On Towers of Light States at Infinite Distance

Kahler Moduli:

1901.08065, 1910.01135 w/ Wolfgang Lerche, Timo Weigand

2011.00024 w/ Daniel Klawer, T. Weigand, Max Wiesner

Complex Structure Moduli (incl. Brane Moduli):

2112.08385 w/ W. Lerche, T. Weigand

2112.07682 w/ T. Weigand

Seung-Joo Lee

ibs Institute for Basic Science

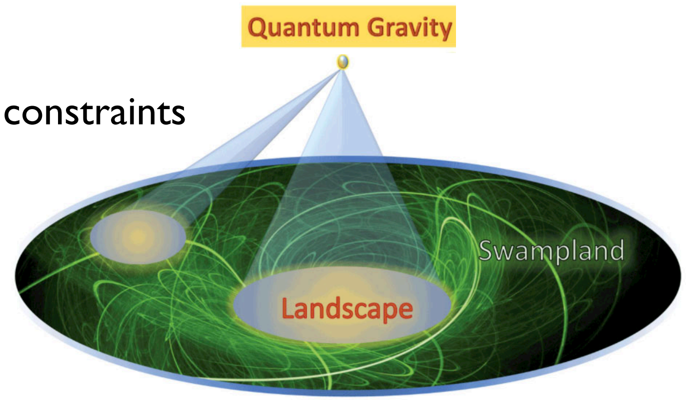
Strings 2022, University of Vienna

22-07-2022

Motivation

Quantum Gravity and String Theory

- **Swampland Conjectures**
  - Which effective field theories (EFTs) couple to Quantum Gravity?
    - **Swampland v.s. Landscape**
  - EFTs in the Landscape subject to **universal** consistency constraints
    - **Swampland Conjectures**
      ⋯⋯▸ general, useful, but **not** fully understood



picture from [Palti '19]

- **Stringy Realization**
  - Quantitative verification of the explicit conjectures
  - Manifestations in string geometry
  - Refinement

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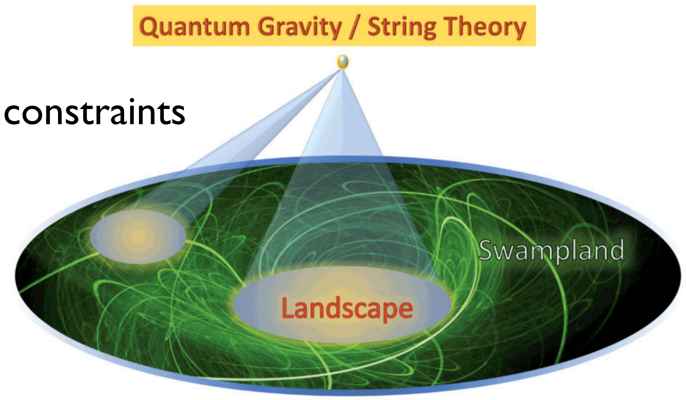

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Swampland Conjectures

Examples

Reviewed e.g. in
[Brennan, Carta, Vafa '17], [Palti '19], [Grana, Herraez '21]
[van Beest, Calderon-Infante, Mirfendereski, Valenzuela '21]

Cobordism Conjecture
[McNamara, Vafa, '19]

AdS Distance Conjecture
[Lust, Palti, Vafa, '19]

No Global Symmetry
[Banks, Dixon, '88], [Harlow, Ooguri, '18]

Completeness
[Polchinski '03]

Distance Conjecture
[Ooguri, Vafa, '06]

Emergent String Conjecture
[S.-J.L., Lerche, Weigand '19]

Trans-Planckian Censorship
[Bedroya, (Brandenberger, Loverde,) Vafa, '19]

sub-Lattice WGC
[Heidenreich, Reece, Rudelius, '16]
[Montero, Shiu, Soler, '16]

Scalar WGC
[Palti, '17] [S.-J.L., Lerche, Weigand '18]
[Heidenreich, Reece, Rudelius,'19]

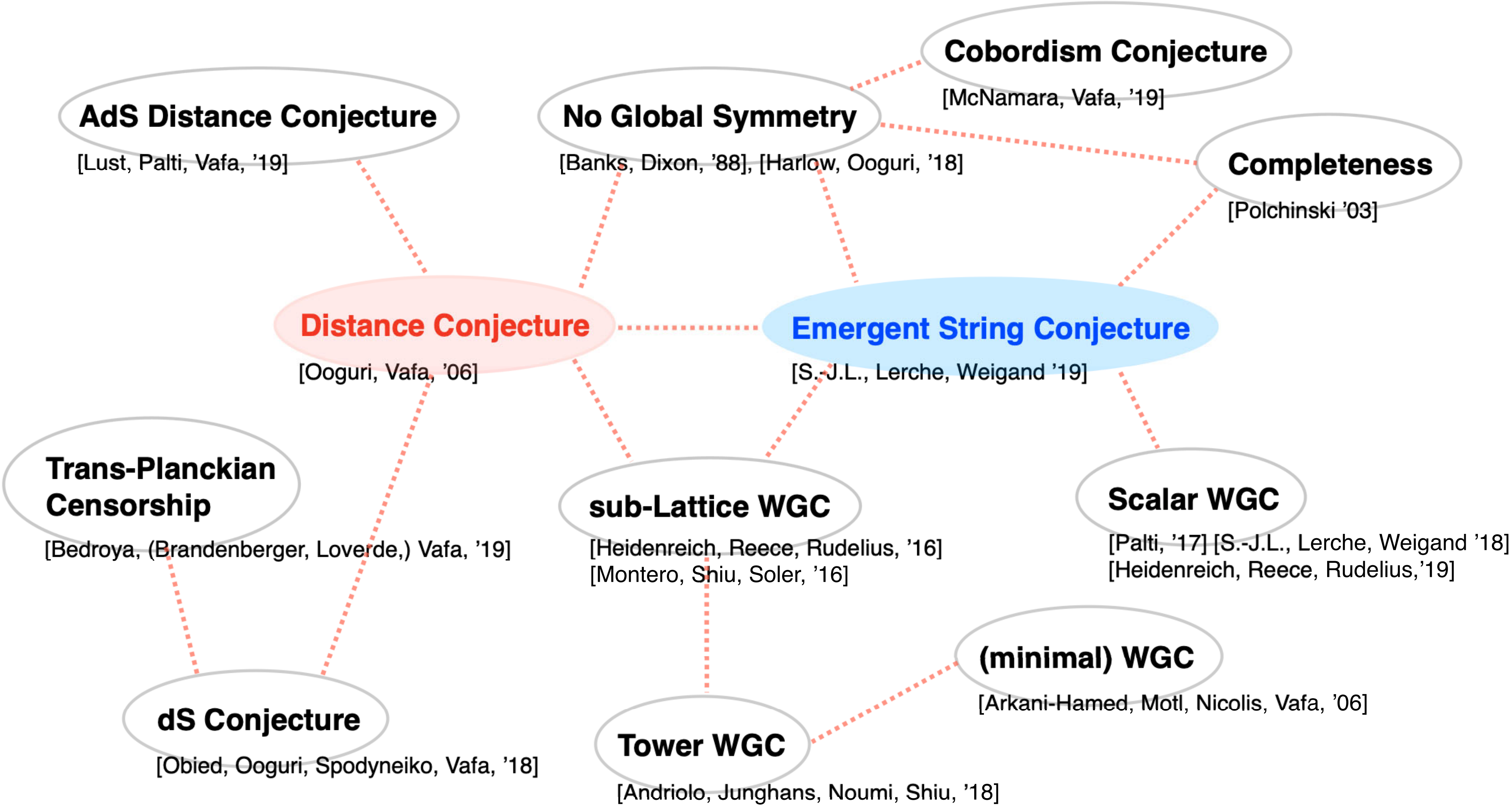
(minimal) WGC
[Arkani-Hamed, Motl, Nicolis, Vafa, '06]

dS Conjecture
[Obied, Ooguri, Spodyneiko, Vafa, '18]

Tower WGC
[Andriolo, Junghans, Noumi, Shiu, '18]

Swampland Conjectures

Charting the Conjectures



String EFTs at Infinite Distance

Prediction of the Distance Conjecture

- **Moduli of String Compactifications**
  - Scalar fields parametrizing the EFTs
  - Universal behaviors of the EFTs at infinite distance?

- **Distance Conjecture** [Ooguri, Vafa '06] (cf.) [Baume, Palti '16], [Klawer, Palti '16]

$$S = \int d^4x \sqrt{-G} \left(M_{\rm Pl}^2 R - (\partial\phi)^2 + \cdots \right)$$

A tower of states become light at infinite distance: $m_0 \sim e^{-\alpha \frac{\Delta\phi}{M_{\rm Pl}}} M_{\rm Pl}$ (for $\Delta\phi > M_{\rm Pl}$)

  - Confirmed in various string setups
  - If true, what is the very nature of the light tower?

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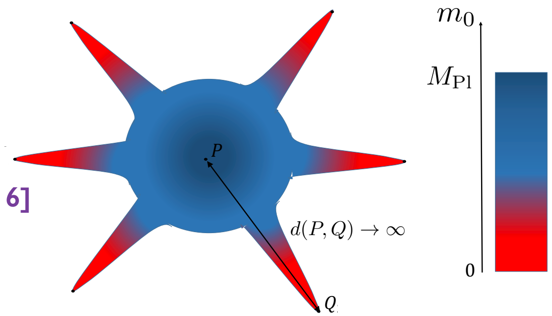

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Nature of the Light Tower

Emergent String Conjecture [S.-J.L., Lerche, Weigand '19]

At infinite distance in moduli space a quantum gravity theory *either* ***decompactifies*** *or* reduces to a ***weakly coupled string theory***

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- Evidence from non-trivial string setups:

Kahler Moduli (SIZE)

"emergence of **unique** critical tensionless string!"

F/M/IIA theory in 6/5/4d [S.-J.L., Lerche, Weigand '18-'20]

IIA/IIB hyper moduli in 4d [(Baume,) Marchesano, Wiesner '19]

M-theory in 4d [Xu '20]

F-theory in 4d, classical & quantum

[S.-J.L., Lerche, Weigand '19], [Klawer, S.-J.L., Weigand, Wiesner '20]

Complex Structure Moduli (SHAPE)

"decompactification via (dual) **KK-like** tower!"

Type II theory in 4d (closed string sector)

[Grimm, Palti, Valenzuela '18], [Grimm, Li, Palti '18],

[Klemm, Joshi '19], [Grimm, Li, Valenzuela '19], …

F-theory in 8d (open string sector)

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Part I.

Kahler Moduli of F-theory

[S.-J.L., Lerche, Weigand '19], [Klawer, S.-J.L., Weigand, Wiesner '20]

- Previous Analysis in F-theory:

  6d N=(1,0) EFTs Talk by Weigand@Strings 19

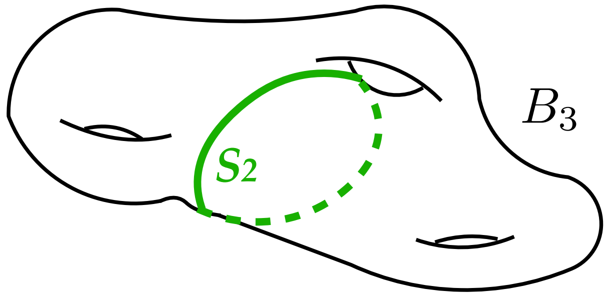
- New Development:

  4d N=1 EFTs (w/ quantum effects incorporated)

F-theory in 4 Dimensions

Couplings via Kahler Moduli

- **4d F-theory**
  - IIB string theory on compact 3-fold B_3
    w/ 7-branes on a divisor S2 ◄·····► 4d gauge fields
    - dilaton profile via an ***elliptic fibration*** $Y_4 \to B_3$
    - fluxes can be turned on



- **Moduli Space**
  - Kahler parameters: $\mathcal{J} = \sum_i \tau^i J_i \quad \in H^2(B_3, \mathbb{R})$
  - Govern the cycle volumes and in turn, the couplings
    - gravity: $(M_{\rm Pl}/M_{\rm IIB})^2 = 4\pi \mathcal{V}_{B_3}$
    - gauge: $1/g^2 = (2\pi)^{-1} \mathcal{V}_{\bf S}$

Characterization of Infinite Distance Limits

Overall vs. Relative

- **Infinite Distance Limits**
  - $\mathcal{J} = \sum_i \tau^i J_i$ with some (all) $\tau^i \to \infty$ $\Rightarrow$ generically $\mathcal{V}_{B_3} \sim \mu^3 \to \infty$
  - Rescaled Kahler parameters $t^i := \mu^{-1}\tau^i$ $\Rightarrow$ finite rescaled volume $\mu^{-3}\mathcal{V}_{B_3} \sim 1$

- **Overall Scaling**
  - All t^i finite:

