Double-beta decay and testing of fundamental symmetries

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UNESCO

United Nations Educational, Scientific and Cultural Organization

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Centre International de Formation et de Recherche Avancées en Physique filiale de l'INPM

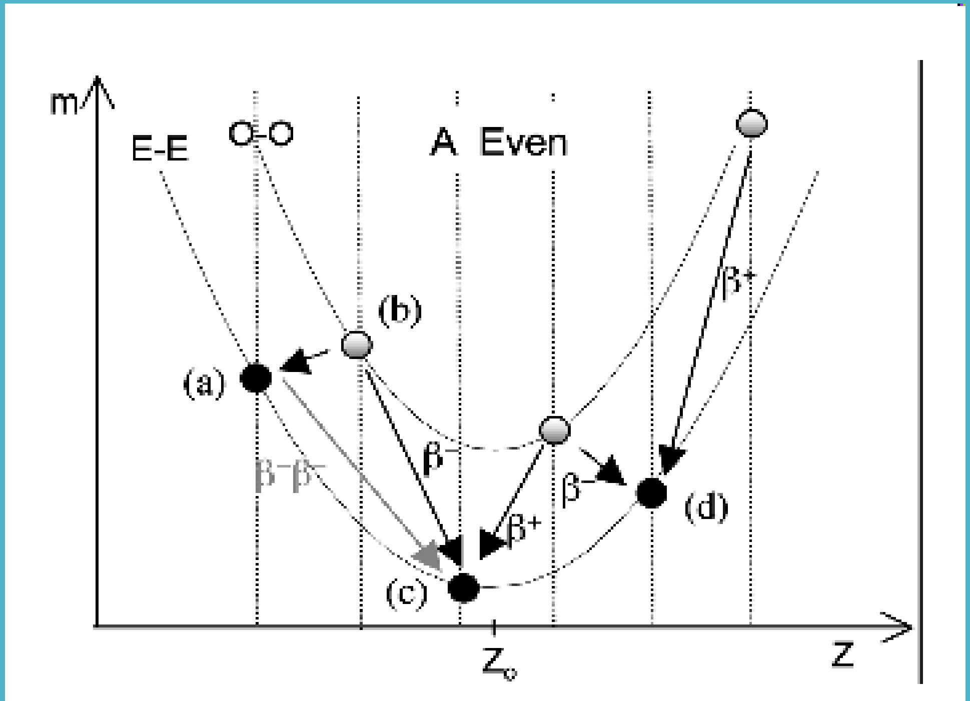
Outline

- Introduction to double-beta decay process
- Challenges in the double-beta decay study
- Double-beta decay potential to search for beyond Standard Model physics
- Search of Lorentz invariance violation (LIV) in $2\nu\beta\beta$ decay
- Conclusions

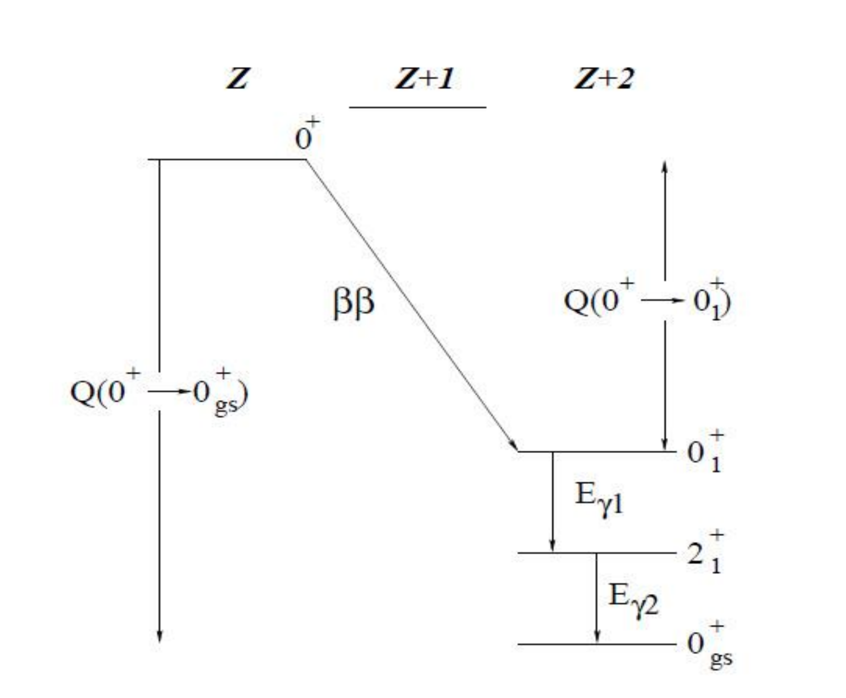
Double Beta Decay

DBD is the rarest known radioactive decay measured until now, by which an e-e nucleus transforms into another e-e nucleus with the same mass A, but with its nuclear charge changed by two units (Z±2)

It occurs whatever single β decay can not occur due to energetical reasons or it is highly forbidden by angular momentum selection rules

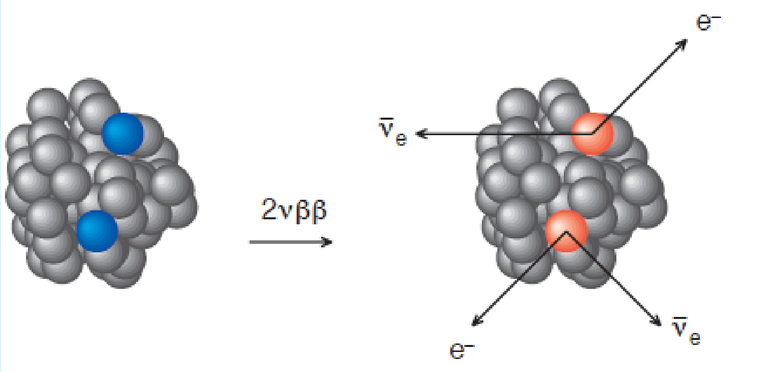


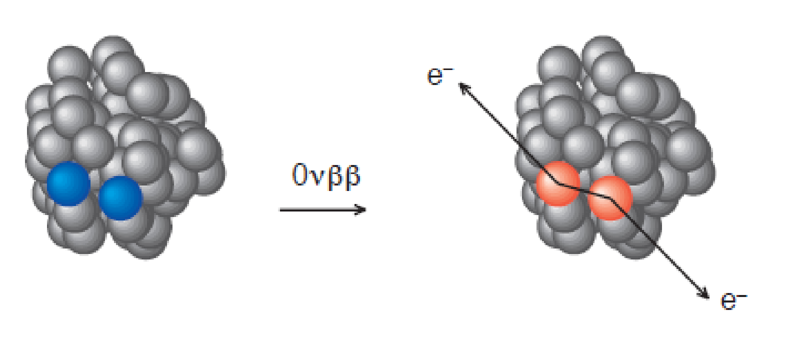
(a) and **(d)** are stable against β decay, but unstable against β β decay: $\beta^-\beta^-$ for (a) and $\beta^+\beta^+$ for (d)



35 isotopes decaying $\beta^-\beta^-$
Several isotopes decaying $\beta^+\beta^+$

Double Beta Decay processes

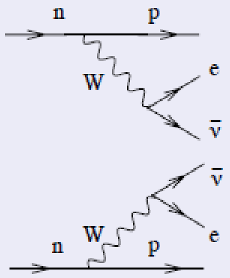




Double-electron decays	$2\nu\beta^-\beta^-$ $0\nu\beta^-\beta^-$
Double-positron	$2\nu\beta^+\beta^+$ $0\nu\beta^+\beta^+$
EC/electron	$2\nu EC\beta^+$ $0\nu EC\beta^+$
EC/EC	$2\nu ECEC$ $0\nu ECEC$

