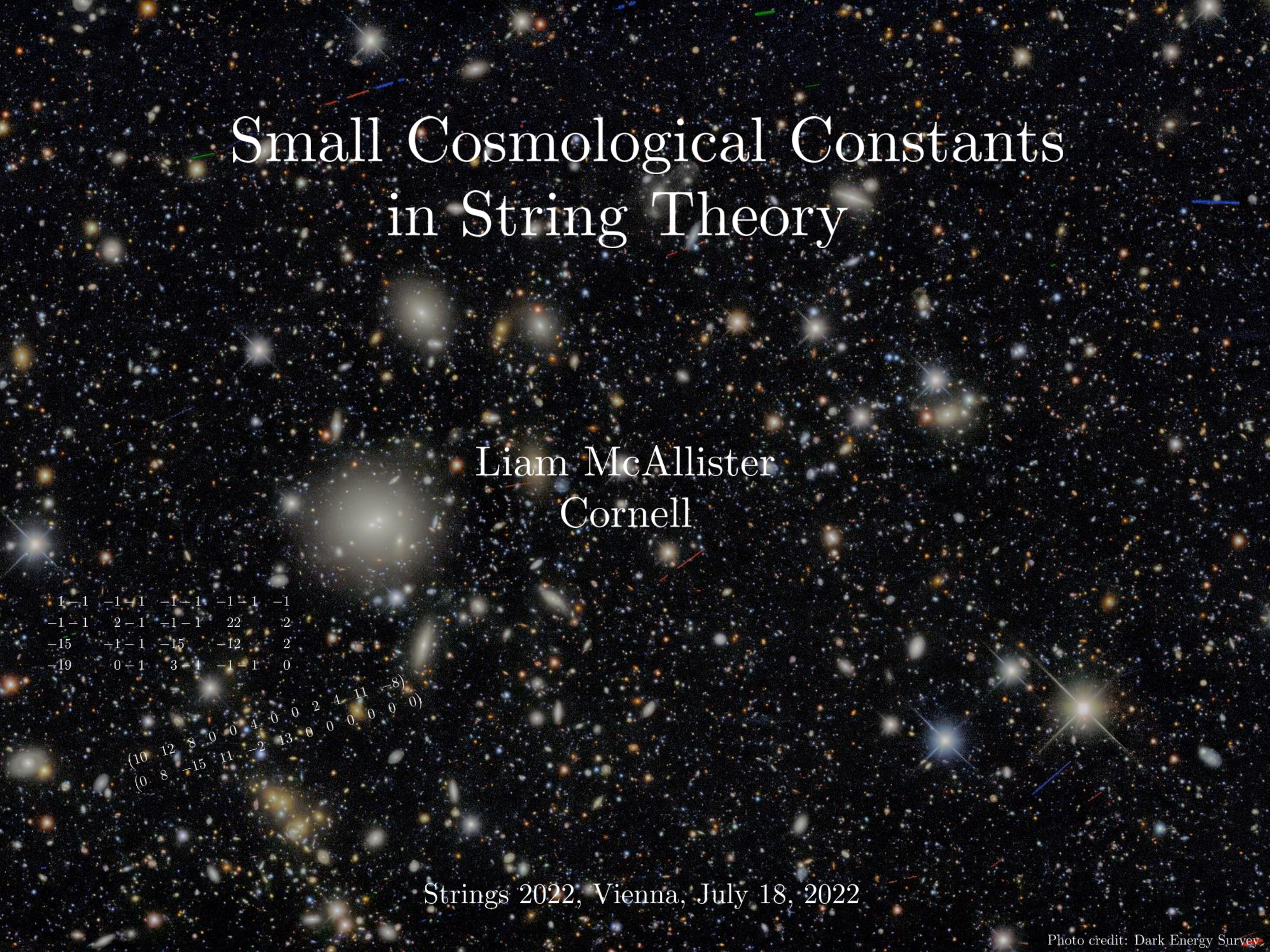


Small Cosmological Constants in String Theory

Liam McAllister
Cornell

Strings 2022, Vienna, July 18, 2022

Photo credit: Dark Energy Survey

Cosmological constant problem

$$\rho_{\text{vac}} \approx 10^{-123}\ M_{\text{pl}}^4$$

Too hard, so consider supersymmetric version.

$$\rho_{\text{vac}} \approx -10^{-123}\ M_{\text{pl}}^4$$

Can we explain how exponentially large SUSY universes arise in a theory with a small fundamental length-scale?

Principle

"It is better to light a lamp(post) than to curse the dark energy."

We work where computations are possible, and make no claim of genericity.

Principle

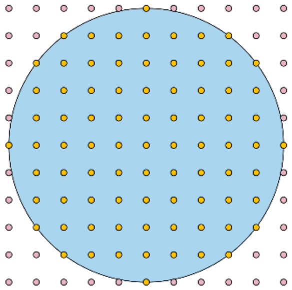
Choose setting where vacuum structure is dominated by well-understood superpotential terms set by integer data — topology and quantized fluxes.

Systematically compute W, and find vacua.

Setting: type IIB flux compactifications on orientifolds of Calabi-Yau threefold hypersurfaces in toric varieties.

We compute the superpotential by exploiting toric structures and purpose-built software.

Small vacuum energy?



Quantized fluxes in a high-dimensional lattice can potentially be chosen to make c.c. small.

Bousso, Polchinski 00   Feng, March-Russell, Sethi, Wilczek 00

N-dimensional Bousso-Polchinski flux landscape:

$$\rho_{\rm vac} = -M_{\rm pl}^4 + Q^i G_{ij} Q^j \,, \qquad \vec{Q} \in \mathbb{Z}^N$$

Fine-tune vast number of terms to precision $\sim 10^{-N}$

Problem:

Finding exponentially small c.c. is exponentially costly.

$\rho_{\rm vac} \lesssim 10^{-123}\, M_{\rm pl}^4$ is out of reach.

Denef, Douglas, 06
Halverson, Ruehle 18

Mechanism for small vacuum energy

We work in low-dimensional lattices ($4 \leq \dim \leq 10$).

We solve a Diophantine problem to find vacua in which all perturbative terms are exactly zero along one dimension.

Demirtas, Kim, L.M., Moritz 19

What remain are naturally exponentially small instantons.

