Recent improvements in parton shower algorithms (for Higgs production)
What is a parton shower?
What is a parton shower?

Illustrated with a dipole shower for final-state emissions

Start with some partonic state
This spans an initial ‘colour dipole’

$Z \rightarrow q\bar{q}$

starting scale of the shower

$q \quad \bar{q}$
What is a parton shower?

Illustrated with a dipole shower for final-state emissions

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This spans an initial ‘colour dipole’

Throw a random number to determine
the scale $\nu_1$ until which ‘nothing happens’

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Throw a random number to determine
the scale $v_1$ until which ‘nothing
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The state splits…
The new gluon is part of two
(independent) dipoles

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This spans an initial ‘colour dipole’

Throw a random number to determine
the scale $v_1$ until which ‘nothing happens’

The state splits…
The new gluon is part of two
(independent) dipoles

Process continues until it reaches a
non-perturbative cut-off scale

End result: set of particles and their four
momenta, from which any (well-defined) observable may be reconstructed
The splitting probability

Emission of a soft gluon: the eikonal Feynman rule

\[ \propto g_s \frac{p^\mu}{p \cdot k} T \otimes \mathcal{M} \]

- \( T \) is a colour-generator
  - Spin dependence is factorised
  - Colour dependence is not

Emission of a collinear particle: Splitting functions \( P_{(ij)a} \)

\[ \propto g_s \frac{1}{p \cdot k} P_{(ij)a}(z) \otimes \mathcal{M}_a \]

- \( \alpha \) is a spin index
  - Colour dependence is factorised
  - Spin dependence is not
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\[ \propto g_s \frac{1}{p \cdot k} P_{(ij),a}(z) \otimes M \]

- \( a \) is a spin index
- Colour dependence is factorised
- Spin dependence is not

To simplify these dependences:
Leading colour and spin-averaged (classical limit)
PS algorithms - matter of making choices

**Evolution variable $\nu$**
Which emissions come first?
$k_t\text{ ordered, angular ordered, virtuality ordered...}$

**Kinematic map**
How to go from $n$ to $n + 1$ partonic state?
*global / local momentum conservation*

**DGLAP v.s. Dipole/Antenna**

<table>
<thead>
<tr>
<th>DGLAP</th>
<th>v.s.</th>
<th>Dipole/Antenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pythia default</td>
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<td>Pythia dipole</td>
</tr>
<tr>
<td>Herwig default</td>
<td>Herwig dipole</td>
<td>Herwig dipole</td>
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<td></td>
<td>Sherpa</td>
<td>Sherpa</td>
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<td>Dire</td>
<td>Dire</td>
</tr>
<tr>
<td></td>
<td>Vincia</td>
<td>Vincia</td>
</tr>
</tbody>
</table>

**Attribution of recoil**
How to select an ‘emitter’?
dipole CM frame, event CM frame
Parton showers: a crucial ingredient

Do an amazing job at describing the phenomenology at colliders (and sometimes even beyond colliders)

Melissa van Beekveld
But differences matter…

VBF production of $h + 2j$

Focus of the talk will lie mostly on this channel…

Colour coherence strongly **suppresses** radiation in central rapidity region

Pythia’s default (global) shower unphysically fills this central region!
Progress in improving the PS accuracy

• Matching to fixed-order
  NLO; i.e. Frixione & Webber [0204244], Nason [0409146], …
  NNLO; i.e. UNNLOPS [1407.3773], MiNNLOps [1908.06987], Geneva [1311.0286], Vincia [2108.07133], …
  NNNLO; Prestel [2106.03206], + Bertone [2202.01082]

• Electroweak corrections
  Vincia [2002.09248, 2108.10786], Pythia [1401.5238], Herwig [2108.10817], …

• Triple collinear / double soft splittings
  Dulat, Höche, Krauss, Prestel [1705.00982, 1705.00742, 1805.03757, 2110.05964]
  Li & Skands [1611.00013], Plätzer & Ruffa [2012.15215], …

• Spin and colour correlations
  Deductor [0706.0017, 1401.6364, 1501.00778, 1902.02105], Herwig [1807.01955]
  PanScales [2011.10054, 2103.16526, 2111.01161], …

• Assessing the logarithmic accuracy of a shower
  Alaric [2208.06057], PanScales [1805.09327, 2002.11114, 2205.02237, 2207.09467], …

\[\text{ggF [2006.04133], VH [2112.04168]}\]

\[\text{colour-singlet (DY) [2102.08390], VH [1909.02026]}\]
Matching for VBF (Powheg-box + PS)

Matching fixes the rapidity distribution of the 3rd jet...

