

#### **Magnetic Dipole Moments**

Magnetic dipole moment of current loop:

$$ec{\mu} = rac{IA}{c} \hat{n}$$

$$= rac{1}{c} rac{ev}{2\pi r} \pi r^2 \hat{n}$$

$$= rac{e}{2mc} mvr \hat{n}$$
 $\vec{\mu} = rac{e}{2mc} \vec{L}$ 

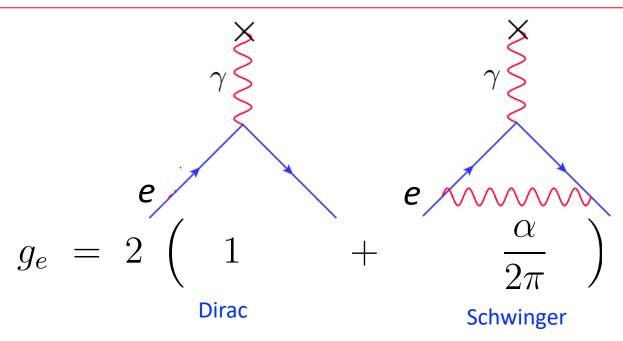
- 1925: Anomalous Zeeman effect could be explained by intrinsic electron spin with magnetic moment
- 1928: Dirac predicts g=2 for fundamental spin 1/2 particle, huge success
- 1933: Stern proposes measurement of  $g_{proton}$ . Pauli "Don't you know the Dirac theory? It's obvious  $g_p=2$ "

# The Magnetic Moment Anomaly

- 1947: Rabi 0.3% discrepancies in ground-state HFS of H,D
- 1947: Kusch and Foley discrepancy in Ga spectroscopy explained if  $g_e = 2.00229(8)$
- 1947: Schwinger calculates correction  $g_e = 2\left(1 + \alpha/2\pi\right) = 2.002324$
- $g_e=2(1+a_e)$  defines magnetic moment anomaly,  $a_e\equiv \frac{(g_e-2)}{2}\approx \frac{\alpha}{2\pi}\approx 0.001162$



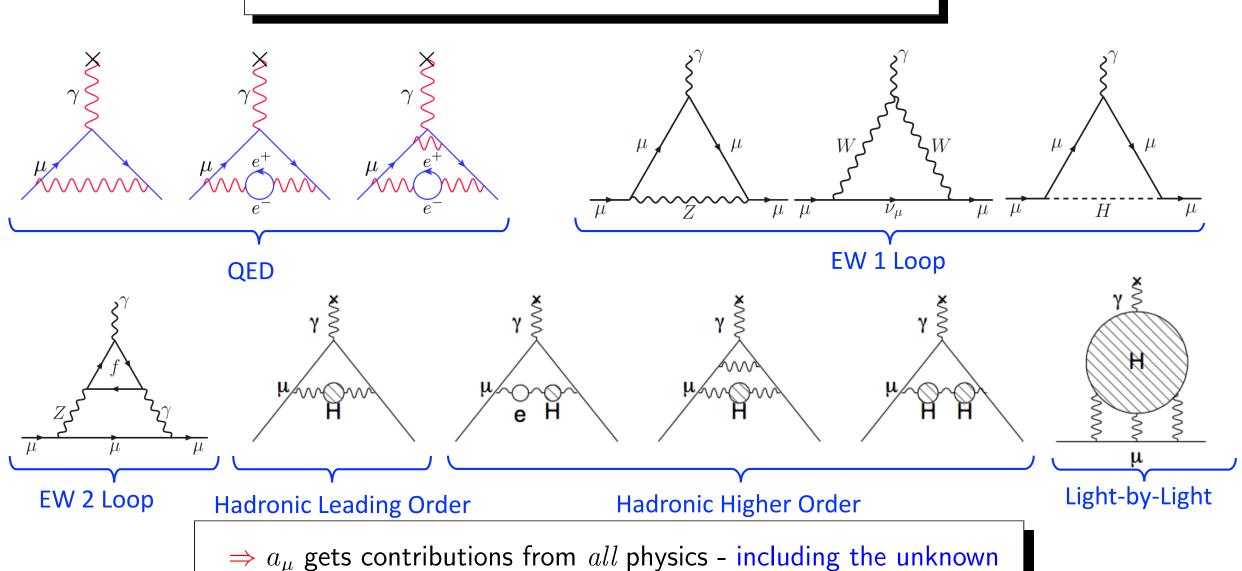
J. Schwinger



- Anomaly  $a_e$  due to radiative corrections from virtual particles in loops
- 1 part in 850 effect, huge success for QED!

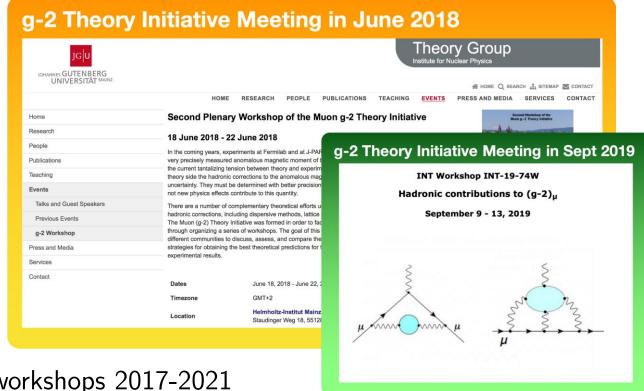
#### Contributions to the Magnetic Moment Anomaly of the Muon

$$a_{\mu}(\mathsf{Standard\ Model}) = a_{\mu}(\mathsf{QED}) + a_{\mu}(\mathsf{Weak}) + a_{\mu}(\mathsf{Hadronic})$$



#### Muon g-2 Theory Initiative: Definitive Standard Model Prediction





- Collaboration of 100+ theorists, held 7 workshops 2017-2021
- ullet Goal: Study all theory inputs, provide definitive SM prediction for  $a_{\mu}$
- T. Aoyama et al., Physics Reports 887, 1-166 (2020)
- 166 pages, 132 authors, 82 institutions, 21 countries, 822 references
- ⇒ We compare experiment result with this recommended value
- → Theory work ongoing, Sept 2022 workshop, update in 2023

### The Standard Model Prediction of a<sub>u</sub>

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a_{\mu}(\text{QED}) = 116 584 718.931 \pm 0.104 