    "*homogeneous decompactification*"
  - A light Kaluza-Klein tower:

    $$\frac{M_{\rm KK}^2}{M_{\rm Pl}^2} \sim \frac{\mathcal{V}_{B_3}^{-1/3}}{\mathcal{V}_{B_3}} \sim \mu^{-4} \to 0$$

- **Strategy**
  - Classify allowed parametric forms of t^i to determine the full Kahler geometry

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and /or

- **Relative Scaling**
  - Some $t^i \to \infty$:

    "__residual__ *infinite distance limits*"
  - Some other $t^i \to 0$

    leading to __shrinking__ *curves*

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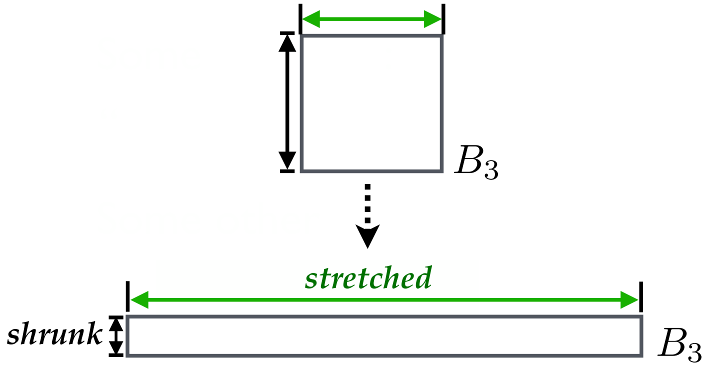

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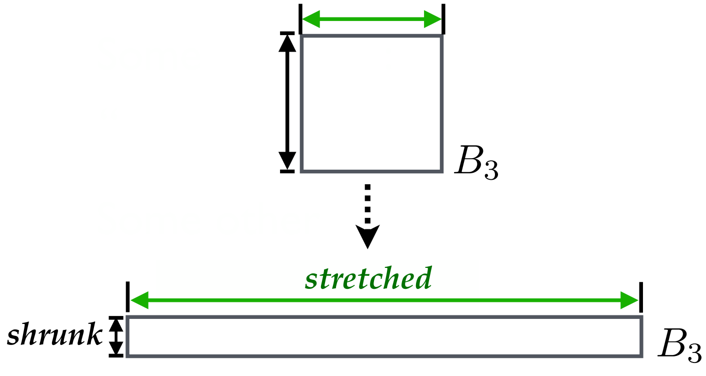

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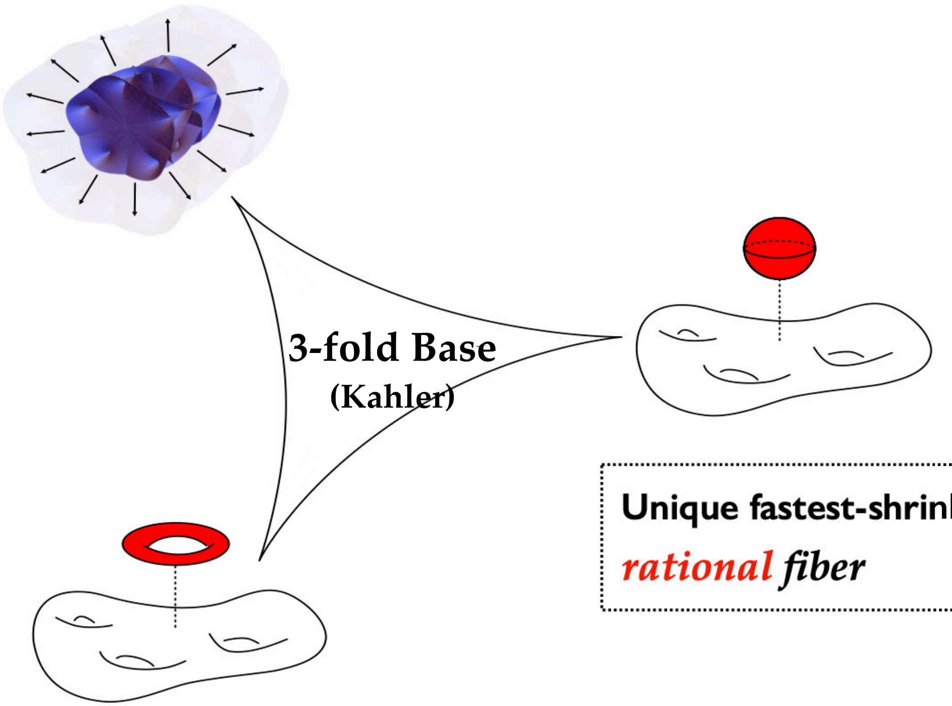
Main Results

- Summary -

Geometry at Infinite Distance

Classification: Kahler geometries [S.-J.L., Lerche, Weigand '19] [Klawer, S.-J.L., Weigand, Wiesner '20]

Apparent expansion of the internal dimensions



Unique fastest-shrinking curve: *rational fiber*

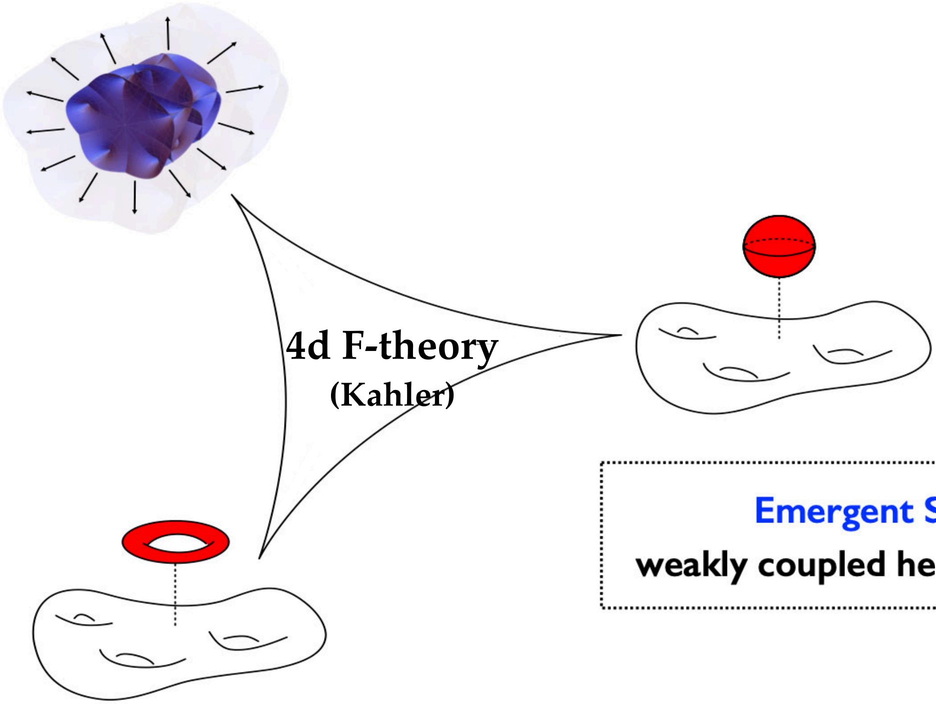
Unique fastest-shrinking curve: *elliptic fiber*

Physics at Infinite Distance

Confirmation: Emergent String Conjecture

[S.-J.L., Lerche, Weigand '19]
[Klawer, S.-J.L., Weigand, Wiesner '20]

Decompactification



Emergent String:
weakly coupled heterotic string

Emergent String:
weakly coupled type II string

Main Results

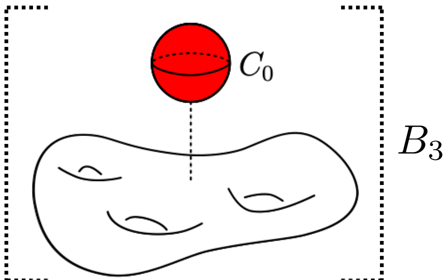
- Elaboration -

Emergent String

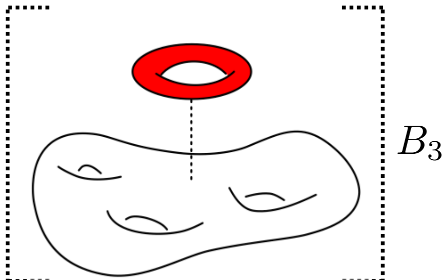
D3 Brane on Shrinking 2-cycle

- **Geometry**

  - B_3 exhibits a unique fastest-shrinking curve fiber C_0 (with genus **0** **or** **1**)



or



- **Physics**

  - D3-brane on C_0 leads to heterotic **or** Type II string with a vanishing tension

$$\frac{M_{\rm str}^2}{M_{\rm Pl}^2} \sim \frac{\mathcal{V}_{C_0}}{\mathcal{V}_{B_3}} \to 0$$

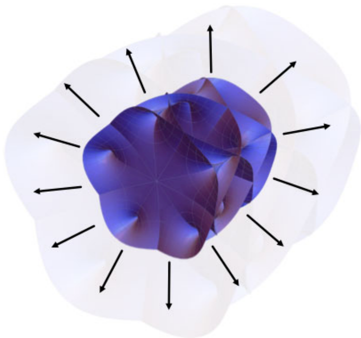
Light tower of string excitations

  - Duality frame well-defined thanks to the uniqueness property

(cf.) to be compared with the KK scale

Decompactification

Expansion of Internal Dimensions



- **Geometry**
  - All other limits:
    - no unique fastest-shrinking curve exists (genus 0 or 1)

- **Physics**
  - Case 1. No shrinking curves present
    - only an overall scaling, i.e., "homogeneous" decompactification
  - Case 2. Multiple fastest-shrinking curves present
    - naively pathological (multiple critical strings)
    - geometry demands, however, that $M_{\text{KK}} << M_{\text{str}}$, signaling decompactification

Light tower of Kaluza-Klein modes

Application

Weak Gravity Conjecture at Weak Gauge Coupling

- **Weak Gravity Conjecture(s)** [Arkani-Hamed, Motl, Nicolis, Vafa '06]; [Heidenreich, Reece, Rudelius '16-'17], [Montero, Shiu, Soler '16]
  - Claims the existence of a ***superextremal*** particle: $g^2 q^2 > m^2/M_{\rm Pl}^2$
  - minimal WGC ....→ one such particle
  - tower WGC ....→ a tower of such particles
  - sublattice WGC ....→ a tower of such particles filling in a charge sublattice

- **Confirmation at Weak Gauge Coupling**
  - Weak gauge coupling as an infinite distance limit
  - in fact, a tensionless *heterotic* string emerges!
  - WS index as a quasi-Jacobi form [S.-J.L., Lerche, Lockhart, Weigand '20][2]
  - Partciel spectrum encoded in the WS index
  - tower/sublattice* WGC via heterotic excitations

* *The sublattice might get shifted for non-generic fluxes*

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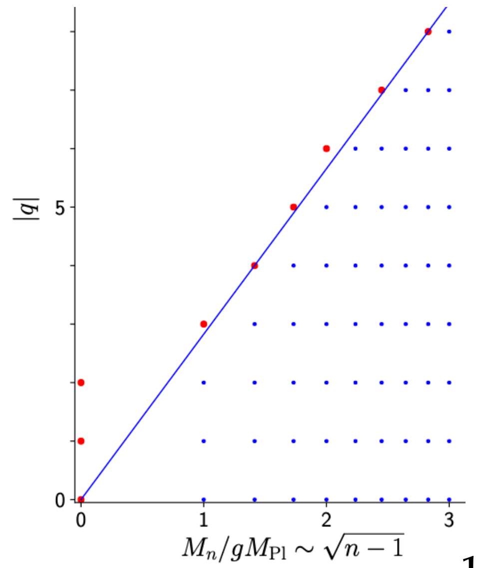

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Part II.

Complex Structure Moduli of F-theory

[S.-J.L., (Lerche,) Weigand '21]

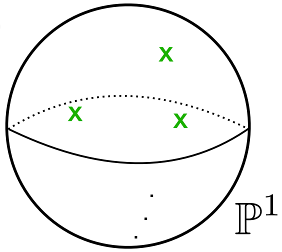
- Previous Results in the literature:
  mostly for the **closed string sector**

- New Development:
  open string sector incorporated systemacially

F-theory in 8 Dimensions

Brane Configuration via Complex Structure

- **8d F-theory**
  - IIB string theory on $\mathbb{P}^1$ with 7-branes at points
  - Dilaton profile via an **elliptic K3**: $y^2 = x^3 + f(s,t)xz^4 + g(s,t)z^6$
  - Discriminant loci: $\Delta(s,t) := 4f^3 + 27g^2 = 0$ ⟷ 7-branes
    - brane moduli as (part of) the K3 complex structure moduli



- **Moduli Space**
  - Complex structure limits of K3s [Kulikov '77], [Persson, Pinkham '81] + [Persson '77], [Friedman, Morrison '81]
    - at infinite distance: *Kulikov Models* of Type II/III ⟶ (where) do we find a light tower of states?

