Review:
Black Hole Microstate Counting in AdS

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SISSA

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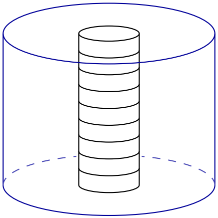
NP-QFT

I will review recent developments

in study of BPS black holes in Anti-de-Sitter space: entropy and microstates



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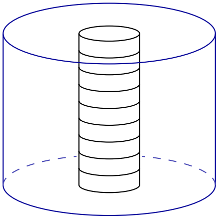

This topic has been extensively studied in *asymptotically-flat* spacetimes

⋆ String theory reproduces the Bekenstein-Hawking entropy [Strominger, Vafa 96]
of BPS black holes in asymptotically-flat spacetimes

This has been refined in a very accurate & impressive way since then

[very long list of people. . .]

- ⋆ The AdS/CFT correspondence provides us with a CONSISTENT and NON-PERTURBATIVE definition of QUANTUM GRAVITY in ANTI-DE-SITTER SPACE,

  in terms of an ordinary QFT at the boundary

- ⋆ It is interesting to study black hole entropy in AdS

- ⋆ AdS_3 and AdS_2 are special. Here AdS_d with $d \geq 4$

Semiclassical Regime for Gravity

⋆ In AdS:

Gravity is weakly coupled
(AdS much larger than Planck scale)

and close to Einstein gravity
(scale of higher-derivative corr.'s much higher than AdS scale)

⋆ In QFT:

large "central charge"
(large N)

QFT is strongly coupled

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Need to take advantage of non-perturbative methods in QFT:

- conformal bootstrap
- integrability (for certain CFT's)
- supersymmetry
- numerics (lattice and/or Montecarlo)
- ...

Black Hole Entropy

$$S_{\text{Bekenstein-Hawking}} = \frac{\text{Horizon Area}}{4\,G_N\,\hbar/c^3}$$

[Bekenstein 72, 73, 74; Hawking 74, 75]

Black hole $=$ Ensemble of states in quantum gravity $\overset{\text{AdS/CFT}}{=}$ Ensemble of states in boundary QFT

$$S_{\text{micro}} = \log N_{\text{micro}} = \frac{\text{Area}}{4\,G_N} + \textit{corrections}:$$

- perturbative
- higher derivative
- non-perturbative (classical sol's)
- non-perturbative (branes)
- ...

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$$S_{\text{micro}} = \log N_{\text{micro}} = \frac{\text{Area}}{4\,G_N} + \textit{corrections}: \begin{cases} \text{perturbative} \\ \text{higher derivative} \\ \text{non-perturbative (classical sol's)} \\ \text{non-perturbative (branes)} \\ \ldots \end{cases}$$

⋆ Some caveats:

- Here consider *large* black holes in AdS
- Boundary QFT captures *all* states in AdS

Strategies

Count states in boundary QFT employing a grand canonical partition function

$$\mathcal{I}(y) = \sum_{\text{states}} y^Q = \sum_{\text{charges } Q} d(Q)\, y^Q$$

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$$d(Q) = \frac{1}{2\pi i} \oint \frac{dy}{y^{Q+1}}\, \mathcal{I}(y) = \oint d\Delta \;\; e^{\log \mathcal{I}(\Delta) - 2\pi i Q \Delta} \qquad\qquad y = e^{2\pi i \Delta}$$

Assuming large degeneracies, saddle-point approximation $\rightarrow$ Legendre transform

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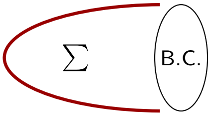
- Euclidean:

$$\mathcal{I} = Z_{M_{d-1} \times S^1} \overset{\text{AdS/CFT}}{=}$$

Euclidean "gravitational path integral" with fixed boundary conditions



Partition function at strong coupling: very hard!

⋆ Employ a SUSY partition function, or index: $$\mathcal{I} = \sum_{\text{states}} (-1)^F \; y^Q$$

Ofter computable *exactly* with localization techniques

Index counts BPS states: applicable to BPS black holes

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⋆ Does an index capture the *full* entropy?

At least at leading order, yes!

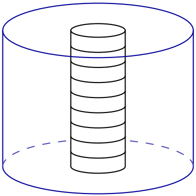
[FB, Hristov, Zaffaroni 16]
[Cabo-Bizet, Cassani, Martelli, Murthy 18]
[Boruch, Heydeman, Iliesiu, Turiaci 22]

Exploit near-horizon AdS_2 with $\mathfrak{su}(1,1|1)$ isometry ($\mathcal{I}$-extremization)

Similar to asyptotically-flat black holes [Sen 09]

Confirmed by analysis of 2d effective action [Boruch, Heydeman, Iliesiu, Turiaci 22]
[see L. Iliesiu's talk]

- Which SUSY partition function?



