Geometric SMEFT

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Based on:

2001.01453 [Helset, AM, Trott], 2007.00565 [Hays, Helset, AM, Trott]
2102.02819 [Corbett, Helset, AM, Trott], 2107.07470 [Corbett, AM, Trott],
2109.555 [AM, Trott]

Higgs2022, Nov 8th, 2022

Motivation

No obvious signs of new light states at LHC — parametrize BSM effects with SM-EFT = SMEFT

$$\mathscr{L} = \mathscr{L}_{SM} + \sum_d \sum_i \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathscr{O}_i^{(d)}(Q, u_c, d_c, L, e_c, H, \cdots, D_\mu)$$

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$$|A|^2 = |A_{SM}|^2 + \frac{2Re(A_{SM}^* A_6)}{\Lambda^2} + \cdots$$

interference piece. State of the art what's SMEFT. Included in MC codes (SMEFTsim, SMEFT@NLO,..)

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Determining Λ is THE goal of the SMEFT strategy — it's the scale where you build the next accelerator

Want to know Λ as well as we can …

Motivation

$$|A|^2 = |A_{SM}|^2 + \frac{2Re(A^*_{SM}A_6)}{\Lambda^2} + \frac{1}{\Lambda^4}\Big(|A_6|^2 + 2Re(A^*_{SM}A_8)\Big) + \cdots$$

interference piece,
usually largest effect

'Higher order'
$\mathcal{O}(1/\Lambda^4)$
corrections

What's the impact from $1/\Lambda^4$ corrections?

SMEFT Warsaw basis: $\mathcal{O}(60)$ operators at dim-6

(flavor universal) $\mathcal{O}(1000)$ operators at dim-8

Higher order effects so should be small… but

- there are instances where interference term isn't present or is suppressed, e.g. helicity mismatch between SM and dim-6

- faster growth with energy, E^4 vs. E^2 : increasingly important when looking at high energy (e.g. tails of some kinematic distribution)

How do we proceed?

Is there a simple estimate, i.e (dim-6)^2 that works?
Do we need to do case by case?

Geometric SMEFT:

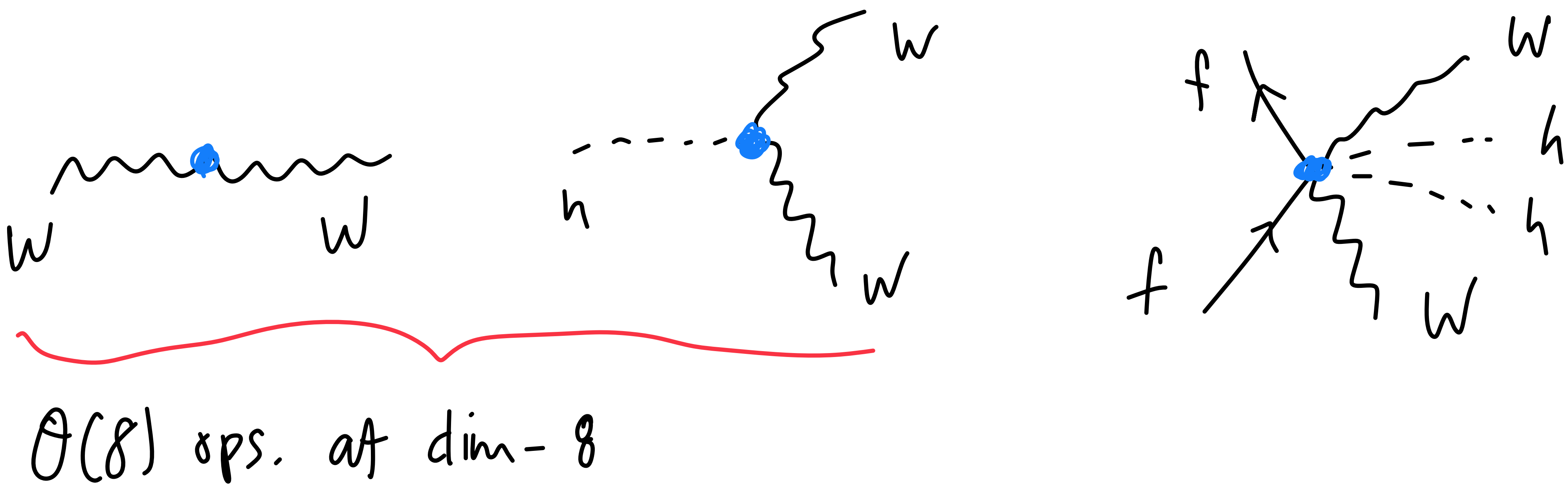
[Helset, AM, Trott 2001.01453]

A reorganization of the SMEFT operators, where

2 and 3-particle interactions can be determined to **all orders in** v/Λ.

Number of operators is small, ~ constant at each order

Ex.)



With fewer operators around, can actually do complete $1/\Lambda^4$ calculations for certain processes.

Use those processes as simple laboratories for truncation error studies

SMEFT operators:

have the form $D^a H^b \bar{\psi}^c \psi^d F^x$

For operator affecting 2,3-pt vertices: restrictions

1.) Can't have too many fields

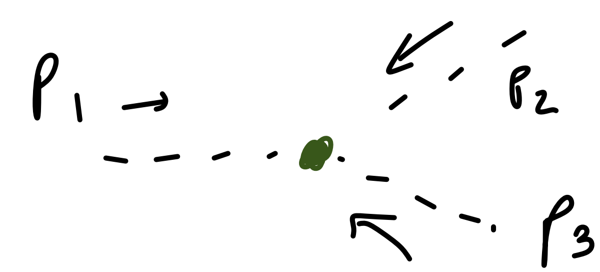
e.g. $(DH^\dagger)(DH)(DH^\dagger)(DH)$ → 4+ fields, can't contribute

2.) Momentum on fields other than H is 'trivial'

e.g. $D_\mu H \, (D^\mu \bar{\psi}) \, \psi$

$$\sim (p_H \cdot p_{\bar{\psi}}) \, H \, \bar{\psi} \, \psi$$

$$\sim \left(\frac{m_\psi^2 - m_H^2 - m_{\bar{\psi}}^2}{2} \right) H \, \bar{\psi} \, \psi$$



$$p_H + p_{\bar{\psi}} + p_\psi = 0$$

Just changes coefficient of $H \, \bar{\psi} \, \psi$: not a new operator structure

Allowed 2, 3-pt structures:

[+ versions with G^A]

$$h_{IJ}(\phi)(D_\mu\phi)^I(D_\mu\phi)^J, \quad g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$$

$$k^A_{IJ}(\phi)(D_\mu\phi)^I(D_\nu\phi)^J\,\mathcal{W}^{\mu\nu}_A, \quad f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho,$$

$$Y(\phi)\bar{\psi}_1\psi_2, \quad L_{I,A}(\phi)\bar{\psi}_1\gamma^\mu\tau_A\psi_2(D_\mu\phi)^I, \quad d_A(\phi)\bar{\psi}_1\sigma^{\mu\nu}\psi_2\mathcal{W}^A_{\mu\nu},$$

Can't have derivatives in them, so only thing left is $H^\dagger H/\Lambda^2 \equiv \phi^2$

Additionally, # of possible EW structures for the functions **saturates**

Ex.) h_{IJ} multiplies two doublets: can either be singlet = δ_{IJ} , or triplet.

Can be worked out to <u>all orders</u> in ϕ!

Allowed 2, 3-pt structures:

[+ versions with G^A]

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Ex.) $$h_{IJ} = \left[1 + \phi^2 C^{(6)}_{H\Box} + \sum_{n=0}^{\infty}\left(\frac{\phi^2}{2}\right)^{n+2}\left(C^{(8+2n)}_{HD} - C^{(8+2n)}_{H,D2}\right)\right]\delta_{IJ} + \frac{\Gamma^I_{A,J}\phi_K\Gamma^K_{A,L}\phi^L}{2}\left(\frac{C^{(6)}_{HD}}{2} + \sum_{n=0}^{\infty}\left(\frac{\phi^2}{2}\right)^{n+1} C^{(8+2n)}_{H,D2}\right)$$

Dim-6 : 2 terms

Dim-8$^+$: 2 terms

Allowed 2, 3-pt structures:

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Dim-6 : 2 terms

Dim-8+: 2 terms

Flat 'metric' in SM, curved in SMEFT. Geometric perspective -> **geoSMEFT**

[Burgess, Lee, Trott '10, Alonso, Jenkins, Manohar '15, '16, Helset, Paraskevas, Trott '18]

More recently [Cohen et al '22, Cheung et al '21, '22, Helset et al '22]

operators small and remains ~fixed for increasing mass dimension

	Mass Dimension				
Field space connection	6	8	10	12	14
$h_{IJ}(\phi)(D_\mu\phi)^I(D^\mu\phi)^J$	2	2	2	2	2
$g_{AB}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\mu\nu}$	3	4	4	4	4
$k_{IJA}(\phi)(D^\mu\phi)^I(D^\nu\phi)^J\mathcal{W}^A_{\mu\nu}$	0	3	4	4	4
$f_{ABC}(\phi)\mathcal{W}^A_{\mu\nu}\mathcal{W}^{B,\nu\rho}\mathcal{W}^{C,\mu}_\rho$	1	2	2	2	2
$Y^u_{pr}(\phi)\bar{Q}u+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^d_{pr}(\phi)\bar{Q}d+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$Y^e_{pr}(\phi)\bar{L}e+$ h.c.	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$	$2\,N_f^2$
$d^{e,pr}_A(\phi)\bar{L}\sigma_{\mu\nu}e\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{u,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}u\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$d^{d,pr}_A(\phi)\bar{Q}\sigma_{\mu\nu}d\mathcal{W}^{\mu\nu}_A+$ h.c.	$4\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$	$6\,N_f^2$
$L^{\psi_R}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	N_f^2	N_f^2	N_f^2	N_f^2	N_f^2
$L^{\psi_L}_{pr,A}(\phi)(D^\mu\phi)^J(\bar{\psi}_{p,L}\gamma_\mu\sigma_A\psi_{r,L})$	$2\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$	$4\,N_f^2$

geoSMEFT at work:

