From stat-mech to microstate counting for extremal and near-extremal black holes

Luca V. Iliesiu

Work to appear with Gustavo-Joaquin Turiaci and Sameer Murthy as well as references to a number of past works also involving Matthew Heydeman, Wenli Zhao, Jan Boruch and Murat Koloğlu.

The gravitational path integral for black hole observables $\stackrel{?}{=}$ Conventional quantum mechanical system

3 related questions for the special case of extremal and near-extremal black holes

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Q2: Are extremal black holes states truly degenerate ($d_{\text{ext BH.}} \sim e^{\frac{\text{Area}}{4G_N}}$)?

Q3: If there truly is a degeneracy, we should have $d_{\text{ext BH.}} \in \mathbb{Z}$.
Can the gravitational path integral reproduce a microstate count, including the integerness of such a count?

3 related questions for the special case of extremal and near-extremal black holes

Q1: What is the energy gap between extremal black hole states and the lightest near-extremal states?

$$E_{\text{gap}} \stackrel{?}{=} \text{ energy scale at which there is a breakdown of black hole thermodynamics?}$$

$$\stackrel{?}{=} \frac{1}{Q^3 \ell_{\text{Pl}}}$$

[Preskill et al `91, Maldacena and Susskind `96, Maldacena and Strominger `97]

Q2: Are extremal black holes states truly degenerate ($d_{\text{ext BH.}} \sim e^{\frac{\text{Area}}{4G_N}}$)?

Q3: If there truly is a degeneracy, we should have $d_{\text{ext BH.}} \in \mathbb{Z}$.

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Q1: What is the energy gap between extremal black hole states and the lightest near-extremal states?

Q2: Are extremal black holes states truly degenerate ($\underbrace{d_{\text{ext BH.}}}_{\text{Protected by what symmetry?}} \sim e^{\frac{\text{Area}}{4G_N}}$)?

Q3: If there truly is a degeneracy, we should have $d_{\text{ext BH.}} \in \mathbb{Z}$.

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What the gravitational path integral *seemingly* tell us for extremal black holes

The extremal entropy including quantum corrections

$$d_{\text{ext BH.}} = \# \exp\left[S_0 + c \log S_0\right] \qquad \text{where} \qquad S_0 = \frac{\text{Area}}{4G_N}$$

[Cardoso, de Wit, Kappeli, Mohaupt `04, Sen `08, `09, Banerjee, Gupta, Sen `10, Banerjee, Gupta, Mandal, Sen `11, Sen `11, …]

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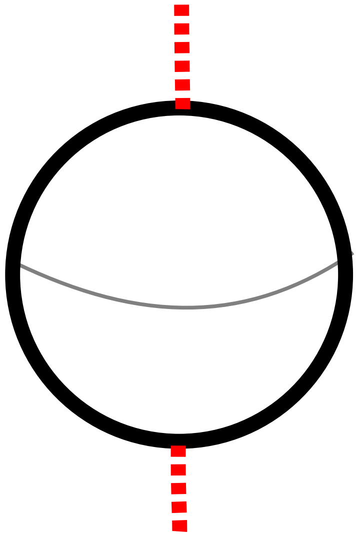
$$= \begin{array}{c}\text{An unexplicably large}\\ \text{degeneracy}\end{array} \quad \text{(in the non-supersymmetric cases)}$$

To answer Q1 and Q2 for both non-supersymmetric and supersymmetric extremal black holes, I want to quickly revisit this computation.

The background

$\mathrm{AdS}_2 \times S^2$



The types of modes that we have to include

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{AdS}_2\times S^2} + h_{\mu\nu}\,, \qquad A_\mu = A_\mu^{\mathrm{AdS}_2\times S^2} + a_\mu\,, \qquad \dots$$

- **Modes with non-zero eigenvalues in the quadratic expansion action**

$$Z_{\text{1-loop}}^{\text{non-zero}} = S_0^{c_{\text{non-zero}}}$$

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$$Z_{\text{1-loop}}^{\text{non-zero}} = S_0^{c_{\text{non-zero}}}$$

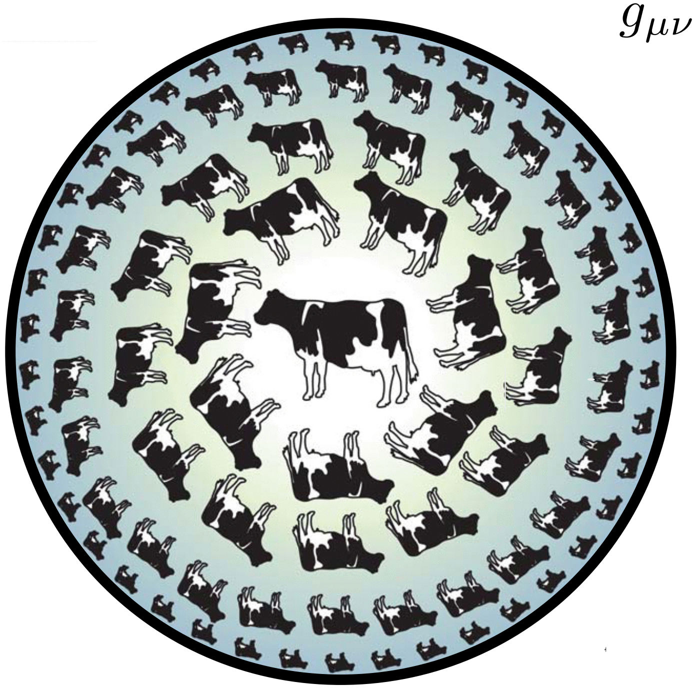
- **Zero modes of the action**

$$Z^{\text{zero-modes}} = \int \underbrace{Dh_{\mu\nu}^{\text{zero-modes}} DA_\mu^{\text{zero-modes}}}_{\text{Large diffeomorphisms \& large gauge transformations}}$$

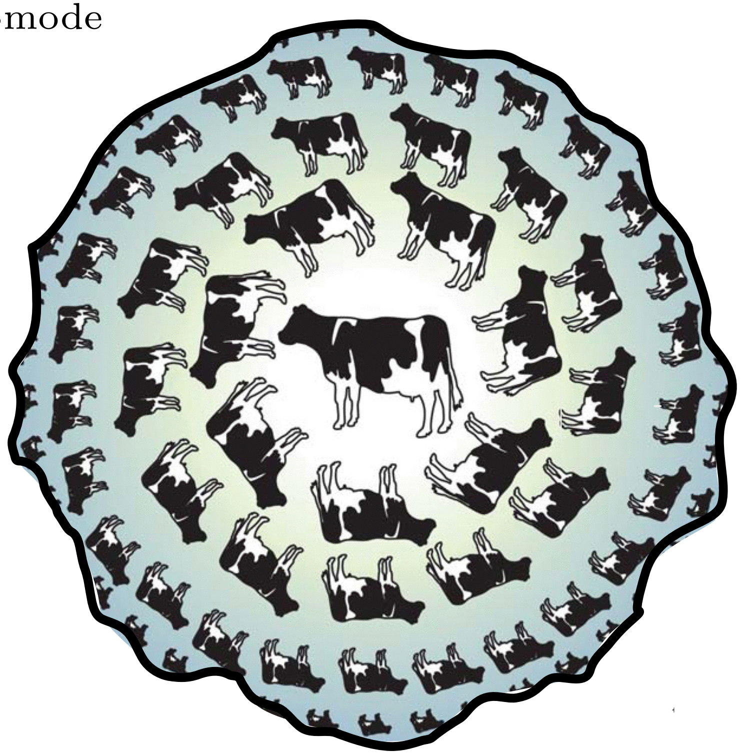
An example of a boundary zero mode: Reissner-Nordstrom black holes

[Sen and collaborators]

$$g_{\mu\nu} = g_{\mu\nu}^{\text{classical}} + h_{\mu\nu}^{\text{zero-mode}}$$



A
bridge between
two communities



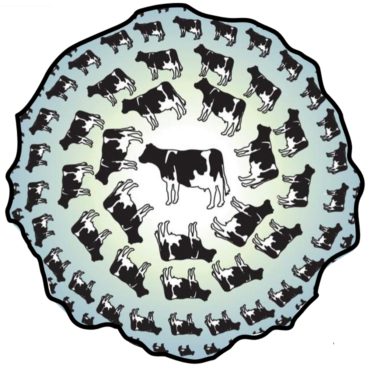
The one-loop determinant

[Sen and collaborators]

$$Z_{\text{1-loop}} = S_0^{c_{\text{non-zero}}} \int Dh_{\mu\nu}^{\text{zero-modes}} =$$

$$c = c_{\text{non-zero}} + c_{\text{zero}} =$$

$$= S_0^{-\frac{1}{180}\overbrace{(964 + n_s + 62n_v + 11n_F)}^{\text{Massless 4D field content}}} \int_{\frac{\text{Large-diffeos}}{\text{Isometries}}}$$



Regularized to 0, finite or ∞ ?

$$\frac{Z_{BH}^{T=0}(Q)}{Z_{BH}(T,Q)} = ?$$

How to regulate the zero modes: going to finite temperature

$$g_{\mu\nu} = g_{\mu\nu}^{\text{extremal}} + \underbrace{g_{\mu\nu}^{T\text{corr.}}} + h_{\mu\nu}$$

Gives a non-zero weight to the
zero-modes that were present at $T = 0$.