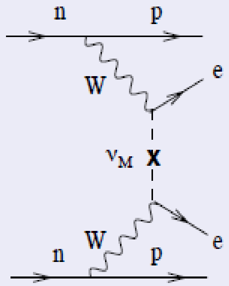
$2\nu\beta\beta$

- $(Z,A) \rightarrow (Z+2,A) + 2e^- + 2\bar{\nu}_e$
- $\Delta L = 0$
- $\left|T_{1/2}^{2\nu}\right|^{-1} = G^{2\nu}(Q_{\beta\beta},Z)\,|M_{2\nu}|^2 \sim \left|10^{20}\ \mathrm{y}\right|^{-1}$



$0\nu\beta\beta$

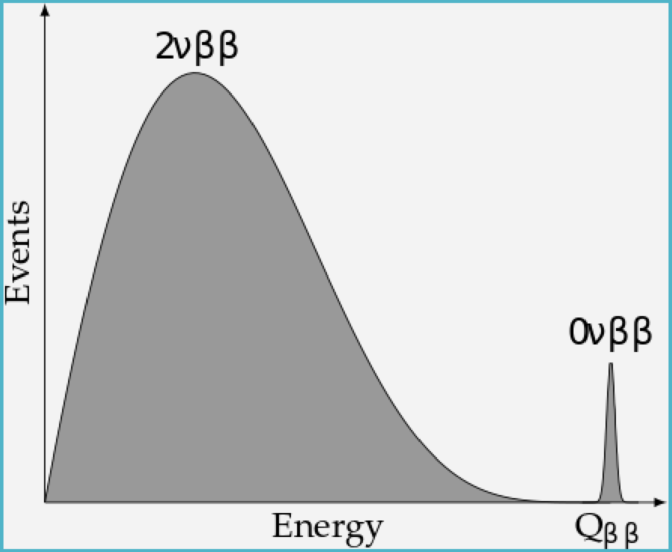
- $(Z,A) \rightarrow (Z+2,A) + 2e^-$
- $\Delta L = 2$
- $\left|T_{1/2}^{0\nu}\right|^{-1} = G^{0\nu}(Q_{\beta\beta},Z)\,|M_{0\nu}|^2\,\langle m_{\beta\beta}^2\rangle \sim \left|10^{25}\ \mathrm{y}\right|^{-1}$
- $\langle m_{\beta\beta}\rangle = \left|\sum_i U_{ei}^2\ m_i\right|$



$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$\nu_e + n \rightarrow e^- + p$$

DBD experimental results



Isotope	$Q_{\beta\beta}$[MeV]	$T^{2\nu}$ [yr] [1]
^{48}Ca	4.272	4.40×10^{19}
^{76}Ge	2.039	1.65×10^{21}
^{82}Se	2.995	9.20×10^{19}
^{96}Zr	3.350	2.30×10^{19}
^{100}Mo	3.034	7.10×10^{18}
^{116}Cd	2.814	2.87×10^{19}
^{128}Te	0.866	2.00×10^{21}
^{130}Te	2.527	6.90×10^{20}
^{136}Xe	2.458	2.19×10^{21}
^{150}Nd	3.371	8.20×10^{18}
^{238}U	1.450	2.00×10^{21}
^{235}Ba(2νECEC)	2.619	~ 1.0×10^{21}
^{100}Mo-$^{100}Ru(0_1)$	1.903	6.70×10^{20}
^{150}Nd-$^{150}Sm(0_1)$	2.630	1.20×10^{20}

DBD experiments in different stages:

a) completed (Gotthard TPC, Heidelberg-Moscow, IGEX, NEMO1,2,3)

b) taking data (COBRA, CUORICINIO-CUORE, EXO, DCBA, GERDA, KamLAND-Zen, MAJORANA, XMASS)

c) proposed/future(CANDLES, MOON, AMoRE, LEGEND, NEXT, SNO+, SuperNEMO, TIN.TIN)

Importance of the DBD study

Neutrino properties:
- character Dirac or Majorana?
- mass scale (absolute mass)
- mass hierarchy
- how many flavors? Sterile neutrinos?

Check of some symmetries: Lepton number, CP, Lorentz

Constrain BSM parameters: associated with different mechanisms/scenarios that may contribute to the neutrinoless DBD occurrence

Double-beta decay lifetimes

$[T^{2\nu}_{1/2}]^{-1} = G^{2\nu}(Q_{\beta\beta}, Z) \quad \text{x} \quad g_A^4 \;\text{x}\; |m_e c^2 M^{2\nu}|^2$ *$2\nu\beta\beta$*

$[T^{0\nu}_{1/2}]^{-1} = \sum_k G^{0\nu}(Q_{\beta\beta}, Z) \quad \text{x} \quad g_A^4 \;\text{x}\; |M_k^{0\nu}|^2 \quad \text{x} \quad <\eta_k>$ *$0\nu\beta\beta$*

$\downarrow \qquad \downarrow \qquad \downarrow$

atomic physics — nuclear physics — particle physics

PSF — NME — BSM

$G^{(2,0)\nu}(E_0, Z)$ phase space factors (PSF)

$M^{(2,0)\nu}$ = nuclear matrix elements (NME)

$$M_{2\nu} = \sum_N \frac{\langle 0_F^+ \| \tau^+\sigma \| 1_N^+\rangle\langle 1_N^+ \| \tau^+\sigma \| 0_I^+\rangle}{\frac{1}{2}W_0 + E_N - E_I}$$

$$M^{0\nu} = M^{0\nu}_{GT} - \left(\frac{g_V}{g_A}\right)^2 M_F^{0\nu} + M_T^{0\nu}$$

$\Sigma_k M^{0\nu}{}_k = |M^{0\nu}{}_\nu|^2 <m_\nu>^2 + |M^{0\nu}{}_N|^2 <m_N>^2 + |M^{0\nu}{}_\lambda|^2 <\eta_\lambda>^2 + |M^{0\nu}{}_q|^2 <\eta_q>^2 + \ldots$

$<\eta_l>$ = BSM parameter depending on the $0\nu\beta\beta$ mechanism $\qquad <\eta_\nu> = <m_\nu>/m_e$

g_A = axial-vector constant

Precise calculations of PSF and NME to predict lifetimes, derive neutrino parameters, extract information on neutrino properties

Challenging issues in double beta decay

1) Theoretical :

- accurate calculation of the NME (a long standing problem, not yet resolved)
- Phase space factors (PSF), electron spectra, angular correlation between electrons
- extraction of the information regarding the ν mass, mass hierarchy,..
- models for the $0\nu\beta\beta$ decay mechanisms, constrain BSM parameters

2) Experimental:

- accurate measurements of $2\nu\beta\beta$ decay, including transitions to excited states, precise determination of electron spectra, angular correlations, etc.
- search for $0\nu\beta\beta$ decay: improvements of experimental set-ups and techniques → large isotopically enriched sources; the reducing of background; detectors with high energy resolution, improved techniques of detection, etc.