With geoSMEFT setup, can set EW inputs to all orders:

$$e,\ g_Z,\ \sin^2\theta_Z \longrightarrow \text{functions of } g, g', h_{IJ}, g_{AB}$$

$$\begin{aligned}
\bar{g}_2 &= g_2\sqrt{g}^{11} = g_2\sqrt{g}^{22}, \\
\bar{g}_Z &= \frac{g_2}{c^2_{\theta_Z}}\left(c_{\bar\theta}\sqrt{g}^{33} - s_{\bar\theta}\sqrt{g}^{34}\right) = \frac{g_1}{s^2_{\theta_Z}}\left(s_{\bar\theta}\sqrt{g}^{44} - c_{\bar\theta}\sqrt{g}^{34}\right), \\
\bar{e} &= g_2\left(s_{\bar\theta}\sqrt{g}^{33} + c_{\bar\theta}\sqrt{g}^{34}\right) = g_1\left(c_{\bar\theta}\sqrt{g}^{44} + s_{\bar\theta}\sqrt{g}^{34}\right),
\end{aligned}$$

couplings

mixing angles

$$s^2_{\theta_Z} = \frac{g_1(\sqrt{g}^{44}s_{\bar\theta} - \sqrt{g}^{34}c_{\bar\theta})}{g_2(\sqrt{g}^{33}c_{\bar\theta} - \sqrt{g}^{34}s_{\bar\theta}) + g_1(\sqrt{g}^{44}s_{\bar\theta} - \sqrt{g}^{34}c_{\bar\theta})},$$

$$s^2_{\bar\theta} = \frac{(g_1\sqrt{g}^{44} - g_2\sqrt{g}^{34})^2}{g_1^2[(\sqrt{g}^{34})^2 + (\sqrt{g}^{44})^2] + g_2^2[(\sqrt{g}^{33})^2 + (\sqrt{g}^{34})^2] - 2g_1g_2\sqrt{g}^{34}(\sqrt{g}^{33} + \sqrt{g}^{44})}.$$

$$\bar{m}_W^2 = \frac{\bar{g}_2^2}{4}\sqrt{h_{11}}^2\bar{v}_T^2, \qquad \bar{m}_Z^2 = \frac{\bar{g}_Z^2}{4}\sqrt{h_{33}}^2\bar{v}_T^2 \qquad \bar{m}_A^2 = 0.$$

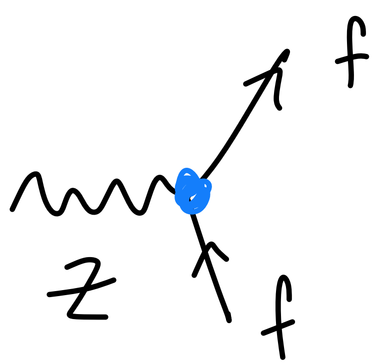
masses

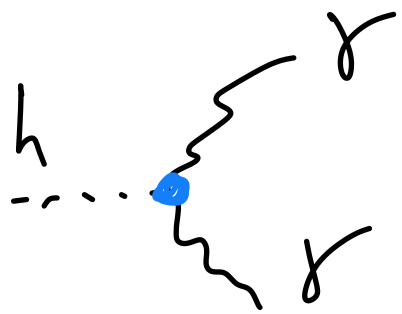
geoSMEFT at work:

SMEFT phenomenology for processes involving 2, 3-pt interactions now doable to any order in v^2/Λ^2

Specifically, $\mathcal{O}(1/\Lambda^4)$ easily calculated for a large set of processes

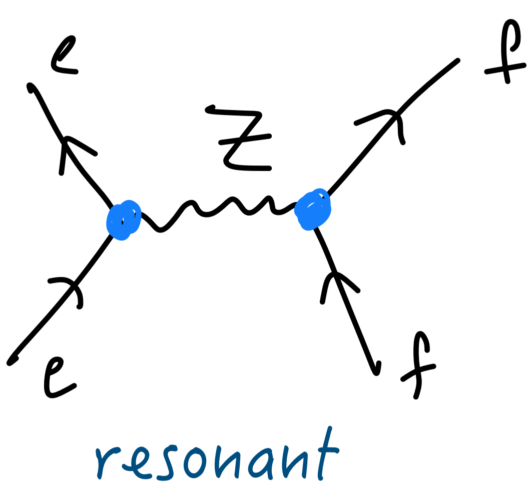
includes

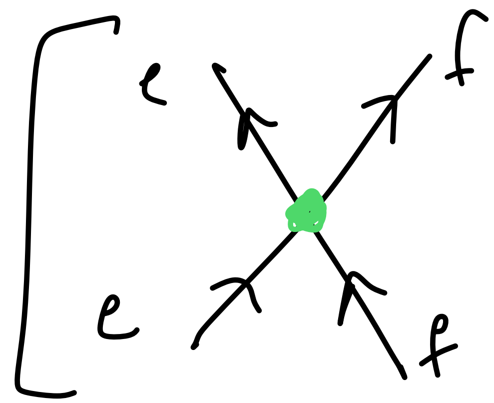




[2007.00565 Hays, Helset, AM, Trott]

and





suppressed by $\dfrac{\Gamma_Z m_Z}{v^2}$

[2102.02819 Corbett, Helset, AM, Trott]

$1 \to 2$ decays: impact on decay widths

e.g) $h \to \gamma\gamma$

defining: $\langle h|\gamma\gamma\rangle_{\mathscr{L}^{(6)}} = \left[\dfrac{g_2^2\tilde{C}^{(6)}_{HB} + g_1^2\tilde{C}^{(6)}_{HW} - g_1 g_2 \tilde{C}^{(6)}_{HWB}}{\left(g_1^2+g_2^2\right)\bar{v}_T}\right]$

$$\tilde{C}^{(6)} = C^{(6)}\frac{v_T^2}{\Lambda^2} \qquad \tilde{C}^{(8)} = C^{(8)}\frac{v_T^4}{\Lambda^4}$$

(dim-6)² estimate: $\left|\mathscr{A}^{h\gamma\gamma}_{SM}\right|^2 + 2\,\mathrm{Re}\left(\mathscr{A}^{h\gamma\gamma}_{SM}\right)\langle h|\gamma\gamma\rangle_{\mathscr{L}^{(6)}} + \langle h|\gamma\gamma\rangle^2_{\mathscr{L}^{(6)}}$

Full $\mathscr{O}(1/\Lambda^4)$ result:

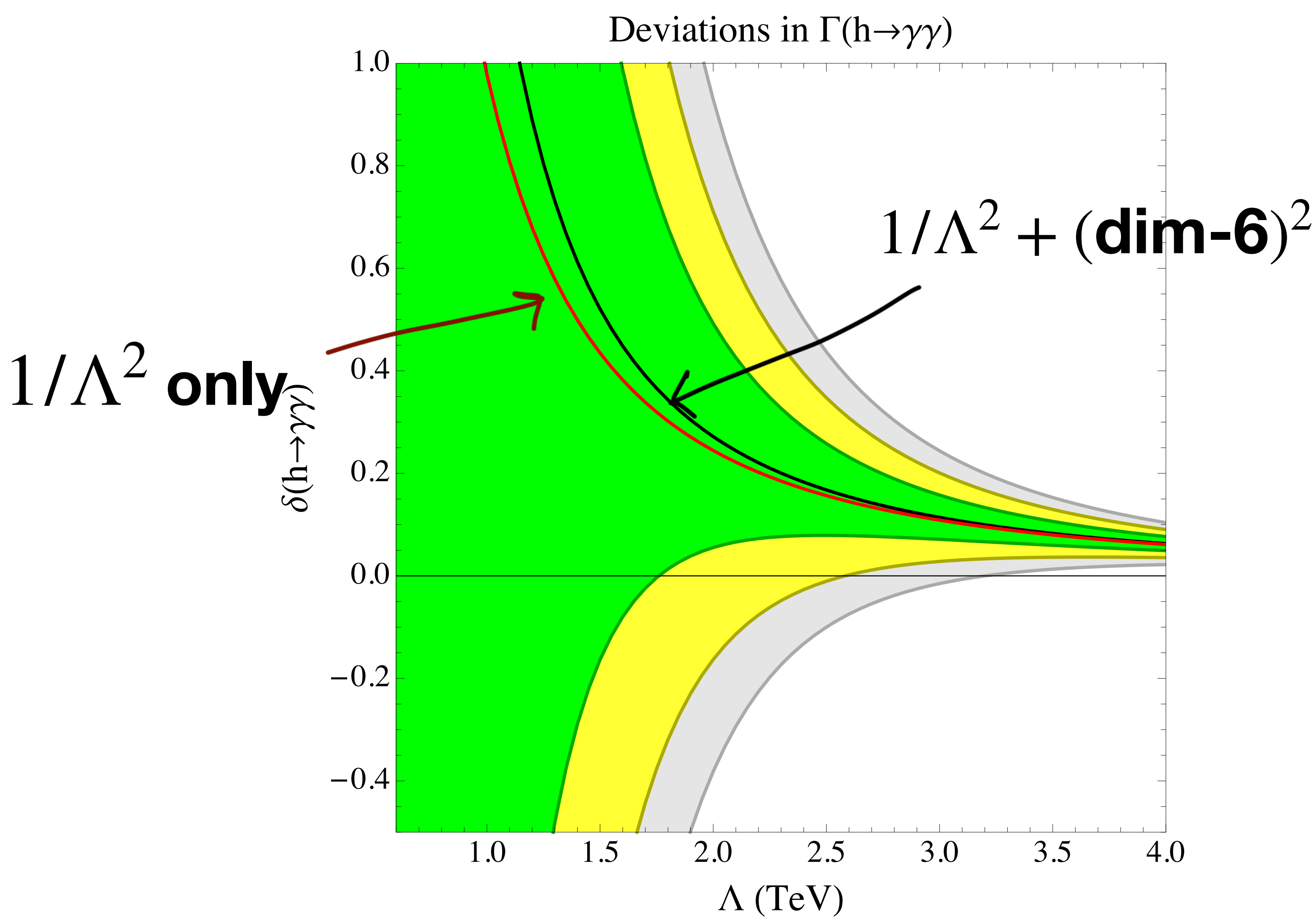
$$\left|\mathscr{A}^{h\gamma\gamma}_{SM}\right|^2 + 2\,\mathrm{Re}\left(\mathscr{A}^{h\gamma\gamma}_{SM}\right)\Bigg(1 + \underbrace{\left\langle \sqrt{h}^{44}\right\rangle_{\mathscr{L}^{(6)}}}_{\tilde{C}^{(6)}_{H\Box},\, \tilde{C}^{(6)}_{HD},\, \tilde{C}^{(8)}_{HD},\, \tilde{C}^{(8)}_{HD2}}\Bigg)\langle h|\gamma\gamma\rangle_{\mathscr{L}^{(6)}} + \left(1 + 4\bar{v}_T\,\mathrm{Re}\left(\mathscr{A}^{h\gamma\gamma}_{SM}\right)\right)\left(\langle h|\gamma\gamma\rangle_{\mathscr{L}^{(6)}}\right)^2$$

$$+2\,\mathrm{Re}\left(\mathscr{A}^{h\gamma\gamma}_{SM}\right)\left[\frac{g_2^2\tilde{C}^{(8)}_{HB} + g_1^2\left(\tilde{C}^{(8)}_{HW} - \tilde{C}^{(8)}_{HW,2}\right) - g_1 g_2\tilde{C}^{(8)}_{HWB}}{\left(g_1^2+g_2^2\right)\bar{v}_T}\right]$$