↓

"Would-be" zero modes

How to regulate the zero modes: going to finite temperature

For Reissner-Nordstrom black holes at fixed charge and angular momentum:

$$Z_{\rm BH}(T,Q) = e^{\overbrace{S_0 - \frac{1}{180}(964+n_s+62n_v+11n_F)}^{\text{The old}}\log S_0}$$

$$\times \int_{\frac{\mathrm{Diff}(S^1)}{SL(2,\mathbb{R})}} D\varepsilon \underbrace{\quad \exp\left[Q^3 \ell_{\rm Pl}\int_0^{1/T} du\{\varepsilon(u),u\}\right]}_{\text{The new:}} (1+O(1/S_0)).$$

A strongly coupled theory of "would-be" zero-modes

=

A failure of semiclassics

[connections to Almheiri, Polchinski`15, Maldacena, Stanford, Yang `16, Almheiri, Kang `16, Moitra, Trivedi and coll. `18, Larsen, Zeng `19, LVI and Turiaci `20, LVI, Turiaci and Murthy `22 and numerous other papers]

The partition function of non-supersymmetric near-extremal black holes

$$Z_{\rm BH}(T,Q) = \overbrace{e^{S_0 - \frac{1}{180}(964+n_s+62n_v+11n_F)\,\log\,S_0}}^{\text{The old}}$$

$$\times\ \underbrace{\left(Q^3 \ell_{\rm Pl} T\right)^{3/2}\ e^{2\pi^2 Q^3 \ell_{\rm Pl} T}\ \left(1+O(1/S_0)\right)}_{\text{The new: Large quantum correction at small temperature}}.$$

The new:
Large quantum correction at small temperature

[putting together computations of Sen and collaborators together with Stanford, Witten `17]

The partition function of non-supersymmetric near-extremal black holes

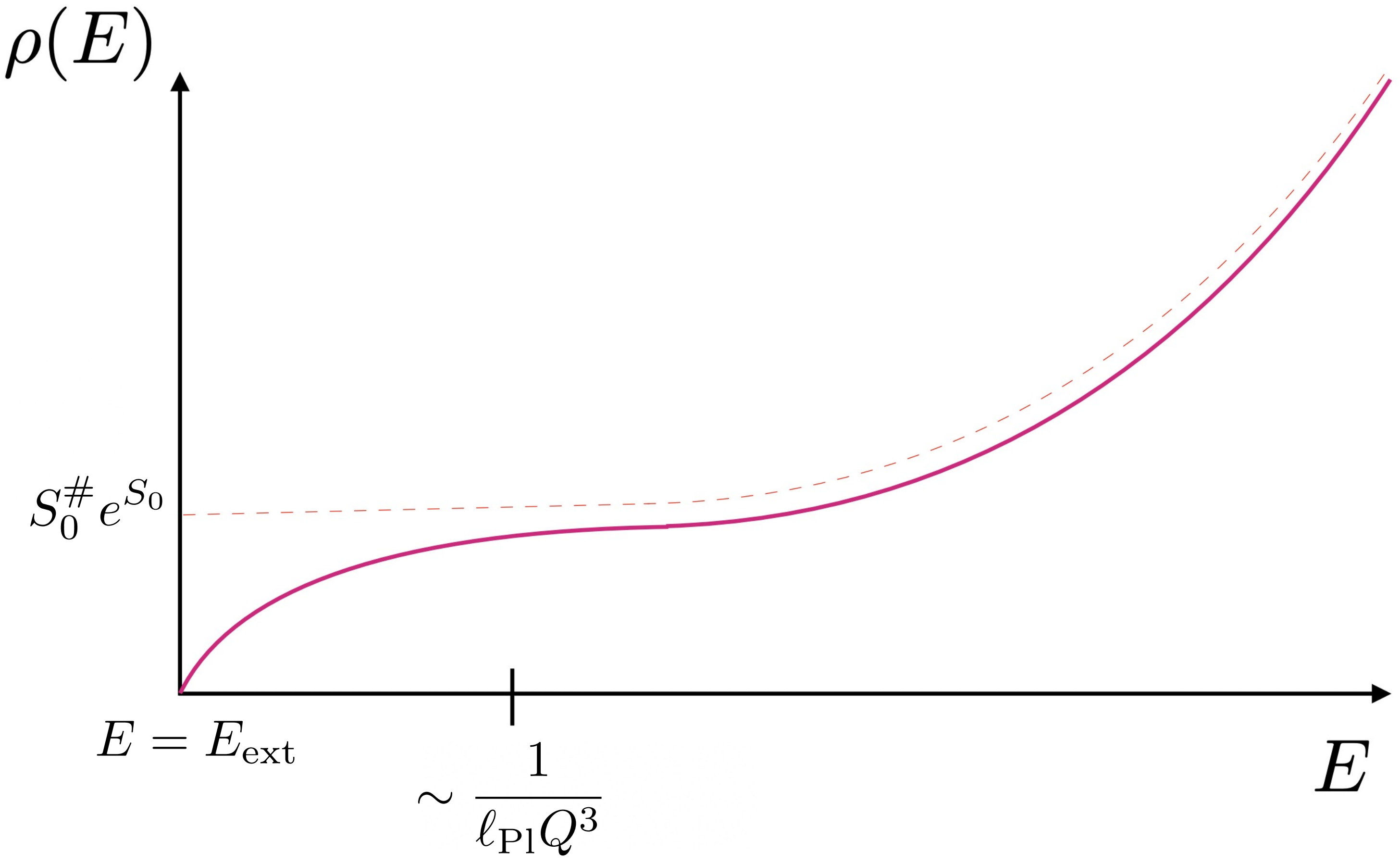
$$\underbrace{\frac{Z_{\text{BH}}^{T=0}(Q)}{Z_{\text{BH}}(T,Q)} \to 0}$$

At least non-perturbatively smaller degeneracy
for extremal **non-supersymmetric** black holes than naively predicted.

The density of states

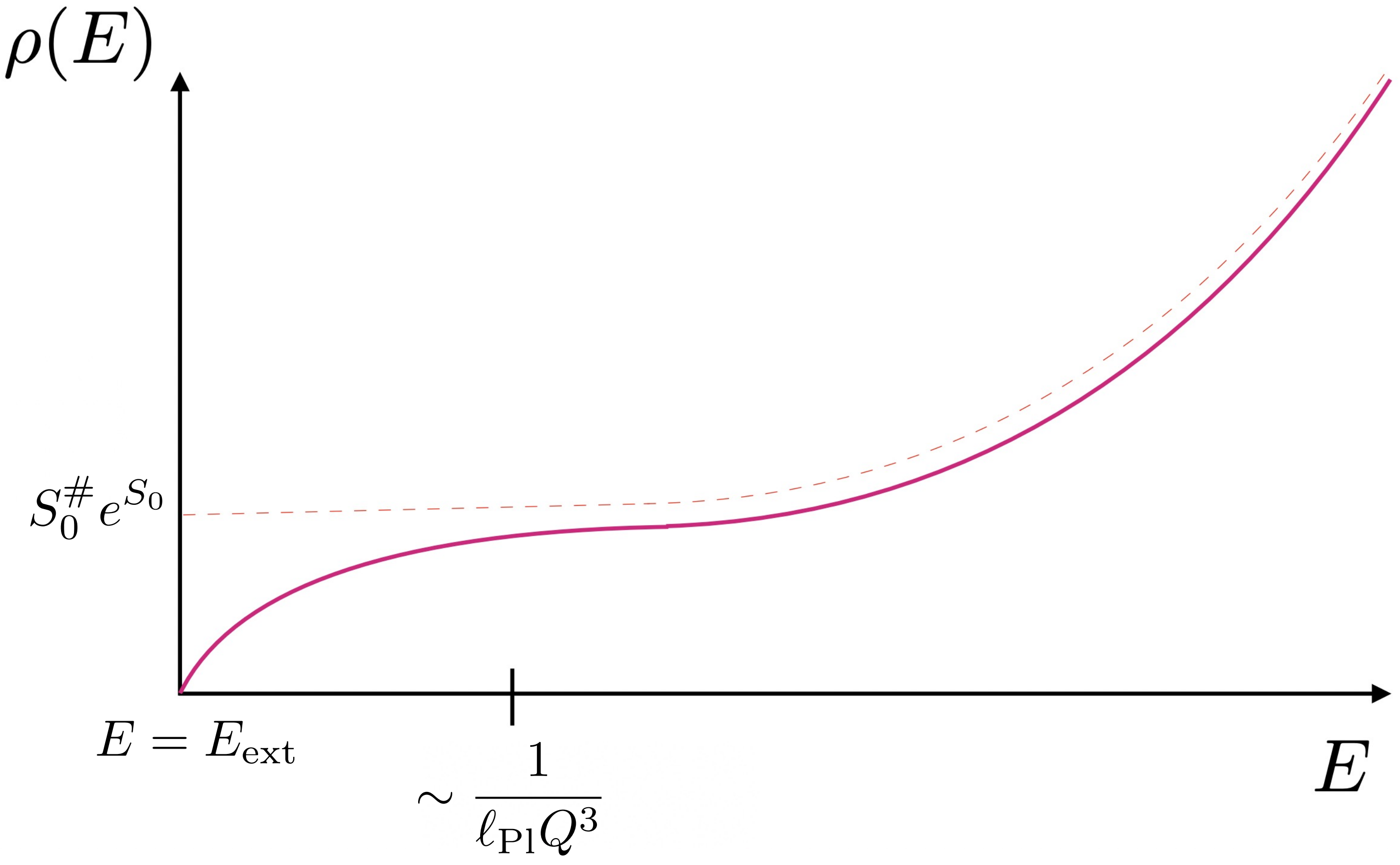
$$Z(\beta) = \int dE \rho(E) e^{-\beta E}$$

[LVI and Turiaci `20, LVI, Turiaci and Murthy `22]



Summary: Q1 and Q2 answered (no supersymmetry)

[LVI and Turiaci `20, LVI, Turiaci and Murthy `22]



Answer to Q1 and Q2: In Einstein-Maxwell theory, no gap and no degeneracy

How about black holes that preserve supersymmetry at extremality?

