Cosmological constant problem

\[ \rho_{\text{vac}} \approx 10^{-123} \ M_{\text{pl}}^4 \]

Too hard, so consider supersymmetric version.

\[ \rho_{\text{vac}} \approx -10^{-123} \ M_{\text{pl}}^4 \]

Can we explain how exponentially large SUSY universes arise in a theory with a small fundamental length-scale?
Principle

“\textit{It is better to light a lamp(post) than to curse the dark energy.}”

We work where computations are possible, and make no claim of genericity.
Principle

Choose setting where vacuum structure is dominated by well-understood superpotential terms set by integer data — topology and quantized fluxes.

Systematically compute $W$, and find vacua.

Setting: type IIB flux compactifications on orientifolds of Calabi-Yau threefold hypersurfaces in toric varieties.

We compute the superpotential by exploiting toric structures and purpose-built software.
Small vacuum energy?

Quantized fluxes in a high-dimensional lattice can potentially be chosen to make c.c. small.

Bousso, Polchinski 00  Feng, March-Russell, Sethi, Wilczek 00

N-dimensional Bousso-Polchinski flux landscape:

\[ \rho_{\text{vac}} = -M_{\text{pl}}^4 + Q^i G_{ij} Q^j , \quad \vec{Q} \in \mathbb{Z}^N \]

Fine-tune vast number of terms to precision \( \sim 10^{-N} \)

Problem:
Finding exponentially small c.c. is exponentially costly.

\[ \rho_{\text{vac}} \lesssim 10^{-123} M_{\text{pl}}^4 \text{ is out of reach.} \]

Denef, Douglas, 06  Halverson, Ruehle 18
Mechanism for small vacuum energy

We work in low-dimensional lattices ($4 \leq \text{dim} \leq 10$).
We solve a Diophantine problem to find vacua in which all perturbative terms are exactly zero along one dimension.

What remain are naturally exponentially small instantons.

By discrete, explicit choices of topology and quantized fluxes, we tune their exponents to polynomial precision,

\[ e.g. \quad W \propto -2 e^{2\pi i \tau \cdot \frac{7}{29}} + 252 e^{2\pi i \tau \cdot \frac{7}{28}} + \ldots \]

\[ \tau = C_0 + \frac{i}{g_s} \]

leading to \( \rho \propto \left( \frac{2}{252} \right)^{2 \times 29} \approx 10^{-122} \).

Distribution is \( d \log \rho \) rather than \( d\rho \)!
Summary

We find solutions of type IIB string theory of the form
\[ \text{AdS}_4 \times X_6 \]
with \( X_6 \) a Calabi-Yau orientifold with three-form flux. The solutions are \( \mathcal{N} = 1 \) supersymmetric. The vacuum energy is exponentially small. Hierarchical scale separation, \( \ell_{\text{AdS}}/\ell_{\text{KK}} > 10^{100} \).

Mechanism: racetrack of worldsheet instantons. Vacuum energy is exponential in integers determined by topological data — Gopakumar-Vafa invariants and flux quanta.

example: \[ \left( \frac{2}{252} \right)^{58} \approx 10^{-122} \]
Collaboration

Demirtas, Kim, L.M., Moritz 19
Demirtas, Kim, L.M., Moritz 20
Demirtas, L.M., Rios-Tascon, 20
Demirtas, Kim, L.M., Moritz, Rios-Tascon, 21
+works in progress with Gendler, Nally

Mehmet Demirtas
Manki Kim
Jakob Moritz
Andres Rios-Tascon
Naomi Gendler
Richard Nally
Plan

I. Computation of the superpotential

II. Control of corrections to the Kähler potential
Type IIB string theory compactified on orientifold, $X$, of a $\text{CY}_3$.