$1 \to 2$ decays: impact on decay widths

e.g) $h \to \gamma\gamma$: Quantify effect by randomly drawing coefficients and comparing dim-6, (dim-6)^2 and full $1/\Lambda^4$ result: for `tree' operators: $\mathcal{O}(1)$ ,`loop' operators: $\mathcal{O}(0.01)$

[Arzt'93], [Einhorn, Wudka '13], [Craig et al '20]

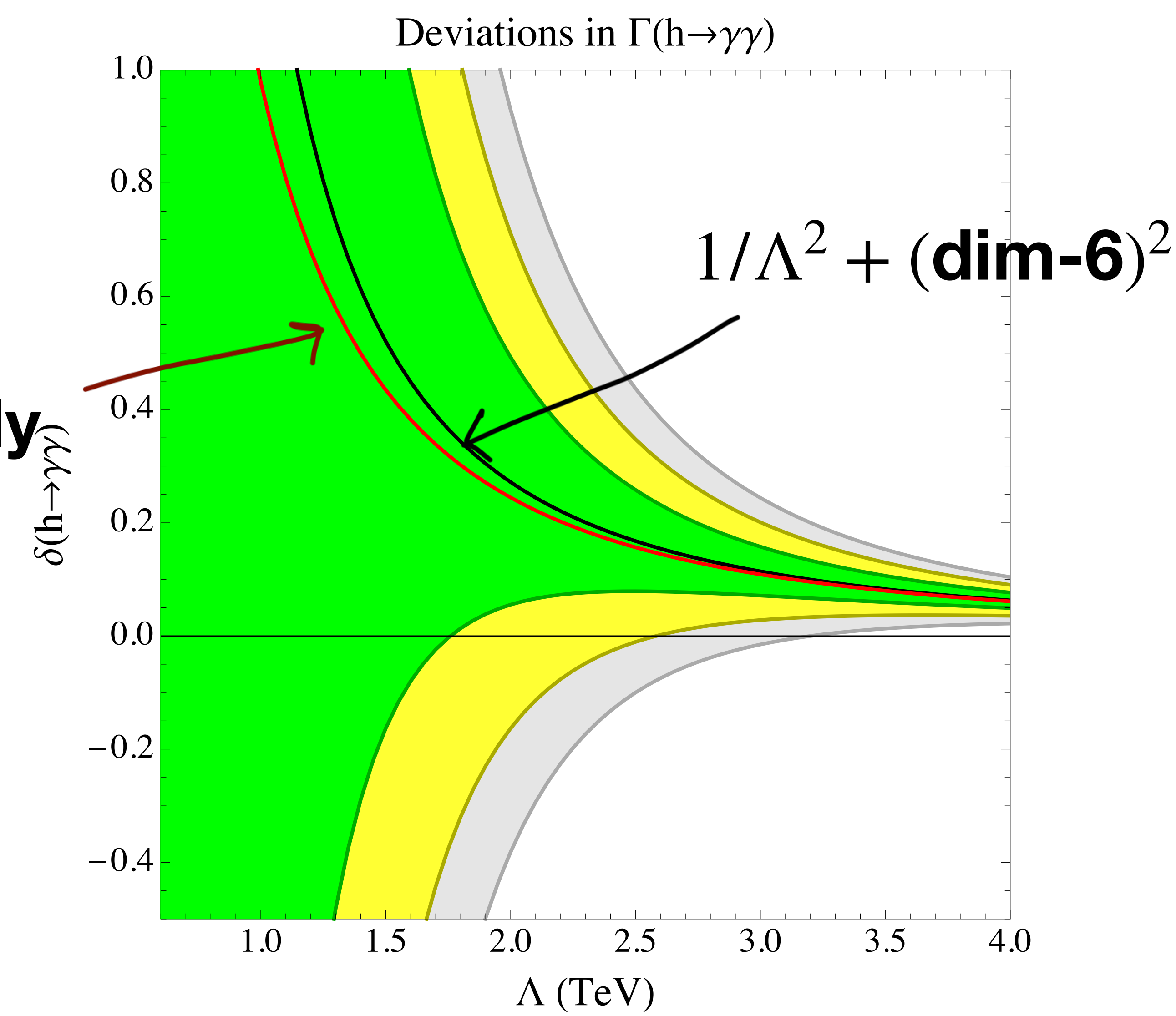


fixing $1/\Lambda^2$, (dim-6)^2 result: contours show range of effects once full $1/\Lambda^4$ effects are included

similar story for $h \to Z\gamma$

$1 \to 2$ decays: impact on decay widths

e.g) $h \to \gamma\gamma$: Quantify effect by randomly drawing coefficients and comparing dim-6, (dim-6)² and full $1/\Lambda^4$ result:
for `tree' operators: $\mathcal{O}(1)$ ,`loop' operators: $\mathcal{O}(0.01)$



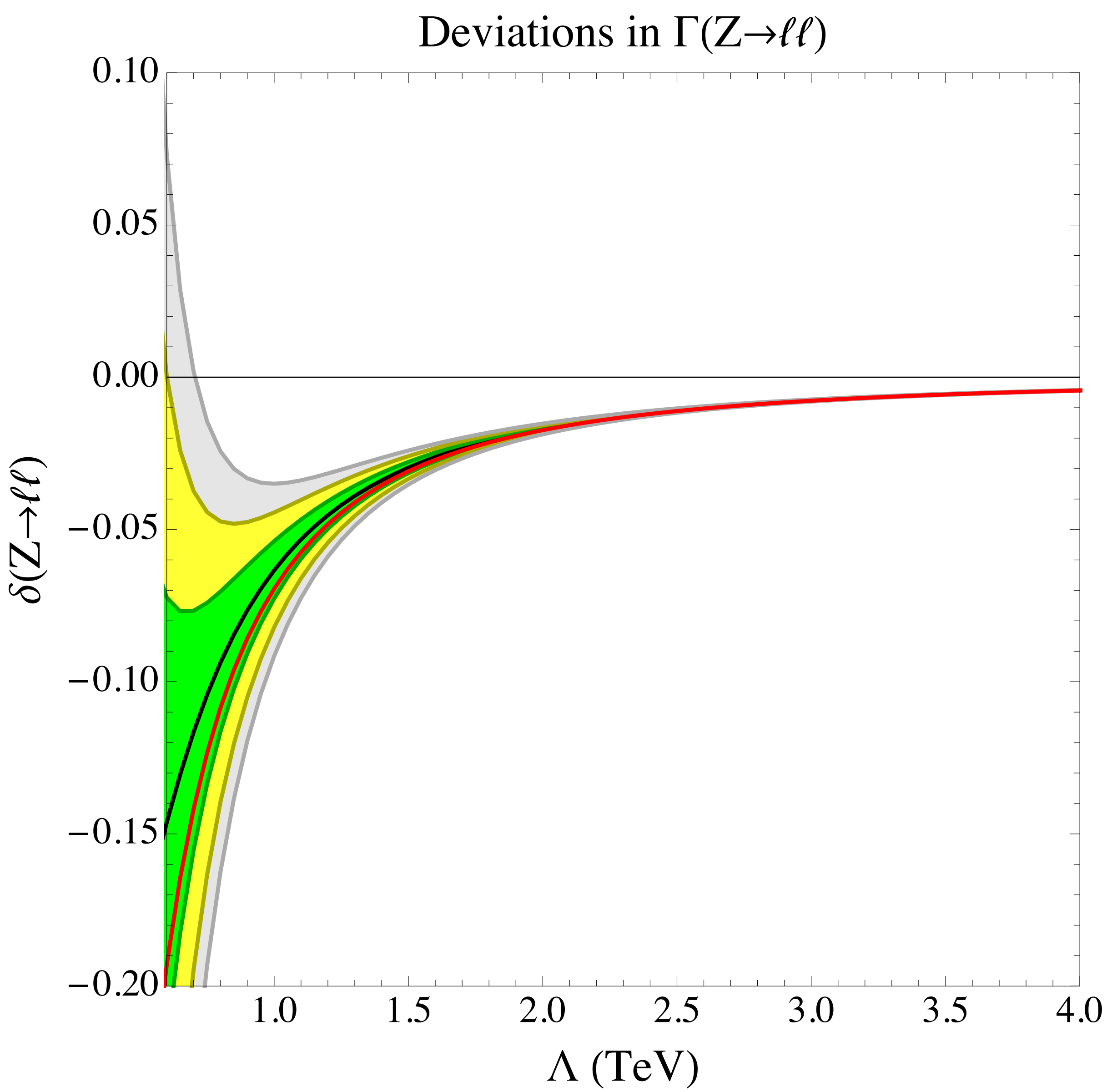
Large effect, $\mathcal{O}(20\%)$ at Λ = 2.5TeV ;

only loop-level operators $(H^\dagger H)\, X^{\mu\nu} X_{\mu\nu}$ enter at dim-6, while tree-level operators $(H^\dagger H)^2\, X^{\mu\nu} X_{\mu\nu}$ enter at dim-8

[model example: kinetically mixed U(1)]

$1 \to 2$ decays: impact on decay widths

e.g.) $Z \to \ell^+\ell^-$: Quantify effect by randomly drawing coefficients and comparing dim-6, (dim-6)^2 and full $1/\Lambda^4$ result:
for `tree' operators: $\mathcal{O}(1)$ ,`loop' operators: $\mathcal{O}(0.01)$

