
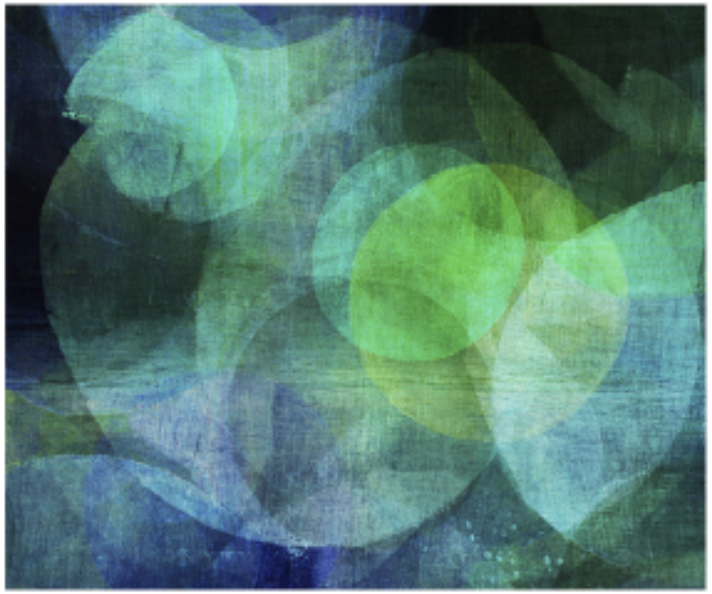

Experimental Particle and Astro-particle Physics

A.Onofre, Physics Department, University of Minho

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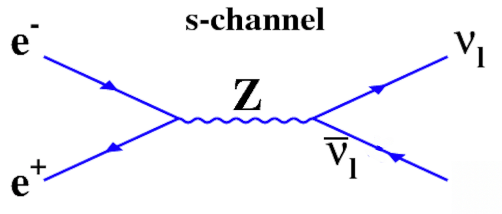
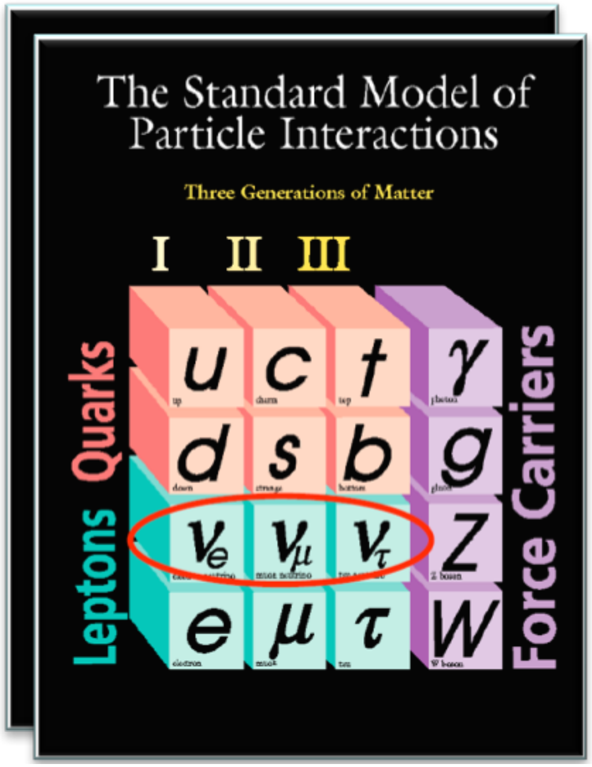
Neutrino Physics

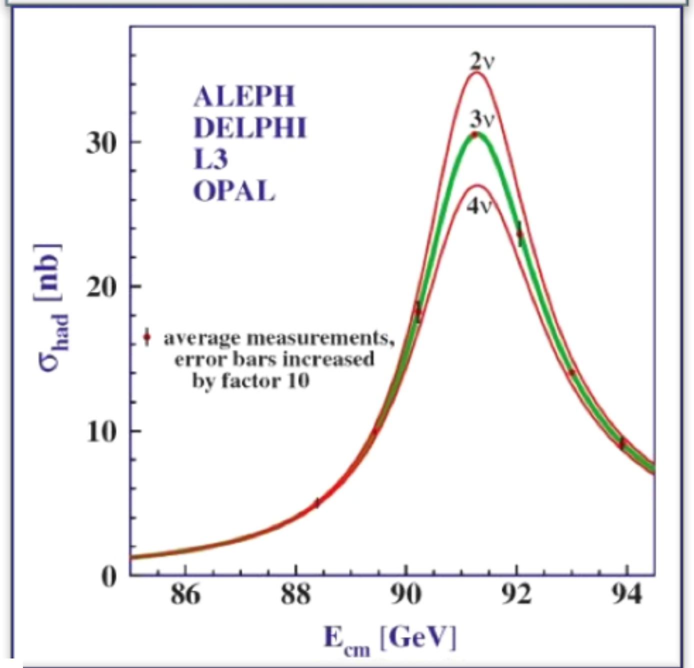
Massless, chargeless leptons => only weak interactions

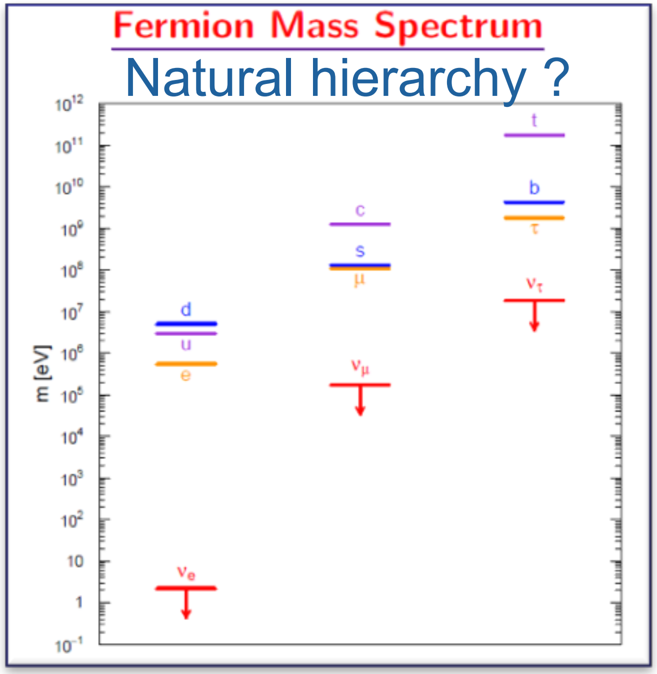
Production at LEP



3 and only 3 ν generations: experimentally verified from Z^0 width measured at LEP (for ν masses <45 GeV/c^2)







Number of Light ν Types

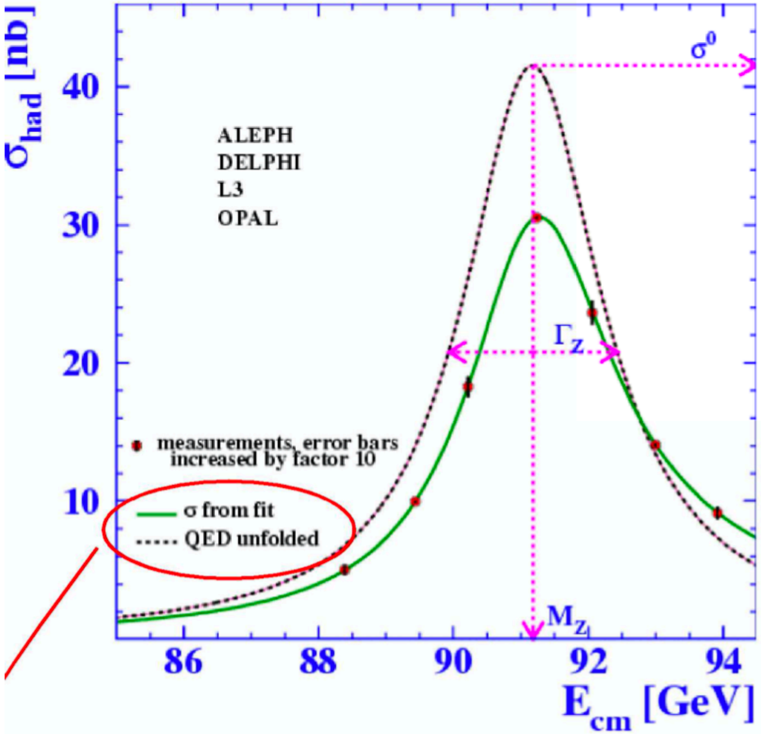
VALUE
2.9840±0.0082

ALEPH, DELPHI, L3, OPAL, SLD and working groups
PRPL 427 257

JP G **37**, 075021 (2010) and 2011 partial update for the 2012 edition (URL: http://pdg.lbl.gov)

Neutrino Physics

Z-Line Shape



Z resonance curve:

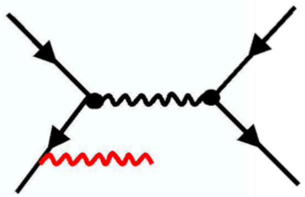
$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

Peak: $$\sigma_0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$

- Resonance position $\rightarrow M_Z$
- Height $\rightarrow \Gamma_e \Gamma_\mu$
- Width $\rightarrow \Gamma_Z$

Initial state Bremsstrahlung corrections

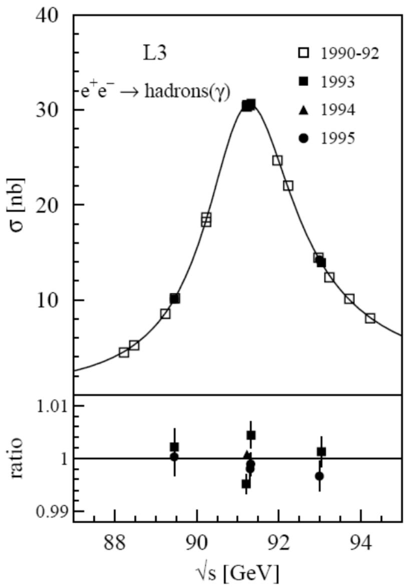
$$\sigma_{ff(\gamma)} = \int_{4m_f^2/s}^{1} G(z)\sigma_{ff}^0(zs)\,dz \qquad z = 1 - \frac{2E_\gamma}{\sqrt{s}}$$



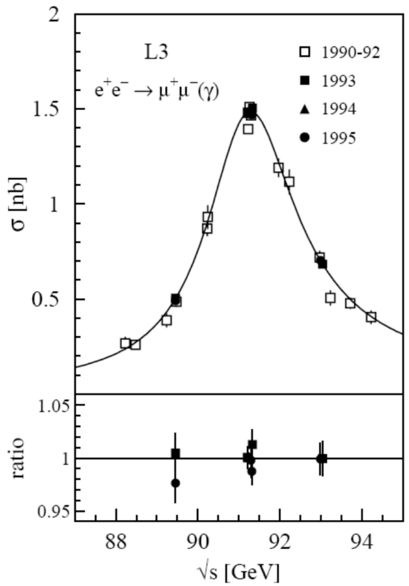
Neutrino Physics

Z-Line Shape

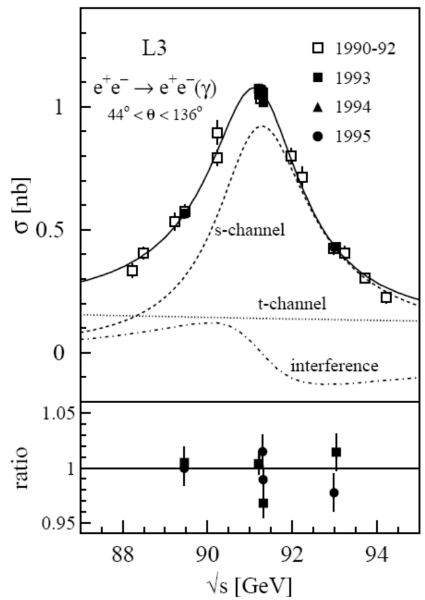
$e^+ e^- \to hadrons$



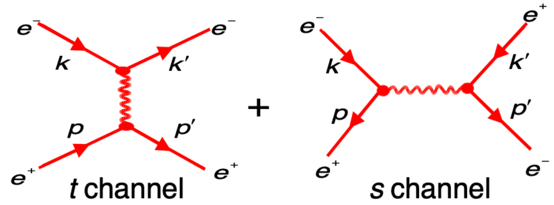
$e^+ e^- \to \mu^+ \mu^-$



$e^+ e^- \to e^+ e^-$



t channel contribution → forward peak



Resonance looks the same, independent of final state: Propagator is the same

Neutrino Physics

Z line shape parameters (LEP average)

M_Z = 91.1876 ± 0.0021 GeV = ±23 ppm (*)

Γ_Z = 2.4952 ± 0.0023 GeV

Γ_{had} = 1.7458 ± 0.0027 GeV

Γ_e = 0.08392 ± 0.00012 GeV

Γ_μ = 0.08399 ± 0.00018 GeV

Γ_τ = 0.08408 ± 0.00022 GeV

±0.09 %

3 leptons are treated independently

test of lepton universality

Γ_Z = 2.4952 ± 0.0023 GeV

Γ_{had} = 1.7444 ± 0.0022 GeV

Γ_e = 0.083985 ± 0.000086 GeV

Assuming lepton universality: $\Gamma_e = \Gamma_\mu = \Gamma_\tau$

Neutrino Physics

Number of Light Neutrinos

In the Standard Model:

$$\Gamma_Z = \Gamma_{had} + 3\cdot\Gamma_\ell + \underbrace{N_\nu \cdot \Gamma_\nu}_{\text{invisible}:\ \Gamma_{inv}} \rightarrow \begin{cases} e^+e^- \to Z \to \nu_e\bar{\nu}_e \\ e^+e^- \to Z \to \nu_\mu\bar{\nu}_\mu \\ e^+e^- \to Z \to \nu_\tau\bar{\nu}_\tau \end{cases}$$

$$\Gamma_{inv} = 0.4990 \pm 0.0015\ \text{GeV}$$

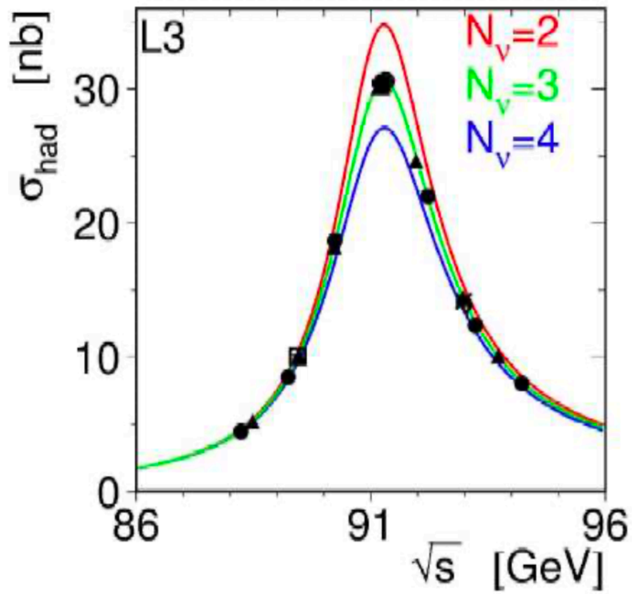
To determine the number of light neutrino generations:

$$N_\nu = \underbrace{\left(\frac{\Gamma_{inv}}{\Gamma_\ell}\right)_{exp}}_{5.9431\pm0.0163} \cdot \underbrace{\left(\frac{\Gamma_\ell}{\Gamma_\nu}\right)_{SM}}_{=1.991\pm0.001}$$