By discrete, explicit choices of topology and quantized fluxes, we tune their exponents to polynomial precision,

$$e.g. \quad W \propto -2\, e^{2\pi i \tau \cdot \frac{7}{29}} + 252\, e^{2\pi i \tau \cdot \frac{7}{28}} + \ldots \qquad \tau = C_0 + \frac{i}{g_s}$$

leading to $\rho \propto \left(\frac{2}{252}\right)^{2\times 29} \approx 10^{-122}$.

Distribution is $\mathrm{d}\log\rho$ rather than $\mathrm{d}\rho$!

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Summary

We find solutions of type IIB string theory of the form

$$\mathrm{AdS}_4 \times X_6$$

with X_6 a Calabi-Yau orientifold with three-form flux.

The solutions are $\mathcal{N} = 1$ supersymmetric.

The vacuum energy is exponentially small.

Hierarchical scale separation, $\ell_{\mathrm{AdS}}/\ell_{\mathrm{KK}} > 10^{100}$.

Mechanism: racetrack of worldsheet instantons.

Vacuum energy is exponential in integers determined by topological data — Gopakumar-Vafa invariants and flux quanta.

$$\text{example:} \quad \left(\frac{2}{252}\right)^{58} \approx 10^{-122}$$

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Collaboration

Demirtas, Kim, L.M., Moritz 19
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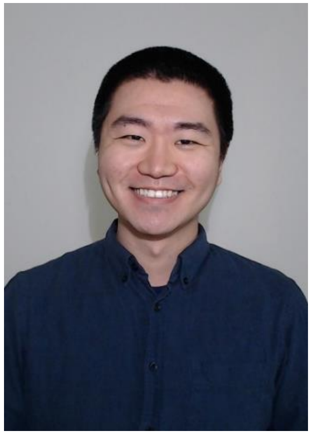
Demirtas, L.M., Rios-Tascon, 20
Demirtas, Kim, L.M., Moritz, Rios-Tascon, 21
+works in progress with Gendler, Nally

Mehmet Demirtas



Manki Kim

Jakob Moritz

Andres Rios-Tascon

Naomi Gendler

Richard Nally

Plan

I. Computation of the superpotential

II. Control of corrections to the Kähler potential

Type IIB string theory compactified on orientifold, X, of a CY_3.

Moduli: axiodilaton τ
complex structure z_a, $a = 1, \dots h^{2,1}(X)$
Kähler: T_i, $i = 1, \dots h^{1,1}(X)$

Choose quantized fluxes $F_3, H_3 \in H^3(X, \mathbb{Z})$. $G_3 := F_3 - \tau H_3$

$$W(\tau, z_a, T_i) = W_{\text{flux}}(\tau, z_a) + W_{\text{np}}(\tau, z_a, T_i)$$

$$W_{\text{flux}} = \int_X G_3 \wedge \Omega$$

Gukov, Vafa, Witten 99

$$= \text{poly}(\tau, z_a) + \sum \exp(\tau, z_a)$$

$$W_{\text{np}} = \sum \text{D-brane instantons}$$

Witten 96

$$= \sum_i \mathcal{A}_i(\tau, z_a) \exp\Big(-2\pi T_i\Big) + \dots .$$

In our solutions, we ensure:

$\text{poly}(\tau, z_a) \equiv 0$ Perturbatively flat vacuum

$$\sum \exp(\tau, z_a) = \mathcal{N}_1 e^{2\pi i p_1 \tau} + \mathcal{N}_2 e^{2\pi i p_2 \tau} + \text{negl.}$$ Racetrack

$\mathcal{N}_1, \mathcal{N}_2 \in \mathbb{Z}, \quad p_1, p_2 \in \mathbb{Q}$

$\mathcal{A}_i(\tau, z_a) = \mathcal{A}_i$ constant Pfaffians

$$W(\tau, z_a, T_i) = W_{\text{flux}}(\tau, z_a) + W_{\text{np}}(\tau, z_a, T_i)$$

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$$\mathcal{N}_1, \mathcal{N}_2 \in \mathbb{Z}, p_1, p_2 \in \mathbb{Q}$$

Racetrack

$\mathcal{A}_i(\tau, z_a) = \mathcal{A}_i$ constant Pfaffians

$$W(\tau, z_a, T_i) = \mathcal{N}_1 e^{2\pi i p_1 \tau} + \mathcal{N}_2 e^{2\pi i p_2 \tau} + \sum_i \mathcal{A}_i \exp\Big(-2\pi T_i\Big)$$

- Purely exponential $\Rightarrow$ vacuum energy naturally exponentially small.
- Determined by the numbers $\mathcal{N}_{1,2},\ p_{1,2},\ \mathcal{A}_i$

$$W(\tau, z_a, T_i) = W_{\rm flux}(\tau, z_a) + W_{\rm np}(\tau, z_a, T_i)$$

$$W_{\rm flux} = \int_X G_3 \wedge \Omega$$

Gukov, Vafa, Witten 99

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- Purely exponential $\Rightarrow$ vacuum energy naturally exponentially small.
- Determined by the numbers $\mathcal{N}_{1,2},\ p_{1,2},\ \mathcal{A}_i$

We find quantized fluxes for which

$$\text{poly}(\tau, z_a) \equiv 0 \qquad \text{perturbatively flat vacuum}$$

$$\text{along a flat valley } z_a = p_a \tau, \quad \vec{p} \in \mathbb{Q}^{h^{2,1}}$$

Complex structure deformations transverse to the valley are heavy.
The along-valley modulus remains massless so far.

