Non-invertible chiral symmetry in 3+1-dimensions

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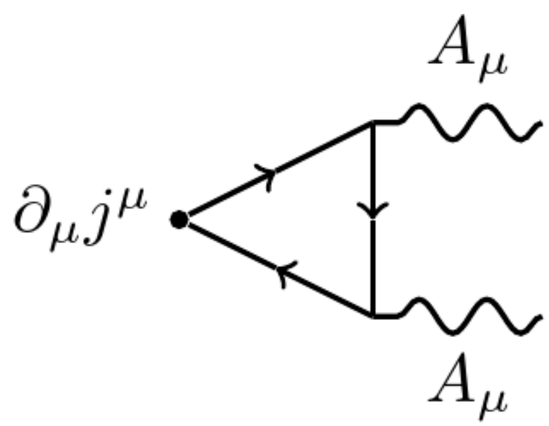
[Kaidi, KO, Zheng arXiv: 2111.01141], [Cordova, KO arXiv: 2205.06243]
c.f. [Choi, Cordova, Hsin, Lam, Shao arXiv:2111.01139], [Choi, Lam, Shao arXiv:2205.05086]

ABJ-anomalous symmetry as symmetry

- ABJ-anomalous symmetry [Adler ’69], [Bell, Jackiw ’69]



$$= \frac{1}{8\pi^2} f_{\mu\nu} \tilde{f}^{\mu\nu}$$

- Usually said that the symmetry is quantumly broken.
- When the gauge group is abelian,

  we can “cure” the symmetry.

[Choi, Lam, Shao ’22], [Cordova, KO ’22]

- Price: non-invertibility

Chiral symmetries in an abelian gauge theory is
a generalized symmetry, called non-invertible symmetry.

Expanding Symmetry

[c.f. talk by Rudelius]
[c.f. talk by Shao in Strings 2021]

- Global symmetry has been generalized over the last decade.
  - Higher-form (higher-group) symmetry

    [Gukov, Kapustin '13], [Kapustin, Thorngren '13]
    [Gaiotto, Kapustin, Seiberg, Willet '14]…
    [Cordova, Dumitrescu, Intriligator' 18], [Benini, Cordova, Hsin '18], …

  - Subsystem symmetry (non-relativistic systems)

    [Vijay, Haah, Fu '16], [You, Devakul, Burnell, Sondhi '18]… [Seiberg Shao '20], …

  - Non-invertible symmetry

Non-invertible symmetries in 3+1d

- Non-invertible symmetries has been known in 1+1d:

  [Verlinde ’88], … [Fuchs Runkel Schweigert ’02], …, [Bhardwaj, Tachikawa ’17], [Chang, Lin, Shao, Wang, Yin ’18], [Thorngren Wang ‘19+’21] …

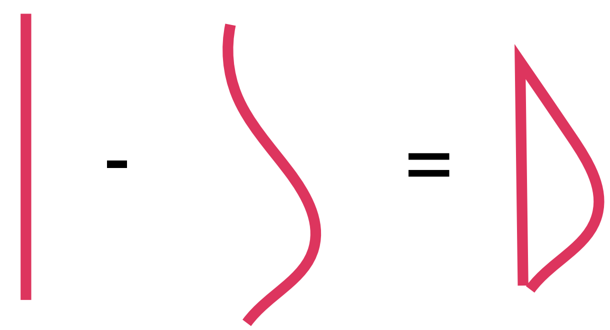
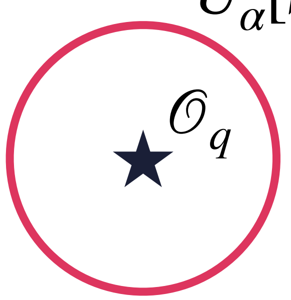
- Ubiquitous in TQFTs.
- More recently, many examples in $d > 2$ non-top. theories:

  [Tachikawa ’17] …, …
  [Ngyuen, Tanizaki, Unsal, Koide, Nagoya, Yamaguch, Choi, Cordova, Hsin, Lam, Shao, Kaidi, KO, Zheng, Roumpedakis, Seifnashri, Bhardwaj, Bottini, Schafer-Nameki, Tiwari, Zafrir, Rudelius, Arias-Tamargo, Rodriguez-Gomez, Hayashi, Antinucci, Galati, ,Rizi , Bashmakov, Del Zotto, Hasan, Damia, Argurio, Tizzano, Garcia-Valdecasas … ’21-’22]