But we again see a huge discrepancy for four-jet observables!
Matching for VBF (Powheg-box + PS)

Matching fixes the rapidity distribution of the 3rd jet...

Important message:
Matching does not magically fix your shower

But we again see a huge discrepancy for four-jet observables!
Good agreement between NLOPS (Herwig dipole [0909.5593] +MC@NLO) and NNLOjet [1802.02445]
Agreement completely lost with higher Higgs $p_T$ cut

Consequence of large scale difference between the two LO dipoles

$Q^2_a \ll Q^2_b$
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- **Assessing the logarithmic accuracy of a shower**
  - Alaric [2208.06057], PanScales [1805.09327, 2002.11114, 2205.02237, 2207.09467], …
Electroweak effects in $h + 2j$

VBF cuts

- $p_T, \text{jet} > 25\text{GeV}$, $|y_{\text{jet}}| < 4.5$
- $m_{jj} > 600\text{GeV}$, $\Delta y_{jj} > 4.5$
- $y_{j1} \cdot y_{j2} < 0$

Designed to decouple the two topologies, but with EW corrections they are interfered.

QED shower has little impact.
Progress in improving the PS accuracy

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Subleading colour corrections - jet veto in $h + 2j$

Non-global observable: sensitive to wide-angle soft gluon emissions in restricted regions of phase space

Soft gluons are sensitive to colour flow of underlying process

i.e.

$qq \rightarrow qqH$

has an octet and a singlet channel
Subleading colour corrections - jet veto in $h + 2j$

Gap survival probability for octet channel (fixed kinematics)

Puzzling agreement between large-$N_c$ and full colour, observed in all channels!

Maybe good news for the large-$N_c$ parton showers, but need to understand what is happening here…

\[
\frac{\alpha_s}{\pi} \ln \frac{p_T}{E_{\text{out}}}
\]
Progress in improving the PS accuracy

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This is all about understanding the impact of different choices made in PS algorithms!
Addressing the accuracy of a parton shower

For a given observable, one may address the question of accuracy systematically

At fixed order

$$\sigma = \sum_n c_n \alpha_s^n = c_0 + c_1 \alpha_s + \ldots$$

At all orders using analytic resummation

$$\Sigma_{\text{NLL}}(\lambda \equiv \alpha_s L) = \exp\left(\frac{1}{\alpha_s} g_1(\lambda) + g_2(\lambda) + \ldots\right)$$

$$\Sigma_{\text{NDL}}(\xi \equiv \alpha_s \ln^2 L) = \h_1(\xi) + \sqrt{\alpha_s} \h_2(\xi) + \ldots$$

A parton shower produces an arbitrary set of final-state particles, which may be recombined into arbitrary observables!

How to address this question of accuracy?
PanScales NLL correctness requirements

**Resummation**
Require single-logarithmic accuracy for suitably defined observables
- global event shapes ($\alpha_s^n L^n$)
- parton distribution / fragmentation functions ($\alpha_s^n L^n$)
- non-global observables ($\alpha_s^n L^n$)
- particle multiplicity ($\alpha_s^n L^{2n-1}$)

**Matrix element tests**
Require correctness of effective matrix elements generated by the shower for well-separated emissions
PanScales NLL correctness requirements

Resummation
Require single-logarithmic accuracy for suitably defined observables
- global event shapes \((\alpha_s^n L^n)\)
- parton distribution / fragmentation functions \((\alpha_s^n L^n)\)
- non-global observables \((\alpha_s^n L^n)\)
- particle multiplicity \((\alpha_s^n L^{2n-1})\)

Tested by taking \[
\frac{\sum^{PS}(\alpha_s L)}{\sum^{NLL/NDL}(\alpha_s L)}
\]
PanScales NLL correctness requirements

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**Tested by taking**

\[
\frac{\Sigma_{PS}(\alpha_s L)}{\Sigma_{NLL/NDL}(\alpha_s L)}
\]