5.9431±0.0163 =1.991±0.001 (small theo. uncertainties from m_{top} M_H)

$$\boxed{N_\nu = 2.9840 \pm 0.0082}$$

No room for new physics: $Z \to$ new



Neutrino Sources

- **Artificial:**
  - nuclear reactors
  - particle accelerators

First detected neutrinos

- **Natural:**
  - **Sun**
  - **Atmosphere**
  - SuperNovae
  - fission in the Earth core (geoNeutrinos)

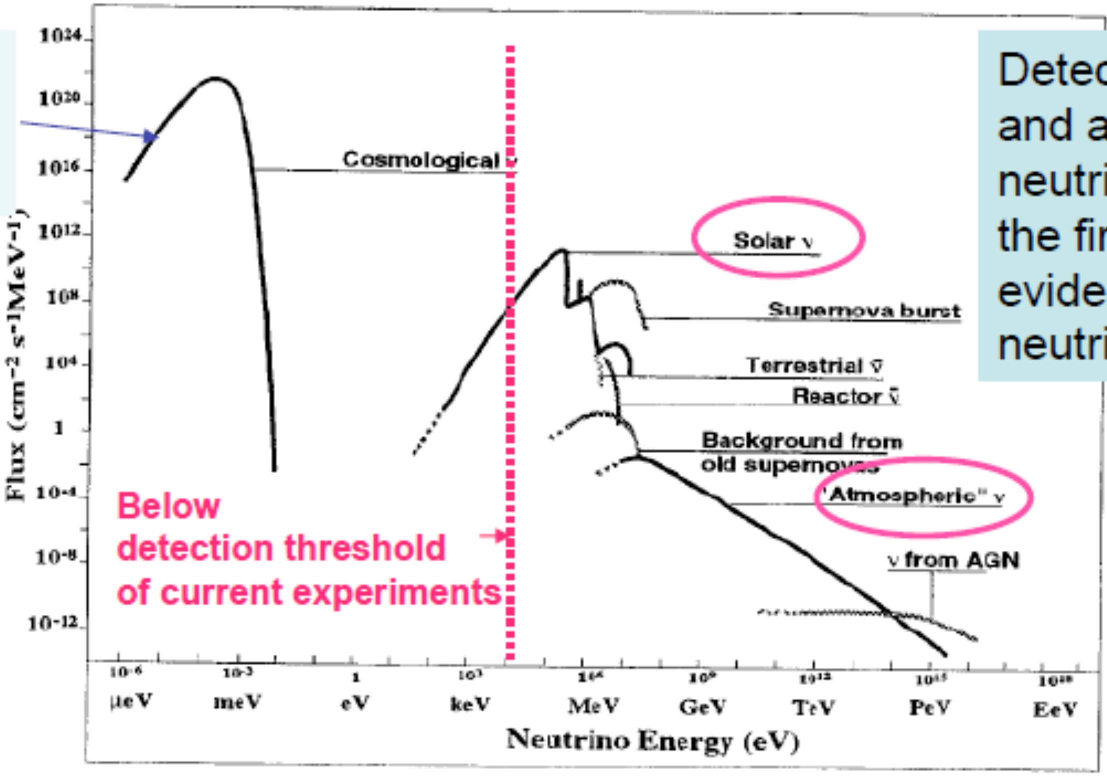
  Expected, but undetected so far,:
  - relic neutrinos from BigBang (~300/cm^3)
  - Astrophysical accelerators (AGN,..), old SN explosions

Neutrinos are everywhere!

Neutrino Flux vs Energy

The Sun is the most intense detected source with a flux on Earth of of 6 10^{10} ν/cm^{2}s

Abundant but challenging detection



Detection of solar and atmospheric neutrino has provided the first compelling evidence of neutrino oscillations

D.Vignaud and M. Spiro, Nucl. Phys., A 654 (1999) 350

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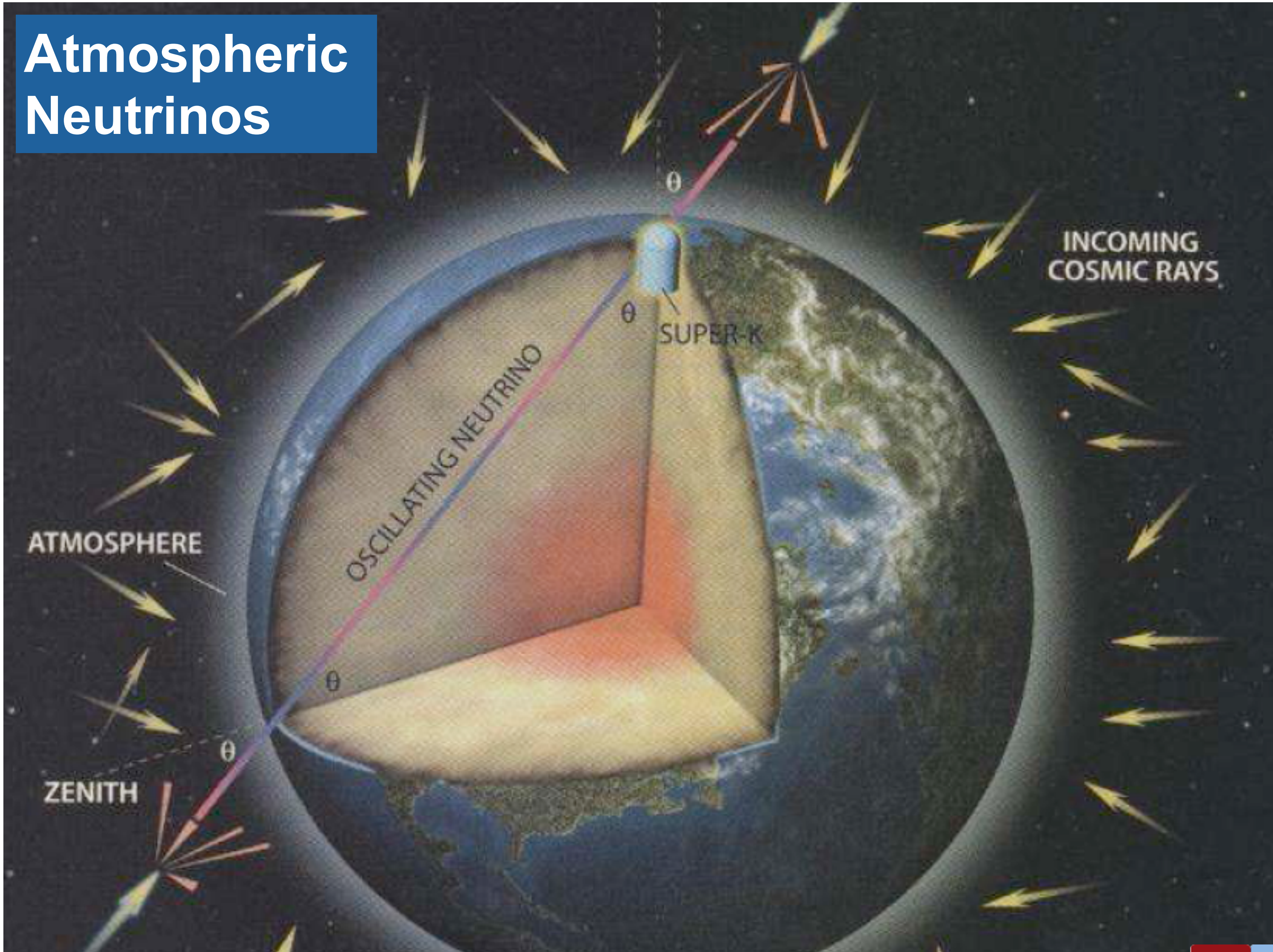
A.Onofre

Atmospheric Neutrinos



Atmospheric neutrinos

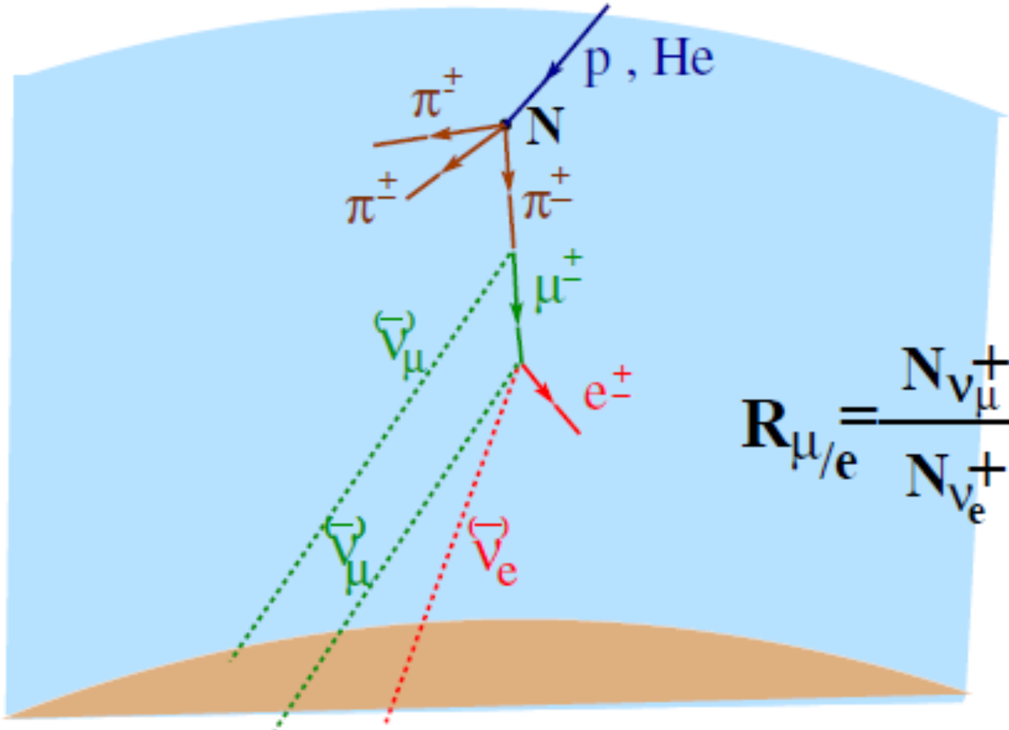
- Atmospheric neutrinos are produced by the interaction of *cosmic rays* (p, He, ...) with the Earth's atmosphere:

$\boxed{1}$ $A_{cr} + A_{air} \rightarrow \pi^{\pm}, K^{\pm}, K^{0}, \ldots$

$\boxed{2}$ $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu},$

$\boxed{3}$ $\mu^{\pm} \rightarrow e^{\pm} + \nu_{e} + \nu_{\mu};$

- at the detector, some ν interacts and produces a **charged lepton**, which is observed.

