Universität Zürich[UZH]

Dalitz-plot analysis techniques (I)

Rafael Silva Coutinho

University of Zurich
May 11th, 2015

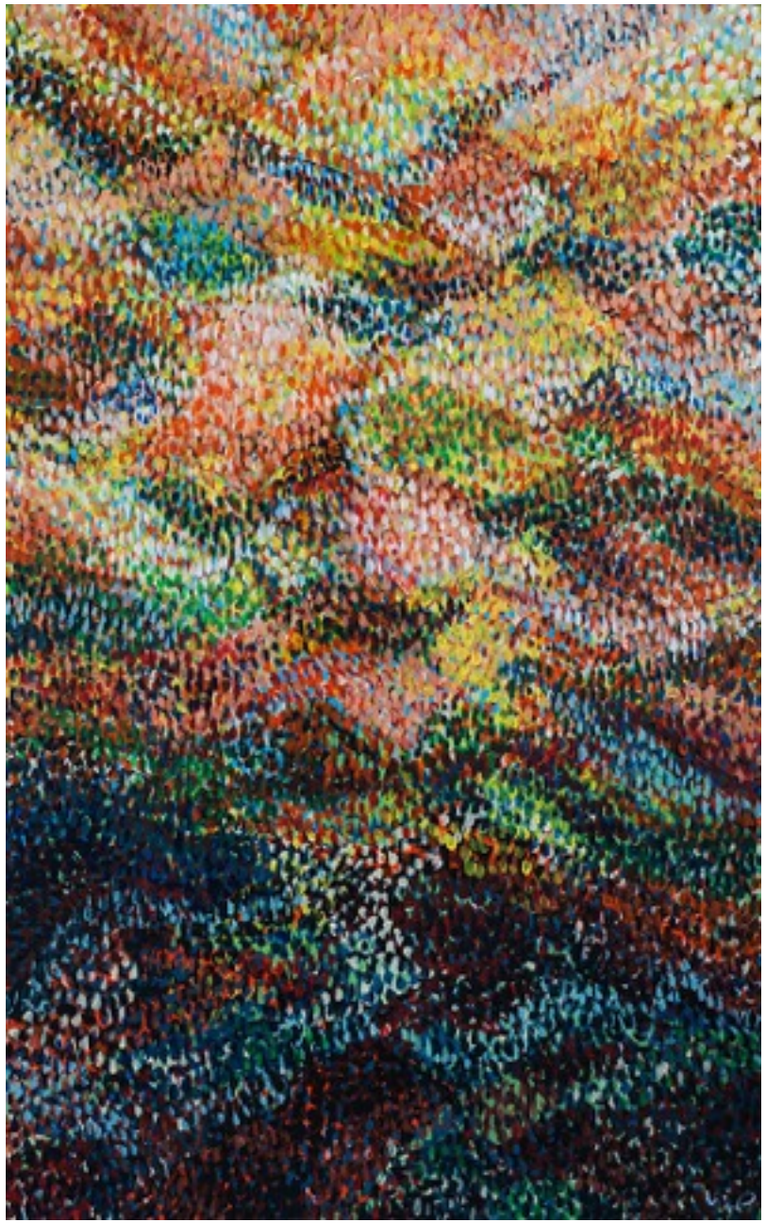
Flavour physics course - Ancillary lecture

Julie Oppermann
1311 - Acrylic on canvas

Ancillary lecture outline

These lectures will give a general overview of the techniques involved in the so-called Dalitz plot analysis:

- Lecture (I):
  - [i] Brief historical introduction
  - [ii] Dalitz plot kinematics
  - [iii] Dynamics description
  - [iv] Isobar Model
- Lecture (II):
  - [i] How to perform a Dalitz-plot fit
  - [ii] Experimental results
  - [iii] Limiting issues / where to go

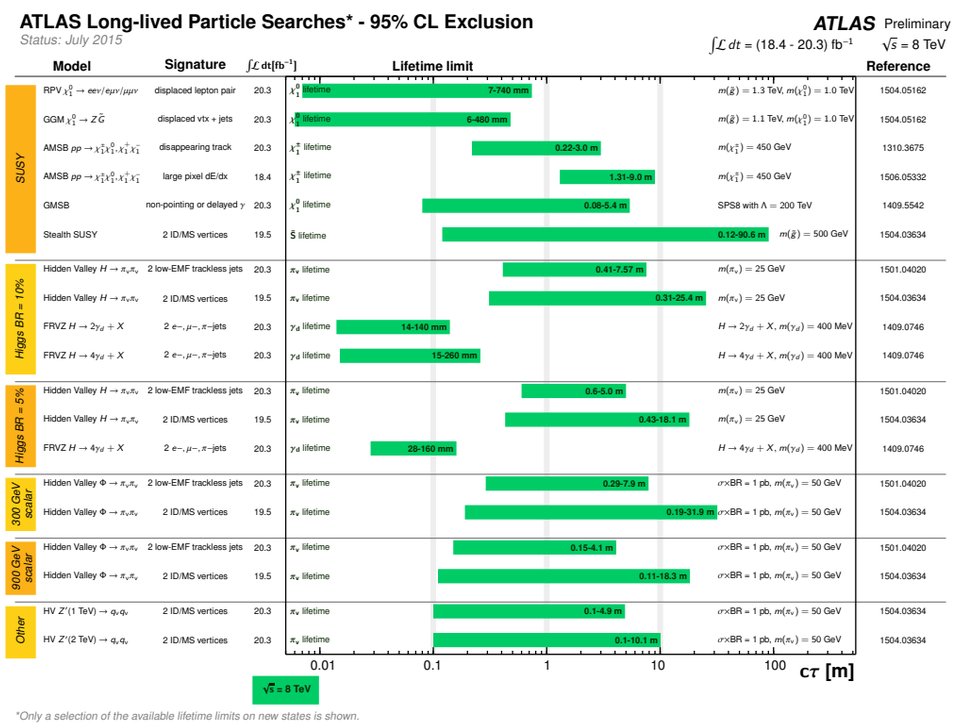


Norm YIP
Ecstasy, No. 1, 2015

Heavy flavours - multi-body framework

Flavour physics is an important scenario to search for indirect anomalous effects beyond the SM

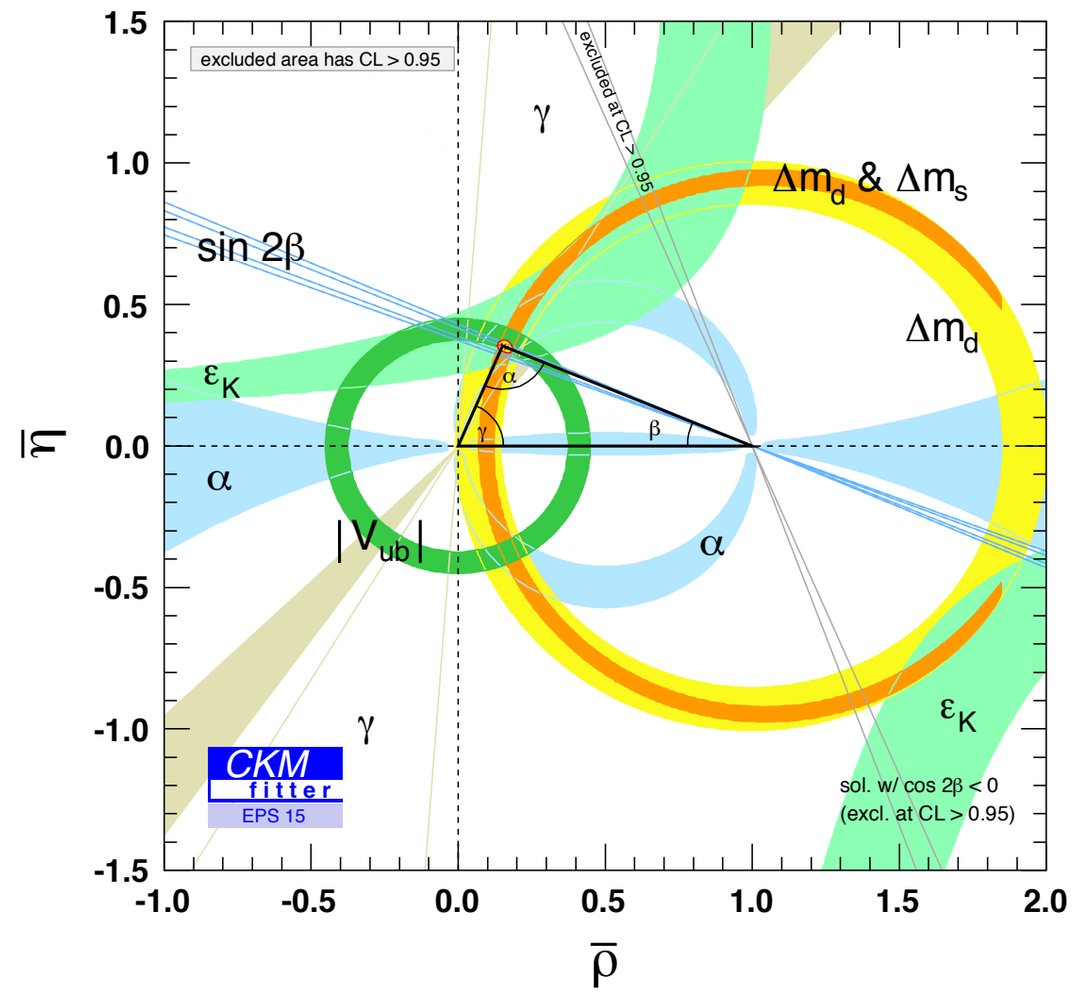
→ Highly sensitivity to effects from new particles (can access higher scales)



Absence of NP signals at ATLAS/CMS

→ flavour physics can indicate the path

CKMfitter Group - Eur. Phys. J. C41, 1-131 (2005)
updated results: http://ckmfitter.in2p3.fr



Consistency measurements are probes of the SM and are model-independent constraints on NP.

KM mechanism is clearly the dominant source of *CP* violation in B's

Heavy flavours - multi-body framework

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Flavour physics is an important scenario to search for indirect anomalous effects beyond the SM

→ Highly sensitivity to effects from new particles (can access higher scales)

(for quark flavour physics)

Stairway to heaven (1970)

Jimmy Page and Robert Plant

"and she's buying the stairway to heaven [NP]"

"and it's whispered that soon if we all call the tune [extra ~~CP~~? FCNC anomalies?], then the piper will lead us to reason"

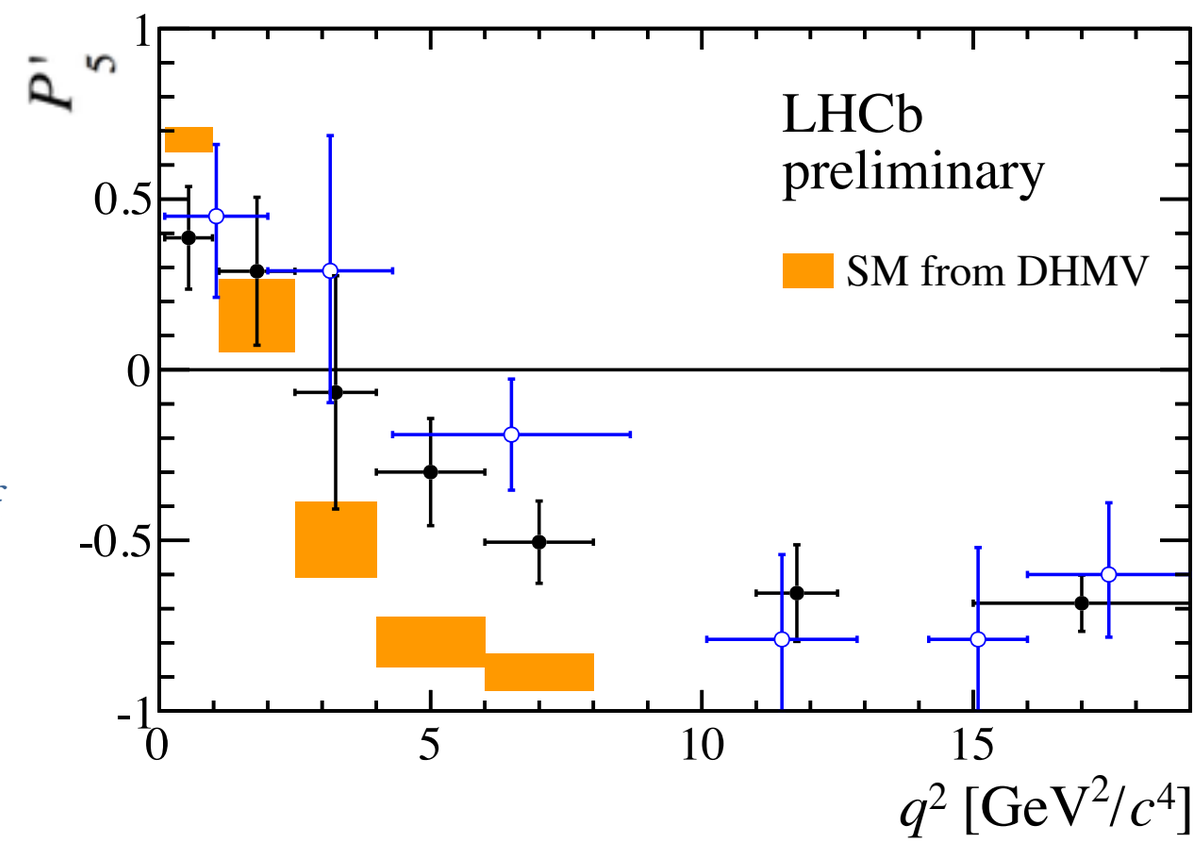
"and a new day will dawn for those who stand long"

Multi-body decays can potentially provide additional information?

