Higgs 2022
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Reasons for HEFT: why we may need more than SMEFT

work in collaboration with:
R. Gómez-Ambrosio,
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A. Salas-Bernárdez,
PRD 106 (2022) 5, 5, 2204.01763 [hep-ph];
2207.09848 [hep-ph]

Outline

1.) Isn't SMEFT enough?

2.) SM, SMEFT, HEFT... and geometry

3.) SMEFT ⇔ HEFT connection: **potential issues**

4.) Conclusions

1) Isn’t SMEFT enough?

- **SM:**
  - Complex doublet H
  - Renormalizable (canonical dim. $D \leq 4$)

$$\mathcal{L}_{SM} = \mathcal{L}_{D\leq 4}$$

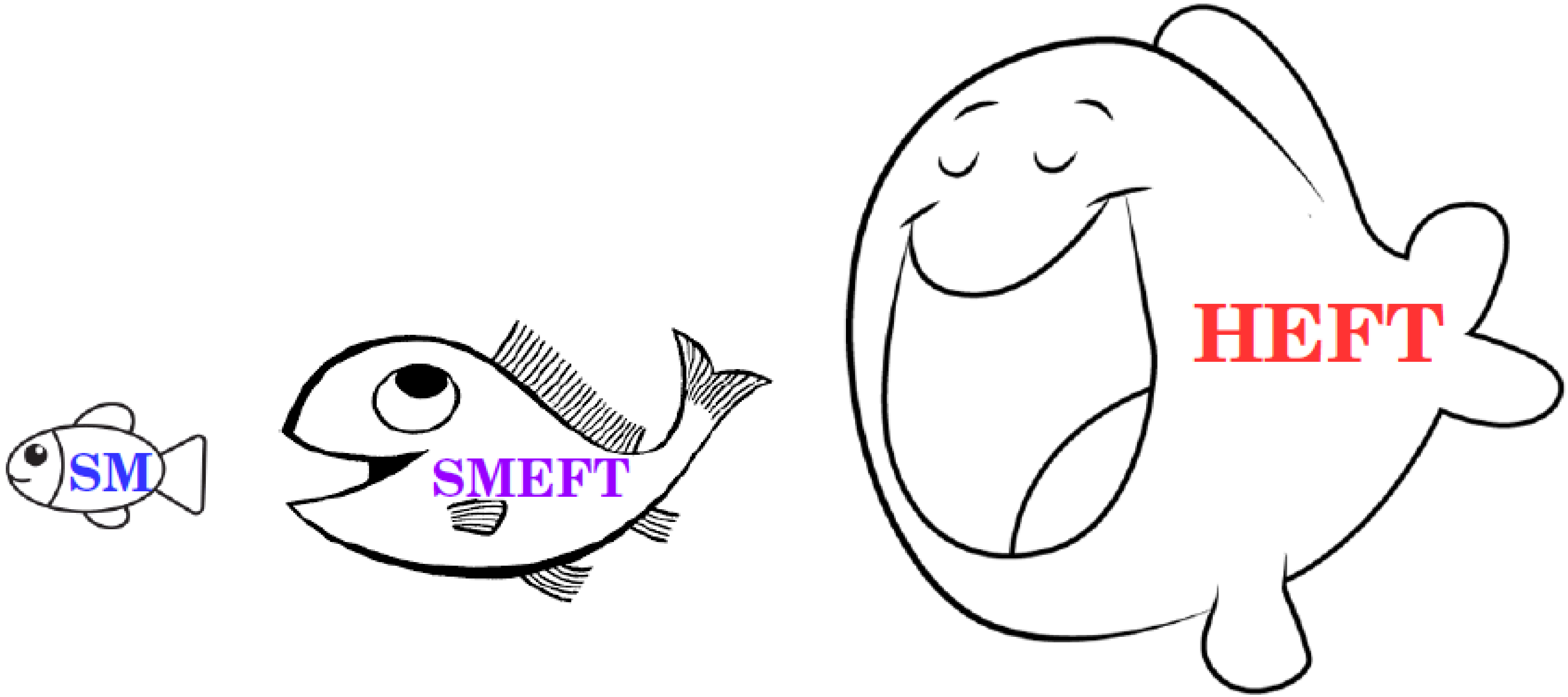
- **SMEFT:**
  - Complex doublet H
  - Non-renormalizable (canonical dim. expan.)

$$\mathscr{L}_{SMEFT} = \mathscr{L}_{SM} + \sum_{n,i} \frac{c_i^{(n)}}{\Lambda^{n-4}} \mathcal{O}_i^{(n)}$$

- **HEFT** (= EWChL = EWET)
  - 3 EW Goldstones + 1 singlet Higgs h (indep.)
  - Non-renormalizable (chiral expan.)

$$\mathcal{L}_{HEFT} = \mathcal{L}_{p^2} + \mathcal{L}_{p^4} + \ldots$$

[w/ $\mathcal{L}_{SM} \subset \mathcal{L}_{p^2}$]



[by R. Gómez-Ambrosio]

What is the standard (misleading) claim?