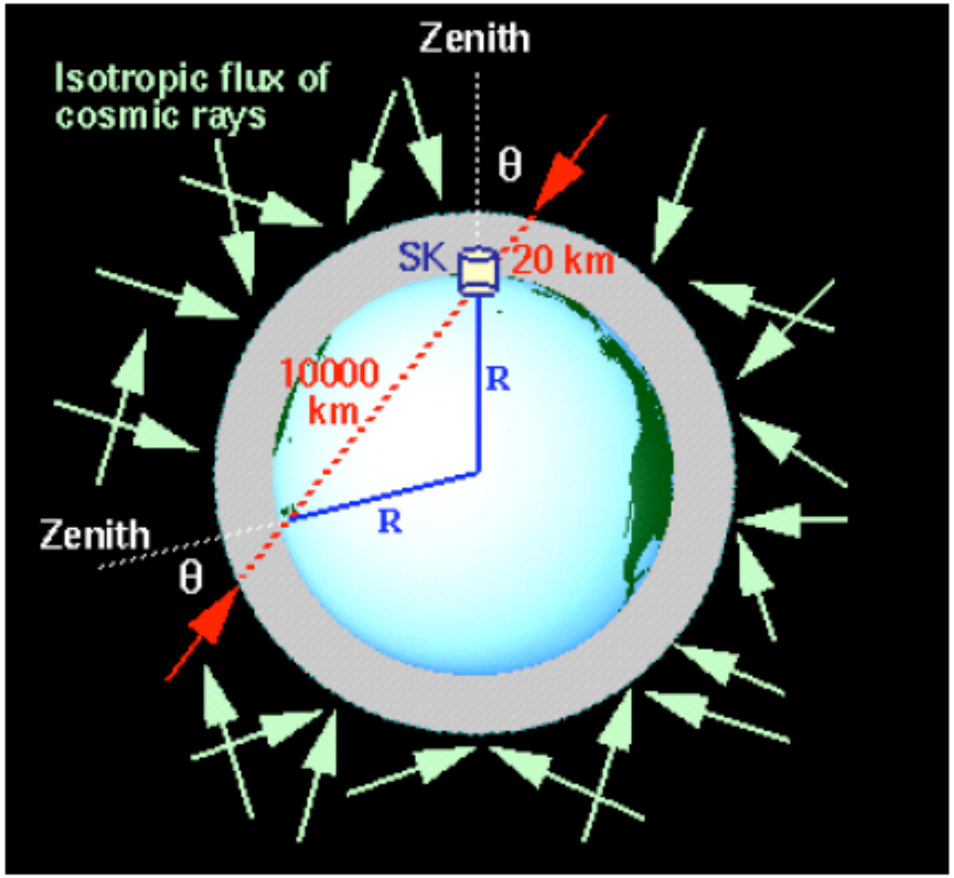


$$R_{\mu/e} = \frac{N_{\nu_\mu} + N_{\bar{\nu}_\mu}}{N_{\nu_e} + N_{\bar{\nu}_e}} \sim 2$$

Cosmic Flux Isotropy

We expect an isotropic Flux of neutrinos at high energies (earth magnetic field deviate path of low-momentum secondaries only : East-West effects)

For E_ν > a few GeV, and a given ν flavour
(Up-going / down-going) ~ 1.0 with <1 % uncertainty



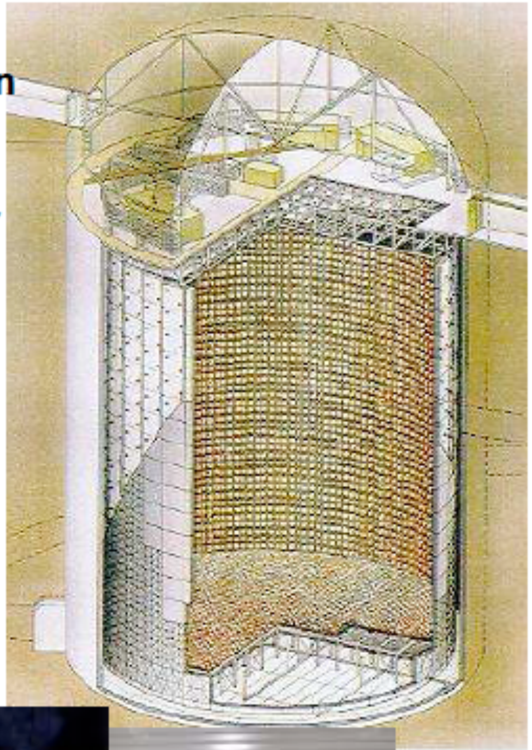
Note the baseline (= distance νproduction-νdetection) spans 3 order of magnitudes!

SUPER-KAMIOKANDE (SuperK)

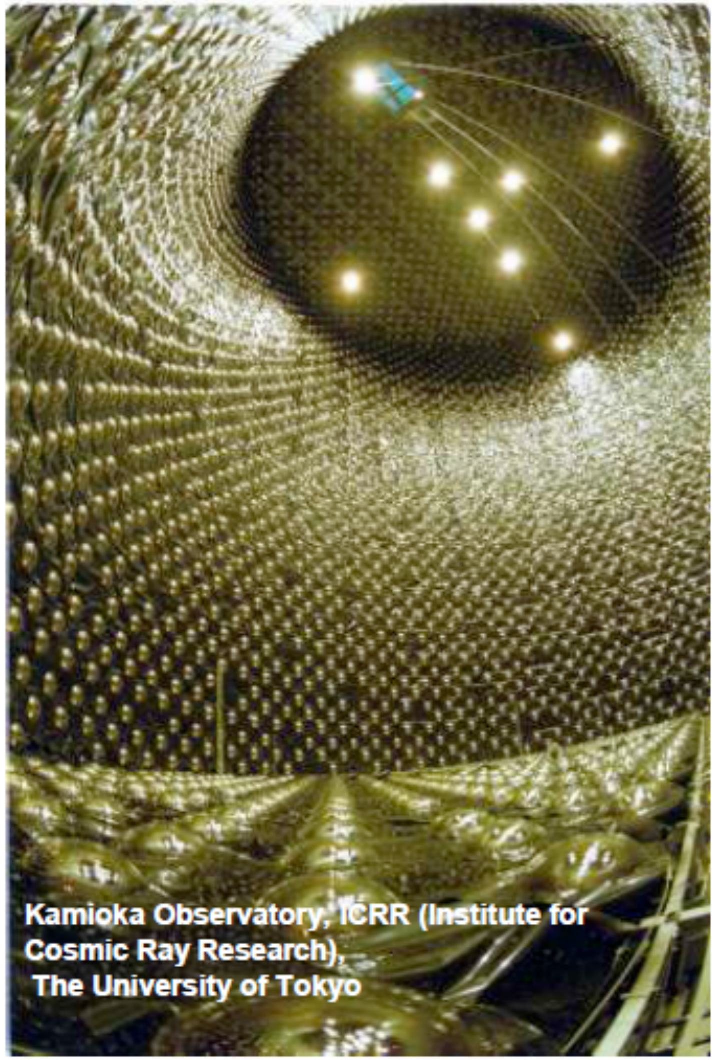
Kamioka Mine in Japan

- 1400m underground

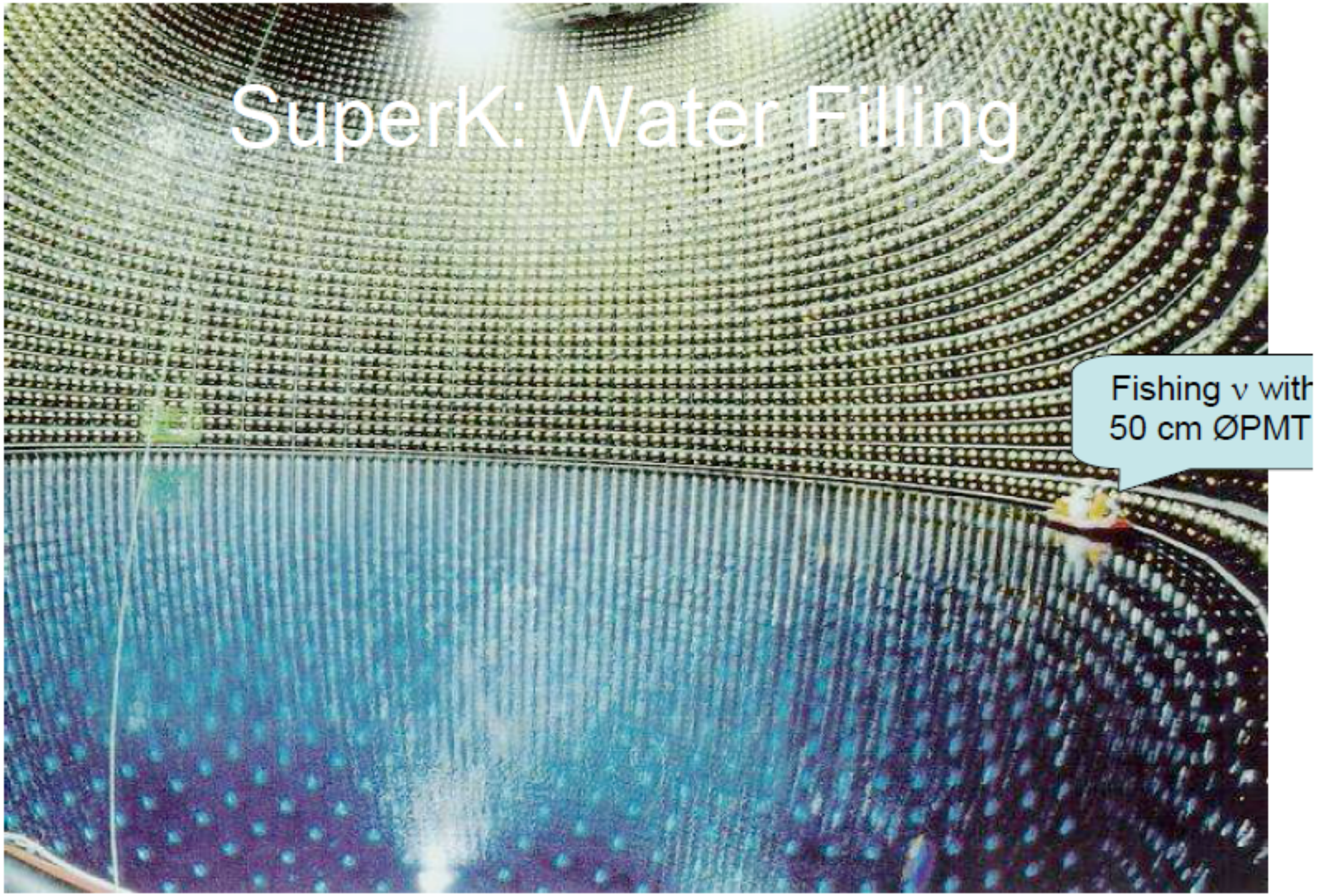
50 ktons of pure water (Fiducial volume for analysis 22.5 ktons)



- 10,000 PMT inner detector
- 2,000 PMT outer detector (cosmic ray veto)

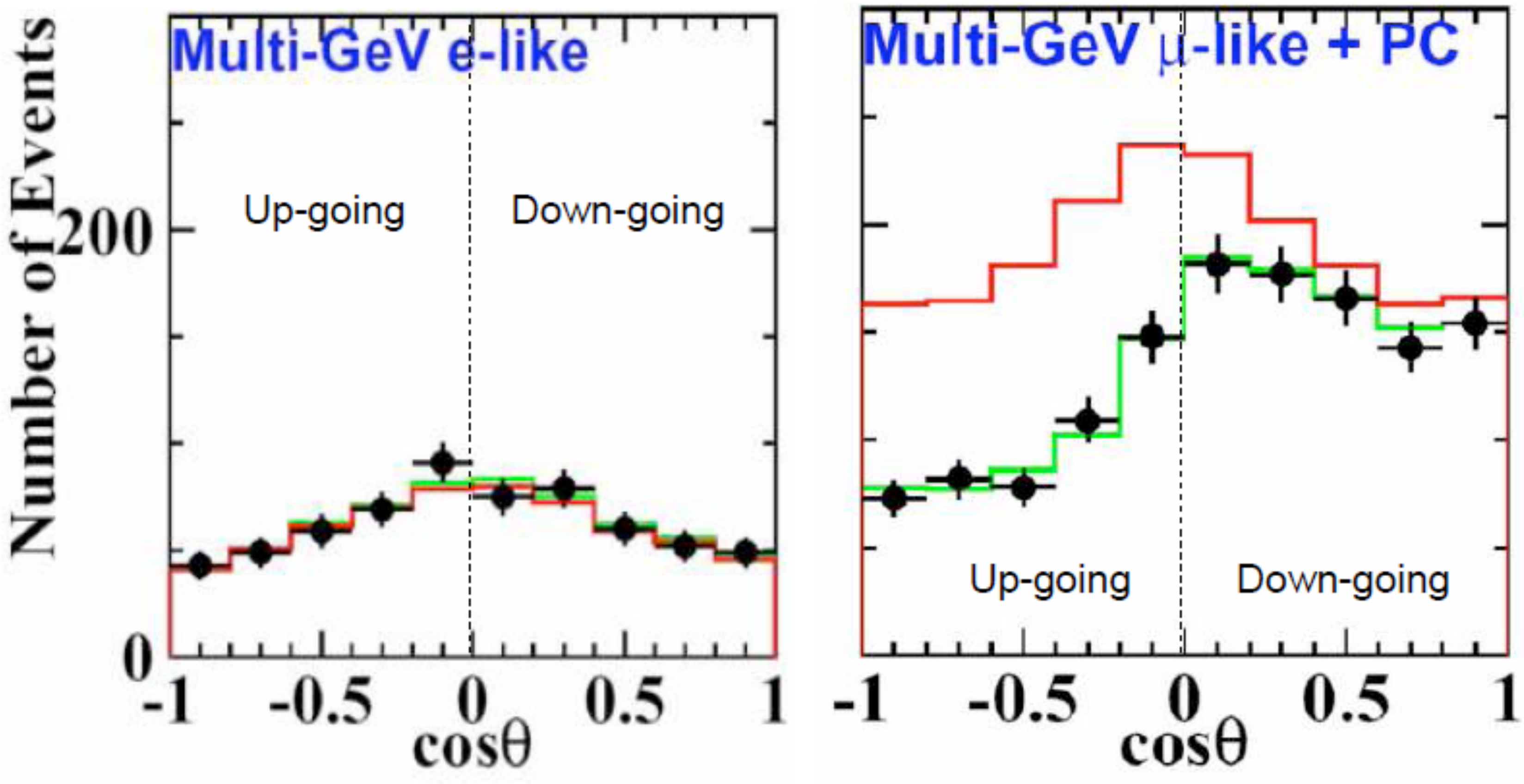


Kamioka Observatory, ICRR (Institute for Cosmic Ray Research),
The University of Tokyo

