Black hole entropy functions and spinning spindles

James Sparks

Mathematical Institute, Oxford

Based on work with Jerome Gauntlett & Dario Martelli

[and Pietro Benetti Genolini, Andrea Boido, Davide Cassani, Chris Couzens, Pietro Ferrero, Juan Pérez Ipiña]

Our programme over the last few years: low-dimensional SUSY AdS/CFT

Highlights:

- New geometric techniques $\Rightarrow$ computation of observables in gravity via extremal problems, without solving Einstein equations
- Allowed to identify dual SCFTs, including for old classes of solutions
- Rich infinite classes of new SCFTs, from branes at geometric singularities/wrapped over cycles, with exact matching of observables
- New supersymmetric compactifications, relation to accelerating black holes, and new perspective on black hole entropy functions

More concretely:

- $\mathrm{AdS}_3 \times \boldsymbol{Y}$ solutions of IIB string theory, $\mathrm{AdS}_2 \times \boldsymbol{Y}$ solutions of M-theory
- Generated by a single type of brane (D3, M2), flux $\boldsymbol{G}$ = differential form
- Supersymmetry with $\boldsymbol{U}(1)_{\boldsymbol{R}}$ symmetry $\Rightarrow$ Killing vector ξ on $\boldsymbol{Y}$

Work around 2007 by [Gauntlett-Kim] *et al* constructed explicit solutions, studied local geometric structure on $\boldsymbol{Y}$, but lacking insight into field theory duals.

From 2018: introduced global geometric techniques, exploiting structure on $\boldsymbol{Y}$.

We introduce a supersymmetric action:

$$\boldsymbol{S} \;=\; \boldsymbol{S}(\boldsymbol{Y}, \boldsymbol{N_a}; \xi)\,,$$

where $\boldsymbol{N_a} = \int_{\boldsymbol{\Sigma_a}} \boldsymbol{G}$ are integer quantized fluxes, over cycles $\boldsymbol{\Sigma_a} \subset \boldsymbol{Y}$.

Key point: depends only on *global* geometric data on $\boldsymbol{Y}$.

By construction, supersymmetric AdS solutions *extremize* $\boldsymbol{S}$, as a function of

$$\boldsymbol{\xi} \;=\; \sum_{\boldsymbol{i}=1}^{\boldsymbol{n}} \boldsymbol{b_i} \frac{\boldsymbol{\partial}}{\boldsymbol{\partial\varphi_i}} \quad \Rightarrow \quad \text{extremize over } \vec{\boldsymbol{b}} \in \mathbb{R}^{\boldsymbol{n}} \quad \Rightarrow \quad \boldsymbol{\xi_*}$$

- We introduced various geometric methods for computing $\boldsymbol{S}$ in closed form, *e.g.* developing some new toric geometry [explicit example later]
- Extremal $\boldsymbol{S}_* = \boldsymbol{S}(\boldsymbol{Y}, \boldsymbol{N_a}; \xi_*)$ = central charge for AdS_3 solutions, entropy for AdS_2 solutions
- $\boldsymbol{\xi}_*$ = superconformal R-symmetry
- Other observables computed similarly, e.g. dimensions of BPS operators

Global description of $\boldsymbol{Y} \Rightarrow$ identification of SCFT duals $\Rightarrow$ exact matching

Brief overview of AdS_3 results:

- $\mathrm{AdS}_3 \times \boldsymbol{Y}_7$ solutions in type IIB, D3-brane flux $\boldsymbol{G}_5$
- Various explicit solutions, *e.g.* [Gauntlett-MacConamhna-Mateos-Waldram]
  - New methods $\Rightarrow$ interpretation as D3-branes wrapped over a spindle
  - New way to preserve supersymmetry (not via a topological twist)
- Infinite classes of D3-branes at Calabi-Yau 3-fold singularities, further wrapped over (orbifold) Riemann surface (no explicit solutions in general)
- $\boldsymbol{S}_* =$ central charge $\boldsymbol{c}$, with general proof that the gravity result matches field theory central charge (via anomaly polynomials & $\boldsymbol{c}$-extremization)

Rest of this talk: $\mathrm{AdS}_2 \times \boldsymbol{Y}_9$ solutions $\Rightarrow$ near horizon of extremal black holes.

- We consider $\boldsymbol{Y}_9$ of fibred form

$$\boldsymbol{X}_7 \hookrightarrow \boldsymbol{Y}_9 \to \boldsymbol{\Sigma}\,,$$

  where $\boldsymbol{\Sigma} =$ (orbifold) Riemann surface

- Near horizon of extremal black hole in $\mathrm{AdS}_4 \times \boldsymbol{X}_7$, horizon $\boldsymbol{\Sigma}$
- Fluxes $\boldsymbol{N_a}$ determine magnetic charges of black hole
- $\boldsymbol{S} = \boldsymbol{S}(\boldsymbol{Y}_9, \boldsymbol{N_a}; \xi) =$ entropy function, similar in spirit to Sen, $\boldsymbol{S}_* =$ entropy

$\mathrm{AdS}_4 \times \boldsymbol{X_7}$ black holes:

- Reduction on $\boldsymbol{X_7} \Rightarrow$ massless $\boldsymbol{U}(1)$ gauge fields $\boldsymbol{A_a}$ in AdS_4
- Different $\boldsymbol{X_7} \Rightarrow$ different SCFT_3, with black holes in the bulk
- Expect general families of supersymmetric black holes with magnetic & electric charges $\boldsymbol{p_a}$, $\boldsymbol{q_a}$, angular momentum $\boldsymbol{J}$, and acceleration
- Acceleration is a new ingredient $\Rightarrow$ different horizon topologies (spindles)

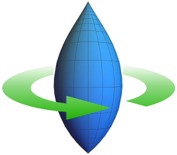
What is known?