LHCb-CONF-2015-002K

LHCb Collaboration

Angular analysis of the $B^0 \to K^{*0}\mu^+\mu^-$



Provides superb laboratory to search for new physics in $b \to sl^+l^-$ FCNC processes: Branching ratio, angular distributions and asymmetries sensitive to NP

Two-body decays physics case

Hadronic 2-body $B \to K\pi$ decays show an interesting A^{CP} pattern:

A^{CP} measurements

	BaBar	Belle
$B^0 \to K^0\pi^0$	$+0.13 \pm 0.13 \pm 0.03$	$+0.14 \pm 0.13 \pm 0.06$
$B^+ \to K^0\pi^+$	$-0.029 \pm 0.039 \pm 0.010$	$-0.011 \pm 0.021 \pm 0.006$
$B^0 \to K^+\pi^-$	$-0.107 \pm 0.016^{+0.006}_{-0.004}$	$-0.069 \pm 0.014 \pm 0.007$
$B^+ \to K^+\pi^0$	$+0.030 \pm 0.039 \pm 0.010$	$+0.043 \pm 0.024 \pm 0.002$

Spectator exchange of d ↔ u: naïvely $B^+ \to K^+\pi^0$ should follow $B^+ \to K^+\pi^-$

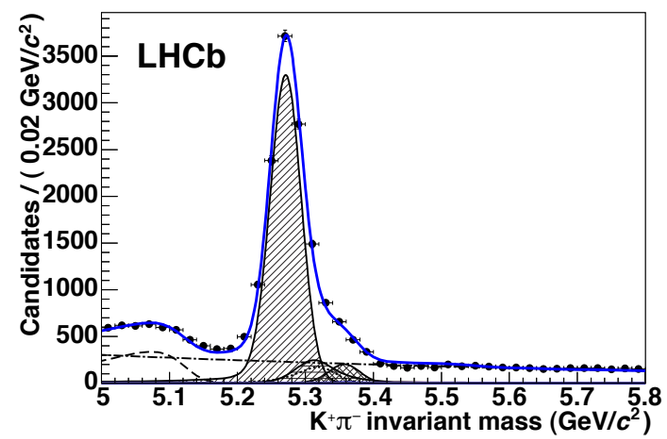
	$B^0 \to K^+\pi^-$	$B^+ \to K^+\pi^0$
LHCb	$-0.088 \pm 0.011 \pm 0.008$	n/a
World average	-0.082 ± 0.006	0.040 ± 0.021

$K\pi$ "puzzle"

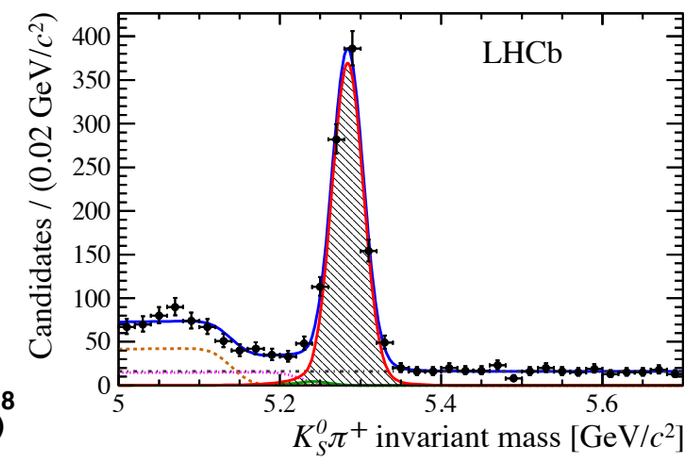
$$\Delta\mathcal{A}^{CP}(K\pi) = 0.122 \pm 0.022 \quad 5\sigma$$

Signature of NP? Need better control on possible QCD effects

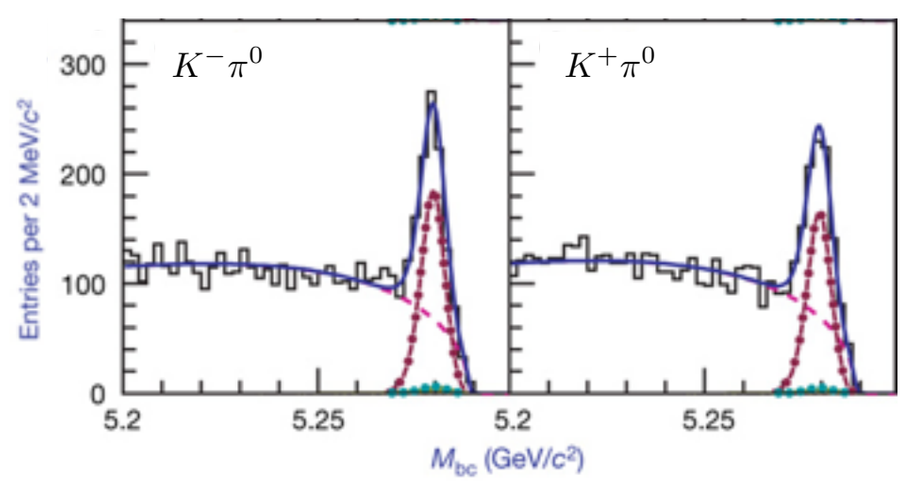
JHEP 1210 (2012) 037
LHCb Collaboration



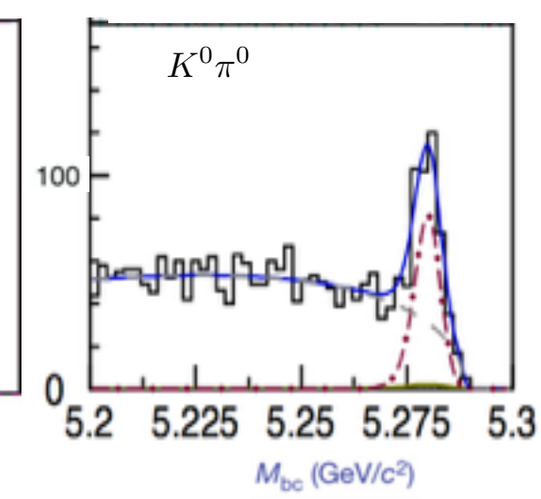
Phys. Lett. B 726 (2013)
LHCb Collaboration



Nature 452 (2008) 332
Belle Collaboration



PRL 99 121601
Belle Collaboration



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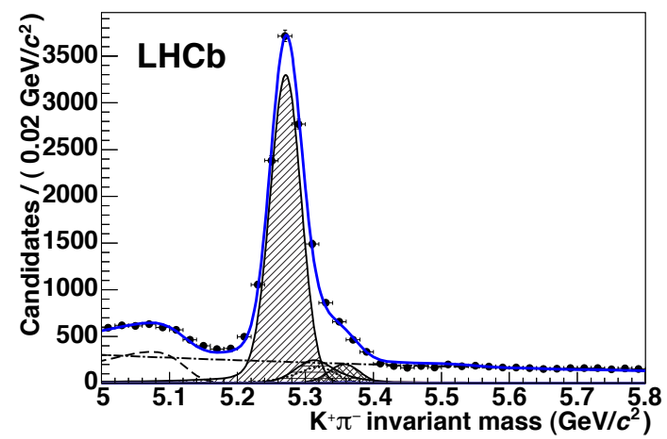

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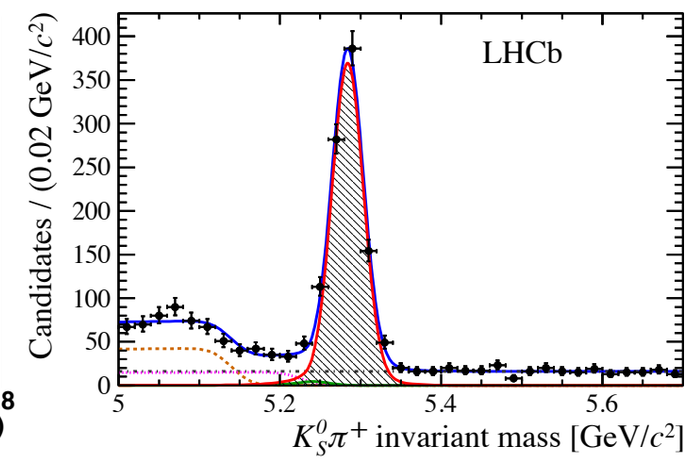
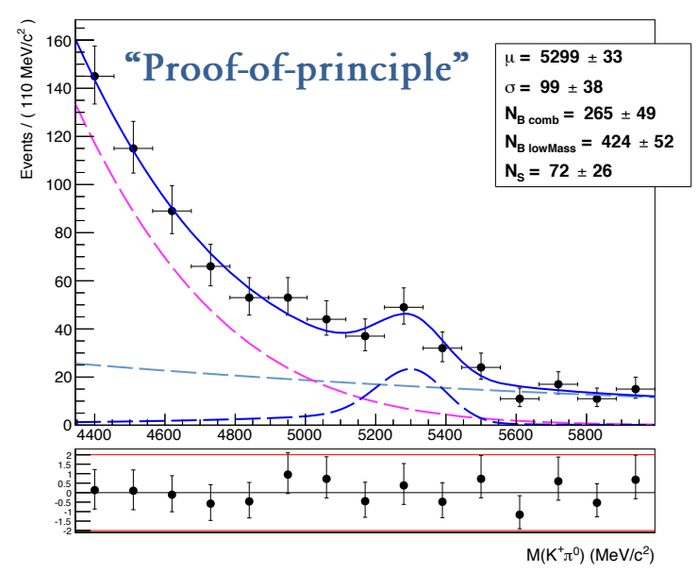
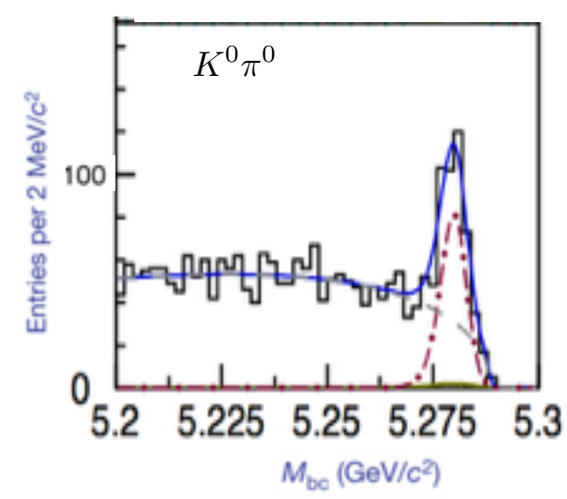


LHCb-CONF-2015-001
LHCb Collaboration



PRL 99 121601
Belle Collaboration



CP violation sensitivity in two-body decays

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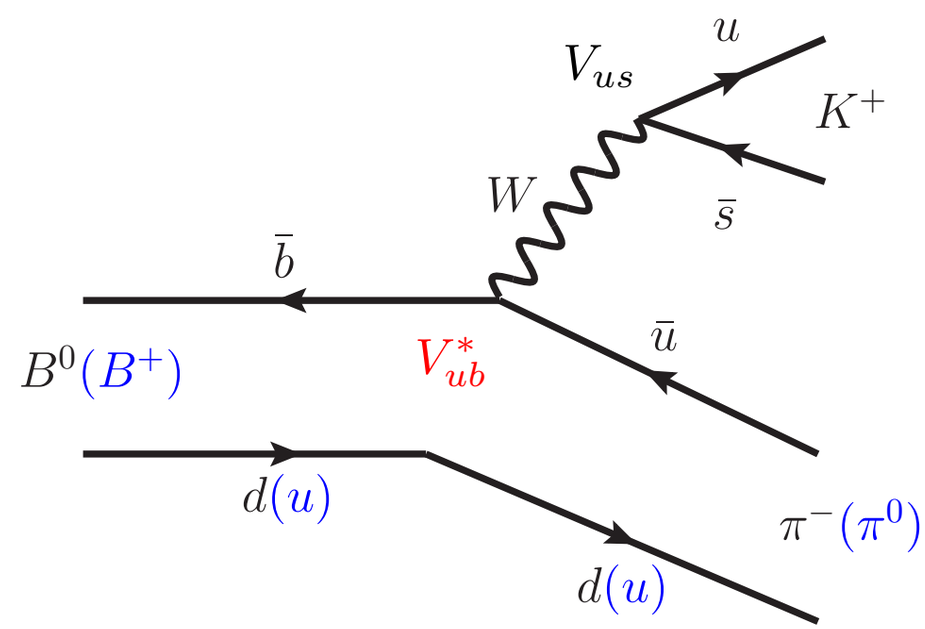
Condition for direct *CP* violation:

$$\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| = \left|\frac{\sum_i A_i e^{i(\delta_i - \phi_i)}}{\sum_i A_i e^{i(\delta_i + \phi_i)}}\right| \neq 1 \quad \Longrightarrow CP \text{ violation}$$

An asymmetry can be obtained as

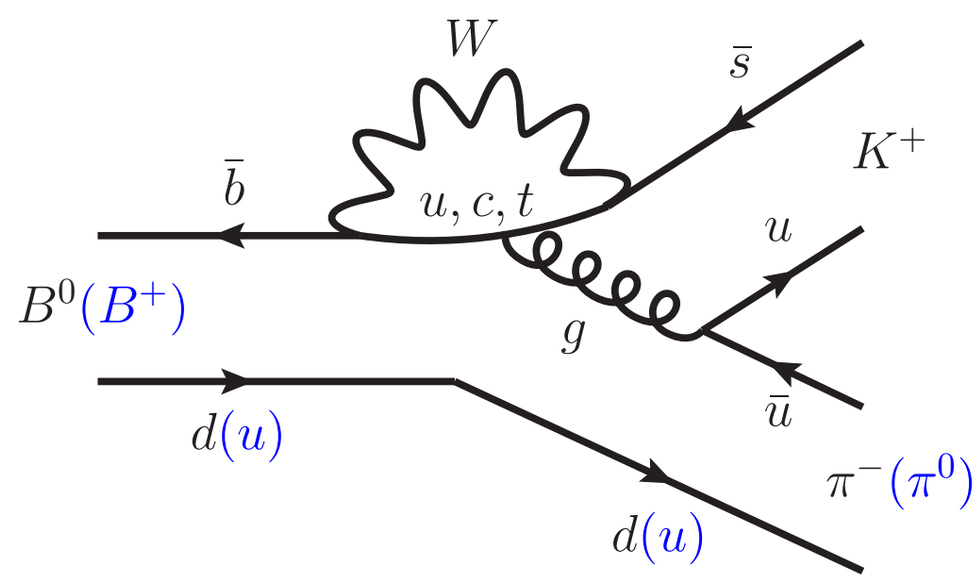
$$\mathcal{A}^{CP} = \frac{|\bar{A}_{\bar{f}}|^2 - |A_f|^2}{|\bar{A}_{\bar{f}}|^2 + |A_f|^2} = \frac{2|A_T||A_P|\sin\delta\sin\phi}{|A_T|^2 + |A_P|^2 + 2|A_T||A_P|\cos\delta\cos\phi}$$

where δ and φ are the *CP* conserving and *CP* violating relative phases between amplitudes.



Limited information available in two-body decays:

- Observables are the Branching ratio and A^{CP};
- Unknowns parameters: A_T, A_P, δ and φ.



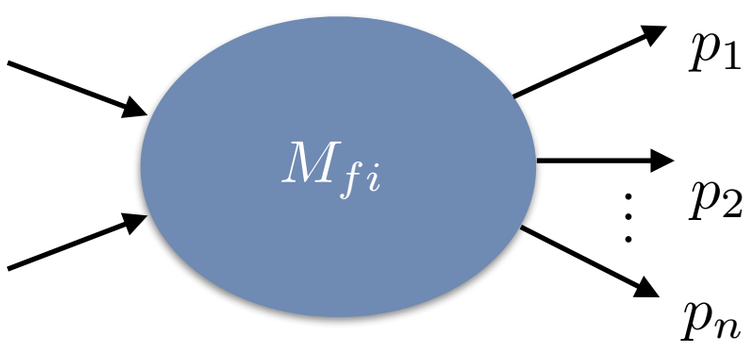
Additional information can be obtained via multi-body decays (n >2).

Interference pattern for multi-channels

Multi-body decays proceed through several intermediate states which interfere

→ The phase-space diagram of this process shows a interference pattern analogous to the double-slit experiment

The coupling of initial and final states is given by the invariant amplitudes of the process

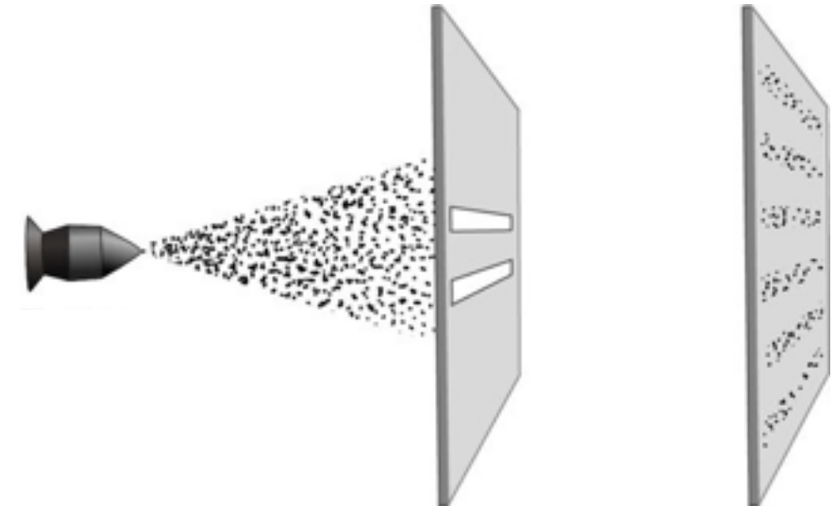


Fermi's Golden rule

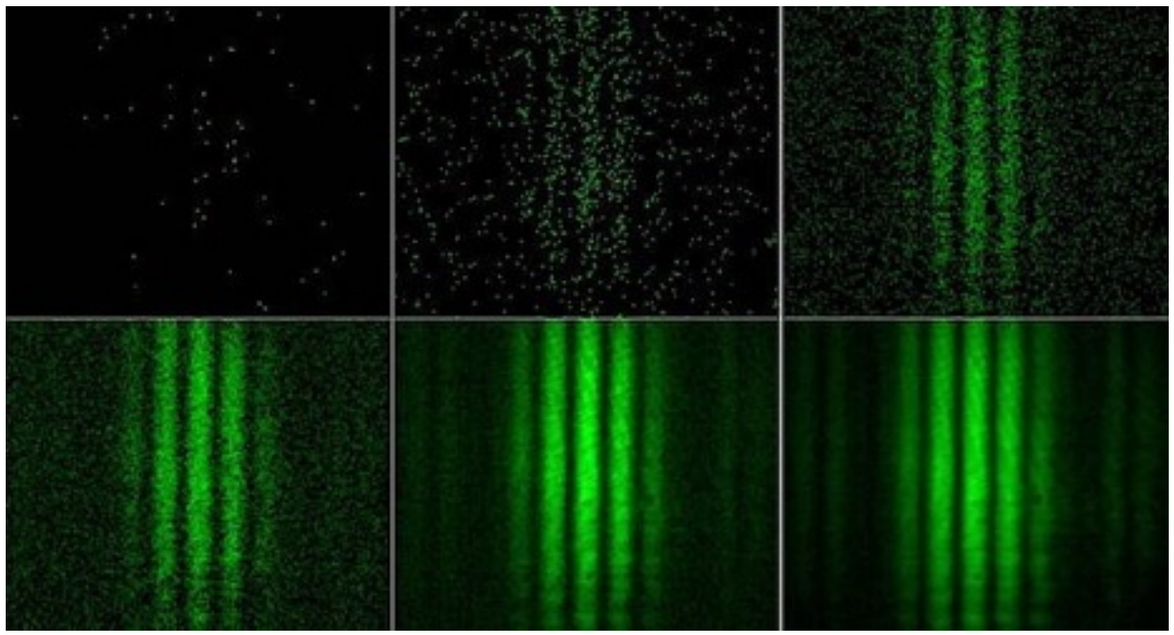
$$d\sigma \propto |M_{fi}|^2 d\phi_N$$

Matrix element

N-body phase-space element



Wave-particle duality of light for the classroom
A. Weis and T. L. Dimitrova



A bit of history: Dalitz plot

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"I visualise geometry better than numbers"

Richard Dalitz (1925-2006)

"On the analysis of tau-meson data and the nature of the tau-meson."

