

Universidade do Minho
Departamento de Física

Experimental Particle and Astroparticle Physics<br>Master Course<br>(Univ.Minho, Univ.Aveiro e Univ.Porto)<br>Examination (duration: 3h00)<br>Justify all your answers!<br>7 December 2021, 14h00-17h00

(1) Consider the scattering process $\mathrm{e}^{-}+\mu^{-} \rightarrow \mathrm{e}^{-}+\mu^{-}$(figure 1 ) in the center-of-mass system, where particles 1 and 2 are moving along the $z$-direction (in opposite directions) with energy E (consider the masses of all particles are negligeable, given E). Evaluate the madelstam variables $s, t$ and $u$ for the scattering.


Figure 1
(2) Consider a three-body decay $M \rightarrow 1+2+3$ (having particles 1,2 and 3 four-momentum $p_{1}, p_{2}$ and $p_{3}$, and masses $m_{1}, m_{2}$ and $m_{3}$, respectively)
a) show how to construct a Dalitz plot using the invariant masses squared, $\mathrm{m}^{2}{ }_{12}$ and $\mathrm{m}^{2}{ }_{23}$, and check if the phase space is uniform across the distribution. What are the minimum and maximum values of $\mathrm{m}^{2}{ }_{12}$ and $\mathrm{m}^{2}{ }_{23}$ ?
b) In figure 2, an unknown particle $X$ is observed to decay according to $X \rightarrow p+K+\pi$. ( $\mathrm{p}=$ proton, $\mathrm{K}=$ kaon and $\mathrm{n}=\mathrm{pion}$ ). What can you tell, from the figure, about the different particles present in the decay?


Figure 2.
(3) Check the commutation relation of momentum and position $\left[p^{\mu}, x^{\nu}\right]=i \hbar g^{\mu \nu}$.
(4) Consider the Klein-Gordon (KG) equation $\mathrm{p}_{\mu} \mathrm{p}^{\mu} \psi=\mathrm{m}_{0}{ }^{2} \mathrm{c}^{2} \mu$ for a free particle with mass $\mathrm{m}_{0}$. Discuss the free particle solutions of the KG equation and construct the four current density. Show how introducing the current density of charge affects the solutions of the KG equation.