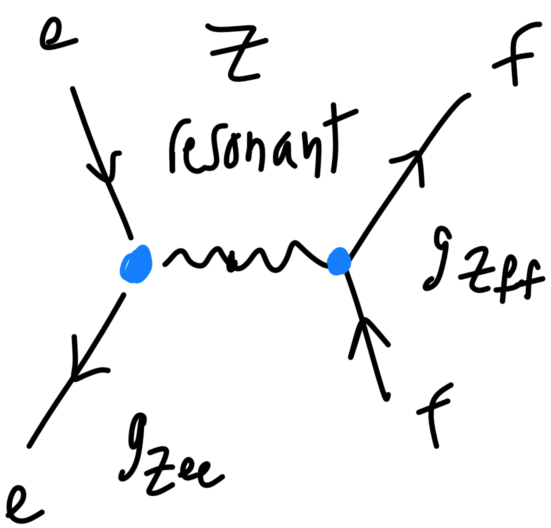


Now tree-level operators present for both dim-6 and dim-8

smaller impact, $\mathcal{O}(\,\%\,)$ at Λ = TeV

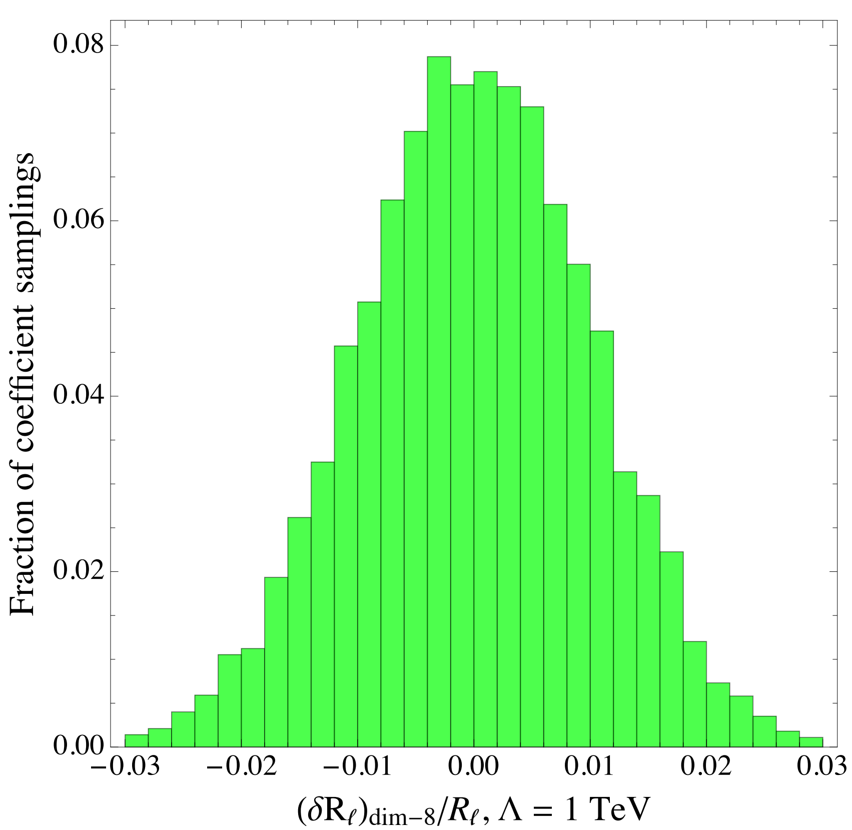
~ (dim6)^2 piece not a bad estimate

Redo classic SMEFT LEP1 analysis to $\mathcal{O}(1/\Lambda^4)$



$$
\begin{aligned}
g_{\rm eff,pr}^{\mathcal{Z},\psi} &= \frac{\bar{g}_Z}{2}\left[(2s^2_{\theta_Z}\,Q_\psi - \sigma_3)\delta_{pr} + \bar{v}_T\langle L_{3,4}^{\psi,pr}\rangle + \sigma_3\bar{v}_T\langle L_{3,3}^{\psi,pr}\rangle\right] \\
&= \langle g_{\rm SM,pr}^{\mathcal{Z},\psi}\rangle + \langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\mathcal{O}(v^2/\Lambda^2)} + \langle g_{\rm eff,pr}^{\mathcal{Z},\psi}\rangle_{\mathcal{O}(v^4/\Lambda^4)} + \cdots .
\end{aligned}
$$

scanning dim-8 coefficients



[2102.02819
Corbett, Helset, AM, Trott]

SMEFT corrections in $\{\hat{m}_W, \hat{m}_Z, \hat{G}_F\}/\{\hat{\alpha}, \hat{m}_Z, \hat{G}_F\}$ scheme			
$\mathcal{O}(\frac{v^4}{\Lambda^4})$	$\langle g_{\rm eff,pp}^{\mathcal{Z},u_R}\rangle$	$\langle g_{\rm eff,pp}^{\mathcal{Z},d_R}\rangle$	$\langle g_{\rm eff,pp}^{\mathcal{Z},\ell_R}\rangle$
$\langle g_{\rm eff}^{\mathcal{Z},\psi}\rangle^2$	14/5.5	-27/-11	-9.1/-3.6
$\tilde{C}_{HB}\,C_{HWB}$	-0.21/0.39	0.10/-0.19	0.31/-0.58
$\tilde{C}^2_{HD}$	0.28/-0.026	-0.14/0.013	-0.42/0.040
$\tilde{C}_{HD}\,\tilde{C}^{(6)}_{H\psi}$	-0.83/-0.19	-0.83/-0.19	-0.83/-0.19
$\tilde{C}_{HD}\,\tilde{C}_{HWB}$	0.59/-0.19	-0.29/0.097	-0.88/0.29
$\tilde{C}_{HD}\langle g_{\rm eff}^{\mathcal{Z},\psi}\rangle$	4.0/0.50	4.0/0.50	4.0/0.50
$(\tilde{C}^{(6)}_{H\psi})^2$	0.62/1.4	-1.2/-2.8	-0.42/-0.93
$\tilde{C}_{HWB}\,\tilde{C}^{(6)}_{H\psi}$	-0.69/0.58	-0.69/0.58	-0.69/0.58
$\tilde{C}^{(6)}_{H\psi}\langle g_{\rm eff}^{\mathcal{Z},\psi}\rangle$	-6.7/-5.8	13/12	4.5/3.9
$\tilde{C}_{HWB}\langle g_{\rm eff}^{\mathcal{Z},\psi}\rangle$	3.7/0.26	3.7/0.26	3.7/0.26
$\tilde{C}_{HW}\,C_{HWB}$	-0.21/0.39	0.10/-0.19	0.31/-0.58
$\tilde{C}^{(8)}_{HD}$	-0.014/0.026	0.0069/-0.013	0.021/-0.040
$\tilde{C}^{(8)}_{HD,2}$	-0.21/0.026	0.10/-0.013	0.31/-0.040
$\tilde{C}^{(8)}_{H\psi}$	0.19/0.19	0.19/0.19	0.19/0.19
$\tilde{C}^{(8)}_{HW,2}$	-0.38/0	0.19/0	0.58/0
$\tilde{C}^{(8)}_{HWB}$	-0.10/0.19	0.051/-0.097	0.15/-0.29
$\delta G^{(8)}$	-0.078/0.15	0.039/-0.075	0.12/-0.22

Combining SM loops with $\mathcal{O}(1/\Lambda^4)$

Can combine with $\mathcal{O}(1/\Lambda^2) \times \text{SM loop}$

$$
\begin{aligned}
\frac{\Gamma^{\hat{m}_W}_{SMEFT}}{\Gamma^{\hat{m}_W}_{\rm SM}} \simeq 1 & - 788 f_1^{\hat{m}_W}, \\
& + 394^2 \, (f_1^{\hat{m}_W})^2 - 351 \, (\tilde{C}^{(6)}_{HW} - \tilde{C}^{(6)}_{HB}) \, f_3^{\hat{m}_W} + 2228 \, \delta G_F^{(6)} \, f_1^{\hat{m}_W}, \\
& + 979 \, \tilde{C}^{(6)}_{HD} (\tilde{C}^{(6)}_{HB} + 0.80 \, \tilde{C}^{(6)}_{HW} - 1.02 \, \tilde{C}^{(6)}_{HWB}) - 788 \left[\left(\tilde{C}^{(6)}_{H\Box} - \frac{\tilde{C}^{(6)}_{HD}}{4} \right) f_1^{\hat{m}_W} + f_2^{\hat{m}_W} \right], \\
& + 2283 \, \tilde{C}^{(6)}_{HWB} (\tilde{C}^{(6)}_{HB} + 0.66 \, \tilde{C}^{(6)}_{HW} - 0.88 \, \tilde{C}^{(6)}_{HWB}) - 1224 \, (f_1^{\hat{m}_W})^2, \\
& - 117 \, \tilde{C}^{(6)}_{HB} - 23 \, \tilde{C}^{(6)}_{HW} + \left[51 + 2 \log \left(\frac{\hat{m}_h^2}{\Lambda^2} \right) \right] \tilde{C}^{(6)}_{HWB} + \left[-0.55 + 3.6 \log \left(\frac{\hat{m}_h^2}{\Lambda^2} \right) \right] \tilde{C}^{(6)}_{W}, \\
& + \left[27 - 28 \log \left(\frac{\hat{m}_h^2}{\Lambda^2} \right) \right] {\rm Re} \, \tilde{\rm C}^{(6)}_{\substack{uB \\ 33}} + 5.5 {\rm Re} \, \tilde{\rm C}^{(6)}_{\substack{uH \\ 33}} + 2 \, \tilde{\rm C}^{(6)}_{\rm H\Box} - \frac{\tilde{\rm C}^{(6)}_{\rm HD}}{2}, \\
& - 3.2 \, \tilde{C}^{(6)}_{HD} - 7.5 \, \tilde{C}^{(6)}_{HWB} - 3 \sqrt{2} \, \delta G_F^{(6)}.
\end{aligned}
$$