- **Goals**
  - Test the Emergent String Conjecture in the open-string sector
  - Clarify the fate of "*non-minimal*" brane stacks

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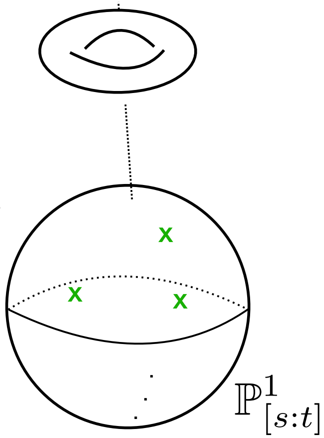

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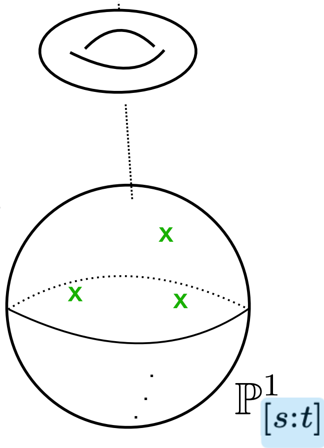

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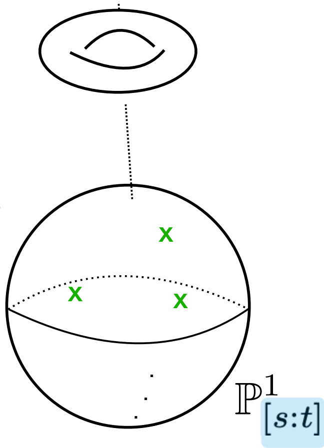

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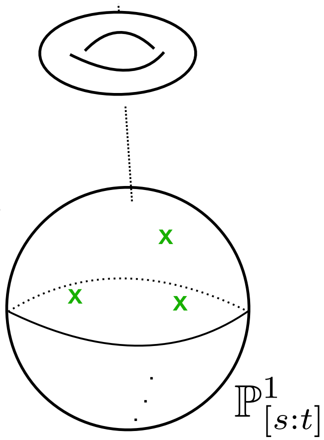

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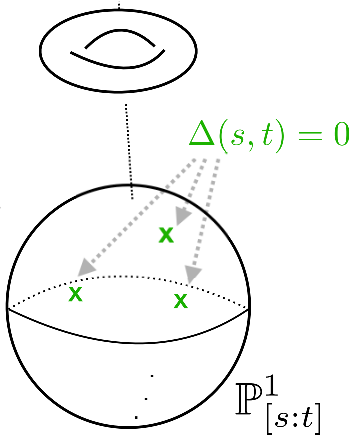

- Moduli Space
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  - Test the Emergent String Conjecture in the open-string sector
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F-theory in 8 Dimensions

Brane Configuration via Complex Structure

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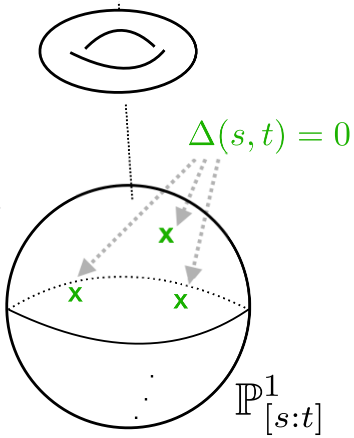

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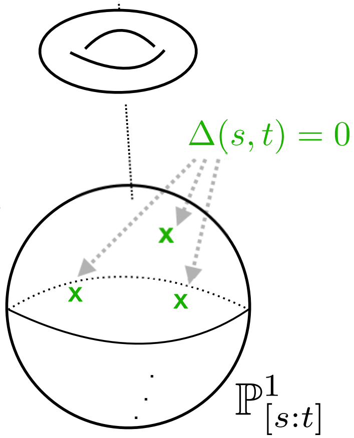

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Physics of Singular Fibers

Kodaira-Neron Classification: Minimal and Non-minimal Fibers

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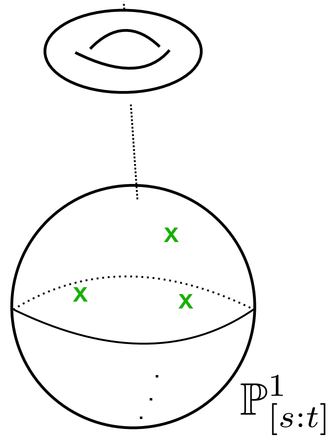

- **Singular Fibers: Minimal & Non-minimal**
  - Codimension-1 **minimal** fibers

    - **finite** enhancements ←······→ Lie gauge algebras G

  - Codimension-1 *non-minimal* fibers

    - typically discarded in model building

  - **Codimension-0** singular fibers

Algebra G	Brane Configuration	ord(f)	ord(g)	ord(Δ)
A_N	A^{N+1}	0	0	$N+1$
D_N	$A^N BC$	2	3	$N+2$
E_6	$A^5 BC^2$	≥ 3	4	8
E_7	$A^6 BC^2$	3	≥ 5	9
E_8	$A^7 BC^2$	≥ 4	5	10

Physics of Singular Fibers

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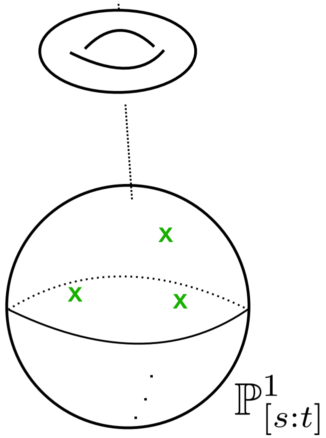

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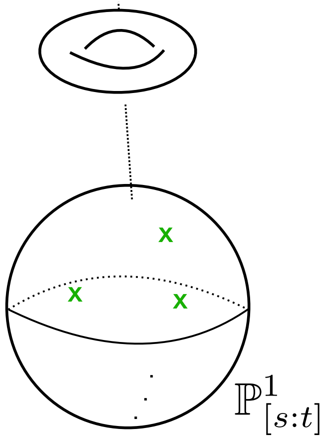

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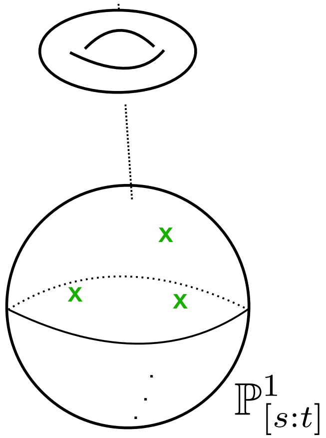

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Infinite Distance Limits (potentially)

What's the physics?

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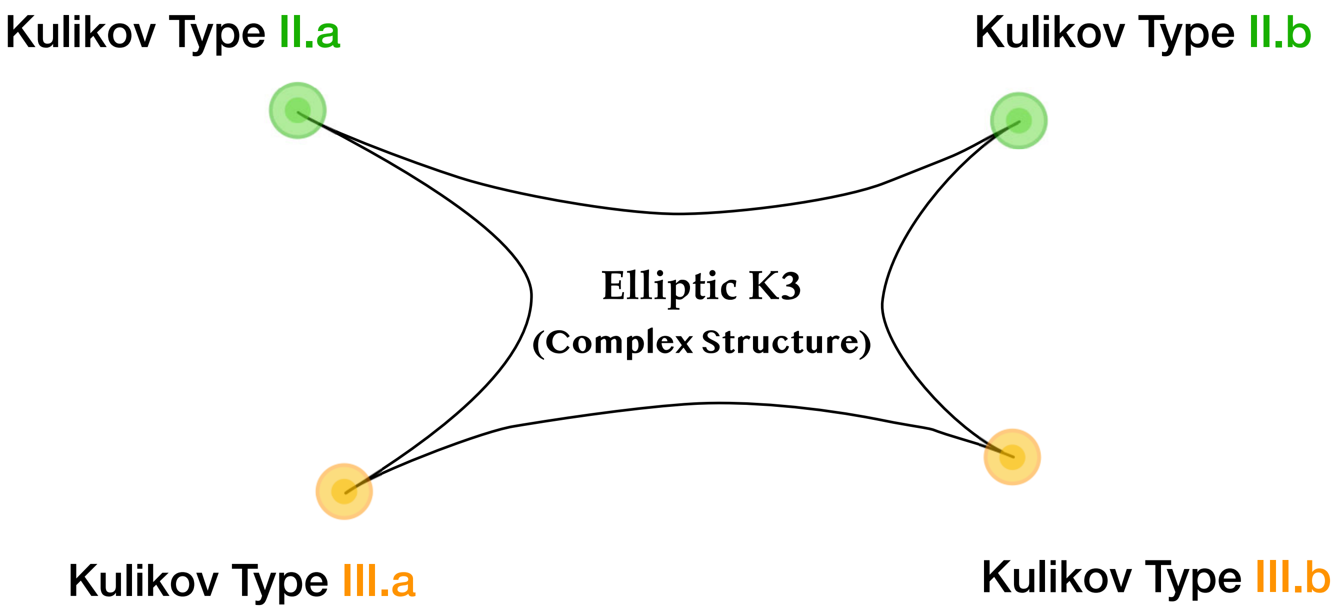
Main Results

- Summary -

Geometry at Infinite Distance

Classification: Kulikov Type II & III Refined [S.-J.L., Weigand '21] cf. [Alexeev, Brunyate, Engel '20]



Infinite Distance Limits
(candidates)

@codim-1: non-minimal fibers

@codim-0: generic singular fibers

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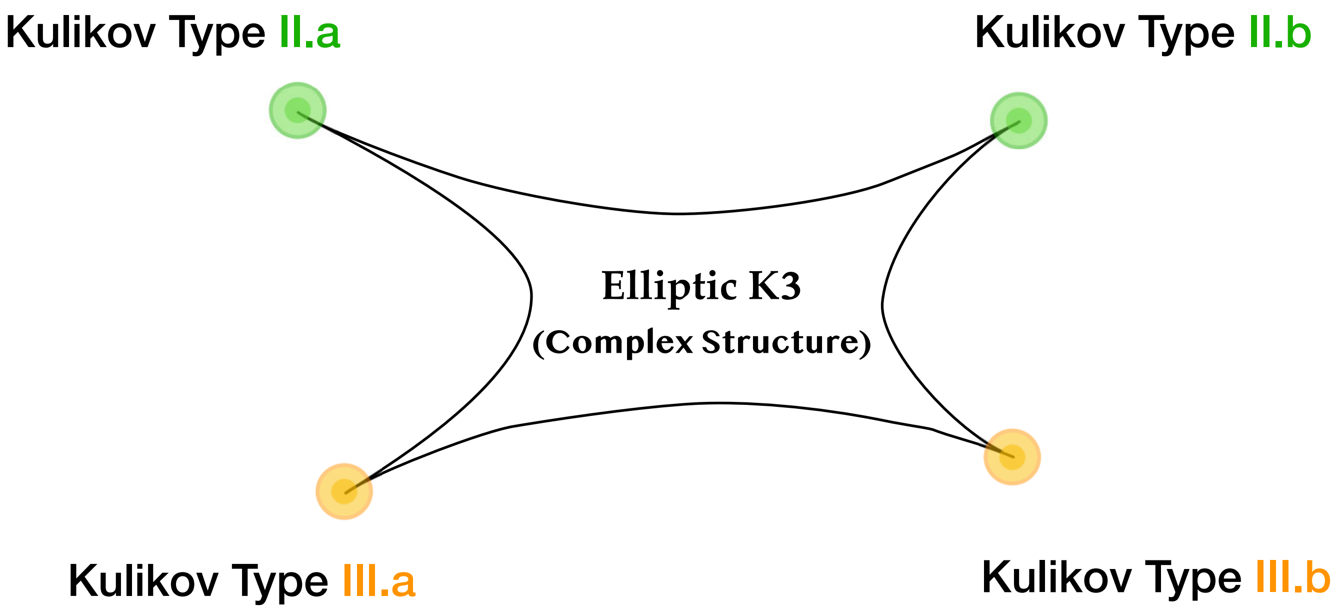

* To be super-precise, certain codim-1 non-minimal fibers can be obtained at finite distance

Infinite Distance Limits* (candidates)	Geometry of Kulikov Models
@codim-1: non-minimal fibers	Type II.a, III.a, III.b
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Physics at Infinite Distance

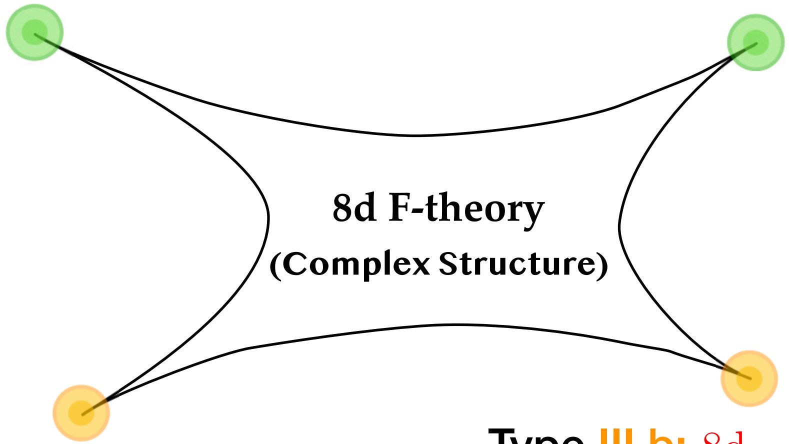
Confirmation: Emergent String Conjecture

