Tolerances for vibration

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Vibration of quadrupoles

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The vertical displacement of a beam caused by a quadrupole vibrating with an amplitude Δy_q and an angular frequency ω_q at the vertical phase advance ϕ_q from the IP:

$$\begin{aligned}\Delta y^* &= \sum_n \sqrt{\beta^* \beta_q} \exp(-nT_0/\tau_y + i\omega_q n T_0) \sin(\phi_q + n\mu_y) k_q \Delta y_q \\ &= \sum_n \sqrt{\beta^* \beta_q} \exp(-n\alpha_y + in\mu_q) \sin(\phi_q + n\mu_y) k_q \Delta y_q \,,\end{aligned} \tag{1}$$

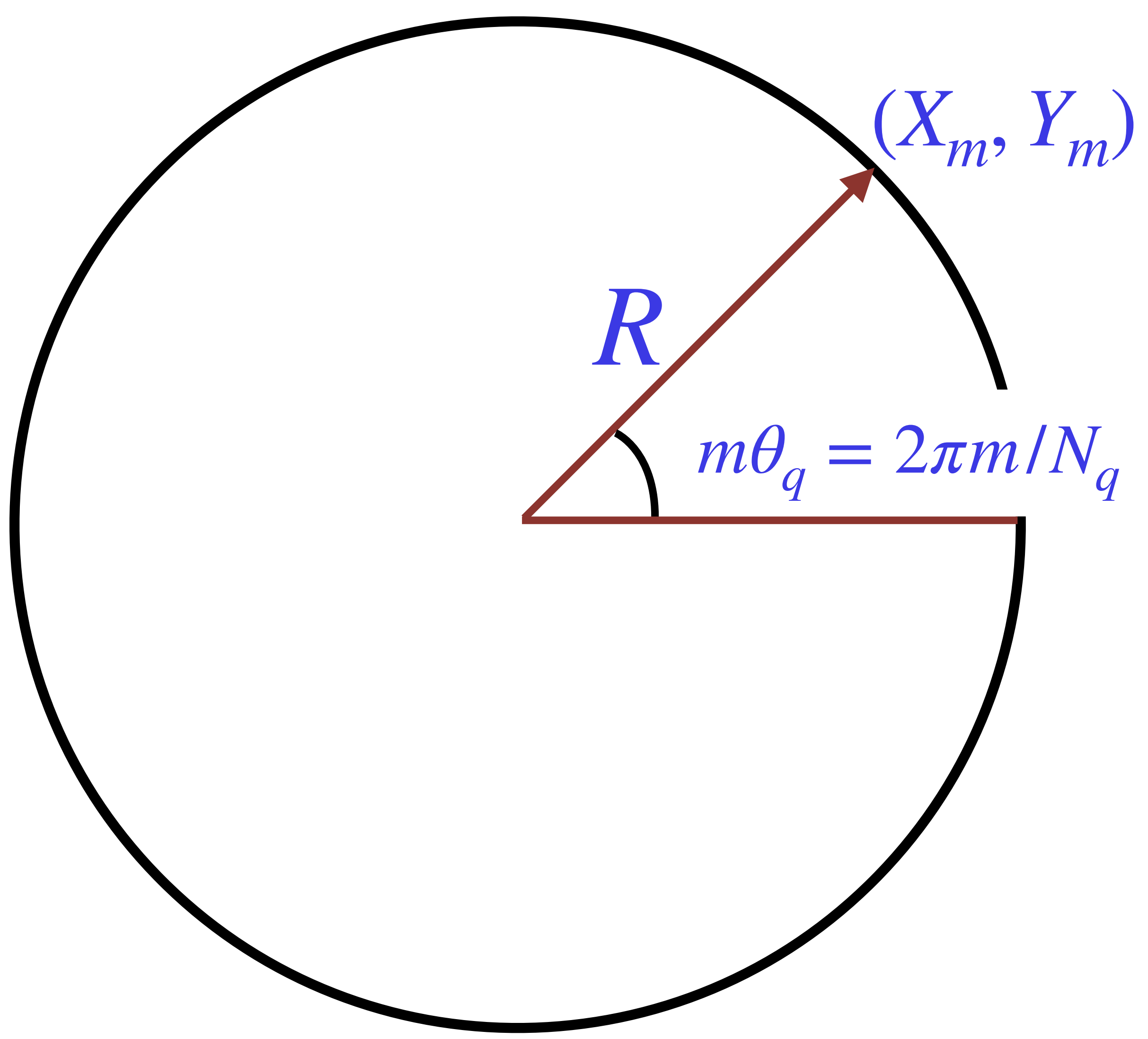
where μ_y, T_0 & τ_y, are the vertical betatron angular tune, the revolution & damping times, and $\alpha_y \equiv T_0/\tau_y$, $\mu_q \equiv \omega_q T_0$. β^*, β_q, k_q are the beta functions at the IP and the quadrupole, and the focusing strength of the quadrupole.

1.1 Vibration due to seismic motion

The vibration amplitude Δy_q can be random to each quad, or coherent due to external seismic motion. First let us evaluate the coherent part by assuming that the quads are distributed over the ring uniformly with the betatron phase $\phi_q = m\Delta\phi_q$, and also physically located over a ring of the radius R with a constant separation azimuthal angle θ_q, *i.e.*,

$$X_m + iY_m = R\exp(im\theta_q)\,, \tag{2}$$

where m runs over 1 through N_q, the number of quads per ring.