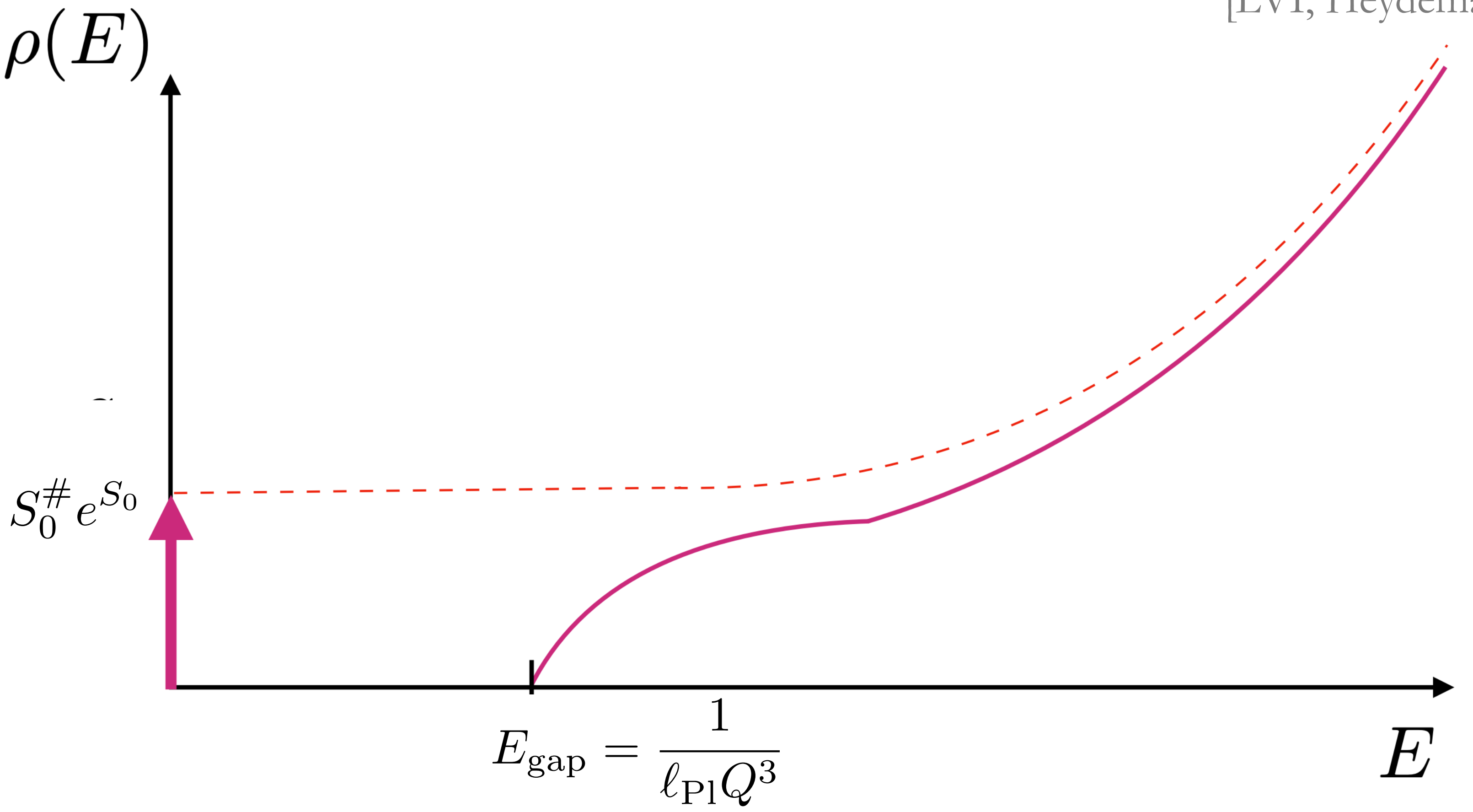
Supersymmetric black holes

For supersymmetric black holes in AdS or flatspace the same one-loop computation and accounting for the ”would-be” zero-modes, leads to wildly different conclusions.

The density of states

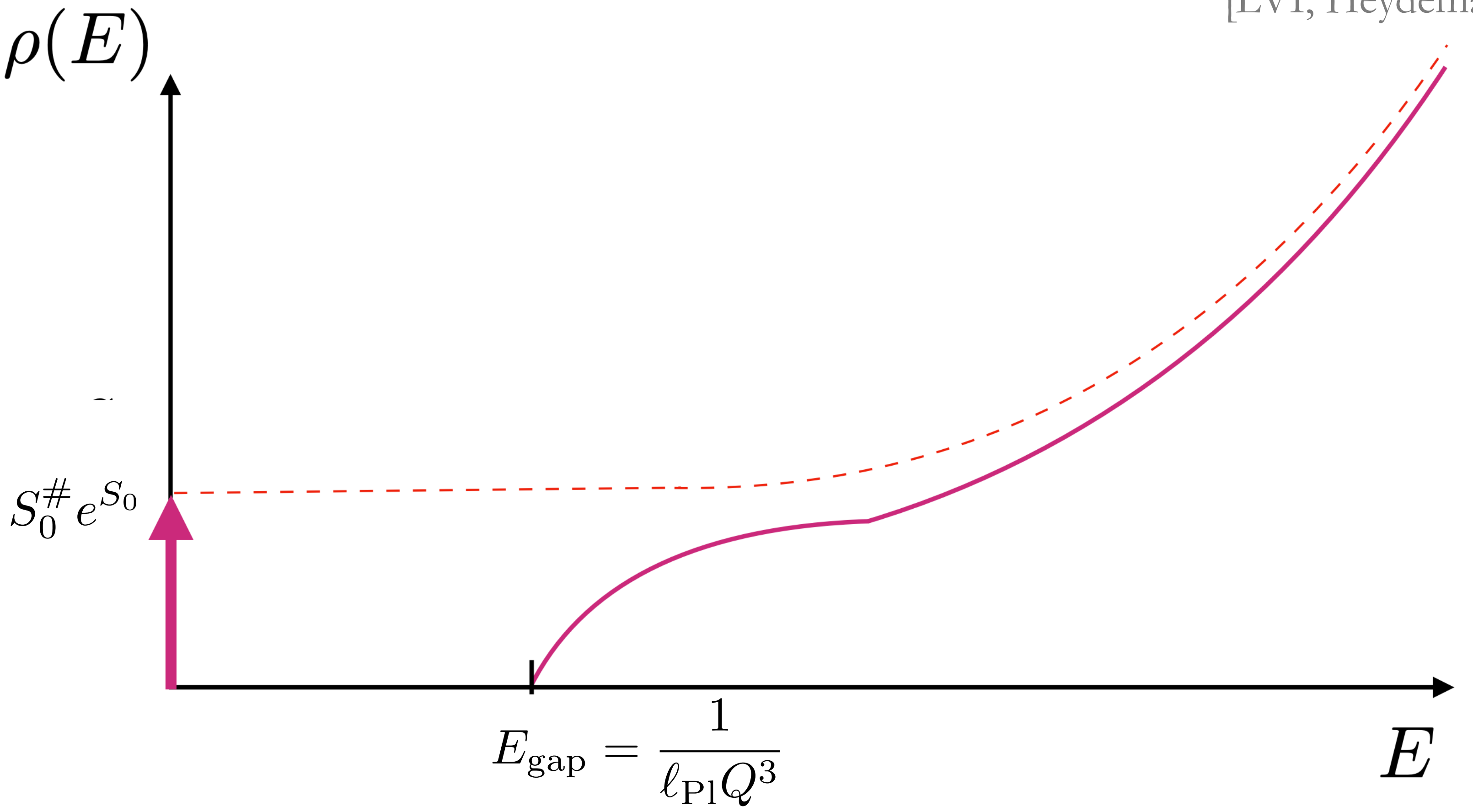
Example: for black holes in 4D flatspace in $\mathcal{N} \geq 2$ supergravity

[LVI, Heydeman, Turiaci and Zhao `20]



Summary: Q1 & Q2 answered (with supersymmetry)

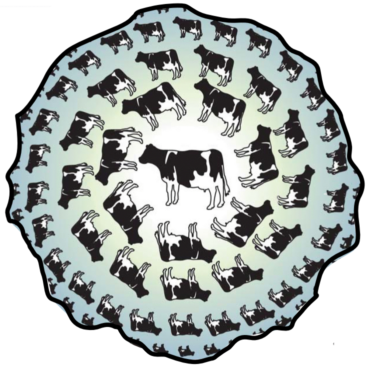
[LVI, Heydeman, Turiaci and Zhao `20]

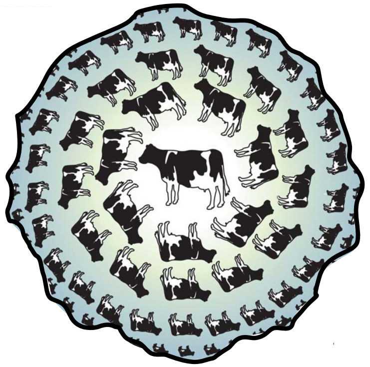


Answer to Q1 and Q2: Thus, in supergravity in flat space, there is a gap and a massive degeneracy, consistent with expectation from microstate counting and with the stringy constructions of the gap.