Holography: black hole solution as an RG flow

AdS/CFT rules $\rightarrow$ read off background for boundary theory

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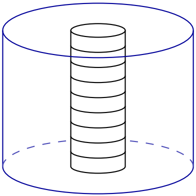

BPS black holes with R-symmetry magnetic charge	BPS Kerr-Newman rotating black holes
(possibly rotating and with electric/magnetic flavor charges)	(possibly with electric and magnetic flavor charges)
↓	↓
topologically twisted indices	superconformal indices

Another interesting class of solutions: spindles

[see J. Sparks' talk]

Magnetically-charged

BPS black holes

in AdS_4

BPS Magnetically-Charged Black Holes in AdS$_4$

⋆ **Spherically symmetric, static, $\frac{1}{16}$-BPS black holes**

[Cacciatori, Klemm 09; Hristov, Vandoren 10]

$$ds^2 = -f(r)\,dt^2 + \frac{1}{f(r)}\,dr^2 + g(r)\,ds^2_{S^2}$$

$$X^i = X^i(r)\,, \quad F^{a=1,2,3,4} = \mathfrak{n}_a\,d\mathrm{vol}_{S^2}\,, \quad \sum \mathfrak{n}_a = 2$$

⋆ Constructed in: 4d $\mathcal{N}=2$ $U(1)^4$ gauge SUGRA (STU model)

or uplift to: 11d supergravity (M-theory) on AdS$_4 \times S^7$

⋆ Magnetic charge for an **R-symmetry** (+ possibly electric flavor charges $\mathfrak{q}_a$ & angular momentum)

Bekenstein-Hawking entropy:

$$S_{BH} = N^{\frac{3}{2}}\,F(\mathfrak{n}_a) \qquad \ell^2_{\text{AdS}}/G_N \sim N^{\frac{3}{2}}$$

⋆ Boundary theory: 3d $\mathcal{N}=8$ ABJM gauge theory $U(N)_1 \times U(N)_{-1}$

Boundary theory is topologically twisted

Topologically Twisted Index

Grand canonical partition function at strong coupling:

protected observable for 3d $\mathcal{N} = 2$ SUSY gauge theories with R-symmetry

$$Z_{\mathsf{TTI}}[y_a, \mathfrak{n}_a] = \mathrm{Tr}_{\mathcal{H}}\ (-1)^F\ e^{-\beta H}\ e^{iJ^a A_a^{\mathsf{bkgd}}}$$

[Gukov, Pei 15]
[FB, Zaffaroni 15]
[Closset, Kim 16]

$\mathcal{H}$: Hilbert space of states on S^2 with R-symmetry background (top. twist)

$\mathcal{Q}^2 = H - m_a^{\mathsf{bkgd}} J^a$ only BPS states with $\mathcal{Q}^2 = 0$ contribute

Complex fugacities: $y_a = e^{i\Delta_a} = e^{i(A_a^{\mathsf{bkgd}} + i\beta m_a^{\mathsf{bkgd}})}$

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⋆ Computable exactly with localization techniques. For ABJM:

$$Z = \frac{1}{(N!)^2} \sum_{\mathfrak{m},\widetilde{\mathfrak{m}}\in\mathbb{Z}^N} \int_{\mathcal{C}} \prod_{i=1}^N \frac{dx_i}{2\pi i x_i}\frac{d\tilde{x}_i}{2\pi i \tilde{x}_i}\, x_i^{\mathfrak{m}_i}\, \tilde{x}_i^{-\widetilde{\mathfrak{m}}_i} \times \prod_{i\neq j}^N \left(1-\frac{x_i}{x_j}\right)\left(1-\frac{\tilde{x}_i}{\tilde{x}_j}\right)\times$$

$$\times \prod_{i,j=1}^N \prod_{a=1,2}\left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j}\, y_a}}{1-\frac{x_i}{\tilde{x}_j}\, y_a}\right)^{\mathfrak{m}_i-\widetilde{\mathfrak{m}}_j-\mathfrak{n}_a+1} \prod_{b=3,4}\left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i}\, y_b}}{1-\frac{\tilde{x}_j}{x_i}\, y_b}\right)^{\widetilde{\mathfrak{m}}_j-\mathfrak{m}_i-\mathfrak{n}_b+1}$$

TT Index at Large

⋆ Compute contour integral as a multi-dimensional residue [FB, Hristov, Zaffaroni 15]

Distribution of poles at large N: "Bethe Ansatz Equations"

$$1 = x_i \prod_{j=1}^{N} \frac{\left(1 - y_3 \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_4 \frac{\tilde{x}_j}{x_i}\right)}{\left(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i}\right)} = \tilde{x}_j \prod_{i=1}^{N} \frac{\left(1 - y_3 \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_4 \frac{\tilde{x}_j}{x_i}\right)}{\left(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i}\right)\left(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i}\right)}$$

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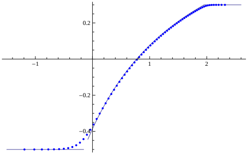
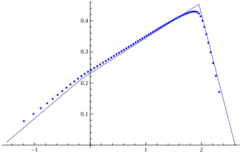
From a numerical analysis, ansatz: $\log x_j = \sqrt{N}\, t_j + i\, v_j$

