

Why measure Higgs pair production?



The Higgs potential

▶ Impressive experimental results on Higgs couplings!



• EW symmetry breaking (in the SM: $\mu^2 = \lambda v^2, m_H = 2\lambda v^2$) $\rightarrow V(H) = \frac{1}{2}m_H^2 H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4$

▶ SM: Higgs self-couplings are fixed by m_H and v



LHC production channels



[Reviews in Physics 5 (2020) 100045]





SM: gg fusion



Borrowed from M. Spira An approximate history (30 years in 30 seconds)



[1] Glover, van der Bij 88; [2] Dawton, Dittmaier, Spira 99; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Meinikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vyonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Malehöfler, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Henrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luison, Wyonidou 17; [15] SPJ, Kuttimali 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campananio, Glaus, Mihilleiture, Spiro, Strichent 18; [16] Grazzini, Heinrich, SPJ, Kerner, Listen, Haren, Linter, Harczelli 18; [17] Glo Florian, Mazzitelli 18; [17] de Gröber, Maier, Rauh 17; [17] Baglio, Campananio, Glaus, Mihilleiture, Spiro, Strichent 18; [18] Grazzini, Heinrich, SPJ, Kerner, Listen, Haren, Linter, Harzelli 18; [17] de Florian, Mazzitelli 18; [17] de Gröber, Maier, Rauh 17; [17] Baglio, Campananio, Glaus, Mihilleiture, Spiro, Strichett 18; [18] Glancian, Degrazi, Glancian, SPJ, Nalveit, Strinkauser, 192, (21) Gavies, Haieren, Histhiran, Steinhauser, Steinhauser, Steinhauser, SH, Allweit, Linter, SH, Allweit, Linter, SH, Allweit, Shen, Yung 19, 19; [27] Davies, Hernen, Mishima, Steinhauser, J, 26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Hernen, Mishima, Steinhauser 19; [28] Baglio, Campananio, Glaus, Mihilleiture, Ronos, Spiro 21; [28] Baglion, Campananio, Glaus, Mihilleiture, Ronos, Spiro 21; [28] Baglion, Campananio, Glaus, Mihilleiture, Ronos, Spiro 21; [29] Calefonto, Degrazis, Glardino, Corbert 18; [21] Davies, Hernen, Mishima, Steinhauser, 19; [28] Baglion, Campananio, Glaus, Mihilleiture, Ronos, Spiro 21; [29] Baglion, Campananio, Glaus, Mihilleiture, Ronos, Spiro 21; [29] Ballofonto, Degrazis, Glardino, Corbert 19; [20] Davies, Hernen, Mishima, Steinhauser 19; 21; [

S. Jones



















Top-mass effects [Grazzini et al. '18]

- ▶ NLO_{m_t} + NNLO ($m_t \to \infty$)
- ► + NNLL [de Florian, Mazzitelli '18]
- With 3 different approximations to the m_t -effects

\sqrt{s}	13 TeV	14 TeV	27 TeV	100 TeV	0.20 6 0.15	NNLO _{B-proj}
NLO [fb]	27.78 ^{+13.8%} -12.8% 28.91 ^{+15.0%}	32.88 ^{+13.5%} _{-12.5%} 34.25 ^{+14.7%}	$127.7^{+11.5\%}_{-10.4\%}$ $134.1^{+12.7\%}_{-10.4\%}$	$1147^{+10.7\%}_{-9.9\%}$ 1220 $^{+11.9\%}$	ц. 4 фр. 0.10- ррр	NNLO _{FTapprox}
NNLO _{NLO-i} [fb]	32.69 ^{+5.3%} _{-7.7%}	38.66 ^{+5.3%} -7.7%	149.3 ^{+4.8%} _{-6.7%}	1337 ^{+4.1%} -5.4%	0.05	
NNLO _{B-proj} [fb] NNLO _{FTapprox} [fb]	$33.42^{+1.3\%}_{-4.8\%}$ $31.05^{+2.2\%}_{-5.0\%}$	$39.58^{+1.4\%}_{-4.7\%}$ $36.69^{+2.1\%}_{-4.9\%}$	$154.2^{+0.7\%}_{-3.8\%}$ $139.9^{+1.3\%}_{-3.9\%}$	$1406^{+0.3\%}_{-2.8\%}$ $1224^{+0.9\%}_{-3.2\%}$	0.00	
M _t unc. NNLO _{FTapprox}	+2.6%	±2.7%	±3.4%	±4.6%	0 1.4 2 1.2	
NNLOFTapprox/NLO	1.110	1.110	1.090	1.007		400 500



