Quasi-Strong-Strong Beam-Beam Simulations for FCC-ee

D. Shatilov (BINP)

FUTURE CIRCULAR COLLIDER
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Introduction

It will rather not be about specific results, but about the code development.

There is an established point of view that the most correct approach for beam-beam interaction is strong-strong simulation, and we should strive for this. On the other hand, this model is very complex, slow and resource-intensive. Is it really so necessary?

Any modeling is a kind of simplified version of reality. And one need to understand well what can be neglected and what cannot be. In this presentation, we will try to show that for the FCC-ee, in most cases, it will be quite sufficient to use a quasi-strong-strong model.

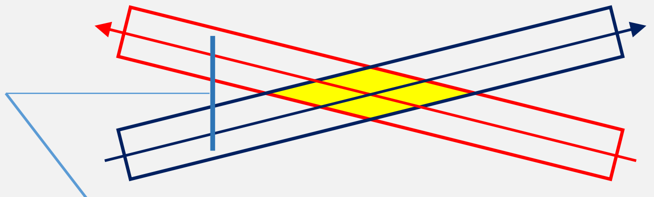
Disruption Parameter

At IPs, both bunches act on each other. Is it necessary to take into account the change in their distribution functions *during collision*?

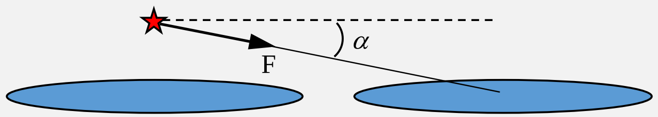
- Beam-beam kicks depend on the distribution of transverse coordinates of the oncoming beam, and [almost] does not depend on the distribution of transverse momenta.
- The kicks change the transverse momenta, not the coordinates. However, during the interaction, $\Delta p_{x,y}$ will have time to transfer into Δx, Δy.
- The magnitude of change in the transverse coordinates during collision is described by the disruption parameter (here $\xi_{x,y}$ refers to one IP):

$$D_{x,y} = 4\pi\xi_{x,y}\frac{\sigma_z}{\beta^*_{x,y}}$$

- In crab waist collision, we have D_x << 1, but large ξ_y and β_y^* << σ_z. Does it mean that D_y >> 1? No, σ_z in the above formula should be replaced by $L_i \approx \beta_y^*$, so we have $D_y \sim 1$.
- Relatively small disruption parameter ($D_{x,y} \leq 1$) means that the distribution of coordinates remains almost unaffected during interaction.
- Examples of $D_{x,y}$ >> 1: linear colliders (ILC, CLIC).



Collision with large Piwinski angle



Vertical plane: kick from the opposite bunch (slice)

If the horizontal distance between slices is greater than a few σ_x, then the force is directed to the center of the opposite slice and falls as 1/r, and $\alpha \propto \sigma_y/\sigma_x$ << 1. The vertical kick: $F_y = F{\cdot}\sin(\alpha)$ << F.

Therefore, the vertical kick is localized in the overlap region of the two beams (marked in yellow).

Simulation Models

Interaction with the opposite bunch

1) **Weak-strong (WS)**

   The opposite (strong) bunch is not affected during long-term (many turns) tracking. This is a simple and fast model. It is always recommended to start with it.

2) **Strong-strong (SS)**

   Both bunches are affected and updated during each collision. This is a complex and time-consuming model, but we must use it when $D_{x,y} >> 1$. Simplified variant (to avoid solving Poisson equation): take into account the barycenter of each slice (transverse displacements) and fit the transverse distribution to Gaussian.

3) **Quasi-strong-strong (QSS)**

   Swap the "weak" and the "strong" bunches every *n-th* turn, and thus update the parameters of the opposite bunch. More realistic and more complex option: simulate two beams simultaneously (in parallel) and exchange data every turn. The opposite bunch is *frozen* (not affected by beam-beam) during collision. This is much faster than SS, but cannot be used when $D_{x,y} >> 1$.

Particle tracking between IP(s)

1) **Linear lattice (constant transport matrix, can be with coupling)**

   Damping and noise can be applied too. It is simple, fast and most flexible. If beam-beam is considered as the major nonlinearity, it is recommended to start with this approach.

2) **Realistic nonlinear lattice**

   This is more time-consuming, but accounts chromaticity, DA and momentum acceptance, interference between beam-beam and lattice-driven resonances (especially when considering misalignments and errors).

