ZH, ttbar.
- Current problems, questions, and next steps.

Basic Equations

Piwinski angle:
$$\phi = \frac{\sigma_z}{\sigma_x} tg\left(\frac{\theta}{2}\right)$$

$$L_{i} = \frac{\sigma_{z}}{\sqrt{1 + \phi^{2}}} \xrightarrow{\theta = 1, \phi = 1} \frac{2\sigma_{x}}{\theta} \approx \beta_{y}^{*}$$

Luminosity:
$$L = \frac{\gamma}{2er_e} \cdot \frac{I_{tot}(\xi_y)}{\beta_y^*} \cdot R_{hg}$$

Collision scheme with large Piwinski angle

$$\xi_{y} = \frac{N_{p} r_{e}}{2\pi \gamma} \cdot \frac{\beta_{y}^{*}}{\sigma_{y} \sigma_{x} \sqrt{1 + \phi^{2}}} \xrightarrow{\theta \square 1, \phi \square 1} \frac{r_{e}}{\pi \gamma \theta} \cdot \frac{N_{p}}{\sigma_{z}} \cdot \sqrt{\frac{\beta_{y}^{*}}{\varepsilon_{y}}}$$

Beam-beam limit in Crab Waist collision scheme can be high, but to obtain it, one needs a small vertical emittance and (possibly) a high linear bunch density.

Linear density is an important parameter for collective instabilities and impedance-related issues, so this is another limitation.

- There is no sense to optimize the luminosity per bunch (or per collision). Attention should only be paid to ξ_y .
- $\sigma_{\rm z}$ is one of the most variable parameters: it depends on many factors, including N_p . Accordingly, the bunch population N_p should be adjusted to obtain the desired ξ_y .
- The number of bunches $n_b \propto 1/N_p$. We usually don't need to worry about this, as the range of valid values is quite wide.

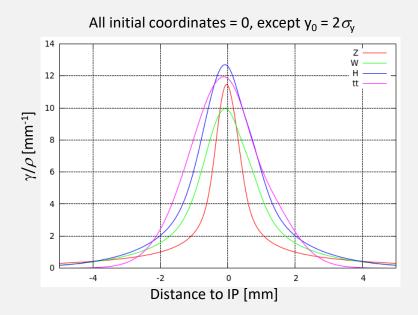
Beamstrahlung

Bending radius in the field of the opposite bunch

surface density

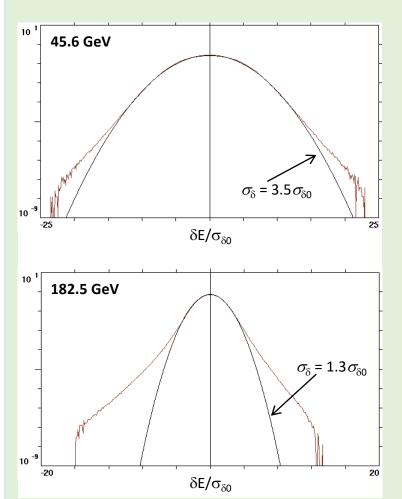
$$\frac{1}{\rho_{\min}} \propto \frac{N_p}{\gamma \sigma_x \sigma_z} \propto \frac{\xi_y}{\sqrt{\beta_x^* \beta_y^*}} \sqrt{\frac{\varepsilon_y}{\varepsilon_x}} \approx 0.002$$

- With increasing energy, beta functions at IP should grow while ξ_v almost does not change => ρ increases.
- Bending radius is not constant along the trajectory, and it depends on the particle coordinates.



Parameters for this plot were taken from the CDR table. At low energy, $\rho_{\rm min}$ < 8 m.

Equilibrium energy distribution



- Critical energy of emitted photons: $u_c \propto \gamma^3/\rho$.
- The factor of increasing the energy spread is higher at low energies. The explanation is that it depends on the ratio of the bending radii in the arcs (SR) and in the IPs (BS).
- For low-energy colliders, ρ_{\min} at IP can be even smaller, but the ring radius is $\underline{\text{much}}$ smaller than in FCC, so the effect of BS is negligible.
- At 45.6 GeV, the energy loss due to BS is ~0.31 MeV per IP, compared to ~36 MeV in the arcs due to SR.
- Long tails at ttbar are produced by single emitted BS photons. The ratio u_c/σ_δ is important here, which grows with γ .
- For asymmetry of the tails, an important parameter is the damping factor during the period of synchrotron oscillations. Therefore, asymmetry grows with γ .