- determination of the $0\nu\beta\beta$ decay mechanisms

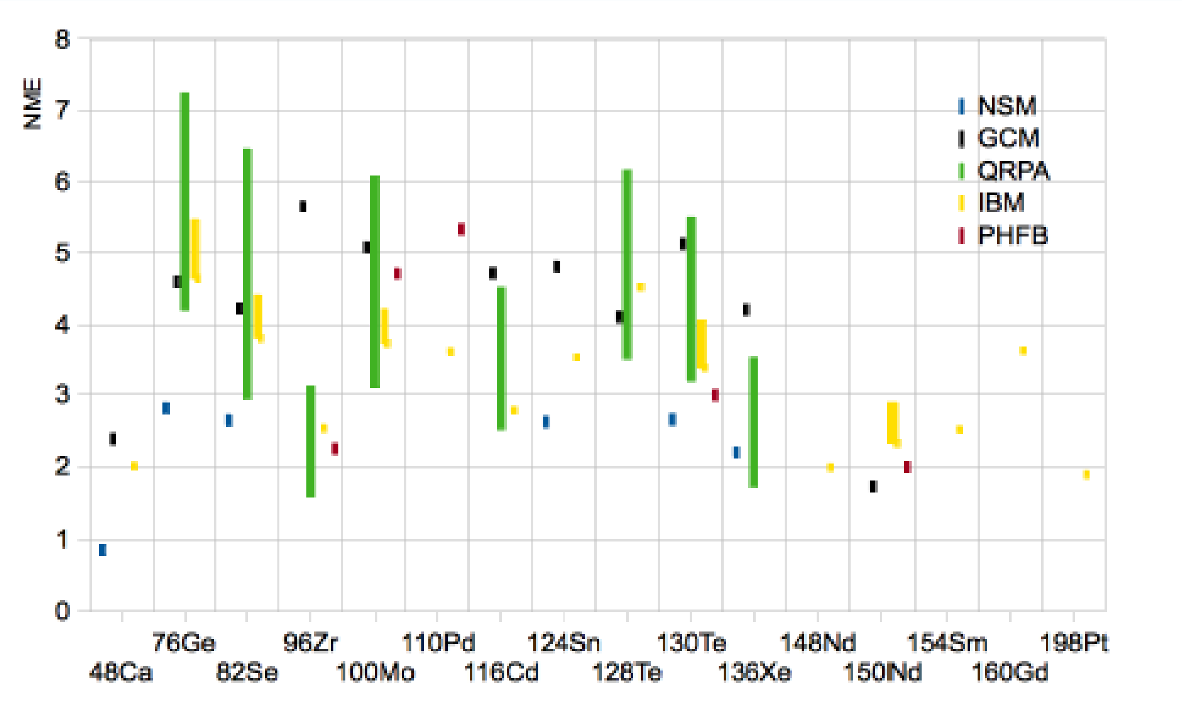
Calculation of the nuclear matrix elements

2νββ

$$M_{GT}^{2\nu} = \sum_j \frac{\langle 0_f^+ | t_- \sigma || 1_j^+ \rangle \langle 1_j^+ || t_- \sigma | 0_i^+ \rangle}{E_j + Q/2 + m_e - E_i}$$

0νββ

$$M^{0\nu} = M_{GT}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 \cdot M_F^{0\nu} - M_T^{0\nu}$$



a) pnQRPA (different versions)

b) interacting Shell model (ISM)

c) IBM-2

d) Generator coordinate method

e) Projected HFB

Origin of differences

- many-body theory (correlations)
- single-particle model space
- effective NN interaction
- Nuclear (input) parameters
  g_A , R, <E>, nuclear form factors

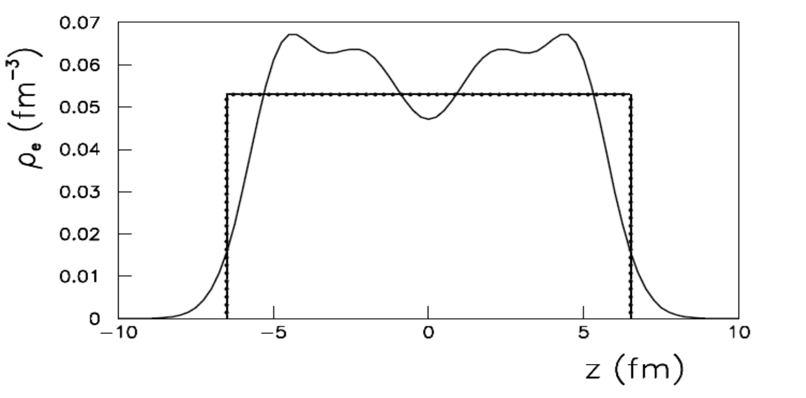
Calculation of the phase space factors for DBD

$$G_{2\nu}^{\beta\beta}(0^+ \to 0^+) = \frac{2\tilde{A}^2}{3\ln 2 g_A^4 (m_e c^2)^2} \int_{m_e c^2}^{Q^{\beta\beta}+m_e c^2} d\epsilon_1 \int_{m_e c^2}^{Q^{\beta\beta}+2m_e c^2-\epsilon_1} d\epsilon_2 \int_0^{Q^{\beta\beta}+2mc_e^2-\epsilon_1-\epsilon_2} d\omega_1 f_{11}^{(0)} w_{2\nu}(\langle K_N\rangle^2+\langle L_N\rangle^2+\langle K_N\rangle\langle L_N\rangle)$$

$$G_{0\nu}^{\beta\beta}(0^+ \to 0^+) = \frac{2}{4g_A^4 R_A^2 \ln 2} \int_{m_e c^2}^{Q^{\beta\beta}+m_e c^2} f_{11}^{(0)} w_{0\nu} d\epsilon_1$$

$$f_{11}^{(0)} = | f^{-1-1} |^2 + | f_{11} |^2 + | f^{-1}{}_1 |^2 + | f_1{}^{-1} |^2$$

$$f^{-1-1} = g_{-1}(\epsilon_1) g_{-1}(\epsilon_2) \; ; \; f_{11} = f_1(\epsilon_1) f_1(\epsilon_2),$$
$$f^{-1}{}_1 = g_{-1}(\epsilon_1) f_1(\epsilon_2) \; ; \; f_1{}^{-1} = f_1(\epsilon_1) g_1(\epsilon_2)$$



$$\frac{dg_\kappa(\epsilon,r)}{dr} = -\frac{\kappa}{r} g_\kappa(\epsilon,r) + \frac{\epsilon - V + m_e c^2}{c\hbar} f_\kappa(\epsilon,r)$$
$$\frac{df_\kappa(\epsilon,r)}{dr} = -\frac{\epsilon - V - m_e c^2}{c\hbar} g_\kappa(\epsilon,r) + \frac{\kappa}{r} f_\kappa(\epsilon,r)$$

$$V(Z,r) = \begin{cases} -\frac{Z\alpha\hbar c}{r}, & r \geq R_A \\ -Z(\alpha\hbar c)\left(\frac{3-(r/R_A)^2}{2R_A}\right), & r < R_A \end{cases}$$

$$V(r) = \alpha\hbar c \int \frac{\rho_e(\vec{r'})}{|\vec{r} - \vec{r'}|} d\vec{r'} \qquad \rho_e(\vec{r}) = 2\sum_i v_i^2 \, | \Psi_i(\vec{r}) |^2$$

Table 1: PSF for $\beta^-\beta^-$ decays to final g.s.

Nucleus	$Q^{\beta^-\beta^-}_{g.s.}$ (MeV)	$G^{\beta^-\beta^-}_{2\nu}(g.s.)$ $(10^{-21}$ yr$^{-1})$ This work	[27]	[23, 24]	[26]	$G^{\beta^-\beta^-}_{0\nu}(g.s.)$ $(10^{-15}$ yr$^{-1})$ This work	[27]	[23, 24]	[26]
^{48}Ca	4.267	15536	15550	16200	16200	24.65	24.81	26.1	26.0
^{76}Ge	2.039	46.47	48.17	53.8	52.6	2.372	2.363	2.62	2.55
^{82}Se	2.996	1573	1596	1830	1740	10.14	10.16	11.4	11.1
^{96}Zr	3.349	6744	6816		7280	20.48	20.58		23.1
^{100}Mo	3.034	3231	3308	3860	3600	15.84	15.92	18.7	45.6
^{110}Pd	2.017	132.5	137.7			4.915	4.815		
^{116}Cd	2.813	2688	2764		2990	16.62	16.70		18.9
^{128}Te	0.8665	0.2149	0.2688	0.35	0.344	0.5783	0.5878	0.748	0.671
^{130}Te	2.528	1442	1529	1970	1940	14.24	14.22	19.4	16.7
^{136}Xe	2.458	1332	1433	2030	1980	14.54	14.58	19.4	17.7
^{150}Nd	3.371	35397	36430	48700	48500	61.94	63.03	85.9	78.4
^{238}U	1.144	98.51	14.57			32.53	33.61		

Stoica, Mirea, Frontiers in Physics **7** (2019)
S. Stoica,Mirea, Phys. Rev. C **88** (2013)
Mirea, Pahomi, Stoica Rom.Rep.Phys. ***67****(2015)*

Table 2 Majorana neutrino mass parameters together with the other components of the $0\nu\beta\beta$ decay halftimes: the $Q_{\beta\beta}$ values, the experimental lifetimes limits, the phase space factors and the nuclear matrix elements.