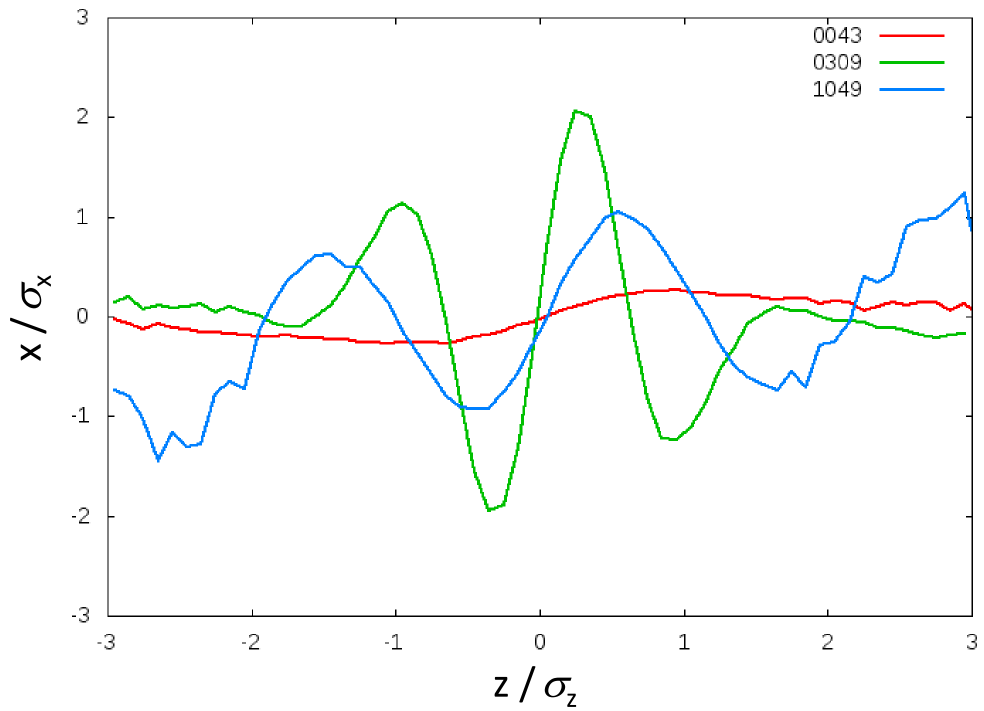
Plus space charge, IBS, electron clouds, impedance, etc.

Crab Waist was discovered in WS, coherent beam-beam instability – in SS, and then confirmed in QSS. In all cases – linear lattice between IPs.

Example: Coherent Beam-Beam Instability

discovered by K. Ohmi in SS simulations

QSS: bunch shape in the horizontal plane at some turns



The shape (barycenter of slices) changes every turn due to betatron and synchrotron oscillations.

QSS simulations:

- Opposite bunch is represented as a sequence of several hundred slices with individual horizontal displacements.
- Two colliding bunches are tracked simultaneously in parallel, and their shapes (transverse emittances and shifts of slices) are updated every turn.
- Particles collide with slices (not slices with slices!) for both bunches. Since D_x is small, the bunch shape in X-Z plane does not change during collision.
- The transverse distribution of slices is assumed to be Gaussian, but $\sigma_{x,y}$ depend on the azimuth.
- This instability leads to an increase in ε_x which has little *direct* effect on the luminosity. But the explicit betatron coupling changes the situation.

Very good agreement was obtained between SS and QSS simulations.

Explicit Betatron Coupling

For more realistic simulations, we need a lattice with many sources of betatron coupling along the ring. For example, enter vertical misalignments of sextupoles, which will result in the "desired" value of vertical emittance.