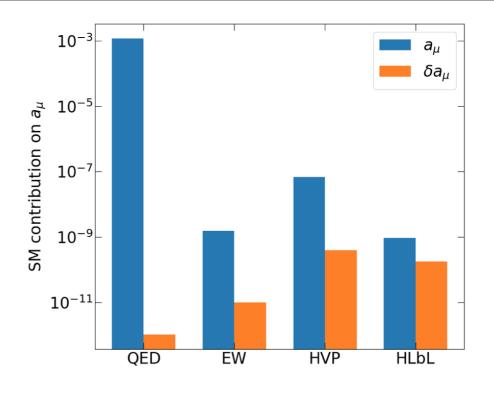
a_{\mu}(\text{HadVP}; e^{+}e^{-}, \text{LO+NLO+NNLO}) = 6 845. \pm 40 

a_{\mu}(\text{Weak}; 2 \text{ loops}) = 153.6 \pm 1.0 

a_{\mu}(\text{Had}; \text{LBL}) = 92. \pm 18 

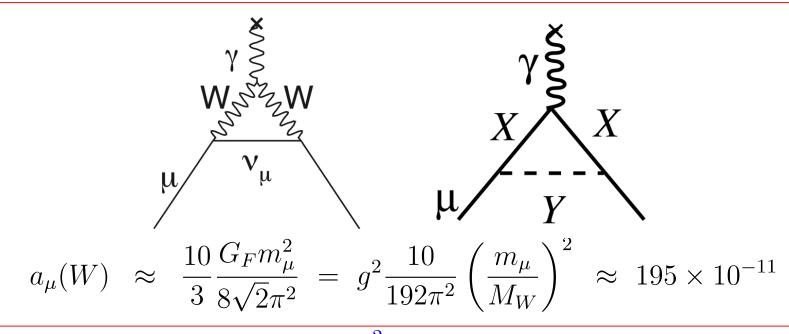
a_{\mu}(\text{Standard Model}) = 116 591 810. \pm 43 \times 10<sup>-11</sup>
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- T. Aoyama et al. (Theory Initiative Whitepaper), "The anomalous magnetic moment of the muon in the Standard Model", Physics Reports 887, 1-166, Dec 2020.
- Uses data-driven approach to HVP
- Prediction has impressive precision: 370 ppb



#### Why measure the magnetic moment anomaly of the muon?

- $a_e = (g_e 2)/2$  determined to 0.24 ppb
- Muons live 2.2  $\mu$ seconds why measure  $a_{\mu}$ ?



- $\Rightarrow$  Contribution of new physics  $\approx \left(\frac{m_{e,\mu}}{M_X}\right)^2$
- $\Rightarrow$  Muon mass 206 times electron mass  $\Rightarrow$  new physics contribution 43,000 times larger
- $\Rightarrow$  New physics contribution of 0.24 ppb on  $a_e \Leftrightarrow 9$  ppm on  $a_\mu$
- $\Rightarrow$  With much lower precision,  $a_{\mu}$  sensitive to much higher mass scales



# Muon g-2 Collaboration

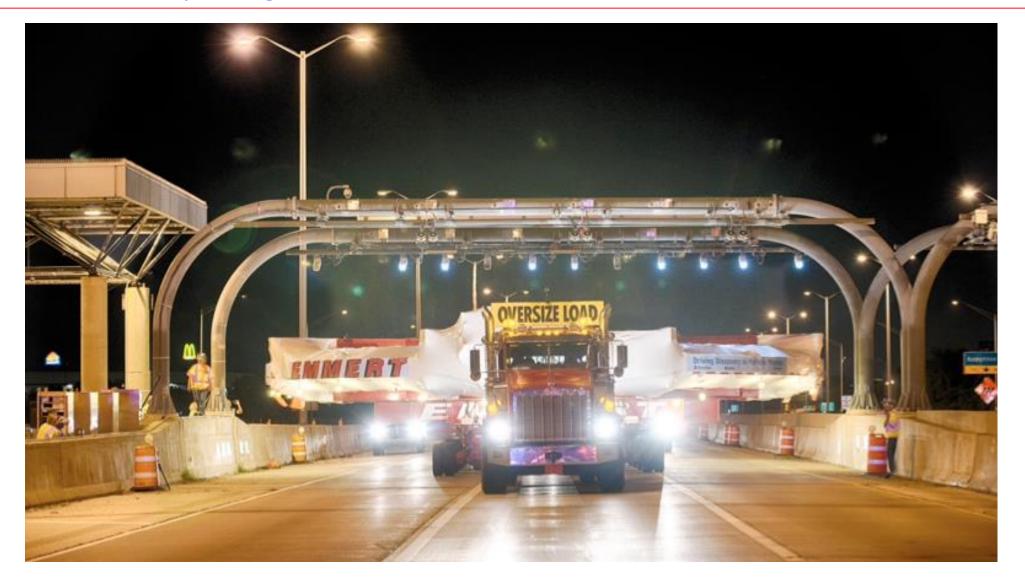
7 Countries, 35 Institutions, 203 Collaborators