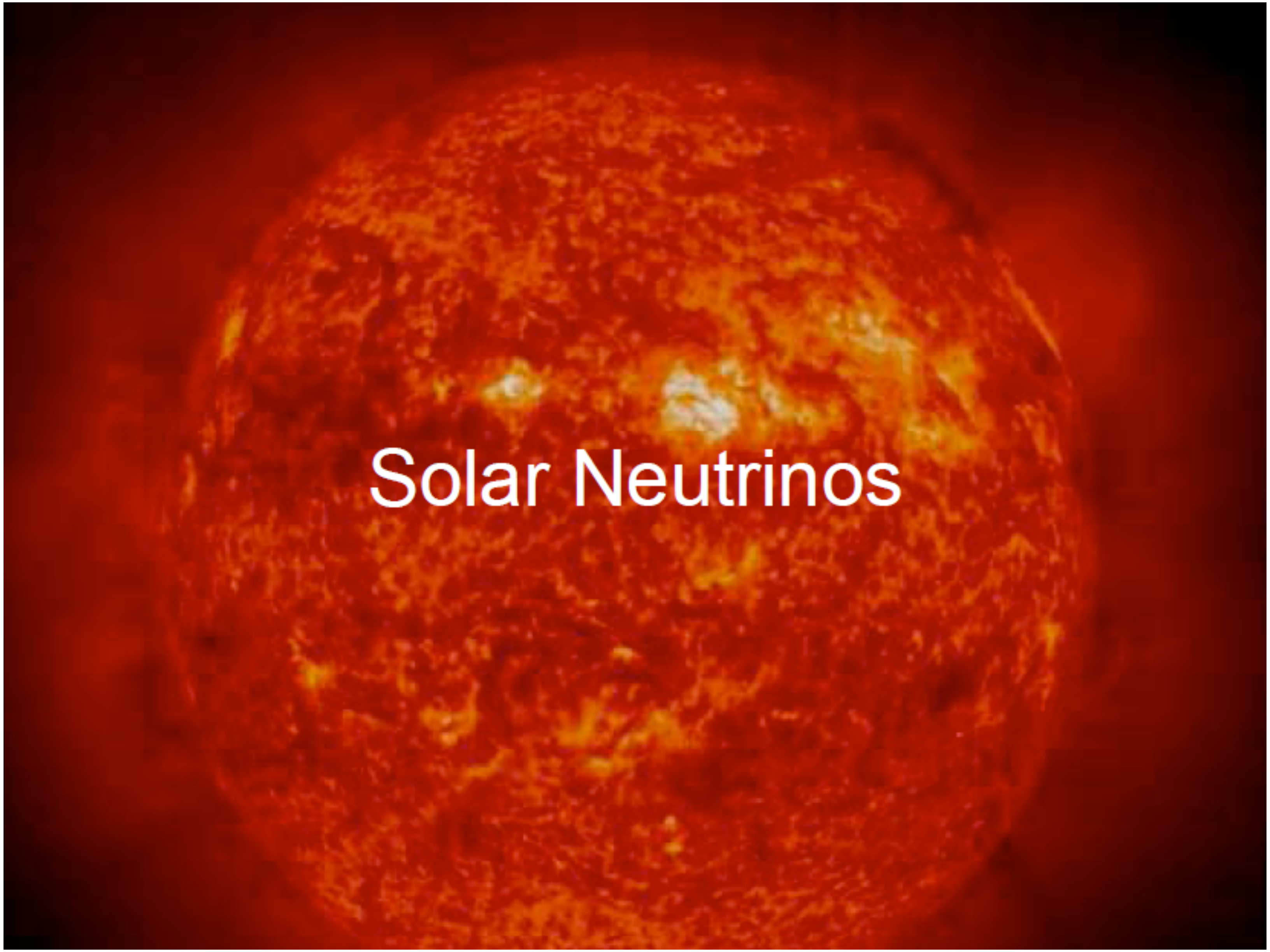


Kamioka Observatory, ICRR (Institute for Cosmic Ray Research), The University of Tokyo 10

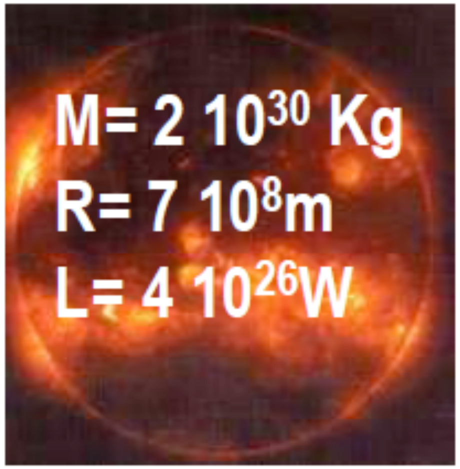
Zenith angle Distribution



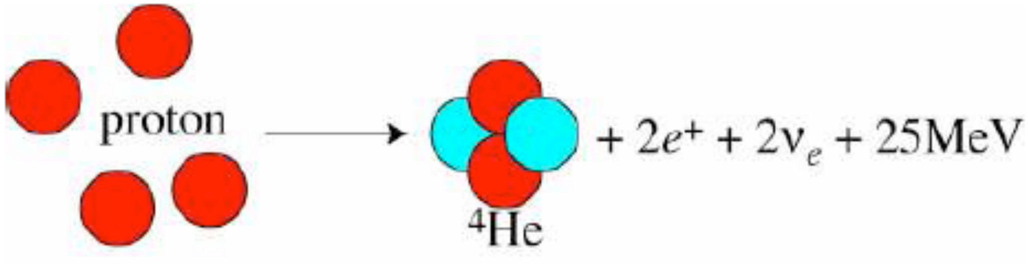
Half of the ν_μ are lost!


Solar Neutrinos

Standard Solar Model (SSM)



Hydrogen fusion in the Sun:



Observables:

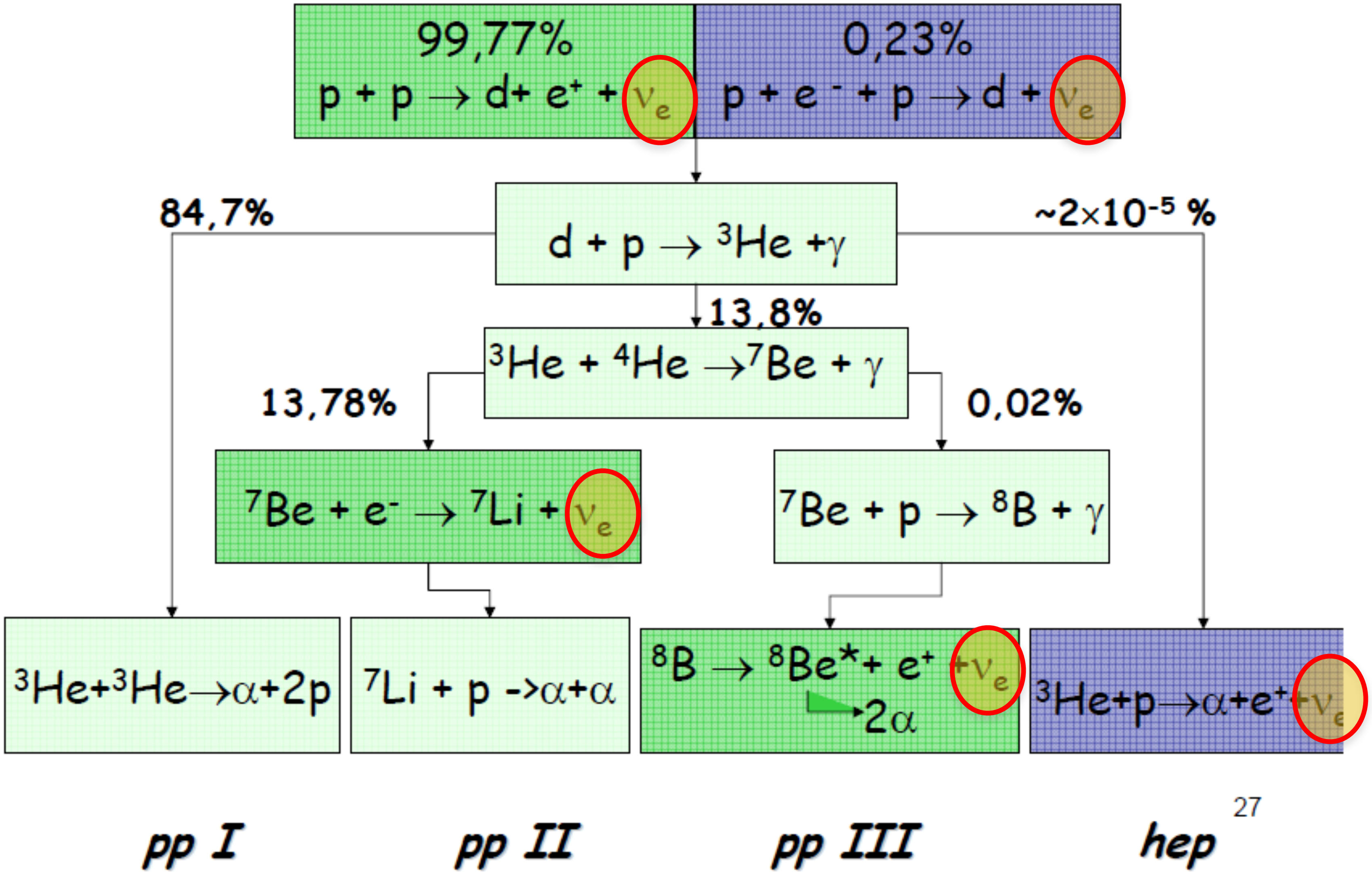
- -Mass
- -Luminosity
- - Radius,
- - Metal content of the photosphere
- - Age

Inferences on solar interior (ρ, P, T)

SSM describes the evolution of an initially homogeneous solar mass M_o up to the sun age t so as to reproduce L_o, R_o and $(Z/X)_{photo}$

$\Rightarrow$ Predicts solar neutrino flux (intensity and spectrum)

The pp-chain


99,77%
$p + p \rightarrow d + e^+ + \nu_e$
0,23%
$p + e^- + p \rightarrow d + \nu_e$
84,7%
$d + p \rightarrow {}^3He + \gamma$
~2×10^{-5} %
13,8%
${}^3He + {}^4He \rightarrow {}^7Be + \gamma$
13,78%
0,02%
${}^7Be + e^- \rightarrow {}^7Li + \nu_e$
${}^7Be + p \rightarrow {}^8B + \gamma$
${}^3He + {}^3He \rightarrow \alpha + 2p$
${}^7Li + p \rightarrow \alpha + \alpha$
${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$
$\rightarrow 2\alpha$
${}^3He + p \rightarrow \alpha + e^+ + \nu_e$
pp I
pp II
pp III
hep
27

Solar Neutrino Energy Spectrum

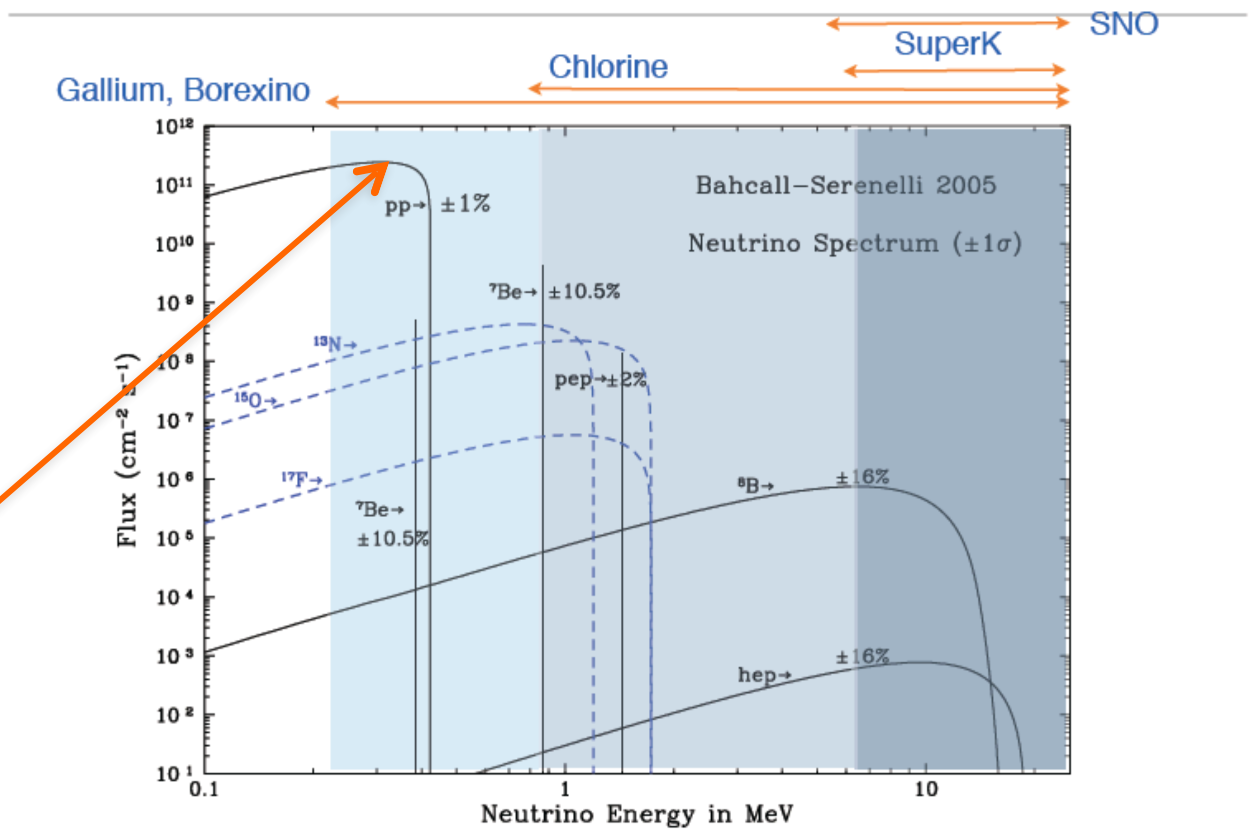
Sun luminosity: $L = 8.6\ 10^{11}$ MeV $cm^{-2}\ s^{-1}$

Total Neutrino flux (only ν_e): $\Phi(\nu_e) = 2 \times L/(26\ \text{MeV}) = 6.6\ 10^{10}\ cm^{-2}\ s^{-1}$

Small theoretical uncertainty (~1%): total flux is constrained by solar luminosity

Spectra and relative abundances have larger uncertainties

Small Uncertainty: Luminosity constrained



Experiments and Detection methods

Solar ν	Important detector parameters
Small x-section	⇒ Big Target Mass, O(kT)
Low energy	⇒ Low Detection Threshold
⇓	
Cosmic rays Background	⇒ Deep underground

➤Radiochemical detectors (integrated flux)

	Start	Method	Thresh.(MeV)
• Homestake	1969-1999	^{37}CL	0.8
• Sage	1990	^{71}Ga	0.2
• Gallex/GNO	1991	^{71}Ga	0.2

➤Real-time detector (differential flux:time,E,θ)

•Kamioka/SuperK	1985	H_2O	5
•SNO	1999	D_2O	5
•Borexino	2007	Liq Scint.	1-2

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C_2Cl_4=tetrachloroethylene

USA

$\nu_e\ {}^{37}Cl \rightarrow {}^{37}Ar\ e^-$

Homestake (1969 ~99)

380,000 l of C_2Cl_4 (615 tons)

Homestake Mine, 1400 m deep

$E\nu > 0.8$ MeV
Sensitive to ^8B + ^7Be

Extract ^{37}Ar once per month by flushing He together with small (known) amount of stable ^{36}Ar to measure extraction efficiency

^{37}Ar is radioactive and decays with half-life of 35 days

UNIT

$$\text{RATE} = \sum(\text{FLUX}) \times (\text{CROSS SECTION})$$

$$\sim 10^{10}\ \text{cm}^{-2}\text{s}^{-1} \times 10^{-46}\ \text{cm}^2$$

$$1\ SNU = 10^{-36}\ \frac{\text{INTERACTIONS PER TARGET}}{\text{ATOM PER SEC}}$$

Predicted rate
8.5± 1.8 SNU

Observed rate
2.56± 0.23 SNU
~0.5 atoms/day!