R. H. Dalitz, Phil. Mag. 44 (1953) 1068

θ^+/τ^+ Puzzle

The decay $\theta^+ \to \pi^+\pi^0$ has been observed for the first time in 1953. Assuming parity conservation:

$$P(\theta^+) = P(\pi^+)P(\pi^0)(-1)^L = (-1)^L$$

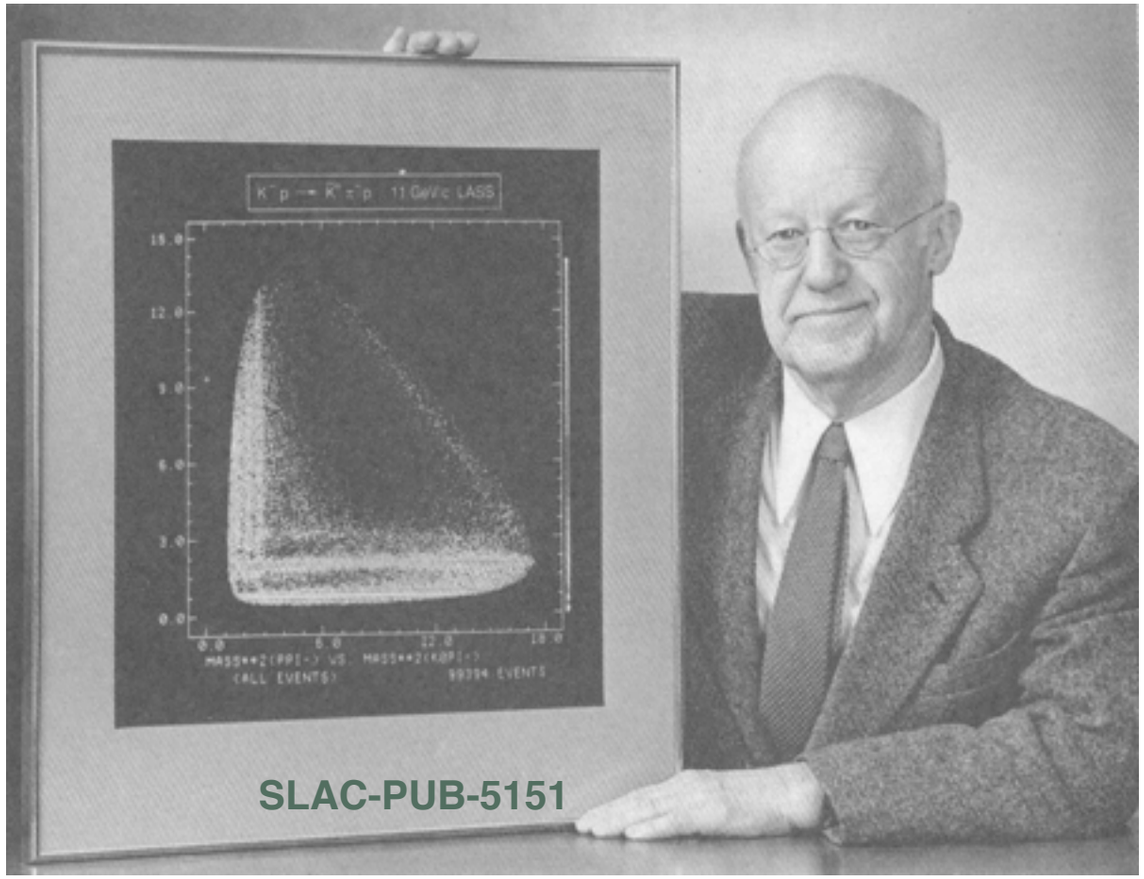
$$J^P = 0^+, 1^-, 2^+, ...$$

The decay $\tau^+ \to \pi^+\pi^-\pi^+$ was also observed in 1949 with mass consistent with theta and

$$P(\tau^+) = P(\pi^+)P(\pi^-)P(\pi^+)(-1)^{l_{12}}(-1)^L$$

$$= -1(-1)^{l_{12}}(-1)^L$$

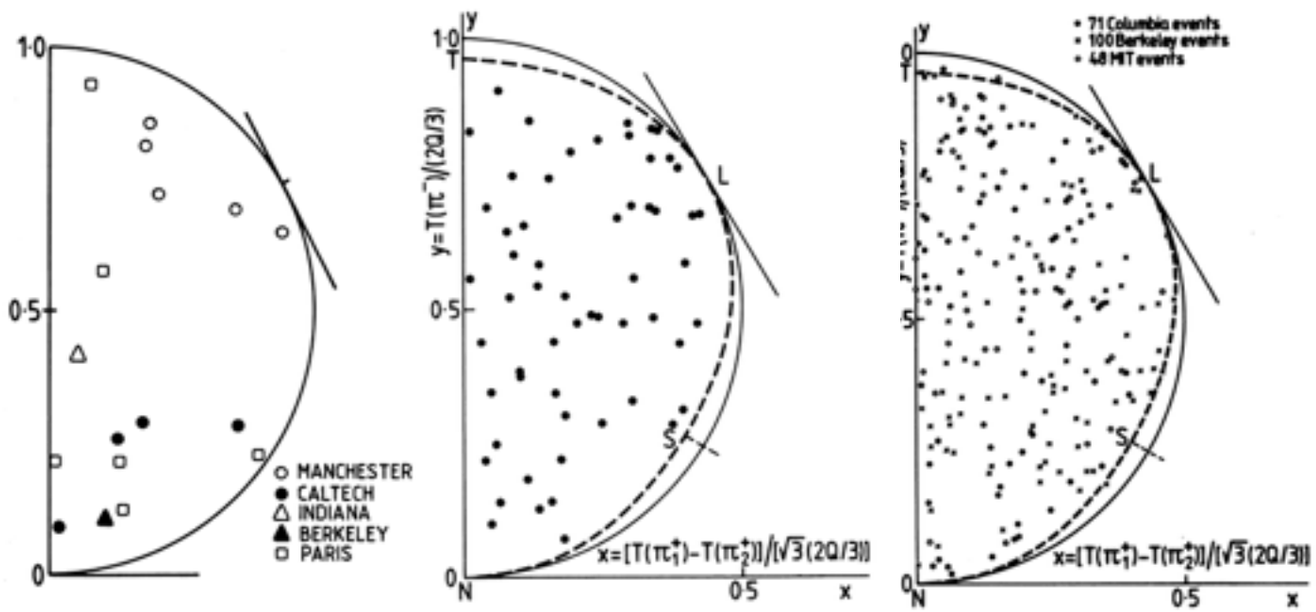
$$J^P = 0^-, 1^\pm, 2^\pm, 3^\pm ...$$

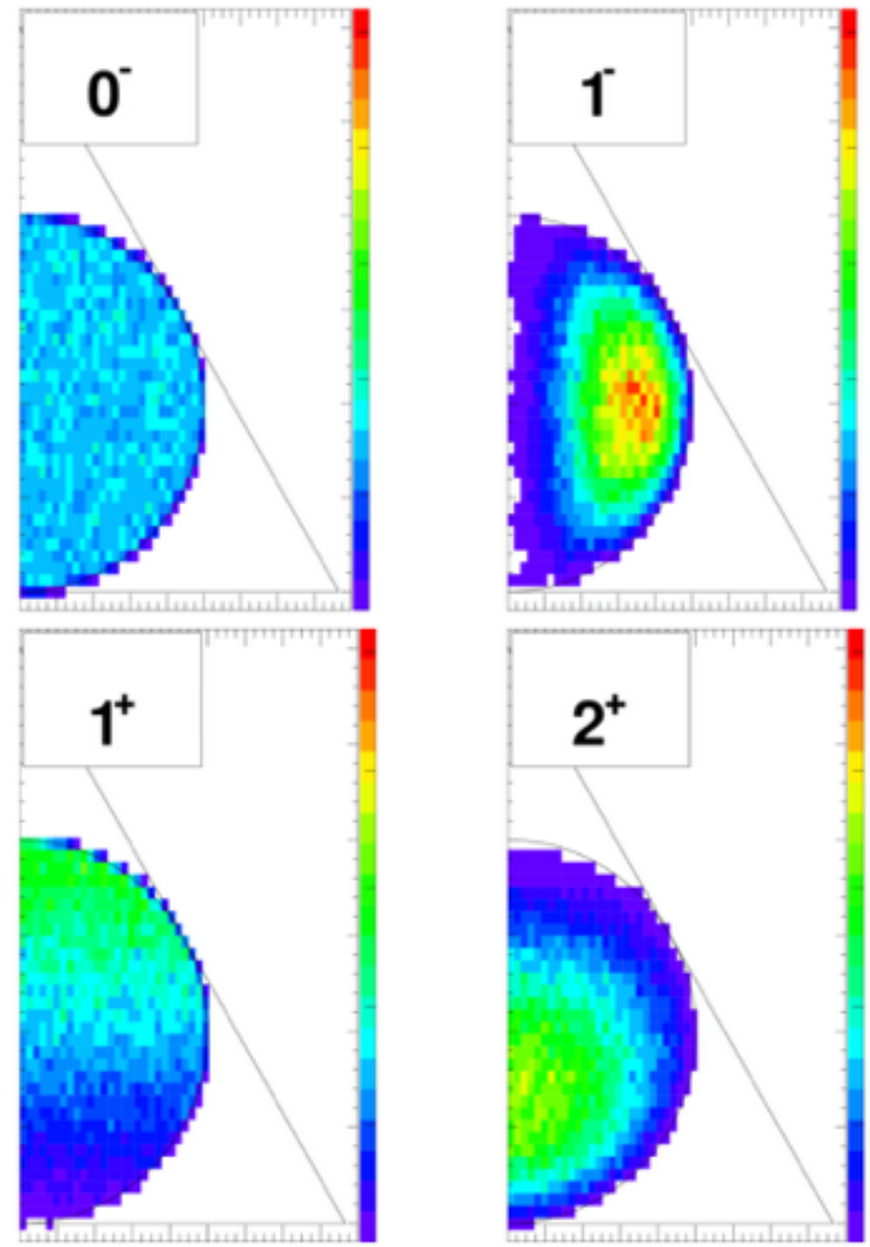


"A work of art" - gift from B. Ritcher, W. Panofsky, S. Drell, D. Leith, D. Aston, W. Dunwmoodie and B. Ratcliff

A bit of history: Dalitz plot

Dalitz noted that since the sum of the pion energies was a constant, $E_1 + E_2 + E_3 = Q$, each event could be specified by two energies and indicated on a two-dimensional plot





- Are τ and θ different states?
- P violation in weak interactions?
- The particles τ and θ have been later associated to the same particle, now known as the K^+ meson.

Three-body decays (spinless) kinematics

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The transition rate in perturbation theory is governed by the Fermi Golden rule, which gives for a particle with mass M decaying into n bodies with masses m_i and four-momenta p_i, the relation

$$\Gamma = \frac{(2\pi)^4}{2M} \int |\mathcal{A}|^2 \delta^4(p - \sum_{i=1}^{n} p_i) \prod_{i=1}^{n} \frac{d^4 p_i}{(2\pi)^3} \delta(p_i^2 - m_i^2)\,,$$

Consider a generic decay of a pseudo-scalar meson at rest, with mass M and four-momentum $P^\mu = (M,0)$, to three particles with masses m_i, four momenta $p^\mu{}_i = (E_i, \mathbf{p}_i)$ and energies E_i, where $i = 1, 2, 3$. Defining Lorentz invariant masses

$$\begin{aligned} m_{ij}^2 &= (p_i^\mu + p_j^\mu)^2 \;=\; m_i^2 + m_j^2 + 2E_iE_j - 2\vec{p}_i \cdot \vec{p}_j \\ &= (P^\mu - p_k^\mu)^2 \;=\; M^2 + m_k^2 - 2ME_k \end{aligned}$$

where the relation

$$m_{12}^2 + m_{13}^2 + m_{23}^2 = M^2 + m_1^2 + m_2^2 + m_3^2$$

constrains the system to two independent values of $m^2{}_{ij}$

Three-body decays (spinless) kinematics

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For n = 3 final state spin-0 particles further constraints can be driven:

Lorentz four-vector	→	12
Meson masses	→	-3
p, E conservation	→	-4
Arbitrary orientation	→	-3
Independent variables		**2**

Graphical representation of the 3-body phase-space: *i.e.* **Dalitz plot!**

(note that this term often abused to refer to phase-space of any multi-body decay)

The conservation of four-momentum of the reaction restricts the events into a closed region of the phase space. The contours of the DP for three-body decays are defined as

$$\Gamma = \frac{1}{2(2\pi)^5 M} \int |\mathcal{A}|^2 \delta^4(p - p_1 - p_2 - p_3) \frac{d\vec{p}_1}{2E_1} \frac{d\vec{p}_2}{2E_2} d^4 p_3 \delta(p_3^2 - m_3^2)$$

which is constrained by the four-dimensional delta function.