# The Big Move: From Brookhaven to Fermilab in 2013

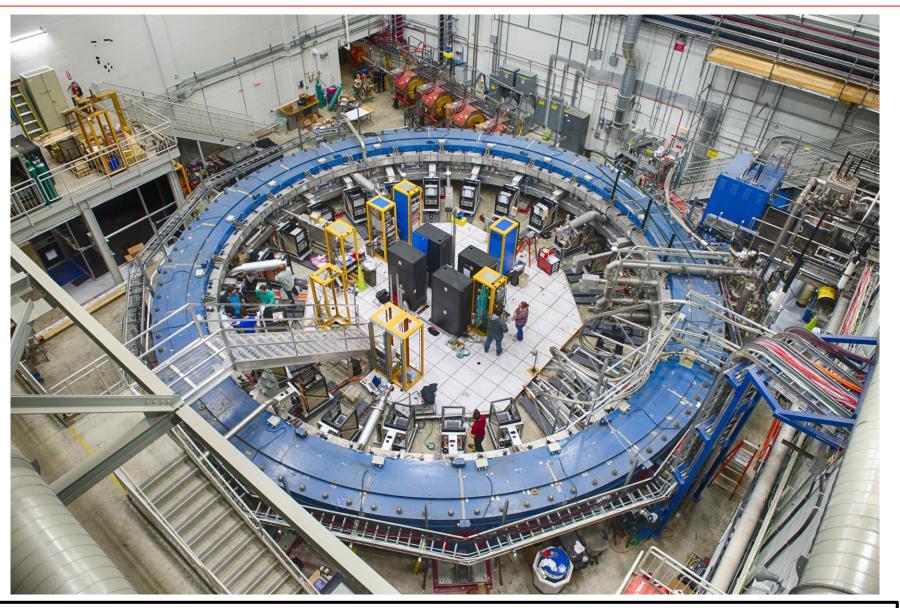


# Transporting the Coils to Fermilab: Shut down 2 interstates!



• Trailer with coils passes toll arches with 6" clearance on each side

### The new Muon g-2 Experiment at Fermilab

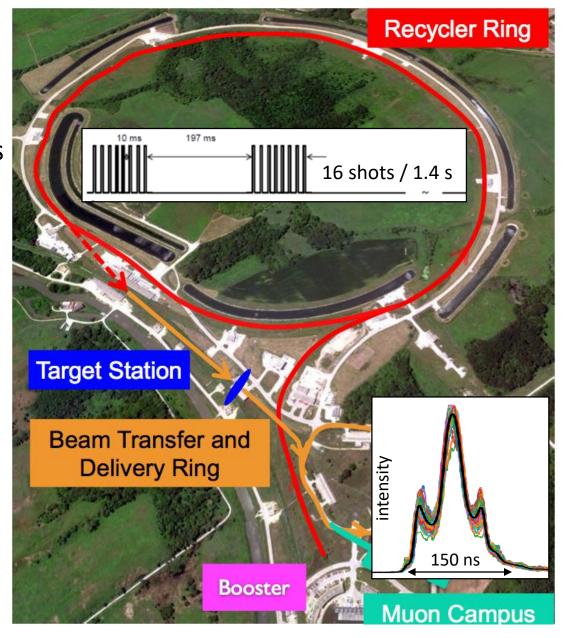


Goal: Measure the muon magnetic moment anomaly  $a_\mu$  to 140 ppb, a fourfold improvement over the 540 ppb precision of Brookhaven

- $\Rightarrow$  Brookhaven statistics limited:  $a_{\mu}^{\rm BNL} = 0.001\,165\,920\,89\,(54)_{\rm stat}\,(33)_{\rm sys}$ 
  - BNL  $\pm 540$  ppb uncertainty  $\Leftrightarrow 9 \times 10^9$  events
- $\Rightarrow$  Fermilab goal factor 21,  $2 \times 10^{11} e^+$

# Fermilab Advantages:

- 4 bunches of  $10^{12}$  protons at 8 GeV
- Hit target,  $p, \pi, \mu$  to delivery ring
- Long decay channel for  $\pi \Rightarrow \mu$
- ullet Only  $\mu$  enter ring, minimal hadronic flash
- $\Rightarrow$  4× higher fill frequency than BNL
- → Muons per fill similar



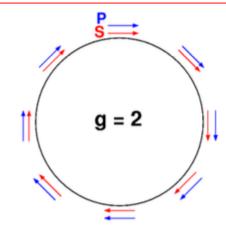
### How do we measure a<sub>...</sub>? Overview of Measurement Technique

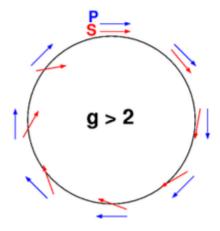
- Inject polarized muons into magnetic storage ring (B=1.45 T) with electric vertical focusing
- Muon cyclotron frequency  $\omega_c \approx 2\pi \times 6.7\,\mathrm{MHz}$
- Muon spin vector precession  $\omega_s \approx 2\pi \times 6.9 \, \text{MHz}$

$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C$$

$$\vec{\omega}_a \approx \frac{e}{mc} \left[ a_{\mu} \vec{B} - \left( a_{\mu} - \left[ \frac{mc}{p} \right]^2 \right) \vec{\beta} \times \vec{E} \right]$$

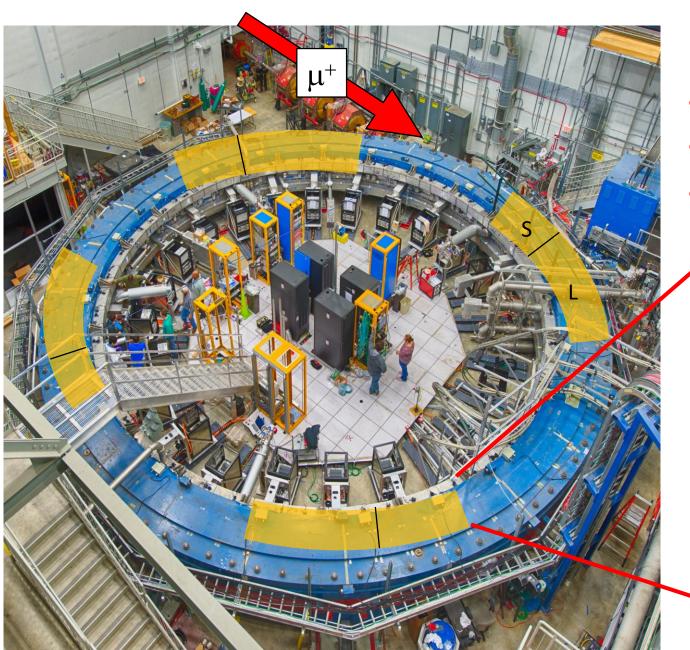
$$\vec{\omega}_a \approx 229 \, \text{kHz}$$





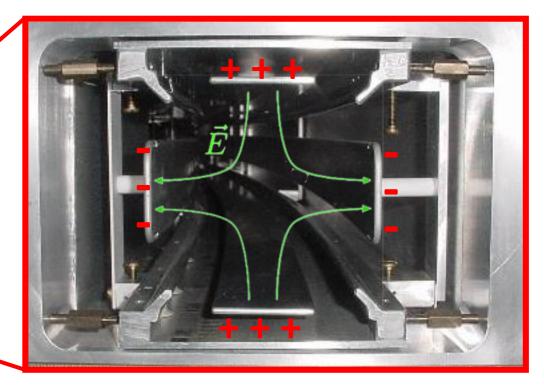
- $\Rightarrow$  Cancel term from electrostatic vertical focusing at  $p_{\mathrm{magic}} = \frac{mc}{\sqrt{a_{\mu}}} \approx$  3.094 GeV/c
- ⇒ Experiment measures two quantities:
  - (1) Muon anomalous precession frequency  $\omega_a$  to  $\pm\,100\,\mathrm{ppb}$  (stat)  $\pm\,70\,\mathrm{ppb}$  (syst)
  - 2) Magnetic field  $ec{B}$  in terms of proton NMR frequency  $\omega_p$  to  $\pm\,70\,\mathrm{ppb}$  (syst)