⋆ Use a continuous distribution of poles:

$v_j(t) - \tilde{v}_j(t)$:



$\rho(t)$:



Partition Function and Entropy

Grand canonical partition function, at leading order in large N:

$$\log Z_{\mathsf{TTI}}(\Delta_a, \mathfrak{n}_a) = -\frac{N^{3/2}}{3}\sqrt{2\Delta_1\Delta_2\Delta_3\Delta_4}\sum_a \frac{\mathfrak{n}_a}{\Delta_a} \;+\; \ldots$$

Here $y_a = e^{i\Delta_a}$, $\quad 0 \leq \Delta_a \leq 2\pi$ and $\sum \Delta_a = 2\pi$.

Bekenstein-Hawking entropy from a (constrained) Legendre transform:

$$\log Z(\Delta_a, \mathfrak{n}_a) - i\Delta_a \mathfrak{q}_a \Big|_{\Delta_a = \mathsf{crit}} \;=\; S_{\mathsf{BH}}(\mathfrak{q}_a, \mathfrak{n}_a)$$

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⋆ $\mathcal{I}$-extremization: Legendre transform is dual to attractor mechanism in AdS_4

[Gauntlett, Martelli, Sparks 19][Hosseini, Zaffaroni 19]

[Kim, Kim 19][van Beest, Cizel, Schafer-Nameki, Sparks 20]

Many Examples in Various Dimensions

⋆ Similar strategies reproduce the Bekenstein-Hawking entropy of various types of BPS black holes in different dimensions, from both twisted and superconformal indices

Magnetically-charged BPS black holes: (VERY partial list!)

- in AdS_4, with addition of electric charges [FB, Hristov, Zaffaroni 16]
  and/or angular momentum [Hristov, Katmadas, Toldo 18; Choi, Hwang 19]
- with exotic horizon Σ_g [FB, Zaffaroni 16][Closset, Kim 16]
- in other theories / geometries (e.g. massive Type IIA on S^6) [Hosseini, Hristov, Passias 17][FB, Khachatryan, Milan 17] [Hosseini, Zaffaroni 16] [Gang, Kim, Pando Zayas 19][Bobev, Crichigno 19]
- in AdS_5 with hyperbolic horizon [Bae, Gang, Lee 19]
- in AdS_6 with toric-Kahler or $\Sigma_{g_1} \times \Sigma_{g_2}$ horizon [Hosseini, Yaakov, Zaffaroni 18] [Crichigno, Jain, Willett 18][Suh 18] + Hristov, Passias, Fluder, Uhlemann]

Many Examples in Various Dimensions

Rotating Kerr-Newman BPS black holes:

- in AdS_4 [Choi, Hwang, Kim 19][Nian, Pando Zayas 19][Choi, Hwang 19]
- in AdS_5 (see later) [many...]
- in AdS_6 (Cardy limit) [Choi, Kim 19]
- in AdS_7 (Cardy limit) [Kantor, Papageorgakis, Richmond 19][Nahmgoong 19]

Perturbative and Higher-Derivative Corrections

⋆ In QFT, we can compute corrections to the TT index at large N.

Analytic computations turn out to be too difficult, [Liu, Pando Zayas, Rathee, Zhao 17]

→ resort to numerical evaluations and fitting: [Bobev, Hong, Reys 22]

$$\log Z_0 = -\frac{N^{3/2}}{3}\sqrt{2\Delta_1\Delta_2\Delta_3\Delta_4}\sum_a \frac{\mathfrak{n}_a}{\Delta_a}$$

$$\log Z = \log Z_0 \ + \ N^{\frac{1}{2}} f_1(\Delta_a, \mathfrak{n}_a) \ - \ \frac{1}{2}\log N \ + \ f_2(\Delta_a, \mathfrak{n}_a) \ + \ \mathcal{O}\big(N^{-\frac{1}{2}}\big)$$

In $SO(8)$-symmetric case $\Delta_a = \frac{\pi}{2}$, for generic horizon Σ_g and internal manifold $S^7/\mathbb{Z}_k$:

$$\log Z \ = \ (1-g)\left[-\frac{\pi\sqrt{2k}}{3}N^{\frac{3}{2}} \ + \ \frac{\pi}{\sqrt{2k}}\frac{k^2+32}{24}N^{\frac{1}{2}} \ - \ \frac{1}{2}\log N\right] \ + \ \dots$$

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Perturbative 1-Loop Correction

[Liu, Pando Zayas, Rathee, Zhao 17]

- ⋆ $\log$ corrections to entropy have been extensively studied for asymptotically-flat black holes [Sen 08]

Window into QG: 1-loop effect from matter fields in near-horizon region

- ⋆ For AdS_4 black holes: contribution from *whole space* [Jeon, Lal 17] [Liu, Pando Zayas, Rathee, Zhao 17]