Moduli: \textit{axiodilaton $\tau$
complex structure $z_a$, $a = 1, \ldots h^{2,1}(X)$
Kähler: $T_i$, $i = 1, \ldots h^{1,1}(X)$}

Choose quantized fluxes $F_3, H_3 \in H^3(X, \mathbb{Z})$. $G_3 := F_3 - \tau H_3$

$$W(\tau, z_a, T_i) = W_{\text{flux}}(\tau, z_a) + W_{\text{np}}(\tau, z_a, T_i)$$

$W_{\text{flux}} = \int_X G_3 \wedge \Omega$

Gukov, Vafa, Witten 99

$W_{\text{np}} = \sum \text{D-brane instantons}$

Witten 96

$$= \text{poly}(\tau, z_a) + \sum \exp(\tau, z_a) = \sum A_i(\tau, z_a) \exp(-2\pi T_i) + \ldots.$$ 

In our solutions, we ensure:

poly($\tau, z_a$) $\equiv 0$ \hspace{1cm} Perturbatively flat vacuum

$$\sum \exp(\tau, z_a) = N_1 e^{2\pi ip_1 \tau} + N_2 e^{2\pi ip_2 \tau} + \text{negl.} \hspace{1cm} \text{Racetrack}$$

$N_1, N_2 \in \mathbb{Z}, \; p_1, p_2 \in \mathbb{Q}$

$A_i(\tau, z_a) = A_i \hspace{1cm} \text{constant Pfaffians}$
\[ W(\tau, z_a, T_i) = W_{\text{flux}}(\tau, z_a) + W_{\text{np}}(\tau, z_a, T_i) \]

\[ W_{\text{flux}} = \int_X G_3 \wedge \Omega \]

Gukov, Vafa, Witten 99

\[ = \text{poly}(\tau, z_a) + \sum \exp(\tau, z_a) \]

\[ W_{\text{np}} = \sum \text{D-brane instantons} \]

Witten 96

\[ = \sum \mathcal{A}_i(\tau, z_a) \exp\left(-2\pi T_i\right) + \ldots. \]

**In our solutions, we ensure:**

\[ \text{poly}(\tau, z_a) \equiv 0 \quad \text{Perturbatively flat vacuum} \]

\[ \sum \exp(\tau, z_a) = \mathcal{N}_1 e^{2\pi i p_1 \tau} + \mathcal{N}_2 e^{2\pi i p_2 \tau} + \text{negl.} \quad \text{Racetrack} \]

\[ \mathcal{N}_1, \mathcal{N}_2 \in \mathbb{Z}, p_1, p_2 \in \mathbb{Q} \]

\[ \mathcal{A}_i(\tau, z_a) = \mathcal{A}_i \quad \text{constant Pfaffians} \]

\[ W(\tau, z_a, T_i) = \mathcal{N}_1 e^{2\pi i p_1 \tau} + \mathcal{N}_2 e^{2\pi i p_2 \tau} + \sum \mathcal{A}_i \exp\left(-2\pi T_i\right) \]

- Purely exponential \( \Rightarrow \) vacuum energy naturally exponentially small.
- Determined by the numbers \( \mathcal{N}_{1,2}, p_{1,2}, \mathcal{A}_i \)
\[ W(\tau, z_a, T_i) = W_{\text{flux}}(\tau, z_a) + W_{\text{np}}(\tau, z_a, T_i) \]

\[ W_{\text{flux}} = \int_X G_3 \wedge \Omega \quad \text{Gukov, Vafa, Witten 99} \]

\[ W_{\text{np}} = \sum \text{D-brane instantons} \quad \text{Witten 96} \]

\[ = \text{poly}(\tau, z_a) + \sum \exp(\tau, z_a) \]

\[ = \sum \mathcal{A}_i(\tau, z_a) \exp\left(-2\pi T_i\right) + \ldots. \]