Then if the quads follow the seismic wave in the ground, the displacement Δy_m of the m-th quadrupole is written as

$$\Delta y_m = u \exp\left(i(k_X X_m + k_Y Y_m - \omega_q t)\right) , \tag{3}$$

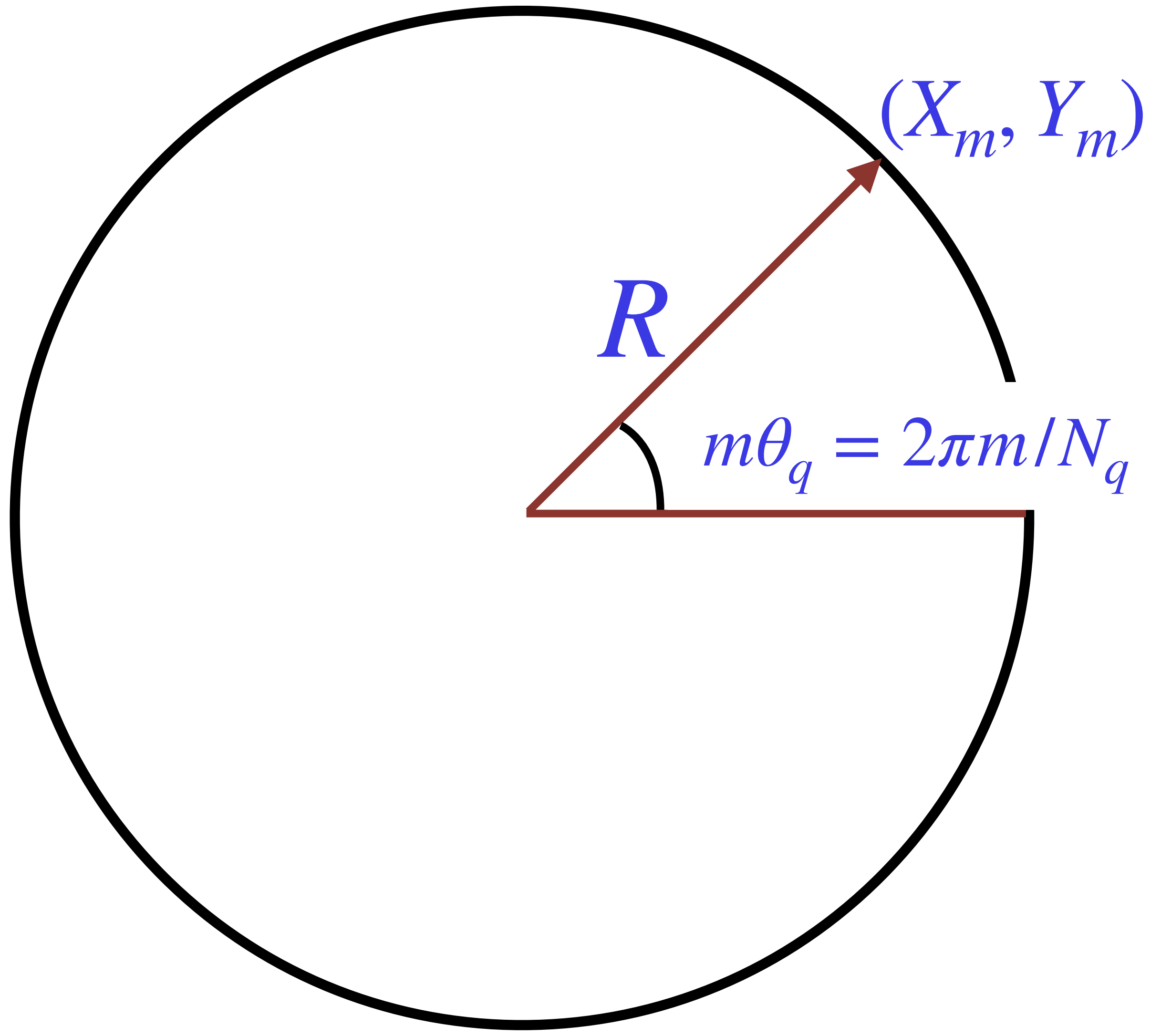
where $k_{X,Y}$ are the components of the seismic wave number vector, and u represents the amplitude. Here we just set $k_X = k$ and $k_Y = 0$ for simplicity without losing generality if the ring is nearly a circle. So we may sum up the term $\sin(\phi_q + n\mu_y)\Delta y_q$ in Eq. (1) over quadrupoles as

$$\begin{aligned} N_q d_s &= \sum_m^{N_q} \sin(\phi_q + n\mu_y)\Delta y_m \\ &= \sum_m^{N_q} \sin(m\Delta\phi_q + n\mu_y) u \exp\left(i(kR\cos m\theta_q - \omega_q t)\right) \\ &= u \sum_{\ell=-\infty}^{\infty} \sum_m^{N_q} \sin(m\Delta\phi_q + n\mu_y) J_\ell(kR) i^\ell \exp(i\ell m\theta_q - i\omega_q t) , \end{aligned} \tag{4}$$

where we have applied $\exp(ix\cos z) = \sum_\ell i^l J_\ell(x) \exp i\ell z$. Although there may be a resonance in Eq. (4) at $\ell \sim \pm\Delta\phi_q/\theta_q$, the index ℓ becomes too large in the case of FCC-ee Z, where $\Delta\phi_q = 83.5\,\mathrm{deg}$, $\theta_q = 360/924 \sim 0.390\,\mathrm{deg}$, and $\ell \sim 214$. As for N_q, we have taken only QD's into account. Thus the coefficient J_ℓ becomes infinitesimal for such a large ℓ, so the resonant effect is negligible.