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Computation in 11d SUGRA on $M_4 \times S^7$: (M_4 is reg. Euclidean black hole)

- in odd dimensions, only zero-modes contribute
- examine fields of 11d SUGRA, including ghosts
- M_4 is non-compact $\rightarrow$ space of L^2 harmonic forms $\mathcal{H}^p_{L^2}(M_4, \mathbb{R})$
  $\dim_{\text{reg}} \mathcal{H}^{p=2}_{L^2} = 2(1-g)$

Reproduces $\log Z_{\Sigma_g \times S^1}(\mathfrak{n}_a, \Delta_a) = \ \dots\ - \dfrac{1-g}{2} \log N\ +\ \dots$

First Higher-Derivative Correction

[Bobev, Charles, Hristov, Reys 20/21]

- ⋆ Add 4-derivative corrections to 4d $\mathcal{N} = 2$ minimal gauged SUGRA

Higher-derivative couplings: Weyl multiplet, T-log multiplet

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- Any solution to 2-∂ action is also a solution of 4-∂ action, and preserves same amount of SUSY (special to AdS_4)

$$\log Z = -\pi\,\mathcal{F}\left(AN^{\frac{3}{2}} + BN^{\frac{1}{2}}\right) + \pi\left(\mathcal{F}-\chi\right)C\,N^{\frac{1}{2}}$$

- $\mathcal{F},\chi$ depend on boundary geometry of asymtpotically-locally-AdS_4 solution

  Mag. AdS BH: $\mathcal{F}=(1-g)$ $\chi=2(1-g)$

- A,B,C depend on theory: fix them with selected localization computations

  $ABJM_k$: $A=\frac{\sqrt{2k}}{3}$ $B=-\frac{k^2+8}{24\sqrt{2k}}$ $C=-\frac{1}{\sqrt{2k}}$

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    $ABJM_k$: $A=\frac{\sqrt{2k}}{3}$ $B=-\frac{k^2+8}{24\sqrt{2k}}$ $C=-\frac{1}{\sqrt{2k}}$

⋆ Grav. evaluation of TT index:

$$\log Z_{\Sigma_g\times S^1} = -(1-g)\frac{\pi\sqrt{2k}}{3}\left(N^{\frac{3}{2}} - \frac{32+k^2}{16k}\,N^{\frac{1}{2}}\right) + \ \ldots$$

⋆ In order to address generic fugacities, need to add vector multiplets

An interesting conjecture

By a very careful numerical analysis, [Bobev, Hong, Reys 22]

conjecture for TT index of ABJM$_k$, to all perturbative orders in $\frac{1}{N^{1/2}}$:

$$\log Z_{S^1\times\Sigma_{\mathfrak{g}\neq 1}} = -\frac{\sqrt{2k\Delta_1\Delta_2\Delta_3\Delta_4}}{3}\sum_{a=1}^{4}\frac{\mathfrak{n}_a}{\Delta_a}\left(\hat{N}_\Delta^{\frac{3}{2}} - \frac{\mathfrak{c}_a}{k}\,\hat{N}_\Delta^{\frac{1}{2}}\right) - \frac{1-\mathfrak{g}}{2}\,\log\hat{N}_\Delta$$

$$+ f_0(k,\Delta,\mathfrak{n}) \quad + \mathcal{O}\big(e^{-\sqrt{Nk}}\big) + \mathcal{O}\big(e^{-\sqrt{N/k}}\big)$$

where
$$\hat{N}_\Delta = N - \frac{k}{24} + \frac{\pi}{12k}\sum_{a=1}^{4}\frac{1}{\Delta_a} \qquad \mathfrak{c}_a = \frac{\prod_{b(\neq a)}(\Delta_b+\Delta_a)}{8\Delta_1\Delta_2\Delta_3\Delta_4}\sum_{b(\neq a)}\Delta_b$$

Kerr-Newman rotating

BPS black holes

in AdS_5

Kerr–Newman BPS black holes in AdS_5

Rotating & electrically-charged $\frac{1}{16}$-BPS black holes in AdS_5 [Gutowski, Reall 04]
[Chong, Cvetic, Lu, Pope 05][Kunduri, Lucietti, Reall 06]

- Constructed in: 5d $\mathcal{N} = 2\ U(1)^3$ gauged SUGRA (STU model)
  or uplift to: 10d type IIB SUGRA on $AdS_5 \times S^5$

- Angular momentum — Here: J_1, J_2
  Electric charges — Charges for $U(1)^3 \subset SO(6)$: R_1, R_2, R_3

- SUSY (1 cplx supercharge $\mathcal{Q}$)

- Bekenstein-Hawking entropy (S^3 horizon):

$$S_{\text{BH}} = \frac{\text{Area}}{4G_N} = \pi\sqrt{R_1 R_2 + R_1 R_3 + R_2 R_3 - 2N^2(J_1 + J_2)}$$