In our solutions, we ensure:

\[ \text{poly}(\tau, z_a) \equiv 0 \quad \text{Perturbatively flat vacuum} \]

\[ \sum \exp(\tau, z_a) = N_1 e^{2\pi i p_1 \tau} + N_2 e^{2\pi i p_2 \tau} + \text{negl.} \quad \text{Racetrack} \]

\[ N_1, N_2 \in \mathbb{Z}, p_1, p_2 \in \mathbb{Q} \]

\[ \mathcal{A}_i(\tau, z_a) = \mathcal{A}_i \quad \text{constant Pfaffians} \]

\[ W(\tau, z_a, T_i) = N_1 e^{2\pi i p_1 \tau} + N_2 e^{2\pi i p_2 \tau} + \sum_i \mathcal{A}_i \exp\left(-2\pi T_i\right) \]

- Purely exponential \( \Rightarrow \) vacuum energy naturally exponentially small.
- Determined by the numbers \( N_{1,2}, p_{1,2}, \mathcal{A}_i \)
We find quantized fluxes for which
\[ \text{poly}(\tau, z_a) \equiv 0 \quad \text{perturbatively flat vacuum} \]
along a flat valley \( z_a = p_a \tau, \quad \vec{p} \in Q^{h^{2,1}} \)

Complex structure deformations transverse to the valley are heavy. The along-valley modulus remains massless so far.
\[ W(\tau, z_a, T_i) = W_{\text{flux}}(\tau, z_a) + W_{\text{np}}(\tau, z_a, T_i) \]

\[ W_{\text{flux}} = \int_X G_3 \wedge \Omega \quad \text{Gukov, Vafa, Witten 99} \]

\[ W_{\text{np}} = \sum \text{D-brane instantons} \quad \text{Witten 96} \]

\[ = \text{poly}(\tau, z_a) + \sum \exp(\tau, z_a) \quad = \sum A_i(\tau, z_a) \exp(-2\pi T_i) + \ldots \]

**Our solutions:**

\[ \text{poly}(\tau, z_a) \equiv 0 \quad \text{Perturbatively flat vacuum} \]

\[ \sum \exp(\tau, z_a) = \mathcal{N}_1 e^{2\pi i p_1 \tau} + \mathcal{N}_2 e^{2\pi i p_2 \tau} + \text{negl.} \quad \text{Racetrack} \]

\[ \mathcal{N}_1, \mathcal{N}_2 \in \mathbb{Z}, p_1, p_2 \in \mathbb{Q} \]

\[ A_i(\tau, z_a) = A_i \quad \text{constant Pfaffians} \]

\[ W(\tau, z_a, T_i) = \mathcal{N}_1 e^{2\pi i p_1 \tau} + \mathcal{N}_2 e^{2\pi i p_2 \tau} + \sum A_i \exp(-2\pi T_i) \]

- Purely exponential \(\Rightarrow\) vacuum energy naturally exponentially small.
- Determined by the numbers \(\mathcal{N}_{1,2}, p_{1,2}, A_i\)
Racetrack Superpotential

$4d \mathcal{N} = 1$ field theory

\[ W(z) = \mathcal{N}_1 e^{-p_1 z} + \mathcal{N}_2 e^{-p_2 z} + \ldots \]

\[ \mathcal{N}_{1,2} \in \mathbb{R}, \quad p_{1,2} > 0 \]

\[ W(z_{\text{min}}) = \frac{\mathcal{N}_2 (p_1 - p_2)}{p_1} \left( -\frac{\mathcal{N}_1 p_1}{\mathcal{N}_2 p_2} \right) \frac{p_2}{p_2 - p_1} \]

\[ \ll 1 \text{ if } |p_1 - p_2| \ll p_2, \quad \mathcal{N}_1 \ll \mathcal{N}_2 \]
Incarnation as Flux Superpotential

prepotential: \( \mathcal{F}(z) = \mathcal{F}_{\text{poly}}(z) + \mathcal{F}_{\text{inst}}(z) \)

\[
\mathcal{F}_{\text{poly}}(z) = -\frac{1}{3!} \tilde{\kappa}_{abc} z^a z^b z^c + \frac{1}{2} \tilde{\alpha}_{ab} z^a z^b + \frac{1}{24} \tilde{c}_a z^b + \frac{\zeta(3) \chi(\tilde{X})}{2(2\pi i)^3}
\]