DT h1 (GeV)

HTL at higher-order

▶ $m_t \to \infty$: integrate out top-quark DOFs & match to SM [Spira '16], [Gerlach, Herren, Steinhauser '18]



- ▶ Range of validity: $250 \text{ GeV} = 2m_H < \sqrt{\hat{s}} \ll 2m_t \sim 350 \text{ GeV}$
- \blacktriangleright Reduces the number of internal scales \rightsquigarrow easier integrals



[Dawson, Dittmaier, Spira '98]







[Chen, Li, Shao, Wang '20]



Kinematic expansions

- $\frac{1}{m_t}$: valid when $\sqrt{\hat{s}} < 2 \cdot m_t$ [Davies et al. '18, '21]
- High- $E, m_H \ll m_t \ll \hat{s}, |\hat{t}|: \sqrt{s} \gtrsim 800 \text{ GeV}$ [Davies et al. '18]
- Small- p_t^H : $\sqrt{s} \lesssim 750~{
 m GeV}$ [Bonciani et al. '18]
- ▶ Large- m_t , and top threshold expansion via Padé ansatz: $\sqrt{s} \lesssim 700 \text{ GeV} [\text{Gröber, Maier, Rauh '18}]$





$HTL \ N^3LO \ _{\rm [Chen, \ Li, \ Shao, \ Wang \ '20]}$

Ingredients:

► N³LO single Higgs

[Anastasiou, Duhr, Dulat, Herzog, Mistlberger '15]

- + 2-loop 4-point functions
 [Banerjee, Borowka, Dhani, Gehrmann, Ravindran '18]
- ► Good perturbative convergence
- ▶ PDF uncertainty > scale uncertainty





$Matched \ N^3LO \ + \ N^3LL \ {}_{\rm [Aijath, \ Shao\ '22]}$

- Threshold resummation at N³LL matched to N³LO calculation by [Chen, Li, Shao, Wang '20]
- ▶ Scale uncertainty below %-level

 \blacktriangleright m_b effects?







► As scale and $\mathcal{O}(1/m_t^2)$ uncertainties are going down, we might need to worry about other sources

$$\overline{m}_t = \frac{m_t}{1 + \frac{4}{3}\frac{\alpha_s(m_t)}{\pi} + K_2 \left(\frac{\alpha_s(m_t)}{\pi}\right)^2 + K_3 \left(\frac{\alpha_s(m_t)}{\pi}\right)^3}$$

ŀ



$$\begin{split} & \kappa_{\lambda} = -10: \quad \sigma_{tot} = 1438(1)^{+10\%}_{-6\%} \text{ fb}, \\ & \kappa_{\lambda} = -5: \quad \sigma_{tot} = 512.8(3)^{+10\%}_{-7\%} \text{ fb}, \\ & \kappa_{\lambda} = -1: \quad \sigma_{tot} = 113.66(7)^{+8\%}_{-9\%} \text{ fb}, \\ & \kappa_{\lambda} = 0: \quad \sigma_{tot} = 61.22(6)^{+6\%}_{-12\%} \text{ fb}, \\ & \kappa_{\lambda} = 1: \quad \sigma_{tot} = 27.73(7)^{+4\%}_{-18\%} \text{ fb}, \\ & \kappa_{\lambda} = 2: \quad \sigma_{tot} = 13.2(1)^{+1\%}_{-22\%} \text{ fb}, \\ & \kappa_{\lambda} = 3: \quad \sigma_{tot} = 12.7(1)^{+9\%}_{-15\%} \text{ fb}, \\ & \kappa_{\lambda} = 5: \quad \sigma_{tot} = 83.2(3)^{+1\%}_{-4\%} \text{ fb}, \\ & \kappa_{\lambda} = 10: \quad \sigma_{tot} = 579(1)^{+12\%}_{-4\%} \text{ fb} \end{split}$$



▶ As scale and $\mathcal{O}(1/m_t^2)$ uncertainties are going down, we might need to worry about other sources



EW corrections

- Nice progress (N³LO HTL) in QCD, but what about EW corrections?
- ► Single (off-shell) $H: \delta_{\rm EW} \sim 5\%$
- ► Top-Yukawa induced EW corrections to *HH* investigated [Mühlleitner, Schlenk,

Spira '22]

► Leading 2-loop Yukawa corrections

[Davies, Mishima, Schönwald, Steinhauser, Zhang '22]





 \rightarrow see talks by M. Spira, H. Zhang



SM: VBF production





- ▶ Inclusive N³LO QCD [Dreyer, Karlberg '18]
- ► Fully-differential NNLO QCD [Dreyer, Karlberg '18] + NLO EW