Demirtas, Kim, L.M., Moritz 19

see also: Giryavets, Kachru, Tripathy, Trivedi 03
Denef, Douglas, Florea 04

$$W(\tau, z_a, T_i) = W_{\rm flux}(\tau, z_a) + W_{\rm np}(\tau, z_a, T_i)$$

$$W_{\rm flux} = \int_X G_3 \wedge \Omega$$

Gukov, Vafa, Witten 99

$$= {\rm poly}(\tau, z_a) + \sum \exp(\tau, z_a)$$

$$W_{\rm np} = \sum \text{D-brane instantons}$$

Witten 96

$$= \sum_i \mathcal{A}_i(\tau, z_a) \exp\Big(-2\pi T_i\Big) + \ldots.$$

Our solutions:

$${\rm poly}(\tau, z_a) \equiv 0 \qquad \text{Perturbatively flat vacuum}$$

$$\sum \exp(\tau, z_a) = \mathcal{N}_1 e^{2\pi i p_1 \tau} + \mathcal{N}_2 e^{2\pi i p_2 \tau} + \text{negl.} \qquad \text{Racetrack}$$

$$\mathcal{N}_1, \mathcal{N}_2 \in \mathbb{Z}, p_1, p_2 \in \mathbb{Q}$$

$$\mathcal{A}_i(\tau, z_a) = \mathcal{A}_i \qquad \text{constant Pfaffians}$$

$$W(\tau, z_a, T_i) = \mathcal{N}_1 e^{2\pi i p_1 \tau} + \mathcal{N}_2 e^{2\pi i p_2 \tau} + \sum_i \mathcal{A}_i \exp\Big(-2\pi T_i\Big)$$

- Purely exponential $\Rightarrow$ vacuum energy naturally exponentially small.
- Determined by the numbers $\mathcal{N}_{1,2},\ p_{1,2},\ \mathcal{A}_i$

Racetrack Superpotential

4d $\mathcal{N} = 1$ field theory

$$W(z) = \mathcal{N}_1 e^{-p_1 z} + \mathcal{N}_2 e^{-p_2 z} + \ldots$$

$$\mathcal{N}_{1,2} \in \mathbb{R}, \quad p_{1,2} > 0$$

$$W(z_{\min}) = \frac{\mathcal{N}_2 \, (p_1 - p_2)}{p_1} \left(- \frac{\mathcal{N}_1 p_1}{\mathcal{N}_2 p_2} \right)^{\frac{p_2}{p_2 - p_1}}$$

$$\ll 1 \text{ if } |p_1 - p_2| \ll p_2, \quad \mathcal{N}_1 \ll \mathcal{N}_2$$

Incarnation as Flux Superpotential

prepotential: $\mathcal{F}(z) = \mathcal{F}_{\text{poly}}(z) + \mathcal{F}_{\text{inst}}(z)$

$$\mathcal{F}_{\text{poly}}(z) = -\frac{1}{3!}\widetilde{\kappa}_{abc} z^a z^b z^c + \frac{1}{2}\tilde{a}_{ab} z^a z^b + \frac{1}{24}\tilde{c}_a z^b + \frac{\zeta(3)\chi(\widetilde{X})}{2(2\pi i)^3}$$

intersection numbers, Chern classes of mirror $\widetilde{X}$

$$\mathcal{F}_{\text{inst}}(z) = -\frac{1}{(2\pi i)^3} \sum_{\tilde{\mathbf{q}} \in \mathcal{M}(\widetilde{X})} \mathrm{GV}_{\tilde{\mathbf{q}}}\, \mathrm{Li}_3\left(e^{2\pi i\, \tilde{\mathbf{q}}\cdot\mathbf{z}}\right)$$

$\mathrm{GV}_{\tilde{\mathbf{q}}}$: genus-0 GV invariants of $\widetilde{X}$

$\mathrm{Li}_k(q) := \sum_{n=1}^{\infty} q^n/n^k$

periods: $\vec{\Pi} = \begin{pmatrix} \int_X \Omega \wedge \beta_A \\ \int_X \Omega \wedge \alpha^A \end{pmatrix} = \begin{pmatrix} \partial_A \mathcal{F} \\ z^A \end{pmatrix}$

$\{\alpha^A, \beta_A\}$, $A = 0, \ldots, h^{2,1}(X)$: basis of $H^3(X, \mathbb{Z})$

$$W_{\text{flux}}(\tau, z^a) = \sqrt{\tfrac{2}{\pi}} \int_X (F_3 - \tau H_3) \wedge \Omega(z) = \sqrt{\tfrac{2}{\pi}}\, \vec{\Pi}^t\, \Sigma\, (\vec{f} - \tau \vec{h})$$

$\Sigma := \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix}$

Task: compute W_{flux} by computing $\mathcal{F}(z)$.

Computing the Prepotential

Principles clear since early days of mirror symmetry.

Hosono, Klemm, Theisen, Yau 94

Our contribution: really doing this in CY_3 with many moduli.

Demirtas, Kim, L.M., Moritz, Rios-Tascon

Setting: hypersurfaces in toric varieties, as classified by Kreuzer and Skarke.

With `INSTANTON`, can access $h^{2,1} \lesssim 10$.

Klemm
Carta, Mininno, Righi, Westphal 21

With our open source software package `CYTools`, can access $h^{2,1} = 491$, and compute GV invariants to very high degree at $h^{2,1} = \mathcal{O}(100)$.

Demirtas, L.M., Rios-Tascon

Flux Superpotential in a Calabi-Yau

$(h^{2,1}, h^{1,1}) = (5, 113)$

We find quantized fluxes,

$$\vec{f} = \begin{pmatrix} 10 & 12 & 8 & 0 & 0 & 4 & 0 & 0 & 2 & 4 & 11 & -8 \end{pmatrix},$$
$$\vec{h} = \begin{pmatrix} 0 & 8 & -15 & 11 & -2 & 13 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

s.t. along $\mathbf{z} = \mathbf{p}\,\tau\,,\quad \mathbf{p} = \begin{pmatrix} \frac{7}{58} & \frac{15}{58} & \frac{101}{116} & \frac{151}{58} & \frac{-13}{116} \end{pmatrix},$

the polynomial part of W_{flux} vanishes *exactly*.