Three-body decays (spinless) kinematics

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The $\delta(p^2_3\, m^2_3)$ term enforces real (on-shell) particles in the final state, in contrast to the possible virtual particles involved in intermediate states. In the centre-of-mass (CM) reference frame, fixing the direction of $\mathbf{p}_1$ and integrating initially in d^4p_3, gives

$$\Gamma = \frac{\pi^2}{2(2\pi)^5 M} \int |\mathcal{A}|^2 \delta_{\cos\theta_{12}} dE_1 dE_2 d\cos\theta_{12}$$

where the angle between $\mathbf{p}_1$ and $\mathbf{p}_2$ is given by

$$\delta_{\cos_{\theta_{12}}} = \delta\left(\cos\theta_{12} - \frac{M^2 + m_1^2 + m_2^2 - m_3^2 - 2M(E_1 + E_2) + 2E_1E_2}{2\vec{p}_1\vec{p}_2}\right)$$

$$E_3 = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos\theta_{12} + m_3^2}$$

Integrating this expression in the cosine results in the description of the decay rate as

$$\boxed{\Gamma = \frac{1}{256\pi^3 M^3} \int |\mathcal{A}|^2\, dm_{ij}^2\, dm_{jk}^2}$$

Three-body decays (spinless) kinematics

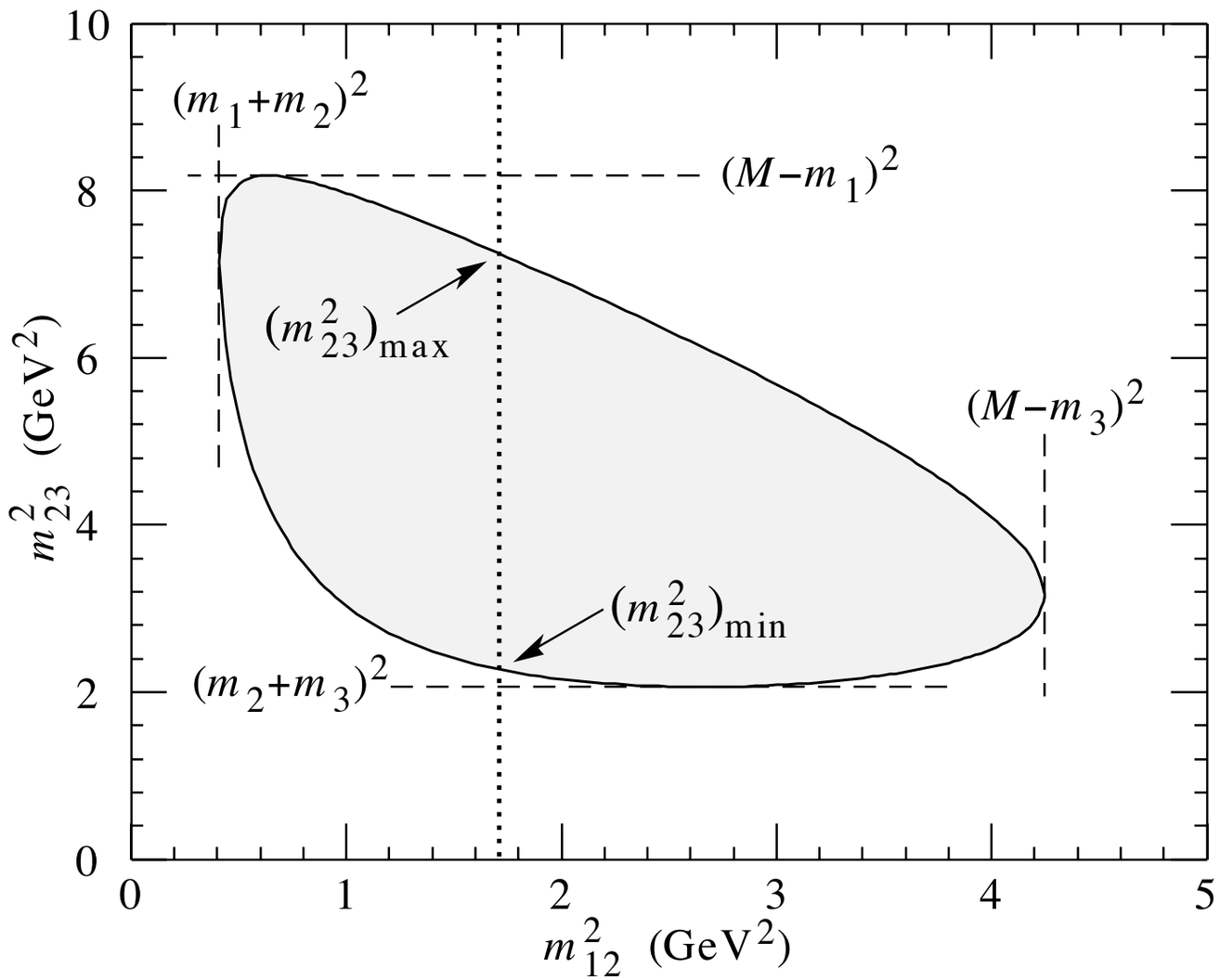
The kinematic boundaries of the Dalitz plot are constrained by the points where $\cos^2\theta_{12} = 1$.

The extrema of this relation within the physical region are underlined in each invariant axis as

$$(m_i + m_j)^2 \leq m_{ij}^2 \leq (M - m_k)^2$$

Interpretation:

- Minimum of m^2_{ij} is attained with $\cos^2\theta_{ij} = 1$, which implies $\theta_{ij} = 0$ and $\theta_{ik} = \theta_{jk} = \pi$
  - Momenta of particles i and j are collinear and opposite to particle k



Particle Data Group Collaboration
PRD 86 (2012) 010001

Three-body decays (spinless) kinematics

An alternative to the conventional parametrisation of the phase space can be obtained by a transformation to a rectangular plane (square DP).

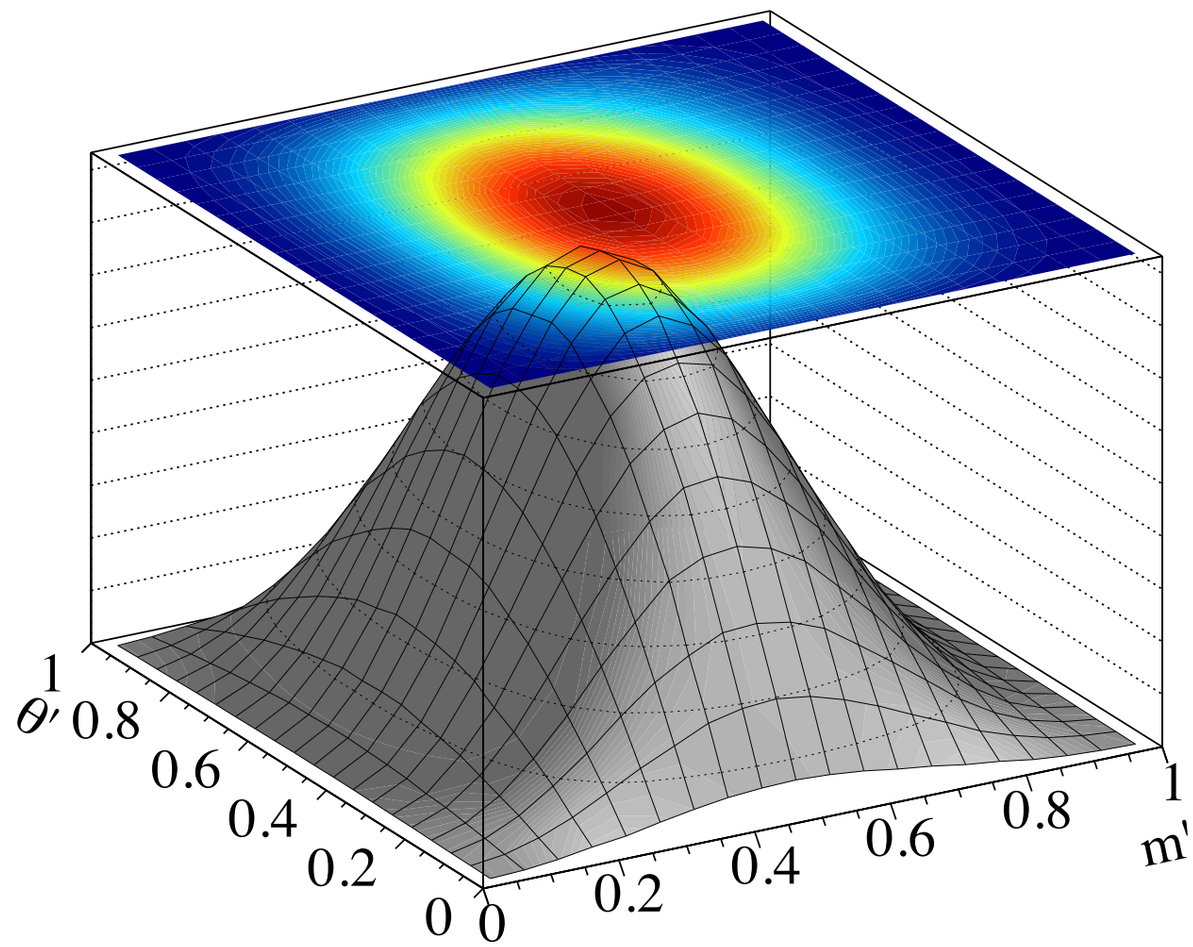
Advantages:

- Enlarge most populated signal region
- Reduce binning dependence for combinatorial background

$$dm_{ij}^2\, dm_{jk}^2 \longrightarrow |\det J|\, dm'\, d\theta'$$

$$m' \equiv \frac{1}{\pi} \arccos\left(2\frac{m_{ij} - m_{ij}^{\min}}{m_{ij}^{\max} - m_{ij}^{\min}} - 1\right)$$

$$\theta' \equiv \frac{1}{\pi}\theta_{ij},$$



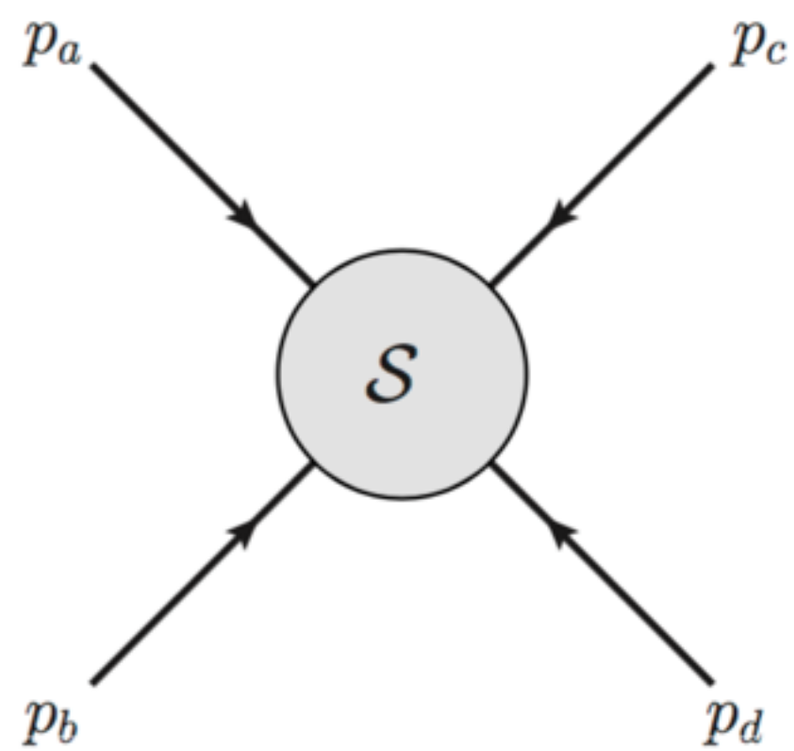
The determinant of the Jacobian is given by

$$|J| = 4\,|\mathbf{p}_{i,j}^*||\mathbf{p}_k^*| \cdot \frac{\partial m_{ij}}{\partial m'} \cdot \frac{\partial \cos\theta_{ij}}{\partial \theta'}$$

Basis of the S-matrix theory

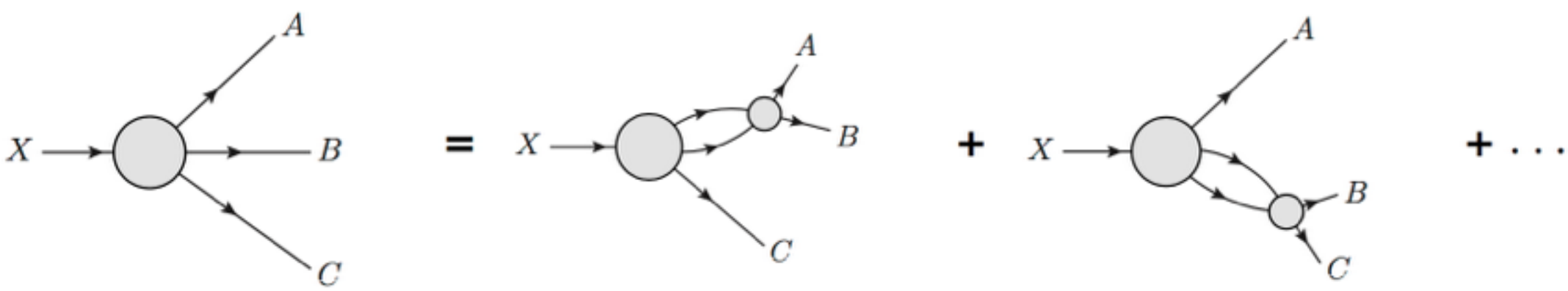
The appearance of structures in the topology of the phase space is related to the nature of the transition amplitude

The S-matrix theory binds the physical observables of all initial states to every possible final state



Analogy:

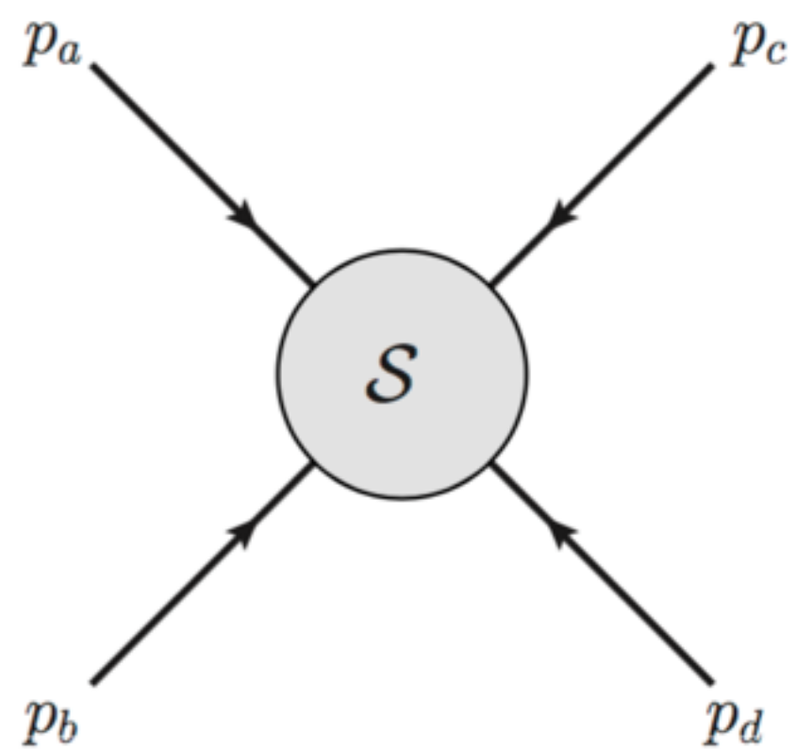
- Wave function of a single particle state (Non-relativistic QM) - modulus square of its matrix elements is related to the probability that any initial state is connected to some particular final state in a scattering process



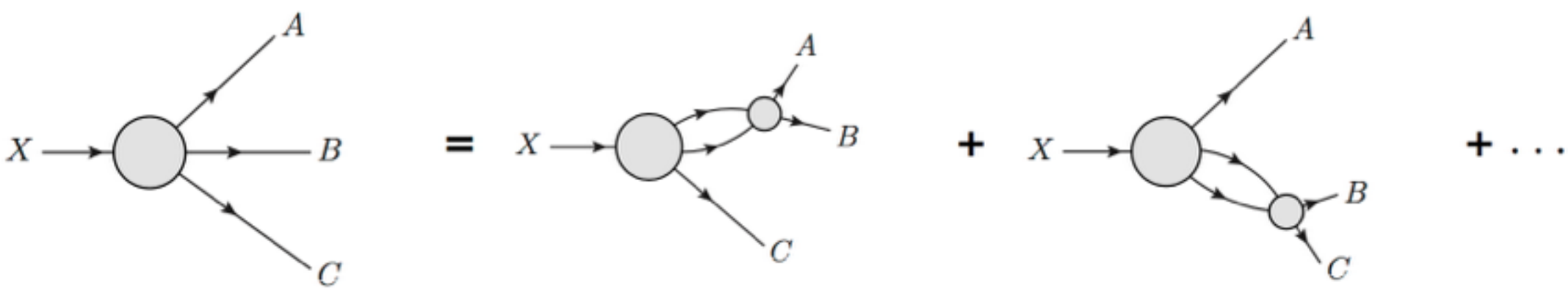
Basis of the S-matrix theory

The scattering amplitude for the initial state transition to a final state is connected to the unitary S-matrix by a scattering amplitude

$$\mathcal{S}_{fi} = \langle f|i\rangle - 2\pi i\langle f|\mathcal{M}|i\rangle = \mathbb{1} - 2\pi i\langle f|\mathcal{M}|i\rangle$$