### Injecting, Kicking, and **Storing** the Polarized Muon Beam

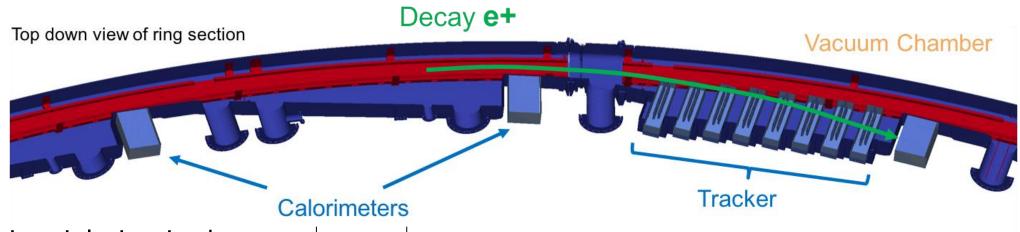


# **Electrostatic Quadrupoles (ESQ)**

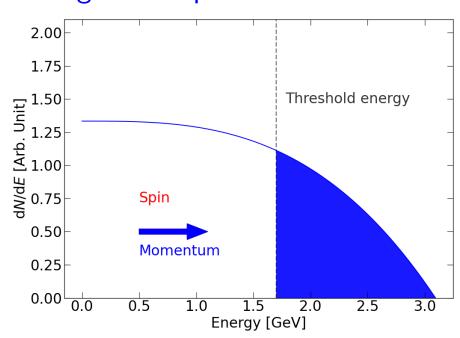
- Electric quadrupole field provides vertical focusing
- Beam: simple harmonic motion about closed orbit
- Quads cover 43% of azimuth

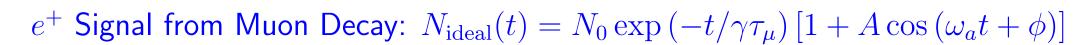


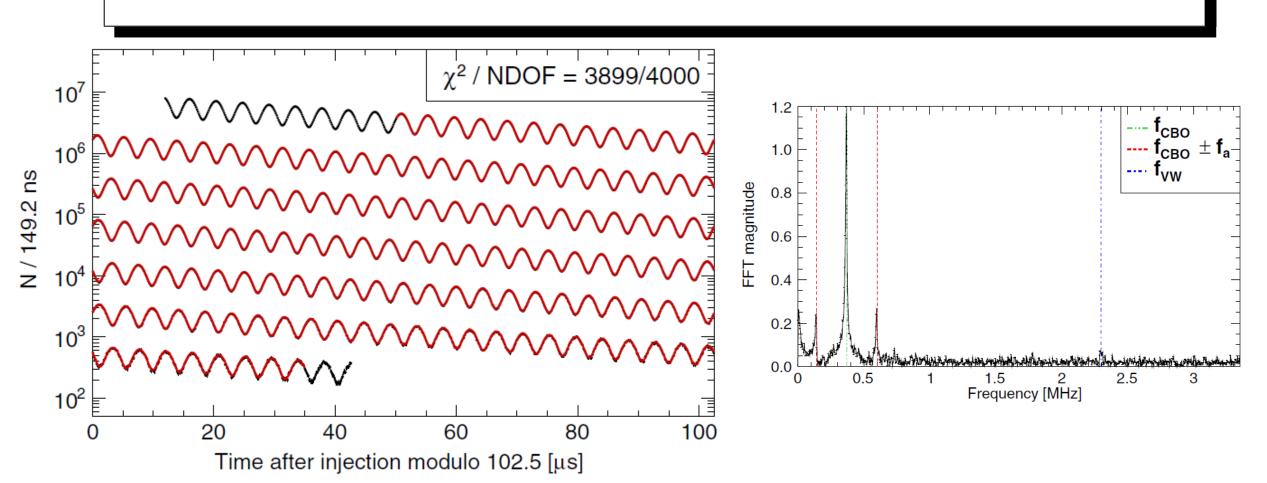
#### Measuring $\omega_a$ : Detecting the $e^+$ from muon decay with calorimeters



- $\Rightarrow$  Parity violation in decay:  $\mu^+ \rightarrow e^+ \bar{\nu}_{\mu} \nu_e$
- $\Rightarrow$  Muon Rest Frame: highest energy decay  $e^+$  emitted in precessing muon spin direction
- $\Rightarrow$  Lab Frame Positron Energy:  $E_{\text{lab}} \approx \gamma E^* \left[ 1 + \cos \left( \omega_a t \right) \right]$
- $\Rightarrow$  Positron detection rate above threshold  $\propto \cos{(\omega_a t)}$ 
  - Reconstruct  $e^+$  energy and time
  - Extrapolate for phase of  $\mu^+$  spin at decay