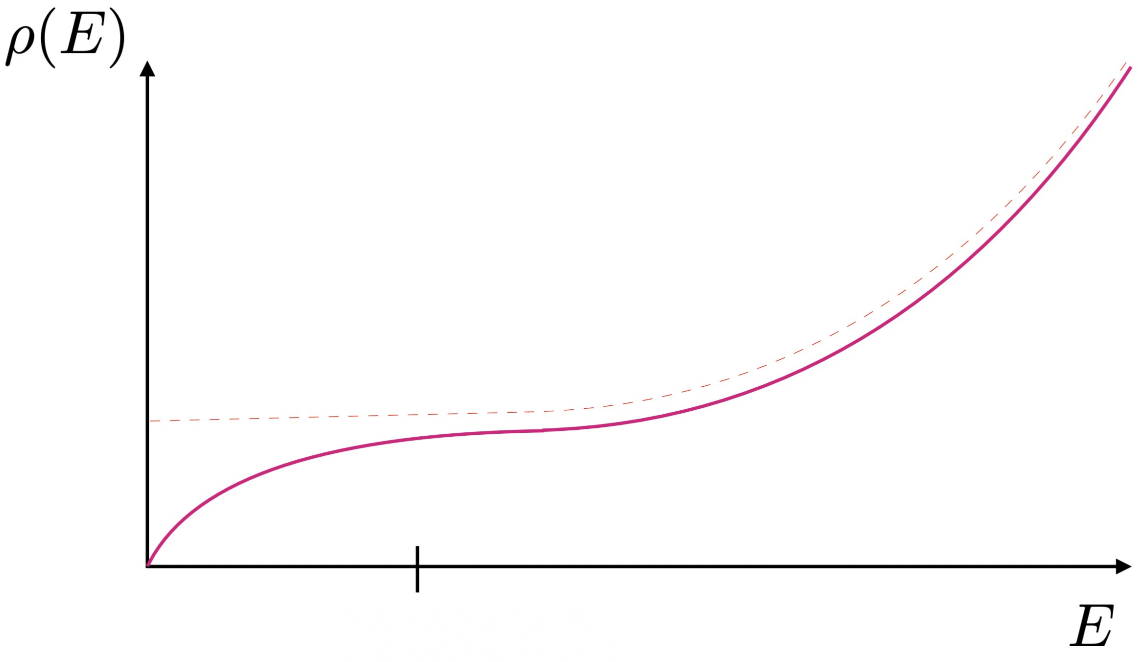
Summary: non-SUSY vs SUSY at zero temperature

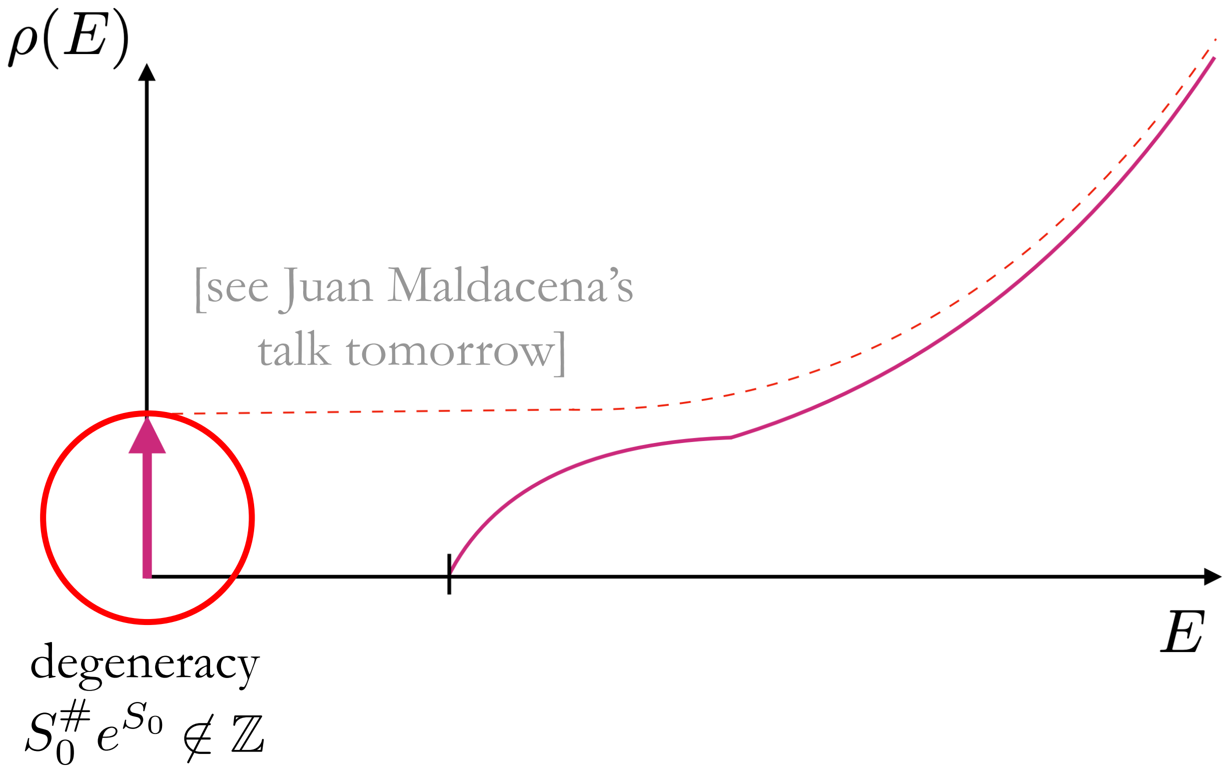
Action = 0 when T = 0

$\int_{\frac{\text{Large diffeos}}{\text{Isometries}}}$  $= 0$

$\int_{\frac{\text{Large super-diffeos}}{\text{Super-isometries}}}$  $= 1$

A non-integer degeneracy





Can we however do better for extremal supersymmetric black holes
and reproduce the degeneracy in a microscopic model?

The basis of our comparison:
1/8-BPS black holes in $\mathcal{N} = 8$ supergravity in 4D flatspace

The microscopic degeneracy in a D1/D5 construction

$$c_{-2,1}(\overbrace{n,\ell}^{\text{Charges of the black hole}}) = \underbrace{d_{\text{micro}}(\Delta)}_{\Delta=4n-\ell^2}$$

[5D black holes degeneracies obtained by Maldacena, Moore, Strominger `99,
related through a 4D/5D lift to 4D degeneracies in Gaiotto, Strominger, Yin `05, Shih, Strominger, Yin `05]

The microscopic degeneracy in a D1/D5 construction

Charges of the black hole

$$c_{-2,1}(\overbrace{n,\ell}) = \underbrace{d_{\text{micro}}(\Delta)}_{\Delta=4n-\ell^2}$$

[Maldacena, Moore, Strominger `99, Gaiotto, Strominger, Yin `05, Shih, Strominger, Yin `05]

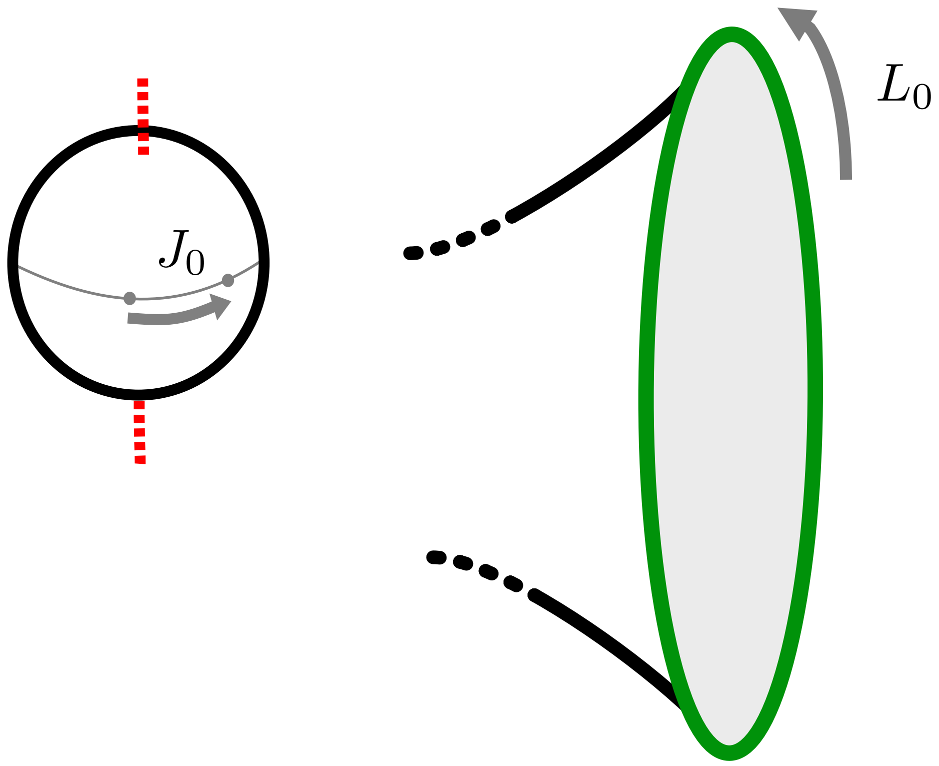
Hardy-Ramanujan-Rademacher expansion for the microscopic degeneracy:

$$d_{\text{micro}}(\Delta) = \sum_{c=1}^{\infty} c^{-9/2} \underbrace{\tilde{I}_{7/2}\left(\frac{\pi\sqrt{\Delta}}{c}\right)}_{\substack{\text{Convergent sum with} \\ \text{successively smaller} \\ \text{exponential terms}}} \overbrace{K_c(\Delta)}^{\substack{\text{Kloosterman sum} \\ \text{over exponential} \\ \text{phases}}}$$

Addressing Q3: Can we reproduce this formula exactly by using the gravitational path integral?

[a lot of progress has been made using localization in supergravity by Dabholkar, Gomes, Gupta, Jeon, Reys, Murthy.]

Close, but was not completed.