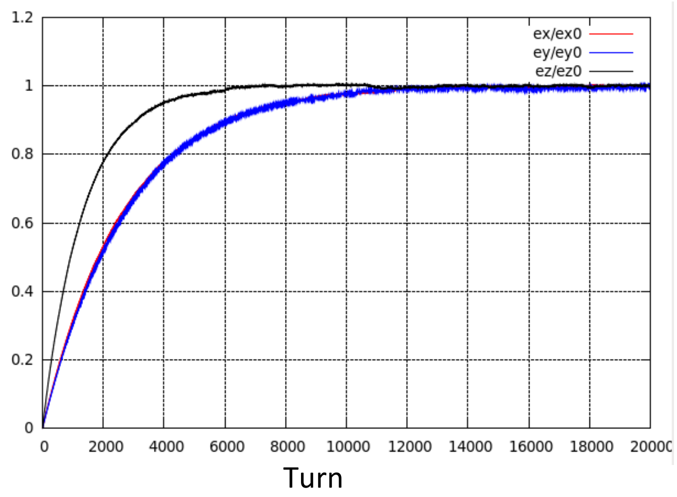
Linear tracking is performed using three matrices: Transport, Damping and Diffusion, which can be obtained from SAD.

Example: a part of Lifetrac's input file:

```
Structure:
  Watch_point
  Map_RF_CS   Damp_RF_CS   Nois_RF_CS    Crab_sext
  Map_CS_IP   Damp_CS_IP   Nois_CS_IP
  Main_IP
  Map_IP_CS   Damp_IP_CS   Nois_IP_CS   -Crab_sext
  Map_CS_RF   Damp_CS_RF   Nois_CS_RF
  RF_Cav
End_structure
```

Vertical emittance is formed partially by the vertical dispersion, and partially by betatron coupling.

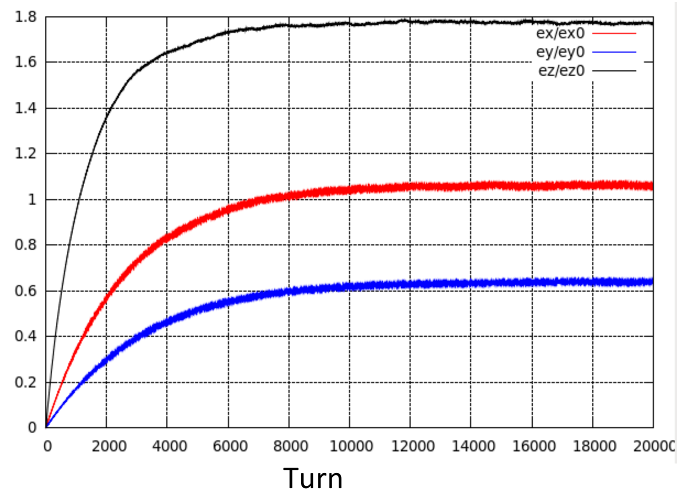
Example: FCC-ee at Z, old lattice as of 2016



Without beam-beam

10^5 particles with initial zero coordinates converge to an equilibrium distribution.

Half-ring, damping time is 5320 turns for x,y and 2660 turns for z.