Let us try this for an arbitrary observable

---

[Dasgupta et al. PRL 125 (2020)]
[Dasgupta et al. JHEP 09 (2018) 033]
PanScales NLL correctness requirements

Resummation
Require single-logarithmic accuracy for suitably defined observables
- global event shapes \( (\alpha_s^n L^n) \)
- parton distribution / fragmentation functions \( (\alpha_s^n L^n) \)
- non-global observables \( (\alpha_s^n L^n) \)
- particle multiplicity \( (\alpha_s^n L^{2n-1}) \)

\[
\sum_{\text{PS}}(\alpha_s L) / \sum_{\text{NLL/NDL}}(\alpha_s L) \]

\[
\lim_{\alpha_s \to 0} \frac{\sum_{\text{PS}}(\alpha_s L)}{\sum_{\text{NLL/NDL}}(\alpha_s L)}
\]

(While keeping the size of \( \lambda = \alpha_s L \) fixed)

Clear deviation from 1 in the \( \alpha_s \to 0 \) limit!
Important issue in dipole showers: attribution of recoil

Standard dipole showers distinguish the emitter from the spectator at $\eta = 0$ in the CM dipole frame.

Boosting back to the event frame...

Leads to an incorrect (and quite unphysical) recoil picture!

Physical attribution of recoil

$g(\tilde{p}_i)\ \bar{q}(\tilde{p}_j)$
What is the impact of this?

Examine this question for **colour-singlet production** at the LHC

<table>
<thead>
<tr>
<th>Kinematic map</th>
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Much like Dire, Vincia, Sherpa and Pythia dipole

\[ k_t \times \exp[|\eta|/2] \]
What is the impact of this?

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**New showers**
What is the impact of this?

Examine this question for **colour-singlet production** at the LHC

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Test different choices for the kinematic map
What is the impact of this?

Examine this question for **colour-singlet production** at the LHC

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Notice difference in attribution of recoil
Transverse momentum of the Z boson

Cumulative distribution
(ratio to analytic prediction)

Both fail the NLL criterion for the transverse momentum of the Z boson!
Transverse momentum of the Z boson

Cumulative distribution (ratio to analytic prediction)

In line with NLL prediction
**Global observables**

- Leading jet transverse momentum ($p_T$), $\alpha_s \to 0$

**Non-global observables**

- Dipole-$k_t$ (local)
- Dipole-$k_t$ (global)
- PanLocal($p_T = 0.5$, antenna)
- PanLocal($p_T = 0.5$, dipole)
- PanGlobal($p_T = 0.5$)

**DGLAP evolution**

- Note: spin correlations and subleading-colour corrections are included.

**Multiplicity**

- $\sqrt{s} = 5$ $m_X$
- $\sqrt{s} = 1000$ $m_X$
Conclusions

• Different shower algorithms show **substantial** differences in their predictions

• Difference between showers often a **dominant** uncertainty in analysis

• Pythia’s default shower does not describe VBF physics, and **should not be used**

  Recommended settings for showers, matching and merging in VBF with Pythia: https://gitlab.com/Pythia8/releases/-/issues/141

• Control over logarithmic accuracy in colour-singlet production (ggF)
  Standard dipole showers (like Dire, Vincia, Sherpa and Pythia’s dipole shower) are **not NLL** accurate!

• Stay tuned for a log-study of VBF
Back up
Mapping between $\lambda$ and physical quantities

<table>
<thead>
<tr>
<th>$Q$ [GeV]</th>
<th>$\alpha_s(Q)$</th>
<th>$p_{t,\text{min}}$ [GeV]</th>
<th>$\xi = \alpha_s L^2$</th>
<th>$\lambda = \alpha_s L$</th>
<th>$\tau$</th>
</tr>
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<tr>
<td>91.2</td>
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<td>1.0</td>
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<td>4.2</td>
<td>−0.61</td>
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<tr>
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<td>3.0</td>
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<td>0.26</td>
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</tr>
</tbody>
</table>
Towards phenomenology - $\Delta \Psi_{12}$