	$Q_{\beta\beta}[MeV]$	$T^{0\nu\beta\beta}_{exp}[yr]$	$G^{0\nu\beta\beta}[yr^{-1}]$	$M^{0\nu\beta\beta}$	$\langle m_\nu \rangle$ $[eV]$
^{48}Ca	4.272	$> 5.8\ 10^{22}$[52]	2.46E-14	0.81-0.90	$< [15.0 - 16.7]$
^{76}Ge	2.039	$> 2.1\ 10^{25}$[38]	2.37E-15	2.81-6.16	$< [0.37 - 0.82]$
^{82}Se	2.995	$> 3.6\ 10^{23}$[53]	1.01E-14	2.64-4.99	$< [1.70 - 3.21]$
^{96}Zr	3.350	$> 9.2\ 10^{21}$[54]	2.05E-14	2.19-5.65	$< [6.59 - 17.0]$
^{100}Mo	3.034	$> 1.1\ 10^{24}$[53]	1.57E-14	3.93-6.07	$< [0.64 - 0.99]$
^{116}Cd	2.814	$> 1.7\ 10^{23}$[56]	1.66E-14	3.29-4.79	$< [2.00 - 2.92]$
^{130}Te	2.527	$> 2.8\ 10^{24}$[57]	1.41E-14	2.65-5.13	$< [0.50 - 0.97]$
^{136}Xe	2.458	$> 1.6\ 10^{25}$[39]	1.45E-14	2.19-4.20	$< [0.25 - 0.48]$
^{150}Nd	3.371	$> 1.8\ 10^{22}$[55]	6.19E-14	1.71-3.16	$< [4.84 - 8.95]$

[23] M. Doi, T. Kotani and E. Takasugi, Prog. Theor. Phys. Suppl. **83**, 1 (1985).

[24] M. Doi and T. Kotani, Prog. Theor. Phys. **87**, 1207 (1992); ibidem **89**, 139 (1993).

[26] J. Suhonen and O. Civitarese, Phys. Rep. **300**, 123 (1998).

[27] J. Kotila and F. Iachello, Phys. Rev. C **85**, 034316 (2012).

Lorentz violation in weak decays

- LV can also be investigated in β and ββ decays
- The general framework characterizing LV is the Standard Model Extension (SME)
- In minimal SME (operators dimension ≤ 4) there are operators that couples to ν_s and affect ν flavor oscillations, ν velocity or ν phase spaces (β, ββ decays)
- There is a q-independent operator (countershaded operator), that doesn't affect ν oscillations, and hence can not be detected in long base-lines (LBL) experiments
- The corresponding coefficient has 4 components (one time-like, $(å^{(3)}{}_{of})_{00}$ and 3 space-like); a non-zero value of $å^{(3)}{}_{of})_{00}$ would produce small deviations in the shape of the electrons spectrum.

In *2νββ* - the electron energy sum spectrum may receive a correction that is maximized at a well-defined energy, depending of the isotope

- the one electron spectra and angular correlation between the electrons can be modified (for experiments with tracking systems that can reconstruct the direction of the two emitted electrons).

In *0νββ* LV Majorana couplings modify the neutrino propagator, introducing novel effects in 0*νββ:*
there is a charge-conjugation-preserving operator that can trigger *0νββ* even if the Majorana m_ν is negligible → lower bounds on the half life $T^{0\nu}{}_{1/2}$ can also be used to constrain the relevant coefficients for LV

- Until now, the most precise tests for LV involving ν_s are perform in ν oscillation experiments.
- Now, deviations due to LV are also investigated in DBD experiments:EXO,NEMO3,GERDA,CUPID.

The coupling of the ν to the countershaded operator modifies the neutrino momentum from the standard expression:

$q^\alpha = (\omega, \mathbf{q}) \rightarrow q^\alpha = (\omega, \mathbf{q} + \mathbf{a}^{(3)}_{of} + \mathring{a}^{(3)}_{of}\,\mathbf{q})$ J.S. Diaz, PRD89(2014)

This deviation modifies the *2νββ transition amplitude and the neutrino dispersion relation.*
The decay rate can be written as a sum of the standard term and a perturbation due to LVββ

$\Gamma^{(2\nu)} = \Gamma_0^{(2\nu)} + d\Gamma^{(2\nu)}$ $T_{1/2}^{(2\nu)} = \Gamma_0^{(2\nu)} / \ln 2$

$\Gamma_0^{(2\nu)} = G_0^{2\nu}(E_0, Z) \times g_A^4 \times |m_e c^2 M^{2\nu}|^2$

$d\Gamma^{(2\nu)} = dG^{2\nu}(E_0, Z) \times g_A^4 \times |m_e c^2 M^{2\nu}|^2$

$G_0^{2\nu} = C \int_0^Q d\varepsilon_1 F(Z, \varepsilon_1) [\varepsilon_1(\varepsilon_1 + 2)]^{1/2} (\varepsilon_1 + 1) \int_0^{Q-\varepsilon 1} d\varepsilon_2 F(Z, \varepsilon_2) [\varepsilon_2(\varepsilon_2 + 2)]^{1/2} (\varepsilon_2 + 1)(Q - \varepsilon_1 - \varepsilon_2)^5$

$dG^{2\nu} = 10\mathring{a}^{(3)}_{of}\, C \int_0^Q d\varepsilon_1 F(Z, \varepsilon_1) [\varepsilon_1(\varepsilon_1 + 2)]^{1/2} (\varepsilon_1 + 1) \int_0^{Q-\varepsilon 1} d\varepsilon_2 F(Z, \varepsilon_2) [\varepsilon_2(\varepsilon_2 + 2)]^{1/2} (\varepsilon_2 + 1)(Q - \varepsilon_1 - \varepsilon_2)^4$

$C = (G_F^4 (\cos_\Theta)^4 m_e)/240\pi^7$ $t_{1,2} = \varepsilon_{1,2} - 1;$ J.S. Diaz, PRD**89**(2014),
EXO collab., arXiv:1601.07266v2[nucl-ex]

$G_0^{2\nu}$, $dG^{2\nu}$ can be calculated in different approximations:

- $F(Z, \varepsilon) = (2\pi y)[1 - \exp(-2\pi y)]^{-1}$, $y = \pm\alpha Z\varepsilon/q$, Primakoff&Rosen, RPP**22**(1959) (approx. A)
- $F(Z, \varepsilon) = 4(2qR_A)^{2(\gamma-1)} |\Gamma(\gamma+iy)|^2 \exp(\pi y) |\Gamma(2\gamma+1)|^{-2}$ Suhonen&Civitarese, PR**301**(1998) (approx. B)
- using exact electron functions obtained by solving Dirac equations Kotila&Iachello,PRC**85**2012;
Stoica, Mirea, Pahomi, PRC**88**(2013), RRP63(2015) (approx. C)

Formalism for Lorentz invariant violation in DBD

$$\Gamma_{\mathrm{SME}} = \Gamma_{\mathrm{SM}} + \delta\Gamma$$

Differential rate for 2νββ decay for ground states to ground states transitions

$$d\Gamma^{2\nu} = [\mathcal{A}^{2\nu} + \mathcal{B}^{2\nu}\cos\theta_{12}]w^{2\nu}d\omega_1 d\varepsilon_1 d\varepsilon_2 d(\cos\theta_{12})$$

$$w_{\mathrm{SM}}^{2\nu} = \frac{g_A^4 G_F^4 |V_{ud}|^4}{64\pi^7}\omega_1^2\omega_2^2 p_1 p_2 \varepsilon_1 \varepsilon_2$$