intersection numbers, Chern classes of mirror \( \tilde{X} \)

\[
\mathcal{F}_{\text{inst}}(z) = -\frac{1}{(2\pi i)^3} \sum_{\tilde{q} \in \mathcal{M}(\tilde{X})} \text{GV}_{\tilde{q}} \text{Li}_3 \left( e^{2\pi i \tilde{q} \cdot z} \right) \quad \text{GV}_{\tilde{q}}: \text{genus-0 GV invariants of } \tilde{X} \quad \text{Li}_k(q) := \sum_{n=1}^{\infty} q^n / n^k
\]

periods: \( \tilde{\Pi} = \begin{pmatrix} \int_X \Omega \wedge \beta_A \\ \int_X \Omega \wedge \alpha^A \end{pmatrix} = \begin{pmatrix} \partial_A \mathcal{F} \\ z^A \end{pmatrix} \) \( \{ \alpha^A, \beta_A \}, A = 0, \ldots, h^{2,1}(X) : \text{basis of } H^3(X, \mathbb{Z}) \)

\[
W_{\text{flux}}(\tau, z^a) = \sqrt{\frac{2}{\pi}} \int_X (F_3 - \tau H_3) \wedge \Omega(z) = \sqrt{\frac{2}{\pi}} \tilde{\Pi}^t \Sigma (\vec{f} - \tau \vec{h}) \quad \Sigma := \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix}
\]

Task: compute \( W_{\text{flux}} \) by computing \( \mathcal{F}(z) \).
Computing the Prepotential

Principles clear since early days of mirror symmetry.  
Hosono, Klemm, Theisen, Yau 94

Our contribution: really doing this in CY₃ with many moduli.  
Demirtas, Kim, L.M., Moritz, Rios-Tascon

Setting: hypersurfaces in toric varieties, as classified by Kreuzer and Skarke.

With INSTANTON, can access $h^{2,1} \lesssim 10$.  
Klemm
Carta, Mininno, Righi, Westphal 21

With our open source software package CYTools, can access $h^{2,1} = 491$,  
and compute GV invariants to very high degree at $h^{2,1} = \mathcal{O}(100)$.  
Demirtas, L.M., Rios-Tascon
Flux Superpotential in a Calabi-Yau

\((h^{2,1}, h^{1,1}) = (5, 113)\)

We find quantized fluxes,

\[
\vec{f} = (10, 12, 8, 0, 0, 4, 0, 0, 2, 4, 11, -8), \quad \vec{h} = (0, 8, -15, 11, -2, 13, 0, 0, 0, 0, 0, 0),
\]

s.t. along \(z = p\tau\), \(p = \left(\frac{7}{58}, \frac{15}{58}, \frac{101}{116}, \frac{151}{58}, -\frac{13}{116}\right)\),

the polynomial part of \(W_{\text{flux}}\) vanishes exactly.

What remain are IIA worldsheet instantons.

The leading ones have GV invariants \(-2\) and \(252\).

\[
W_{\text{flux}}(\tau) = \frac{2}{(2\pi)^{5/2}} \left(-2e^{2\pi i\tau \cdot \frac{7}{29}} + 252e^{2\pi i\tau \cdot \frac{7}{28}} + 104e^{2\pi i\tau \cdot \frac{43}{116}} + \ldots\right)
\]

Moduli are stabilized at \(g_s \approx 2\pi \cdot \frac{1}{28 \log(1/W_0)} \approx 0.011\) and

\[
\langle W_{\text{flux}} \rangle \approx 0.526 \times \left(\frac{2}{252}\right)^2 \approx 6.46 \times 10^{-62}.
\]
Story so far

General form: \[ W(\tau, z_a, T_i) = W_{\text{flux}}(\tau, z_a) + W_{\text{np}}(\tau, z_a, T_i) \]

Chose fluxes s.t.: \[ W_{\text{flux}}(\tau, z_a) = \text{poly}(\tau, z_a) + \sum \exp(\tau, z_a) \]