- Low energy: scattering is dominated by resonances, that in the absence of significant overlaps, are identified as distinct enhancements in the cross-section
- High energies: smoother behaviour related to crossed channel Regge exchange (not covered here)



Mandelstam hypothesis (physical region)

Consider the elastic scattering of two spinless particles in the s-channel:

$$p_1 + p_2 \to p_1' + p_2' \qquad \text{s-channel} \quad \text{(total energy in the CM system)}$$

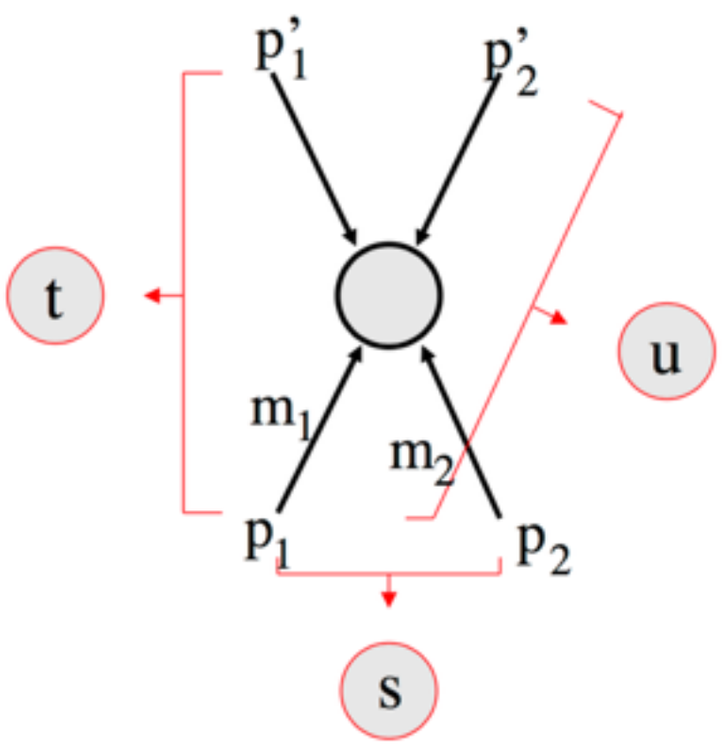
$$p_1 + p_1' \to p_2 + p_2' \qquad \text{t-channel} \quad \text{(four-momentum transferred)}$$

$$p_1 + p_2' \to p_2 + p_1' \qquad \text{u-channel}$$

$$4m^2 \leq s$$

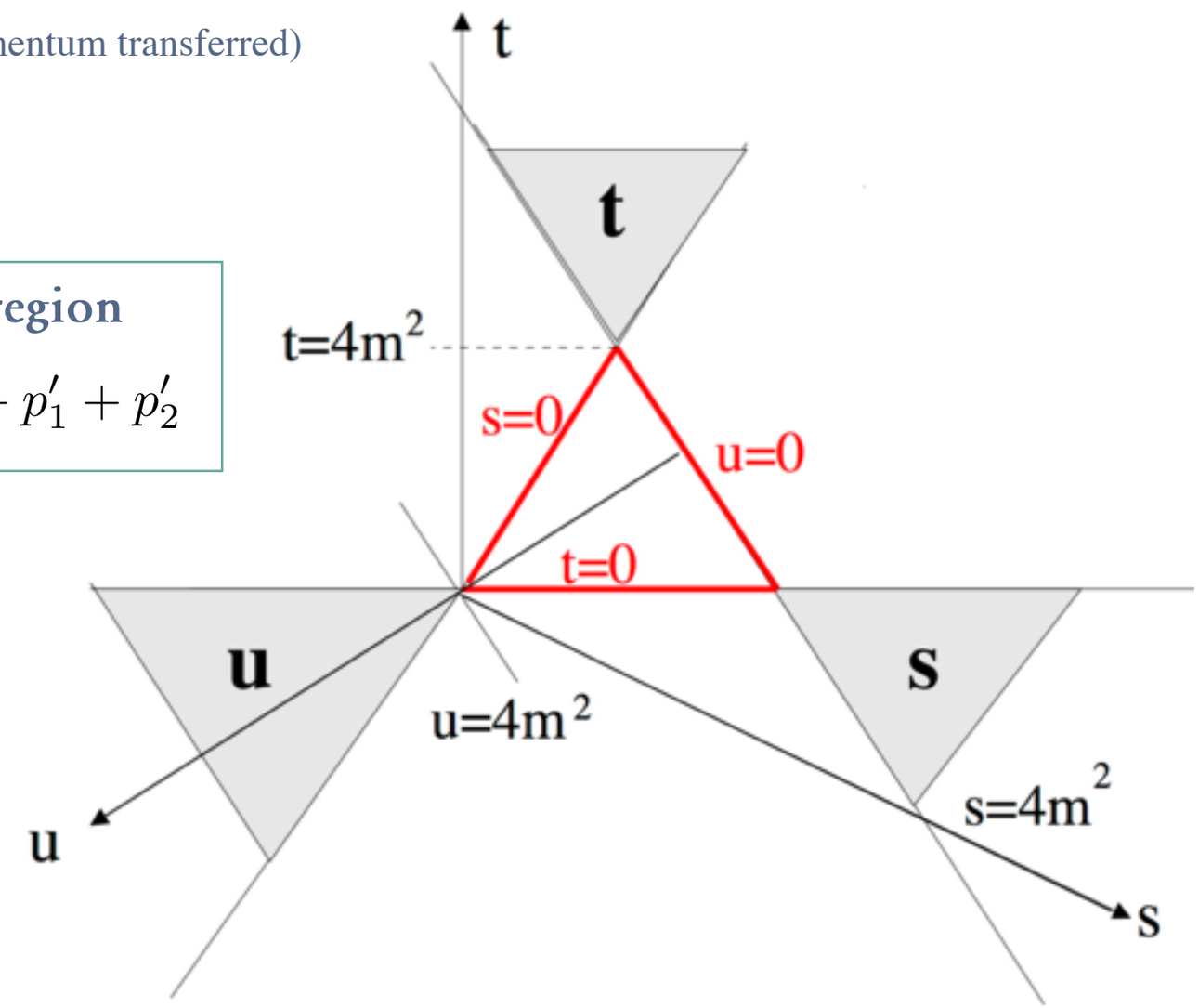
$$t_{\min} \leq t \leq 0$$

$$t_{\min} = -4q^2 = 4m^2 - s$$



Decay region

$$p_1 \to p_2 + p_1' + p_2'$$



Scattering amplitude for 2-body

The determination of the scattering amplitude is the main goal of an amplitude analysis

(i) **The differential cross-section on the plot is measure. How to obtain the scattering amplitude for this or any case?**

Consider the Fermi Golden rule for two-body scattering:

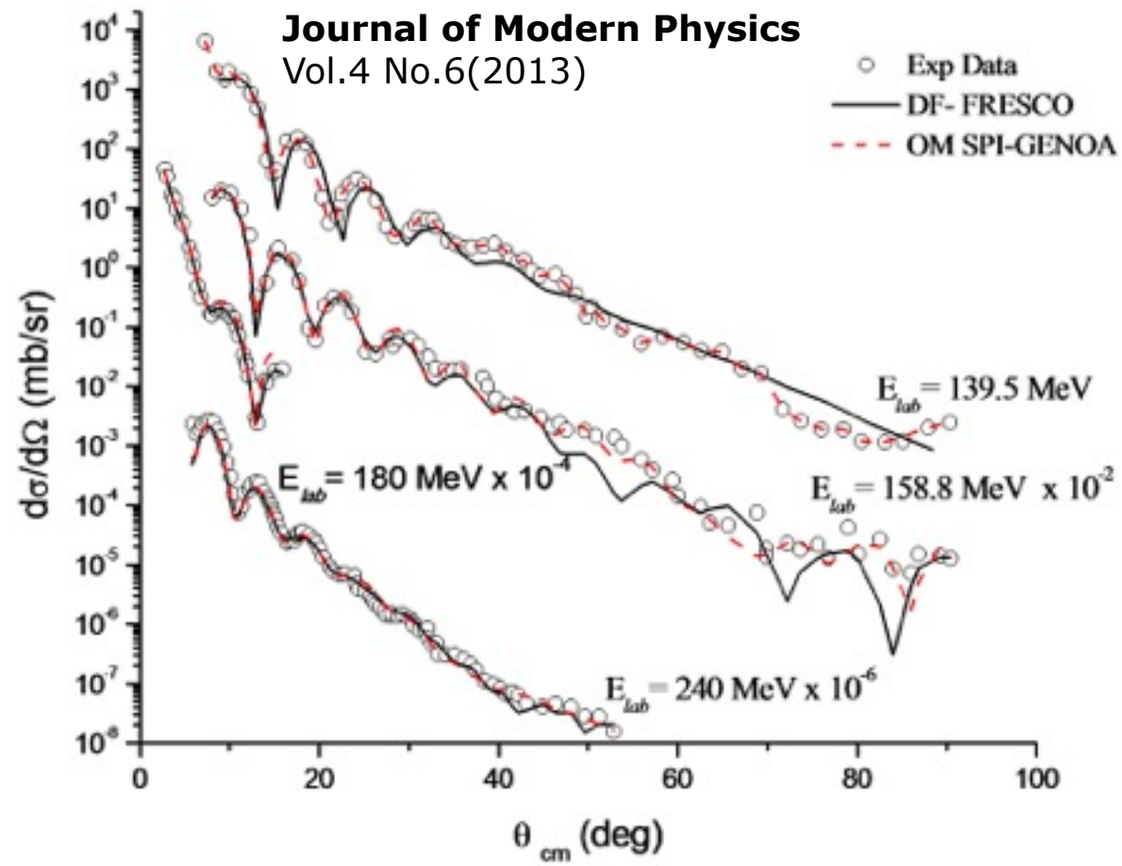
$$\frac{d\sigma_{\text{elastic}}}{d\Omega} = \frac{1}{64\pi^2 s}|\mathcal{M}(s,z)|^2$$

where:

$$z = \cos\theta \quad \text{from} \quad t = 2p^2(\cos\theta - 1)$$

It is convenient to expand the scattering amplitude in a series of partial waves

$$|\mathcal{M}(s,z)| = 16\pi \sum_{l=0}^{\infty}(2l+1)f_l(s)P_l(z)$$



Performing an angular integration using the property that the Legendre functions are a complete basis between -1 and 1:

$$\sigma_{\text{elastic}}(s) = \frac{16\pi}{s}\sum_{l=0}^{\infty}(2l+1)|f_l(s)|^2$$

Unitarity and analyticity of the S-matrix

Universität Zürich[UZH]

The unitarity of the S-matrix imposes important restrictions on the scattering amplitude

Transition probabilities are given by

$$\sum_n P_n = 1$$

Unitarity of scattering-matrix

$$P_n = |\langle n \mid \mathcal{S} \mid i \rangle|^2$$

$$\sum_n \langle i \mid \mathcal{S}^\dagger \mid n \rangle \langle n \mid \mathcal{S} \mid i \rangle = 1$$

$$\boxed{\mathcal{S}^\dagger \mathcal{S} = \mathcal{I}}$$

As we've seen before:

$$\mathcal{S} = \mathcal{I} + i\mathcal{T}$$

$$\boxed{\mathcal{T} - \mathcal{T}^\dagger = i\mathcal{T}^\dagger \mathcal{T}}$$

Optical Theorem! (Prove it!)

Optical Theorem: connects the imaginary part of the elastic amplitude to the cross-section of all processes evaluated at z = 1

i.e.: In the regime of elastic unitarity:

$$\sigma_{\text{tot}}(s) = \sigma_{\text{elastic}}(s)$$

For each partial wave the following relation is valid

$$\Im\left[f_l(s)\right] = \frac{2p}{\sqrt{s}}|f_l(s)|^2 = \rho(s)|f_l(s)|^2$$

The general solution for partial wave amplitudes

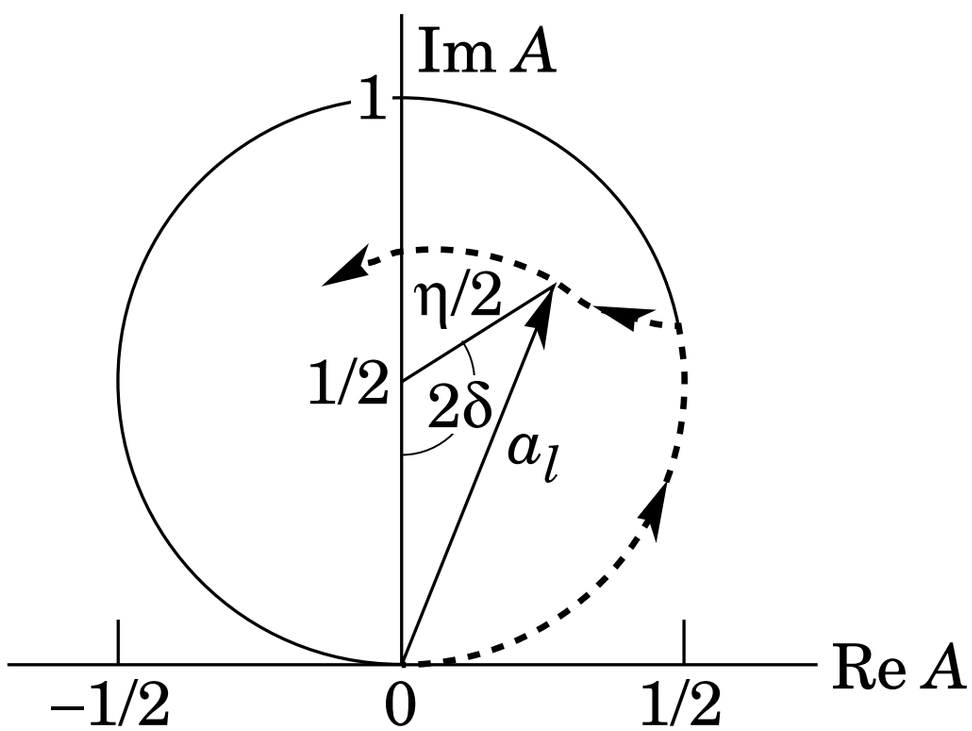
$$f_l(s) = \frac{1}{\rho(s)} \frac{\eta_l e^{2i\delta_l(s)} - 1}{2i}$$