What remain are IIA worldsheet instantons.
The leading ones have GV invariants -2 and 252.

higher power of $\exp(-1/g_s)$

$$W_{\text{flux}}(\tau) = \frac{2}{(2\pi)^{5/2}}\left(-2\,e^{2\pi i\tau\cdot\frac{7}{29}} + 252\,e^{2\pi i\tau\cdot\frac{7}{28}} + 104\,e^{2\pi i\tau\cdot\frac{43}{116}} + \ldots\right)$$

Moduli are stabilized at $g_s \approx 2\pi\cdot\frac{7}{28}\frac{1}{\log(1/W_0)} \approx 0.011$ and

$$\langle W_{\text{flux}}\rangle \approx 0.526\times\left(\frac{2}{252}\right)^{29} \approx 6.46\times 10^{-62}\,.$$

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Story so far

General form: $$W(\tau, z_a, T_i) = W_{\rm flux}(\tau, z_a) + W_{\rm np}(\tau, z_a, T_i)$$

Chose fluxes s.t.: $$W_{\rm flux}(\tau, z_a) = \text{poly}(\tau, z_a) + \sum \exp(\tau, z_a)$$

Chose CY_3 s.t.: $$W_{\rm flux}(\tau, z_a) = \text{poly}(\tau, z_a) + \mathcal{N}_1 e^{2\pi i p_1 \tau} + \mathcal{N}_2 e^{2\pi i p_2 \tau} + \text{negl.}$$

$$W(\tau, z_a, T_i) = \mathcal{N}_1 e^{2\pi i p_1 \tau} + \mathcal{N}_2 e^{2\pi i p_2 \tau} + W_{\rm np}(\tau, z_a, T_i)$$

Next: need to compute $W_{\rm np}(\tau, z_a, T_i)$ and stabilize the Kähler moduli.

Superpotential for Kähler moduli

General form: $$W_{\rm np}(\tau, z_a, T_i) = \sum_D \mathcal{A}_D(\tau, z_a) \exp\Big(-\tfrac{2\pi}{c_D} T_D\Big)$$

We find CY_3 in which:

$$W_{\rm np}(\tau, z_a, T_i) = \sum_{D_I} \mathcal{A}_{D_I} \exp\Big(-\tfrac{2\pi}{c_I} T_I\Big) + \text{negl.}$$

where the sum runs over prime toric divisors D_I, $I \in \{1, \ldots h^{1,1}+4\}$,
with Pfaffians $\mathcal{A}_{D_I}$ that are constants, with no dependence on τ, z_a.

We select cases where:

- at least $h^{1,1}$ of the D_I are rigid
- their F-theory uplifts $\widehat{D}_I$ have trivial intermediate Jacobian $\mathcal{J}$
- in an orientifold where all seven-branes lie in $\mathfrak{so}(8)$ stacks

$$D_I \text{ rigid} \Leftrightarrow h^\bullet(\widehat{D}_I) = (1,0,0,0)$$

Witten 96a: Nonperturbative superpotentials in string theory

$$h^{2,1}(\widehat{D}_I) = 0 \Rightarrow \mathcal{J} \text{ trivial} \Rightarrow \mathcal{A}_{D_I} \text{ constant}$$

Witten 96b: Five-brane effective action in M theory

Find toric uplift to F-theory, compute $h^{2,1}(\widehat{D}_I)$ by stratification Kim 21

Finding a vacuum

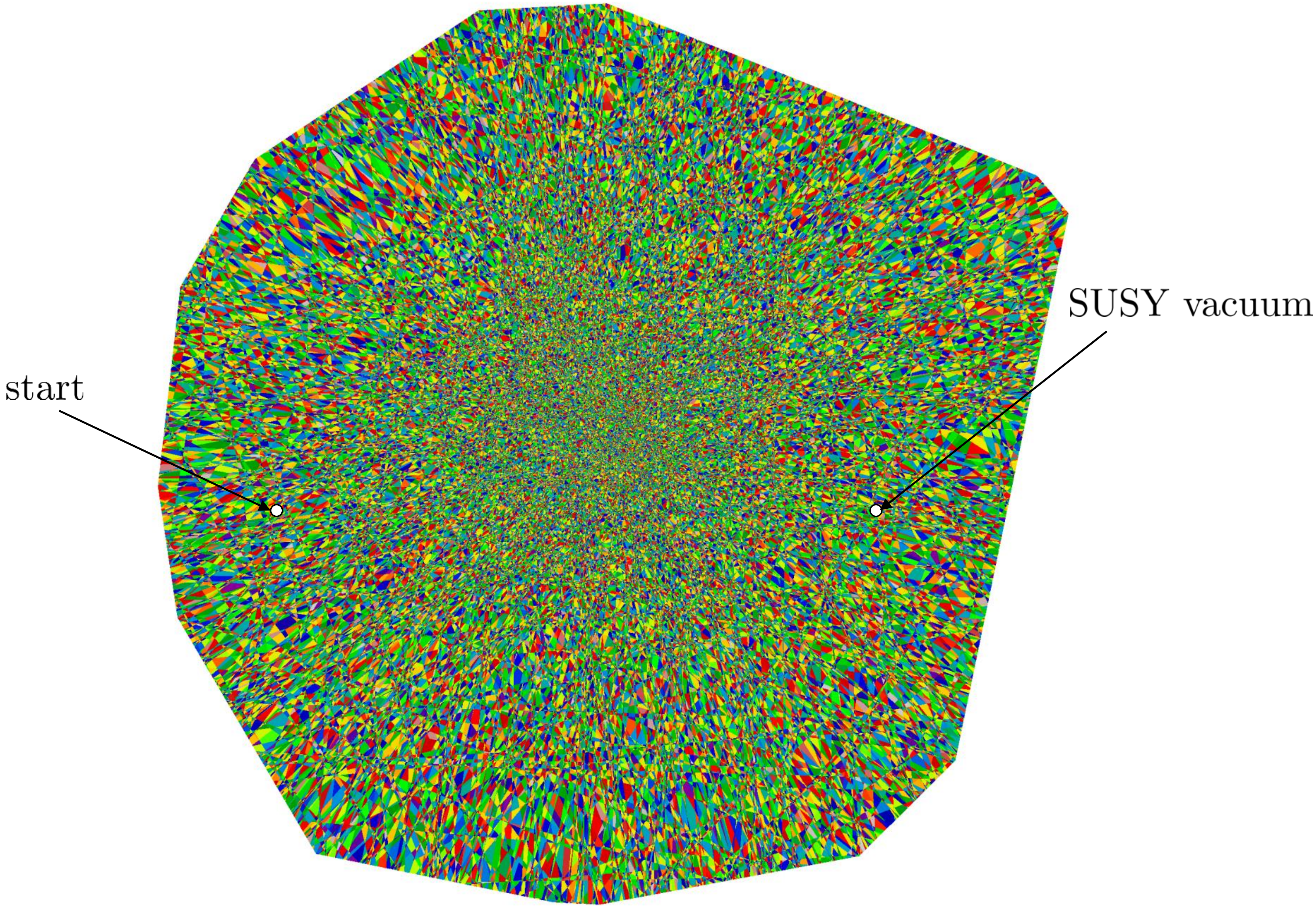
1. Compute the superpotential in many compactifications.