Chose CY3 s.t.: \[ W_{\text{flux}}(\tau, z_a) = \text{poly}(\tau, z_a) + \mathcal{N}_1 e^{2\pi i p_1 \tau} + \mathcal{N}_2 e^{2\pi i p_2 \tau} + \text{negl.} \]

\[ W(\tau, z_a, T_i) = \mathcal{N}_1 e^{2\pi i p_1 \tau} + \mathcal{N}_2 e^{2\pi i p_2 \tau} + W_{\text{np}}(\tau, z_a, T_i) \]

Next: need to compute \( W_{\text{np}}(\tau, z_a, T_i) \) and stabilize the Kähler moduli.
Superpotential for Kähler moduli

General form: \[ W_{np}(\tau, z_a, T_i) = \sum_{D} A_D(\tau, z_a) \exp\left(-\frac{2\pi}{c_D} T_D\right) \]

We find CY$_3$ in which:
\[ W_{np}(\tau, z_a, T_i) = \sum_{D_I} A_{D_I} \exp\left(-\frac{2\pi}{c_I} T_I\right) + \text{negl.} \]

where the sum runs over prime toric divisors $D_I$, $I \in \{1, \ldots h^{1,1} + 4\}$, with Pfaffians $A_{D_I}$ that are constants, with no dependence on $\tau, z_a$.

We select cases where:

- at least $h^{1,1}$ of the $D_I$ are rigid
- their F-theory uplifts $\hat{D}_I$ have trivial intermediate Jacobian $J$
- in an orientifold where all seven-branes lie in $so(8)$ stacks

\[ D_I \text{ rigid} \Leftrightarrow h^\bullet(\hat{D}_I) = (1, 0, 0, 0) \]

\[ h^{2,1}(\hat{D}_I) = 0 \Rightarrow J \text{ trivial} \Rightarrow A_{D_I} \text{ constant} \]

Witten 96a: Nonperturbative superpotentials in string theory

Witten 96b: Five-brane effective action in M theory

Find toric uplift to F-theory, compute $h^{2,1}(\hat{D}_I)$ by stratification

Kim 21
Finding a vacuum

1. Compute the superpotential in many compactifications.

2. Find a case with desired structure.

3. Search the Kähler moduli space for a SUSY vacuum.

4. Determine whether vacuum survives $g_s$ and $\alpha'$ corrections.
2d cross-section of Kähler cone in $h^{1,1} = 491$ threefold
2d cross-section of Kähler cone in $h^{1,1} = 491$ threefold
Control of superpotential

- We have shown by explicit computation that superpotential is

\[ W = N_1 e^{2\pi i p_1 \tau} + N_2 e^{2\pi i p_2 \tau} + \sum_I A_{D_I} \exp\left(-\frac{2\pi}{c_I} T_I\right) + \text{negl.} \]

  - \( p_1, p_2 \in \mathbb{Q} \) set by fluxes
  - \( N_1, N_2 \in \mathbb{Z} \) set by fluxes and by GV invariants of \( \hat{X} \)
  - \( c_I \) set by choice of orientifold action

- Ensured that \( A_{D_I} \) are \textbf{nonzero numbers} by standard zero-mode counting.
  - not identically zero by imposing rigidity of \( D_I \)
  - constant by imposing \( h^{2,1}(\hat{D}_I) = 0 \)

Their numerical values have small effects: \( \Delta T_I / T_I \sim \log(\Delta A_D) / \log(W_0) \)

We checked that our vacua persist for \( A_{D_I} \in [10^{-4}, 10^4] \)

- \( W_{\text{ED}(-1)} = \sum_{k=1}^{\infty} B_k(z) e^{2\pi i k \tau} \) negligible due to \( g_s \ll 1 \)

Totally explicit computation, up to the numerical values of the \( A_{D_I} \).