$$\exp(i(\boldsymbol{k} \cdot \boldsymbol{x} - \omega_q t))$$

$$\boldsymbol{k} = (k_X, k_Y) = (k, 0)$$



The term $\ell = 0$ in Eq. (4) gives

$$d_{s0} = uJ_0(kR)\frac{\sin(\mu_y/2)\sin(n\mu_y + (\mu_y - \Delta\phi_q)/2)}{\sin(\Delta\phi_q/2)}\,. \tag{5}$$

We know $J_0(x) \leq 1$, and the rests of the rhs of Eq. (5) are not far from 1. Then magnitude of the coherent component looks smaller than the random component:

$$|d_s| \ll \sqrt{N_q}u\,. \tag{6}$$

$$= \sum_n \sqrt{\beta^*\beta_q}\exp(-n\alpha_y + in\mu_q)\sin(\phi_q + n\mu_y)k_q\Delta y_q\,,$$

1.2 Resonance with the betatron frequency

Then the expectation value of the vibration of the beam at the IP,$\langle|\Delta y^*|^2\rangle$ is obtained by averaging Eq. ,(1) over ϕ_q as

$$\begin{aligned}\langle|\Delta y^*|^2\rangle &= \frac{1}{2\pi}\int|\Delta y^*|^2 d\phi_q \\ &= \frac{\beta^*\beta_q k_q^2\langle\Delta y_q^2\rangle}{4}\frac{\exp(\alpha)(\cosh\alpha - \cos\mu_q\cos\mu_y)}{(\cosh\alpha - \cos(\mu_q - \mu_y))(\cosh\alpha - \cos(\mu_q + \mu_y))}\,.\end{aligned} \tag{7}$$

Thus the vibration at the IP has resonances at $\mu_q = \pm\mu_y + 2m\pi$ with an integer m.

At each resonance, by assuming the spectrum of $\langle \Delta y_q^2 \rangle$ is uniform, the vibration at the IP can be evaluated as:

$$\langle |\Delta y^*|^2 \rangle = \frac{\beta^* \beta_q k_q^2}{8\alpha T_0} \sum_m S((\pm\mu_y \pm 2m\pi)/T_0)\,, \tag{8}$$

where $S(\omega)$ is the power spectrum density of $\langle \Delta y_q^2(\omega) \rangle$, and we have assumed $\cos\mu_q \cos\mu_y \sim 1/2$ and $\alpha \ll 1$.

A measurement of ground vibration tells that[1],

$$S(\omega) = \sigma\omega^{-4} \sim 10^{-15} \left(\frac{\omega}{2\pi\mathrm{Hz}}\right)^{-4} \mathrm{m}^2/\mathrm{Hz}\,, \tag{9}$$

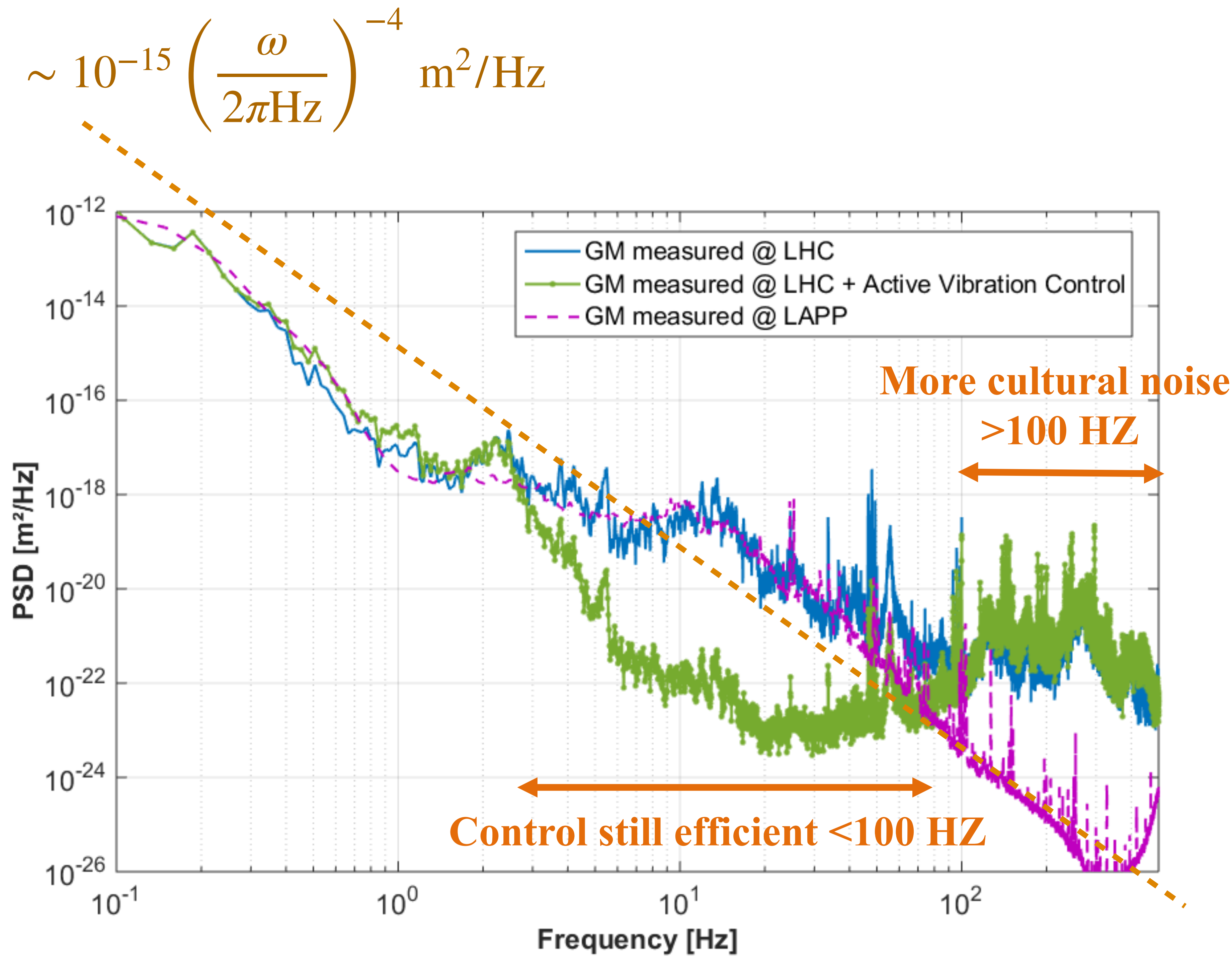
with a coefficient σ, then among the resonances only the lowest one $m \sim \mu_y/2\pi$ will matter. In the case of FCC-ee, it is at