- Angular momenta, charges and entropy scale $\sim N^2 \sim \frac{\ell^3_{\text{AdS}}}{G_{\text{N}}}$

4d Superconformal Index

[Romelsberger 05; Kinney, Maldacena, Minwalla, Raju 05]

- Dual boundary theory: 4d $\mathcal{N} = 4$ $SU(N)$ SYM

⋆ Superconformal index:

Counts (with sign) BPS states on S^3 = protected operators on flat space

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- Dual boundary theory: 4d $\mathcal{N} = 4$ $SU(N)$ SYM

⋆ Superconformal index:

Counts (with sign) BPS states on S^3 = protected operators on flat space

Index of $\mathcal{N} = 4$ SYM:

$$\mathcal{I}(p, q, y_1, y_2) = \mathrm{Tr}\,(-1)^F\, e^{-\beta\{\mathcal{Q}, \mathcal{Q}^\dagger\}}\, p^{J_1 + \frac{1}{2}R_3}\, q^{J_2 + \frac{1}{2}R_3}\, y_1^{\frac{1}{2}(R_1 - R_3)}\, y_2^{\frac{1}{2}(R_2 - R_3)}$$

Write: $p = e^{2\pi i \tau}$ $\quad q = e^{2\pi i \sigma}$ $\quad y_a = e^{2\pi i \Delta_a}$

Introduce Δ_3 such that: $\Delta_1 + \Delta_2 + \Delta_3 - \tau - \sigma \in \mathbb{Z}$

⋆ Equals the Euclidean partition function on $S^3 \times S^1$ with background flat connections:

$$\mathcal{I} = Z_{S^3 \times S^1}(\tau, \sigma, \Delta_1, \Delta_2)$$

★ *Exact* integral formula: [Aharony, Marsano, Minwalla, Papadodimas, Van Raamsdonk 03] [Sundborg 99][Romelsberger 05][Kinney, Maldacena, Minwalla, Raju 05]

$$\mathcal{I} = \kappa_N \oint_{\mathbb{T}^{\mathrm{rk}(G)}} \prod_{i=1}^{\mathrm{rk}(G)} \frac{dz_i}{2\pi i z_i} \times \frac{\prod_{a=1}^{3} \prod_{\rho \in \mathfrak{R}_{\mathsf{adj}}} \widetilde{\Gamma}\big(\rho(u) + \Delta_a; \tau, \sigma\big)}{\prod_{\alpha \in \mathfrak{g}} \widetilde{\Gamma}\big(\alpha(u); \tau, \sigma\big)}$$

with

$$z = e^{2\pi i u} \qquad \kappa_N = \frac{(p;p)_\infty^{\mathrm{rk}(G)} (q;q)_\infty^{\mathrm{rk}(G)}}{|\mathsf{Weyl}_G|} \qquad \widetilde{\Gamma}(u;\tau,\sigma) = \prod_{m,n=0}^{\infty} \frac{1 - p^{m+1} q^{n+1}/z}{1 - p^m q^n z}$$

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★ Taking the large N limit turns out to be tricky...

One saddle point with $u_i \in \mathbb{R}$ found long ago [Kinney, Maldacena, Minwalla, Raju 05]
it describes gas of gravitons in AdS_5, but not black holes

- Many different approaches have been devised by now

Cardy Limit

[Choi, Kim, Kim, Nahmgoong 18][Honda 19][Arabi Ardehali 19]

⋆ Integrand simplifies in a (high temperature) Cardy limit:

- angular chemical potentials $\tau, \sigma \to 0$ (with τ/σ fixed)
- electric chemical potentials $\mathbb{Im}\,\Delta_a \to 0$ with $\mathbb{Re}\,\Delta_a$ fixed

$$\mathcal{I} = Z_{S^3 \times S^1} \underset{\tau,\sigma \to 0}{\simeq} \int d^{\mathrm{rk}(G)} u \;\; e^{\frac{i\pi}{6\tau\sigma} V_2(u) + \frac{i\pi(\tau+\sigma)}{2\tau\sigma} V_1(u)}$$

$V_{1,2}(u)$: piecewise polynomial functions, that depends on gauge/matter rep.

In a suitable range of $\mathbb{Re}\,\Delta_a$'s, V_2 has a local minumum at $u = 0$.

[Di Pietro, Komargodski 14][Arabi Ardehali 15][Di Pietro, Honda 16]

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⋆ Next, take large N limit:

(here $[x] = x - \lceil x \rceil$)

$$\log \mathcal{I} = -i\pi N^2 \frac{[\Delta_1][\Delta_2][\Delta_3]}{\tau\sigma} + \dots$$

⋆ (Constrained) Legendre transform reproduces Bekenstein-Hawking entropy:

$$S_{\mathsf{BH}} = \log \mathcal{I} - 2\pi i \left(\sum X_a \tfrac{R_a}{2} + 2\tau J \right) \Big|_{\substack{\text{constrained}\\ \text{extremum}}}$$

Cardy limit captures limit of: charges $\gg$ central charge

Cardy limit of $\mathcal{N}=1$ theories can be studied at generic N, and in greater detail.