- Exciting emerging area of research!
- No need to go to exotic examples:

  Infinite non-invertible symmetry in massless QED

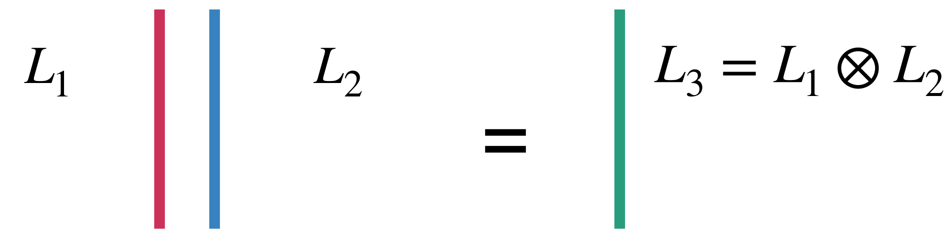
Topological Symmetry operator

- $U(1)$ Symmetry ➡ Symmetry op. $U_\alpha[\Sigma] = e^{i\alpha Q[\Sigma]} = e^{i\alpha \int_\Sigma *j}$
  on codim-1 surface Σ
- $U_\alpha[\Sigma]$ is topological :  $= \int_{\text{interior}} d * j = 0$
- 

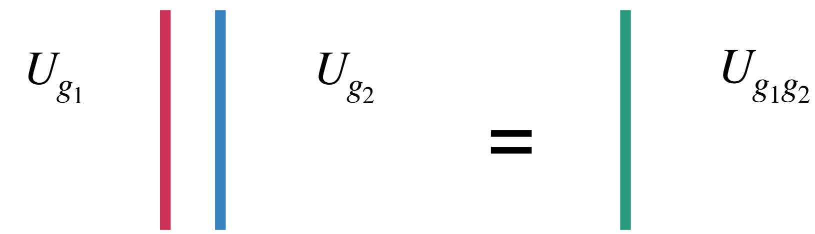
  $= e^{i\alpha q}\mathcal{O}_q$: $U(1)$ action
- For general G, $g \in G$ ➡ $U_g[\Sigma]$

Fusion product

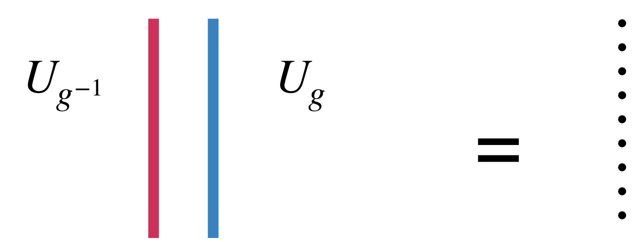
- Given two topological surface operators, one can "fuse" them:



- For top. operators for conventional symmetry,
$U_{g_1} \otimes U_{g_2} = U_{g_1 g_2}$



- $U_g[\Sigma]$ has its **inverse**:



conventional symmetry ⇔ **codim-1 invertible** topological operator

Higher form symmetry

[Gaiotto, Kapustin, Seiberg, Willet '14]

(Ordinary) symmetry ⇔ codim-1 invertible topological operators

⬇ Generalize

"p-form symmetry" ⇔ codim-(p+1) invertible topological operators

- Acts on p-dim extended operator instead of local (point) operators.
- Precise formulation of "center symmetry" in a gauge theory. (p=1)
- Magnetic one-form symmetry in abelian gauge theory:
  - $f = dA, j_{\mu\nu} \propto \epsilon_{\mu\nu\rho\sigma} f^{\rho\sigma} \rightarrow \partial^{\mu} j_{\mu\nu} = 0$
  - Symmetry op. $U_{\alpha}^{(1)}(\Sigma) := e^{i\alpha \int_{\Sigma} f/2\pi}$

Non-invertible top. operator

(Ordinary) symmetry ⇔ codim-1 invertible topological operators

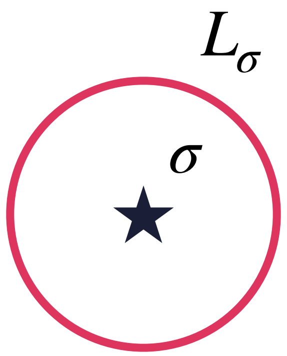
Generalize

“non-invertible symmetry” ⇔ general topological operators

- RG-flow invariant [Chang, Lin, Shao, Wang, Yin ’18], …
- $O_i \otimes O_j = \sum_k O_k$: beyond group

[Verlinde ’88], …

- E.g.: 1+1d Ising CFT: L_ϵ: $\mathbb{Z}_2$-line (invertible), L_σ: KW duality line
- $L_\epsilon \otimes L_\epsilon = 1, \quad L_\sigma \otimes L_\epsilon = L_\sigma, \quad L_\sigma \otimes L_\sigma = 1 + L_\epsilon$
- L_σ is non-invertible.