cf. Alexandrov, Firat, Kim, Sen, Stefański 22
Control of Kähler potential

Recall: \[ V_F = -3e^K |W|^2 \]

\[ = -3e^{K_0 + \delta K} |W|^2 \]

⇒ need only show \( \delta K \) small enough not to destroy vacuum, not that \( \delta K \lesssim 10^{-100} \).
Control of Kähler potential

• Einstein-frame volumes are large
\[ \text{Re}(T_i) \approx \frac{c_i}{2\pi} \log(W_0^{-1}) \]

• String-frame volumes are order-unity
\[ g_s \propto \frac{2\pi}{\log(W_0^{-1})} \ll 1 \]

• Weak string coupling \( \Rightarrow \) leading corrections are at string tree level, to all orders in \( \alpha' \)

• Specifically: worldsheet instanton corrections to \( \mathcal{F} \) of parent \( \mathcal{N} = 2 \text{ CY}_3 \).

We are able to compute these corrections directly, because we can compute periods in \( \text{CY}_3 \) with \( h^{2,1} \gg 1 \).
Worldsheet instantons from small curves

$$
\delta F \propto \sum_{\mathbf{q}} \sum_{n \in \mathbb{N}} \GV_{n \mathbf{q}} e^{-2\pi n \mathbf{q} \cdot \mathbf{t}}
$$

\(\mathbf{q}\): curve classes in \(X\) \hspace{1cm} t: Kähler parameters in vacuum

$$
\xi_n(\mathbf{q}) := \GV_{n \mathbf{q}} e^{-2\pi n \mathbf{q} \cdot \mathbf{t}}
$$

To test convergence, we compute \(\xi_n(\mathbf{q})\) for small-volume curves.

**Two kinds of curves:**

*nilpotent:* \(\GV_{k \mathbf{q}} = 0 \ \forall k > k_{\text{max}} \in \mathbb{N}\)

*potent:* infinite series

Nilpotent curves are safely collapsible.
They give finitely many polylogs, which we **include explicitly**.

We then examine thousands of rays of potent curves.
Convergence of worldsheet instanton sum

Along multiples of a potent curve \( C \),

\[
\begin{align*}
\text{GV}(C) &= 3 \\
\text{GV}(2C) &= -6 \\
\text{GV}(3C) &= 27 \\
\text{GV}(4C) &= -192 \\
\text{GV}(5C) &= 1695 \\
\text{GV}(6C) &= -17064 \\
\text{GV}(7C) &= 188454 \\
\text{GV}(8C) &= -2228160 \\
\text{GV}(9C) &= 27748899 \\
\text{GV}(10C) &= -360012150. \\
\text{GV}(100C) &= -91461158123783137122697397476857357418750633461367914322579026697369512751047337367692277761351484717813209296148860000. 
\end{align*}
\]

Exponential increase with very stable rate.

So for large enough \( t \), \( \xi_n := \text{GV}_{n\mathbf{q}} e^{-2\pi n \mathbf{q} \cdot \mathbf{t}} \) will decay exponentially with \( n \).

e.g. quintic: \( t_{\text{min}}^{(1)} = \frac{1}{2\pi} \log(2875) \approx 1.27, \quad t_{\text{true}}^{\text{min}} \approx 1.208 \).
Convergence of worldsheet instanton sum

Along multiples of a potent curve $\mathcal{C}$, (example with $h^{1,1} = 113$)

\[
\begin{align*}
GV(\mathcal{C}) &= 3 \\
GV(2\mathcal{C}) &= -6 \\
GV(3\mathcal{C}) &= 27 \\
GV(4\mathcal{C}) &= -192 \\
GV(5\mathcal{C}) &= 1695 \\
GV(6\mathcal{C}) &= -17064 \\
GV(7\mathcal{C}) &= 188454 \\
GV(8\mathcal{C}) &= -2228160 \\
GV(9\mathcal{C}) &= 27748899 \\
GV(10\mathcal{C}) &= -360012150. \\
GV(100\mathcal{C}) &= -914611581237831371226973974768573574187506334613679143 \\
&\quad 22579026697369512751047337367692277761351484717813209296148860000.
\end{align*}
\]

Exponential increase with very stable rate.