$$\omega/2\pi = \omega_r/2\pi \sim (1.2, 1.8)\,\mathrm{kHz}\,, \tag{10}$$

corresponding to $[\mu_y/2\pi] \sim (0.4, 0.6)$, resulting in

$$S(\omega_r) \sim (4.8, 0.95) \times 10^{-28}\,\mathrm{m}^2/\mathrm{Hz}\,. \tag{11}$$

[1] https://indico.cern.ch/event/694811/contributions/2863859/attachments/1595533/2526938/2018_02_06_FCCee_MDI_workshop_Serluca.pdf



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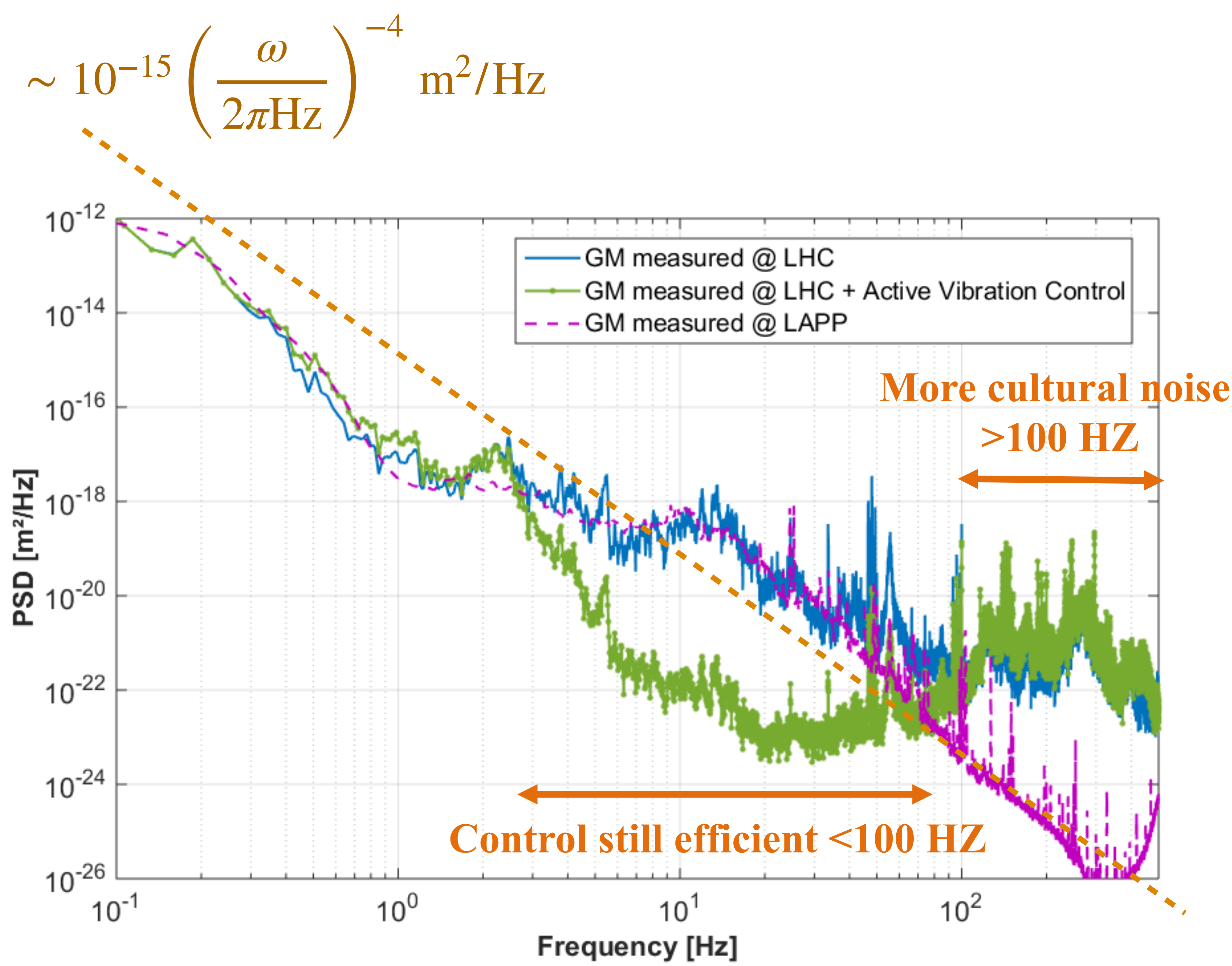
If we plugin numbers at FCC-ee Z:

$$\begin{aligned} \beta^* &= 0.8\,\mathrm{mm}, & \langle\beta\rangle &= 515\,\mathrm{m}, \\ \langle k_q^2\rangle^{1/2} &= 0.058\,/\mathrm{m}, & \langle\beta k_q^2\rangle^{1/2} &= 16.6\,/\mathrm{m}, \\ T_0 &= 300\,\mu\mathrm{s}, & \alpha &= 4\times10^{-4}\,\mathrm{s} \end{aligned} \tag{12}$$

into Eq, (8) and multiply the number of all defocusing quadrupoles N_q=924, we get

$$\sqrt{\Delta y^{*2}} \sim 78\,\mathrm{pm}\,, \tag{13}$$

which is well smaller than the IP vertical beam size, $\sim 37\,\mathrm{nm}$.



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1.3 Non-resonant vibration

Next let us look at the off-resonant contribution of Eq. (7), If we roughly approximtate the tune-dependent term by 1, the integrated power spectrum in a range $\omega \geq \omega_c$ is given as

$$\begin{aligned} \sqrt{\Delta y^{*2}} &= \frac{N_q \beta^* \beta_q k_q^2}{4} \int_{\omega_c}^{\infty} S(\omega) \frac{d\omega}{2\pi} \\ &= \frac{N_q \beta^* \beta_q k_q^2 \sigma}{24\pi\omega_c^3} \,. \end{aligned} \tag{14}$$

In the case for the previous measurement, we estimate $\sigma \sim 1.6 \times 10^{-12}\,\mathrm{m^2/Hz}$, then

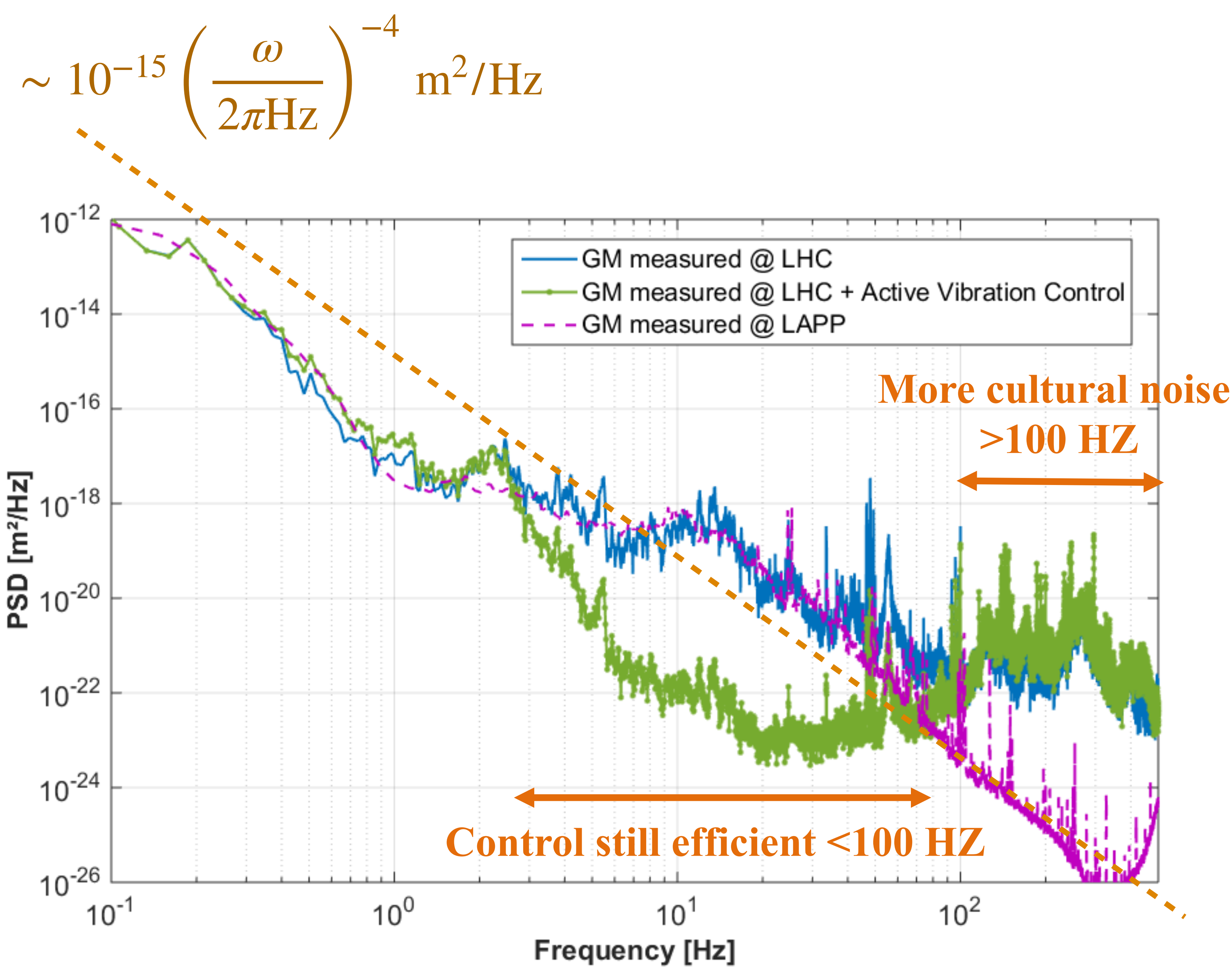
$$\sqrt{\Delta y^{*2}} \sim 32.3\,\mathrm{nm} \tag{15}$$