ε, ω = electron and neutrino energies;
p = electron momenta; θ = angle between electrons;
K, N = kinematic factors

$$\mathcal{A}^{2\nu} = \frac{1}{4}a(\varepsilon_1, \varepsilon_2)|M_{2\nu}|^2\tilde{A}^2 \times \left[(\langle K_N\rangle + \langle L_N\rangle)^2 + \frac{1}{3}(\langle K_N\rangle - \langle L_N\rangle)^2\right]$$

$$\mathcal{B}^{2\nu} = \frac{1}{4}b(\varepsilon_1, \varepsilon_2)|M_{2\nu}|^2\tilde{A}^2 \times \left[(\langle K_N\rangle + \langle L_N\rangle)^2 - \frac{1}{9}(\langle K_N\rangle - \langle L_N\rangle)^2\right]$$

$$\langle K_N\rangle = \frac{1}{\varepsilon_1 + \omega_1 + \langle E_N\rangle - E_I} + \frac{1}{\varepsilon_2 + \omega_2 + \langle E_N\rangle - E_I}$$

$$\langle L_N\rangle = \frac{1}{\varepsilon_1 + \omega_2 + \langle E_N\rangle - E_I} + \frac{1}{\varepsilon_2 + \omega_1 + \langle E_N\rangle - E_I}$$

$$\frac{d\Gamma^{2\nu}_{SM}}{d(\cos\theta_{12})} = \frac{1}{2}\Gamma^{2\nu}_{SM}[1 + \kappa^{2\nu}_{SM}\cos\theta_{12}]$$

$$\kappa^{2\nu}_{SM} = \frac{\Lambda^{2\nu}_{SM}}{\Gamma^{2\nu}_{SM}}.$$

angular correlation coefficient

$$\frac{\Gamma^{2\nu}_{SM}}{\ln 2} = g_A^4 |m_e M_{2\nu}|^2 G^{2\nu}_{SM}, \qquad \frac{\Lambda^{2\nu}_{SM}}{\ln 2} = g_A^4 |m_e M_{2\nu}|^2 H^{2\nu}_{SM}$$

$$d^3q = 4\pi\omega^2 d\omega \quad \rightarrow \quad d^3q = 4\pi(\omega^2 + 2\omega \mathring{a}^{(3)}_{of})d\omega$$

$$\Gamma_{SME} = \Gamma_{SM} + \delta\Gamma$$

$$\Gamma^{2\nu}_{SME} = \Gamma^{2\nu}_{00} + \Gamma^{2\nu}_{01} + \Gamma^{2\nu}_{10}, \qquad \Lambda^{2\nu}_{SME} = \Lambda^{2\nu}_{00} + \Lambda^{2\nu}_{01} + \Lambda^{2\nu}_{10}$$

$$G_{SME} = G_{SM} + \delta G,$$
$$H_{SME} = H_{SM} + \delta H$$

$$\frac{d\Gamma^{2\nu}_{SME}}{d(\cos\theta_{12})} = \frac{1}{2}\Gamma^{2\nu}_{SME}[1 + \kappa^{2\nu}_{SME}\cos\theta_{12}]$$

$$\kappa^{2\nu}_{SME} = \frac{\Lambda^{2\nu}_{SME}}{\Gamma^{2\nu}_{SME}}$$

$$\frac{d\Gamma_{\rm SM}}{d\varepsilon_1 d(\cos\theta_{12})} = C\frac{dG_{\rm SM}}{d\varepsilon_1}[1+\alpha_{\rm SM}\cos\theta_{12}]$$

$$\alpha_{\rm SM} \equiv (dH_{00}^{2\nu}/d\varepsilon_1)/(dG_{00}^{2\nu}/d\varepsilon_1)$$

$$\frac{d\Gamma_{\rm SME}}{d\varepsilon_1 d(\cos\theta_{12})} = C\frac{dG_{\rm SM}}{d\varepsilon_1}\times\left[1+\mathring{a}_{\rm of}^{(3)}\chi^{(1)}(\epsilon_1) \ + \left(\alpha_{\rm SM}+\mathring{a}_{\rm of}^{(3)}\frac{d(\delta H)/d\varepsilon_1}{dG_{\rm SM}/d\varepsilon_1}\right)\cos\theta_{12}\right]$$

$$\frac{d\Gamma_{\rm SME}^{2\nu}}{dK} = C\frac{dG_{00}^{2\nu}}{dK}(1+\mathring{a}_{\rm of}^{(3)}\chi^{(+)}(K))$$

$$\chi^{(+)} = \frac{d(\delta G^{2\nu})}{dK}\bigg/\frac{dG_{00}^{2\nu}}{dK}$$

$$\frac{d\Gamma_{\rm SME}^{2\nu}}{d\varepsilon_1} = C\frac{dG_{00}^{2\nu}}{d\varepsilon_1}(1+\mathring{a}_{\rm of}^{(3)}\chi^{(1)}(\varepsilon_1))$$

$$\chi^{(1)} = \frac{d(\delta G^{2\nu})}{d\varepsilon_1}\bigg/\frac{dG_{00}^{2\nu}}{d\varepsilon_1}$$

$$\alpha_{\rm SME} = \alpha_{\rm SM} + \mathring{a}_{\rm of}^{(3)}\frac{d(\delta H^{2\nu})/d\varepsilon_1}{dG_{00}^{2\nu}/d\varepsilon_1}$$

$$\left\{\begin{matrix} G_{\text{SM}} \\ \delta G \end{matrix}\right\} = \frac{\tilde{A}^2 G_F^2 |V_{\text{ud}}|^2 m_e^9}{96\pi^7 \ln 2} \frac{1}{m_e^{11}} \int_{m_e}^{E_I - E_F - m_e} d\varepsilon_1 \varepsilon_1 p_1 \int_{m_e}^{E_I - E_F - \varepsilon_1} d\varepsilon_2 \varepsilon_2 p_2$$

$$\times \int_0^{E_I - E_F - \varepsilon_1 - \varepsilon_2} d\omega_1 \omega_2^2 a(\varepsilon_1, \varepsilon_2) [\langle K_N \rangle^2 + \langle L_N \rangle^2 + \langle K_N \rangle \langle L_N \rangle] \left\{\begin{matrix} \omega_1^2 \\ 4\mathring{a}_{\text{of}}^{(3)} \omega_1 \end{matrix}\right\},$$

$$\left\{\begin{matrix} H_{\text{SM}} \\ \delta H \end{matrix}\right\} = \frac{\tilde{A}^2 G_F^2 |V_{\text{ud}}|^2 m_e^9}{96\pi^7 \ln 2} \frac{1}{m_e^{11}} \int_{m_e}^{E_I - E_F - m_e} d\varepsilon_1 \varepsilon_1 p_1 \int_{m_e}^{E_I - E_F - \varepsilon_1} d\varepsilon_2 \varepsilon_2 p_2$$

$$\times \int_0^{E_I - E_F - \varepsilon_1 - \varepsilon_2} d\omega_1 \omega_2^2 b(\varepsilon_1, \varepsilon_2) \left[\frac{2}{3} \langle K_N \rangle^2 + \frac{2}{3} \langle L_N \rangle^2 + \frac{5}{3} \langle K_N \rangle \langle L_N \rangle\right] \left\{\begin{matrix} \omega_1^2 \\ 4\mathring{a}_{\text{of}}^{(3)} \omega_1 \end{matrix}\right\}$$

$$\frac{dG_{2\nu}^{(0)}}{d\epsilon_1}$$

$$\frac{dG_{2\nu}^{(0)}}{d(\epsilon_1 + \epsilon_2 - 2m_e c^2)}$$

$$\alpha(\epsilon_1) = \frac{dG_{2\nu}^{(1)}/d\epsilon_1}{dG_{2\nu}^{(0)}/d\epsilon_1}$$

Results and discussions

Summed energy spectra of electrons in DBD

Summed energy spectra of electrons in the approximations A, B and C

A= NR approx. is inadequate in precise electron spectra analyses

B = approx. (analytical Fermi function); non-inclusion of FNS and screening effects: differences up to 30% as compared with “exact” Fermi function

C = exact Fermi functions, screening effect, “realistic” Coulomb-type potential

Nitescu, Ghinescu, Mirea, Stoica, JPG 47 (2020)