$= 0$

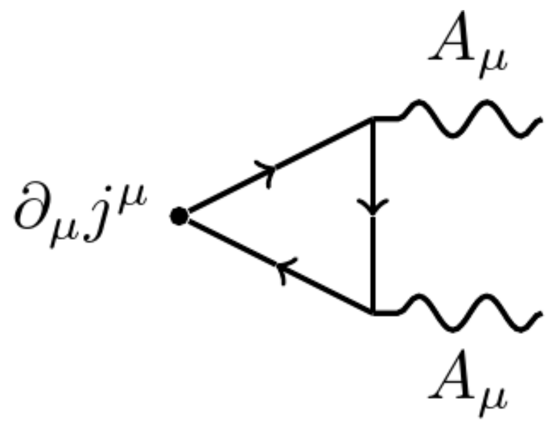
Non-invertible Chiral Symmetry

Massless QED chiral symmetry

- QED: $U(1)$ gauge field (f) + Weyl massless fermions e_+, e_-

- chiral $U(1)$ (global) sym: $e_- \to \mathrm{e}^{\mathrm{i}\alpha} e_-$: ABJ-anomalous

  [Adler '69], [Bell, Jackiw '69]



$$= \frac{1}{8\pi^2} f_{\mu\nu} \tilde{f}^{\mu\nu}$$

- $j \to j - *\mathrm{CS},\ d\mathrm{CS} = \dfrac{1}{8\pi^2} f \wedge f$?

  The CS term is not gauge invariant.

Non-invertible Chiral Symmetry

[Kaidi, Ohmori, Zheng '21], [Choi, Lam, Shao '22], [Cordova, KO '22]

- Set the parameter rational: $\alpha = 2\pi \frac{p}{q}$

  The "naive" symmetry operator: $\mathrm{U}_{2\pi p/q}[\Sigma_3] = \mathrm{e}^{\mathrm{i}\frac{2\pi p}{q}\int_{\Sigma_3} j}$

- The "naive modification" leads to

  " $e^{\mathrm{i}\frac{2\pi p}{q}\int_{\Sigma_3} *(j+*\mathrm{CS}(f))} = \mathrm{U}_{2\pi p/q}\, e^{-\mathrm{i}\frac{2\pi p}{q}\int_{\Sigma_3}\mathrm{CS}(f)}$ "

- We are wanting fractional CS term of EM field:

  response action for fractional hall state (FQH) with $\nu = \frac{p}{q}$

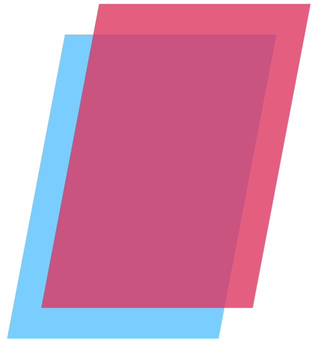
- We can cancel the ABJ-anomaly by FQH state on the defect!

Non-invertible Chiral Symmetry

[Kaidi, Ohmori, Zheng '21], [Choi, Lam, Shao '22], [Cordova, KO '22]

- The ill-quantized response of FQH is reproduced by a 2+1d TQFT, which is the IR fixed point of FQH system.

- 2+1d TQFT $\mathrm{A}^{q,p}$ for filling $\nu = \dfrac{p}{q}$: [Hsin, Lam, Seiberg '18]
  a universal part of FQH TQFT that coupled to EM.
  e.g. $\mathrm{A}^{q,1} = \mathrm{U}(1)_q$ dynamical CS theory.



- Then, the modified defect is
  $\mathrm{D}_{p/q}[\Sigma_3] = \mathrm{U}_{2\pi p/q}[\Sigma_3] \otimes \mathrm{A}^{q,p}[\Sigma_3, f]$: topological

- $\mathrm{A}^{q,p}$ is coupled with bulk EM field strength f,
  as its $\mathbb{Z}_q$ one-form symmetry background, as the actual FQH does.

$\mathrm{D}_{p/q}$ defines the non-invertible chiral symmetry.