Momentum acceptance determines the maximum allowable critical energy for BS photons, which in turn is proportional to $\xi_{\rm v}$ (and hence luminosity).

Dependencies and Limitations

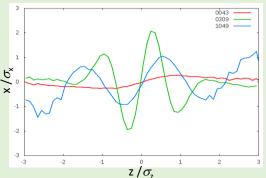
• When σ_{δ} and therefore σ_{z} are mainly determined by BS (i.e. at Z, WW, ZH, but *not* at ttbar), the following rule can be applied for dependencies on N_{p} :

$$\{\sigma_{\delta}, \sigma_{z}, \xi_{y}, L\} \propto \sqrt{N_{p}}, \quad \xi_{x} = const$$

- Since $\xi_{x,y}$ and BS depend on the bunch density (that is, on σ_z), and σ_z itself strongly depends on BS, a positive feedback arises, which can lead to a 3D flip-flop. Possible triggers are:
 - 1) Asymmetry in N_p
 - 2) Asymmetry in $\sigma_{\rm v}$
 - 3) Asymmetry in longitudinal shifts (transient beam loading)
 - 4) Asymmetry in the working points (betatron tunes)
- To avoid this instability, it is necessary to constantly maintain a high degree of symmetry in each pair of colliding bunches. And a special procedure is required to gradually increase N_p bootstrapping.

Coherent beam-beam instability (TMCI)

Bunch shape at some turns



Excited coherent modes are associated with synchro-betatron resonances:

$$2v_{x}-2mv_{z}=n, \qquad m\leq \phi$$

If ϕ is not too large, we can solve the problem by choosing

$$v_x > 0.5 + \phi v_z$$

We are close to this requirement at ZH and are fulfilling it at ttbar.

An important parameter for this instability is the ratio ξ_x/ν_z , which needs to be minimized.

$$\xi_{x} = \frac{N_{p} r_{e}}{2\pi\gamma} \cdot \frac{\beta_{x}^{*}}{\sigma_{x}^{2} \left(1 + \phi^{2}\right)} \xrightarrow{\theta \square 1, \ \phi \square 1} \frac{2r_{e}}{\pi\gamma\theta^{2}} \cdot \frac{N_{p} \beta_{x}^{*}}{\sigma_{z}^{2}}$$

Mitigation of instability:

- 1) Decrease in β_x^*
- Increase in the momentum compaction factor (but there is a side effect: increased emittances) – only at Z and WW
- 3) Decrease in RF voltage only at Z
- 4) Proper choice of the working point

Parameter Optimization at Z

- Recent simulations (Y. Zhand, M. Zobov) have shown that when impedances are taken into account, coherent beam-beam instability is enhanced. To solve the problem, momentum compaction factor was increased by switching from 60°/60° to 90°/90° long cell optics in arcs (more details in the presentation by K. Oide). This also helps to mitigate collective instabilities.
- The negative consequences of increasing α_p (increase in \mathcal{E}_y and in L_i) are weakened at this energy, but to obtain the "old" luminosity, it is necessary to slightly increase the linear charge density this will probably be impossible due to other restrictions.
- Low RF frequency (400 MHz) is preferable to mitigate the coherent beam-beam instability (due to smaller v_7), electron clouds and ion instabilities (due to greater bunch spacing).
- In the "old" optics with 4 IPs, in order to suppress the coherent beam-beam instability, it was required to reduce β_x^* from 15 to 10 cm. As the α_p has increased, this may not be necessary, but should be checked.
- As it is now seen, the main problem is associated with misalignments and errors, which (even after correction) can lead to a significant decrease in the momentum acceptance. An acceptable bunch population and luminosity depend on how successfully we can solve this problem.