Asymptotically
$AdS_2 \times S^2$
with
$PSU(1,1|2)$

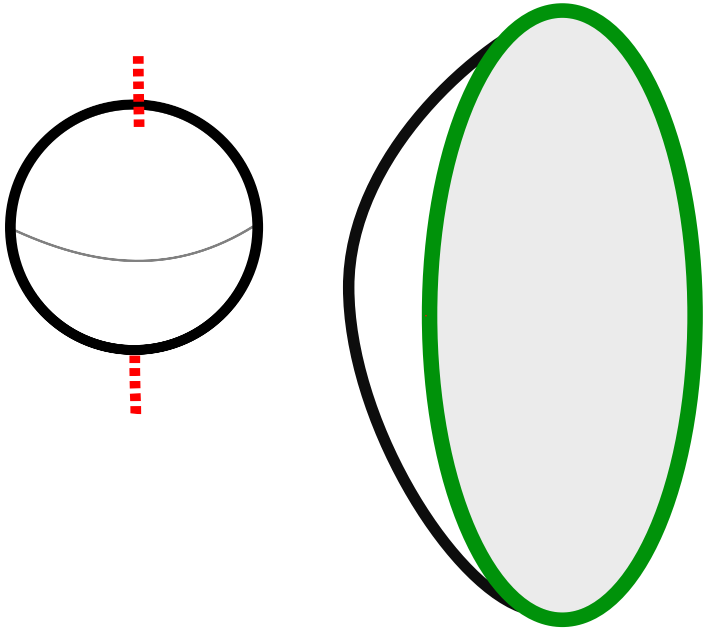
$Q_{\text{eq}}^2 = L_0 - J_0$

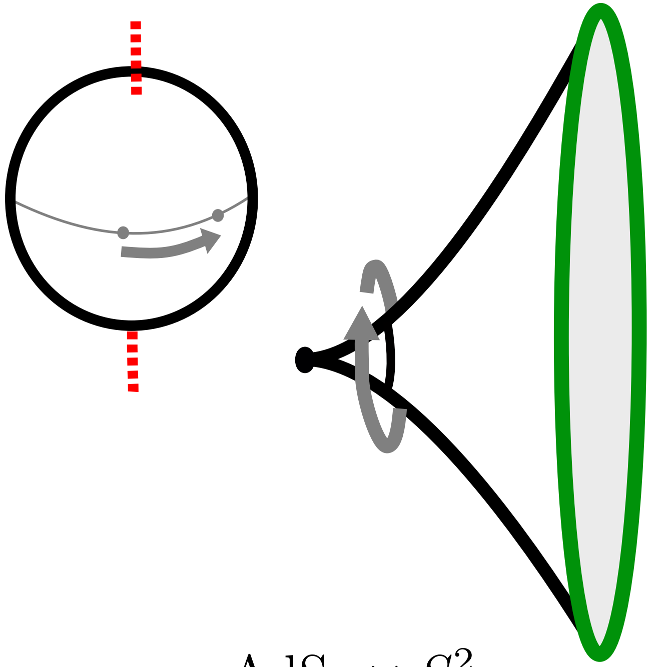
$S_{SUGRA}(\lambda) = S_{SUGRA} + \lambda Q_{\text{eq}} \mathcal{V}$

$Z_{\text{grav}}(\infty) = Z_{\text{grav}}(0)$

[Banerjee, Banerjee, Gupta, Mandal, Sen `09, Dabholkar, Gomes, Murthy `10, `11 and `14, de Wit, Reys, Murthy `18]

The path integral localizes to $Q_{\text{eq}} \underbrace{\psi}_{\text{all physical fermions in the theory}} = 0$



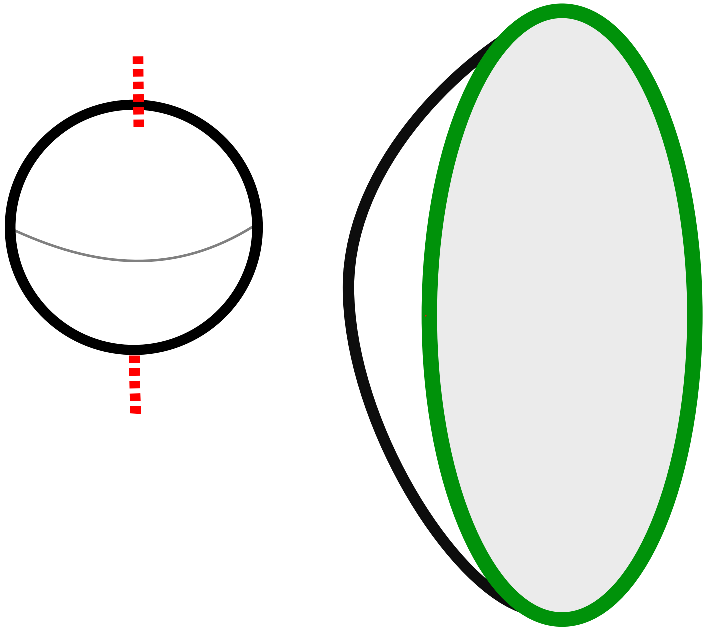
$\mathrm{AdS}_2 \times S^2$ $+$ 

$\dfrac{\mathrm{AdS}_2 \times S^2}{\mathbb{Z}_c}$

The path integral localizes to $Q_{\mathrm{eq}} \underbrace{\psi}_{\substack{\text{all physical fermions}\\ \text{in the theory}}} = 0\,.$

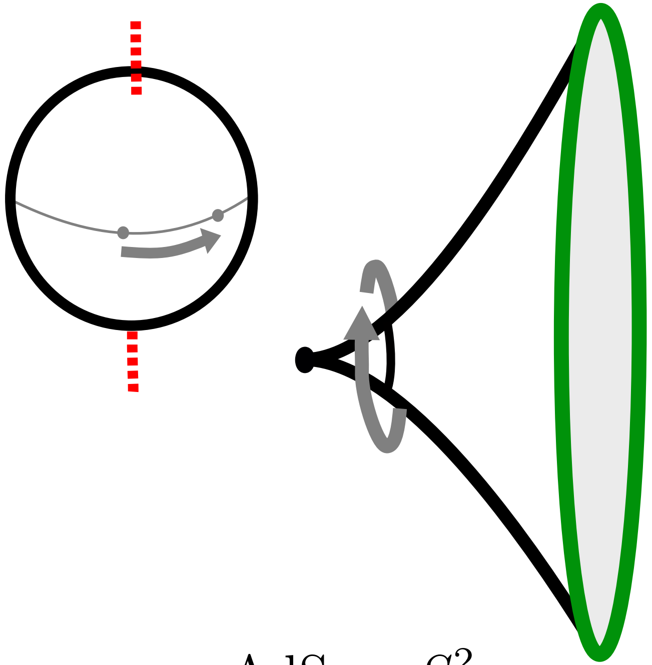
We will take the perspective that the gravitational path integral requires a sum over topologies.

[Dabholkar, Gomes, Murthy `10, `11 and `14]



$\mathrm{AdS}_2 \times S^2$

$+$



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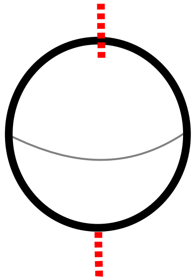
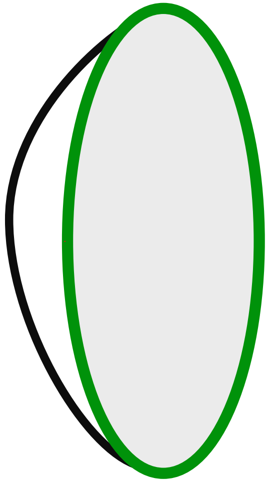
The path integral localizes to $Q_{\mathrm{eq}} \underbrace{\psi}_{\text{all physical fermions in the theory}} = 0$

The scalars in the vector supermultiplets have an entire localization locus which we parametrize by ϕ^I with $I = \underbrace{0}_{\text{scalar of graviphoton}},\ 1,\ \ldots,\ n_v$

[Dabholkar, Gomes, Murthy `10, Gupta, Murthy `12]

Evaluating the localized gravitational path integral





The on-shell localizing action

$$= \int \prod_{I=0}^{n_v} [d\phi^I] \; \exp\Big(-\underbrace{\pi \, q_I \, \phi^I}_{\text{EM bdy. term}} + 4\pi \, \underbrace{\mathrm{Im}\, F(\phi^I)}_{\mathcal{N}=8 \text{ prepotential}} \Big) \, Z_{\text{1-loop}} \overbrace{(\phi^0)}^{\text{Related to scale of } AdS_2 \times S^2}$$

[Dabholkar, Gomes, Murthy `10, `11 and `14, Murthy and Reys `13]

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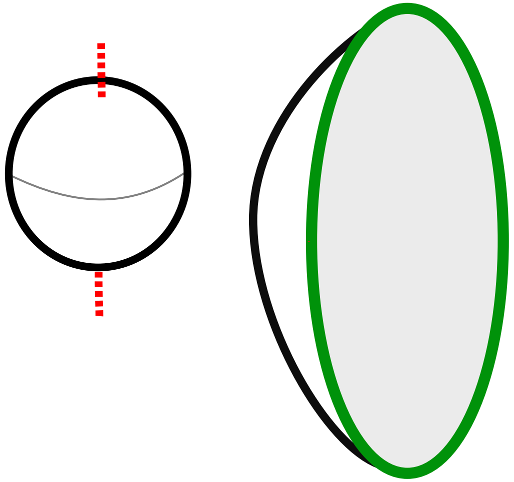

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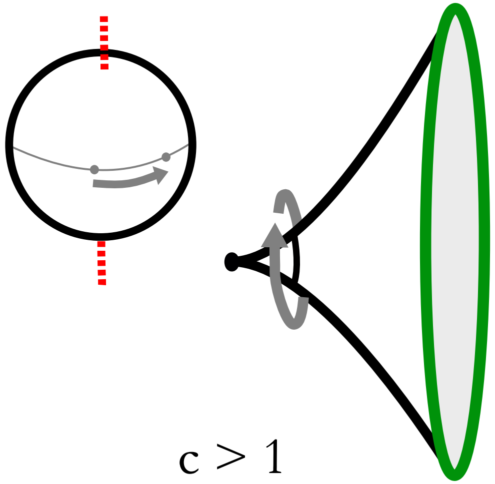

c > 1

$$= \int \prod_{I=0}^{n_v} [d\phi^I]_c \; \exp\Big(\frac{1}{c} \Big[- \pi\, q_I\, \phi^I + 4\pi\, \mathrm{Im}\, F(\phi^I) \Big] \Big) \; \underbrace{Z_{\text{1-loop, c}}(\phi^0)}_{\text{non-trivial c-dependence}} \; \underbrace{Z_{\text{top},c}(\Delta)}_{\text{Topological term}}$$

$$= \sqrt{c} K_c(\Delta)$$

[Dabholkar, Gomes, Murthy `14]

In $\mathcal{N}=8$ supergravity the prepotential is quadratic in $\phi^1, \ldots, \phi^{n_v}$

$$Z_{\text{BH}}^{T=0}(\Delta) = \sum_{c=1}^{\infty} K_c(\Delta) \int \overbrace{\frac{d\phi^0}{\phi_0^{3/2}}}^{\substack{\text{scalar of} \\ \text{graviphoton}}} \underbrace{Z_{\text{1-loop},\, c}(\phi^0)}_{\substack{\text{The missing} \\ \text{ingredient}}} e^{-\frac{\pi\Delta\phi^0}{4c} - \frac{\pi}{c\phi^0}}$$

[progress was made but the contribution of all supermultiplets had not been computed by Gupta, Ito and Jeon `15, Murthy and Reys `15, Murthy and Jeon `18]

Once again, $Z_{\text{1-loop},\,c}(\rho^0) = Z^{\text{bulk}}_{\text{1-loop},\,c}(\rho^0)\, Z_{\text{zero-modes}}(\rho^0)$

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$$Z^{\text{bulk}}_{\text{1-loop},\, c}(\phi^0) = \prod_{\mathcal{M} \in \mathcal{N}=2 \text{ multiplets}} \underbrace{Z^{\mathcal{M}}_{\text{1-loop},\, c}(\phi^0)}$$

[LVI, Murthy, Turiaci `22]

(Weyl, spin-3/2, vector, hyper, chiral)

Beautifully computed using Atiyah-Bott fixed-point formula.

