Measuring the effective range of the $X(3872)$

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Weinberg's criterion

- Weinberg's criterion relates the "elementariness" Z of a state to the behavior of the amplitude close to threshold
- Up to orders $O(1/m_\pi)$, the amplitude at threshold is given by

$$t(k) = \frac{1}{-\kappa_0 + \frac{1}{2} r_0 k^2 - ik + O(k^4/m_\pi^3)}$$

  which leads to

$$r_0 = -\frac{Z}{1-Z}\kappa^{-1} + O(1/m_\pi)$$

  where $\kappa = \sqrt{2\mu B}$ the binding momentum
- Bethe + Smorodinsky show that $O(1/m_\pi) > 0$ under general assumptions, so that $Z = 0$ (molecule) implies $r_0 > 0$

Conditions for Weinberg

- This argument applies to single channel scattering (e.g. deuteron)
- The $X(3872)$ is NOT a single channel problem!
- However, it lies veryveryvery close to the $D^0\bar{D}^{*0}$ threshold, it makes sense to focus on just this channel
- How to take rid of the other ones?

Coupled channel problem

Let's study the scattering in the channel $0 \to 0$ close to its threshold, with $E_\ell^{\text{thr}} < \cdots < E_0^{\text{thr}} < \cdots < E_H^{\text{thr}}$

$$t_{0\to 0}^{-1}(E) \simeq f(k_\ell^2, \ldots, k_0^2, \ldots, k_H^2) - i g_\ell k_\ell \cdots - i g_0 k_0 \cdots - i g_H k_H$$

with $k_i = \sqrt{2\mu_i (E - E_i^{\text{thr}})}$

- The term with $-i g_\ell k_\ell$ is smooth and imaginary, it doesn't mix with f, we can set it to zero
- what about $-i g_H k_H$? It is real, since we are below the H-threshold. It mixes with f, if I put it to zero I will modify badly the properties of the pole

Coupled channel problem

Let's study the scattering in the channel $i \to i$ close to its threshold, with $E_1^{\text{thr}} < \cdots < E_i^{\text{thr}} < \cdots < E_H^{\text{thr}}$

$$t_{ii}^{-1}(E) \simeq \tilde{f}(k_i^2) - i g_i k_i$$

- I absorb all the heavier channels in $\tilde{f}(k_i^2)$
- Does it mean that those are controlling the physics at $\sim E_i^{\text{thr}}$?
  No, this is just an effective amplitude parametrization, not my full theory.
  I only want to extract κ_0 and r_0

Isospin violation (1)

What's the situation with the $X(3872)$?
(For simplicity, $E^{\text{thr}}_{D^0\bar{D}^{*0}} = 0$)

$$t^{-1}(E) \propto E - m_X^0 + \frac{i}{2} g_{\text{LHCb}} \left(\sqrt{2\mu E} + \sqrt{2\mu_+(E-\delta)} \right) + \frac{i}{2} \left(\Gamma_\rho^0(E) + \Gamma_\omega^0(E) + \Gamma_0^0 \right)$$

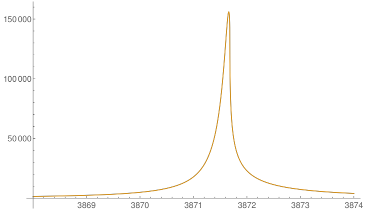
- All Γs open safely below $E^{\text{thr}}_{D^0\bar{D}^{*0}}, \Rightarrow \Gamma = 0$
- What about the charged channel? In the isospin limit, $\delta = 0$ and one can assume scattering in $I = 0$ only
- However, $B \sim O(100 \text{ keV}) \ll \delta \simeq 8.2$ MeV

Isospin violation (2)

Isospin is badly broken. The question we want to ask is:
"Is the X a $D^0 \bar{D}^{*0}$ molecule" rather than
"Is the X a $D\bar{D}^*$ molecule with $I = 0$ in the isospin limit"

If we want to answer the first question, expanding $\sqrt{2\mu_+(E-\delta)}$ at $E = 0$ and get r_0 is the right thing to do.

The fact that this is fine can be seen by comparing the original curve with the one where $\sqrt{2\mu_+(E-\delta)}$ is replaced by the expansion



Effective range (1)

From the LHCb Flatté we get

$$r_0 = -\frac{2}{\mu g_{\text{LHCb}}} - \sqrt{\frac{\mu_+}{2\mu^2\delta}} \simeq -5.34 \text{ fm}\,.$$

for the central value $g_{\text{LHCb}} = 0.108$.
Incidentally, this leads to $Z = 14\%$ as found by LHCb
However, if $g_{\text{LHCb}} \gg 1$ is compatible with errors,

$$r_0 = -\sqrt{\frac{\mu_+}{2\mu^2\delta}} \simeq -1.56 \text{ fm}\,.$$

still negative.

Effective range (2)

Does it mean that the properties of the charged threshold are driving the moleculariness of the X? **No**, it means that the parametrization we are using seems to do so, but what we do is

1. switching off the open widths (imaginary parts)
2. taking the derivative in $E = 0$

which is a well-defined procedure.

One should study other parametrizations and apply the same procedure to see if the sign of r_0 stays unchanged.

Conclusions

We aim at extracting the effective range of the $D^0\bar{D}^{*0}$ scattering, using a generic lineshape parametrization

When applied to the Flatté amplitude published by LHCb, we get

$$-5.34\ \text{fm} \lesssim r_0 \lesssim -1.56\ \text{fm}$$

within errors.

Negative values of r_0 point to the existence of a bare compact state in the X wave function