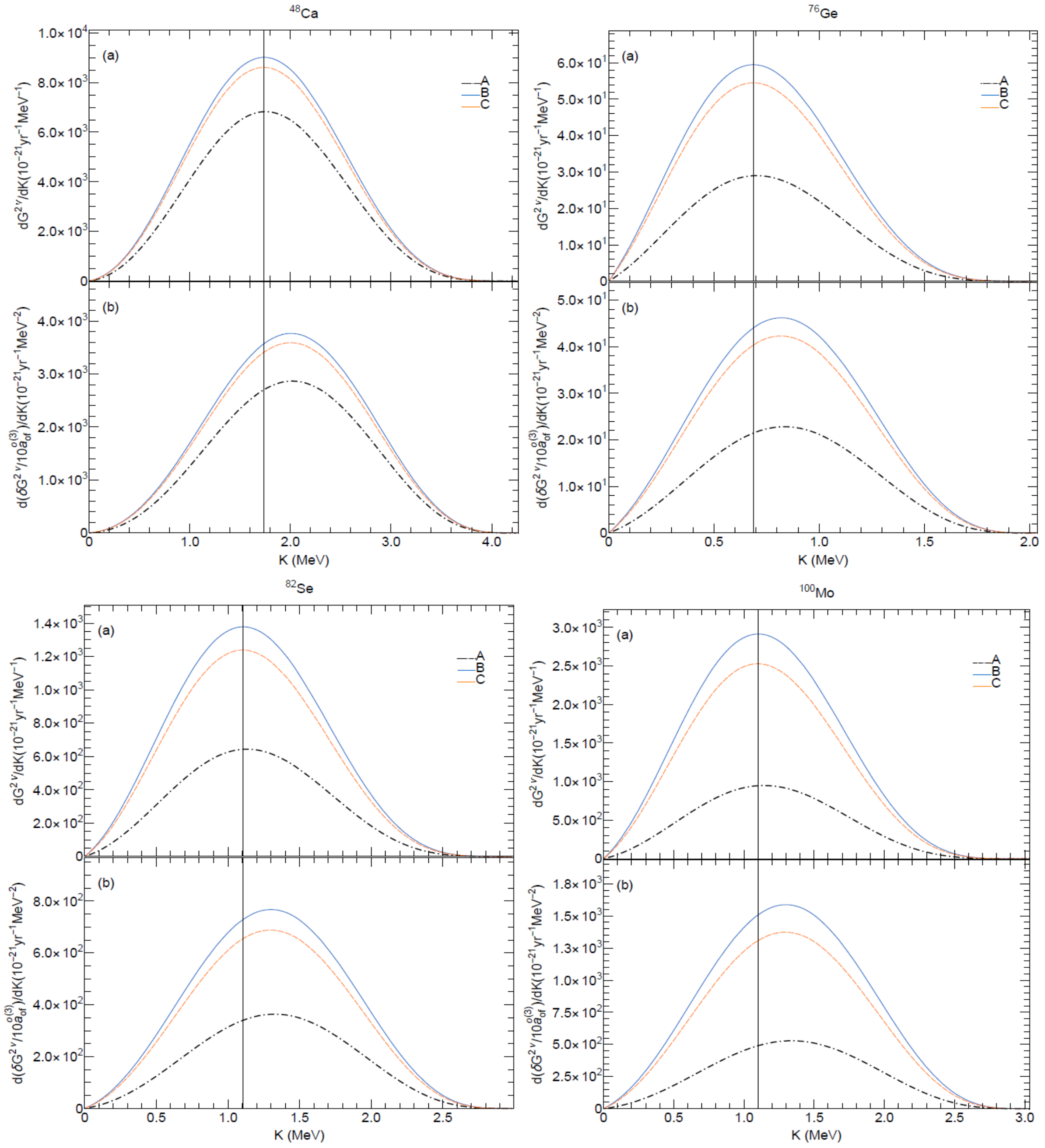


Figure 1. Summed energy spectra of electrons in the standard $2\nu\beta\beta$ decay (a) and

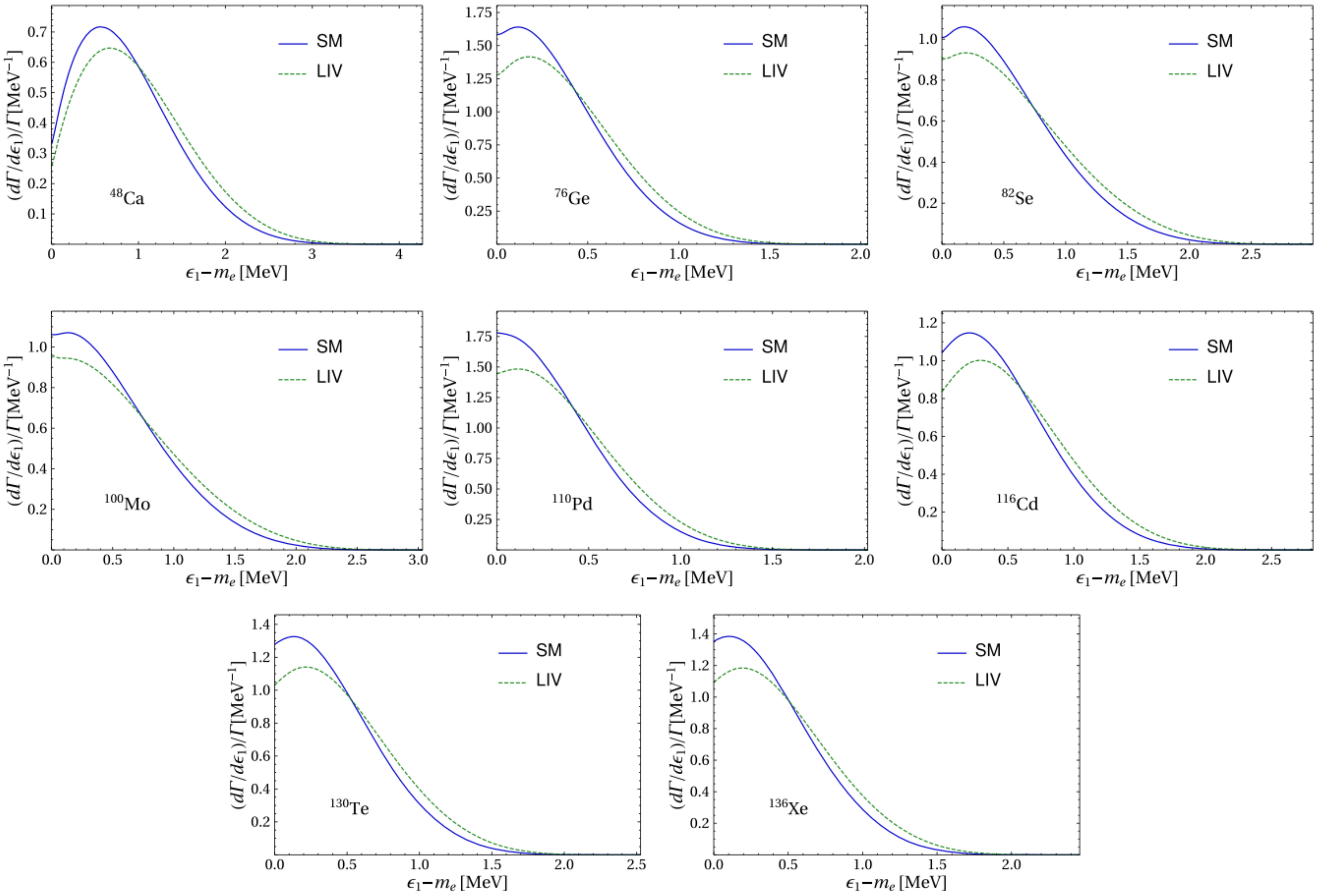


FIG. 1. Normalized $2\nu\beta\beta$ single-electron spectra within the SM with the solid line, and the first order contribution in $\mathring{a}^{(3)}_{\mathrm{of}}$ due to LIV with the dashed line. See text for the assumption on the hypothesis used.