⏰😥

Complicated expression in ϕ^0 and c .

Once again, $Z_{\text{1-loop},\,c}(\rho^0) = Z^{\text{bulk}}_{\text{1-loop},\,c}(\rho^0)\, Z_{\text{zero-modes}}(\rho^0)$

Simple ϕ^0-dependence and completely c-independent!

$$Z^{\text{bulk}}_{\text{1-loop},\,c}(\phi^0) = \prod_{\mathcal{M}\in\mathcal{N}=2 \text{ multiplets}} Z^{\mathcal{M}}_{\text{1-loop},\,c}(\phi^0) \;=\; \overbrace{(\phi^0)^5}$$

(Weyl, spin-3/2, vector, hyper, chiral)

[LVI, Murthy, Turiaci `22]

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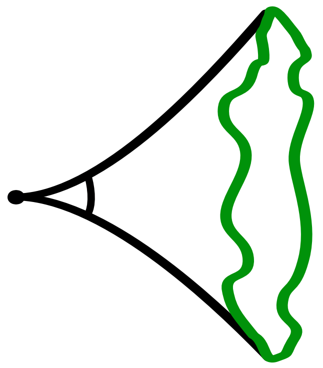
$$Z_{\text{zero-modes},\,c}(\phi^0) = \int_{\frac{\text{Large superdiffeos}}{\text{Superisometries}}}$$



$$= \frac{1}{c\phi_0}$$

[LVI, Kologlu, Turiaci `21, LVI, Murthy, Turiaci `22]

Putting everything together we have,

$$Z_{\mathrm{BH}}^{T=0}(\Delta) = \sum_{c=1}^{\infty} K_c(\Delta) \int \frac{d\phi^0}{(\phi^0)^{3/2}} \underbrace{(\phi^0)^5}_{\text{bulk one-loop determinant}} \overbrace{\frac{1}{c\phi^0}}^{\text{boundary zero-modes}} e^{-\frac{\pi \Delta \phi^0}{4c} - \frac{\pi}{c\phi^0}}$$

[LVI, Murthy, Turiaci `22]

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$$= \sum_{c=1}^{\infty} \underbrace{c^{-9/2}}_{\substack{\text{Account}\\ \text{of the ``would-be'' zero-modes}\\ \text{is crucial}}} K_c(\Delta)\ \tilde{I}_{7/2}\Big(\frac{\pi\sqrt{\Delta}}{c}\Big)$$

[LVI, Murthy, Turiaci `22]

Answer to Q3:

$$Z_{\mathrm{BH}}^{T=0}(\Delta) = d_{\mathrm{micro}}(\Delta)$$

An exact integer from the gravitational path integral

Concluding remarks and outlook: AdS_2 holography revisited

In higher dimensional holography, the scale of quantum gravity corrections is controlled by $\ell_{\mathrm{AdS}}/\ell_{\mathrm{Pl}}$

In AdS_2 holography, there are two scales $S_0^{1/2} \sim \ell_{\mathrm{AdS}}/\ell_{\mathrm{Pl}} \sim \ell_{S^2}/\ell_{\mathrm{Pl}}$ and T/E_{gap}

Concluding remarks and outlook: AdS$_2$ holography revisited

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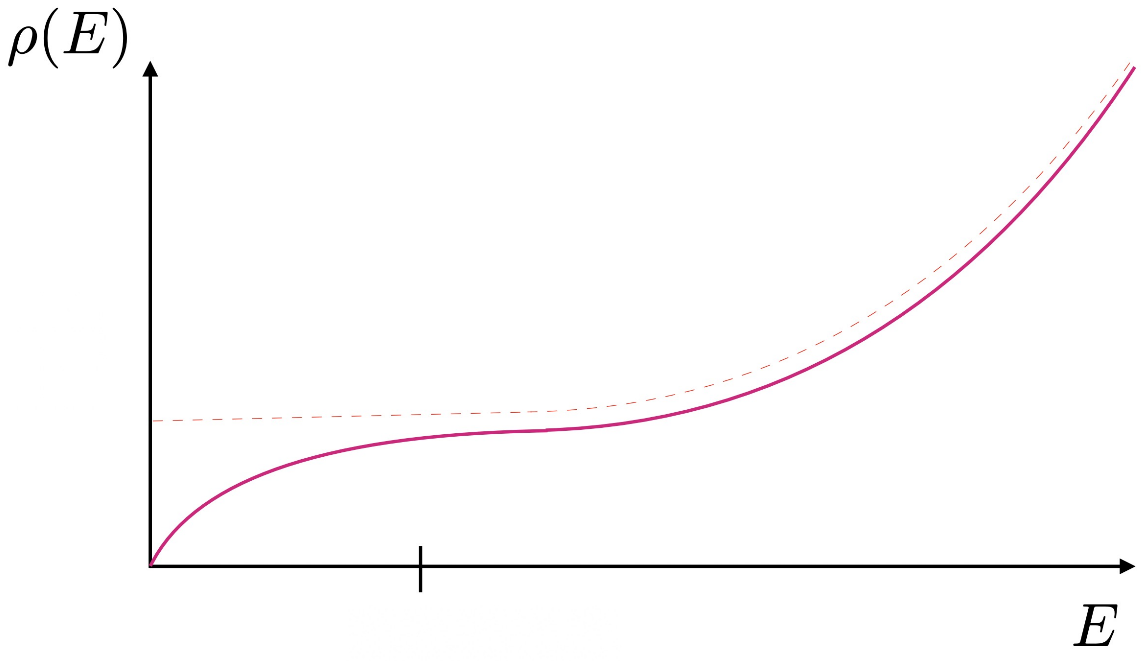
In AdS$_2$ holography, there are two scales $S_0^{1/2} \sim \ell_{\text{AdS}}/\ell_{\text{Pl}} \sim \ell_{S^2}/\ell_{\text{Pl}}$ and T/E_{gap}

The statistical mechanics of near-extremal black holes



The presence of the "would-be" zero modes

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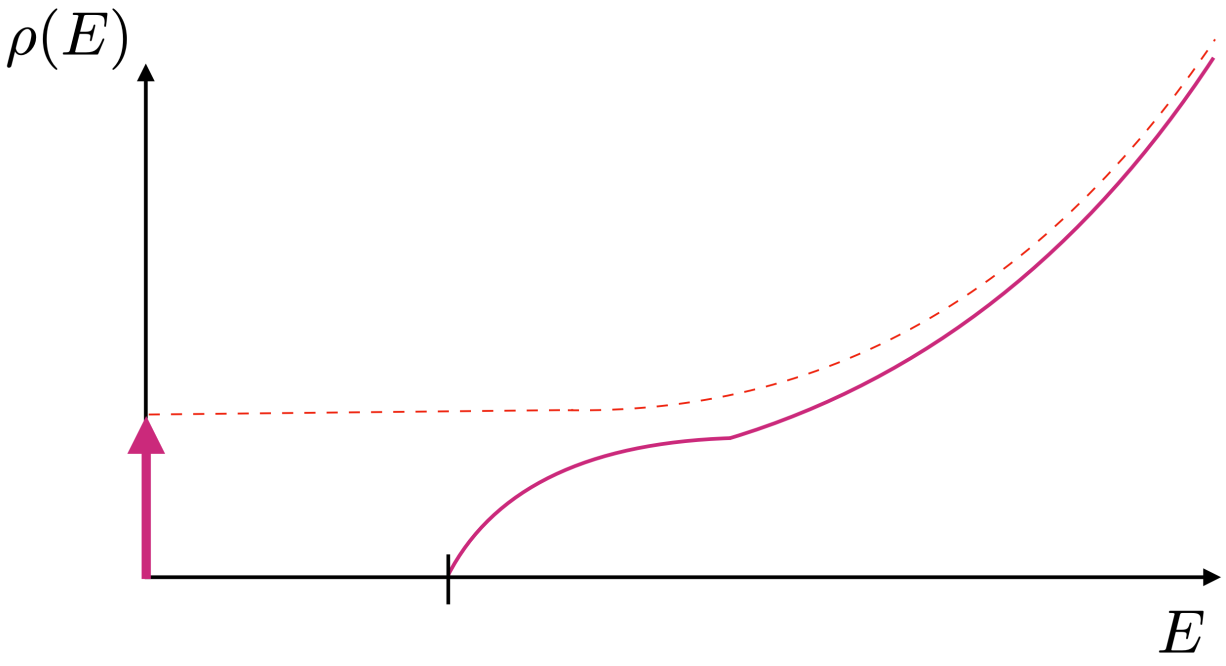


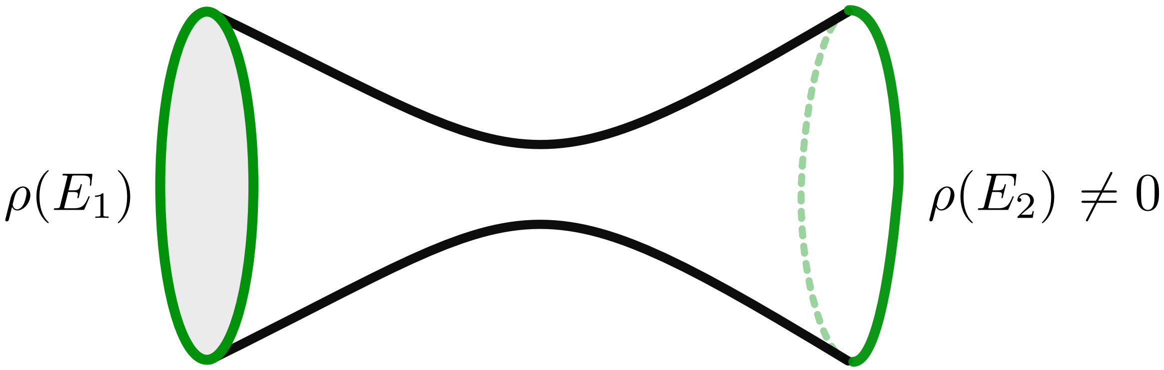
A non-perturbatively dense spectrum even at low-energies

Concluding remarks and outlook:
The outstanding problem near-extremality

To make this a conventional QM we'd like to make the spectrum discrete.