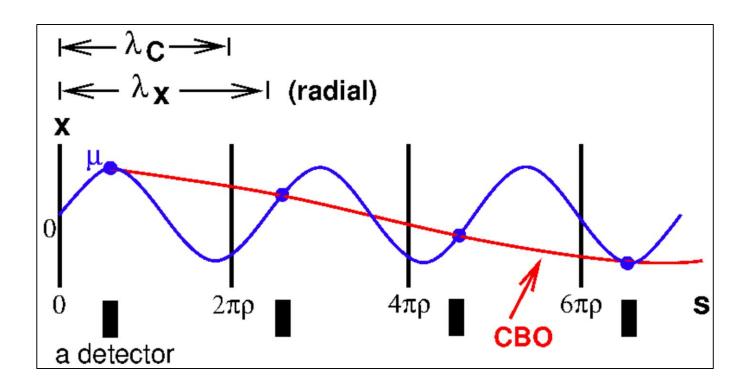


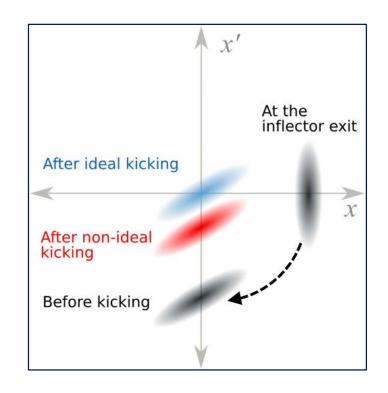




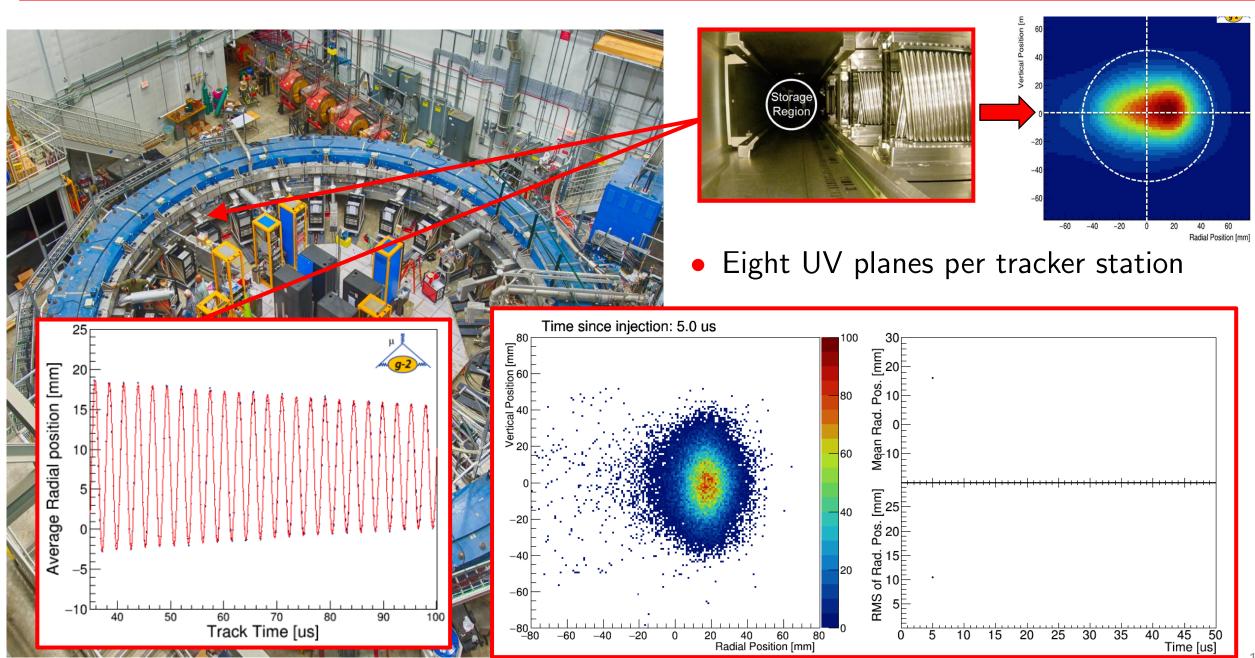
• FFT of residual to fit shows many features: 5 parameter entirely inadequate

#### Importance of Beam Dynamics: Coherent Betatron Oscillations (CBO)

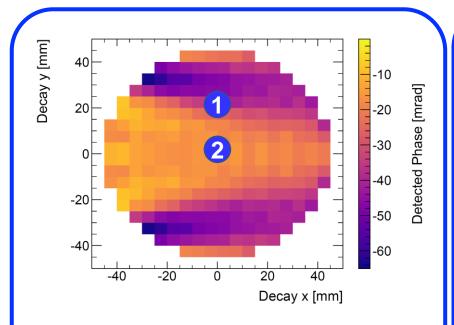




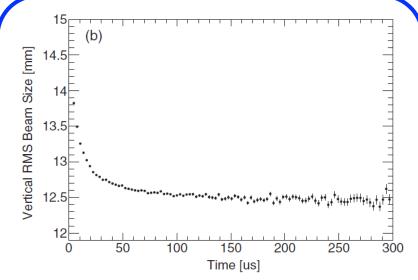
- ullet Calo acceptance depends on muon radius at decay: coherent beam motion modulates  $e^+$  time spectrum
- Radial betatron wavelength (blue line)  $\neq$  circumference (cyclotron wavelength)
- ullet Red line: apparent radial breathing in and out of beam at alias frequency  $f_{\mathsf{CBO}} = f_{\mathsf{cyclotron}} f_{\mathsf{betatron}}$
- Effect dephases gradually, nearly cancels when all detectors added together

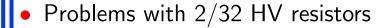


# Run-1 Challenge: Phase Acceptance Correction: C<sub>pa</sub>



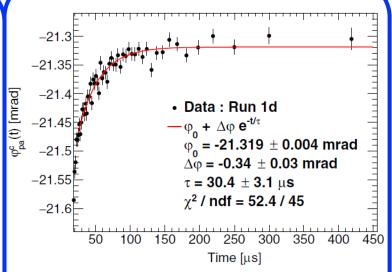
- Detector acceptance couples g-2 phase of detected  $e^+$  to parent muon decay position (x,y, $\phi$ )
- Wiggle plots for  $(1) \neq (2)$





Beam vertical width changed during fill

• 
$$\Delta\omega_a = \frac{d\phi}{dt} = \frac{d\phi}{dY_{rms}} \frac{dY_{rms}}{dt} \neq 0$$



- Account for  $\phi(t)$  in fit
- Took a year to understand

• 
$$C_{pa} = (-158 \pm 75) \, \text{ppb}$$

# Extracting $\omega_a^m$ : measured g-2 frequency

Incorporating beam dynamics, detector effects, muon losses, fit function becomes:

$$N(t) = N_0 \cdot N_x(t) \cdot N_y(t) \cdot \Lambda(t) \cdot e^{-t/\gamma \tau_{\mu}} \cdot \left[1 + A_0 \cdot A_x(t) \cdot \cos\left(\omega_a^m t + \phi_0 \cdot \phi_x(t)\right)\right]$$

$$N_{x}(t) = 1 + e^{-t/\tau_{\text{CBO}}} A_{N,x,1} \cos(\omega_{\text{CBO}} t + \phi_{N,x,1})$$

$$+ e^{-2t/\tau_{\text{CBO}}} A_{N,x,2} \cos(2\omega_{\text{CBO}} t + \phi_{N,x,2})$$