[S.-J.L., Weigand '21]
[S.-J.L., Lerche, Weigand '21]

Type II.a: $8\text{d} \to 10\text{d} \quad (T_{\text{het}} \to \infty)$

$\hat{E}_9 \oplus \hat{E}_9 / \sim$

Type II.b: $8\text{d emergent string} \quad (g_{IIB} \to 0)$



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Physics of Light Particle Tower	**Infinite Distance Limits*** (candidates)	**Geometry** of Kulikov Models
decompactification	@codim-1: non-minimal fibers	Type II.a, III.a, III.b
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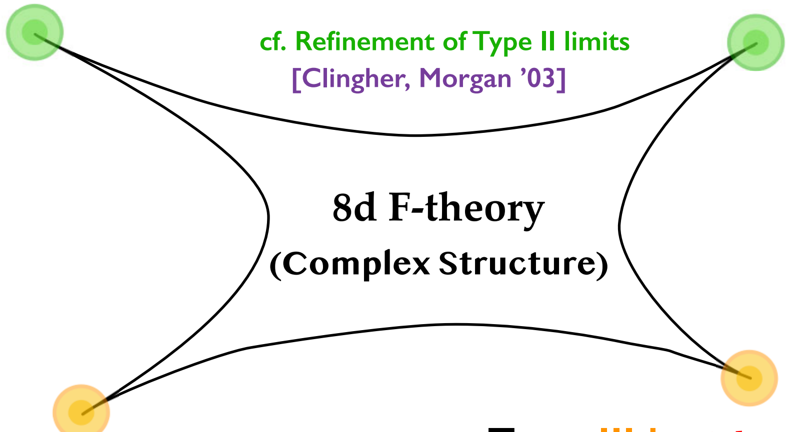
cf. [Morrison, Vafa '96]

Type II.b: $8\mathrm{d}$ emergent string $\quad (g_{IIB} \to 0)$

cf. [Aspinwall, Morrison '96]

cf. Refinement of Type II limits [Clingher, Morgan '03]



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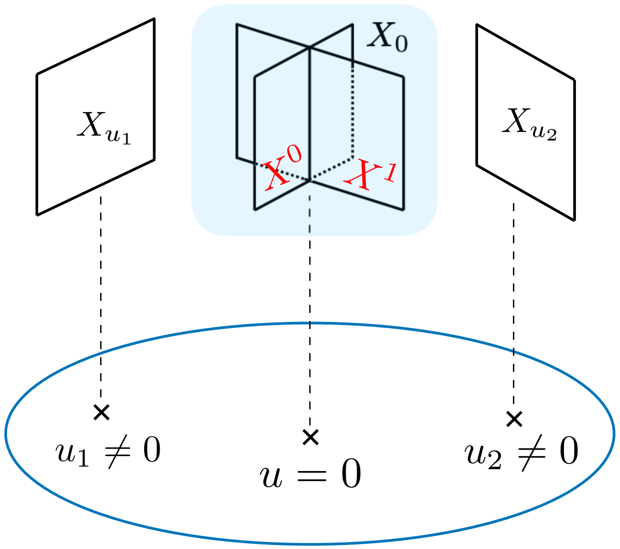
Rudiments

- Geometry of Kulikov Models -

Degeneration of K3

Kulikov Models

- **Degeneration of K3 Surface**
  - Family of K3 surfaces X_u
    - parameter $u \in D := \{u \in \mathbb{C}\,;\, |u| < 1\}$
  - (Semi-stable) degeneration at $u = 0$: $X_0 = \bigcup_{i=0}^{n} X^i$



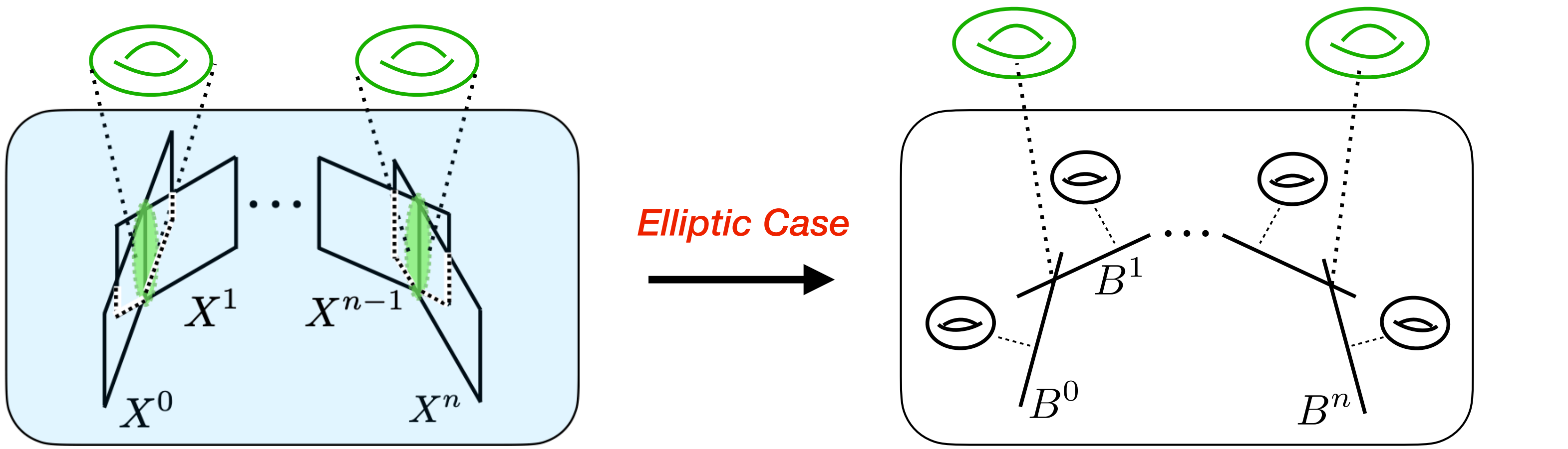
- **Kulikov Model**
  - Criteria
    - reduced, normal-crossing & trivial canonical bundle
  - Achievable [Kulikov '77], [Persson, Pinkham '81]
    - by base changes ($u \to u^k$) & birational transformations

⇢ We *can* transform any F-theory geometry into a Kulikov form!

Infinite Distance Degeneration of K3

Kulikov Models of Type II/III

- **Classification of Kulikov Models** [Kulikov '77], [Persson '77], [Friedman, Morrison '81]
  - Type I at finite distance
  - Type II/III at infinite distance
    - Type II: $X^i \cap X^{i+1}$ is elliptic & 2 transcendental 2-tori shrink ($\gamma_{j=1,2}$)
    - Type III: $X^i \cap X^j$ is rational & 1 transcendental 2-torus shrinks (γ_1)



Degenerate K3 $(u{=}0)$: $X_0 = \bigcup_{i=0}^{n} X^i$

Components elliptic: $X^i \to B^i (\simeq \mathbb{P}^1)$

Infinite Distance Degeneration of K3

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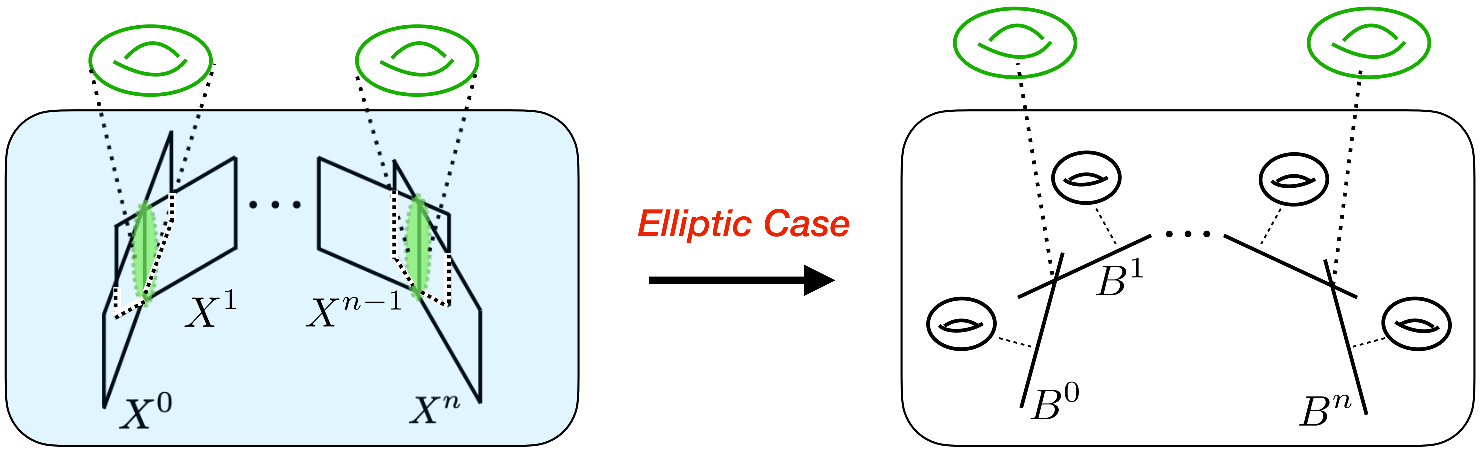

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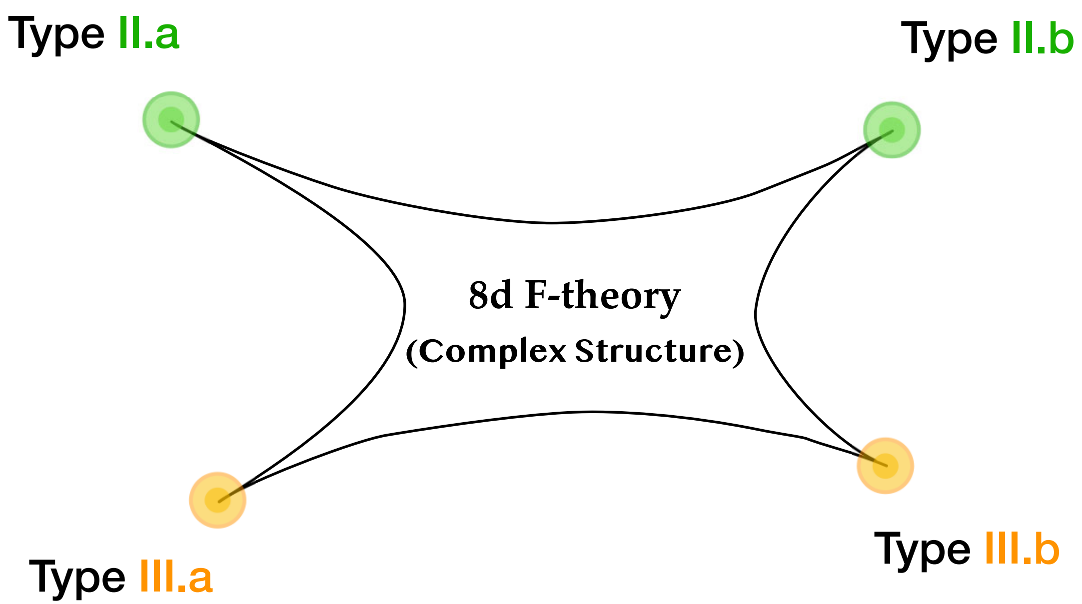
Main Results

- Elaboration -

Classification of Infinite Distance Limits

Geometry and Physics



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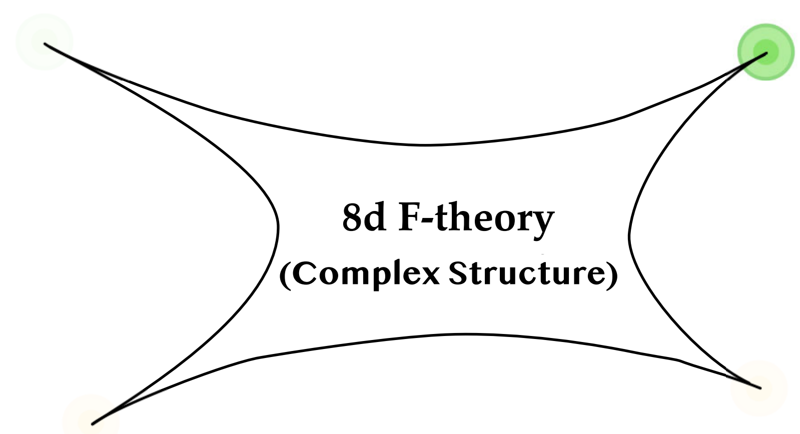
Classification of Infinite Distance Limits