for $\omega_c = 2\pi \times 1\,\mathrm{Hz}$. The assumption here is that below the critical frequency ω_c, an orbit feedback suppresses the beam oscillation perfectly. Thus the expected vibration reaches to the vertical beam size at the IP.

Among the vibration, the dominant contribution is from the defocusing final quads "QC1%1/2", which reaches

$$\sqrt{\Delta y^{*2}}_{\mathrm{QC1\%1/2}} \sim 31.9\,\mathrm{nm}\,. \tag{16}$$

Thus suppressing the vibration of the final quads and a feedback system works beyond 1 Hz will be crucial.



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Maintaining the collision

1.4 Beam-beam deflection (SLC, B-factories)

If two beams have relative vertical offset at the IP by Δy^*, each beam receive a beam-beam kick at the IP:

$$\Delta y^* = \pm \frac{2\pi\xi_y}{\beta_y^*}\Delta y^* \,, \tag{17}$$

where ξ_y is the vertical beam-beam parameter. If we plugin the numbers for Z:

$$\xi_y = 0.152 \,, \qquad \beta_y^* = 0.8\,\mathrm{mm}, \qquad \Delta y^* = \frac{1}{10}\sigma_y^* = 3.4\,\mathrm{nm} \,, \tag{18}$$

the beam-beam kick becomes

$$\Delta p_y^* = 4.08\,\mu\mathrm{rad} \,. \tag{19}$$

If we have BPMs for both beams at $\pm 2\,$m from the IP, this kick is well larger than the resolution of the BPMs, at least for the average over the bunches.

Thus the beam-beam deflection has been the primary method to detect and correct the relative offset of two beams at the IP for a linear or double ring colliders.