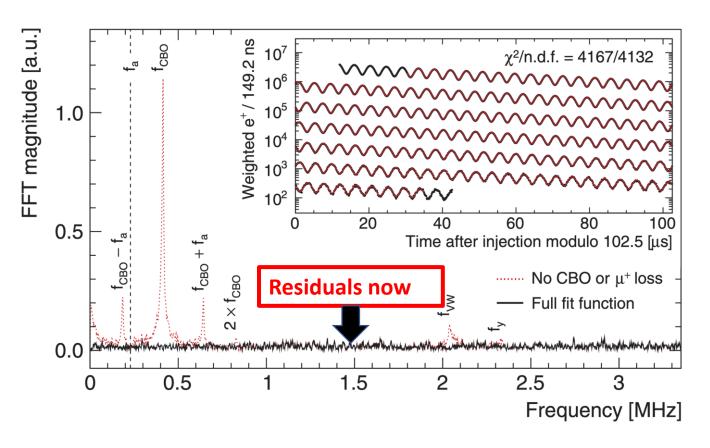
$$N_{y}(t) = 1 + e^{-t/\tau_{y}} A_{N,y,1} \cos(\omega_{y} t + \phi_{N,y,1})$$

$$+ e^{-2t/\tau_{y}} A_{N,y,2} \cos(\omega_{VW} t + \phi_{N,y,2})$$

$$\Lambda(t) = 1 - K_{\text{loss}} \int_{0}^{t} e^{t'/\gamma\tau_{\mu}} L(t') dt'$$

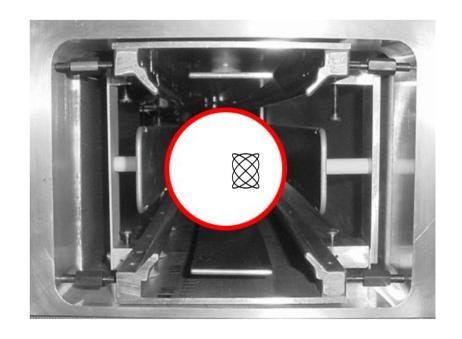
$$A_{x}(t) = 1 + e^{-t/\tau_{\text{CBO}}} A_{A,x,1} \cos(\omega_{\text{CBO}} t + \phi_{A,x,1})$$

$$\phi_{x}(t) = 1 + e^{-t/\tau_{\text{CBO}}} A_{\phi,x,1} \cos(\omega_{\text{CBO}} t + \phi_{\phi,x,1})$$



### Detour back in time: Importance of Magnetic Field Homogeneity





- ullet Muons occupy volume determined by vertical and radial  $oldsymbol{B}$  fields, betatron oscillations
- Muon spin precesses  $\omega_a$  according to  ${m B}$  in small volume  $\Rightarrow \omega_a({\vec r}) \approx a_\mu \left[\frac{eB({\vec r})}{m_\mu}\right]$
- Need B field weighted by stored muon distribution
- Reasons for homogeneous field:
  - Stable beam dynamics, adiabaticity
  - Smaller uncertainty on  $\tilde{\omega}_p$  from convolution of muon distribution with field
  - Easier to measure

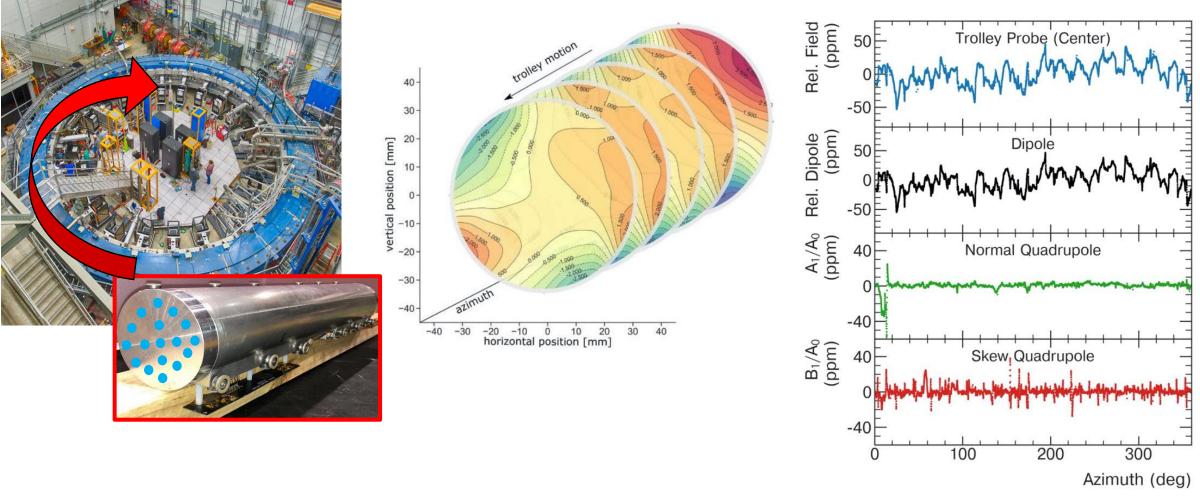
#### Measuring the Magnetic Field B using Pulsed Proton NMR in Water

$$\omega_a \approx a_\mu \left[\frac{eB}{m_\mu}\right] \qquad a_\mu = \frac{\omega_a}{\omega_p'} \frac{2\mu_p'}{\hbar} \frac{m_\mu}{e} \longrightarrow \frac{\omega_a}{\tilde{\omega}_p'(T_r)} \times \frac{\mu_p'(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$

$$\Rightarrow \text{ Extract } B \text{ using NMR:} \quad \hbar \omega_p'(T_r) = 2 \mu_p'(T_r) B$$

- Magnetic moment of proton in spherical water sample  $\mu_p'(34.7^{\circ}C)$  measured to 10.5 ppb
- $\Rightarrow$  Want NMR precession frequency protons in spherical water  $\omega_p'(T_r)$  in storage volume while muons stored
- Some Problems:
  - Can't have NMR probes in storage volume at same time/place as muons!
  - Whatever we use to measure B-field perturbs the local field!  $\Rightarrow$  measured B-field different than what muons see!
- Calibration/corrections necessary to go from raw magnetometer frequency  $\omega_{\rm raw}$  to equivalent  $\omega_p'(T_r)$
- Essential steps:
  - Develop calibration probe whose NMR frequency  $\omega_{cp}$  can be related to  $\omega_p'(T_r)$
  - Transfer calibration to device (NMR Trolley) that measures field inside muon storage volume  $\Rightarrow \omega_p'(x,y,\phi,t_0)$
  - Use NMR probes outside storage volume to monitor field while muons stored  $\omega_p'(x,y,\phi,t)$