Unitarity and analyticity of the S-matrix

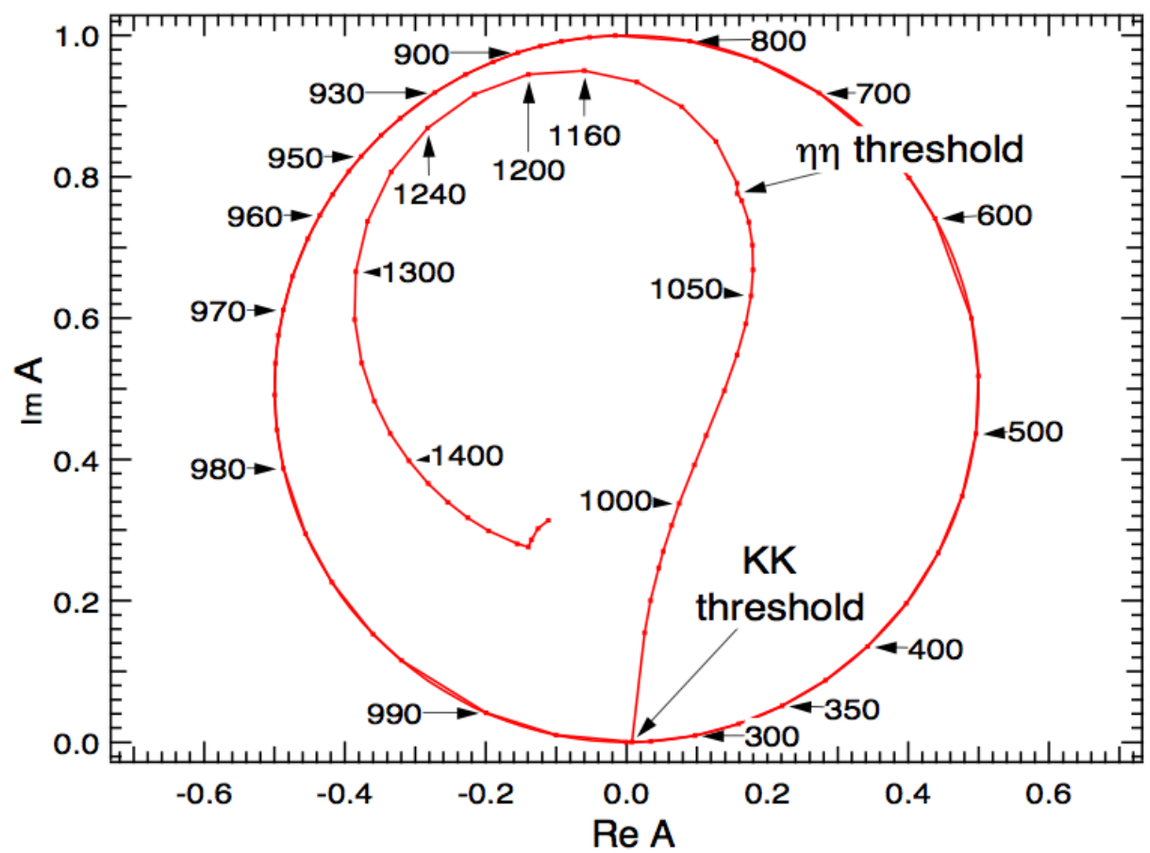
Universität Zürich[UZH]

This methodology formulates elastic scattering as a physical phase shift in the partial wave, originating from the transmission through the interaction region

Evolution of this complex number with energy can be visualised as a trajectory in the Argand plane:



In general the physically allowed region should remain at (elastic) or within this circle (inelastic)



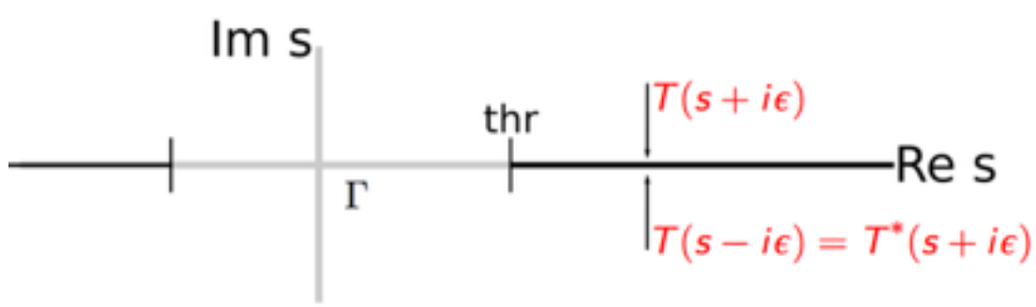
Nils Tornqvist, arXiv:hep-ph/9608464

Unitarity and analyticity of the S-matrix

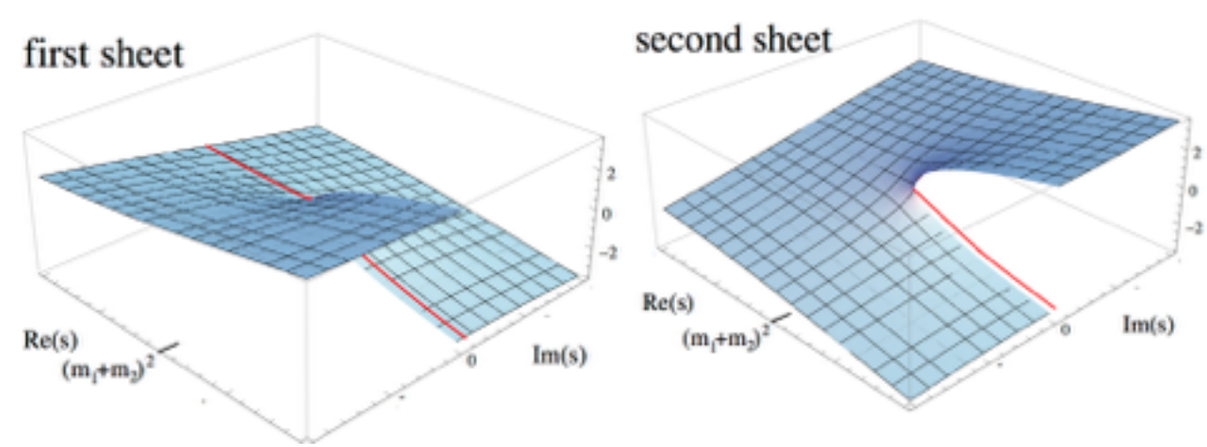
Note that the Mandelstam phase-space factor in the amplitude structure has a clear imaginary dependence. In a nutshell, the analyticity of the amplitude functions lead to

- Analyticity implements causality
- Analytic continuation into the complex s-plane provides strong constraint on shape
- Shape of analytic function defined by its singularities (cuts, poles ..)

There is a discontinuity along the unitarity cut



Particle Data Group Collaboration
PRD 86 (2012) 010001



The singularities on the first Riemann sheet (known as physical region) correspond to the zeros of the S-matrix whilst the **poles** on the second sheet are associated to **resonances**.

Resonant states - initial considerations

Naïve definition: short-lived particles similar to excited spectral lines of atoms

The relativistic wave function of unstable particle is proportional to: $e^{-iMt}e^{-\Gamma t/2}$

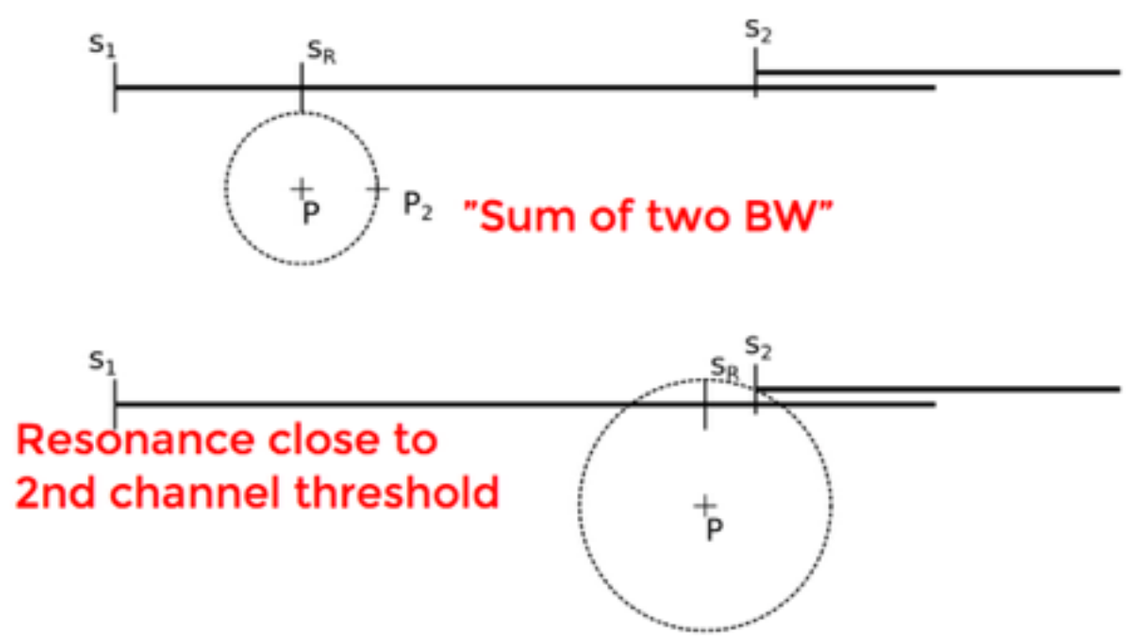
(Dependence is justified by the large uncertainty on the energy associated to the short lifetime)

The propagator for these particles are obtained through a Fourier transformation into the energy space, which results in the so-named **Breit-Wigner formula (BW)** [Prove it!]

In the partial wave representation, the amplitude reads

$$f_l(s) \sim \frac{1}{(M^2 - s) - i\rho\Gamma}$$

This approximation is valid in the region near the pole at $s = M^2 - i\rho\Gamma$ particularly for a single channel and small values of gamma



Dalitz plot analysis - Isobar Model

Universität Zürich[UZH]

Amplitude analysis most commonly performed in the "Isobar Model" framework

Total amplitude is approximated as coherent sum of quasi-two-body contributions:

CP violating — Strong dynamics *CP* conserving

$$\mathcal{A}(m_{ij}^2, m_{jk}^2) = \sum_{r=l}^{N} c_l F_l(m_{ij}^2, m_{jk}^2)$$

c_l : complex coefficients describing the relative magnitude and phase of the different isobars

F_l: dynamical amplitudes that contain the lineshape and spin-dependence of the hadronic part

$$F_j(L, s, t) = R_j(s) \times X_L(|\vec{p}|r) \times X_L(|\vec{q}|r) \times T_j(L, \vec{p}, \vec{q})$$

Resonance mass term (e.g. Breit–Wigner) — Barrier factors - *p*, *q*: momenta of bachelor and resonance — Angular probability distribution

Maximum likelihood fit procedure:

$$\mathcal{L} = \prod_{i}^{N_c} \left[\sum_{k} N_k \mathcal{P}_k(m_{12}^2, m_{23}^2) \right]$$

Model dependent approach

See Lecture (II)

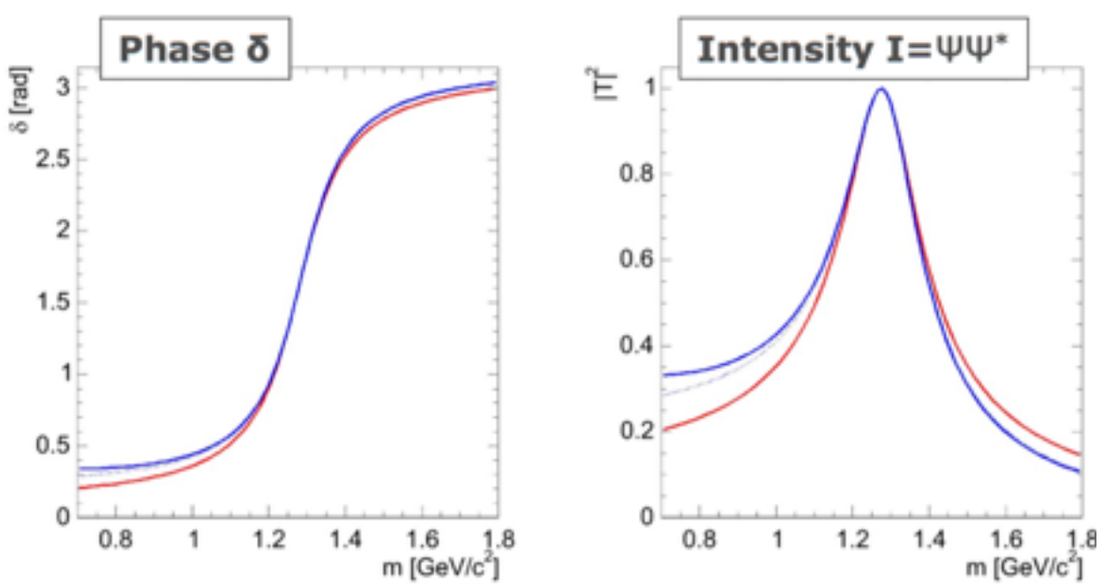
Alternative approaches: **partial wave analysis**

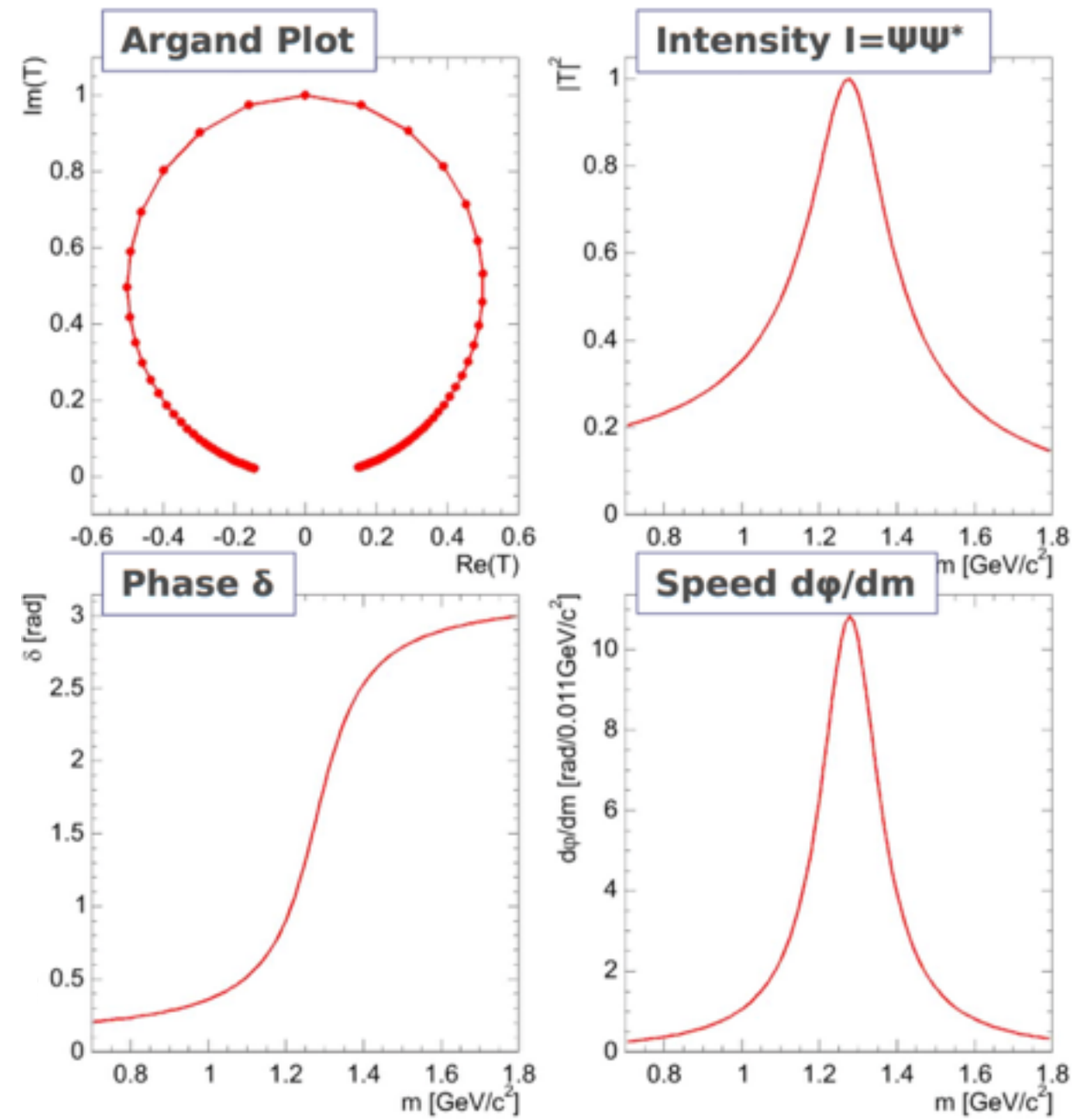
Mass line shape (RBW)

The simplified modelling of single-channel single-pole resonances using a BW can be extended to the relativistic case as

$$R(s) = \frac{1}{(m_0^2 - s) - i m_0 \Gamma}$$

$$\Gamma = \Gamma_0 \left(\frac{q}{q_0}\right)^{2L+1} \left(\frac{m_0}{\sqrt{s}}\right) X_L^2(|\vec{q}|r)$$





Blatt-Weisskopf barrier factors

Near to the threshold, the conservation of angular momentum implies that the amplitude is proportional to $\Gamma_r \propto q^{2l+1} = \rho q^{2l}$

Issue: Correction needed to prevent the amplitude from diverging at infinity

Empirical centrifugal barrier: Blatt-Weisskopf

$$X_{L=0}(z) = 1$$

$$X_{L=1}(z) = \sqrt{\frac{1+z_0^2}{1+z^2}}$$

$$X_{L=2}(z) = \sqrt{\frac{z_0^4+3z_0^2+9}{z^4+3z^2+9}}$$

where z_0 represents the value of z when the invariant mass is equal to the pole mass of the resonance.