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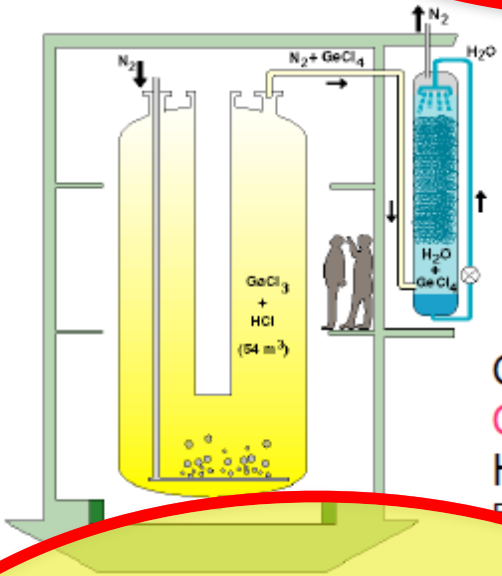
1SNU=1 neutrino interaction per second for 10^{+36} atoms of target

Italy, Gran Sasso

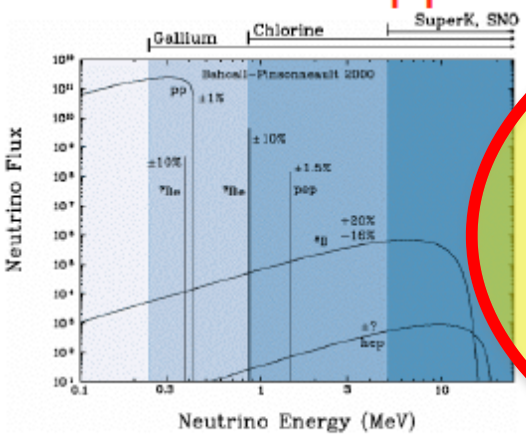
Gallium Experiments

$\nu_e \; ^{71}Ga \rightarrow \; ^{71}Ge \; e^-$



Gallex/GNO Calibrated with High intensity ^{51}Cr ν source

Eν > 0.23 MeV
Sensitive to pp



http://www.sns.ias.edu/~jnb/

- Observed (Data): 68.1 ± 3.75 SNU (GALLEX + GNO + SAGE)
- Predicted (SSM): 131^{+12}_{-10} SNU
- **Data / SSM = 0.52 ± 0.03**

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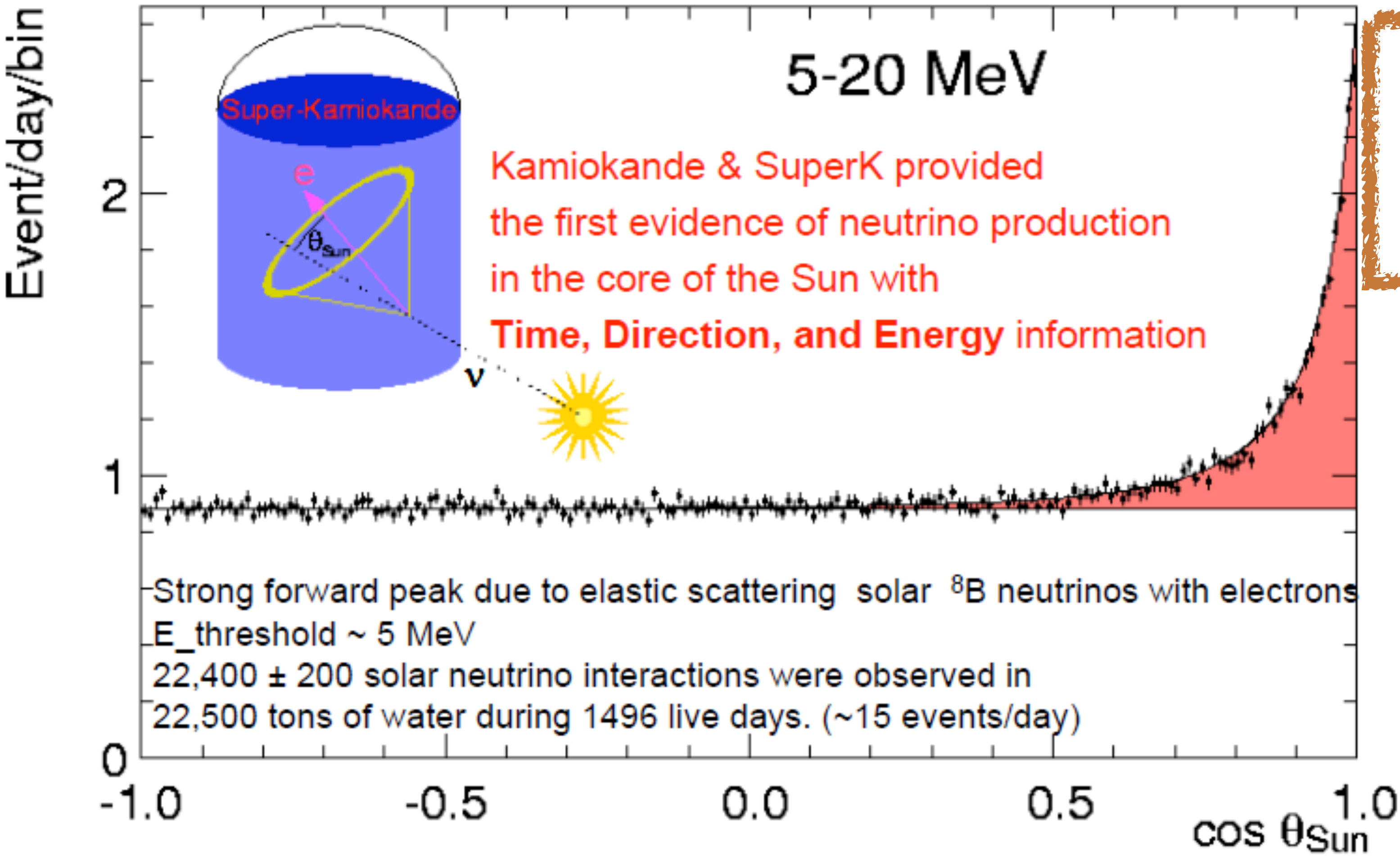
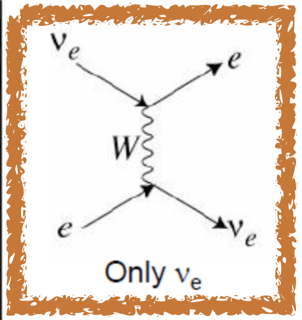
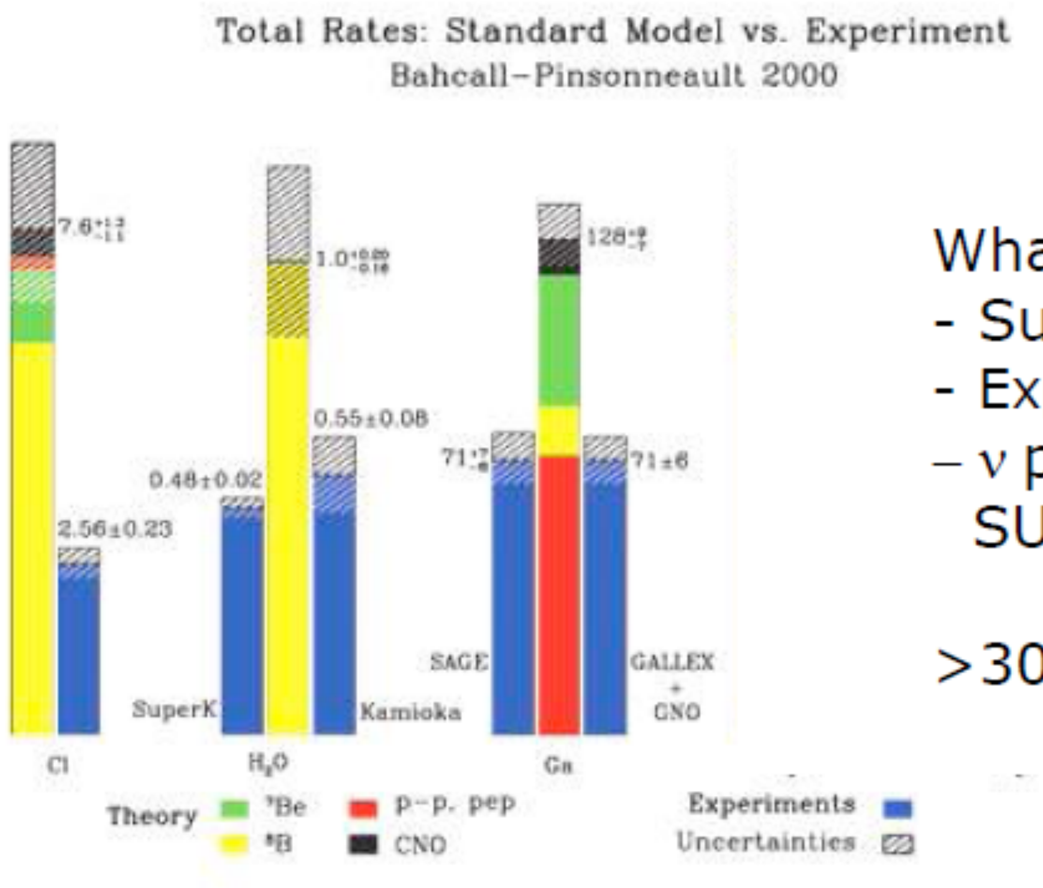
SuperK

$\nu_e \; e^- \rightarrow \nu_e \; e^-)$





SOLAR Neutrino PROBLEM



What can be wrong?
- Sun model
- Experiments
– ν propagation from SUN to Earth

>30 years of debate!

A ν trick?

ν decay? Now excluded by SN1987A

$\gamma\,\tau = (E\nu\ /\ m\nu)\ \tau\ > 8$ min

Best bet: $\nu_e \rightarrow \nu_x$ oscillation

Flux suppression could have the right energy dependence according to chosen oscillation mechanism and parameters (Δm^2,$\sin^2 2\theta$)

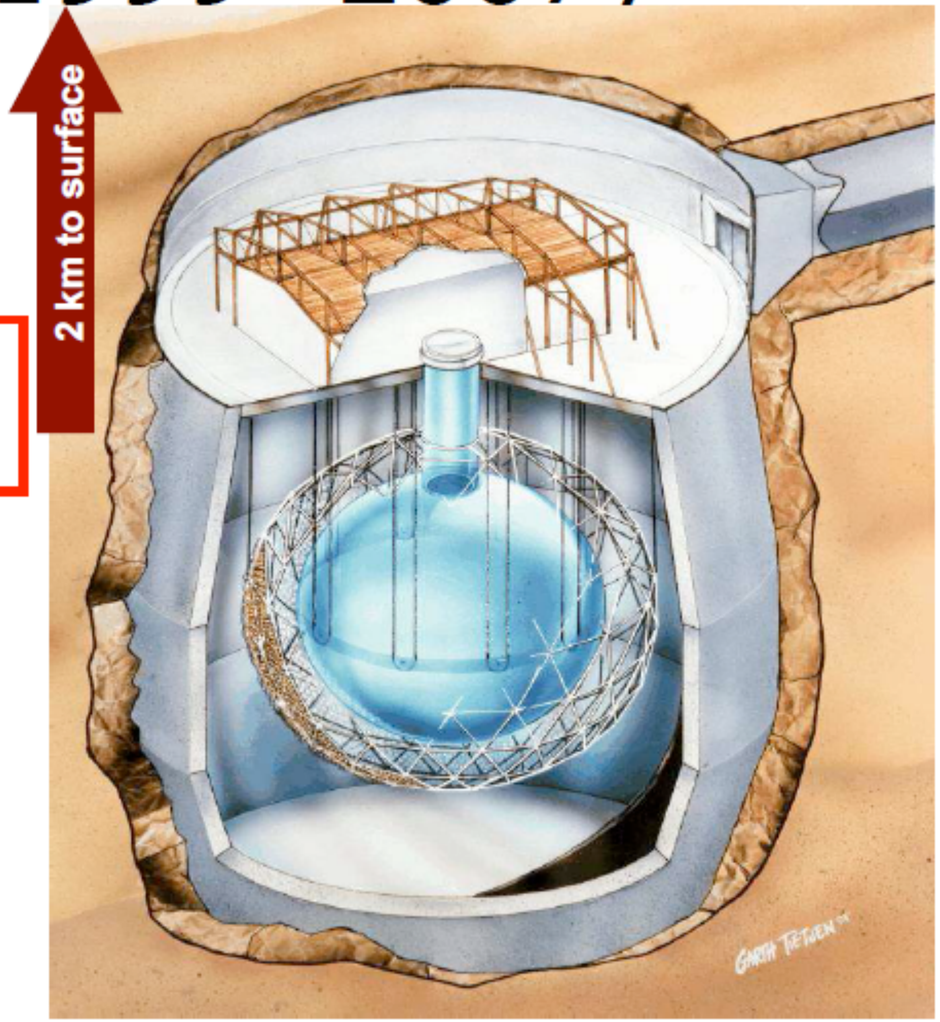
Confirmation could come from an experiment equally sensitive to all ν flavor, via detection of NC interactions: SNO