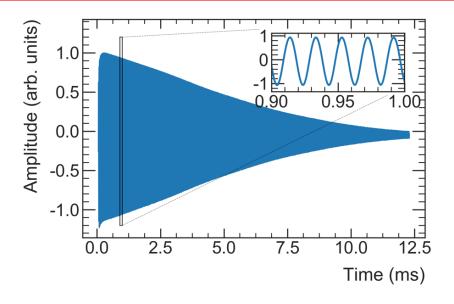
### Mapping the field in the storage volume every 3 days with the trolley:



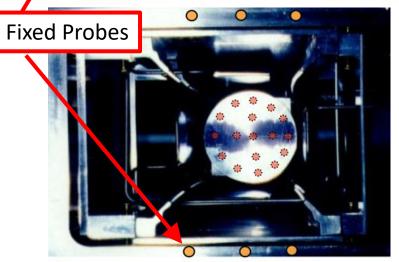
- ullet Trolley with 17 NMR probes maps magnetic field in muon storage volume every pprox 3 days
- 9000× 17 data points (every 5 mm)
- Takes a few hours

# During Muon Storage: track the field with the fixed probes



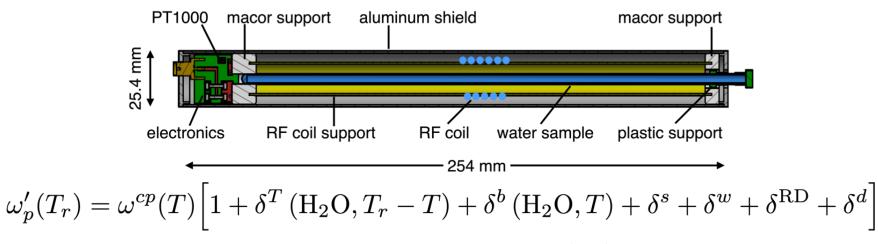




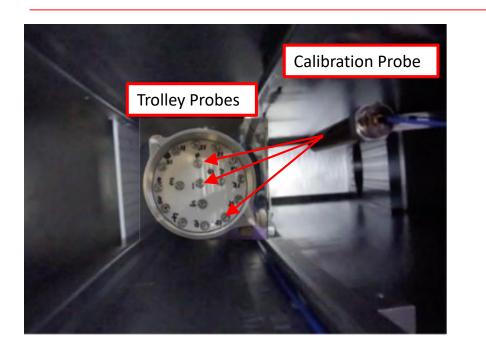


- 378 fixed NMR probes, above/below storage volume
- NMR probe stations every 5°, read every 1.4 seconds
- Determine offset between fixed probes and trolley when it passes by
- ⇒ Infer what trolley would read while muons stored

### The Absolute Calibration Probe (CP) and Calibrating the Trolley



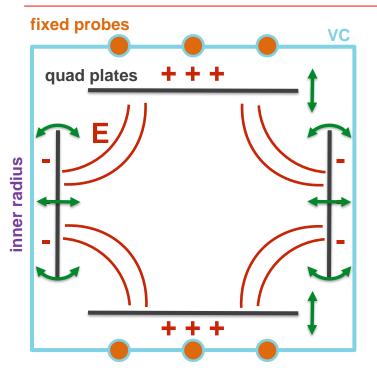
- Determine corrections relating  $\omega^{cp}$  to  $\omega_p'(T_r)$  to 20 ppb
- Cross-checked with <sup>3</sup>He probe to 38 ppb





- Alternately measure B in same location with trolley, CP
- Relates  $\omega_{\text{trolley}}(\vec{r})$  to  $\omega^{CP}(\vec{r})$  to  $\omega_p'(T_r, \vec{r})$
- Takes 4-8 hours to calibrate single trolley probe
- Results consistent to 30 ppb

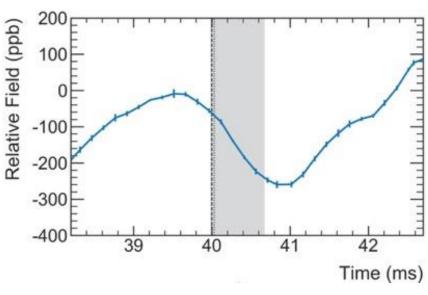
# Largest Field Systematic Uncertainty: Field Transients from the Electrostatic Quads: B<sub>q</sub>





- When muons injected, electrostatic quads are pulsed
- Impulse causes motion of quad plates
- Moving conductor in B → magnetic field perturbation
- Not seen by trolley
- Seen by fixed probes but attenuated, phase shifted
- Must measure separately
- Inserted NMR probes between pulsing quads





#### Table of Muon g-2 Run-1 Statistical and Systematic Uncertainties

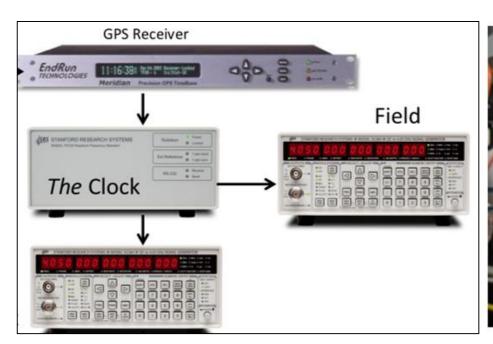
Quantity	Correction terms (ppb)	Uncertainty (ppb)
$\omega_a^m$ (statistical)		434
$\omega_a^m$ (systematic)	• • •	56
$C_e$	489	53
$C_p$	180	13
$C_{ml}$	-11	5
$C_{pa}$	-158	75
$f_{\text{calib}}\langle \omega_p(x, y, \phi) \times M(x, y, \phi) \rangle$	• • •	56
$B_k$	-27	37
$B_q^{"}$	-17	92
$\mu_p'(34.7^{\circ})/\mu_e$	• • •	10
$m_{\mu}/m_{e}$	• • •	22
$g_e/2$	• • •	0
Total systematic	• • •	157
Total fundamental factors		25
Totals	544	462