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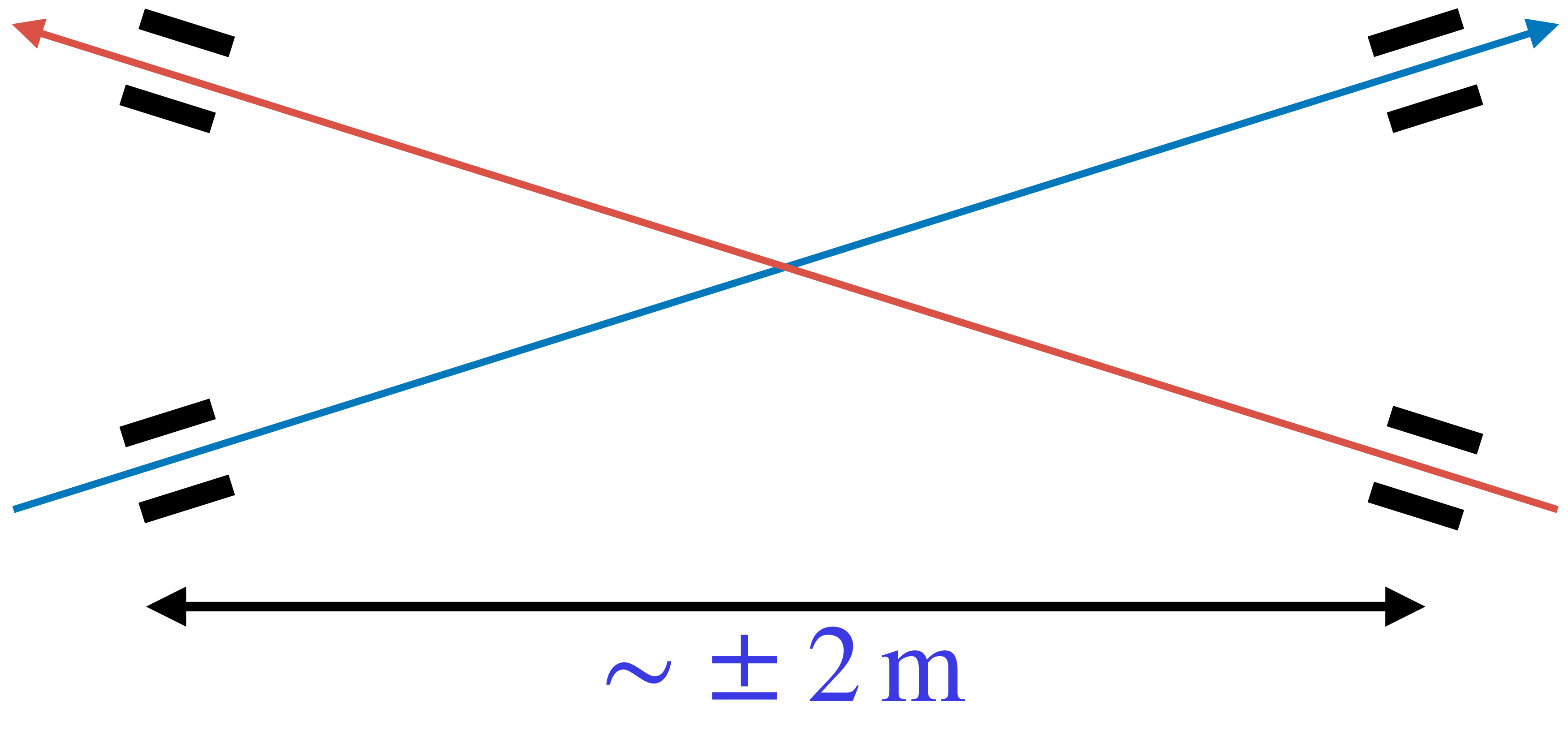
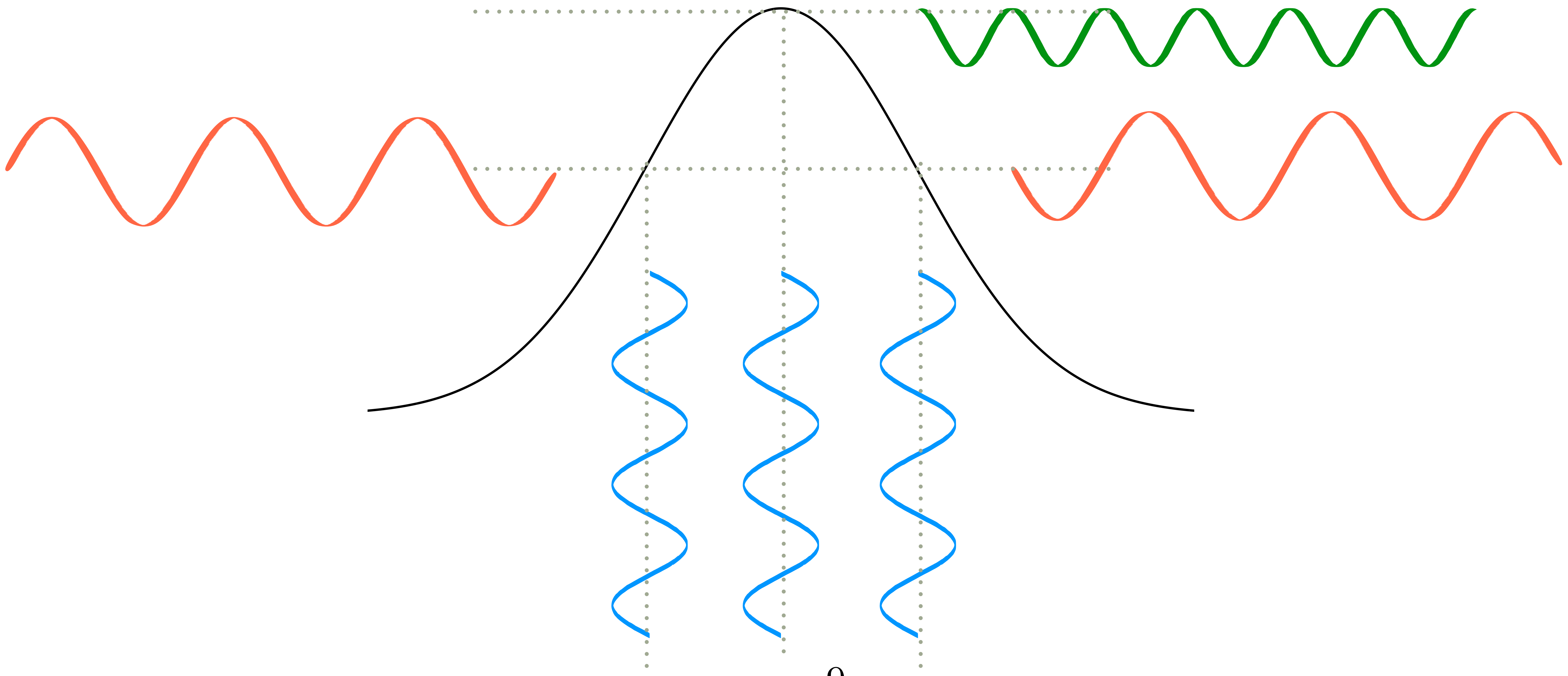


By combining the readings of four BPMs at the both sides of IP for both beams, it is possible to extract the beam-beam deflection.

Maintaining the collision (2)

1.5 Dithering

As the horizontal beam-beam parameter is very small in low energies ($\xi_x = 0.004$ at Z), the beam-beam deflection is not appropriate for the detection of horizon offset at the IP. In such a case, a method called *dithering*, developed at PEP-II, is applicable, It shakes one beam with a single frequency, then detect the modulation of luminosity at that frequency, then by nullifying that component, the optimum offset is obtained.