Geometry and Physics

Type II.b: 8d emergent string $(g_{IIB} \to 0)$



May assume generic I_n fibers: *rescale e.g. as* $(f, g, \Delta) = (u^4 f', u^6 g', u^{12+n}\Delta') \longrightarrow (f', g', u^n \Delta')$

Divergence of $j(\tau) \sim f^3/\Delta$ implies $g_{IIB} \to 0$

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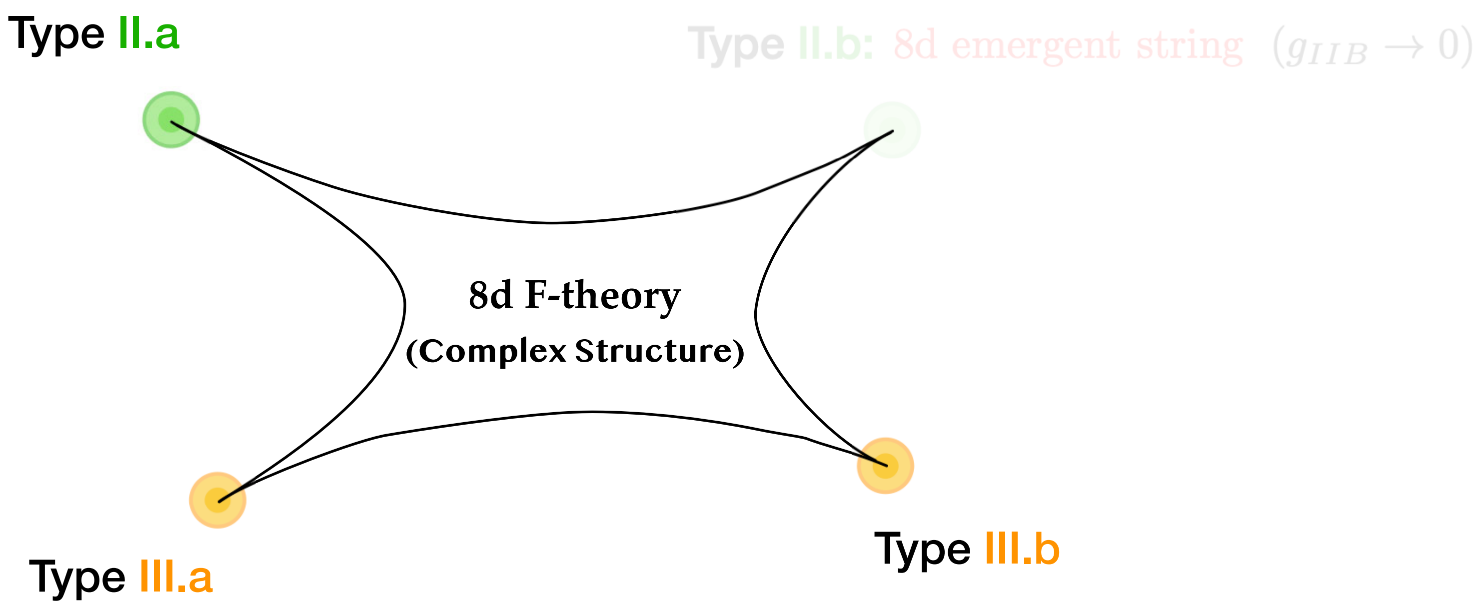

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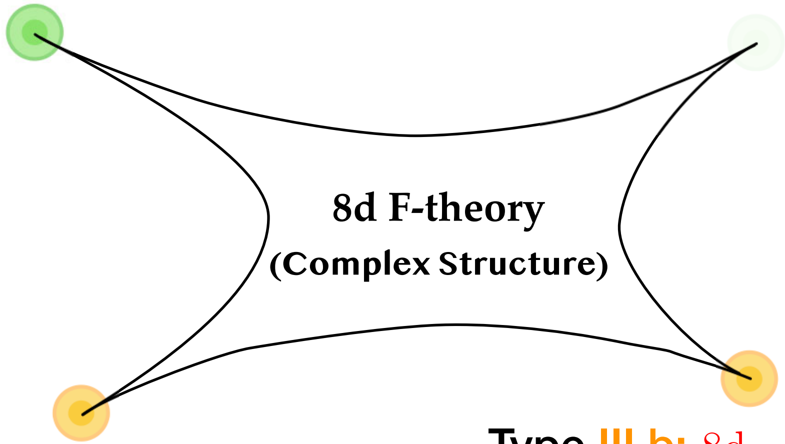

Type III.a

Type III.b: $8\text{d} \to 10\text{d}$ $\boxed{(g_{IIB} \to 0,}\ U_{IIB} \to 0)$

"both codim-1 & codim-0 singularities"

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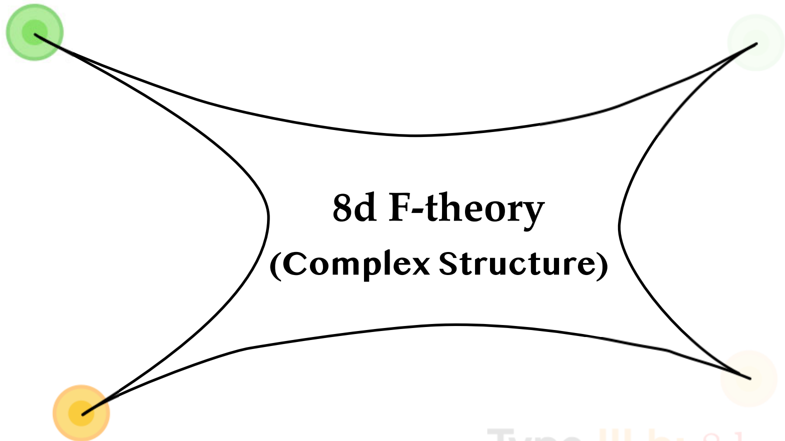

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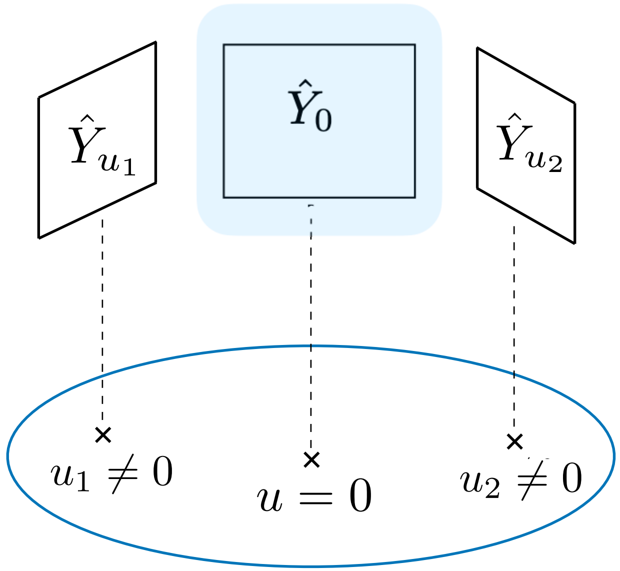
Keep Blowing Up

Geometric Strategy [S.-J.L., Weigand '21]



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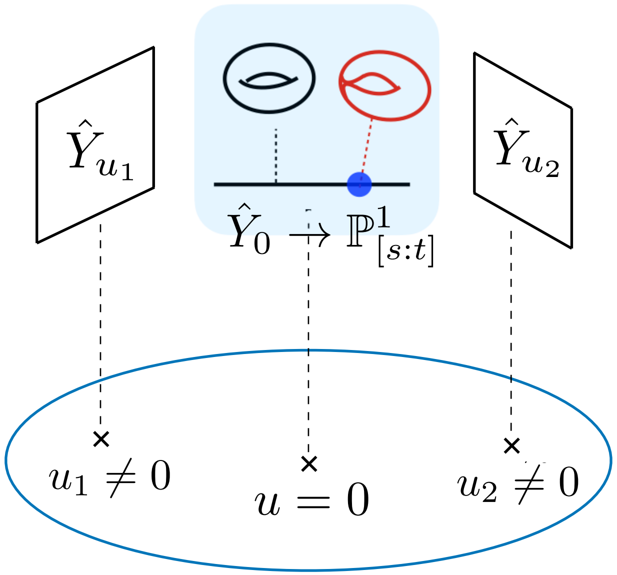

Smooth fibers generically

Non-minimal* fiber @s=0

$$\mathrm{ord}_{\hat{Y}_0}(f, g, \Delta)|_{\bullet} = \begin{cases} (\geq 4, \geq 6, \quad 12) \\ (\quad 4, \quad 6, > 12) \end{cases}$$

* $(>4, >6, >12)$ fiber can always reduce to either minimal fiber or one of the two types listed

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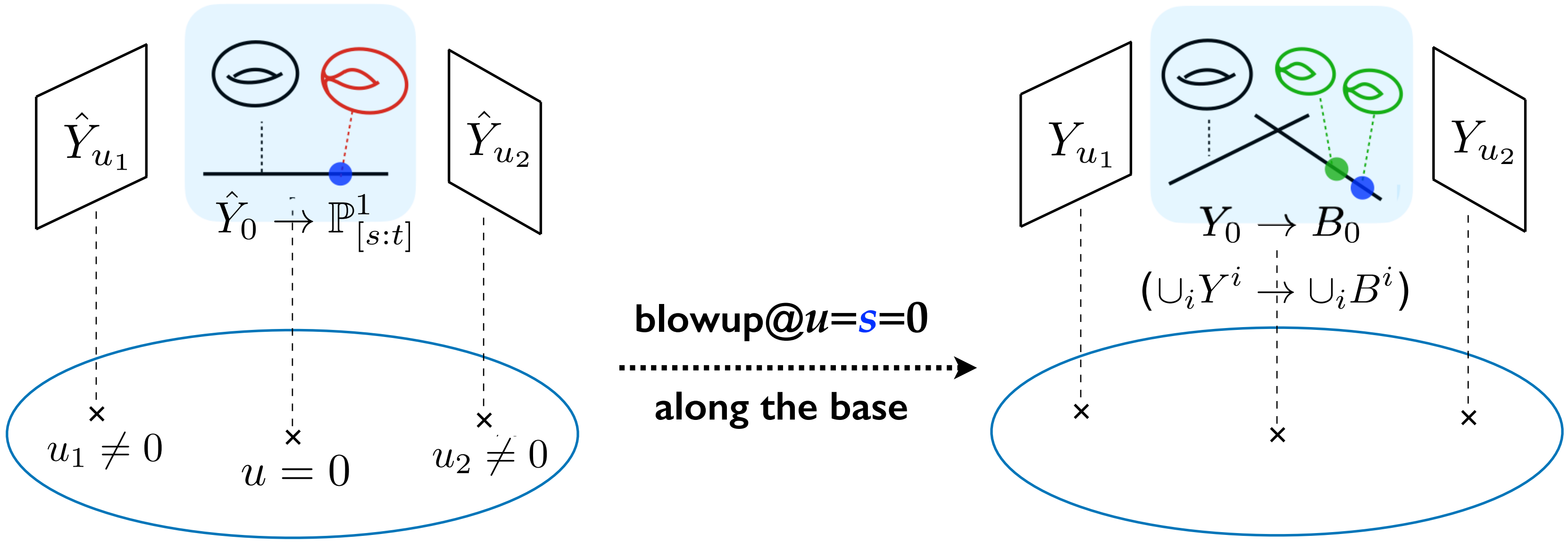

Smooth fibers generically
Non-minimal* fiber @s=0

Smooth fibers generically
Minimal singular fibers (even@s=0)