$\mathcal{O}(1/\Lambda^2)$

$\mathcal{O}(1/\Lambda^4)$

$\mathcal{O}(1/\Lambda^2 \times \text{loop})$

where

$$
\begin{aligned}
\delta G_F^{(6)} &= \frac{1}{\sqrt{2}} \left(\tilde{C}^{(3)}_{\substack{Hl \\ ee}} + \tilde{C}^{(3)}_{\substack{Hl \\ \mu\mu}} - \frac{1}{2} (\tilde{C}'_{\substack{ll \\ \mu ee \mu}} + \tilde{C}'_{\substack{ll \\ e \mu\mu e}}) \right), \\
f_1^{\hat{m}_W} &= \left[\tilde{C}^{(6)}_{HB} + 0.29 \, \tilde{C}^{(6)}_{HW} - 0.54 \, \tilde{C}^{(6)}_{HWB} \right], \\
f_2^{\hat{m}_W} &= \left[\tilde{C}^{(8)}_{HB} + 0.29 \, (\tilde{C}^{(8)}_{HW} + \tilde{C}^{(8)}_{HW,2}) - 0.54 \, \tilde{C}^{(8)}_{HWB} \right], \\
f_3^{\hat{m}_W} &= \left[\tilde{C}^{(6)}_{HW} - \tilde{C}^{(6)}_{HB} - 0.66 \, \tilde{C}^{(6)}_{HWB} \right],
\end{aligned}
$$

Combined result informs on how assumptions about coefficients affect uncertainty

[Corbett, AM, Trott 2107.07470]

Combining SM loops with $\mathcal{O}(1/\Lambda^4)$

Can combine with $\mathcal{O}(1/\Lambda^2) \times \mathrm{SM\,loop}$

Coefficient choice: i.e. $C_{GH}^{(6)}$ vs. $g_3^2\, C_{GH}^{(6)}$ intertwines loop and SMEFT expansions!

$\mathcal{O}(1/\Lambda^2)$

$\mathcal{O}(1/\Lambda^4)$

$\mathcal{O}(1/\Lambda^2 \times \mathrm{loop})$

$$
\begin{aligned}
\frac{\Gamma^{\hat{m}_W}_{SMEFT}}{\Gamma^{\hat{m}_W}_{\mathrm{SM}}} \simeq\ & 1 - 788\, f_1^{\hat{m}_W}, \\
& + 394^2\, (f_1^{\hat{m}_W})^2 - 351\, (\tilde{C}^{(6)}_{HW} - \tilde{C}^{(6)}_{HB})\, f_3^{\hat{m}_W} + 2228\, \delta G_F^{(6)}\, f_1^{\hat{m}_W}, \\
& + 979\, \tilde{C}^{(6)}_{HD} (\tilde{C}^{(6)}_{HB} + 0.80\, \tilde{C}^{(6)}_{HW} - 1.02\, \tilde{C}^{(6)}_{HWB}) - 788 \left[\left(\tilde{C}^{(6)}_{H\Box} - \frac{\tilde{C}^{(6)}_{HD}}{4} \right) f_1^{\hat{m}_W} + f_2^{\hat{m}_W} \right], \\
& + 2283\, \tilde{C}^{(6)}_{HWB} (\tilde{C}^{(6)}_{HB} + 0.66\, \tilde{C}^{(6)}_{HW} - 0.88\, \tilde{C}^{(6)}_{HWB}) - 1224\, (f_1^{\hat{m}_W})^2, \\
& - 117\, \tilde{C}^{(6)}_{HB} - 23\, \tilde{C}^{(6)}_{HW} + \left[51 + 2 \log\left(\frac{\hat{m}_h^2}{\Lambda^2} \right) \right] \tilde{C}^{(6)}_{HWB} + \left[-0.55 + 3.6 \log\left(\frac{\hat{m}_h^2}{\Lambda^2} \right) \right] \tilde{C}^{(6)}_{W}, \\
& + \left[27 - 28 \log\left(\frac{\hat{m}_h^2}{\Lambda^2} \right) \right] \mathrm{Re}\, \tilde{C}^{(6)}_{\substack{uB \\ 33}} + 5.5 \mathrm{Re}\, \tilde{C}^{(6)}_{\substack{uH \\ 33}} + 2\, \tilde{C}^{(6)}_{H\Box} - \frac{\tilde{C}^{(6)}_{HD}}{2}, \\
& - 3.2\, \tilde{C}^{(6)}_{HD} - 7.5\, \tilde{C}^{(6)}_{HWB} - 3\sqrt{2}\, \delta G_F^{(6)}.
\end{aligned}
$$

where

$$
\begin{aligned}
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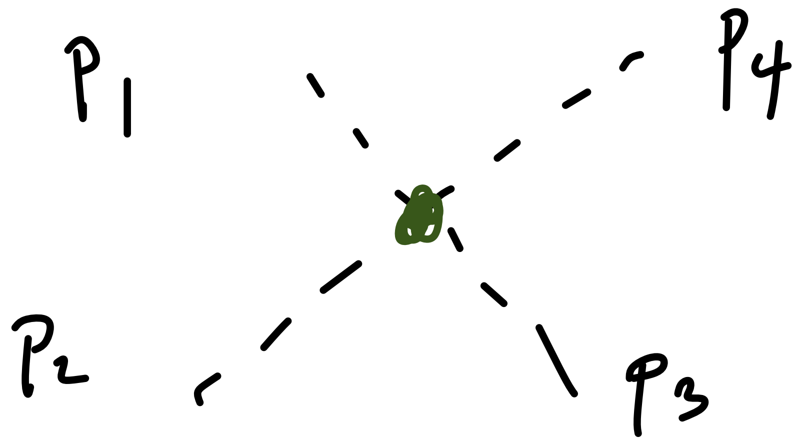
Combined result informs on how assumptions about coefficients affect uncertainty

[Corbett, AM, Trott 2107.07470]

4-pt interactions: can we go ‘full metric’?



Key part of 2- and 3-pt result is that special kinematics made all momentum products trivial

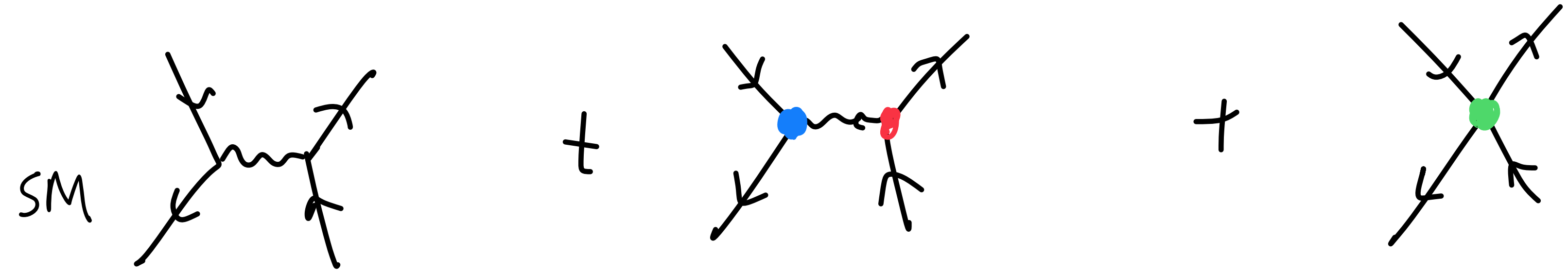
No longer true at $\geq$ 4-pt interactions, i.e. for 4-pt: $\mathcal{O} \sim s^n\, t^m$

$\longrightarrow$ infinite set of higher derivative operators can contribute

Effects must be added in by hand. But — dim-8 effects enter $\mathcal{O}(1/\Lambda^4)$ by interfering with SM, therefore need to match SM helicity/color/flavor structure

In practice means # of `by-hand’ operators is small for many relevant n = 4 processes

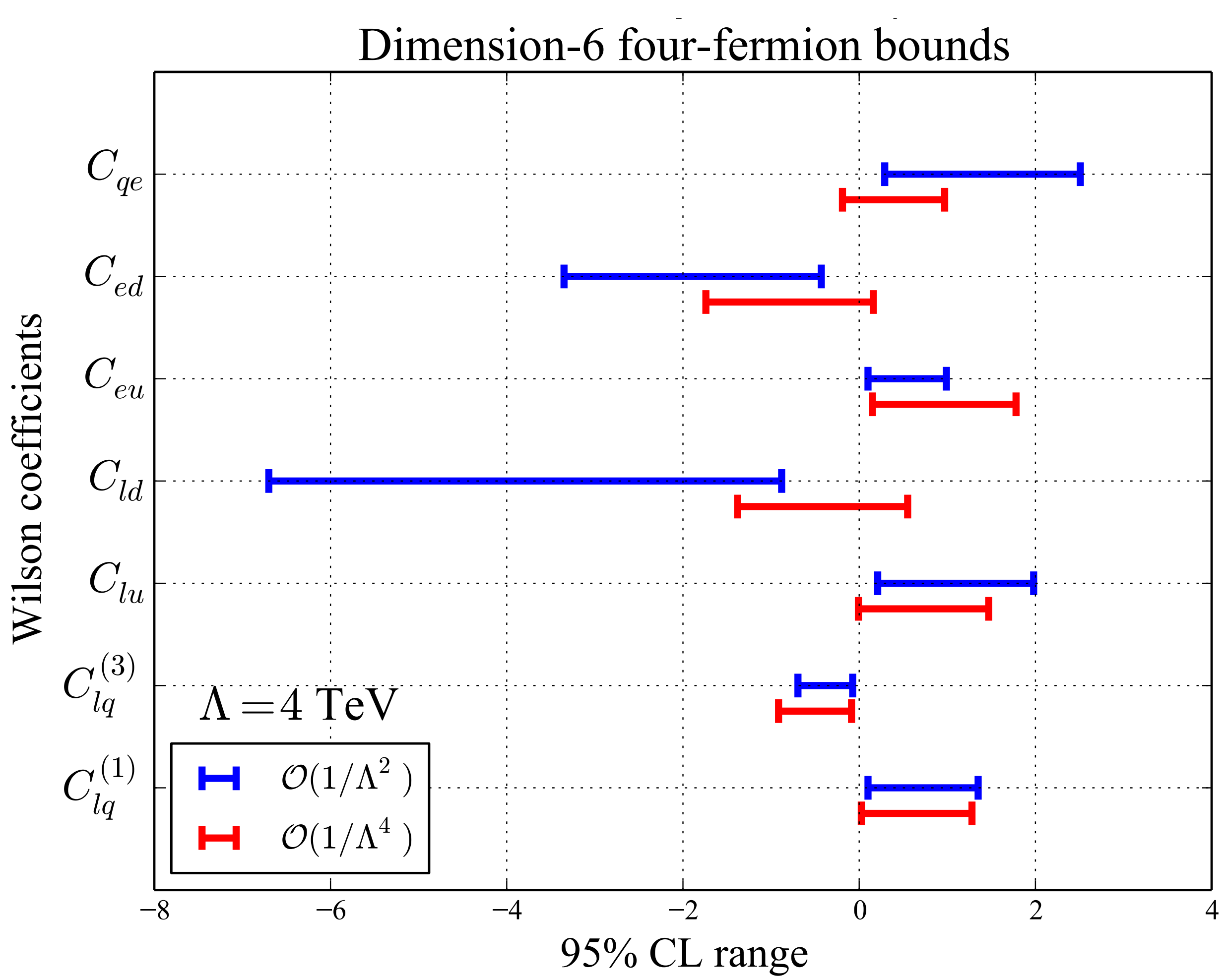
Ex. $pp \to \ell^+\ell^-, \ell^\pm\nu$ to $\mathcal{O}(1/\Lambda^4)$