pp, $\sqrt{s} = 13.6$ TeV, Toy PDFs, anti-$k_t(R = 0.4)$
Born: $d\phi \rightarrow Z$, $M_Z = 91.1876$ GeV, $y_2 = 0$
$20 < p_t < 30$ GeV, $0.3 < p_T/p_t < 0.5$, $y_{max} = 2.5$, $|\Delta y_{12}| > 1.0$

$$\frac{1}{N} \frac{dN}{d|\Delta \phi_{12}|}$$

Spread of NLL showers (Dipole-$k_t$ global is contained)

$$\alpha_s(x, \mu_{R,0}) \left(1 + \frac{K\alpha_s(x, \mu_{R,0})}{2\pi} + 2\alpha_s(x, \mu_{R,0})b_0(1-z)\ln x_r\right)$$
Towards phenomenology - $\Delta \Psi_{12}$

Dipole-kt global now falls outside the spread

More asymptotic regime

Less double-soft contamination
A parton shower orders emissions

The evolution variable $\nu$ tells us which emissions come first, and which later in the showering process

We use the definition $\nu \approx k_t e^{-\beta_{PS} |\eta|}$
Local kinematic map

\[ p_i = a_i \tilde{p}_i + b_i \tilde{p}_j + f k_\perp \]

\[ p_j = a_j \tilde{p}_i + b_j \tilde{p}_j + (1 - f) k_\perp \]

\[ p_k = a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp \]

Mapping coefficients depend on

- Evolution variable \( \ln v \)
- Rapidity \( \eta \)

Dipole: step function for \( f \)
Antenna: smooth transition for \( f \)
**Global kinematic map**

\[ p_i = a_i \tilde{p}_i \]

\[ p_j = b_j \tilde{p}_j \]

\[ p_k = a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp \]

Boost (part of) event after each emission to restore momentum conservation.

Choice: global in some/all \( + / - \) and \( \perp \) components.
A standard dipole shower: \textit{dipole-}k_t

1. Evolution variable: transverse momentum ($k_t$)

2. Kinematic map:
      For every emission the momentum is locally conserved
      This means that the e.g. the Z-boson $p_t$ almost never gets rescaled
      $\rightarrow$ not in line with the NLL prediction Plätzer, Gieseke [0909.5593], Nagy, Soper [0912.4534]
   
   b) Global Plätzer, Gieseke [0909.5593], Höche, Prestel [1506.05057] [\textit{Pythia8 (global ISR), Deductor and Alaric have different solutions}]
      The Z-boson absorbs the $k_t$ imbalance induced by the global map through a boost
      Claimed to fix the Z-$p_t$ distribution

3. Attribution of recoil: dipole CM frame
Introducing NLL-accurate showers for pp

**PanGlobal**

1. Evolution variable
   \[ v \approx k_t e^{-\beta_{PS} |\eta|} \] with \( 0 \leq \beta_{PS} < 1 \)
   \( (\beta_{PS} = 0 \) is standard \( k_t \)-ordering)\

2. Kinematic map
   Global \( \perp \)
   Local \( +/- \)
   Transverse-momentum imbalance is absorbed by the hard system (Z/h)

3. Attribution of recoil
   hard-system CM frame

**PanLocal**

1. Evolution variable
   \[ v \approx k_t e^{-\beta_{PS} |\eta|} \] with \( 0 < \beta_{PS} < 1 \)

2. Kinematic map
   Local \( \perp \)
   Local \( +/- \)
   Initial-state particles that gain a \( k_t \) component are realigned with the beam axis with a boost

3. Attribution of recoil
   hard-system CM frame
PanGlobal details

- Kinematic map
  \[ p_k = a_k \tilde{p}_i + b_k \tilde{p}_j + k_\perp, \quad \text{(emitted particle)} \]
  \[ p_i = a_i \tilde{p}_i \]
  \[ p_j = b_j \tilde{p}_j, \]

- Constraining the coefficients:
  1. \[ p_k^2 = p_i^2 = p_j^2 = 0 \]
  2. Relate \( a_k \) and \( b_k \) to the shower variables

- The transverse-momentum imbalance is absorbed into the FS partons in a two-step process:
  1. Rescale IS partons such that \( r^2 Q_{\text{in}}^2 = Q_{\text{out}}^2, \quad r^2 = r_ar_b \)
  2. Boost (part of the) FS such that \( \Lambda(Q_{\text{out}}) = r Q_{\text{in}} \)
PanGlobal boost