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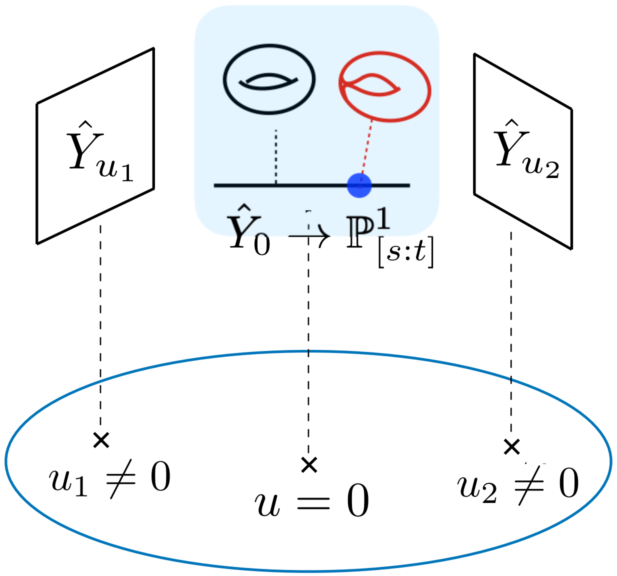
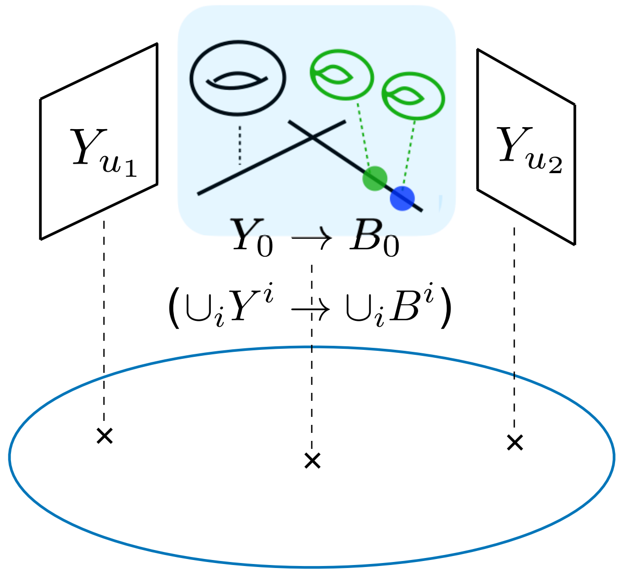


blowup@u=s=0

along the base



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blowup
along the fiber

Kulikov Model

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Keep Blowing Up

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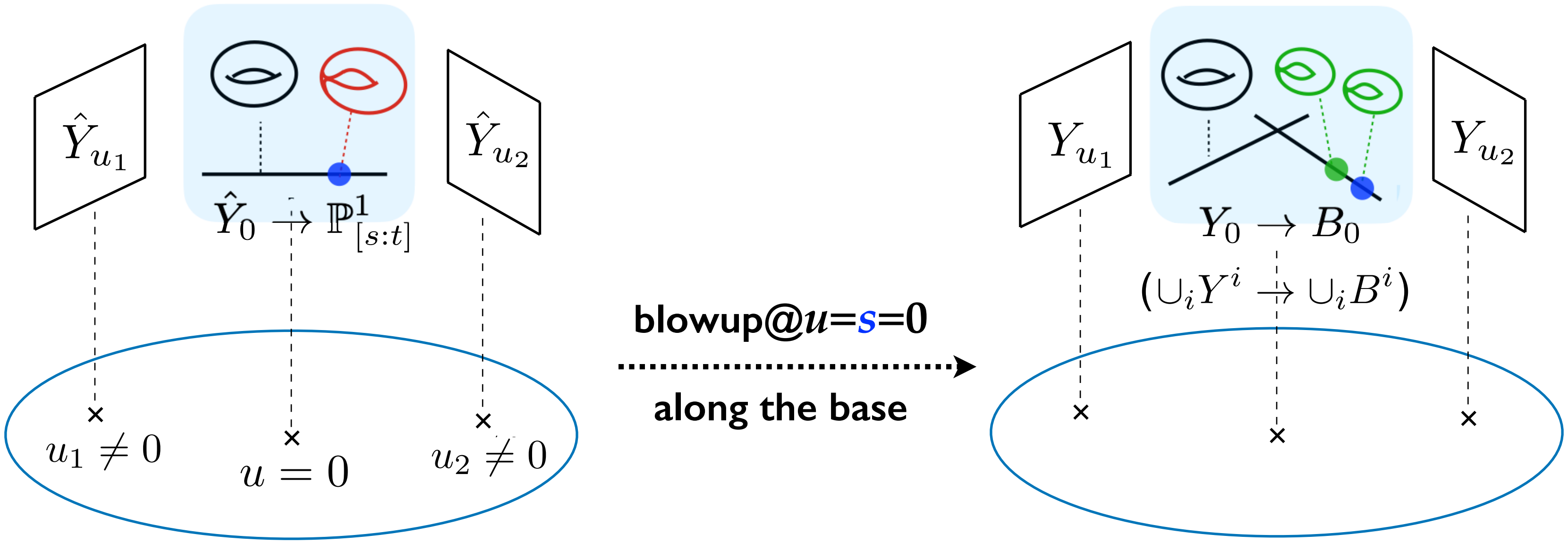


Smooth fibers generically
Non-minimal* fiber @s=0

Smooth fibers generically
Minimal singular fibers (even@s=0)

$$\mathrm{ord}_{\hat{Y}_0}(f,g,\Delta)|_{\bullet} = \begin{cases} (\geq 4, \geq 6, \; 12) \to \text{Type II.a} \\ (\;4, \;\; 6, > 12) \to \text{Type III.a} \end{cases}$$

blowup along the fiber

Kulikov Model

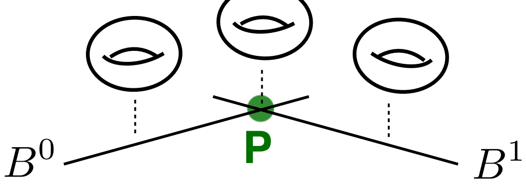
* $(>4, >6, >12)$ fiber can always reduce to either minimal fiber or one of the two types listed

Physics of Type II.a Kulikov Models

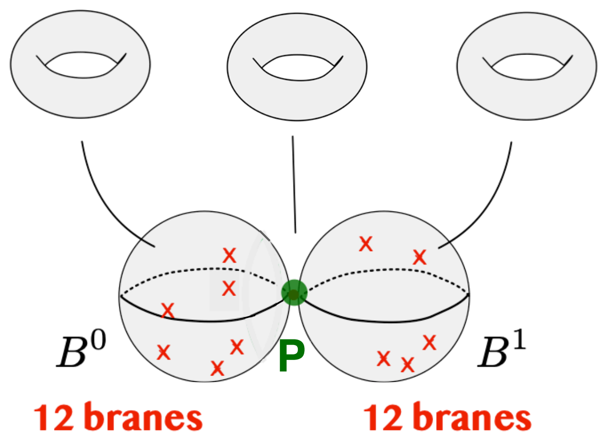
Decompactification (cf.) [Morrison, Vafa '96]

Type II.a

Smooth fibers generically (I_0)



$Y^{0,1}$ are rational elliptic surfaces (aka **dP₉**) intersecting at their common elliptic fiber



- Vanishing 2-tori

  $\gamma_1 = S_A \times \Sigma$ and $\gamma_2 = S_B \times \Sigma$

- Light tower

  1. M-theory

     - M2 branes on $\gamma_{j=1,2}$

  2. F-theory

     - (1,0) and (0,1) strings on Σ

     allowed by (trivial) monodromy of I_0

- Physics

  Decompactification to 10d

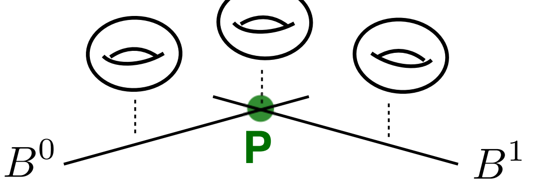
  (cf.) dual heterotic torus: $T_{\text{het}} \to \infty$

Physics of Type II.a Kulikov Models

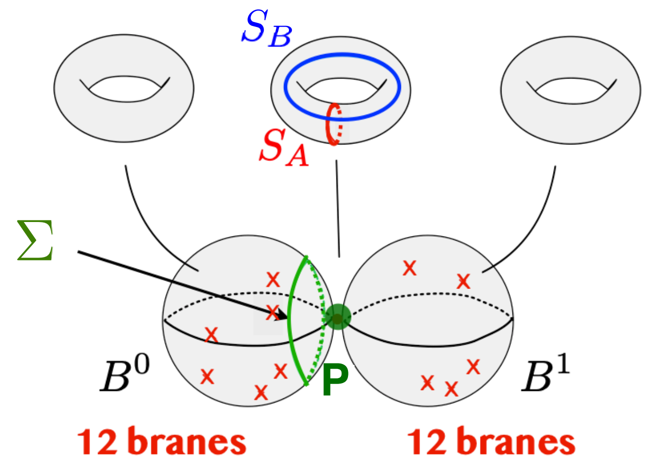
Decompactification (cf.) [Morrison, Vafa '96]

Type II.a

Smooth fibers generically (I_0)



$Y^{0,1}$ are rational elliptic surfaces (aka **dP₉**)
intersecting at their common elliptic fiber



- **Vanishing 2-tori**

  $\gamma_1 = S_A \times \Sigma$ and $\gamma_2 = S_B \times \Sigma$

- **Light tower**

  1. **M-theory**

     - M2 branes on $\gamma_{j=1,2}$

  2. **F-theory**

     - (1,0) and (0,1) strings on Σ

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- **Physics**

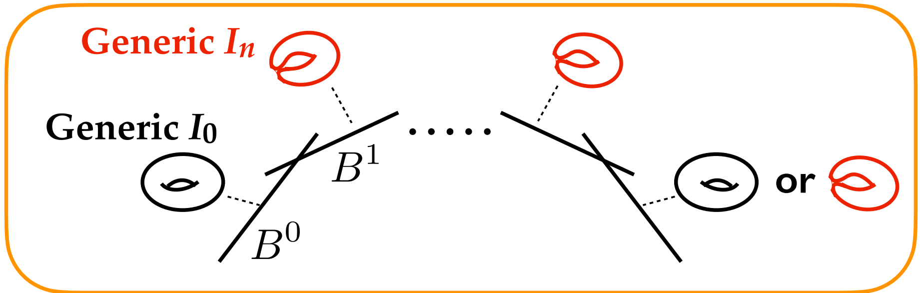
  Decompactification to 10d

  (cf.) dual heterotic torus: $T_{\text{het}} \to \infty$

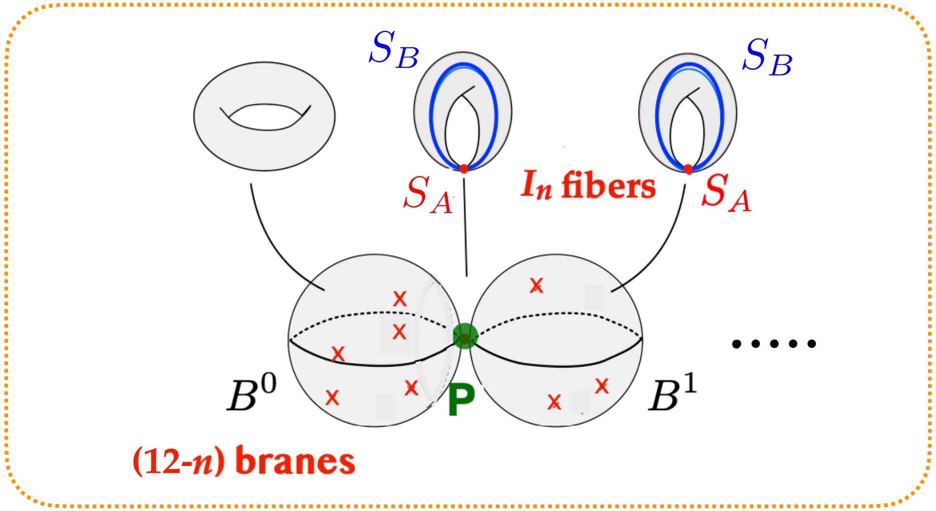
Physics of Type III.a Kulikov Models

Decompactification

Type III.a



At least one end is rational elliptic (**dP_9** surface), intersecting a component with generic $I_{n>0}$ fibers



- Vanishing 2-torus

  $\gamma_1 = S_A \times \Sigma$

- Light tower

  1. M-theory

     - M2 branes on γ_1

  2. F-theory

     - (1,0) string on Σ

     allowed by the monodromy of $I_{n>0}$

- Physics

  - Partial decompactification to 9d

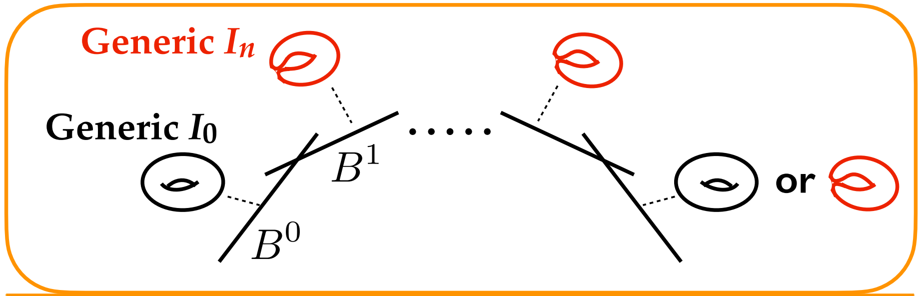
  (cf.) Dual heterotic torus:

  confirmed for $E_7 \times E_8$ *models!*

Physics of Type III.a Kulikov Models

Decompactification

Type III.a



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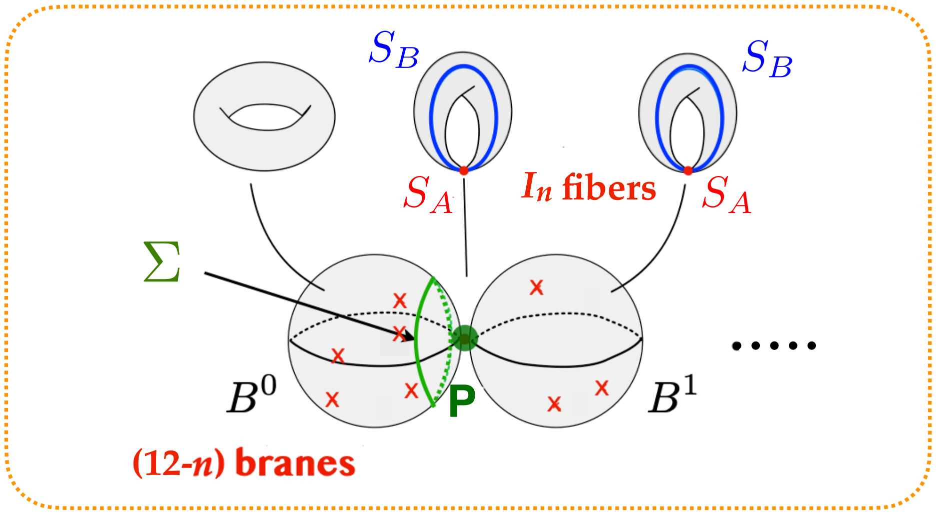