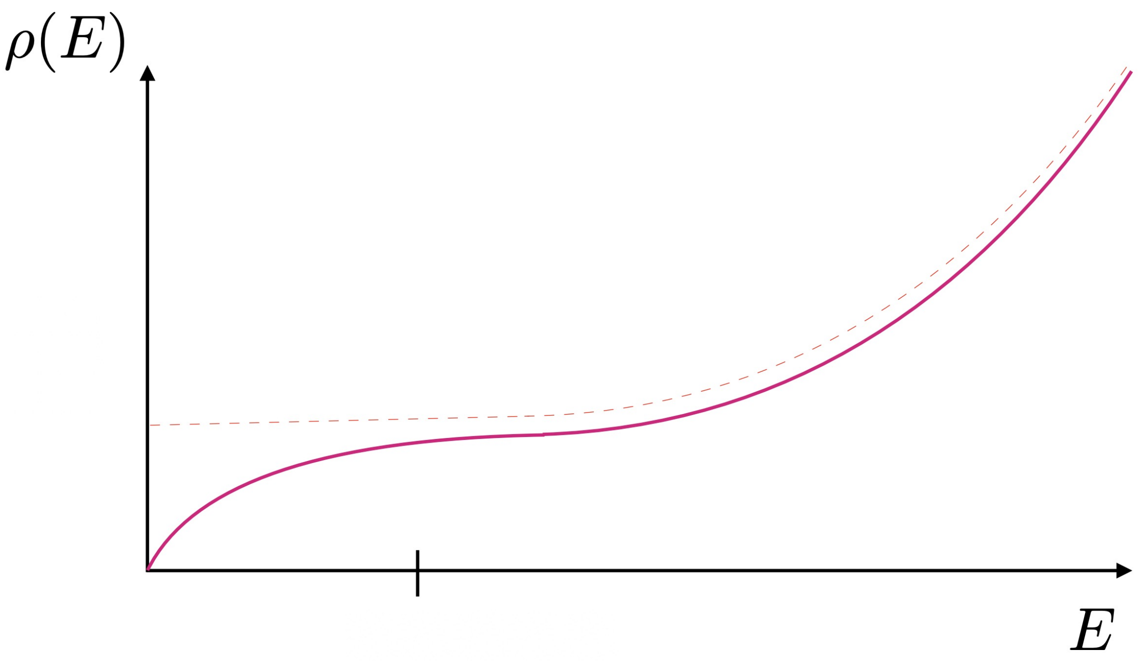
Concluding remarks and outlook:
The outstanding problem near-extremality

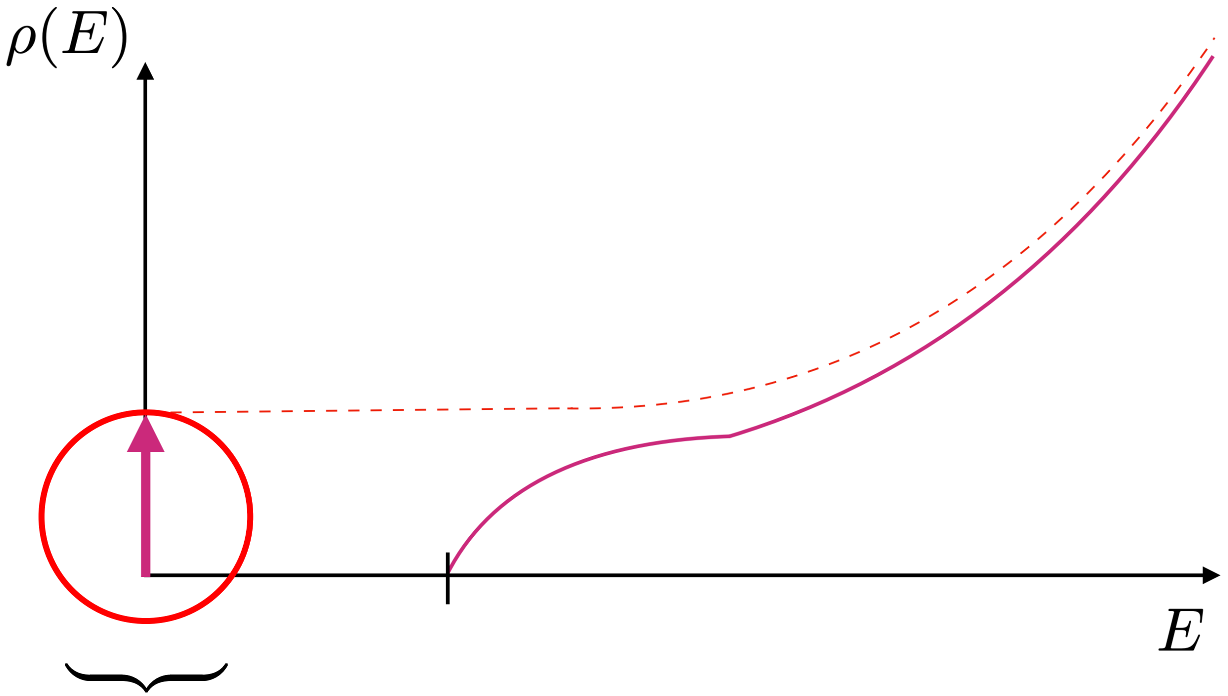
To make this a conventional QM we'd like to make the spectrum discrete.



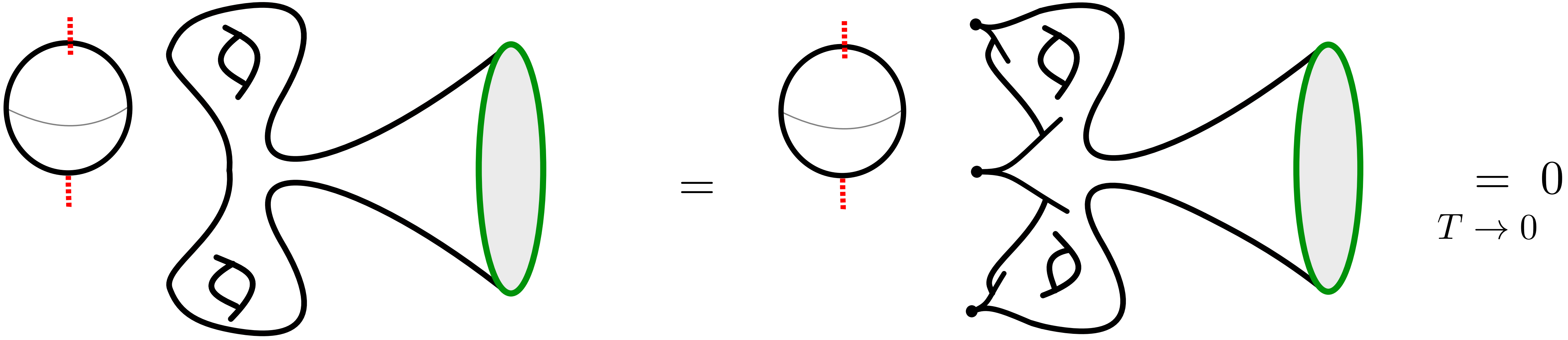
While the sum over topologies might tell us information about the spectral statistics of the theory [see Factorization and Ensembles panel tonight], without a full control over stringy and non-perturbative corrections there is no way to guess this discrete, and most likely chaotic, spectrum from the bulk perspective.

Concluding remarks and outlook:
All hope is not lost, the AdS_2 groundstate is different

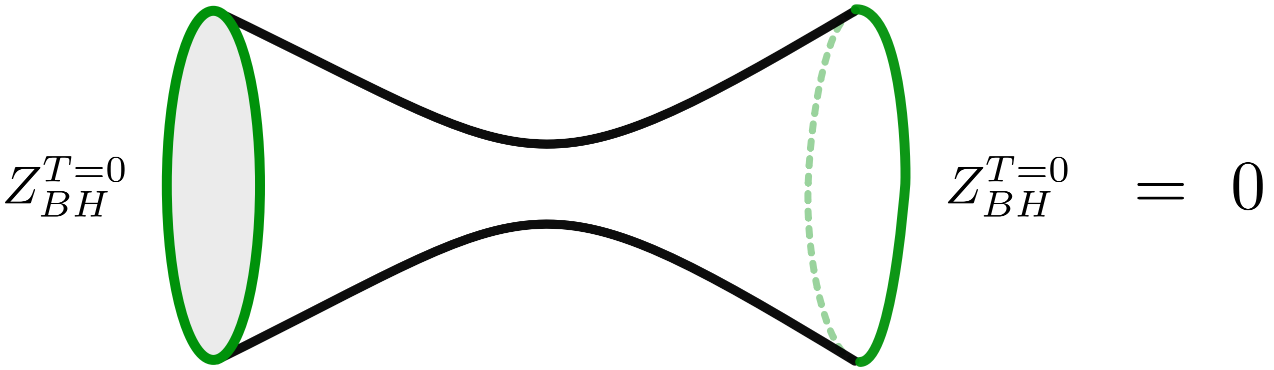




The sum over topologies is absolutely necessary,
but is nevertheless under control.