2. Find a case with desired structure.

3. Search the Kähler moduli space for a SUSY vacuum.

4. Determine whether vacuum survives g_s and α' corrections.

2d cross-section of Kähler cone in $h^{1,1} = 491$ threefold



Demirtas, L.M., Rios-Tascon, 20

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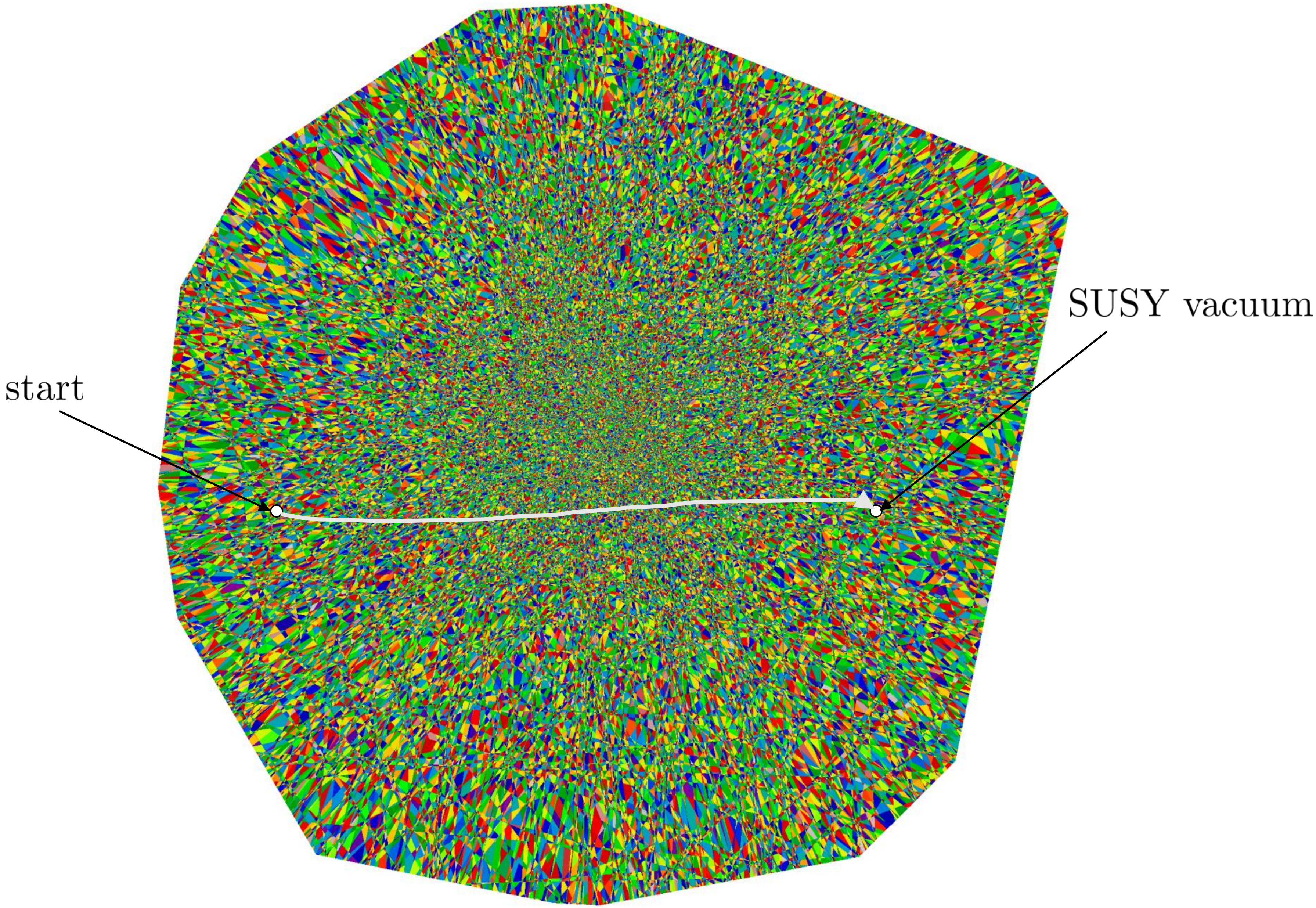

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Control of superpotential

- We have shown by explicit computation that superpotential is

$$W = \mathcal{N}_1 e^{2\pi i p_1 \tau} + \mathcal{N}_2 e^{2\pi i p_2 \tau} + \sum_I \mathcal{A}_{D_I} \exp\Big(-\tfrac{2\pi}{c_I} T_I\Big) + \text{negl.}$$

  - $p_1, p_2 \in \mathbb{Q}$ set by fluxes
  - $\mathcal{N}_1, \mathcal{N}_2 \in \mathbb{Z}$ set by fluxes and by GV invariants of $\widetilde{X}$
  - c_I set by choice of orientifold action

- Ensured that $\mathcal{A}_{D_I}$ are nonzero numbers by standard zero-mode counting.
  - not identically zero by imposing rigidity of D_I
  - constant by imposing $h^{2,1}(\widehat{D}_I) = 0$

  Their numerical values have small effects: $\Delta T_I / T_I \sim \log(\Delta \mathcal{A}_D)/\log(W_0)$
  We checked that our vacua persist for $\mathcal{A}_{D_I} \in [10^{-4}, 10^4]$

- $W_{\mathrm{ED}(-1)} = \sum_{k=1}^{\infty} B_k(z) e^{2\pi i k \tau}$ negligible due to $g_s \ll 1$

Totally explicit computation, up to the numerical values of the $\mathcal{A}_{D_I}$.

cf. Alexandrov, Fırat, Kim, Sen, Stefański 22

Control of Kähler potential

Recall: $V_F = -3e^{K}|W|^2$

$$= -3e^{K_0+\delta K}|W|^2$$

$\Rightarrow$ need only show δK small enough not to destroy vacuum,

not that $\delta K \lesssim 10^{-100}$.