Fusion rule

- A fusion rule:

$$\left(\mathsf{D}_{-1/q} \otimes \mathsf{D}_{1/q}\right)[\Sigma_3] = \left(\mathsf{A}^{q,-1} \otimes \mathsf{A}^{q,1}\right)[\Sigma_3, f]$$

$$= \sum_{S \in H_2(\Sigma_3, \mathbb{Z}_q)} U^{(1)}_{2\pi/q}[S]\, e^{2\pi \mathrm{i}/q\ T(S)}$$

: "condensation operator"

…[Gaiotto, Johnson-Freyd '19],
…, [Roumpedakis, Seifnashri, Shao '22]

- Ubiquitous in non-invertible symmetry in 3+1 dimenions.

[Kaidi, KO, Zheng '21], [Choi, Cordova, Hsin, Lam, Shao '21]

Symmetry action on S^3

- $\mathsf{D}_{p/q}[M_3] = \mathsf{C}_{p/q}[M_3] \times \mathsf{A}^{q,p}[M_3, f]$

- How $\mathsf{D}_{p/q}$ acts on the Hilbert space $\mathscr{H}_{M_3}$ on space mfd. M_3?

- Simplest case: $M_3 = S^3$

  $Z(\mathsf{A}^{q,p})[S^3, f] = 1/\sqrt{q}$, ➡ $\mathsf{D}_{p/q}[S^3] \propto \mathsf{U}_{p/q}$

- Chiral charge is preserved on S^3 or flat limit $\mathbb{R}^3$.
  as if it were a conventional symmetry. : fermion helicity conservation

  c.f. [Harlow, Ooguri '18] , [Choi, Lam, Shao '22]

- No instanton on $S^3 \times \mathbb{R}$, or flat limit $\mathbb{R}^4$ for abelian gauge group.

Symmetry action on general manifold

- $\mathsf{D}_{p/q}[M_3] = \mathsf{U}_{p/q}[M_3] \times \mathsf{A}^{q,p}[M_3, f]$

- $M_3 = S^2 \times S^1$: $\mathcal{H}_{S^2\times S^1} = \oplus_m \mathcal{H}_m$, $m = \int_{S^2} \frac{f}{2\pi}$.

  $$Z(\mathsf{A}^{q,p})[S^2 \times S^1, m] = \begin{cases} 0 & m \neq 0 \mod q \\ 1 & m = 0 \mod q \end{cases}$$: non-invertible.

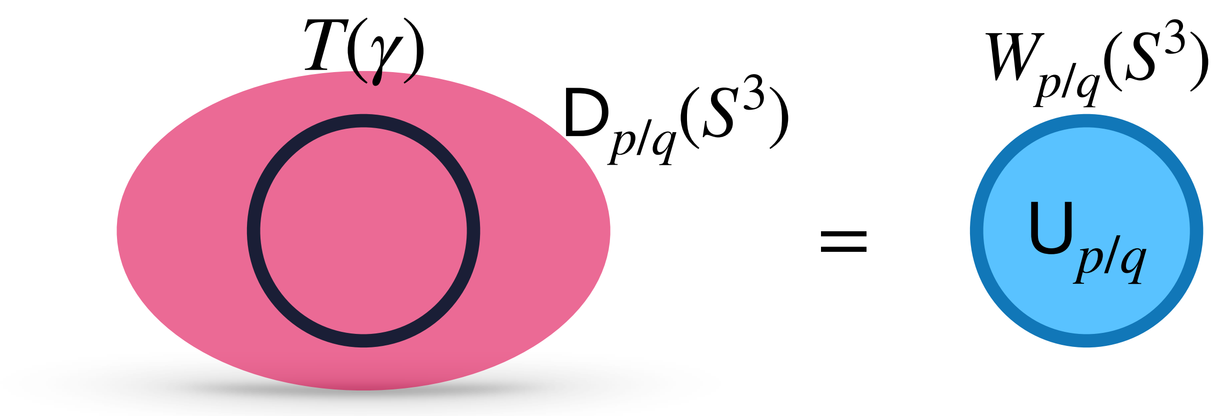
- $\mathsf{D}_{p/q}[S^1 \times S^2] = \mathsf{U}_{2\pi p/q} \times P\,(m = 0 \mod q)$.
  Chiral charge is preserved modulo m on $\mathcal{H}_m$.

- Magnetic catalysis.