Key message: mass dependence of the width of the resonance

Additional remarks:

(i) Alternative parametrisation of the Blatt-Weisskopf is found in the literature

(ii) In this formulation, the BW function can be expressed as

$$1/\cot\delta - i \quad \text{or} \quad \sin\delta e^{i\delta}$$

Angular distribution

In contrast to scalar resonances which have no preferential direction for the daughters, vector and higher spin states present non-trivial angular distributions

The Lorentz-invariant decay amplitudes in the spin-1 case are defined as

$$\mathcal{A}(R \to P_i P_j) = X_L^R \epsilon_\mu(m)(p_j - p_i)^\mu$$

$$\mathcal{A}(B \to R P_k) = X_L^B \epsilon_\nu(m) p_k^\nu$$

Examining the product of these amplitudes and summing over the polarisation

$$X_L^R X_L^B (-2\vec{p}_i \cdot \vec{p}_k)$$

The generalisation of this angular term for arbitrary integer spin has been developed by Zemach in terms of Legendre polynomials

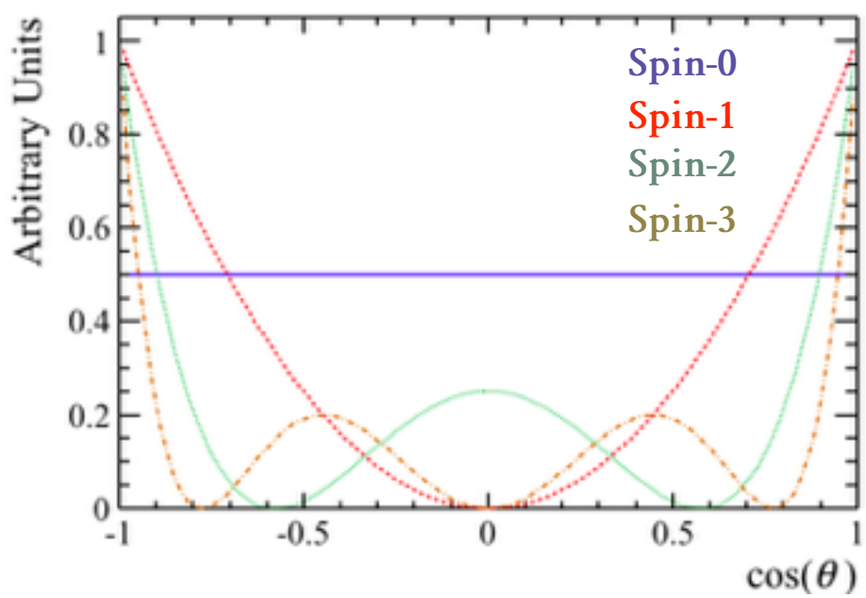
$$(-2\vec{p}_i \cdot \vec{p}_k)^J P_J(\cos\theta_{ik})$$

Some examples are shown in the following slides

Dalitz plot analysis features

Intensity along bands indicates magnitude and the spin of the given resonance

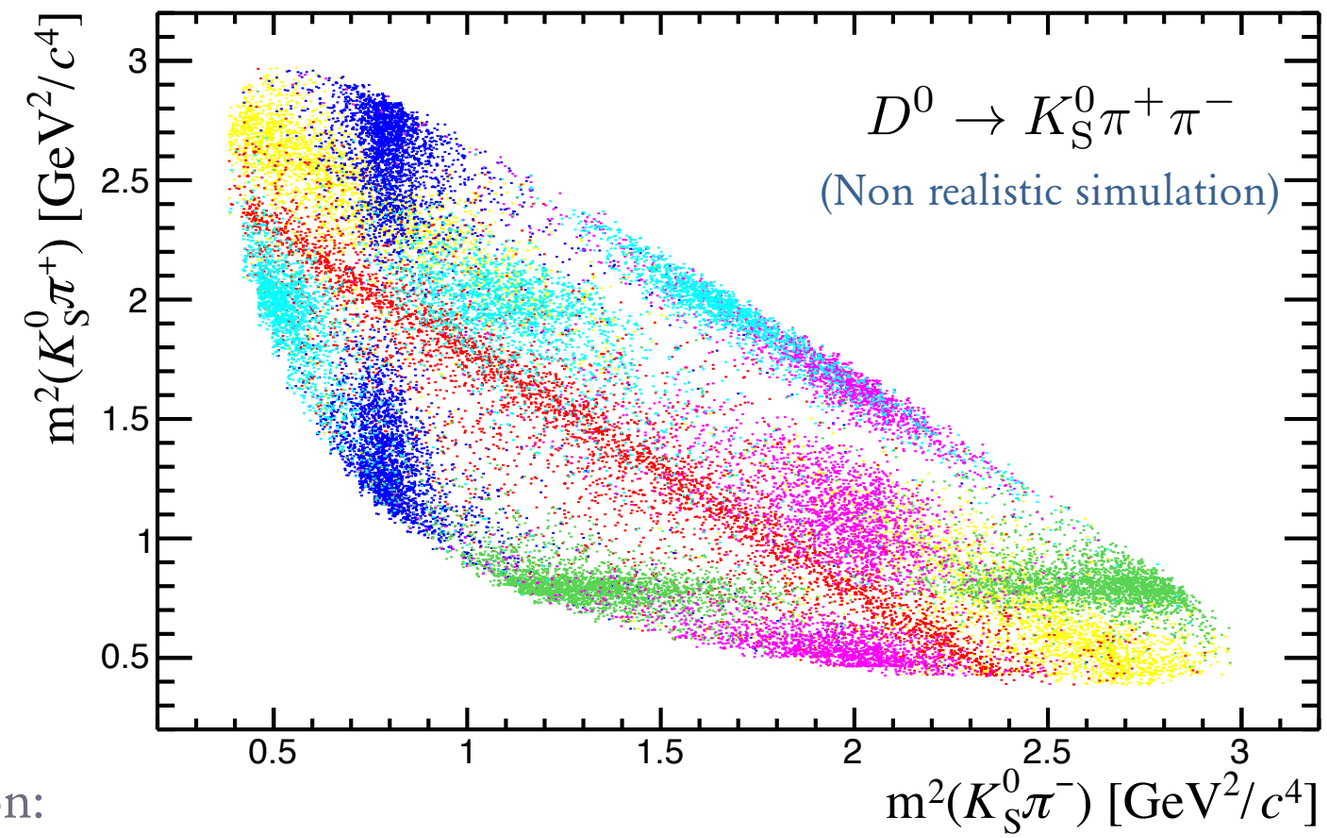
Angular distribution of resonance related to Legendre Polynomials



Amplitude analysis can provide information:

- Relative phases between states
- Sensitivity to CP violating effects
- Resolve ambiguities in weak phases
- Hadron spectroscopy

Toy simulation using Laura++ package: https://laura.hepforge.org



Resonant DP illustration:

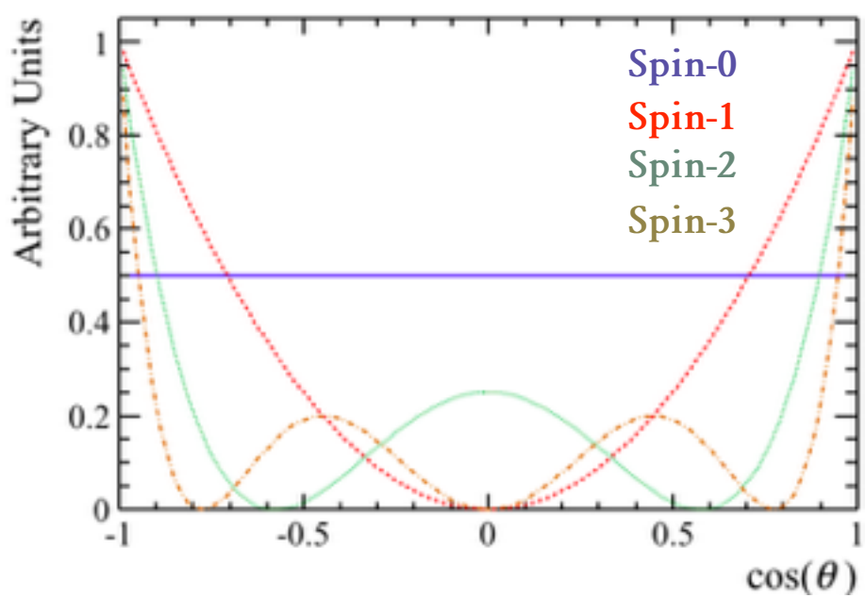
- $K^*(892)$ [vector]: green and blue
- $K^*_2(1430)$ [tensor]: magenta and cyan
- $\rho(770)$ [vector]: yellow
- $f_0(980)$ [scalar]: red

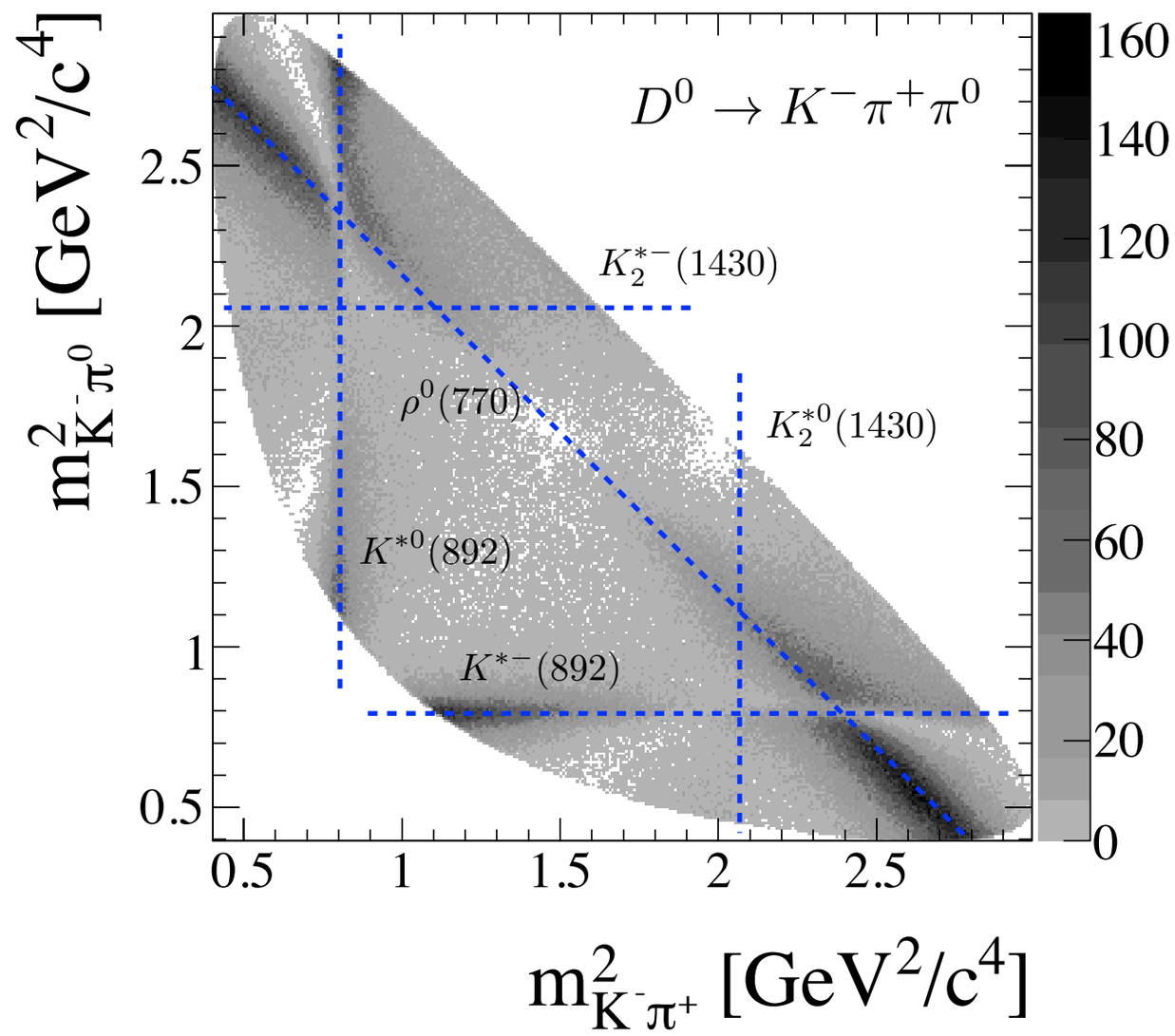
Dalitz plot analysis features

Intensity along bands indicates magnitude and the spin of the given resonance

Phys.Rev.Lett.103:211801,2009
BaBar Collaboration

Angular distribution of resonance related to Legendre Polynomials





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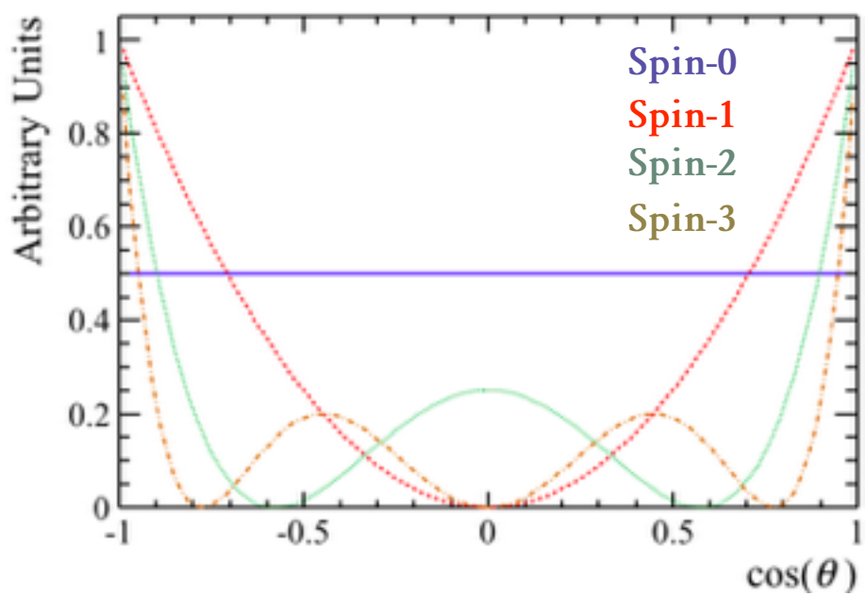