Sudbary Neutrino Observatory (Ontario, 1999~2007)

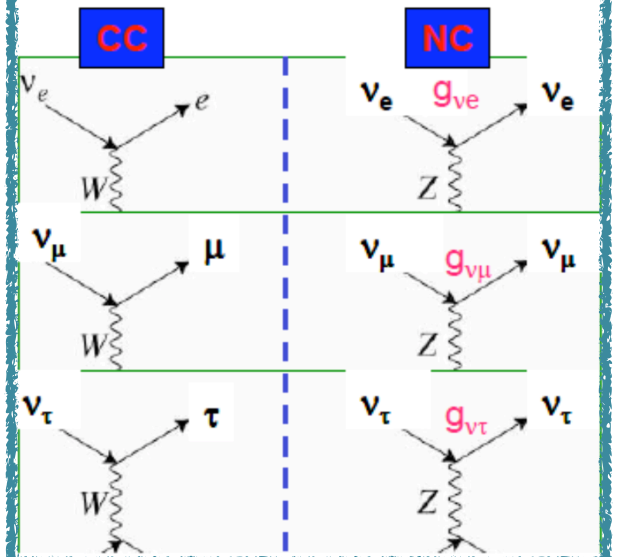
1 Kton D_2O

SNO can determine both:
$\Phi(\nu_e)$ and $\Phi(\nu_e + \nu_\mu + \nu_\tau)$

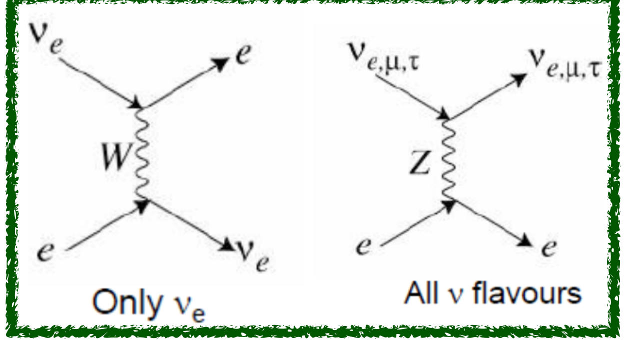
Threshold energy for neutrino detection 5MeV
$\Rightarrow$Sensitive to ^8B neutrinos



Universality: $g_{\nu e} = g_{\nu\mu} = g_{\nu\tau}$



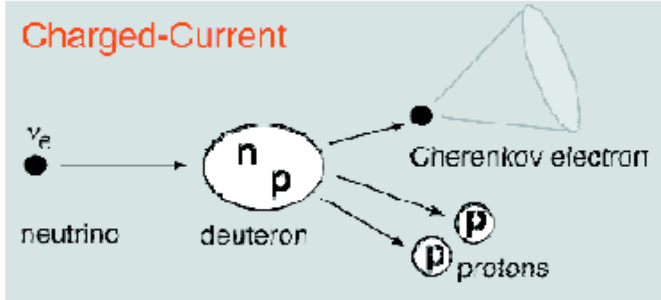
Elastic Scattering (ES)



ν Detection at SNO

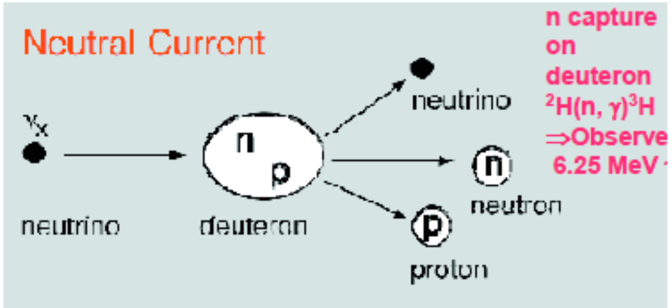
CC $\nu_e + d \rightarrow p + p + e^-$

- Measurement of ν_e energy spectrum
- Weak directionality



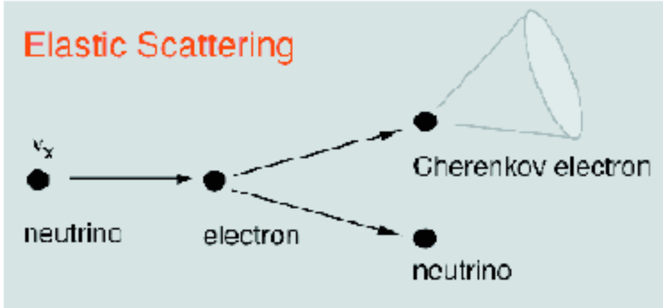
NC $\nu_x + d \rightarrow p + n + \nu_x$

- Measure total ^8B ν
- Equally sensitive to ALL ν
- $\sigma(\nu_e) = \sigma(\nu_\mu) = \sigma(\nu_\tau)$



ES $\nu_x + e^- \rightarrow \nu_x + e^-$

- Low Statistics
- $\sigma(\nu_e) \approx 7\ \sigma(\nu_\mu) \approx 7\ \sigma(\nu_\tau)$
- Strong directionality



First SNO RESULTS (April 2002)

- The measured total B neutrino flux is in excellent agreement with the SSM prediction.

SSM is right

- Only 1/3 of the B-neutrinos survive as ν_e

All Experiments are right!

$\Rightarrow$ 2/3 of the produced ν_e transform into active neutrinos (ν_μ or ν_τ, indicated as $\phi_{\mu\tau}$)

Evidence of flavour transformation!

(independent of SSM)

$\phi_{CC}(\nu_e) = \phi_e$

$\phi_{NC}(\nu_x) = \phi_e + \phi_{\mu\tau}$

$\phi_{ES}(\nu_x) = \phi_e + 0.15\phi_{\mu\tau}$

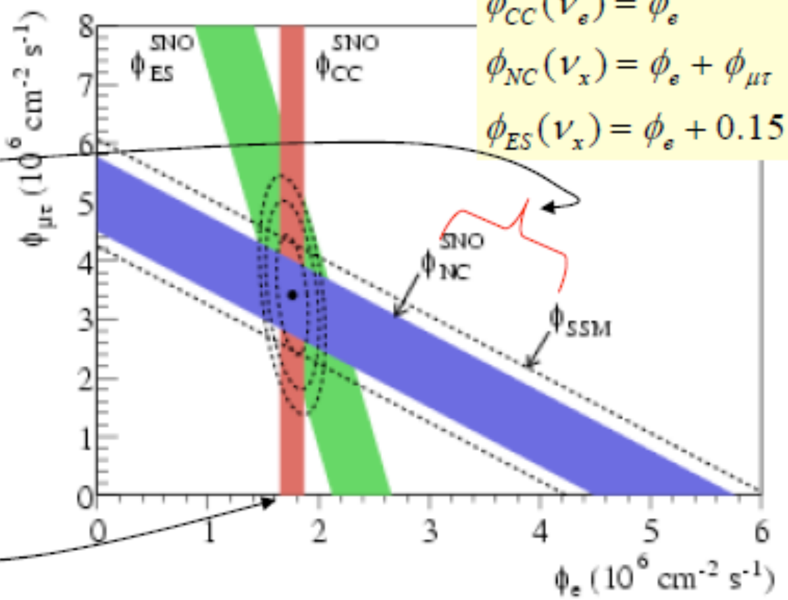
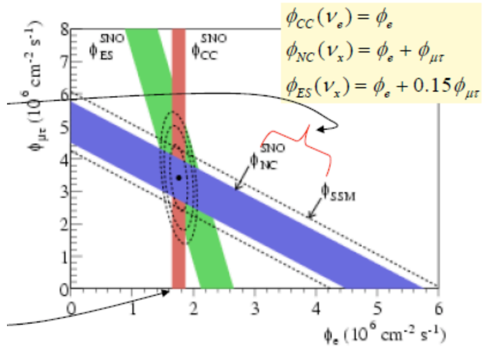


FIG. 3: Flux of ^{8}B solar neutrinos which are μ or τ flavor vs flux of electron neutrinos deduced from the three neutrino reactions in SNO. The diagonal bands show the total ^{8}B flux as predicted by the SSM [11] (dashed lines) and that measured with the NC reaction in SNO (solid band). The intercepts of these bands with the axes represent the $\pm 1\sigma$ errors. The bands intersect at the fit values for ϕ_e and $\phi_{\mu\tau}$, indicating that the combined flux results are consistent with neutrino flavor transformation assuming no distortion in the ^{8}B neutrino energy spectrum.

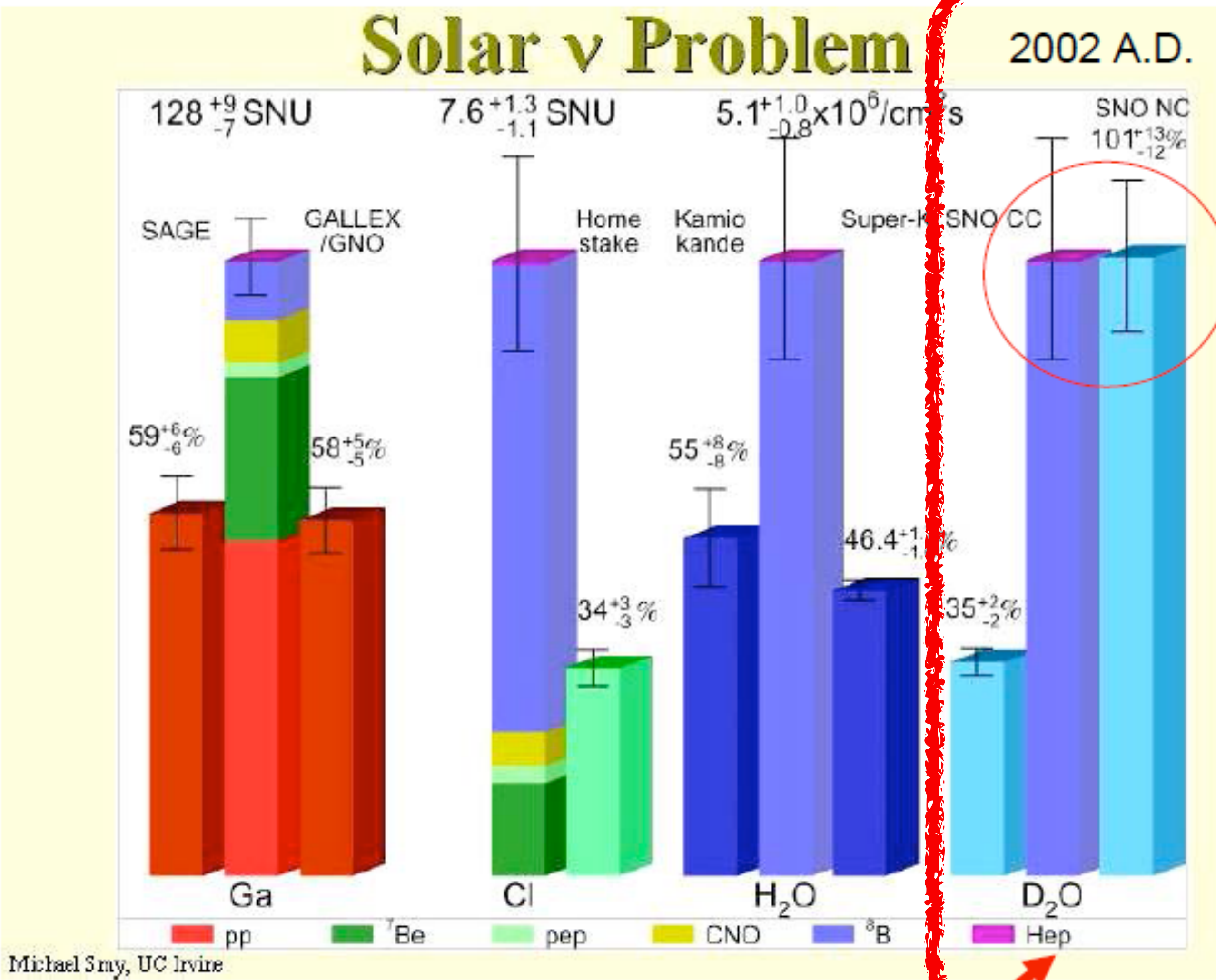
Using the measurement of the three independent reaction channels, SNO was able to disentangle the individual fluxes of neutrinos. Their measurement of the neutrino fluxes was, in units of $10^{-8} cm^{-2} s^{-1}$,

$$\begin{aligned} \phi_{CC} = & \quad \phi(\nu_e) = & 1.76 \pm 0.01 \\ \phi_{ES} = & \quad \phi(\nu_e) + 0.15(\phi(\nu_\mu) + \phi(\nu_\tau)) = & 2.39 \pm 0.26 \\ \phi_{NC} = & \quad \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau) = & 5.09 \pm 0.63 \end{aligned}$$



The numbers are striking. The total flux of muon and tau neutrinos from the Sun $(\phi(\nu_\mu) + \phi(\nu_\tau))$ is $(3.33 \pm 0.63) \times 10^{-8}\text{cm}^{-2}\text{s}^{-1}$, roughly 3 times *larger* than the flux of ν_e. Since we know the Sun only produces electron neutrinos, the only conclusion is that neutrinos must change flavour between the Sun and the Earth. Furthur, the SSM predicts a total flux of neutrinos with energies greater than 2 MeV (the deuteron break-up energy) of

$$\phi_{SSM} = (5.05 \pm 1.01) \times 10^{-8}\text{cm}^{-2}\text{s}^{-1} \tag{9}$$



SNO solves it!

What Does All This Means?