- **Vanishing 2-torus**

  $\gamma_1 = S_A \times \Sigma$

- **Light tower**

  1. <u>M-theory</u>

     - M2 branes on γ_1

  2. <u>F-theory</u>

     - (1,0) string on Σ

  allowed by the monodromy of $I_{n>0}$

- **Physics**

  - Partial decompactification to 9d

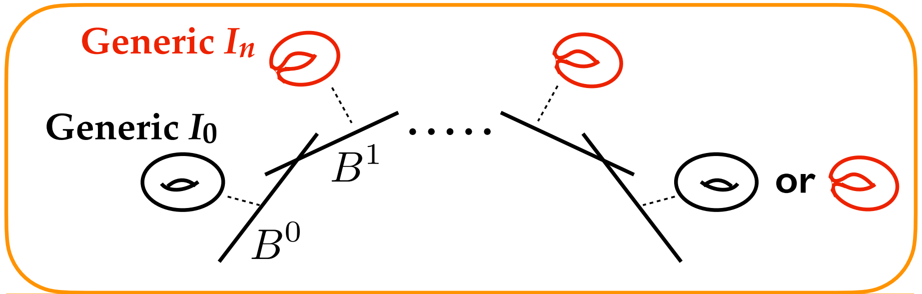
  (cf.) Dual heterotic torus: $T_{\text{het}} \sim U_{\text{het}} \to \infty$?

  confirmed for $E_7 \times E_8$ *models!*

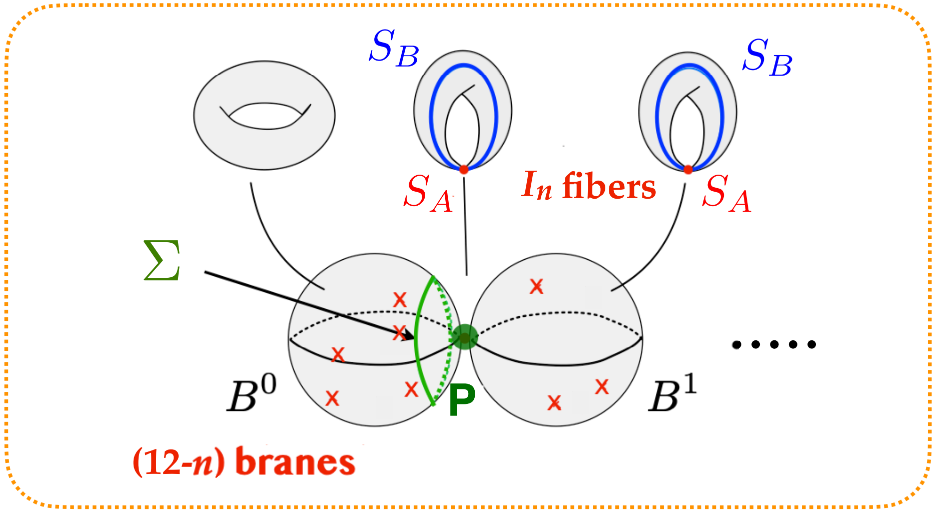
Physics of Type III.a Kulikov Models

Decompactification

Type III.a



At least one end is rational elliptic (**dP9** surface), intersecting a component with generic $I_{n>0}$ fibers



- **Vanishing 2-torus**
  $\gamma_1 = S_A \times \Sigma$
- **Light tower**
  1. **M-theory**
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     - (1,0) string on Σ
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- **Physics**
  - Partial decompactification to 9d
  (cf.) Dual heterotic torus: $T_{\text{het}} \sim U_{\text{het}} \to \infty$?
  confirmed for $\mathrm{E}_7 \times \mathrm{E}_8$ *models!*

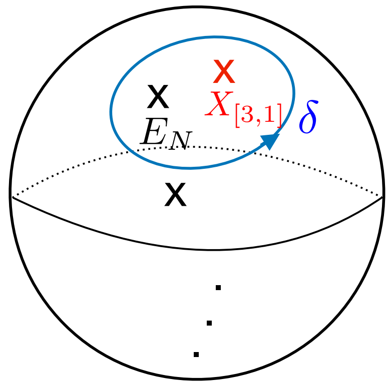
Light Tower: ***another*** F-theoretic Interpretation
- Affine extension $E_{9-n} \longrightarrow \hat{E}_{9-n}$

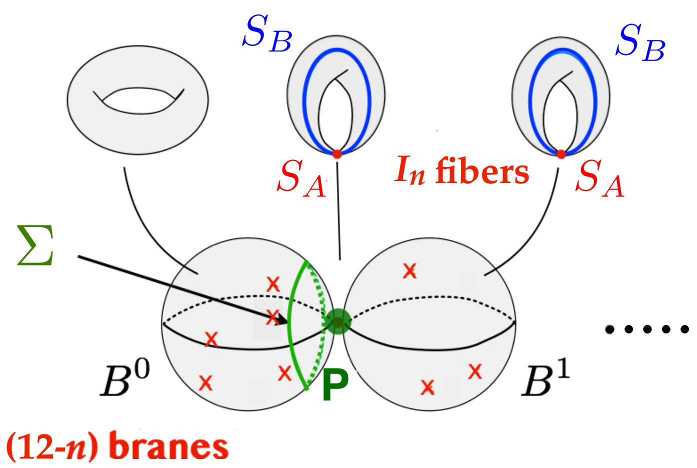
Affinization of Gauge Algebra

Monodromy Analysis

- **Affine Extensions** [DeWolfe, Hauer, Iqbal, Zwiebach '98] (cf.) [Gaberdiel, Zwiebach '97]
  - $E_N \xrightarrow{\oplus X_{[3,1]}} \hat{E}_N$ w/ $M_{\hat{E}_N} = \begin{pmatrix} 1 & 9-N \\ 0 & 1 \end{pmatrix}$
  - $M_{\hat{E}_N}\delta = \delta$ for $\delta = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

    ⇢ BPS state massless when E_N and $X_{[3,1]}$ coalesce
  - δ corresponds to the imaginary root of the affine extension





$(12-n)$ branes in B^0 away from **P**

$$M_{\hat{E}_{9-n}} \cdot M_{A_{n-1}} = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -n \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

I_n fiber at **P**

(cf.) generalizes to CHL vacua [Cvetic, Dierigl, Lin, Zhang]; heterotic perspective [Collazuol, Grana, Herraez]

Conclusions

Summary

Towers of Light States at Infinite Distance

- The **Emergent String Conjecture**

At Infinite Distance:

the EFT either decompactifies or reduces to a weakly-coupled string theory

⇩

The Nature of Light Tower:

either Kaluza-Klein or String excitations

- Every **equi-dim'l** limit at infinite distance as a weakly-coupled string theory
  - **String duality** at work for **every** such limit

    (cf.) no *membrane limits* [Alvarez-Garcia, Klawer, Weigand '21]
- String vacua can provide microscopic intuitions
  - In this talk: model-independent analysis of geometric **F-theory vacua**

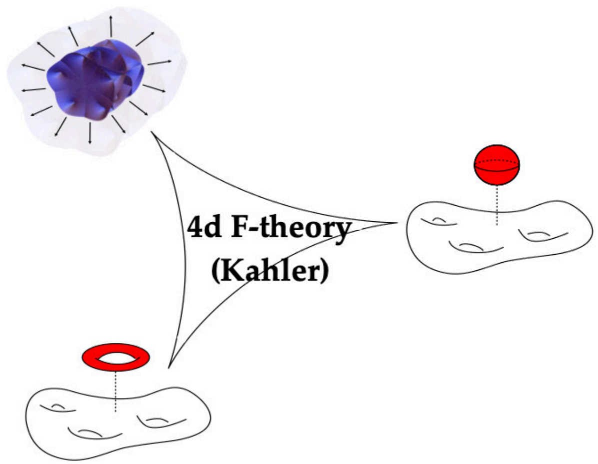
Summary

F-theory at Infinite Distance and Light Towers

- **Kahler Moduli: 4d F-theory**
  - Geometric classification of infinite distance physics:

    (a) Unique *fastest-shrinking 2-cycle*: **weakly-coupled string**

    (b) No *or* **multiple** *such 2-cycles*: **decompactification**
  - Verification for string EFTs w/ 4 real superchrages

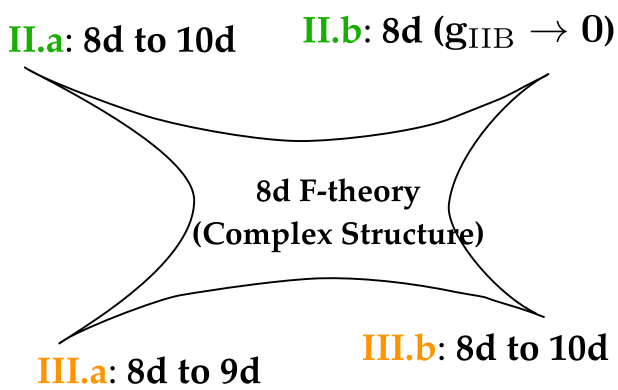


- **Complex Structure Moduli: 8d F-theory**
  - Geometric classification of infinite distance physics:

    (a) *Kulikov Type* **II.b**: **weakly-coupled string**

    (b) *Kulikov Type* **II.a**/**III.a**/**III.b**: **decompactification**
  - **Non-minimal** brane stacks as a "**brane moduli**" limit

    - Decompactification (**affine extension** of gauge algebra)



Outlook

Remarks on Future Directions

- **Lower Dim'l F-theory**

	8d EFTs		6d EFTs		4d EFTs
# (Supercharges)	16	1/2 ⟶	8	1/2 ⟶	4
Internal Spaces	unique $B_1 = \mathbb{P}^1$		many $B_2 = \mathbb{P}^2,\ \mathbb{F}_n,$ blowups		plethora $B_3 = \cdots$
Kahler Limits	trivial		non-trivial [S.-J.L., Lerche, Weigand '18][2]		wild [S.-J.L., Lerche, Weigand '19] [Klawer, S.-J.L., Weigand, Wiesner '20]
Brane Limits	non-trivial [S.-J.L., Weigand '21] [S.-J.L., Lerche, Weigand '21]		wild wip w/ Alvarez-Garcia, Weigand		

- **Bottom-up Inspiration for the Emergent String Conjecture?**
  - Relevant ideas [Lanza, Marchesano, Martucci, Valenzuela '20-22], …

Outlook

Remarks on Future Directions

- **Lower Dim'l F-theory**

	8d EFTs		6d EFTs		4d EFTs
# (Supercharges)	16	1/2 →	8	1/2 →	4
Internal Spaces	unique $B_1 = \mathbb{P}^1$		many $B_2 = \mathbb{P}^2,\ \mathbb{F}_n,$ blowups		plethora $B_3 = \cdots$
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- **Bottom-up Inspiration for the Emergent String Conjecture?**
  - Relevant ideas [Lanza, Marchesano, Martucci, Valenzuela '20-22], …

Thank You

Back Up Slides

Kahler Forms at Infinite Distance

Classification [S.-J.L., Lerche, Weigand '19] [Klawer, S.-J.L., Weigand, Wiesner '20]

$$\mathcal{J} = \sum_i \tau^i J_i = \mu\Big(\lambda J_0 + \sum_\alpha \frac{a^\alpha}{\lambda^2} J_\alpha + \sum_r b^r J_r\Big)$$

with $a^\alpha \precsim 1\,,\ b^r \precsim \lambda$

OR

$$\mathcal{J} = \sum_i \tau^i J_i = \mu\big(\lambda J_0 + \sum_\kappa c^\kappa J_\kappa\big)$$

with $c^\kappa \precsim \lambda$

$$J_0^3 = 0$$

$$J_0^2 \cdot J_\alpha \neq 0\,,\ \forall\alpha$$

$$J_0^2 \cdot J_\kappa = 0\,,\ \forall\kappa \quad \& \quad \exists\kappa' \text{ s.t. } J_0 \cdot J_\kappa \cdot J_\kappa' \neq 0$$

$$J_0^2 \cdot J_r = 0\,,\ \forall r \quad \& \quad J_0 \cdot J_r \cdot J_i = 0\,,\ \forall r\ ,\ \forall i = r', \kappa$$