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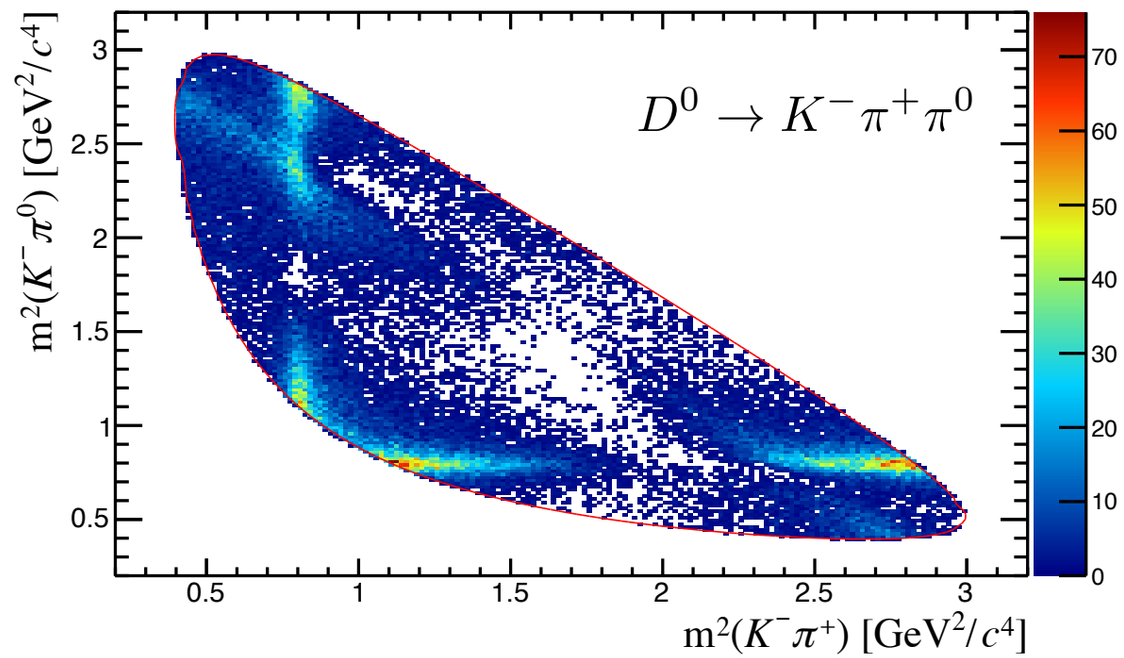
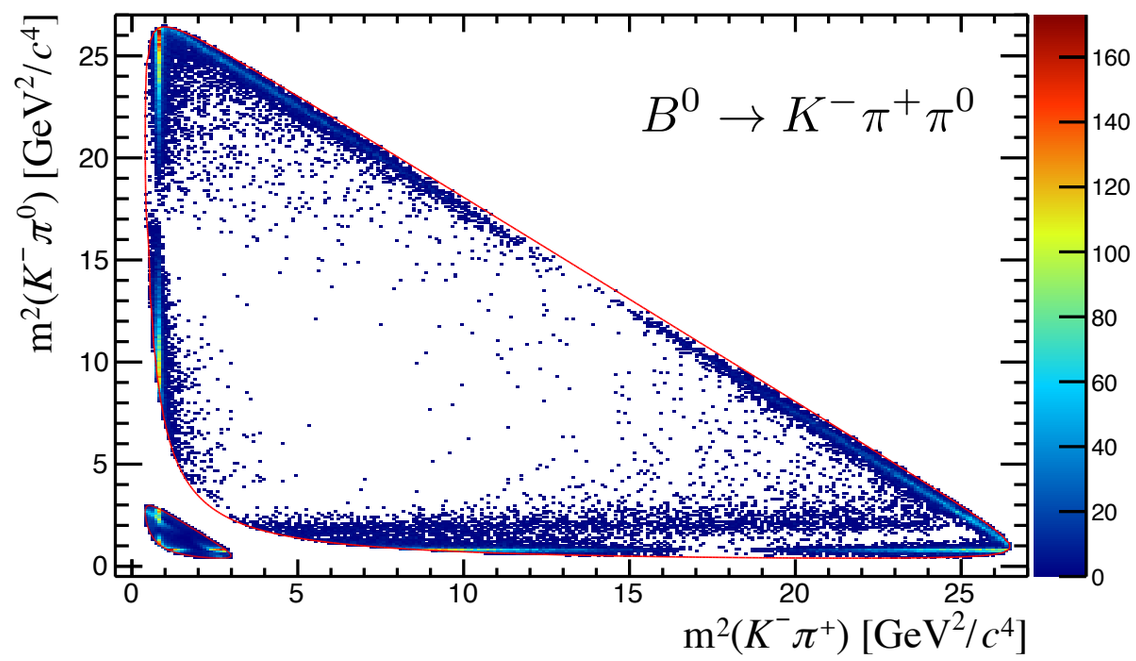


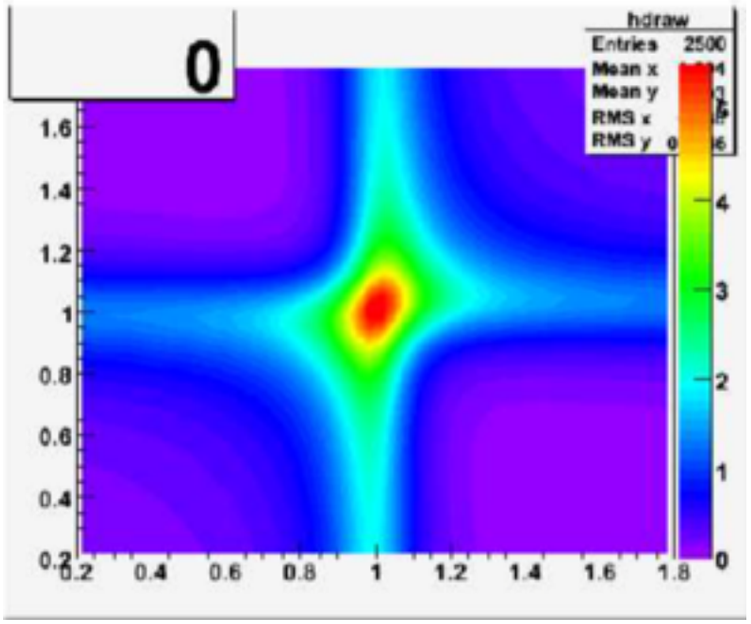


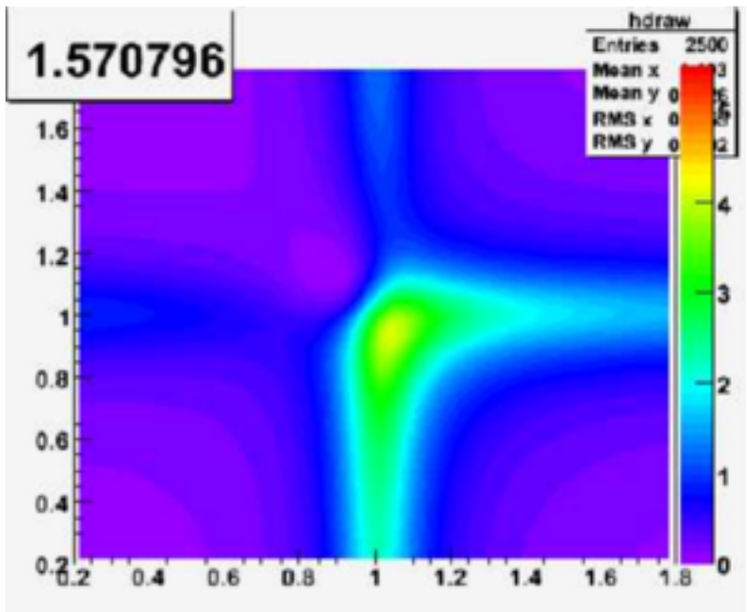
Interference effects

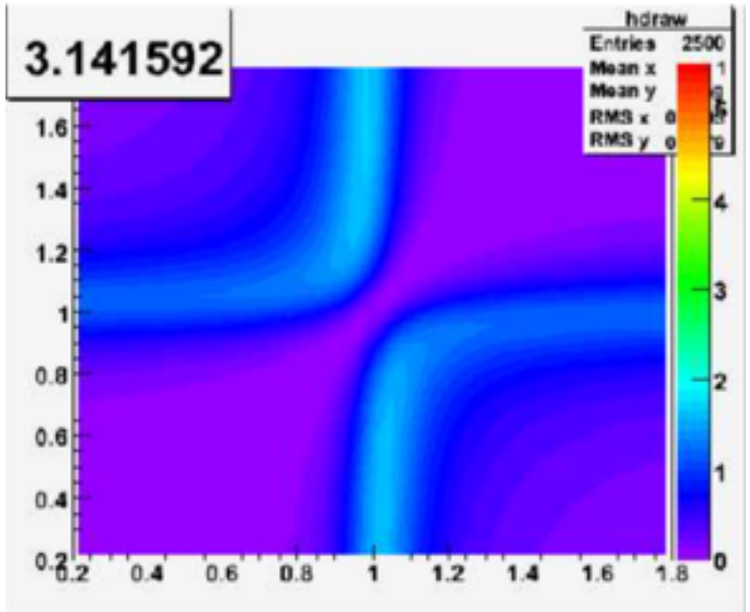
In dominant single-channels, the distribution of events in the DP is well-defined in certain energy regions. However, in most scenarios the decay amplitude is more complicated and consists of many overlapping/interfering resonances.

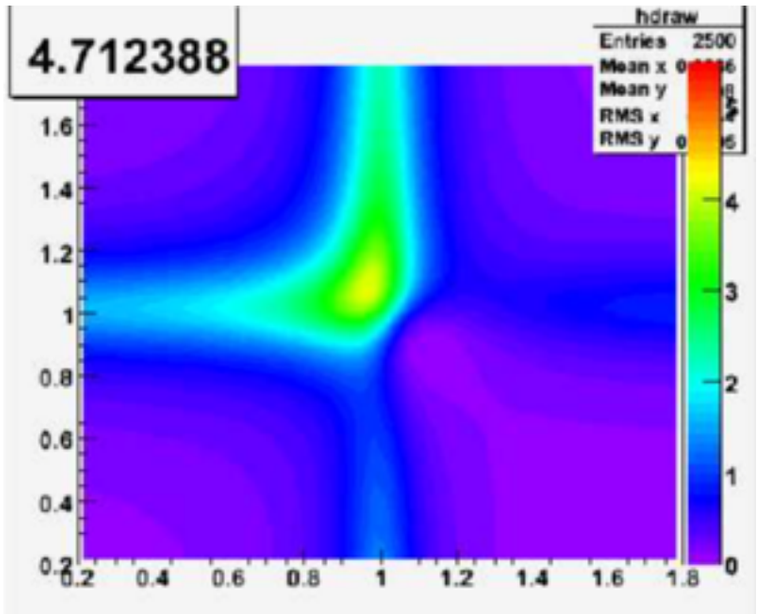
$$|\mathcal{A}|^2 = |a_1 e^{i\delta_1} F_1(m_{ij}^2) + a_2 e^{i\delta_2} F_2(m_{jk}^2)|^2$$

$$\propto |F_1|^2 + r^2|F_2|^2 + 2r\Re\left(F_1 F_2^*\right)\cos\delta - 2r\Im\left(F_1 F_2^*\right)\sin\delta$$

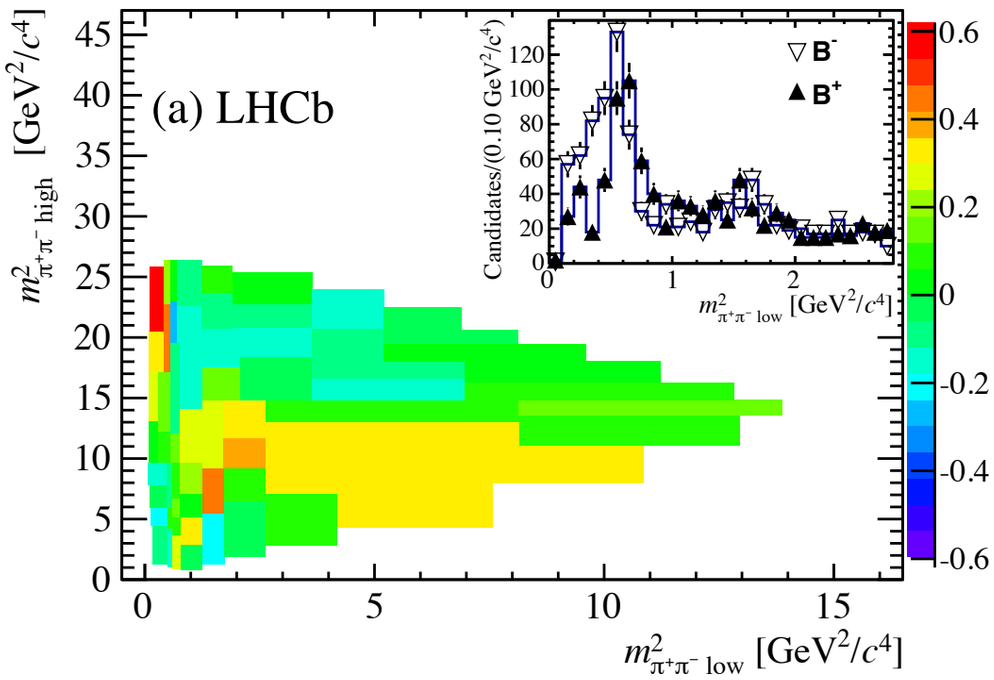


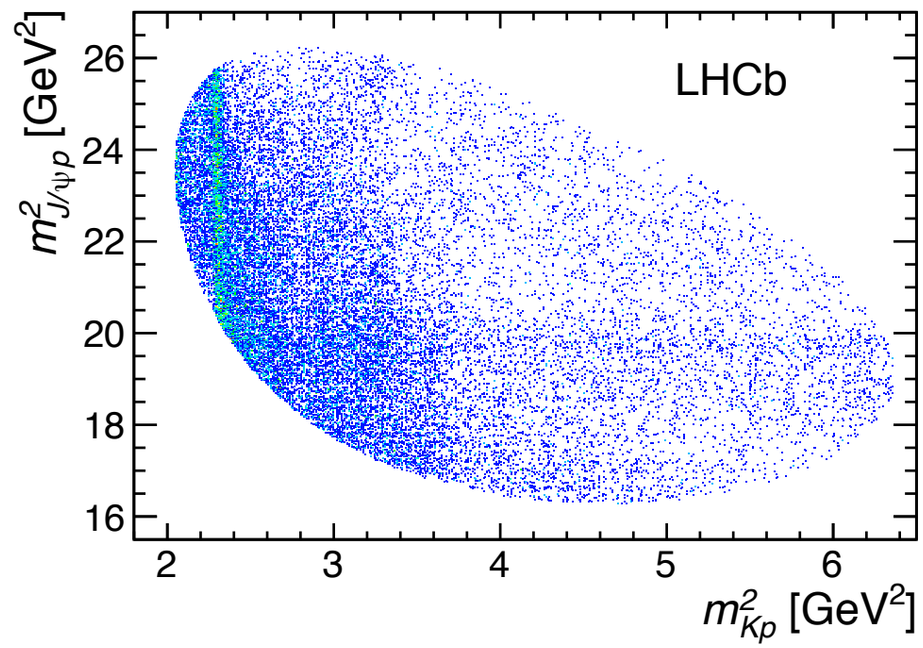




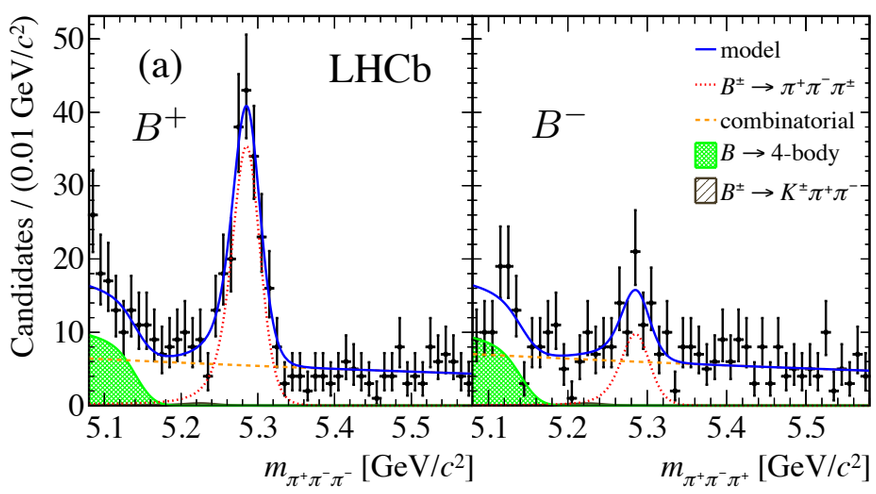


Prelude to Lecture (II)





$m^2_{\pi^+\pi^- \text{ low}} < 0.4\,\text{GeV}^2/c^4$ and $m^2_{\pi^+\pi^- \text{ high}} > 15\,\text{GeV}^2/c^4$



$$\mathcal{A}^{CP}_{\text{reg}}(B^\pm \to \pi^\pm\pi^+\pi^-) = 0.584 \pm 0.082 \pm 0.027 \pm 0.007$$

Argand diagram: replace Breit-Wigner amplitude