3pt — in geoSMEFT

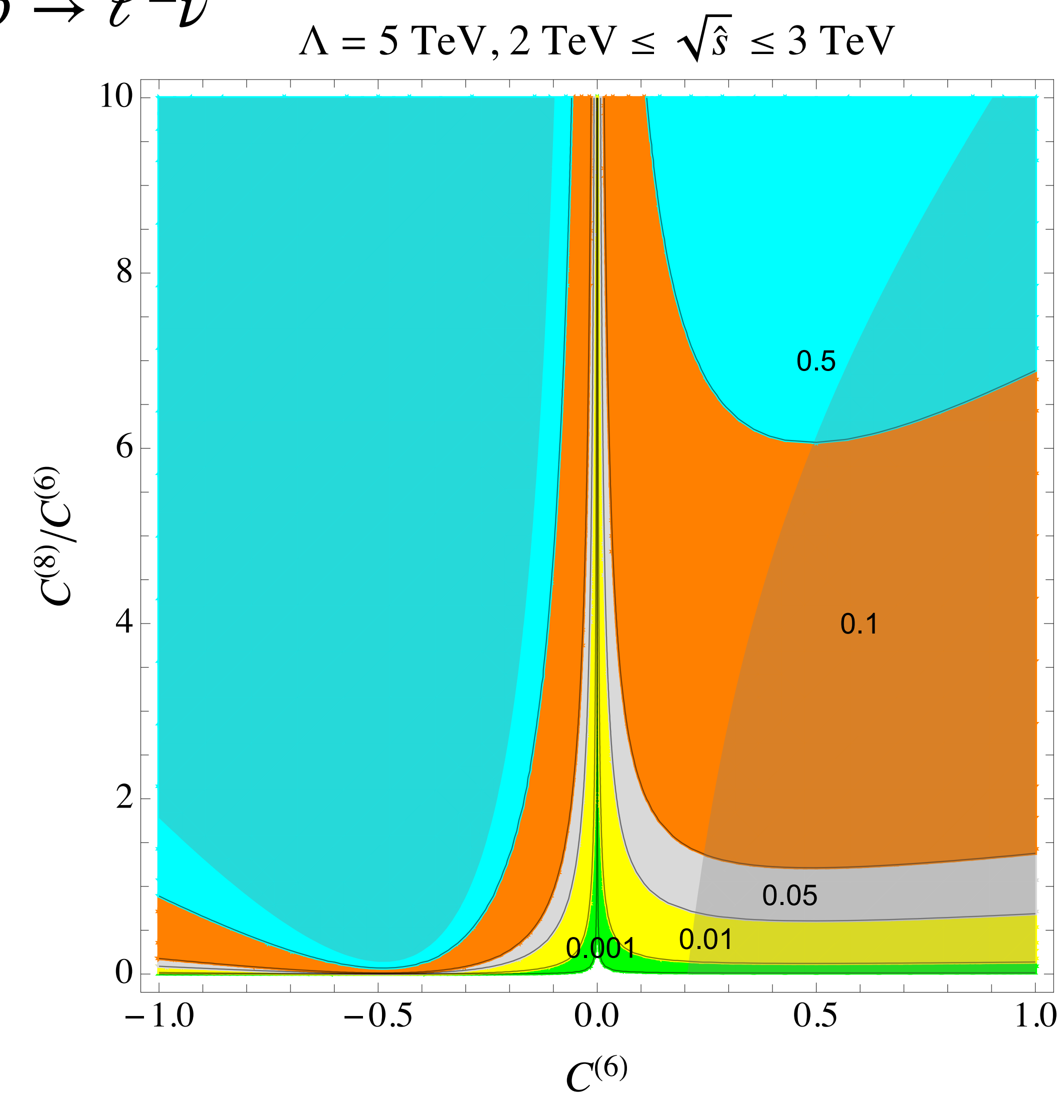
new at 4-pt, $\mathcal{O}(10)$ operators at $1/\Lambda^4$

$pp \to \ell^+\ell^-$



[Boughezal el al 2106.05337]

$pp \to \ell^\pm\nu$



[Kim, AM 2203.11976]

[see also Boughezal et al 2207.01703, Allwicher et al 2207.10714]

Roadmap for truncation studies

Library of process known to $\mathcal{O}(1/\Lambda^4)$

$1 \to 2$, Resonant $2 \to 2$: $gg \to h \to \gamma\gamma$, $e^+e^- \to Z \to \bar{f}f$

Drell-Yan, $pp \to Wh$ known; Zh, diboson in progress

[Corbett, Kim, AM]

I've focused on 'bottom up' analyses, but top down also important

[Dawson et al 2110.06929, 2205.01561]

New process:

- geoSMEFT pieces have same kinematics at dim 6 and 8

∴ can capture many effects by reweighing:

In MG already via SMEFTsim/ SMEFT@NLO

$\sigma(SM \times$ **dim-6)**

$$\frac{\textbf{couplings at } 1/\Lambda^4}{\textbf{couplings at } 1/\Lambda^2}$$

Known analytically

- Only need to add contact terms/novel kinematics

So where does this leave us?

- geoSMEFT: approach where 2 and 3 particle vertices sensitive to a minimal # of operators, # ~ constant with mass dimension. Physics with 2-, 3-particle vertices doable to any order in v/Λ (tree level)

- Can study select processes to $1/\Lambda^4$, use them to form guidelines for how to include truncation error more generally in SMEFT studies

Lots to do:

- Encapsulate what we've learned into a truncation uncertainty/uncertainties to hand off to experimentalists

[ex. AM, Trott 2109.05595]

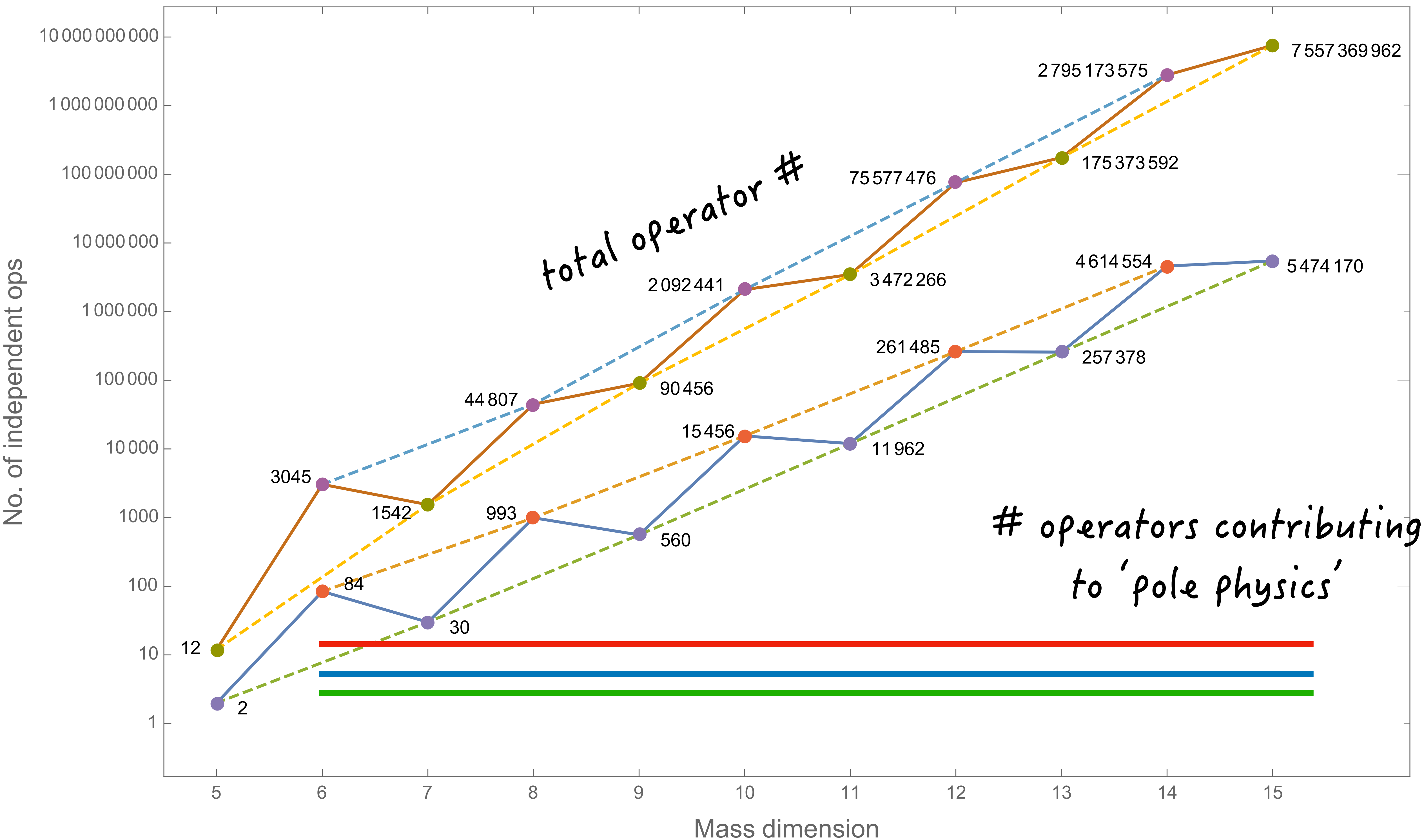
- How to pin down new coefficients (e.g. remove flat directions)?