Geometry of Type III Kulikov Models

Refined Classification

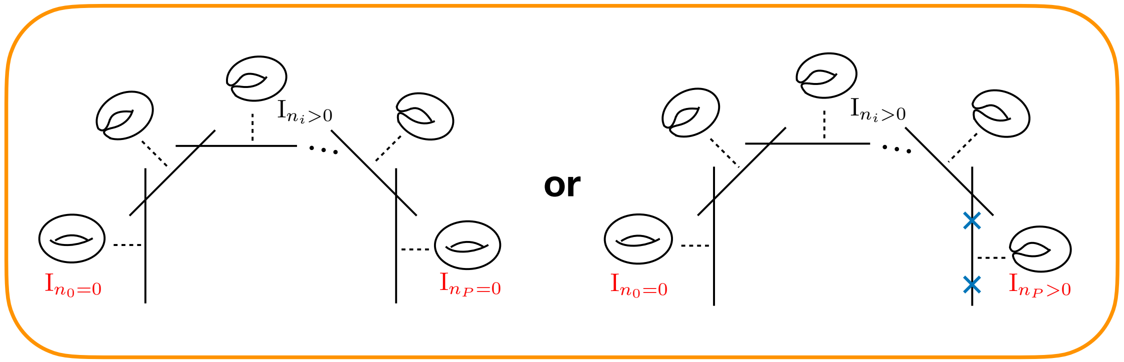
Theorem [S.-J.L., Weigand '21] (see also [Alexeev, Brunyate, Engel '20]):

The Weierstrass models associated with Type III Kulikov models of elliptic K3s (up to birational transf. & base change) take the chain form,

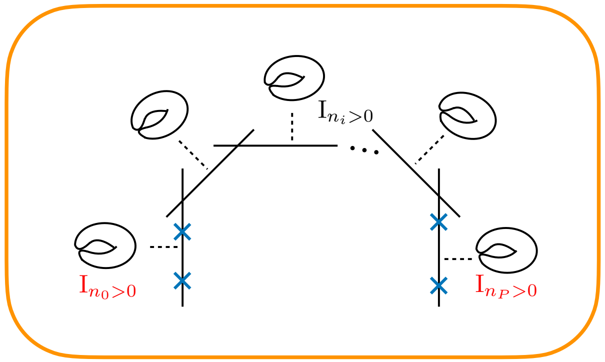
$$Y_0 = \bigcup_{i=0}^{P} Y^i \quad , \quad P \geq 2$$

with each component elliptic over the rational base B^i. The models fall into either of the two types:

Type III.a



Type III.b



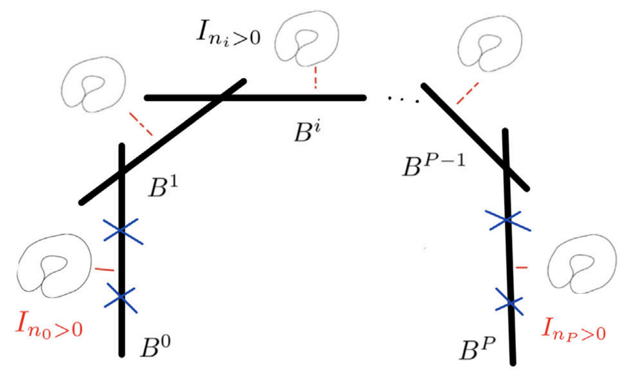
where ***special*** fibers on the $\mathrm{I}_{n_i>0}$ components can only be of

- A-type: $\mathrm{ord}_{Y^i}(f, g, \Delta)|_{\mathrm{pt}\in B^i} = (0, 0, k)$
- D-type: $\mathrm{ord}_{Y^i}(f, g, \Delta)|_{\mathrm{pt}\in B^i} = (2, 3, k)$, only allowed in the end components (as a pair)

Physics of Type III.b Kulikov Models

Decompactification [S.-J.L., Lerche, Weigand '21]

Type III.b



Every surface is an $\mathrm{I}_{n>0}$ component

- **Vanishing 2-torus**

  $\gamma_1 = S_A \times \Sigma$

- **Light tower**

  1. M-theory
     - M2 branes on γ_1
     - M2 on S_A gives a tensionless string
  2. F-theory
     - weakly-coupled IIB string globally defined
     - collision of O-planes required on top
       (*as vanishing orders should enhance to* (4,6))
     - complex structure of IIB torus degenerates

- **Physics**

  - Decompactification to 10d

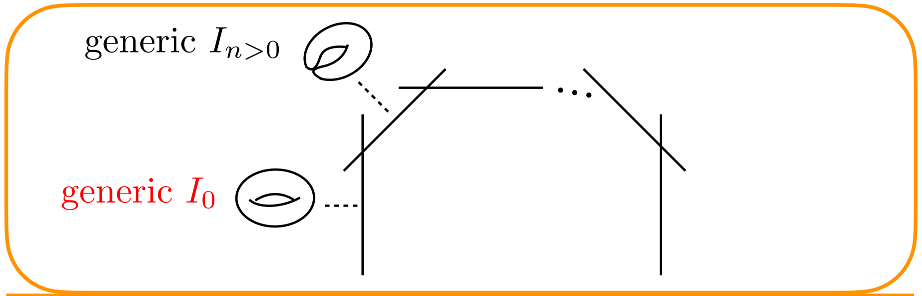
  (cf.) the other light tower ***not*** an M2 tower

  (*needs to be inferred indirectly*)

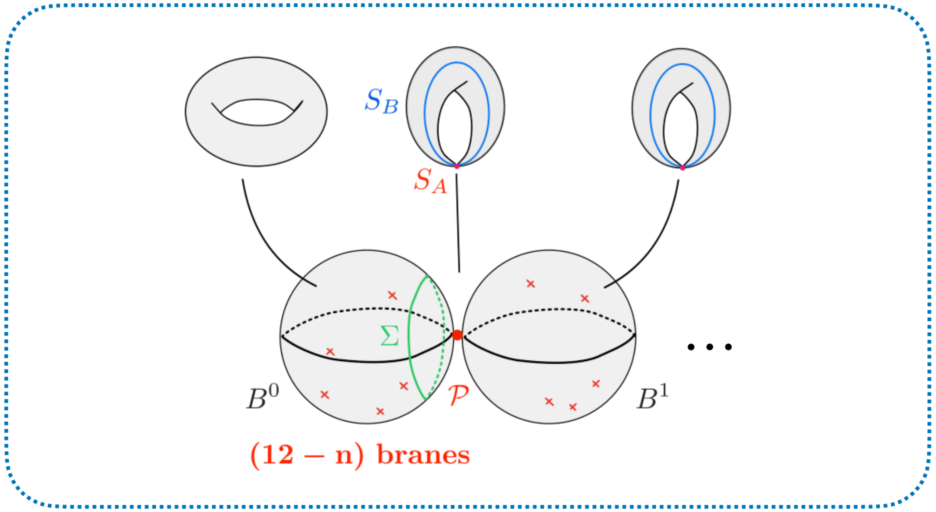
Physics of Type III.a Kulikov Models

Non-abelian Gauge Algebras [S.-J.L., Lerche, Weigand '21]

Type III.a



At least one end is an I_0 component (dP₉ surface), which intersects an $\mathrm{I}_{n>0}$ component



- **Asymptotic Symmetry Algebra**

  $G_\infty = H \oplus \hat{E}_a \oplus \hat{E}_b \quad \text{or} \quad H \oplus \hat{E}_a$

- **9d Gauge Algebra**

  $G_{9\text{d}} = H \oplus E_a \oplus E_b \quad \text{or} \quad H \oplus E_a$

- **Maximal Non-abelian Algebras in 9d**

  $G_\infty^{\max} = A_{17-a-b} \oplus \hat{E}_a \oplus \hat{E}_b \quad \text{or} \quad D_{17-a} \oplus \hat{E}_a$

  ⬇

  $G_{9\text{d}}^{\max} = A_{17-a-b} \oplus E_a \oplus E_b \quad \text{or} \quad D_{17-a} \oplus E_a$

- agree with the heterotic analysis

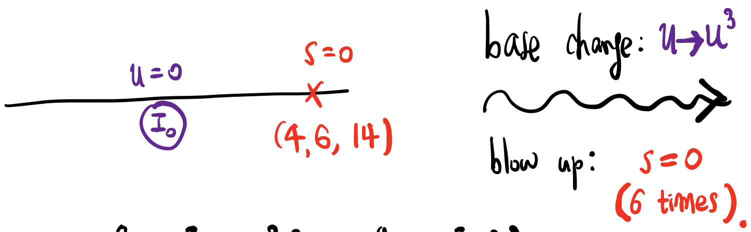
[de Freitas, Font, Fraiman, Grana, Nunez '20]
(cf). [Collazuol, Grana, Herraez '22]

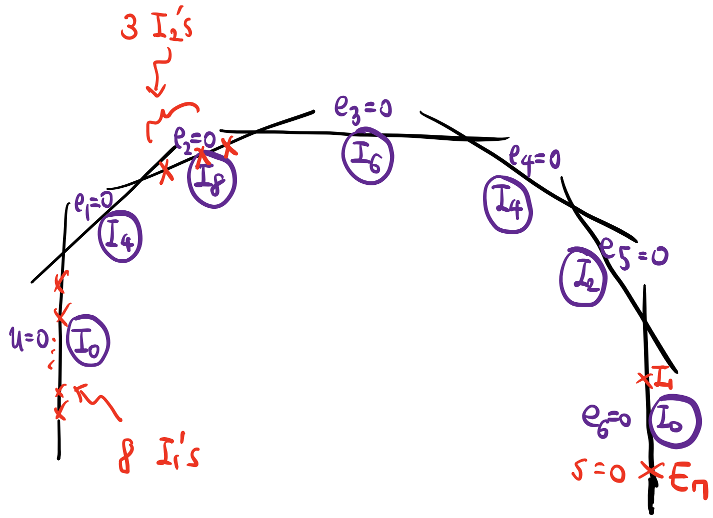
- evidence for our 9d decompactification proposal!

Example: an Illustration

Type III.a via Non-minimal Fiber [S.-J.L., Weigand '21]

- **Non-Minimality with (4, 6, >12) Vanishing**
  - An illustrative example
    - model of a non-minimal fiber w/ $ord(f, g, \Delta) = (4, 6, 14)$ that turns into Type III.a





$$f = -3s^3(2s^5 + 2s^3t^2 + st^4 + 2t^5u^2)$$

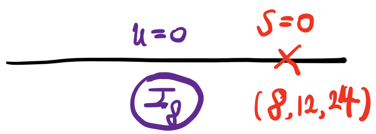
$$g = 2s^5(s^7 + 3s^5t^2 + 3s^3t^4 + st^6 + 3t^7u^2)$$

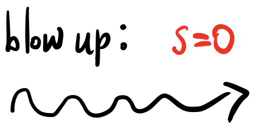
$$\Delta = s^9(\underbrace{\cdots}_{\text{order-5 vanishing @ } s=u=0.})$$

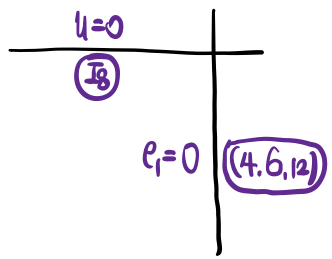
Example: an Alert

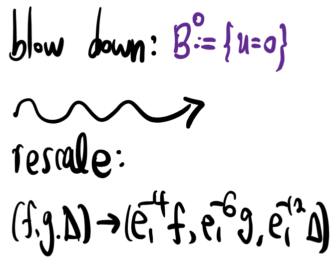
Type I via Non-minimal Fiber [S.-J.L., Weigand '21]

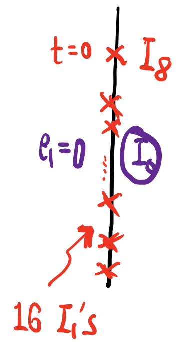
- **Non-Minimality with (>4, >6, >12) Vanishing**
  - Non-minimal fibers may secretly sit at finite distance
  - An alerting example
    - model of a non-minimal fiber w/ $ord(f, g, \Delta) = (8, 12, 24)$ that is secretly of Type I











$$f = u^8 t^8 - 3 s^8$$

$$g = u^{12} t^{12} - 2 s^{12}$$

$$\Delta = t^8 u^8 (108 s^{16} - 108 s^{12} t^4 u^4 - 36 s^8 t^8 u^8 + 31 t^{16} u^{16})$$