So for large enough $t$, $\xi_n := GV_{nq} e^{-2\pi n q \cdot t}$ will decay exponentially with $n$.

In our vacua, is $t$ large enough?
Convergence of worldsheet instanton sum

\[ \xi_n := GV_{nq} e^{-2\pi n \cdot q \cdot t} \]
Convergence of worldsheet instanton sum

\[ \xi_n := GV_{nq} e^{-2\pi n q \cdot t} \]

largest correction: \( \mathcal{O}(10^{-5}) \)
**KKLT vacua**

Kachru, Kallosh, Linde, and Trivedi made two claims in their 2003 paper:

Claim 1: in type IIB compactifications with \( W = W_{\text{flux}}(\tau, z_a) + W_{\text{np}}(\tau, z_a, T_i) \), if \( \langle W_{\text{flux}} \rangle \ll 1 \), there can exist well-controlled SUSY AdS\(_4\) vacua.

Claim 2: given such an AdS\(_4\) vacuum in a compactification with:
   - a warped deformed conifold region \( \text{Klebanov, Strassler 00} \)
   - containing one or more anti-D3-branes \( \text{Kachru, Pearson, Verlinde 01} \)
   - in a suitable parameter regime, there can exist metastable dS\(_4\) vacua.

We have now given strong evidence for the first claim.
KKLT status summary

No evidence of obstructions or inconsistencies.

Still awaiting complete explicit compactifications.

AdS₄ ✓
Demirtas, Kim, L.M., Moritz, Rios-Tascon 21

dS₄ work in progress

talk by L.M. at Strings 2019, Brussels
Conclusions

We have constructed SUSY AdS$_4$ vacua in CY$_3$ orientifolds. Exponentially small $W_{\text{flux}}$ from quantized fluxes. Explicit computation of $W_{\text{ED3}}$ stabilizing the Kähler moduli. Stabilization at $g_s \ll 1$, large Einstein-frame volume. Small cosmological constants, giant scale separation. Methods can be applied to build a corner of the landscape. Search for de Sitter vacua is work in progress.
Thanks!
Towards de Sitter vacua?

We have built tools to compute $\mathcal{N} = 1$ EFTs in a regime of type IIB flux compactifications where we can explicitly enumerate all necessary integer data, and can search for vacua in which our approximations are self-consistent.

Our main results so far concern $\mathcal{N} = 1$ SUSY AdS$_4$ vacua, where

- polynomial tuning of topological parameters provides a mechanism that makes $\rho_{\text{vac}}$ exponentially small.
- unbroken SUSY protects $\rho_{\text{vac}}$ from quantum corrections.

But the EFTs appear rich enough to support de Sitter vacua in which SUSY is broken \textit{spontaneously:}

- not by anti-D3-branes, but by competition among superpotential terms.

Establishing self-consistent approximations and parametric control is unsurprisingly more involved than for SUSY AdS$_4$.

Demirtas, Gendler, Kim, L.M., Moritz, Nally, Rios-Tascon, work in progress.
Comments on related work

Would be valuable to find the dual CFT$_3$.

A candidate dual suggested in Lüst, Vafa, Wiesner, Xu 22 was shown there to have parametrically incorrect central charge.

It was suggested in Lüst et al. that superpotential terms stabilizing the Kähler moduli “will not materialize”.

But since we have directly exhibited all the necessary terms, a simpler reconciliation is that their proposed dual is not the correct one.

Worth weighing evidence on both sides.
An example with \((h^{2,1}, h^{1,1}) = (5, 113)\)

The vertices of \(\Delta\) are the columns of
\[
\begin{pmatrix}
1 & -3 & -3 & 0 & 0 & 0 & -5 & -2 \\
0 & -2 & -1 & 0 & 0 & 1 & -3 & -1 \\
0 & 0 & -1 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & -1 & -1
\end{pmatrix}.
\]

There are \(h^{1,1} + 1 = 25_{\mathfrak{so}(8)} + 89_{\text{ED3}}\) rigid prime divisors with \(h^{2,1}(\hat{D}_I) = 0\).