- ⋆ In 2d, Cardy limit follows from modular invariance
- ⋆ In 4d no modular invariance, yet central charges control the Cardy limit

  Obtained by reduction on S^1, careful treatment of
  3d EFT of massive and zero modes, involves $SU(N)_N$

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  Obtained by reduction on S^1, careful treatment of
  3d EFT of massive and zero modes, involves $SU(N)_N$

- E.g., for the so-called "R-charge index", or "index on 2nd sheet":

$$\mathcal{I} = \mathrm{Tr}\ e^{-i\pi R}\ e^{-\beta\{\mathcal{Q},\mathcal{Q}^\dagger\}}\ e^{2\pi i\tau\left(J_1+\frac{1}{2}R\right)}\ e^{2\pi i\sigma\left(J_2+\frac{1}{2}R\right)}$$

where R is an R-symmetry (with mild constraints), [Cassani, Komargodski 21]

[see also: Kim, Kim, Song 19; Cabo-Bizet, Cassani, Martelli, Murthy 19; Lezcano, Hong, Liu, Pando Zayas 20]
[Amariti, Fazzi, Segati 21; Jejjala, Lei, van Leuven, Li 21; Cabo-Bizet 21][Arabi Ardehali, Murthy 21]

$$\log\mathcal{I} = \pi i\,\frac{(\tau+\sigma+1)^3}{24\tau\sigma}\,\mathrm{Tr}\,R^3 - \pi i\,\frac{(\tau+\sigma+1)(\tau^2+\sigma^2-1)}{24\tau\sigma}\,\mathrm{Tr}\,R$$
$$+\log\left|G_{\text{1-form}}\right| + \mathcal{O}\left(e^{-\frac{\#}{\tau}}\right)$$

- $\mathcal{N}=4$ SYM: Asymptotic behaviour for $\tau,\sigma\to\mathbb{Q}$ [Arabi Ardehali, Murthy 21]

⋆ Relax any limit on fugacities, only large N

This analysis reveals interesting *non-perturbative* corrections

Various tools:

- Bethe Ansatz formula [Closset, Kim, Willet 17; FB, Milan 18] [Lanir, Nedelin, Sela 19; Lezcano, Pando Zayas 19] [Arabi Ardehali, Hong, Liu 19; FB, Colombo, Soltani, Zaffaroni, Zhang 20]
- Non-analytic extension [Cabo-Bizet, Murthy 19] [Cabi-Bizet, Cassani, Martelli, Murthy 20; Cabo-Bizet 20]
- Direct saddle-point approximation [Choi, Jeong, Kim, Lee 21]
- Truncation of plethystic expansion [Copetti, Grassi, Komargodski, Tizzano 20] [Choi, Jeong, Kim 21]
- Giant graviton expansion [Imamura 21/22; Gaiotto, Lee 21] [Murthy 22; Lee 22]

Bethe Ansatz Formula for 4d Superconformal Index

Alternative formula: (set $\tau = \sigma$)

[Closset, Kim, Willett 17]
[FB, Milan 18]
[FB, Rizi 21]

$$\mathcal{I} = \sum_{u \,\in\, \mathfrak{M}_{\mathsf{BAE}}} \mathcal{Z}(u; \Delta, \tau, \tau)\; H(u; \Delta, \tau)^{-1}$$

1. $\mathfrak{M}_{\mathsf{BAE}}$ are solutions to "Bethe Ansatz Equations" for $\mathrm{rk}(G)$ complexified holonomies $[u_i]$ living on a complex torus T^2_τ of modular parameter τ:

$\mathfrak{M}_{\mathsf{BAE}}$:
$SU(N)$ $\mathcal{N}=4$ SYM

$$Q_i(u) = \prod_{a=1}^{3} \prod_{j=1}^{N} \frac{\theta(\Delta_a - u_{ij}; \tau)}{\theta(\Delta_a + u_{ij}; \tau)} = 1$$

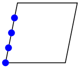
$u_{ij} =$
$u_i - u_j \neq 0$

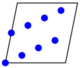
Equations are defined on T^2_τ and are invariant under $SL(2,\mathbb{Z})$

2. $\mathcal{Z}$: same integrand as in integral formula H: Jacobian $H = \det_{ij} \frac{\partial Q_i}{\partial u_j}$

⋆ ∃ *Discrete* family of exact solutions [Hosseini, Nedelin, Zaffaroni 16][Hong, Liu 18]

labelled by $\{m, r\}$ with $m \cdot n = N$ and $r \in \mathbb{Z}_n$

- BASIC SOLUTION $\{1, 0\}$: $u_j \sim \frac{\tau}{N}\, j$ 

- Other solutions – forming $SL(2, \mathbb{Z})$ orbits: *e.g.* 

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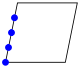
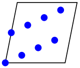
- BASIC SOLUTION $\{1, 0\}$: $u_j \sim \frac{\tau}{N}\, j$ 