- Symmetry action on arbitrary M_3

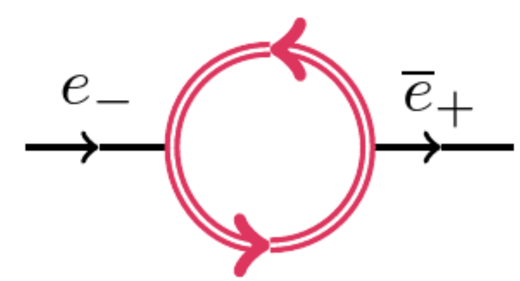
Breaking by monopole bubbles

- The non-invertible symmetry also acts on ’t Hooft operator:



- Dynamical monopole → Explicit chiral symmetry breaking.

- Such a term is suppressed by $e^{-\#MR} \sim e^{-S_{inst}(g_{UV})}$,
  where M:monopole mass, R: monopole core size.



  Monopole loop effect = UV instanton effect.

- The same non-invertible symmetry in Maxwell-axion: SSB phase.
  Potential generated by monopole loops. [Fan, Fraser, Reece, Stout ’18]

Summary

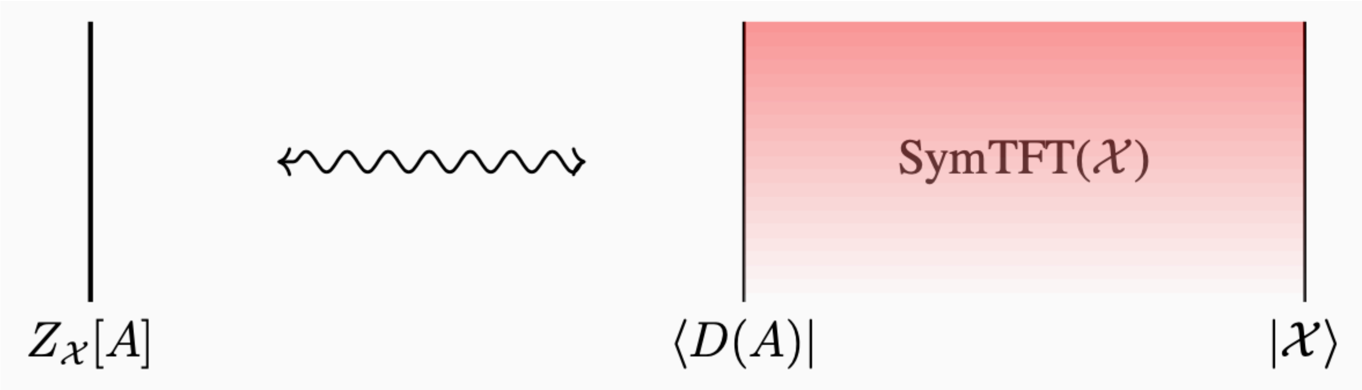
- Many examples of non-invertible symmetry in $d > 1+1$
- One of them: ABJ-anomalous chiral sym. in abelian gauge theory.
  - $\alpha = 2\pi p/q$
  - $\mathrm{D}_{p/q}[\Sigma_3] = \mathrm{e}^{\mathrm{i}\frac{2\pi p}{q}\int_{\Sigma_3} j} \otimes$ (Fractional Hall State TQFT): topological
  - Symmetry action on general spacial manifold
  - Dynamical monopole can break the chiral symmetry
    $\rightarrow$ exponentially suppressed breaking.

Non-invertible symmetry provides
a renewed understanding of chiral symmetry!

Future directions

- Better formal understanding

  - Symmetry TFT: TQFT_{d+1} governing finite symmetry in QFT_d : "topological sector of holography"



[Witten '98]…
[Freed, Teleman '12]
[Gaiotto, Kulp '20]
[Apruzzi, Bonetti, García-Etxebarria, Hosseini, Schafer-Nameki '21]
[talk by García-Etxebarria]

Construction for 3+1 d Kramers-Wannier duality symmetry
[Kaidi, KO, Zheng WIP]

- Concrete applications

[Choi, Lam, Shao '22]

  - Phenomenology? New selection rule? New naturalness?

  - Swampland

[Rudelius's talk]

[Rudelius Shao '20],
[Ben Heidenreich, McNamara, Montero, Reece, Rudelius '21],
[Arias-Tamargo, 'Rodriguez-Gomez '22]

Thank you for your attention!

Summary

- Many examples of non-invertible symmetry in $d > 1+1$
- One of them: ABJ-anomalous chiral sym. in abelian gauge theory.
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