What does this mean? Suppose we label the mass states as ν_1, ν_2 and ν_3 and that they have different, but close, masses. Everytime we create an electron in a weak interaction we will create one of these mass eigenstates (ensuring the energy and momentum is conserved at the weak interaction vertex as we do so). Suppose that we create these with different probabilities (i.e. 10% of the time we create a ν_1 etc). If we could resolve the mass of each state, we could follow each mass state as it propagates. However, the neutrino masses are too small to experimentally resolve them. We know we created one of them, but not which one, so what we create, at the weak interaction vertex, is a coherent superposition of the ν_i mass states - this coherent superposition we call the *electron* neutrino :

$$|\nu_e>= U_{e1}|\nu_1> + U_{e2}|\nu_2> + U_{e3}|\nu_3> \qquad (1)$$

This will lead to an oscillation probability

$$P(\nu_x \rightarrow \nu_y) = sin^2(2\theta) sin^2(1.27\Delta m^2 \frac{L(km)}{E(GeV)})$$

Relevant parameters:

The mass squared difference, Δm^2

The angle θ (mixing angle)

L/E

Two Flavour Neutrino Oscilations

The ground rules are : the eigenstates of the Hamiltonian are $|\nu_1>$ and $|\nu_2>$ with eigenvalues m_1 and m_2 for neutrinos at rest. A neutrino of type j with momentum p is an energy eigenstate with eigenvalues $E_j = \sqrt{m_j^2 + p^2}$. Neutrinos are produced in weak interactions in weak eigenstates of definite lepton number ($|\nu_e>$, $|\nu_\mu>$ or $|\nu_\tau>$) that are *not* energy eigenstates. These two sets of states are related to each other by a unitary matrix. which we can write as U where, in two dimensions,

$$U = \begin{pmatrix} U_{\alpha 1} & U_{\alpha 2} \\ U_{\beta 1} & U_{\beta 2} \end{pmatrix} \tag{14}$$

Suppose that we generate a neutrino beam with some amount of neutrino flavours ν_e and ν_μ. Then in terms of the mass states ν_1 and ν_2 we can write

$$\begin{pmatrix} |\nu_e> \\ |\nu_\mu> \end{pmatrix} = U \begin{pmatrix} |\nu_1> \\ |\nu_2> \end{pmatrix} = \begin{pmatrix} U_{\alpha 1} & U_{\alpha 2} \\ U_{\beta 1} & U_{\beta 2} \end{pmatrix} \begin{pmatrix} |\nu_1> \\ |\nu_2> \end{pmatrix} \tag{15}$$

More compactly we can write the flavour state ν_α as a linear combination,

$$|\nu_\alpha> = \sum_{k=1,2} U_{\alpha k} |\nu_k> \tag{16}$$

Two Flavour Neutrino Oscilations

Suppose we generate a neutrino beam containing a flavour state $|\nu_\alpha(0,0)>$ which describes a neutrino generated with a definite flavour α at space-time point $(x,t)=(0,0)$. Suppose we aim the neutrinos along the x-axis and let them propagate in a free space towards a detector some distance L away.

The $\nu_{1,2}$ propagate according to the time-dependent Schrodinger Equation with no potentials

$$i\frac{\partial}{\partial t}|\nu_i(x,t)>=E|\nu_i(x,t)>=-\frac{1}{2m_i}\frac{\partial^2}{\partial x^2}|\nu_i(x,t)> \quad i\exists 1,2 \tag{17}$$

The solution to this equation is a plane-wave :

$$|\nu_k(x,t)>=e^{-i(E_k t-p_k x)}|\nu_k(0,0)>=e^{-i\phi_k}|\nu_k(0,0)> \tag{18}$$

where $p_k=(t,\mathbf{p})$ is the 4-momentum of the neutrino mass state $|\nu_k>$ and $x=(t,\mathbf{x})$ is the 4-space vector.

At some later space-time point (x,t) then the flavour state α will be

$$|\nu_\alpha(x,t)>=\sum_{k=1,2}U_{\alpha k}|\nu_k(x,t)>=\sum_{k=1,2}U_{\alpha k}e^{-i\phi_k}|\nu_k(0,0)> \tag{19}$$

Inverting the mixing matrix we can write

$$|\nu_k(0,0)>=\sum_\gamma U^*_{\gamma k}|\nu_\gamma(0,0)> \tag{20}$$

Two Flavour Neutrino Oscilations

Substituting Equation 20 into Equation 19 we then write the flavour state $|\nu_\alpha>$ at space-time point (x,t) in terms of the flavour states at the generation point

$$|\nu_\alpha(x,t)>=\sum_{k=1,2} U_{\alpha k} e^{-i\phi_k} \sum_\gamma U^*_{\gamma k} |\nu_\gamma(0,0)>=\sum_\gamma \sum_k U^*_{\gamma k} e^{-i\phi_k} U_{\alpha k} |\nu_\gamma(0,0)> \tag{21}$$

and so the transition amplitude for detecting a neutrino of flavour β at space-time point (t,x) given that we generated a neutrino of flavour α at space-time point $(0,0)$ is

$$\begin{aligned} A(\nu_\alpha(0,0)\rightarrow\nu_\beta(x,t)) &= <\nu_\beta(x,t)|\nu_\alpha(0,0)> \\ &= \sum_\gamma \sum_k U_{\gamma k} e^{i\phi_k} U^*_{\beta k} <\nu_\gamma(0,0)|\nu_\alpha(0,0)> \\ &= \sum_k U_{\alpha k} e^{i\phi_k} U^*_{\beta k} \end{aligned}$$

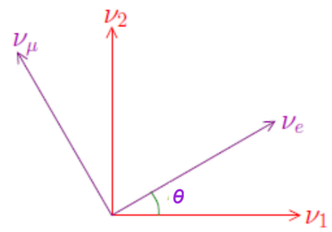
where the last step comes from the orthogonality of the flavour states, $<\nu_\gamma(0,0)|\nu_\alpha(0,0)>=\delta_{\gamma\alpha}$.

The oscillation probability is the coherent sum

$$\begin{aligned} P(\nu_\alpha\rightarrow\nu_\beta) = |A(\nu_\alpha(0,0)\rightarrow\nu_\beta(x,t))|^2 &= |\sum_k U_{\alpha k} e^{i\phi_k} U^*_{\beta k}|^2 \\ &= \sum_k U_{\alpha k} e^{i\phi_k} U^*_{\beta k} \sum_j U^*_{\alpha j} e^{-i\phi_j} U_{\beta j} \\ &= \sum_j \sum_k U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j} e^{-i(\phi_j-\phi_k)} \end{aligned}$$

Two Flavour Neutrino Oscilations

In the case of 2-dimensions, there is only one unitary matrix - the 2x2 rotation matrix which which rotates a vector in the flavour basis into a vector in the mass basis :



$$U = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$$

so that

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$\boxed{\begin{aligned} |\nu_e\rangle &= \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle \end{aligned}}$$

where θ is an unspecified parameter known as the *mixing angle*. This will have to be measured by an experiment. Using this matrix, we can find work out the oscillation probability in a somewhat more transparent form. The sum is over 4 elements with combinations of $k\exists(1,2)$ and $j\exists(1,2)$:

Two Flavour Neutrino Oscilations

- (k=1,j=1) : $U_{\alpha1}U^*_{\beta1}U^*_{\alpha1}U_{\beta1}e^{-i(\phi_1-\phi_1)} = |U_{\beta1}|^2|U_{\alpha1}|^2$
- (k=1, j=2) : $U_{\alpha1}U^*_{\beta1}U^*_{\alpha2}U_{\beta2}e^{-i(\phi_2-\phi_1)}$
- (k=2, j=1) : $U_{\alpha2}U^*_{\beta2}U^*_{\alpha1}U_{\beta1}e^{-i(\phi_1-\phi_2)}$
- (k=2, j=2) : $U_{\alpha2}U^*_{\beta2}U^*_{\alpha2}U_{\beta2}e^{-i(\phi_2-\phi_2)} = |U_{\beta2}|^2|U_{\alpha2}|^2$

So the oscillation probability is

$$
\begin{aligned}
P(\nu_\alpha \to \nu_\beta) &= (|U_{\beta1}|^2|U_{\alpha1}|^2 + |U_{\beta2}|^2|U_{\alpha2}|^2) + U_{\alpha1}U^*_{\beta1}U_{\alpha2}U^*_{\beta2}(e^{i(\phi_2-\phi_1)} + e^{-i(\phi_2-\phi_1)}) \\
&= (|U_{\beta1}|^2|U_{\alpha1}|^2 + |U_{\beta2}|^2|U_{\alpha2}|^2) + 2U_{\alpha1}U^*_{\beta1}U_{\alpha2}U^*_{\beta2}cos(\phi_2-\phi_1) \\
&= (sin^2\theta cos^2\theta + cos^2\theta sin^2\theta) + 2(cos\theta)(-sin\theta)(sin\theta)(cos\theta)cos(\phi_2-\phi_1) \\
&= 2cos^2\theta sin^2\theta(1 - cos(\phi_2-\phi_1)) \\
&= 2sin^2(2\theta)sin^2(\frac{\phi_2-\phi_1}{2})
\end{aligned}
$$

where in the last two steps I have used the trigonometric identites $cos\theta sin\theta = \frac{1}{2}sin(2\theta)$ and $2sin^2(\theta) = 1 - cos(2\theta)$.

Two Flavour Neutrino Oscilations

At this point we need to do something with the phase difference $\phi_2 - \phi_1$. Recall that

$$\phi_i = E_i t - p_i x \tag{40}$$

The phase difference is, then,

$$\phi_2 - \phi_1 = (E_2 - E_1)t - (p_2 - p_1)x \tag{41}$$

If we assume that the neutrinos are relativistic (a reasonable assumption), then $t = x = L$ (where L is the conventional measure of the distance between source and detector) and

$$p_i = \sqrt{E_i^2 - m_i^2} = E_i\sqrt{1 - \frac{m_i^2}{E_i^2}} \approx E_i(1 - \frac{m_i^2}{2E_i^2}) \tag{42}$$

so

$$\phi_2 - \phi_1 = (\frac{m_1^2}{2E_1} - \frac{m_2^2}{2E_2})L \tag{43}$$

$$\phi_2 - \phi_1 = (\frac{m_1^2}{2E_1} - \frac{m_2^2}{2E_2})L = \frac{\Delta m^2 L}{2E} \tag{44}$$

where $\Delta m^2 = m_1^2 - m_2^2$ and $E_1 = E_2 = E$.
Substituting back into the probability equation we get

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta)sin^2(\frac{\Delta m^2 L}{4E_\nu}) \tag{45}$$

Two Flavour Neutrino Oscilations

and if we agree to measure L in units of kilometres and E in units of GeV and pay attention to all the $\hbar$ and c we've left out we end up with

$$\boxed{P(\nu_e \rightarrow \nu_\mu) = sin^2(2\theta) sin^2(1.27\Delta m^2 \frac{L}{E_\nu})} \tag{46}$$

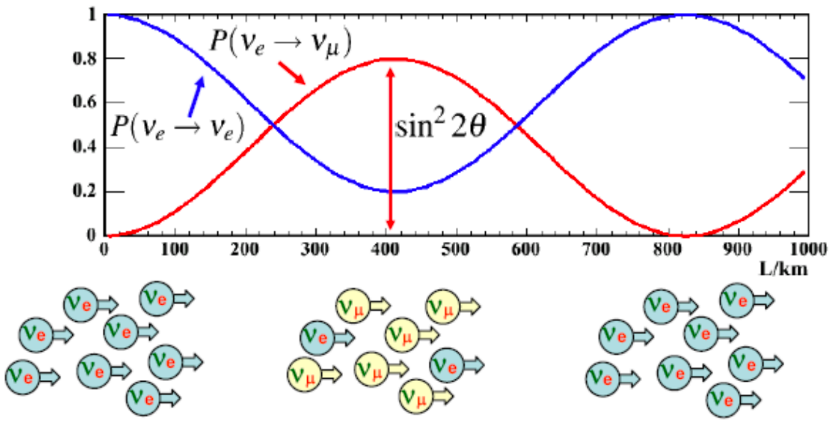
This is the probability that one generates a ν_e but detects ν_μ and is called the oscillation probability. The corresponding survival probability is the chance of generating a ν_e and detecting a ν_e : $P(\nu_e \rightarrow \nu_e) = 1 - P(\nu_e \rightarrow \nu_\mu)$.

$$\boxed{P(\nu_e \rightarrow \nu_e) = 1 - P(\nu_e \rightarrow \nu_\mu) = 1 - \ sin^2(2\theta) sin^2(1.27\Delta m^2 \frac{L}{E_\nu})}$$

Two Flavour Neutrino Oscillations

A plot of this function is shown in Figure 7 for a particular set of parameters : $\Delta m^2 = 3\times 10^{-3} eV^2$, $sin^2(2\theta) = 0.8$ and $E_\nu = 1$GeV. At $L = 0$, the oscillation probability is zero and the corresponding survival probability is one. As L increases the oscillatios begin to switch on until $1.27\Delta m^2 \frac{L}{E} = \frac{\pi}{2}$ or $L = 400$ km. At this point the oscillation is a maximum. However, the mixing angle is just $sin^2(2\theta) = 0.8$ so at maximal mixing, only 80% of the initial neutrinos have oscillated away. As L increases furthur, the oscillation dies down until, around $L = 820$ km, the beam is entirely composed of the initial neutrino flavour. If $sin^2(2\theta) = 1.0$, the oscillations would be referred to as *maximal*, meaning that at some point on the path to the detector 100% of the neutrinos have oscillated.

•e.g. $\Delta m^2 = 0.003\,\mathrm{eV}^2$, $\sin^2 2\theta = 0.8$, $E_\nu = 1\,\mathrm{GeV}$

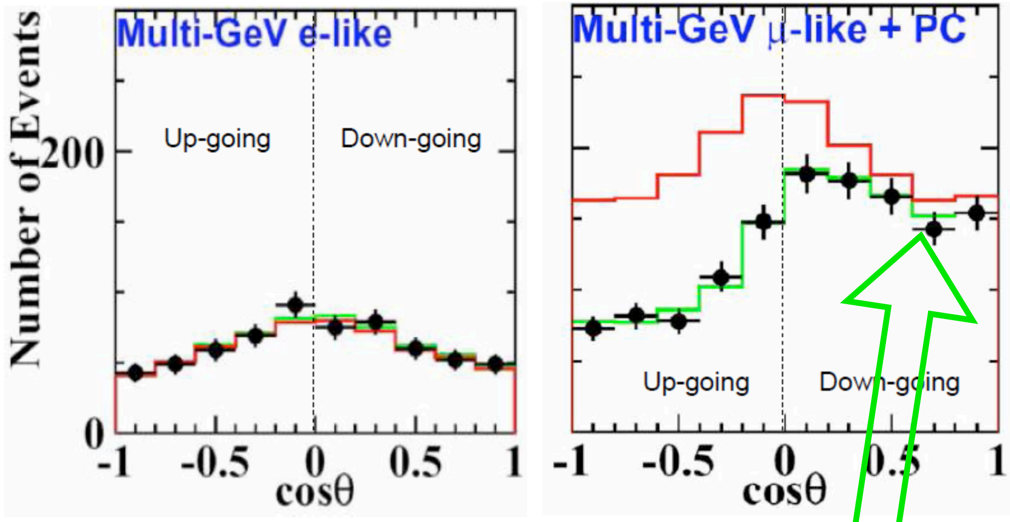


$$P(\nu_e \to \nu_e) = 1 - sin^2(2\theta) sin^2(1.27\Delta m^2 \frac{L}{E_\nu})$$

$$P(\nu_e \to \nu_\mu) = sin^2(2\theta) sin^2(1.27\Delta m^2 \frac{L}{E_\nu})$$

Figure 7: The oscillation probability as a function of the baseline, L, for a given set of parameters : $\Delta m^2 = 3 \times 10^{-3} eV^2$, $sin^2(2\theta) = 0.8$ and $E_\nu = 1$GeV.