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An example of vertical bump at IP

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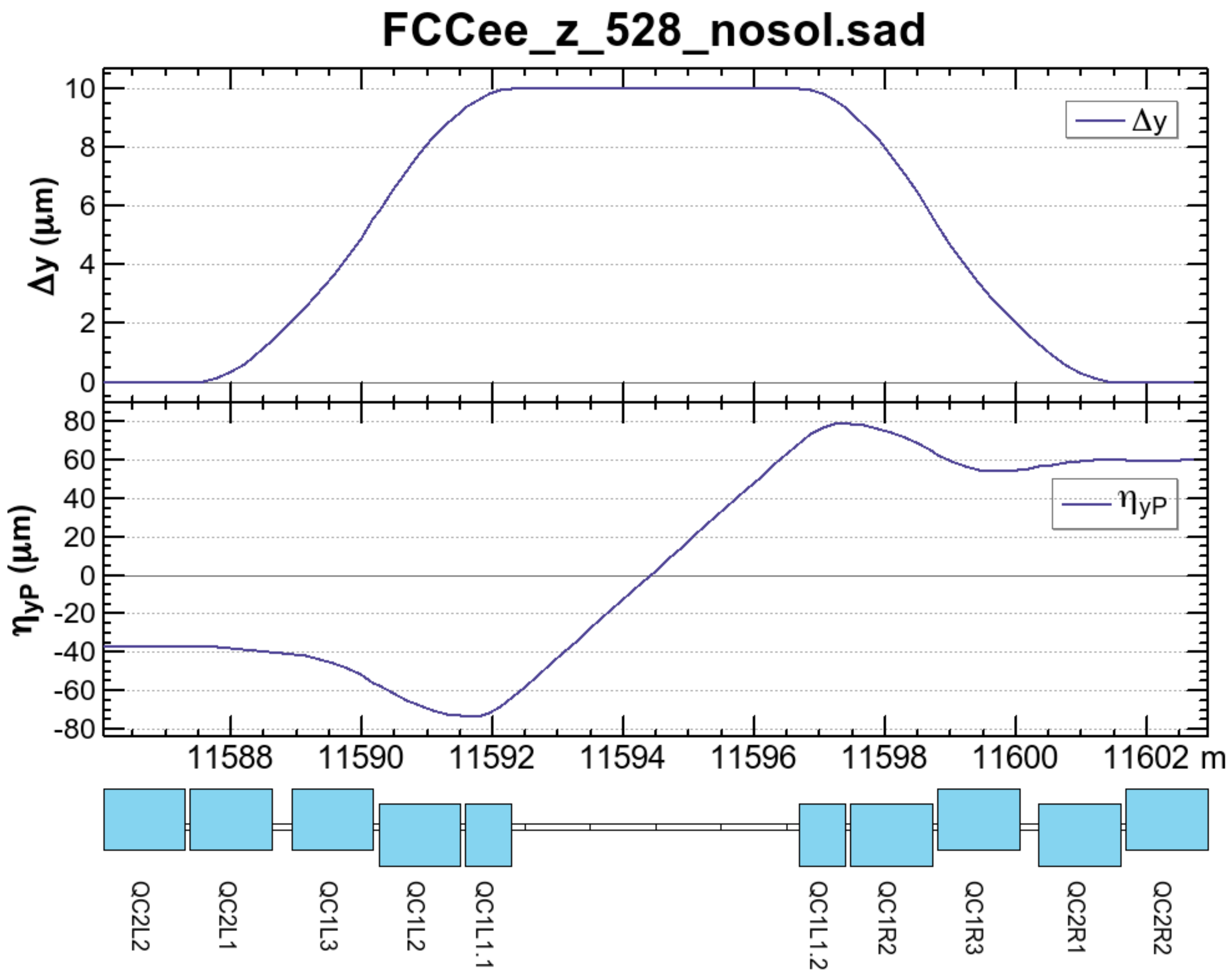
- A simplest vertical bump orbit to control the IP offset can be produced by the skew dipole corrector winding of QC{12}{LR}1.
- This example does not close the dispersion.
- However, the associated vertical emittance generated by the dispersion leak is only 2.6 am by the 10 μm vertical offset at the IP. So the dispersion leak is not a practical issue.
- If this corrector is used for the IP feedback, its frequency response can be an issue, due to reduction by the beam pipe.



Summary

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Tolerances for the vibration of quadrupoles are evaluated for three cases:

- A seismic wave has smaller effects than random motion of each quadrupole for an equal amplitude.
- Resonance with the betatron frequency: weak, as the betatron frequency is in the range of kHz.
- Non-resonant, incoherent vibration of each quad produces 30 nm vertical motion at the IP for ≧1 Hz.
  - Mostly by the final quads QC1.
  - Assuming each quad follows the ground motion measured at LHC & LAPP.
  - No amplification of the mechanical motion of the girders has been assumed.
  - Below a frequency $\lesssim$ 10 Hz, a vertical orbit feedback is required.
- IP vertical offset can be detected by the beam-beam deflection.
  - For horizontal except for tt, dithering method can be user to maximize the luminosity.
- A simple vertical bump orbit can correct the IP offset easily.
  - Frequency response can be an issue.