[Dreyer, Karlberg, Lang, Pellen '20]

$\sigma_{ m LO}^{ m full}$	$\delta_{ m NLO~QCD}^{ m full}$	$\delta_{\rm NNLO~QCD}^{\rm VBF}$	$\delta_{\rm NLO \; EW}^{\rm full}$ ϵ	$\tau_{\rm NNLO~QCD \times NLO~EW}$	$\delta_{\rm NNLO \ QCD}^{\rm NF}$ [fb]
$0.78444(9)\substack{+0.0825\\-0.0694}$	-0.07110(13)	-0.0115(5)	-0.0476(2)	$0.6684(5)^{+0.002}_{-0.0004}$	-0.001766(7)
$^{+10.5\%}_{-8.8\%}$	-9.1%	-1.5%	-6.1%	$-14.8\%^{+0.3\%}_{-0.06\%}$	-0.23%



► NNLO QCD implemented in public MC: proVBFHH [Cacciari,

Dreyer, Karlberg, Salam, Zanderighi, (Tancredi)]



[Dreyer, Karlberg, Lang, Pellen '20]



► NNLO QCD implemented in public MC: proVBFHH [Cacciari,

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HH backgrounds

▶ bbH



- Included at LO (in ggF NNLOPS) [ATLAS 2112.11876] with additional 100% uncertainty
- ▶ NLO QCD corrections (HTL) [Deutschmann, Maltoni, Wiesemann, Zaro '18]
- ► Complete NLO (QCD&EW) corrections known [Pagani, Shao, Zaro '20]
- ▶ Amplitudes for $b\bar{b}H$ in the 5FS known at NNLO [Badger, Hartanto, Krys, Zoia '21]

►
$$t\bar{t} (W^+W^-b\bar{b})$$

 Typically simulated at NLO QCD (Powheg): large theory uncertainty

 MiNNLO_{PS} [Mazzitelli, Monni, Nason, Re, Wiesemann, Zanderighi '21]





HH in Effective Field Theories



► SMEFT:

• $H \equiv \mathrm{SU}(2)_L \times U(1)_Y$ doublet

► Canonical dimension counting $(\sim 1/\Lambda^n)$

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{i} \frac{C_{i}^{(6)}}{\Lambda^{2}} \mathcal{O}_{i}^{(6)} + \mathcal{O}(\frac{1}{\Lambda^{3}})$$

► HEFT:

 $\blacktriangleright \ H \equiv \text{EW singlet}$

• Chiral dimension counting d_{χ} (\equiv loop counting)

$$\mathcal{L}_{\text{HEFT}} = \mathcal{L}_{(d_{\chi}=2)} + \sum_{L=1}^{\infty} \sum_{i} \left(\frac{1}{16\pi^2}\right)^L c_i^{(L)} O_i^{(L)}$$



 \dots in hh

16 / 25

► SMEFT:

$$\begin{split} \Delta \mathcal{L}_{\text{SMEFT}}^{(\text{Warsaw})} &= \frac{C_{H,\Box}}{\Lambda^2} (\phi^{\dagger}\phi) \Box (\phi^{\dagger}\phi) + \frac{C_{HD}}{\Lambda^2} (\phi^{\dagger}D_{\mu}\phi)^* (\phi^{\dagger}D^{\mu}\phi) \\ &+ \frac{C_H}{\Lambda^2} (\phi^{\dagger}\phi)^3 + \left(\frac{C_{uH}}{\Lambda^2} \phi^{\dagger}\phi \bar{q}_L \phi^c t_R + h.c.\right) + \frac{C_{HG}}{\Lambda^2} \phi^{\dagger}\phi G^a_{\mu\nu} G^{\mu\nu,a} \\ &+ \frac{\bar{C}_{uG}}{\Lambda^2} (\bar{q}_L \sigma^{\mu\nu} T^a G^a_{\mu\nu} \tilde{\phi} t_R + h.c.) \end{split}$$

$$\bullet \text{ HEFT:} \\ \Delta \mathcal{L}_{\text{HEFT}} &= -c_{hhh} \frac{m_h^2}{2v} h^3 \\ &- m_t \left(c_t \frac{h}{v} + c_{tt} \frac{h^2}{v^2}\right) \bar{t} t \\ &+ \frac{\alpha_s}{8\pi} \left(c_{ggh} \frac{h}{v} + c_{gghh} \frac{h^2}{v^2}\right) G^a_{\mu\nu} G^{a,\mu\nu} \end{split}$$