Super-Kamiokande Results



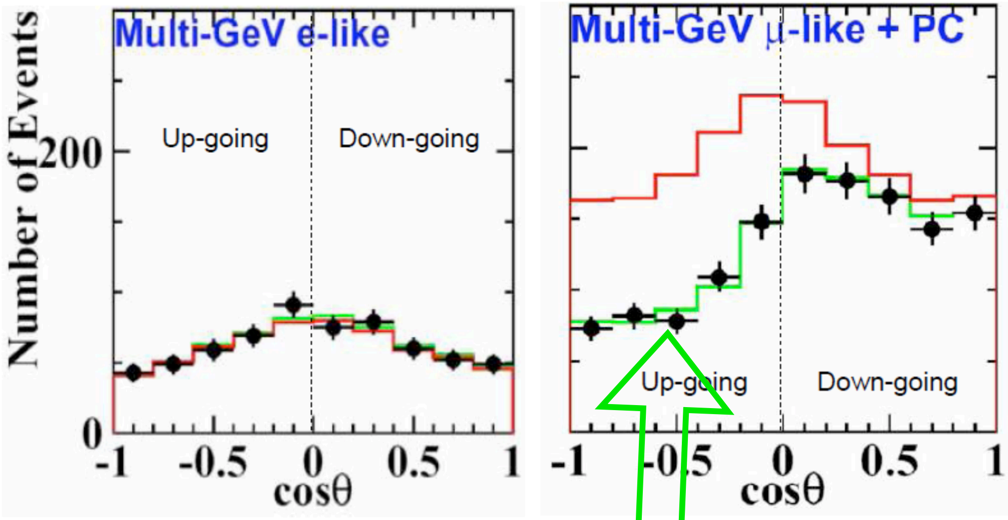
$$P(\nu_\mu \rightarrow \nu_\tau) = sin^2(2\theta_{atm}) sin^2(1.27\Delta m^2_{atm} \frac{L}{E_\nu}) \tag{53}$$

where θ_{atm} and Δm^2_{atm} are the mixing angle and squared mass difference for the atmospheric neutrinos respectively. Let us suppose that Δm^2_{atm} is around $1 \times 10^{-3} eV^2$. If L/E is small, then $sin^2(1.27\Delta m^2_{atm}\frac{L}{E_\nu})$ is too small for the oscillations to have started. Suppose that the multi-GeV plot has neutrino energy of about 1 GeV. The baseline for downward going neutrino is on the order of 10 km, so

$$1.27\Delta m^2_{atm}\frac{L}{E_\nu} = 1.27 \times 10^{-3} \times 10(km)/1(GeV) = 0.00127 \tag{54}$$

Hence $P(\nu_\mu \rightarrow \nu_\tau) = sin^2(2\theta_{atm}) sin^2(0.00127) <= sin^2(0.00127) = 1.6 \times 10^{-6}$. This can explain the downward going muon-like behaviour - the baseline isn't long enough for the relevant oscillations

Super-Kamiokande Results



to have started. However, as the zenith angle sweeps around from zero degrees to 180 degrees, the distance neutrinos travel to the detector (see Figure 5) sweeps from around 10 km all the way to around 13000 km. At a baseline of 13000 km,

$$1.27\Delta m^2_{atm}\frac{L}{E_\nu} = 1.27 \times 10^{-3} \times 13000(km)/1(GeV) = 16.51 \tag{55}$$

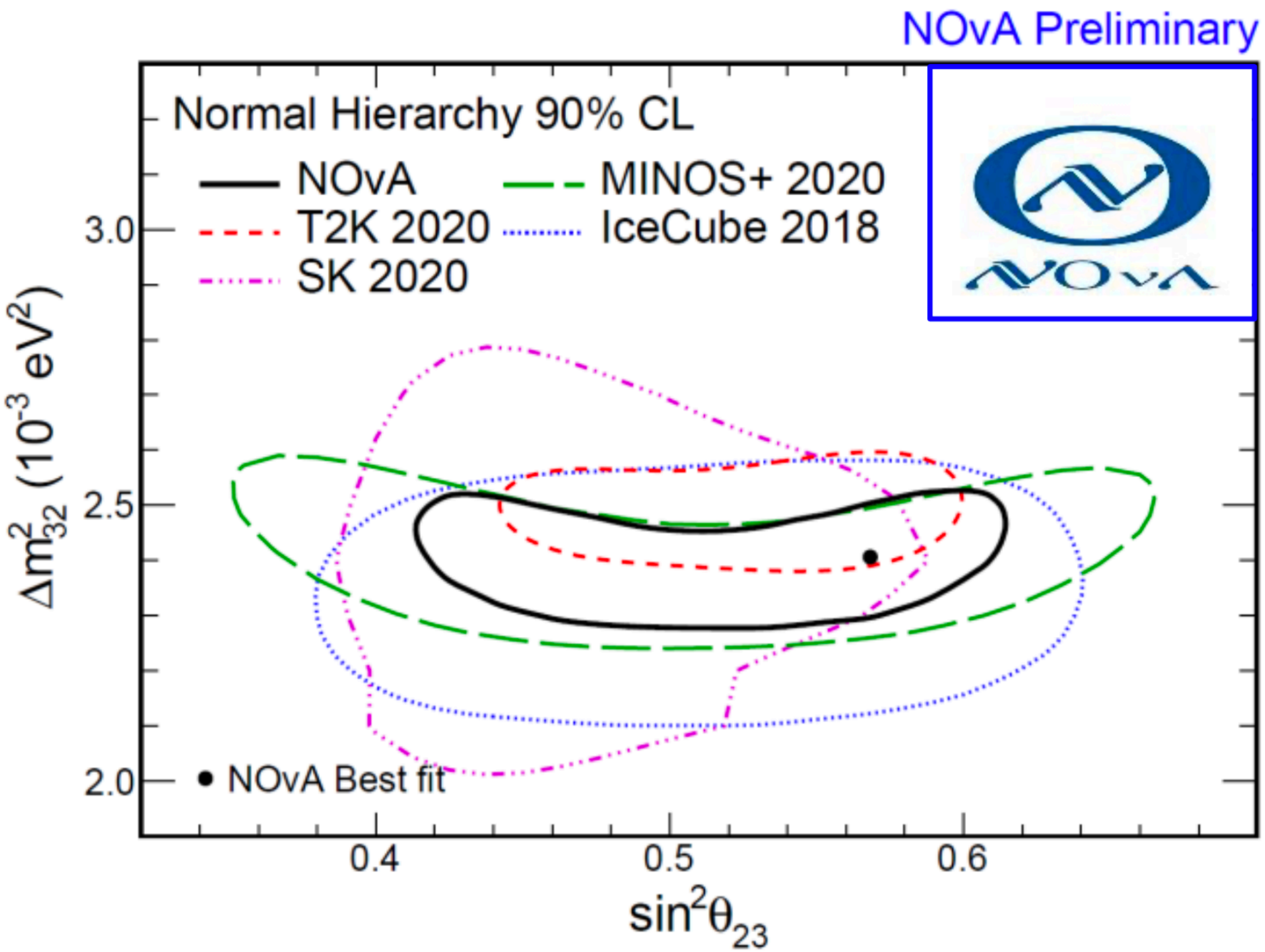
Here $P(\nu_\mu \to \nu_\tau) = sin^2(2\theta_{atm})sin^2(16.51) <= sin^2(16.51) = 0.51$. This explains the upward-going muon behaviour. About 50% have oscillated away which seems to agree with the data. In this case the frequency of oscillation is so fast that the $sin^2(1.27\Delta m^2_{atm}\frac{L}{E_\nu})$ term just averages to 0.5 and so, $P(\nu_\mu \to \nu_\tau) \approx 0.5sin^(2\theta_{atm})$. This also seems to suggest that $sin^(2\theta_{atm}) \approx 1.0$ or that the mixing angle is 45 degrees.

In fact, after proper analysis we find that

$$\Delta m^2_{atm} = 3 \times 10^{-3}\mathrm{eV}^2 \qquad sin^2(2\theta_{atm}) = 1.0 \tag{56}$$

and that the oscillation is almost completely $\nu_\mu \to \nu_\tau$.

Phenomenology of ν Oscillations



$$\sin^2(\theta_{23}) = 0.57^{+0.04}_{-0.03}$$
$$\Delta m^2_{32} = (2.41 \pm 0.07) \times 10^{-3}\ \text{eV}^2\ \text{(NO)}$$

MNS matrix

$$U_{MNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Standard parameterization of Maki-Nakagawa-Sakata matrix

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$s_{13} = \sin\theta_{13}$
$c_{13} = \cos\theta_{13}$

Atmospheric SOLAR

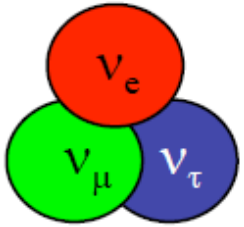
Solar & atmospheric ν oscillations easily accommodated within 3 generations.

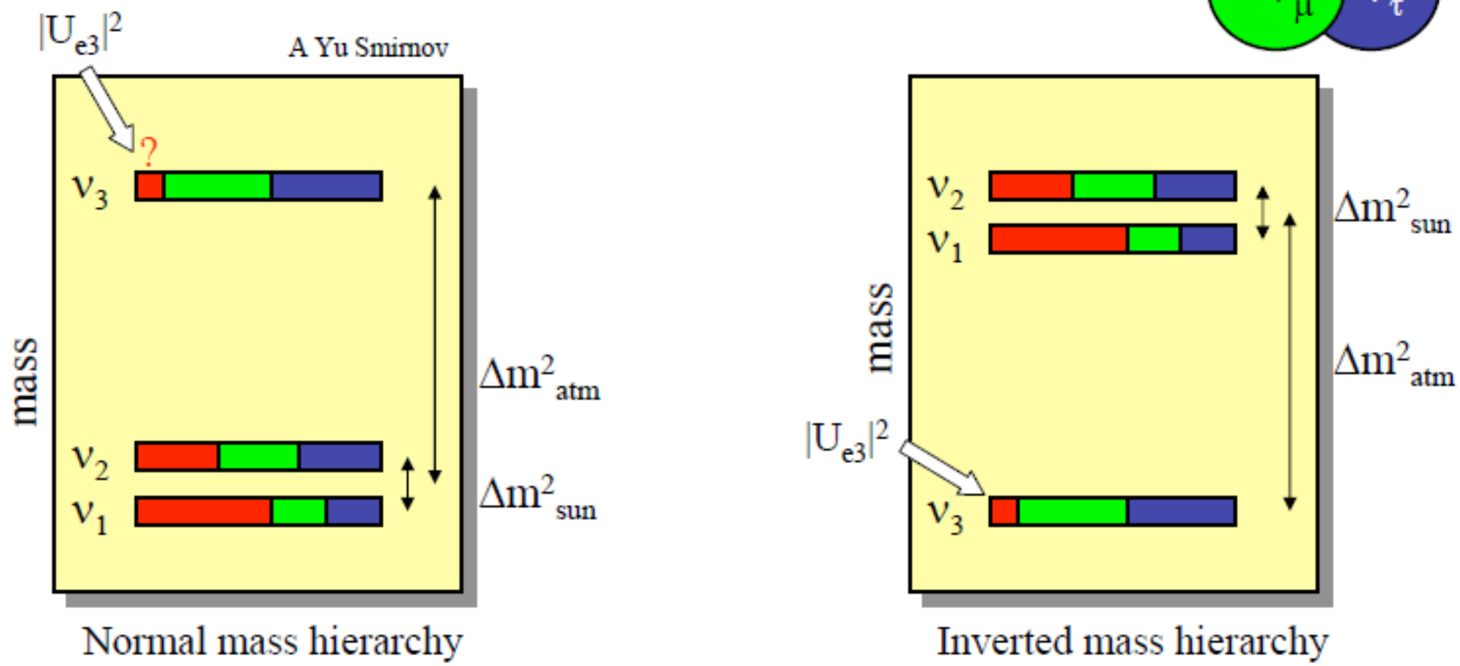
Because of small $\sin^2 2\theta_{13}$, **solar & atmospheric ν oscillations almost decouple**

θ_{23} (atmospheric) $\cong 45^0$
θ_{12} (solar) $\cong 30^0$
θ_{13} (reactor) $< 13^0$
δ?

$$U_{MNS}: \begin{pmatrix} \sim \frac{\sqrt{2}}{2} & \sim -\frac{\sqrt{2}}{2} & \sin\theta_{13}\, e^{i\delta} \\ \sim \frac{1}{2} & \sim \frac{1}{2} & \sim -\frac{\sqrt{2}}{2} \\ \sim \frac{1}{2} & \sim \frac{1}{2} & \sim \frac{\sqrt{2}}{2} \end{pmatrix}$$

Mass spectrum and mixing





We do not know yet:

- Absolute mass scale
- Type of the mass hierarchy: Normal, Inverted
- U_{e3} = ? We know only that it is smaller than the other angles

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Solar Neutrinos