[Alioli et al 2003.11615, Boughezal et al 2104.03979, 2207.01703]

Thank you!

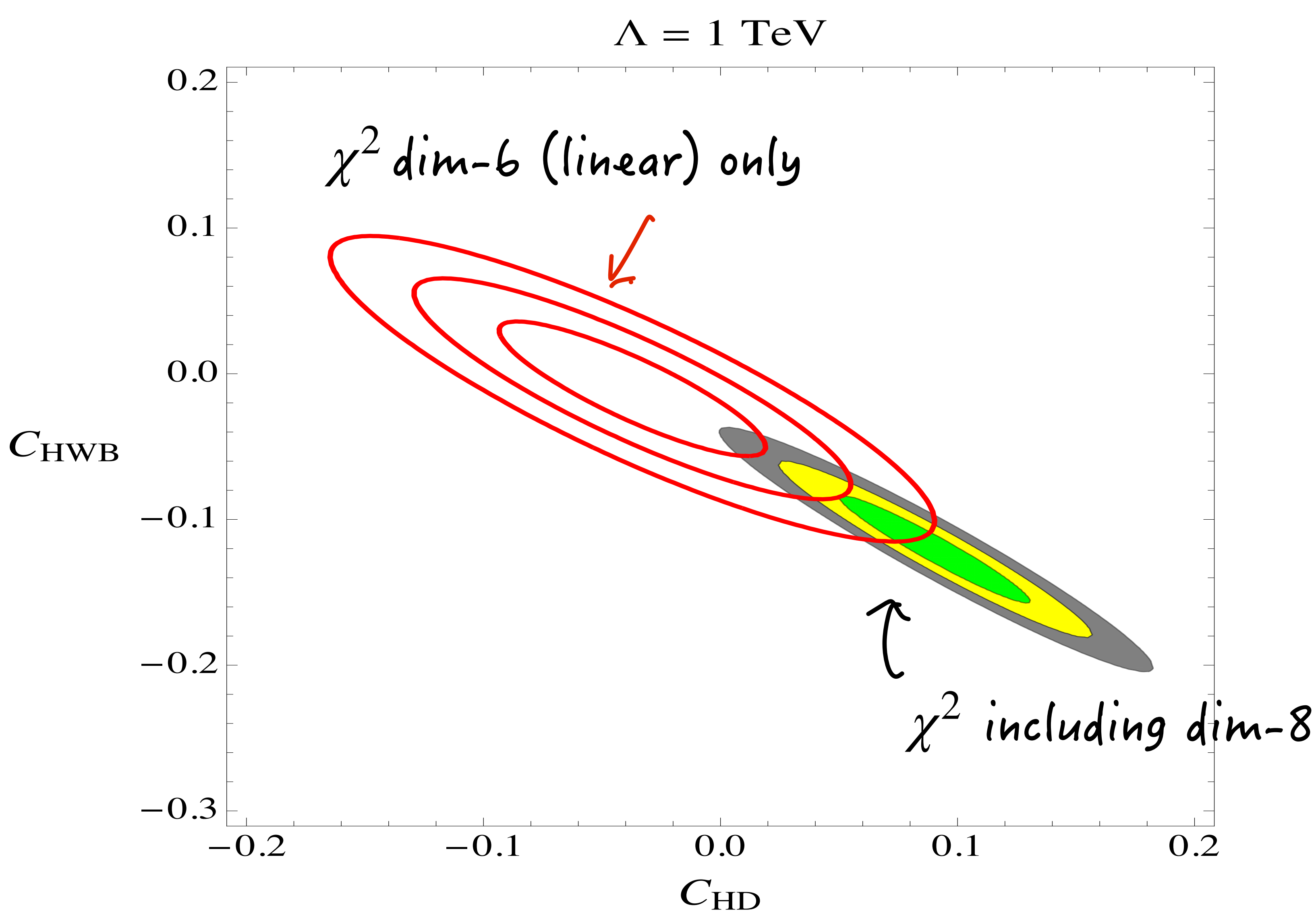
Geometric SMEFT:

[Henning et al 1512.03433]



Redo classic SMEFT LEP1 analysis to $\mathcal{O}(1/\Lambda^4)$

E.g. try classic S-T plot: Zero all dimension-6 operators except C_{HD} ~ T, C_{HWB} ~ S but leave all dimension-8 on. Set all dimension-8 coefficients to 1 (tree) or 0.01 (loop) and fix Λ, then compare χ^2 ellipses with and without dimension-8 terms



can repeat for other 2-d projections

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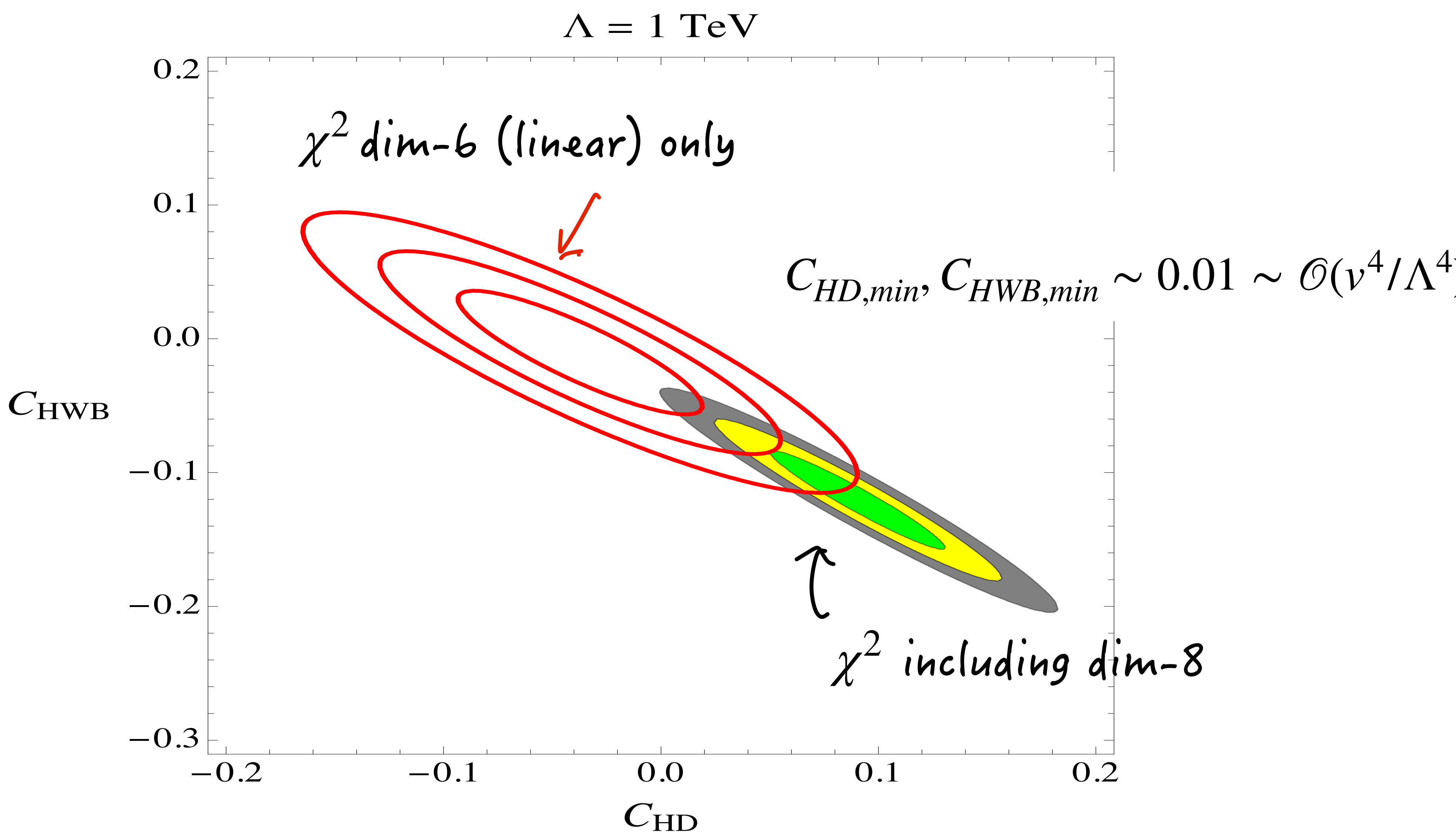


can repeat for other 2-d projections

Kinetic mixing model

Try a specific UV model: kinetically mixed U(1)

$$\Delta\mathscr{L} = -\frac{1}{4}K_{\mu\nu}K^{\mu\nu} + \frac{1}{2}m_K^2 K_\mu K^\mu - \frac{k}{2}B^{\mu\nu}K_{\mu\nu}$$

integrate out to dim-8 (tree level only)

$$\Delta\mathscr{L} = -\frac{k^2}{2m_K^2}j_\mu j^\mu + \frac{k^2 - k^4}{2m_K^4}\left(\partial^2 j_\mu\right)j^\mu + \frac{g_1^2 k^4}{4m_K^4}\left(H^\dagger H\right)j_\mu j^\mu$$

where

$$j_\mu = \sum_\psi \left(-g_1 \mathbf{y}_\psi\right)\bar{\psi}\gamma_\mu\psi + \left(-\frac{1}{2}g_1\right)H^\dagger i D_\mu H$$

Kinetic mixing model

dim-6

$H^2\psi^2 D$

$C_{H\ell}^{1,(6)}$	$-\frac{y_\ell g_1^2}{2m_K^2} b_1$
$C_{He}^{(6)}$	$-\frac{y_e g_1^2}{2m_K^2} b_1$
$C_{Hq}^{1,(6)}$	$-\frac{y_q g_1^2}{2m_K^2} b_1$
$C_{Hu}^{(6)}$	$-\frac{y_u g_1^2}{2m_K^2} b_1$
$C_{Hd}^{(6)}$	$-\frac{y_d g_1^2}{2m_K^2} b_1$

$H^4 D^2$

$C_{H\Box}^{(6)}$	$-\frac{g_1^2 k^2}{8m_K^2}$
$C_{HD}^{(6)}$	$-\frac{g_1^2 k^2}{2m_K^2}$