HEFT NNLO' QCD [de Florian, Fabre, Heinrich, Mazzitelli, LS '21] 18 / 26

▶ NLO_{mt} [Borowka et al. '16],[Buchalla, Capozi, Celis, Heinrich, LS '18] + NNLO $(m_t \rightarrow \infty)$ [de Florian, Fabre, Mazzitelli '16]

$$\begin{split} \sigma_{\rm BSM} / \sigma_{\rm SM} &= a_1 \, c_t^4 + a_2 \, c_{tt}^2 + a_3 \, c_t^2 c_{hhh}^2 + a_4 \, c_{ggh}^2 c_{hhh}^2 + a_5 \, c_{gghh}^2 + a_6 \, c_{tt} c_t^2 + a_7 \, c_t^3 c_{hhh} \\ &+ a_8 \, c_{tt} c_t \, c_{hhh} + a_9 \, c_{tt} c_{ggh} c_{hhh} + a_{10} \, c_{tt} c_{gghh} + a_{11} \, c_t^2 c_{ggh} c_{hhh} + a_{12} \, c_t^2 c_{gghh} \\ &+ a_{13} \, c_t c_{hhh}^2 c_{ggh} + a_{14} \, c_{tchhh} c_{gghh} + a_{15} \, c_{ggh} c_{hhh} c_{gghh} + a_{10} \, c_t^2 c_{ggh}^2 \\ &+ a_{17} \, c_t c_{tt} c_{ggh} + a_{18} \, c_t c_{ggh}^2 c_{hhh} + a_{19} \, c_t c_{ggh} c_{gghh} + a_{20} \, c_t^2 c_{ggh}^2 \\ &+ a_{21} \, c_{tt} c_{ggh}^2 + a_{22} \, c_{ggh}^3 c_{hhh} + a_{23} \, c_{ggh}^2 c_{gghh} + a_{24} \, c_{dgh}^4 + a_{25} \, c_{ggh}^3 c_t \end{split}$$





HEFT NNLO' QCD [de Florian, Fabre, Heinrich, Mazzitelli, LS '21] 18 / 26





Dim-8 operators for HH [Cappati, Covarelli, Torrielli, Zaro '22]





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SMEFT@NLO [Degrande, Durieux, Maltoni, Mimasu, Vryonidou, Zhang '20] 20 / 26

- Automated NLO SMEFT implementation (not limited to HH)
- ▶ Dimension-6 operators (Warsaw basis)
- ▶ Interface to MadGraph [Alwall et al. '14]



SMEFT NLO_{m_t} [Heinrich, Lang, LS '22]



 \triangleright At amplitude-squared level:

$$\sigma \simeq \begin{cases} \sigma_{\rm SM} + \sigma_{\rm SM \times dim6} & (a) \\ \sigma_{(\rm SM+dim6) \times (\rm SM+dim6)} & (b) \\ \sigma_{(\rm SM+dim6) \times (\rm SM+dim6)} + \sigma_{\rm SM \times dim6^2} & (c) \\ \sigma_{(\rm SM+dim6+dim6^2) \times (\rm SM+dim6+dim6^2)} & (d) \end{cases}$$



SMEFT NLO_{m_t [Heinrich, Lang, LS '22]}







[Gomez-Ambrosio, Llanes-Estrada, Salas-Bernardez, Sanz-Cillero '22] Flare function

 $SMEFT \leftrightarrow HEFT$

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2} \partial_{\mu} h \partial^{\mu} h - V(h) + \frac{1}{2} \mathcal{F}(h) \partial_{\mu} w^{i} \partial^{\mu} w^{j} \left(\delta_{ij} + \frac{w_{i} w_{j}}{v^{2} - w^{2}} \right)$$
$$\mathcal{F}(h) = 1 + \sum a_{n} \left(\frac{h}{v} \right)^{n}$$

$$\mathcal{L}_{\text{SMEFT}} = \frac{v^2}{4} \mathcal{F}(h_1) \langle D_{\mu} U^{\dagger} D^{\mu} U \rangle + \frac{1}{2} (\partial_{\mu} h_1)^2 - V(h) - \frac{c_{H\square}((v+h_1)^3 - v^3))}{3\Lambda^2} V'(h_1)$$
$$\mathcal{F}(h_1) = 1 + \left(\frac{h_1}{v}\right) \left(2 + 2\frac{c_{H\square}v^2}{\Lambda^2}\right) + \dots + \left(\frac{h_1}{v}\right)^4 \left(2\frac{c_{H\square}v^2}{3\Lambda^2}\right)$$