Never boost partons created by the shower, only give recoil to the ‘hard system’ $H$

• Just boost the $Z/h$

$$\Lambda(\tilde{p}_H) = r_a\tilde{p}_a + r_b\tilde{p}_b - p_k - \sum_{f \notin H} \tilde{p}_f$$

• We furthermore require

1. The mass of the hard system is preserved
2. The rapidity of the hard system is preserved

This fixes $r_a$ and $r_b$
### General global observables

#### NLL accuracy tests - $pp \rightarrow Z$

<table>
<thead>
<tr>
<th>Observable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{p,0}$</td>
<td>$S_{p,j,\beta} = \sum_{i \in \text{jets}} p_{\perp,i} e^{-\beta</td>
</tr>
<tr>
<td>$S_{j,0}$</td>
<td>$M_{j,\beta} = \max_{i \in \text{jets}} [p_{\perp,i} e^{-\beta</td>
</tr>
<tr>
<td>$M_{j,0}$</td>
<td></td>
</tr>
<tr>
<td>$S_{p,\frac{1}{2}}$</td>
<td>PanLocal ($\beta_{PS} = \frac{1}{2}, \text{dip.}$)</td>
</tr>
<tr>
<td>$S_{j,\frac{1}{2}}$</td>
<td>PanLocal ($\beta_{PS} = \frac{1}{2}, \text{ant.}$)</td>
</tr>
<tr>
<td>$M_{j,\frac{1}{2}}$</td>
<td>PanGlobal ($\beta_{PS} = 0$)</td>
</tr>
<tr>
<td>$S_{p,1}$</td>
<td>PanGlobal ($\beta_{PS} = \frac{1}{2}$)</td>
</tr>
<tr>
<td>$S_{j,1}$</td>
<td></td>
</tr>
<tr>
<td>$M_{j,1}$</td>
<td></td>
</tr>
</tbody>
</table>

$$\lim_{\alpha_s \rightarrow 0} \left[ \frac{\Sigma_{PS}}{\Sigma_{NLL}} - 1 \right] \text{ for } \lambda = \alpha_s L = -\frac{1}{2}$$
Global event shapes for $y_Z \neq 0$

NLL accuracy tests - $pp \rightarrow Z, y_Z = 2$

\[ \lim_{\alpha_s \rightarrow 0} \left[ \frac{\Sigma_{PS}}{\Sigma_{NLL}} - 1 \right] \text{ for } \lambda = \alpha_s L = -\frac{1}{2} \]
The Sudakov suppression is compensated by azimuthal cancellations at small $p_t$

Leads to a power-law fall-off

$$\frac{d\Sigma}{dp_t^2} = \int_0^\infty \frac{db}{2} J_0(bp_tZ) \Sigma_V(b_0/b)$$

Parisi, Petronzio [NPB 154 (1979) 427-440]
Parton distribution functions

\[ \frac{1}{\sigma} \frac{d\sigma_i}{dx} = \frac{1}{f_i(\hat{x}, m_Z^2)} \int_1^1 \frac{dz}{\hat{z}} D_{i\alpha}(z, \alpha_s L) f_i \left( \frac{\hat{x}}{z}, p_t^{cut} \right) \delta \left( \frac{\hat{z}}{z} - x \right) \]
Non-global observable: rapidity gap

\[ \Delta \eta \]

\[
\begin{align*}
\text{Dipole-} k_t & (\text{local}) \\
\text{Dipole-} k_t & (\text{global}) \\
\text{PanLocal}(\beta_{FS} = \frac{1}{2}, \text{dipole}) \\
\text{PanLocal}(\beta_{FS} = \frac{1}{2}, \text{antenna}) \\
\text{PanGlobal}(\beta_{FS} = 0) \\
\text{PanGlobal}(\beta_{FS} = \frac{1}{2})
\end{align*}
\]
Particle multiplicity

\[ \sqrt{s} = 5 \, M_{Z,H} \quad \text{vs.} \quad \sqrt{s} = 1000 \, M_{Z,H} \]