Control of Kähler potential

- Einstein-frame volumes are large $\quad \mathrm{Re}(T_i) \approx \frac{c_i}{2\pi} \log(W_0^{-1})$
- String-frame volumes are order-unity $\quad g_s \propto \frac{2\pi}{\log(W_0^{-1})} \ll 1$
- Weak string coupling $\Rightarrow$ leading corrections are at string tree level, to all orders in α'
- Specifically: worldsheet instanton corrections to $\mathcal{F}$ of parent $\mathcal{N} = 2$ CY_3.

We are able to compute these corrections directly, because we can compute periods in CY_3 with $h^{2,1} \gg 1$.

Worldsheet instantons from small curves

$$\delta\mathcal{F} \propto \sum_{\mathbf{q}} \sum_{n\in\mathbb{N}} \mathrm{GV}_{n\mathbf{q}}\, e^{-2\pi n\, \mathbf{q}\cdot\mathbf{t}}$$

$\mathbf{q}$: curve classes in X

$\mathbf{t}$: Kähler parameters in vacuum

$$\xi_n(\mathbf{q}) := \mathrm{GV}_{n\mathbf{q}}\, e^{-2\pi n\, \mathbf{q}\cdot\mathbf{t}}$$

To test convergence, we compute $\xi_n(\mathbf{q})$ for small-volume curves.

Two kinds of curves:

nilpotent: $\mathrm{GV}_{k\mathbf{q}} = 0\ \forall k > k_{\mathrm{max}} \in \mathbb{N}$

potent: infinite series

Nilpotent curves are safely collapsible.
They give finitely many polylogs, which we include explicitly.

We then examine thousands of rays of potent curves.

Convergence of worldsheet instanton sum

Along multiples of a potent curve $\mathcal{C}$, (example with $h^{1,1} = 113$)

$$\begin{aligned}
\mathrm{GV}(\mathcal{C}) &= 3 \\
\mathrm{GV}(2\mathcal{C}) &= -6 \\
\mathrm{GV}(3\mathcal{C}) &= 27 \\
\mathrm{GV}(4\mathcal{C}) &= -192 \\
\mathrm{GV}(5\mathcal{C}) &= 1695 \\
\mathrm{GV}(6\mathcal{C}) &= -17064 \\
\mathrm{GV}(7\mathcal{C}) &= 188454 \\
\mathrm{GV}(8\mathcal{C}) &= -2228160 \\
\mathrm{GV}(9\mathcal{C}) &= 27748899 \\
\mathrm{GV}(10\mathcal{C}) &= -360012150\,.
\end{aligned}$$

$$\mathrm{GV}(100\mathcal{C}) = -9146115812378313712269739747685735741875063346136791432257902669736951275104733736769227776135148471781320929614886 0000\,.$$

Exponential increase with very stable rate.

So for *large enough* $\mathbf{t}$, $\xi_n := \mathrm{GV}_{n\mathbf{q}}\, e^{-2\pi n\, \mathbf{q}\cdot\mathbf{t}}$ will decay exponentially with n.

e.g. quintic: $\mathbf{t}^{(1)}_{\mathrm{min}} = \frac{1}{2\pi}\log(2875) \approx 1.27,$ $\quad \mathbf{t}^{\mathrm{true}}_{\mathrm{min}} \approx 1.208$

Candelas, De La Ossa, Green, Parkes 90

Convergence of worldsheet instanton sum

Along multiples of a potent curve $\mathcal{C}$, (example with $h^{1,1} = 113$)

$$
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$$
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$$

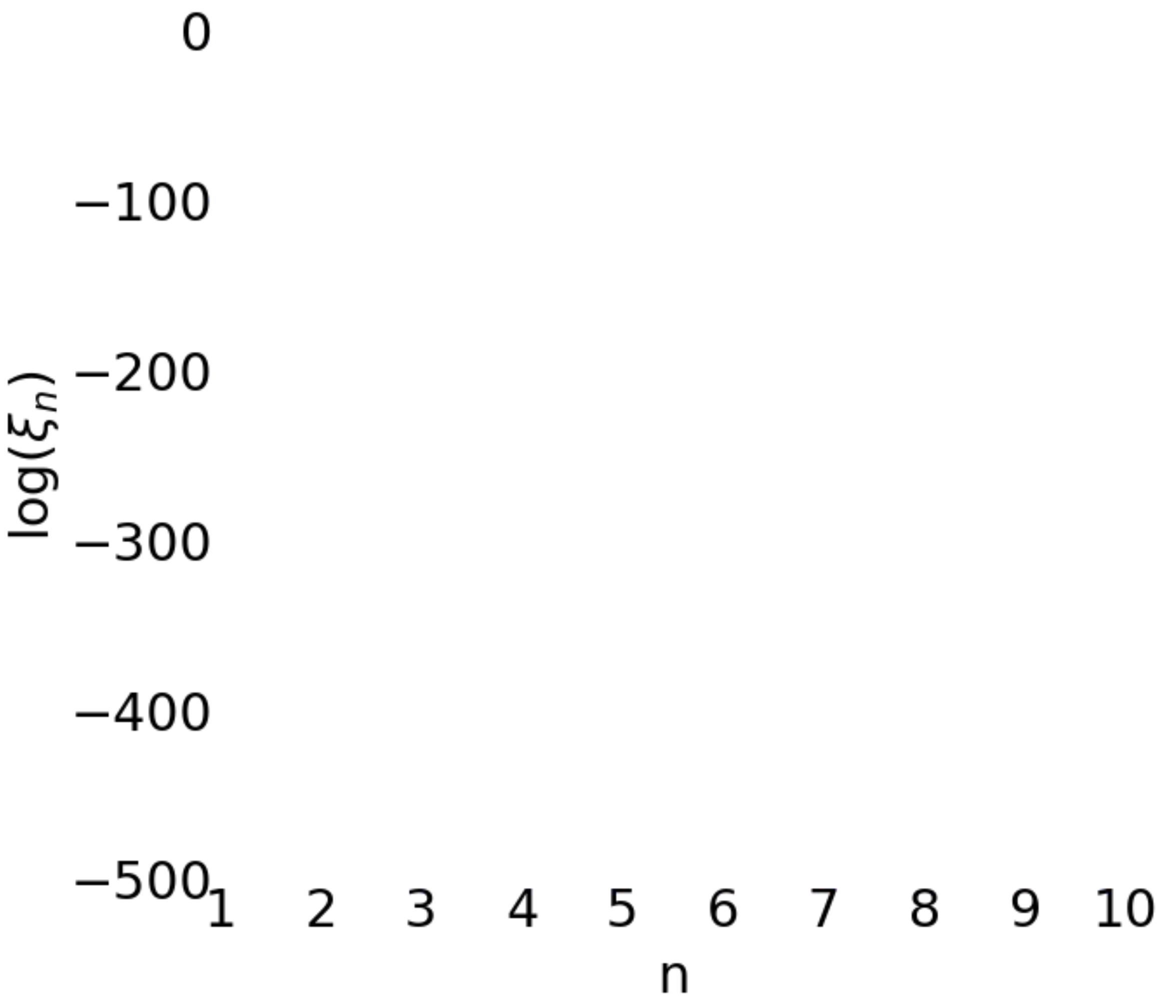
Exponential increase with very stable rate.