Data set	$\tilde{\omega}_p'(T_r)/2\pi$ (Hz)	Uncertainty (ppb)
Run-1a	61,791,871.2	115
Run-1b	61,791,937.8	127
Run-1c	61,791,845.4	125
Run-1d	61,792,003.4	108
	Average over all data s	ets
Field Measurements		56
ESQ Transient		92
Kicker Transient		37
Total		114

- Took three years to analyze Run 1 (2018)
- Field measurement uncertainty: calibration, trolley maps, tracking uncertainty, muon convolution, ...
- $\omega_a^m(\text{systematic})$ : pileup, gain correction, modeling CBO decoherence
- Results stable vs fit start/stop times, individual calorimeters,
   Run 1a, b, c, d (different quad and kicker settings)

#### Ready to Unblind

- Both  $\omega_a$  and  $\omega_p$  share common clock
- $\omega_a$  clock hardware "blinded"
- Obscures timebase for the "wiggle" plot
- Blinding factor set by people outside collaboration, stored in envelopes
- Unblinding: yields  $\omega_a \Rightarrow a_\mu$



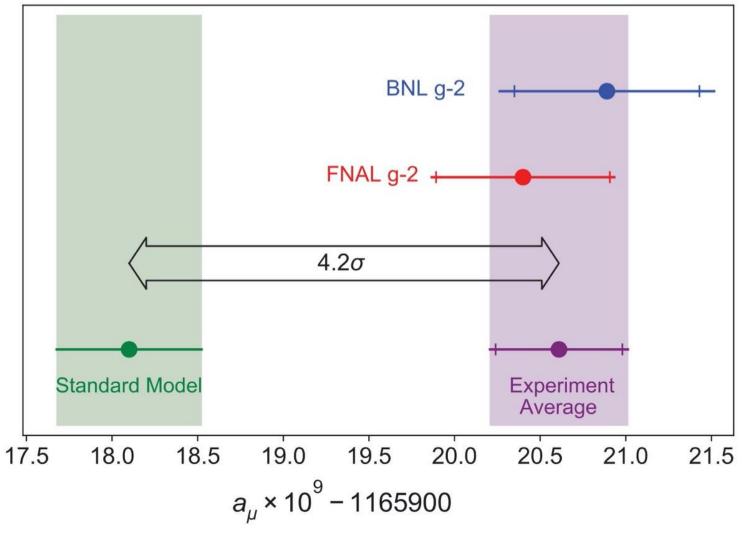






Clock stability monitored weekly by non-collaborators

#### Final Result: Run-1 Muon g-2

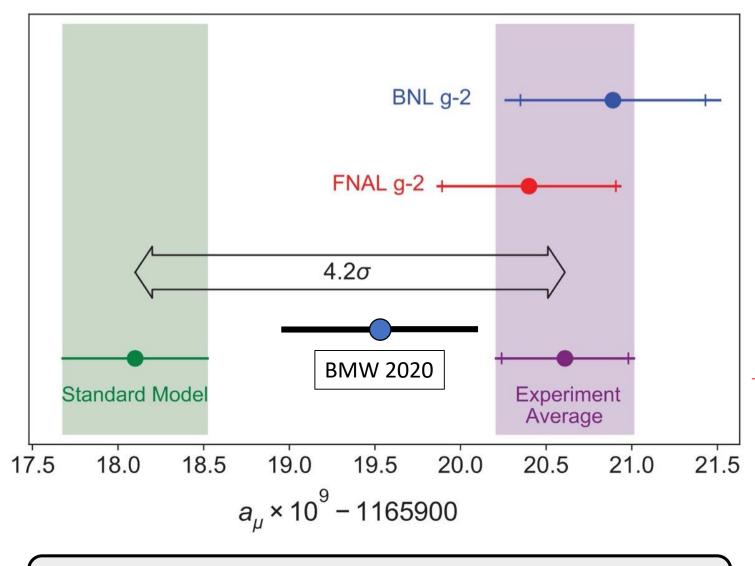


$$a_{\mu}({\rm Exp}) - a_{\mu}({\rm SM}) = (251 \pm 59) \times 10^{-11}$$
  
Significance: 4.2 $\sigma$ 

$$a_{\mu}(\text{FNAL}) = 116\,592\,040(54) \times 10^{-11}$$
 (0.46 ppm)  
 $a_{\mu}(\text{Exp}) = 116\,592\,061(41) \times 10^{-11}$  (0.35 ppm)

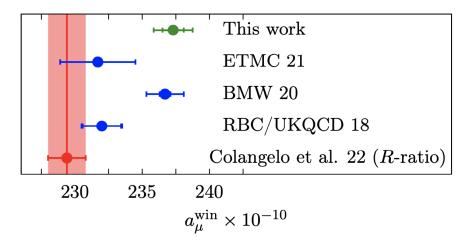
- FNAL results consistent with BNL
- Statistical uncertainty 434 ppb
- Systematic uncertainty 157 ppb
- Fund. constants uncertainty 25 ppb
- $\Rightarrow$  Discrepancy large compared to weak  $(251 \pm 59 \text{ vs } 153.6) \times 10^{-11}$

#### What about the BMW20 Result? (Sz. Borsanyi et al., Nature 593, 51-55, Apr 7, 2021)



$$a_{\mu}(\text{Exp}) - a_{\mu}(\text{BMW2020}) = (107 \pm 71) \times 10^{-11}$$

- BMW 2020: First lattice QCD estimate of HadVP below a percent
- Huge accomplishment!
- Too late to include in WP result
- In tension with dispersive approach
- Members of BMW in Theory Initiative
- → Important to resolve



M. Ce et al., (Mainz) arXiv:2206.06582v1

- FNAL result consistent with BNL
- ullet Combined result differs from SM prediction by 4.2  $\sigma$
- Run-1 is 6% of final data set
  - ullet  $\mu^-$  data run not approved
  - Taking more  $\mu^+$  data
  - g-2 Theory Initiative workshop Sep 2022
    - Update in 2023
    - HVP(lattice) below 0.5% by 2025?
    - HVP: MUonE, other exp inputs
    - Hlbl(dispersive)  $\lesssim 10\%$  by 2025?
    - Hlbl(lattice)  $\lesssim 10\%$  by 2025?
    - HVP(lattice) below 0.5% by 2025?
- Looking forward to result from J-PARC g-2