- ▶ With correlations between flare function coefficients
- ► Connection to geometry: scalar loop corrections ~ curvature of the scalar manifold metric [Guo et al. '15], [Alonso et al. '16]











Available MC tools

HEFT

- \blacktriangleright LO and NLO $m_t \rightarrow \infty$ HPAIR [Gröber, Mühlleitner, Spira, Streicher '15]
- ▶ Full top-mass dependent NLO QCD corrections to $gg \rightarrow hh$ [Borowka et al '16], [Baglio et al '18]
 - ... incorporated within HEFT [Buchalla, Celis, Capozi, Heinrich, LS '18]
 - ... and in Powheg-BOX-V2/ggHH [Heinrich, Jones, Kerner, LS '20]
- ▶ NNLO' (NLO full- m_t + NNLO $m_t \rightarrow \infty$) predictions [de Florian,

Fabre, Heinrich, Mazzitelli, LS '21]

SMEFT

- \blacktriangleright LO and NLO $m_t \rightarrow \infty$ HPAIR [Gröber, Mühlleitner, Spira, Streicher '15]
- SMEFT@NLO & MG5_aMC@NLO [Degrande, Durieux, Maltoni, Mimasu, Vryonidou, Zhang '20]
- ► NLO full-*m_t* available in Powheg-BOX-V2/ggHH_SMEFT with various truncation options [Heinrich, Lang, LS '22]



HH: LHC Higgs Working Group

24 / 26

Upcoming Pub-Note [Alasfar, Cadamuro, Dimitriadi, Ferrari, Gröber, Heinrich

Carlson, Lang, Sjölin, Ördek, Sánchez, LS, 22xx.xxxx]

- ► Updated BMs from [Heinrich, Capozi '18]
- Review of uncertainty sources
- ▶ Set of NLO (full- m_t) A_i coefficients (incl. and diff.) w. full correlations, and scale variations
- ▶ Speed-up of event generation by reweighting SM samples

benchmark (* = modified)	c_{hhh}	c_t	c_{tt}	c_{ggh}	c_{gghh}
SM	1	1	0	0	0
1*	5.105	1.1	0	0	0
2*	6.842	1.033	$\frac{1}{6}$	$-\frac{1}{3}$	0
3*	2.21	1.05	$-\frac{1}{3}$	0.5	0.25^{*}
4*	2.79	0.9	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$
5	3.95	1.17	$-\frac{1}{3}$	$\frac{1}{6}$	$-\frac{1}{2}$
6*	-0.684	0.9	$-\frac{1}{6}$	0.5	0.25
7	-0.10	0.94	1	$\frac{1}{6}$	$-\frac{1}{6}$



HH: LHC Higgs Working Group

24 /-26

H: LHC HIggs WOIKIng GIOUP

Carlson, Lang, Sjölin, Ördek, Sánchez, LS, 22xx.xxxxx

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Though I was asked to present results in SM and EFT's only, there are very many results from BSM models as well!

- ► 2HDM triple-Higgs coupling [Arco, Heinemeyer, Herrero '20, '21, '22]
- ▶ 2HDM: GW and *c*_{HHH} [Biekötter et al. '21, '22]
- ► HH with an extra scalar singlet [Abouabid et al '21],[Adhikari, Lane, Lewis, Sullivan '22] → see talk by I. Lewis
- ► Radiative corrections to c_{hhh} in the 2HDM Bahl, Braathen, Weiglein '22]
- ► SFOEWPT \leftrightarrow 2HDM-EFT [Anisha, Biermann, Englert, Mühlleitner '22]
- c_{hhh} in CP-violating NMSSM [Borschensky, Dao, Gabelmann,

Mühlleitner, Rzehak '22] \rightarrow see talk by H. Rzehak



Summary

- ▶ Much progress on theoretical front in recent years
 - ▶ ggF: NLO (full QCD), $N^{3}LO$ (HTL)
 - ▶ VBF: N³LO (incl.), NNLO QCD + NLO EW (diff.), non-factorisable contributions
- ▶ Leaps on both the theory and the experimental fronts!
- ▶ Two different EFT approaches:
 - ▶ SMEFT: linear realisation, $H \in$ doublet, Wilson coefficients naturally small, partial correlations
 - ▶ HEFT: non-linear, $H \in$ singlet, Wilson coefficients formally $\sim \mathcal{O}(1)$, no relations between e.g. c_{ggh} and c_{gghh}
- ▶ hh is a nice playground to study differences between these EFT's (e.g. whether the Higgs sector is realised (non-)linearly)
- ▶ Many other interesting developments (EW corrections, *m_t*-scheme, higher-*D*-operator constraints, generic EFT considerations...)