So for *large enough* $\mathbf{t}$, $\xi_n := \mathrm{GV}_{n\mathbf{q}}\, e^{-2\pi n\, \mathbf{q}\cdot\mathbf{t}}$ will decay exponentially with n.

In our vacua, is $\mathbf{t}$ large enough?

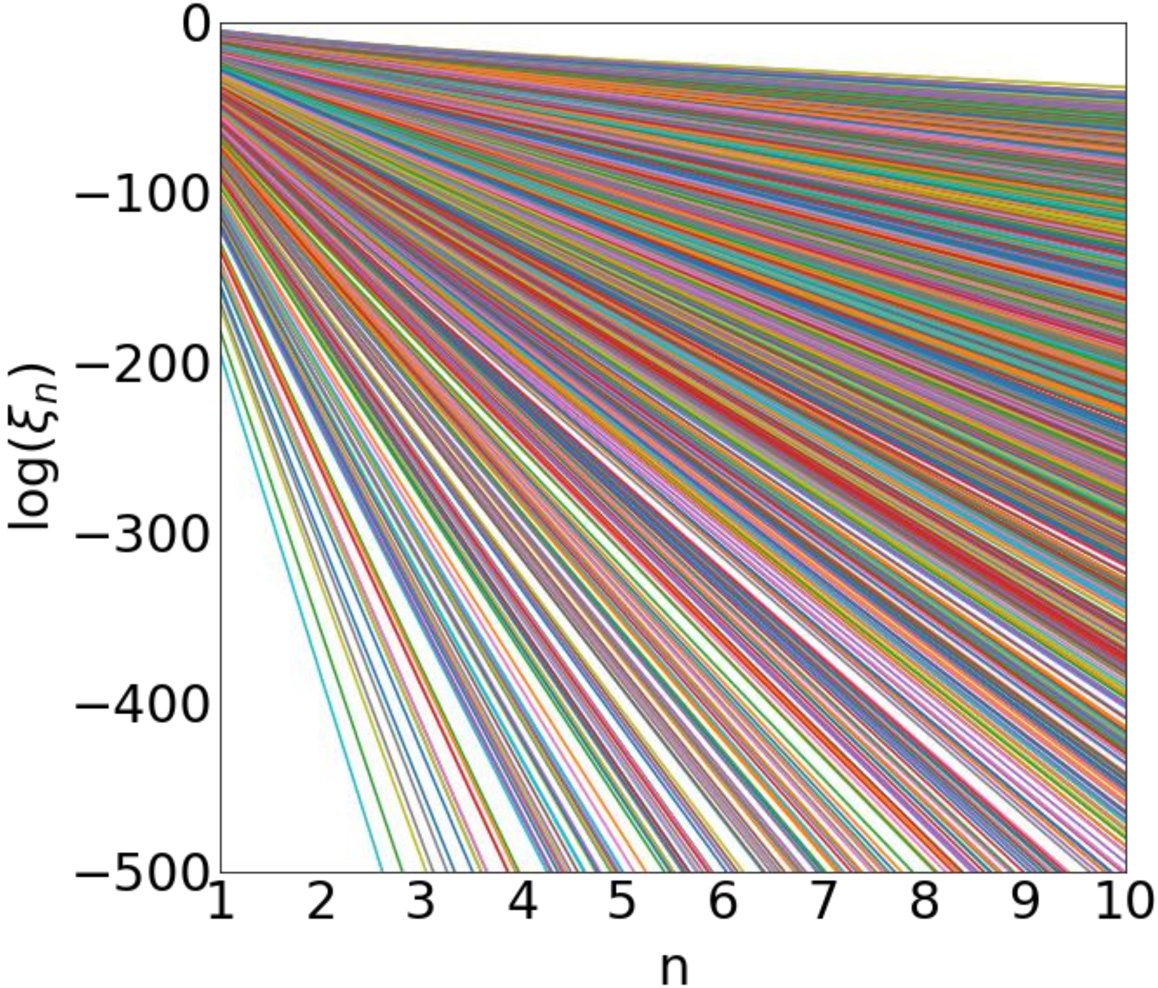
Convergence of worldsheet instanton sum

$$\xi_n := \mathrm{GV}_{n\mathbf{q}}\, e^{-2\pi n\, \mathbf{q}\cdot\mathbf{t}}$$



Convergence of worldsheet instanton sum

$$\xi_n := \mathrm{GV}_{n\mathbf{q}}\, e^{-2\pi n\, \mathbf{q}\cdot\mathbf{t}}$$



largest correction: $\mathcal{O}(10^{-5})$

KKLT vacua

Kachru, Kallosh, Linde, and Trivedi made two claims in their 2003 paper:

Claim 1: in type IIB compactifications with $W = W_{\text{flux}}(\tau, z_a) + W_{\text{np}}(\tau, z_a, T_i)$,
if $\langle W_{\text{flux}} \rangle \ll 1$, there can exist well-controlled SUSY AdS_4 vacua.

Claim 2: given such an AdS_4 vacuum in a compactification with:
a warped deformed conifold region Klebanov, Strassler 00
containing one or more anti-D3-branes, Kachru, Pearson, Verlinde 01
in a suitable parameter regime, there can exist metastable dS_4 vacua.

We have now given strong evidence for the first claim.

KKLT status summary

No evidence of obstructions or inconsistencies.

Still awaiting complete explicit compactifications.

AdS_4 ✓

Demirtas, Kim, L.M., Moritz, Rios-Tascon 21

dS_4 work in progress

talk by L.M. at Strings 2019, Brussels

Conclusions

We have constructed SUSY AdS_4 vacua in CY_3 orientifolds.