$\psi^4 : (\bar{L}L)(\bar{L}L)$

$C_{\ell\ell}^{(6)}$	$-\frac{1}{8}\frac{g_1^2 k^2}{m_K^2}$
$C_{qq}^{1,(6)}$	$-\frac{1}{72}\frac{g_1^2 k^2}{m_K^2}$
$C_{\ell q}^{1,(6)}$	$\frac{1}{12}\frac{g_1^2 k^2}{m_K^2}$

...

dim-8

$H^4\psi^2 D$

$C_{H\ell}^{1,(8)}$	$\frac{y_\ell g_1^4}{4\,m_K^4}\,k^4 - \frac{g_1^2\,y_\ell}{m_K^4}(k^2-k^4)(2\lambda+\frac{g_1^2+g_2^2}{4})$
$C_{He}^{1,(8)}$	$\frac{y_e g_1^4}{4\,m_K^4}\,k^4 - \frac{g_1^2\,y_e}{m_K^4}(k^2-k^4)(2\lambda+\frac{g_1^2+g_2^2}{4})$
$C_{Hq}^{1,(8)}$	$\frac{y_q g_1^4}{4\,m_K^4}\,k^4 - \frac{g_1^2\,y_q}{m_K^4}(k^2-k^4)(2\lambda+\frac{g_1^2+g_2^2}{4})$
$C_{Hu}^{1,(8)}$	$\frac{y_u g_1^4}{4\,m_K^4}\,k^4 - \frac{g_1^2\,y_u}{m_K^4}(k^2-k^4)(2\lambda+\frac{g_1^2+g_2^2}{4})$
$C_{Hd}^{1,(8)}$	$\frac{y_d g_1^4}{4\,m_K^4}\,k^4 - \frac{g_1^2\,y_d}{m_K^4}(k^2-k^4)(2\lambda+\frac{g_1^2+g_2^2}{4})$
$C_{H\ell}^{2,(8)}$	$-\frac{g_1^2\,g_2^2}{16\,m_K^4}(k^2-k^4)$
$C_{Hq}^{2,(8)}$	$-\frac{g_1^2\,g_2^2}{16\,m_K^4}(k^2-k^4)$
$C_{H\ell}^{3,(8)}$	$-\frac{g_1^2\,g_2^2}{16\,m_K^4}(k^2-k^4)$
$C_{Hq}^{3,(8)}$	$-\frac{g_1^2\,g_2^2}{16\,m_K^4}(k^2-k^4)$

$H^6 D^2$

$C_{H,D2}^{(8)}$	$\frac{g_1^4\,k^4}{8\,m_K^4} - \frac{g_1^2\,g_2^2}{2\,m_K^4}(k^2-k^4)$
$C_{HD}^{(8)}$	$\frac{3\,g_1^4\,k^4}{16\,m_K^4} - \frac{g_1^2\,g_2^2}{2\,m_K^4}(k^2-k^4)$

$X^2 H^4$

$C_{HB}^{(8)}$	$-\frac{g_1^4}{16\,m_K^4}(k^2-k^4)$
$C_{HW}^{(8)}$	$\frac{g_1^2\,g_2^2}{16\,m_K^4}(k^2-k^4)$

No operators that impact $h \to \gamma\gamma$

operators impacting $h \to \gamma\gamma$ present

$\therefore$ at dim-6 level, no effect, while there is an effect if we go to full $1/\Lambda^4$

Example: $L_{I,A}(\phi)\bar{\psi}_1\gamma^\mu\tau_A\psi_2(D_\mu\phi)^I$

contributing operators

$$\mathcal{Q}^{1,(6+2n)}_{\substack{H\psi\\pr}} = (H^\dagger H)^n H^\dagger \, i\overleftrightarrow{D}^\mu H \bar{\psi}_p\gamma_\mu\psi_r,$$

$$\mathcal{Q}^{3,(6+2n)}_{\substack{H\psi\\pr}} = (H^\dagger H)^n \, H^\dagger \, i\overleftrightarrow{D}^\mu_a H \bar{\psi}_p\gamma_\mu\sigma_a\psi_r,$$

higher dim. versions of "class 7" operators

$$\mathcal{Q}^{2,(8+2n)}_{\substack{H\psi\\pr}} = (H^\dagger H)^n (H^\dagger\sigma_a H) \, H^\dagger \, i\overleftrightarrow{D}^\mu H \bar{\psi}_p\gamma_\mu\sigma_a\psi_r,$$

$$\mathcal{Q}^{\epsilon,(8+2n)}_{\substack{H\psi\\pr}} = \epsilon^a_{bc} \, (H^\dagger H)^n \, (H^\dagger\sigma_c H) \, H^\dagger \, i\overleftrightarrow{D}^\mu_b H \bar{\psi}_p\gamma_\mu\sigma_a\psi_r.$$

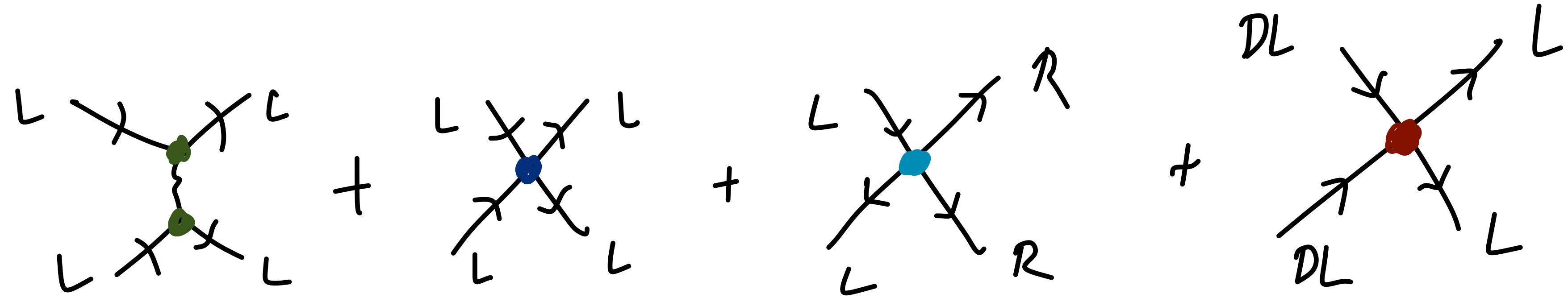
new effects from $d \geq 8$

compact form for connection:

$$\begin{aligned} L^{\psi,pr}_{J,A} = & -(\phi\,\gamma_4)_J\delta_{A4}\sum_{n=0}^{\infty} C^{1,(6+2n)}_{\substack{H\psi\\pr}}\left(\frac{\phi^2}{2}\right)^n - (\phi\,\gamma_A)_J(1-\delta_{A4})\sum_{n=0}^{\infty} C^{3,(6+2n)}_{\substack{H\psi_L\\pr}}\left(\frac{\phi^2}{2}\right)^n \\ & + \frac{1}{2}(\phi\,\gamma_4)_J(1-\delta_{A4})\left(\phi_K\Gamma^K_{A,L}\phi^L\right)\sum_{n=0}^{\infty} C^{2,(8+2n)}_{\substack{H\psi_L\\pr}}\left(\frac{\phi^2}{2}\right)^n \\ & + \frac{\epsilon^A_{BC}}{2}(\phi\,\gamma_B)_J\left(\phi_K\Gamma^K_{C,L}\phi^L\right)\sum_{n=0}^{\infty} C^{\epsilon,(8+2n)}_{\substack{H\psi_L\\pr}}\left(\frac{\phi^2}{2}\right)^n \end{aligned}$$

What about G_F?

G_F involves more than quadratic terms:



However, since G_F derived at muon mass scale $(D \sim m_\mu \ll \Lambda)$ and SM term is from L⁴, # of higher dimensional contributions is dramatically reduced

$$C^{(8+2n)}_{4\ell,2}\left(H^\dagger H\right)^{1+n}\left(\bar{\ell}_2\gamma^\mu\sigma^i\ell_2\right)\left(\bar{\ell}_1\gamma_\mu\sigma_i\ell_1\right) \qquad iC^{(8+2n)}_{4\ell,5}\epsilon_{ijk}\left(H^\dagger H\right)^n\left(H^\dagger\sigma^i H\right)\left(\bar{\ell}_2\gamma^\mu\sigma_j\ell_2\right)\left(\bar{\ell}_1\gamma_\mu\sigma_k\ell_1\right)$$

All orders result is possible even for contact terms:

$$\mathscr{G}^{4pt}_F = \frac{1}{\bar{v}_T^2}\left(\tilde{C}^{(6)}_{\mu cc\mu} + \tilde{C}^{(6)}_{\mu\mu\mu e} + \frac{\tilde{C}^{(8+2n)}_{4\ell,2}}{2^n} + \frac{\tilde{C}^{(8+2n)}_{4\ell,5}}{2^n}\right)$$

[Hays, Helset, Martin, Trott 2007.00565]

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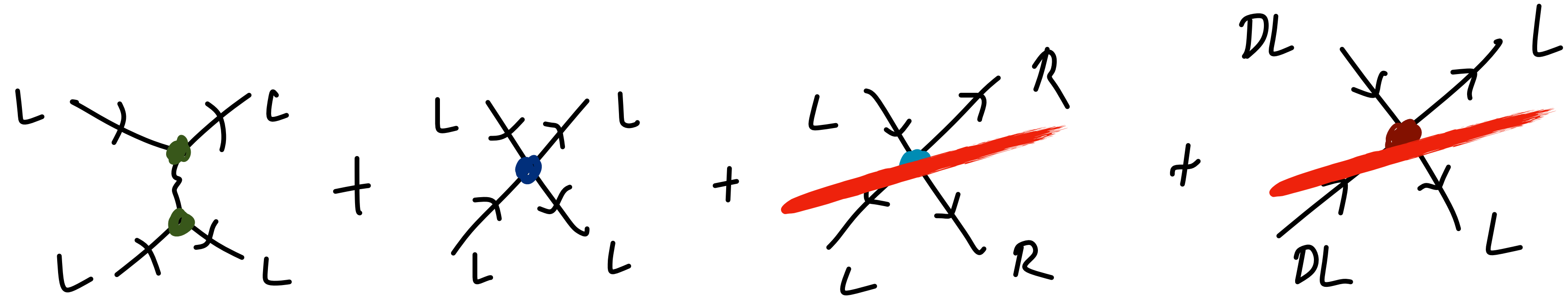

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