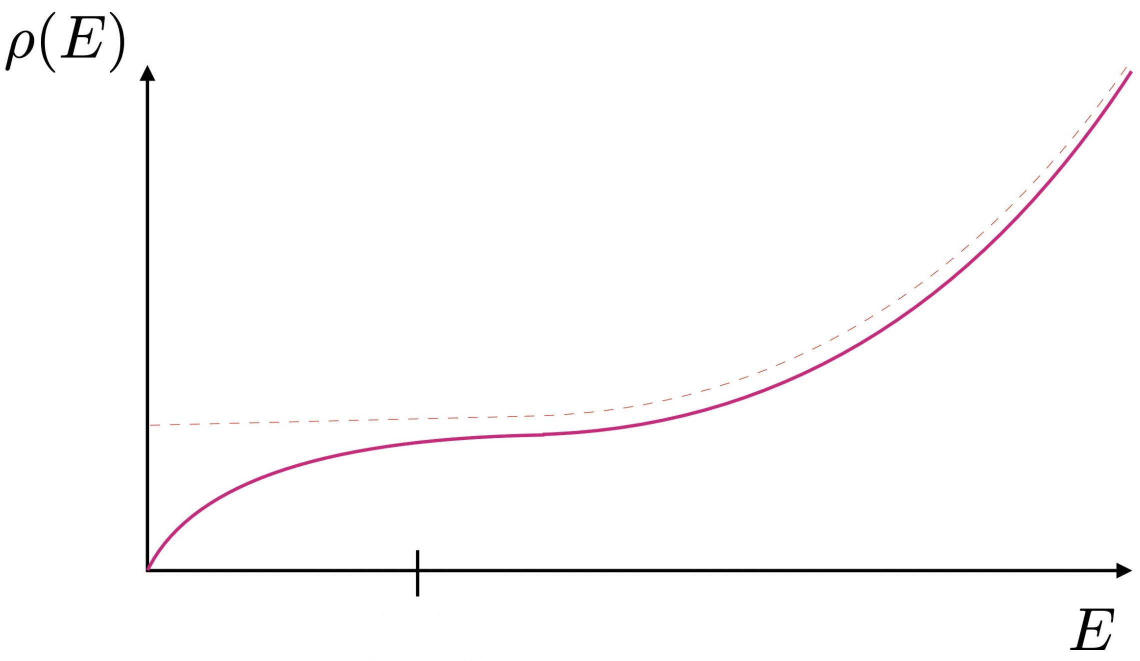
The topological expansion is under control for both the degeneracy and BPS index. In both cases, there is no factorization puzzle since we find that both the degeneracy and BPS index have no connected contributions.

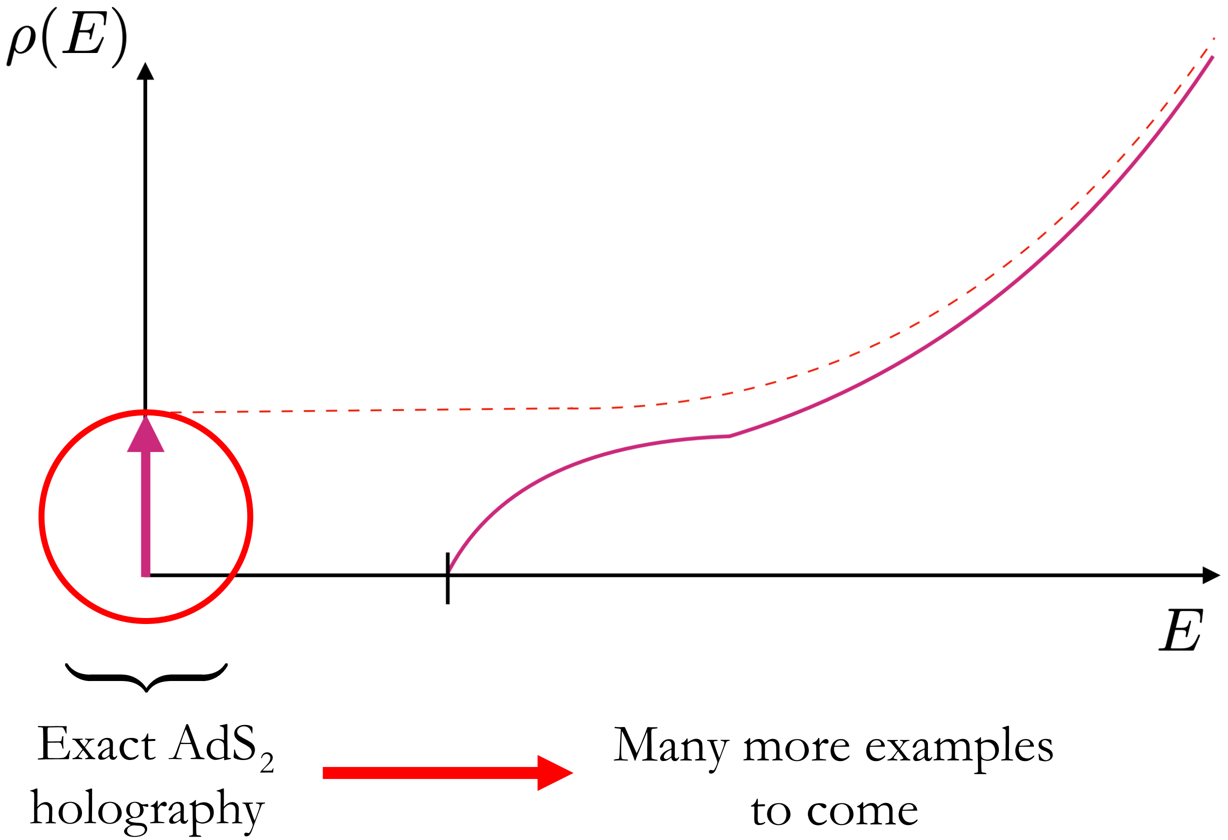


[LVI, Koloğlu, Turiaci `21]

Concluding remarks:
All hope is not lost, the AdS_2 groundstate is different

Thank you!
Questions?





Appendix

$$Z^{\text{bulk}}_{\text{1-loop}} = \sqrt{\frac{\det K_f}{\det K_b}} = \sqrt{\frac{\det_{\Psi} H}{\det_{\Phi} H}}$$

$$\text{heat kernel of multiplet}(t) = \text{ind} D_{10}(t) = \text{Tr}_{\text{Ker}\, D_{10}} e^{tH} - \text{Tr}_{\text{Coker}\, D_{10}} e^{tH}$$

c=1: $$\mathrm{ind}(D_{10})(t) = \sum_{\{x|\tilde{x}(t)=x\}} \frac{\mathrm{Tr}_\Phi\left(e^{tH}\right) - \mathrm{Tr}_\Psi\left(e^{tH}\right)}{\det\left(1 - \partial\tilde{x}/\partial x\right)}$$

c>1: $$\mathrm{ind}_c(D_{10})(t) = \frac{1}{|\mathbb{Z}_c|} \sum_{\gamma\in\mathbb{Z}_c} \sum_{\{x|\tilde{x}(\gamma,t)=x\}} \frac{\mathrm{Tr}_\Phi\left(\gamma e^{tH}\right) - \mathrm{Tr}_\Psi\left(\gamma e^{tH}\right)}{\det\left(1 - \partial\tilde{x}/\partial x\right)}$$

where $\tilde{x} =$ center of AdS & N and S poles of S^2

We first arrange all fields and ghosts into a cohomology complex, of the form $(\Phi,\ \Psi,\ Q_{\mathrm{eq}}\Phi,\ Q_{\mathrm{eq}}\Psi)$

Near the fixed points, the space looks locally like $\mathbb{R}^4$ so we only need to look at irreps of Φ and Ψ, including all fields and ghosts, under SO(4) in order to evaluate the indices using the appropriate characters.

$$\begin{aligned}
\text{Vector}: &\qquad \mathrm{ind}_{\mathrm{vect.}}(D_{10})(t) = \frac{2(q+q^{-1})-4}{(1-q)^2\,(1-q^{-1})^2}\,, \\
\text{Hyper}: &\qquad \mathrm{ind}_{\mathrm{hyper}}(D_{10})(t) = -\frac{2(q+q^{-1})-4}{(1-q)^2\,(1-q^{-1})^2}\,, \\
\text{Weyl}: &\qquad \mathrm{ind}_{\mathrm{Weyl}}(D_{10})(t) = \frac{2(q^2+q^{-2})-6(q+q^{-1})+8}{(1-q)^2\,(1-q^{-1})^2}\,, \\
\text{Spin-}\frac{3}{2}: &\qquad \mathrm{ind}_{\mathrm{spin}\ 3/2}(D_{10})(t) = \frac{-2(q^2+q^{-2})+4(q+q^{-1})-4}{(1-q)^2\,(1-q^{-1})^2}\,, \\
\text{Chiral}: &\qquad \mathrm{ind}_{\mathrm{chiral}}(D_{10})(t) = 0\,.
\end{aligned}$$

Putting the $\mathcal{N}=2$ supergravity multiplets together into an $\mathcal{N}=8$ supergravity Weyl multiplet, we have:

Independent of t!

$$\mathcal{N}=8\text{ Weyl}\sim\text{Weyl}+6(\text{spin-}\frac{3}{2}+\text{vect.})+15\text{ vect.}+10\text{ hyper}:\qquad \text{ind}^{\mathcal{N}=8}_{\text{Weyl}}(D_{10})(t)=\overbrace{-10}\,.$$

Simple ϕ^0-dependence and completely c-independent!

$$Z^{\text{bulk}}_{\text{1-loop},\,c}(\phi^0)=\prod_{\mathcal{M}\in\mathcal{N}=2\text{ multiplets}}Z^{\mathcal{M}}_{\text{1-loop},\,c}(\phi^0)=\overbrace{(\phi^0)^5}$$

(Weyl, spin-3/2, vector, hyper, chiral)