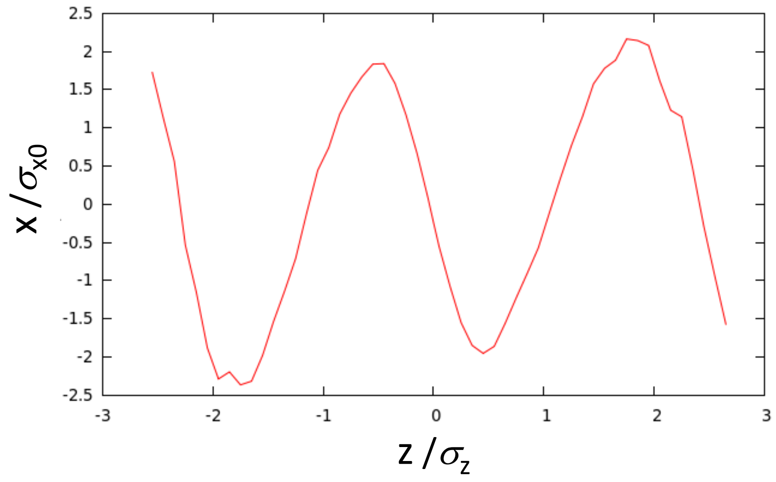
WS with beam-beam

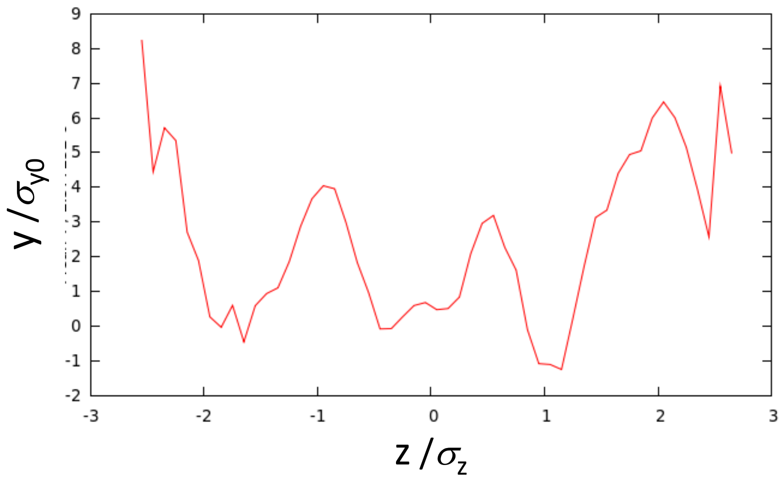
Energy spread increased due to beamstrahlung.

ε_x slightly increased, but ε_y decreased, since $\xi_y > \xi_x$ and the w.p. moves away from the coupling resonance.

Coherent Beam-Beam Instability with Betatron Coupling

One of the turns

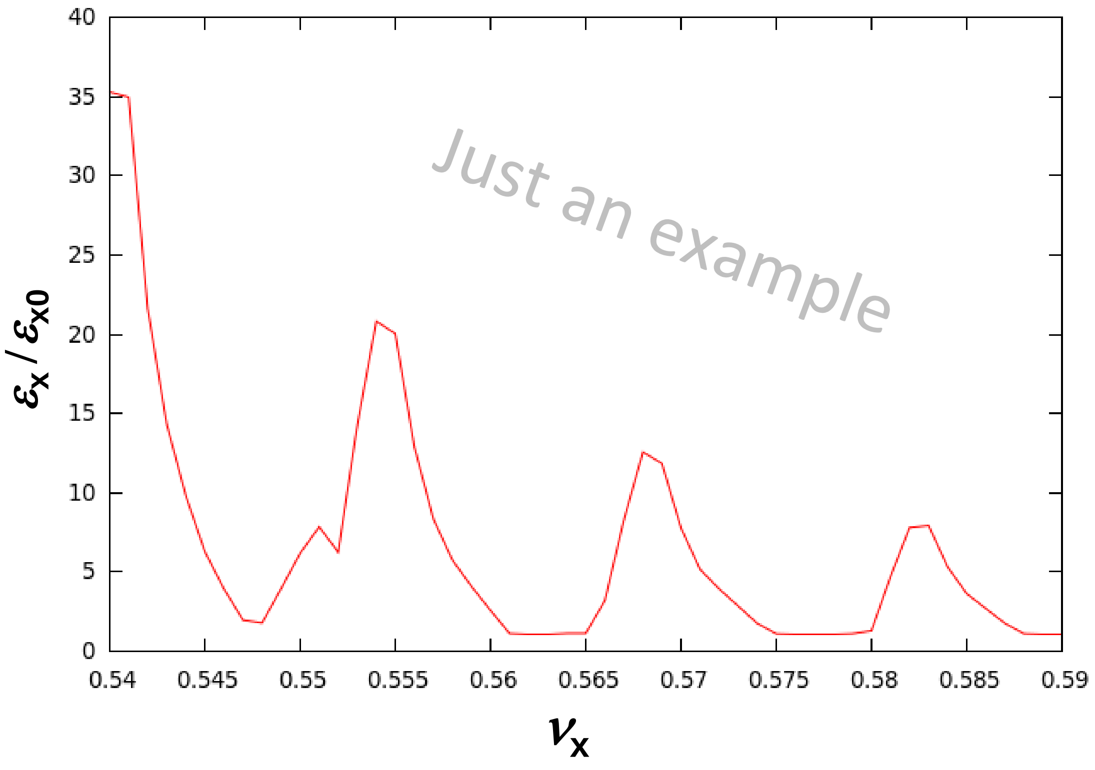




- The bunch shape wriggles in the horizontal plane. Due to betatron coupling, these waves also appear in the vertical plane.
- Dependence of the vertical kick on vertical displacement is much stronger than the dependence of the horizontal kick on horizontal displacement. Wriggles in the vertical plane are amplified by the vertical beam-beam kicks.
- These zigzags are pumped into the vertical emittance more efficiently than into the horizontal one! Possible reasons:
  - Difference between betatron tunes.
  - Large vertical tune spread.
- As a result, the vertical emittance blowup turned out to be much stronger than the horizontal!