- Dipole-\( k_t \) (local IF)
- Dipole-\( k_t \) (global IF)
- PanGlobal (\( \beta = 0 \))
- PanGlobal (\( \beta = 1/2 \))
- PanLocal (\( \beta = 1/2, \text{dip.} \))
- PanLocal (\( \beta = 1/2, \text{ant.} \))

\[ \lim_{\alpha_s \to 0} \frac{N_{\text{shower}} - N_{\text{NDL}}}{N_{\text{NDL}} - N_{\text{DL}}} \quad \text{for} \quad \xi = \alpha_s \ln^2(M_{Z,H}/k_{t,\text{cut}}) = 5 \]
PanLocal issue for $\beta_{PS} = 0$

- Recoil is taken from the first gluon even when emissions are separated in rapidity

- Separation of dipole in event CM frame is not enough to cure dipole-showers with local maps from locality issue, the transverse momentum ordering is problematic here

- Only when emissions are ordered in angle ($\beta_{PS} > 0$) we solve this

- Then commensurate $k_t$ emissions are ordered in angle, so they take their recoil from the hard system (after boost)
Issue for $\beta_{PS} = 1$

- For IF dipoles, momentum of first emission is rescaled by $b_j = 1 - \beta_k$ in map.
- For $\beta = 1$ this equates to $1 - \frac{\tilde{s}_i}{\tilde{s}_{ij}Q}$ and becomes independent of $\bar{\eta}$.
- Consider change in first emitted parton:
  
  \[ p_{k,1} = \tilde{p}_j \rightarrow b_j p_{k,1} = \left(1 - \frac{\tilde{s}_i}{\tilde{s}_{ij}Q}\right)p_{k,1} \]

  - With $\frac{\tilde{s}_i}{\tilde{s}_{ij}} = \frac{2\tilde{p}_i \cdot Q}{2\tilde{p}_i \cdot \tilde{p}_j} = \frac{1}{b_{k,1}}$ and $b_{k,1} = \beta_{k,1} = \frac{\nu_1}{Q}$.

  \[
  \frac{k_{\perp,1}}{k_{\perp,1 \text{ after } 2}} = \left(1 - \frac{\nu_2}{\nu_1}\right)
  \]
Colour tests

Test of the differential matrix element

Here primary $\bar{q}q$ Lund plane and the new $g$ Lund leaf

LC = leading colour (standard)
FC = full colour

CFFE = standard colour treatment

Segment and NODS two ways to improve the colour handling in the PanScales showers
Colour tests

\[ I_{FC}^{Zq_1} = \int \frac{d\Omega}{2\pi} \left| \frac{\mathcal{M}_{q\bar{q}_1g_2}}{\mathcal{M}_{q\bar{q}_1g_1}} \right|^2 \]

Test of the integrated rate of emissions
Spin tests

Two collinear emissions

\[ \frac{d\sigma}{d\Delta \psi_{ij}} \propto a_0 \left( 1 + \frac{a_2}{a_0} \cos(2\Delta \psi_{ij}) \right) \]
Spin tests

One collinear, one soft emission

\[ \frac{d\sigma}{d\Delta \psi_{ij}} \propto a_0 \left( 1 + \frac{a_2}{a_0} \cos(2\Delta \psi_{ij}) \right) \]
Spin tests

\[ \frac{d\sigma}{d\Delta \psi_{13}} \propto a_0 \left( 1 + \frac{a_2}{a_0} \cos(2\Delta \psi_{13}) + \frac{b_2}{a_0} \sin(2\Delta \psi_{13}) \right) \]
Consider $M_{R,0}$, max $p_{\perp}$ of emissions in the right hemisphere (sensitive to super-leading logs at $\mathcal{O}(\alpha_s^3)$)

- Take toy-model approach with only soft primary emissions and fixed coupling.

- Take difference between CEASAR result and toy shower $\delta F_n(L)$, $n = \text{order in } \alpha_s$, where
  \[
  F = \sum_1^n \alpha_s^n F_n
  \]
  has terms of $\alpha_s^n L^m$ with $m \leq n$.

- Clearly a discrepancy at fixed-order for standard dipole showers.

- Vanishes at all orders because it is numerically comparable to the NNLL terms -> orange points.
Super-leading logarithms

- Discrepancy not there for PanScales family of showers