Exponentially small W_{flux} from quantized fluxes.

Explicit computation of W_{ED3} stabilizing the Kähler moduli.

Stabilization at $g_s \ll 1$, large Einstein-frame volume.

Small cosmological constants, giant scale separation.

Methods can be applied to build a corner of the landscape.

Search for de Sitter vacua is work in progress.


Thanks!

Towards de Sitter vacua?

We have built tools to compute $\mathcal{N}=1$ EFTs
in a regime of type IIB flux compactifications where we can explicitly enumerate all necessary integer data,
and can search for vacua in which our approximations are self-consistent.

Our main results so far concern $\mathcal{N}=1$ SUSY AdS_4 vacua, where

- polynomial tuning of topological parameters provides a mechanism that makes ρ_{vac} exponentially small.
- unbroken SUSY protects ρ_{vac} from quantum corrections.

But the EFTs appear rich enough to support de Sitter vacua in which SUSY is broken spontaneously:

not by anti-D3-branes, but by competition among superpotential terms.

Establishing self-consistent approximations and parametric control is unsurprisingly more involved than for SUSY AdS_4.

Demirtas, Gendler, Kim, L.M., Moritz, Nally, Rios-Tascon, work in progress.

Comments on related work

Would be valuable to find the dual CFT_3.

A candidate dual suggested in Lüst, Vafa, Wiesner, Xu 22
was shown there to have parametrically incorrect central charge.

It was suggested in Lüst et al. that superpotential terms stabilizing the Kähler moduli "will not materialize".

But since we have directly exhibited all the necessary terms, a simpler reconciliation is that their proposed dual is not the correct one.

Worth weighing evidence on both sides.

An example with $(h^{2,1}, h^{1,1}) = (5, 113)$

The vertices of Δ are the columns of

$$\begin{pmatrix} 1 & -3 & -3 & 0 & 0 & 0 & -5 & -2 \\ 0 & -2 & -1 & 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & -1 \end{pmatrix} .$$

$\mathrm{Vol}(\mathrm{ED3}) \approx 22$
$\mathrm{Vol}(\mathfrak{so}(8)) \approx 132$
$\mathrm{Vol}(X) \approx 8.1 \times 10^5$

There are $h^{1,1} + 1 = 25_{\mathfrak{so}(8)} + 89_{\mathrm{ED3}}$ rigid prime divisors with $h^{2,1}(\widehat{D}_I) = 0$. The fluxes

$$\vec{f} = \begin{pmatrix} 10 & 12 & 8 & 0 & 0 & 4 & 0 & 0 & 2 & 4 & 11 & -8 \end{pmatrix} ,$$
$$\vec{h} = \begin{pmatrix} 0 & 8 & -15 & 11 & -2 & 13 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} ,$$

carry D3-brane charge 56. The D3-brane tadpole is 60, so there are 4 D3-branes. The leading instantons have GV invariants $\mathscr{N}_{\tilde{\mathbf{q}}} = (-2, 252)$ and

$$W_{\text{flux}}(\tau) = \sqrt{\tfrac{2}{\pi}} \frac{1}{(2\pi i)^2} \left(-2\, e^{2\pi i \tau \cdot \frac{7}{29}} + 252\, e^{2\pi i \tau \cdot \frac{7}{28}} \right) + \mathcal{O}\left(e^{2\pi i \tau \cdot \frac{43}{116}} \right) ,$$

which stabilizes the moduli at $g_s \approx 0.011$ and

$$W_0 := \langle |W_{\text{flux}}| \rangle \approx 0.526 \times \left(\frac{2}{252} \right)^{29} M_{\text{pl}}^3 \approx 6.46 \times 10^{-62} M_{\text{pl}}^3 .$$

We find a supersymmetric AdS$_4$ vacuum with volume $\mathcal{V}_{\text{string}} \approx 945$ and vacuum energy

$$V_0 = -3 M_{\text{pl}}^{-2} e^{\mathcal{K}} |W|^2 \approx -1.68 \times 10^{-144} M_{\text{pl}}^4 .$$

cf. $\left(\frac{2}{252} \right)^{58} \approx 10^{-122}$

Comments on Computability

Although the effort involved in tuning integers is polynomial, some computational advances were required in order to enumerate the possible integers.

Had to be able to:

- Construct orientifolds
- Construct F-theory uplifts
- Compute intersection numbers, Kähler cones, GV invariants
- Enumerate floppable and non-floppable curves
- all at $h^{1,1} \gtrsim 100$
- and find quantized fluxes giving small W_0
- and do so automatically, on a large scale.

But with those data in hand, we can work polynomially hard and find exponentially small C.C.

The final answers are expressed in terms of integers, and much can be verified by hand.

Comments on Computability

The flux landscape is low-dimensional,
so the statistical analysis of Denef and Douglas predicts
that W_0 values as small as we find should not exist. Denef, Douglas, 04

They approximated the fluxes as *continuous*.
But we are imposing one exact (integer) equation, $W^{\text{flux}}_{\text{perturbative}} = 0$.
So our solutions are *measure zero* in their ensemble.
However, in any finite landscape of vacua, ours are a finite fraction.

Racetrack spectra

$(h^{2,1}, h^{1,1}) = (5, 113)$; $[7/29 : 7/28]$

$(h^{2,1}, h^{1,1}) = (5, 113)$; $[34/280 : 35/280]$

$(h^{2,1}, h^{1,1}) = (5, 81)$; $[9/24 : 10/24]$

$(h^{2,1}, h^{1,1}) = (7, 51)$; $[8/30 : 9/30]$

$(h^{2,1}, h^{1,1}) = (4, 214)$; $[32/110 : 33/110]$

Worked out and tested in full detail in the paper, as complete examples.

We easily generate vast numbers of examples with milder racetracks, e.g. $[2/4, 3/4]$, but have not gone through full processing of these.

see also: Alvarez-Garcia, Blumenhagen, Brinkmann, Schlechter 20
Honma and Otsuka 21
Marchesano, Prieto, Wiesner 21
Broeckel, Cicoli, Maharana, Singh, Sinha 21
Bastian, Grimm, van de Heisteeg 21
Carta, Mininno, Shukla 22
Blumenhagen, Gligovic, Kaddachi, 22