Single electron spectra

Nitescu, Ghinescu, Stoica
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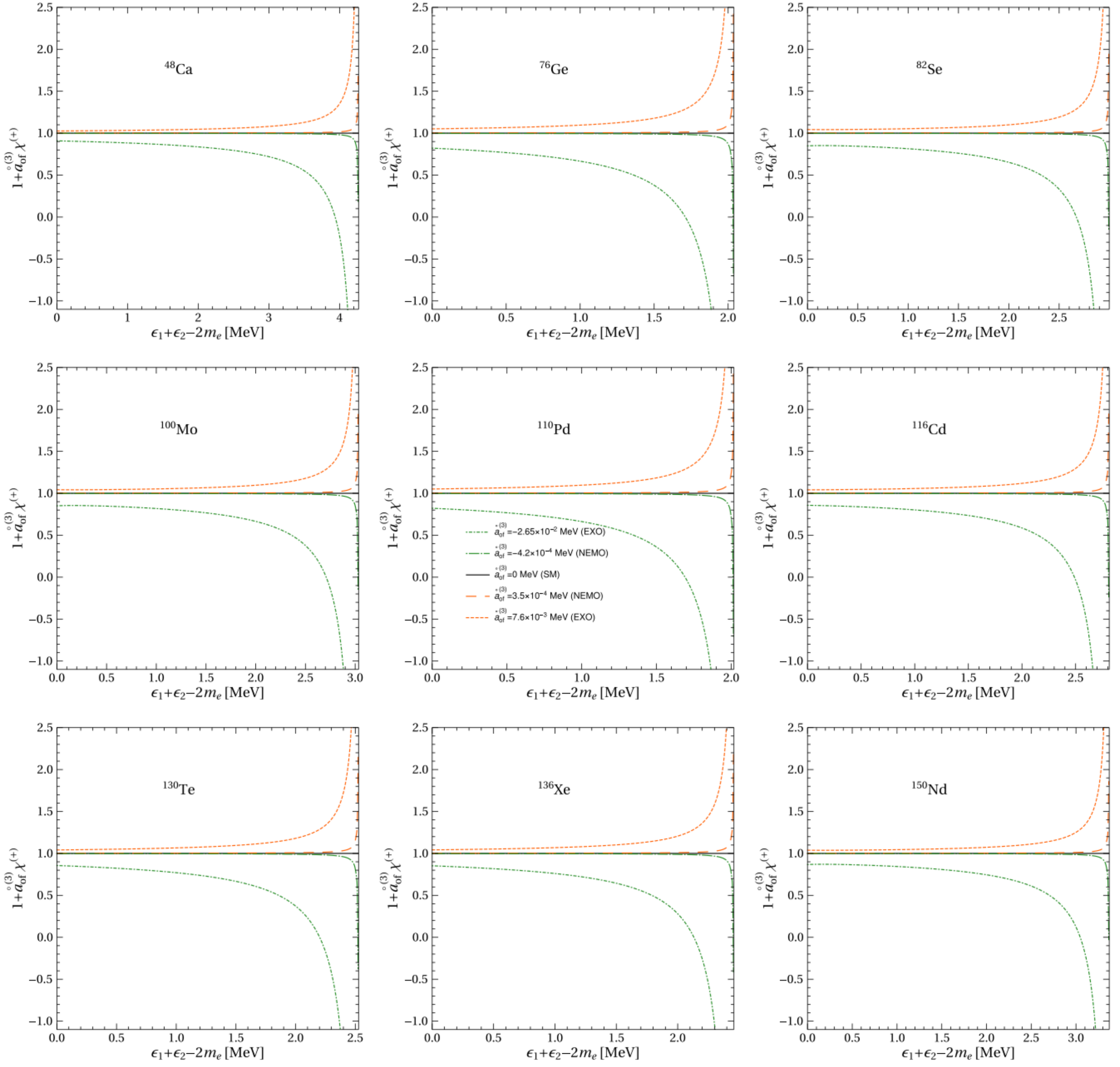

FIG. 4. The quantity $\chi^{(+)}(K)$ depicted for current limits of $\overset{\circ}{a}{}^{(3)}_{\rm of}$. The same conventions as in Fig. 3 are used.

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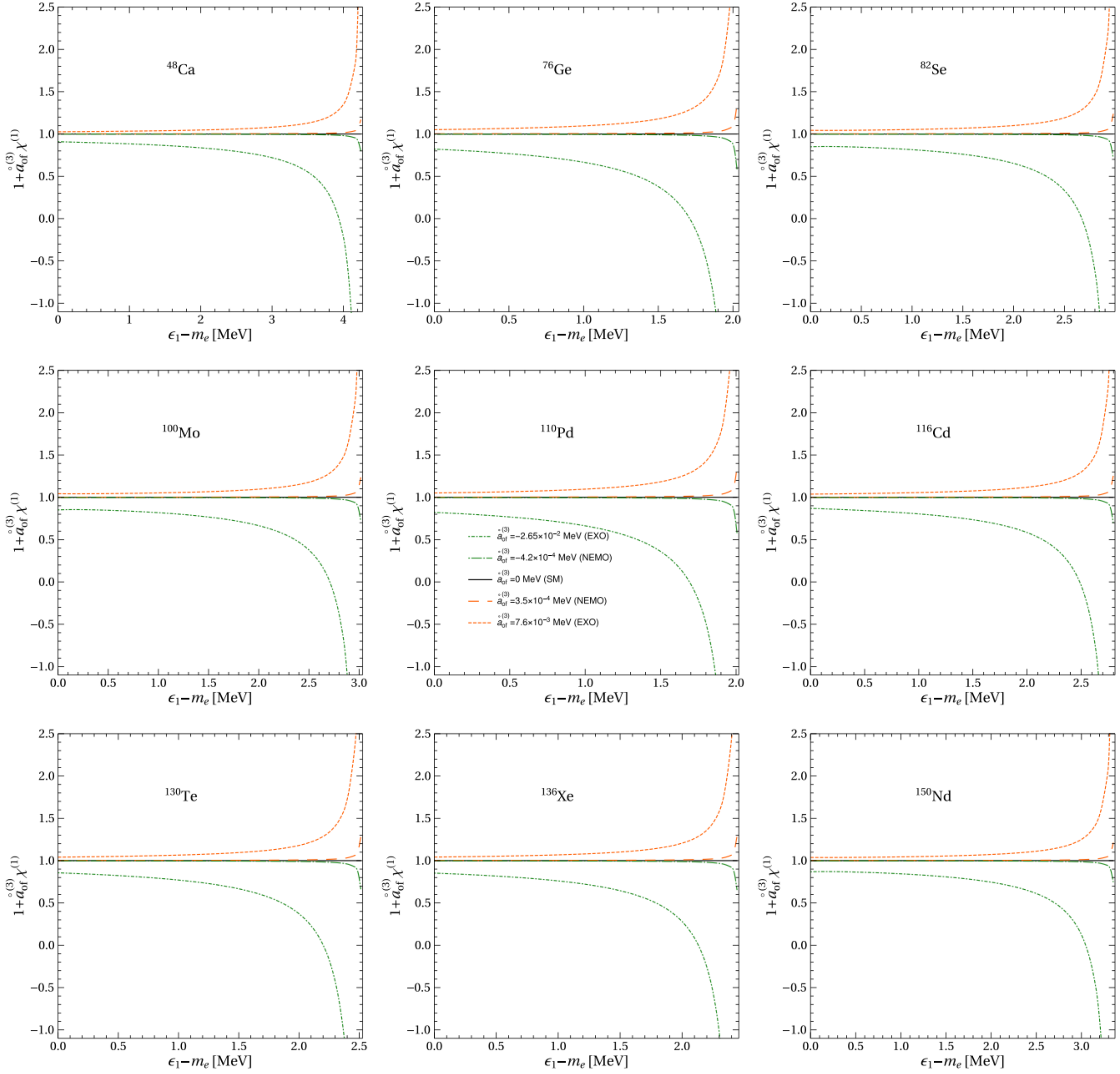

FIG. 3. The quantity $\chi^{(1)}(\epsilon_1)$ depicted for current limits of $\overset{\circ}{a}{}^{(3)}_{\mathrm{of}}$ (dashed for upper limit and dot-dashed for lower limit). The solid line at $\chi^{(1)}(\epsilon_1) = 0$ represents the SM prediction.