The fluxes
\[
\vec{f} = (10, 12, 8, 0, 0, 4, 0, 0, 2, 4, 11, -8), \\
\vec{h} = (0, 8, -15, 11, -2, 13, 0, 0, 0, 0, 0, 0),
\]
carry D3-brane charge 56. The D3-brane tadpole is 60, so there are 4 D3-branes.

The leading instantons have GV invariants \(\mathcal{N}_q = (-2, 252)\) and

\[
W_{\text{flux}}(\tau) = \sqrt{\frac{2}{\pi}} \frac{1}{(2\pi i)^2} \left( -2 e^{2\pi i \tau \cdot \frac{7}{29}} + 252 e^{2\pi i \tau \cdot \frac{7}{28}} \right) + \mathcal{O} \left( e^{2\pi i \tau \cdot \frac{43}{116}} \right),
\]

which stabilizes the moduli at \(g_s \approx 0.011\) and

\[
W_0 := \langle |W_{\text{flux}}| \rangle \approx 0.526 \times \left( \frac{2}{252} \right)^{29} M_{\text{pl}}^3 \approx 6.46 \times 10^{-62} M_{\text{pl}}^3.
\]

We find a supersymmetric AdS\(_4\) vacuum with volume \(V_{\text{string}} \approx 945\) and vacuum energy
\[
V_0 = -3 M_{\text{pl}}^{-2} e^{K} |W|^2 \approx -1.68 \times 10^{-144} M_{\text{pl}}^4.
\]

\(\text{cf. } \left( \frac{2}{252} \right)^{58} \approx 10^{-122}\)
Comments on Computability

Although the effort involved in tuning integers is polynomial, some computational advances were required in order to enumerate the possible integers.

Had to be able to:
- Construct orientifolds
- Construct F-theory uplifts
- Compute intersection numbers, Kähler cones, GV invariants
- Enumerate floppable and non-floppable curves
- all at $h^{1,1} \gtrsim 100$
- and find quantized fluxes giving small $W_0$
- and do so automatically, on a large scale.

But with those data in hand, we can work polynomially hard and find exponentially small C.C.

The final answers are expressed in terms of integers, and much can be verified by hand.
Comments on Computability

The flux landscape is low-dimensional, so the statistical analysis of Denef and Douglas predicts that $W_0$ values as small as we find should not exist.  

They approximated the fluxes as \textit{continuous}. But we are imposing one exact (integer) equation, $W_{\text{perturbative}}^{\text{flux}} = 0$. So our solutions are \textit{measure zero} in their ensemble. However, in any finite landscape of vacua, ours are a finite fraction.
Racetrack spectra

\( (h^{2,1}, h^{1,1}) = (5, 113); \ [7/29 : 7/28] \)

\( (h^{2,1}, h^{1,1}) = (5, 113); \ [34/280 : 35/280] \)

\( (h^{2,1}, h^{1,1}) = (5, 81); \ [9/24 : 10/24] \)

\( (h^{2,1}, h^{1,1}) = (7, 51); \ [8/30 : 9/30] \)

\( (h^{2,1}, h^{1,1}) = (4, 214); \ [32/110 : 33/110] \)

Worked out and tested in full detail in the paper, as complete examples.

We easily generate vast numbers of examples with milder racetracks, e.g. \([2/4, 3/4]\), but have not gone through full processing of these.

see also: Alvarez-Garcia, Blumenhagen, Brinkmann, Schlechter 20
Honma and Otsuka 21
Marchesano, Prieto, Wiesner 21
Broeckel, Cicoli, Maharana, Singh, Sinha 21
Bastian, Grimm, van de Heisteeg 21
Carta, Mininno, Shukla 22
Blumenhagen, Gligovic, Kaddachi, 22