- Other solutions – forming $SL(2, \mathbb{Z})$ orbits: *e.g.* 

★ Contrib. of BASIC SOLUTION reproduces Bekenstein-Hawking entropy:

[FB, Milan 18]

$$\lim_{N\to\infty} \mathcal{I}(\tau, \Delta_1, \Delta_2)\Big|_{\substack{\text{BASIC}\\ \text{SOLUTION}}} \simeq \exp\left(-i\pi N^2 \, \frac{[\Delta_1]_\tau \, [\Delta_2]_\tau \, [\Delta_3]_\tau}{\tau^2} \right)$$

Non-Perturbative Corrections from QFT

Expansion of the index at large N: $$\mathcal{I} = \sum_{\text{solutions} \in \mathfrak{M}_{\text{BAE}}} e^{\mathcal{O}(N^2) + \cdots}$$

It looks like a semiclassical expansion

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⋆ Large N contribution of $\{m, r\}$ solutions (with fixed m, r): (cfr. Cardy limit)

$$\log \mathcal{I}_{\{m,r\}} = \underbrace{-\frac{i\pi N^2}{m} \frac{[m\Delta_1]_{\check{\tau}}[m\Delta_2]_{\check{\tau}}[m\Delta_3]_{\check{\tau}}}{(m\tau + r)^2}}_{\text{on-shell action}} + \underbrace{\log N}_{\text{1-loop ?}} + \mathcal{O}(1)$$

$$+ \underbrace{\sum e^{\frac{2\pi i N}{m} \frac{[m\Delta_a]_{\check{\tau}}}{\check{\tau}} + \dots}}_{\text{Euclidean D3-branes}} + \dots$$

where $\check{\tau} = m\tau + r$ [Lezcano, Hong, Liu, Pando Zayas 20][Aharony, FB, Mamroud, Milan 21]

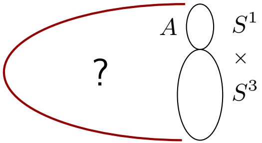
“Classical” Non-Perturbative Corrections

Fill-in bulk geometry

for given boundary conditions

[Witten 98; Dijkgraaf, Maldacena, Moore, Verlinde 00]

[Maloney, Witten 07]



- $\exists$ infinite family of complex Euclidean solutions (including orbifolds) of 10d type IIB supergravity

  ⋆ SUSY, but *not* extremal,

  with correct boundary conditions

  [Cabo-Bizet, Cassani, Martelli, Murthy 18]

  [Aharony, FB, Mamroud, Milan 21]

- (Renormalized) on-shell action F_{grav}

  Reproduces $\mathcal{O}(N^2)$ contribution to $\log \mathcal{I}_{\{m,r\}}$

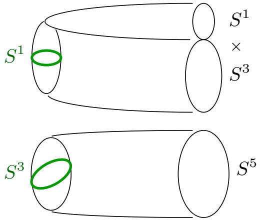
"Stringy" Non-Perturbative Corrections

[Aharony, FB, Mamroud, Milan 21]

- ⋆ A class of non-perturbative corrections from Euclidean SUSY D3-branes wrapped on 10d geometry at the horizon

  On-shell action:

$$S_{\mathrm{D3}} = 2\pi N \, \frac{\Delta_a^{\mathrm{g}}}{\tau^{\mathrm{g}}} \qquad \text{or} \qquad S_{\mathrm{D3}} = 2\pi N \, \frac{\Delta_a^{\mathrm{g}}}{\sigma^{\mathrm{g}}}$$



- ⋆ Effect of D3-brane corrections:

$$\mathcal{I} = Z_{S^3\times S^1} \;\simeq\; e^{-F_{\mathrm{grav}}} + \sum_k e^{-F_{\mathrm{grav}}}\, e^{ikS_{\mathrm{D3}}} \;\simeq\; \exp\Big\{ -\underbrace{F_{\mathrm{grav}}}_{\mathcal{O}(N^2)} + \sum_k \underbrace{e^{ikS_{\mathrm{D3}}}}_{\mathcal{O}(e^{-N})} \Big\}$$

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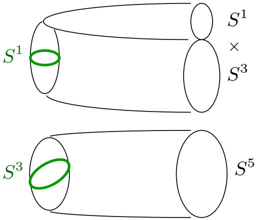

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⋆ Criterium to retain a complex saddle:

$$\mathbb{I}\mathrm{m}\, S_{\mathrm{D3}} > 0 \qquad \text{for all (SUSY) D3-brane embeddings}$$

Violation implies "D3-brane condensation" towards some other saddle point.
Expected to signal that complex saddle point does *not* contribute to the integral.
Matches with expansion of index.