Nitescu, Ghinescu, Stoica
PRD**103**(2021); **105**(2022)

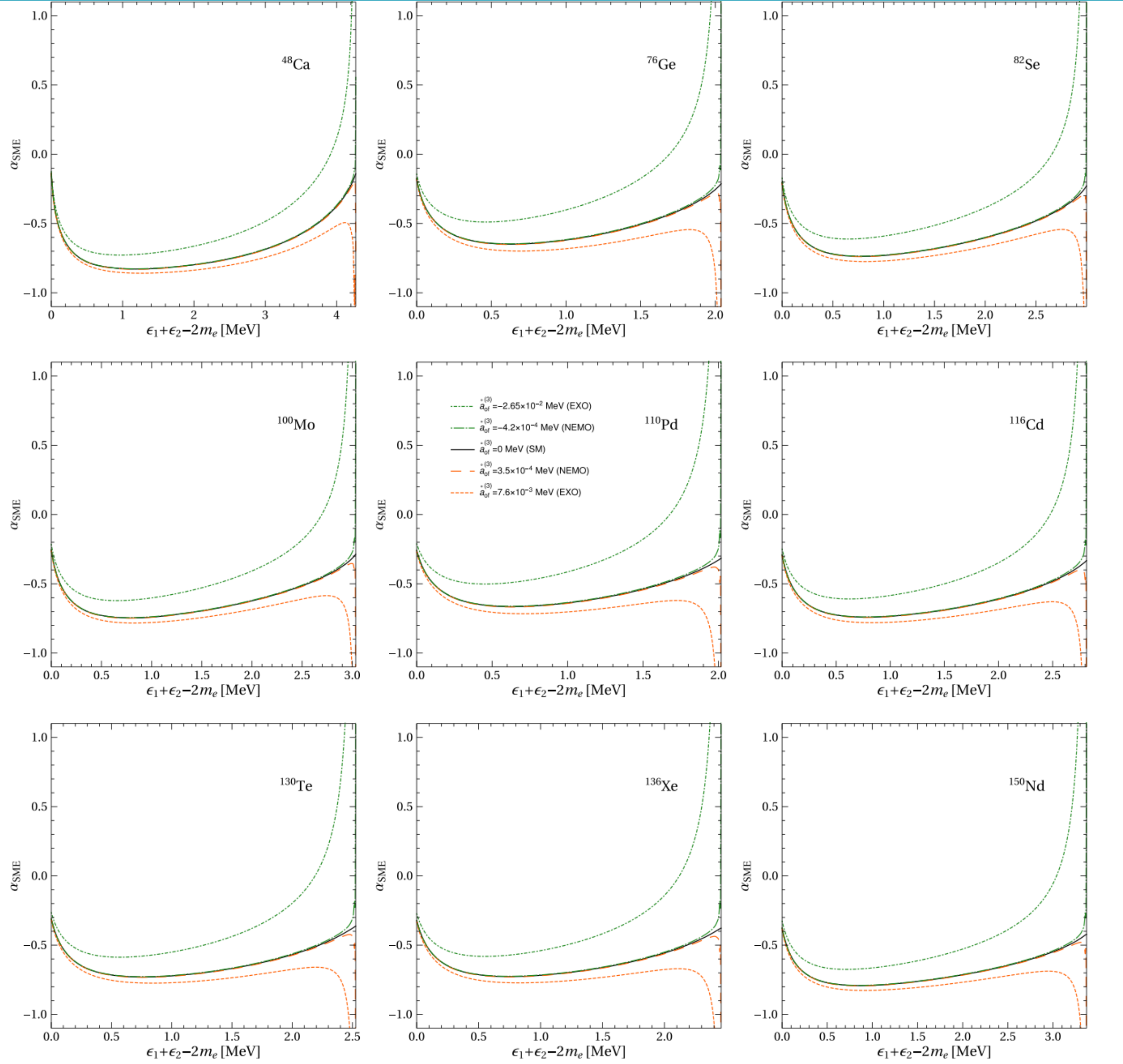

FIG. 5. The angular correlation spectrum plotted for the current limits of $\mathring{a}_{\rm of}^{(3)}$. The same conventions as in Fig. 3 are used.

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Angular correlation coefficient

$$\kappa_{SM}^{2\nu} = \frac{\Lambda_{SM}^{2\nu}}{\Gamma_{SM}^{2\nu}}$$

$$\kappa_{SME}^{2\nu} = \kappa_{SM}^{2\nu} + \mathring{a}_{of}^{(3)} \frac{\delta G_1^{2\nu}}{G_0^{2\nu}}$$

$$k_{SME}^{2\nu} = -0.6676 - 4.285 \times \mathring{a}_{of}^{(3)}$$

On the other hand, the angular correlation coefficient can be determined experimentally via forward-backward asymmetry

$$\mathcal{A}^{2\nu} \equiv \frac{\int_{-1}^{0} \frac{d\Gamma^{2\nu}}{dx} dx - \int_{0}^{1} \frac{d\Gamma^{2\nu}}{dx} dx}{\Gamma} = \frac{N_+ - N_-}{N_+ + N_-} = \frac{1}{2} k_{SM}^{2\nu}$$

Comments from PRD103(2021)

where $x = \cos\theta_{12}$ and $N_-(N_+)$ are the $2\nu\beta\beta$ events with the angle θ_{12} smaller (larger) than $\pi/2$. For a number of $N = 5 \times 10^5$ events at NEMO-3 [12] and considering only the statistical errors, the angular correlation coefficient is measurable with the uncertainty $k_{SM}^{2\nu} = 0.6676 \pm 0.0027$. Without a statistically significant deviation from the SM expectation, we obtain a bound $|\mathring{a}_{of}^{(3)}| \lesssim 1.04 \times 10^{-3}$ MeV at 90% CL. This is only a rough estimation, and dedicated experimental analysis, including the systematic uncertainties, is necessary for a better one. We note that this estimation lies between the $\mathring{a}_{of}^{(3)}$ limits reported by NEMO-3 and EXO-200, which were obtained from the analysis of the summed energy spectra of electrons. We note here that if

in a future experiment the number of $2\nu\beta\beta$ events would increase by 3 orders of magnitude (as planned for example in the SuperNEMO experiment), our estimation yields $|\mathring{a}_{of}^{(3)}| \lesssim 3.3 \times 10^{-5}$ MeV at 90% CL, which is comparable with the limits obtained from tritium decay experiments [8]. Thus, we predict good perspectives for searching for LIV effects in future DBD experiments, due to the significant increase of statistics.

Conclusions

- There is an extensive theoretical and experimental effort for studying DBD process particularly due to its broad potential to test/search BSM physics.

• The interest comes from the information that this process can provide about fundamental properties of neutrinos, conservation of some symmetries (LNC, CP, LIV) and strength of BSM parameters associated with possible scenarios of occurrence of $0u\beta\beta$ decay mode.

• Theoretically the effort is focused to the accurately computation of the NME and PSF, mainly for $0\nu\beta\beta$ decay, and for understanding the mechanism of its occurrence.

• The NME and PSF calculations enter now into a precision era and the goal is to provide experimentalists with accurate values of these quantities.

• Although the NME calculation brings the largest uncertainties in the DBD predictions and data interpretation, improved PSF calculation has proved to be quite necessary for addressing other issues related to the DBD study, including BSM physics, search of Lorentz invariance violation.

• The next planned DBD experiments are very promising for new discoveries in neutrino physics and check of fundamental symmetries.