Parameter Optimization at WW

Arc optics: $60^{\circ}/60^{\circ} \Rightarrow 90^{\circ}/90^{\circ}$, long cell

At this energy, the $60^{\circ}/60^{\circ}$ optics is optimal, but we decided to switch to $90^{\circ}/90^{\circ}$ long cell – more details in the presentation by K. Oide.

Drawback: peak luminosity drops by about 20%.

Benefits:

- Same arc optics as at Z, simplifies transition from Z to W. The integrated luminosity may not decrease.
- Do not need anymore 60°/60° cell: reduces the number of sextupoles and slightly increases the filling factor.
- Improves overall coherent stability.
- Increases the synchrotron tune (this is important for the energy calibration).

RF options

For energy calibration by resonant depolarization, the synchrotron modulation index is important:

$$\zeta = v_0 \sigma_{\delta} / v_z$$

- In the CDR, with v_z = 0.05, we get ζ =2.4, which is too large. And now we have increased σ_δ , since the arc radius has decreased. But increase in α_p helps.
- With $U_{RF} = 750$ MV and 400 MHz (as in the CDR) we get $v_z = 0.067$ and $\zeta = 1.9$. With $U_{RF} = 1$ GV, we get $v_z = 0.08$ and $\zeta = 1.57$. The optimum RF voltage must be determined by agreement between RF and depolarization requirements.
- Higher RF frequency can be useful. For example, with 600 MHz and 700 MV we get v_z = 0.079. There should be no obstacles from the side of coherent instability.

Parameter Optimization at ZH

- At this energy, we have to switch to 90°/90° short cell optics to get small emittances.
- Resonant depolarization is not possible here, so we do not need large v_z .
- Piwinski angle is not very large, so we can choose $v_x \approx 0.5 + \phi v_z$ and avoid coherent beam-beam instability.
- Change in RF frequency and/or RF voltage will affect ϕ and v_z to the same extent, therefore will not affect the above condition.
- The only requirement for the RF system is to provide more RF acceptance than the momentum acceptance of nonlinear lattice.
- As for all energies, luminosity depends on momentum acceptance in the presence of misalignments and errors.

Parameter Optimization at ttbar

Luminosity is limited by BS lifetime:

$$\tau_{bs} \propto \exp\left(\frac{2\alpha\eta\rho}{3r_e\gamma^2}\right) \cdot \frac{\rho\sqrt{\eta\rho}}{L_i \cdot \gamma^2}$$

 α – fine structure constant

 η – momentum acceptance

 ρ – bending radius of a trajectory at the IP

 L_i – length of interaction area

The major tool for increasing the lifetime is making ρ larger. For flat beams, ρ is inversely proportional to the surface charge density:

$$\frac{1}{\rho} \propto \frac{N_p}{\gamma \sigma_x \sigma_z} \propto \frac{\xi_y}{L_i} \sqrt{\frac{\varepsilon_y}{\beta_y^*}} \propto L \sqrt{\frac{\varepsilon_y}{\beta_y^*}}$$
(assuming $L_i \approx \beta_y^*$)

- We need to increase ρ with large luminosity => small emittances (90°/90° short cell optics) and increase in L_i (i.e. in σ_x) and β_y^* .
- Since ε_x should be small, σ_x is controlled by β_x^* which was increased to 1 m.
- Asymmetrical momentum acceptance to match the actual energy distribution (K. Oide).
- The only requirement for the RF system is to provide more RF acceptance than the momentum acceptance of nonlinear lattice. The bunch length does not matter! But we should keep $N_p \propto \sigma_z$.

Problems: Misalignments, Errors and Corrections

- Misalignments and errors can lead to a significant decrease in the momentum acceptance.
 This limits the luminosity per IP (even in the case of ideal super-periodicity).
- The full beam-beam footprint from 2 or 4 IPs can cross a number of strong resonances, e.g. 1/2, 1/3, etc. The width of these resonances depends on the level of symmetry breaking, which depends on the magnitude of misalignments and the quality of corrections.
- Ways to solve the problem: improve the quality of corrections, reduce the magnitude of misalignments (can be expensive!). Further optimization of working points may be required.
- Optimal bunch population, number of bunches, number of IPs (2 or 4) and luminosity it
 all depends on the above.
- Potential problems associated with impedances, collective instabilities, etc. may also require revision of some parameters.