Perturbative and Higher-Derivative Corrections

Perturbative and higher-derivative corrections poorly understood in this example

⋆ $\log N$ term in BASIC SOLUTION [David, Lezcano, Nian, Pando Zayas 21]

Can be reproduced by Kerr/CFT applied to near-horizon

Computation in Lorentizian signature and microcanonical ensemble

Suggests no contribution from the bulk

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⋆ $\mathcal{O}(1)$ terms

Absence of $\mathcal{O}(N)$: first higher-derivative correction vanishes [Melo, Santos 20]

Expected 1-loop contribution from gas of gravitons in black hole background

[*cfr.* Kinney, Maldacena, Minwalla, Raju 05]

Perturbative series truncates (compatible with Cardy limit). Why?

An Interesting Class of Matrix Models

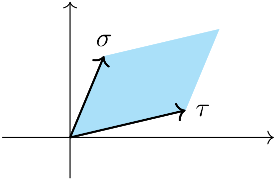
- ⋆ The integral formula for the index can be recast in the form:

$$\mathcal{I} = \frac{1}{N!} \int d^N u_a \prod_{a\neq b} \Big(1 - e^{\frac{2\pi i}{\tau} u_{ab}}\Big) \exp\Big[-\sum_{a\neq b}\Big(V_\sigma(u_{ab}) + V_\tau(u_{ab})\Big)\Big]$$

with V_σ periodic in $u \to u + \sigma$, V_τ periodic in $u \to u + \tau$

[Choi, Jeong, Kim, Lee 21]

At large N, family of saddle points with uniform distribution of eigenvalues on a *parallelogram* in the u-plane



- ⋆ Saddle are labelled by $r, s \in \mathbb{Z}$
  Generalize the $(m = 1, r)$ solutions to BAE's to $\tau \neq \sigma$
  $\exists$ only in certain ranges of τ, σ, Δ_a $\rightarrow$ agreement with D3-brane stability
  At leading order, contributions agree.

In all approaches, there seems to be other potential contributions:

- BAEs: continuous branches of solutions [Arabi Ardehali, Hong, Liu 19]
- Saddle point: multi-cut solutions with filling fractions

Poorly understood.

Hint of new BPS black hole solutions?

⋆ **Truncation of the Plethystic Expansion** [Copetti, Grassi, Komargodski, Tizzano 20]

$$\mathcal{I} = \int_{U \in SU(N)} [DU] \;\; \exp\left(\;\sum_{k=1}^{\infty} \frac{1}{k} \; f(y_a^k, p^k, q^k) \;\; \mathrm{Tr}\, U^k \, \mathrm{Tr}\, U^{-k} \; \right)$$

Truncation to one or two terms:

at large N reproduces fairly well Hawking-Page transition between AdS_5 and black hole

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⋆ Giant graviton expansion [Imamura 21/22; Gaiotto, Lee 21; Murthy 22; Lee 22]

$$\mathcal{I}(\Delta_a, \tau_i) = \mathcal{I}_{\mathsf{KK}}(\Delta_a, \tau_i) \sum_{n_1,n_2,n_3=0}^{\infty} e^{-N\sum_a \Delta_a n_a} \;\; Z_{n_1,n_2,n_3}(\Delta_a, \tau_i)$$

where $\quad Z_{n_1,n_2,n_3} = \oint_{\mathcal{C}} \prod_{I=1}^{3} \prod_{a=1}^{n_I} du_a^{(I)} \left(\prod_{I=1}^{3} Z_I^{\mathsf{4d}} \right) \left(\prod_{I\neq J} Z_{I,J}^{\mathsf{2d}} \right)$ [Imamura 21]

$U(n_1)\times U(n_2)\times U(n_3)$ gauge theory with bifundamentals

- Bulk: n_1, n_2, n_3 D3-branes wrapping three intersecting S^3's in S^5
- Index: $\quad \mathcal{I}(y) = 1 + \;\#y\; + \ldots + \#y^N \; + \ldots$

↑ trace relations

At large N, reproduces black hole entropy [Choi, Kim, Lee, Lee 22]

Quantum mechanics of BPS and near-BPS black holes
can be studied in great detail using near-horizon AdS_2 region
and Schwarzian-like effective field theory [Iliesiu, Turiaci 20]

[Heydeman, Iliesiu, Turiaci, Zhao 20; Boruch, Heydaman, Iliesiu, Turiaci 22]

[Lin, Maldacena, Rozenberg, Shan 22]

[See L. Iliesiu and J. Maldacena's talks]

Can it be reproduced from the boundary QFT?

Some Open Questions

- AdS Black hole entropy beyond SUSY [see Larsen, Nian, Zeng 19]

  Near-horizon JT-like gravity from field theory?

- Classification of Euclidean/Lorentzian BPS gravitational saddles

  Index: potential new contributions

  Gravity: indications of hairy or multi-center BPS black holes

  [Markeviciute, Santos 18]

  [Monten, Toldo 21; Cai, Liu 22]

- Resummation of contributions and phase transitions

  [see Copetti, Grassi, Komargodski, Tizzano 20]