- For $\boldsymbol{X}_7 = \boldsymbol{S}^7$ and horizon $\boldsymbol{\Sigma} = \boldsymbol{S}^2$, general dyonic rotating black holes may be constructed in STU gauged supergravity, entropy matched to a dual microstate counting [Benini-Hristov-Zaffaroni]
- For general $\boldsymbol{X}_7$, class of rotating, accelerating black holes with single electric & magnetic charge constructed in [Ferrero-Gauntlett-Ipiña-Martelli-JFS]

It seems hopeless to construct general black hole solutions explicitly.

But can we compute their entropy, and understand dual microstate counting?

Well-known in GR that acceleration $\Rightarrow$ conical deficit angles along horizon $\boldsymbol{\Sigma}$.



Provided these are *quantized* ($2\boldsymbol{\pi}/\boldsymbol{n}_{\pm}$ at each "pole"), so $\boldsymbol{\Sigma}$ = orbifold known as a *spindle*, $\mathrm{AdS}_4 \times \boldsymbol{X}_7$ solution has no conical deficit angles!

This has to do with supersymmetry relating acceleration to magnetic charge, and the latter twists $\boldsymbol{X}_7$ over the black hole, removing the deficit angles.

General lesson: lower-dimensional singular solutions can have non-singular uplifts!

Our supersymmetric action $\boldsymbol{S}(\boldsymbol{Y}_9, \boldsymbol{N_a}; \vec{\boldsymbol{b}})$ allows to compute black hole entropy:

- With zero electric charge & angular momentum (although ∃ conjecture for how to include these)
- Magnetic charges $\boldsymbol{p_a} = \boldsymbol{N_a}/\boldsymbol{N}$, where $\boldsymbol{N} = \int_{\boldsymbol{X}_7} \boldsymbol{G}_7$ = number of M2-branes
- Recall $\boldsymbol{Y}_9$ is $\boldsymbol{X}_7$ fibred over horizon $\boldsymbol{\Sigma}$. We have a general "gravitational block" formula for $\boldsymbol{S}$, with contributions coming from each "pole" of $\boldsymbol{\Sigma}$
- This remarkably agrees with guesses for entropy functions e.g. for STU black holes, but is both *derived*, and much more general!

Example:

$$S = \frac{1}{b_0}\left(\frac{1}{\sqrt{\mathrm{Vol}(X_7)(\vec{b}_+)}} - \frac{\sigma}{\sqrt{\mathrm{Vol}(X_7)(\vec{b}_-)}}\right)\frac{8\pi^3 N^{3/2}}{3\sqrt{6}}$$

where $\sigma = \pm 1$ and

- b_0 = component of R-symmetry vector $\vec{b}$ rotating horizon Σ
- $\vec{b}_\pm = \vec{b} \mp \frac{b_0}{n_\pm}\vec{v}_\pm$ ($\vec{v}_\pm$ determined by magnetic charges p_a/fibration)

Can also be written as

$$S = \frac{4}{b_0}\left[\mathcal{F}_{S^3}(\vec{b}_+) - \sigma\,\mathcal{F}_{S^3}(\vec{b}_-)\right]$$

Relation to other approaches to black hole entropy & thermodynamics?

Problem: Euclidean on-shell action of extremal black holes not well-defined.

Solution: look at *complex* family of supersymmetric but non-extremal black hole solutions, turning on chemical potential $\boldsymbol{\omega}$ for rotation.

Remarkably, for the families of explicit black hole solutions we find

$$\boldsymbol{S}\,|_{\boldsymbol{b}_0=\boldsymbol{\omega}/2\boldsymbol{\pi}\mathbf{i}} \;=\; -\text{Euclidean action of black hole!}$$

- Relates near horizon (IR) to bulk and boundary (UV)
- Extremizing right hand side gives Legendre transform $\Rightarrow$ entropy

We have a conjecture for how this should work more generally.

Summary:

- New geometric approach to computing observables in gravity
- Infinite classes of AdS_3 & AdS_2 solutions $\Rightarrow$ new 2d SCFTs with exact matching, near horizon limits of extremal black holes
- New compactifications on spindles, and relation to acceleration in GR

Some questions:

- Black strings for which the AdS_3 solutions are near horizon limits?
- Field theories on spindles/general singular spaces? Microstate counting?
- Entropy function $\Leftrightarrow$ Euclidean black hole action via fixed point theorem?