Coherent Beam-Beam Instability: Tune Scan

Horizontal emittance vs. betatron tune



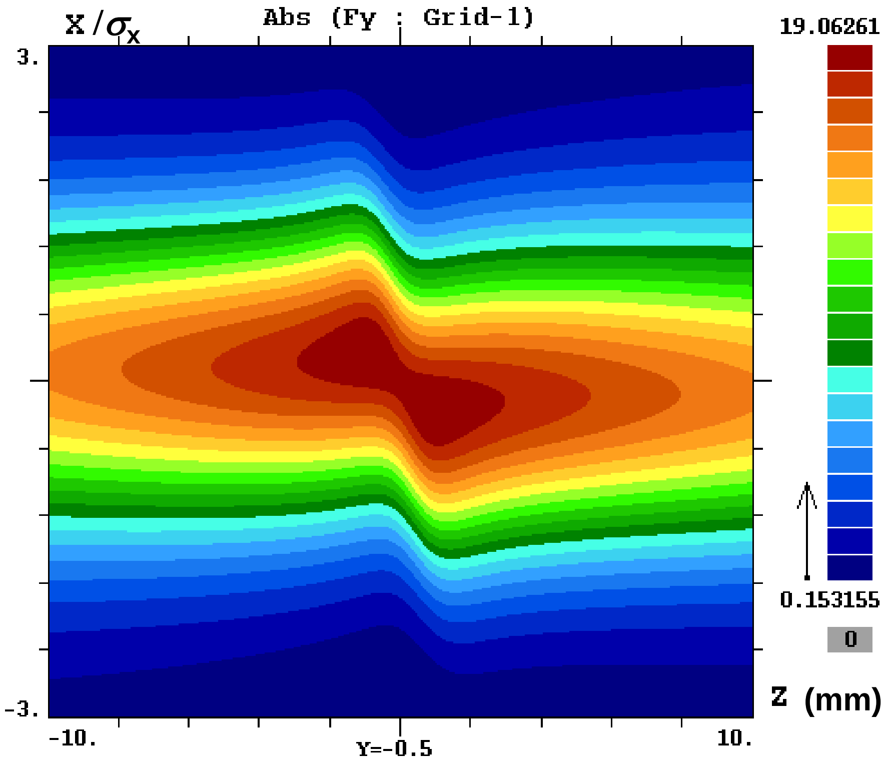
Scanning in a model with an explicit betatron coupling is difficult, since for each point one will have to re-select the sextupole offsets in order to obtain the design ε_y, and then get all matrices from SAD.

Therefore, a simple model without betatron coupling is used, which provides the correct values for ε_x.

The vertical emittance and luminosity in such a model will be incorrect and do not need to be paid attention to. It is only important for us to identify the areas in which there is no instability.

There are many scans to be performed to optimize the parameters, and there are many points in each scan. And each point is tens of thousands of turns (several damping times). Hence, computation speed matters, which means the advantage of QSS model.

Non-Gaussian Strong Bunch



Vertical kick from a crabbed bunch

ICFA BDN 52, p.42 (2010)

So far, we have assumed that the density distribution in the slices is Gaussian. In fact, this is not the case, especially in the crab waist collision scheme.

For an arbitrary distribution, one need to build grids and calculate the kicks by interpolating between nodes. Here different approaches are possible, but they are equally applicable to all beam-beam models: WS, SS and QSS.

Here is an example of a grid for crabbed bunch. As it turned out, this has a positive effect: the suppression of resonances is slightly improved and the luminosity is slightly increased.

On the other hand, there are many effects that we do not yet take into account, and which *slightly* worsen the situation. So, for simplicity, un-crabbed strong bunches are usually used in simulations.

Summary

- Disruption parameters in FCC-ee are not large, which allows the use of a quasi-strong-strong model instead of a strong-strong one. The advantages are simplicity and speed.
- However, since $D_y \sim 1$, strong-strong model also is needed just in case, and to confirm the results. But probably not for massive computations.
- There are many other effects to consider, which can lead to unexpected results when interfering with beam-beam. Probably a QSS model will also be sufficient to consider and study this.