"To this day LHC data is consistent with a Higgs boson doublet as is introduced in the Standard Model.
As a consequence, the possibility of nonlinear effects does not currently draw major interest"

What is the implicit claim?

"Why should we care about nonlinear effects?
Small experimental deviations from SM $\Rightarrow$ SMEFT will be good enough"

I hope I may convince you these statements are false (*)

(*) Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, PRD 106 (2022) 5, 5; arXiv: 2207.09848 [hep-ph]

The SM is falsified
by finding a non-zero Wilson Coefficient

How is the SMEFT falsified?

SMEFT vs HEFT

* A deviation from the SM, if small enough, can always be parametrised by the Warsaw basis

SMEFT vs HEFT

* A deviation from the SM, if small enough, can always be parametrised by the Warsaw basis

FALSE

SMEFT vs HEFT

* A deviation from the SM, if small enough, can always be parametrised by the Warsaw basis

NOT STRICTLY TRUE

2) SM, SMEFT, HEFT… and geometry

Geometry of the scalar field

Recent works highlighting the EFT geometry

* R. Alonso, E. E. Jenkins, and A. V. Manohar,
  * “A Geometric Formulation of Higgs Effective Field Theory: Measuring the Curvature of Scalar Field Space,” Phys. Lett. B754 (2016) 335–342, arXiv:1511.00724 [hep-ph].
  * “Sigma Models with Negative Curvature,” Phys.Lett.B756,358(2016),arXiv:1602.00706 [hep-ph].
  * “Geometry of the Scalar Sector,” JHEP 08 (2016) 101, arXiv:1605.03602 [hep-ph].” (Cohen et al., 2021, p. 95)
* T. Cohen, N. Craig, X. Lu, and D. Sutherland:
  * “Is SMEFT Enough?”, JHEP 03, 237, arXiv:2008.08597 [hep-ph].
  * “Unitarity Violation and the Geometry of Higgs EFTs”, (2021), arXiv:2108.03240 [hep- ph].

we now know that HEFT and SMEFT can be understood geometrically

Old works in 80’s:

Boulware,Brown, Annals Phys. 138 (1982) 392

[thanks to M. Knecht for calling out my attention to this]

• Beautiful geometric connection to scalar loop corrections [(*)] provided by the curvature [(x)] of the scalar manifold metric $g_{ij}(\phi) = \begin{bmatrix} F(h)^2 g_{ab}(\varphi) & 0 \\ 0 & 1 \end{bmatrix}$, with $\mathcal{L} = \frac{1}{2} g_{ij} D_\mu \phi^i D^\mu \phi^j$

$$\mathcal{R}_4 = \left(1 - v^2 (F')^2\right) F^2 = \left(1 - \mathcal{K}^2/4\right) \mathcal{F}_C \,,$$

$$\mathcal{R}_2 = \left(1 - v^2 (F')^2\right) - \frac{v^2 F'' F}{(N_\varphi - 1)} = \left(1 - \mathcal{K}^2/4\right) - \frac{\mathcal{F}_C \Omega}{8} \,,$$

$$\mathcal{R}_0 = 2\mathcal{F}_C^{-1} \mathcal{R}_2 - \mathcal{F}_C^{-2} \mathcal{R}_4 \,,$$

$$F = \mathcal{F}_C^{1/2} \qquad N_\varphi = 3$$

with Λ^{-2} = the Riemann $\mathbb{R}_{ijmn} \propto \mathscr{R}_{0,2,4} / v^2$ *(loosely speaking, the curvature $\mathbb{R}$)*

- NDA gives you the suppression of individual diagrams ~ $1/(4\pi v)^2$

  but the full loop suppression is ~ $g^2 \mathbb{R} / (4\pi)^2$ & ~ $\mathbb{R}^2 / (4\pi)^2$

  EFT as an expansion $\mathcal{M} \sim R\, \mathbf{p}^2 + \frac{R^2 \mathbf{p}^4}{(4\pi)^2} + \frac{R^3 \mathbf{p}^6}{(4\pi)^4} + \ldots$ in the curvature?

- **SM:** $\mathbb{R}_{ijmn} = 0$ ➔ No $O(p^4)$ renormalization

(*) Guo,Ruiz-Femenia,SC, PRD92 (2015) 074005

(x) Alonso,Jenkins,Manohar, PLB754 (2016) 335; PLB756 (2016) 358; JHEP 1608 (2016) 101

3) SMEFT⇔HEFT connection:

potential issues

SMEFT ⇔ HEFT connection

"Two" EW EFT candidates

- Standard Model Effective Field Theory (SMEFT)

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_{n=5}^{\infty} \sum_i \frac{c_i^{(n)}}{\Lambda^{n-4}} \mathcal{O}_i^{(n)}(H)\ .$$

- Higgs Effective Field Theory (HEFT):
  Chiral Lagrangian

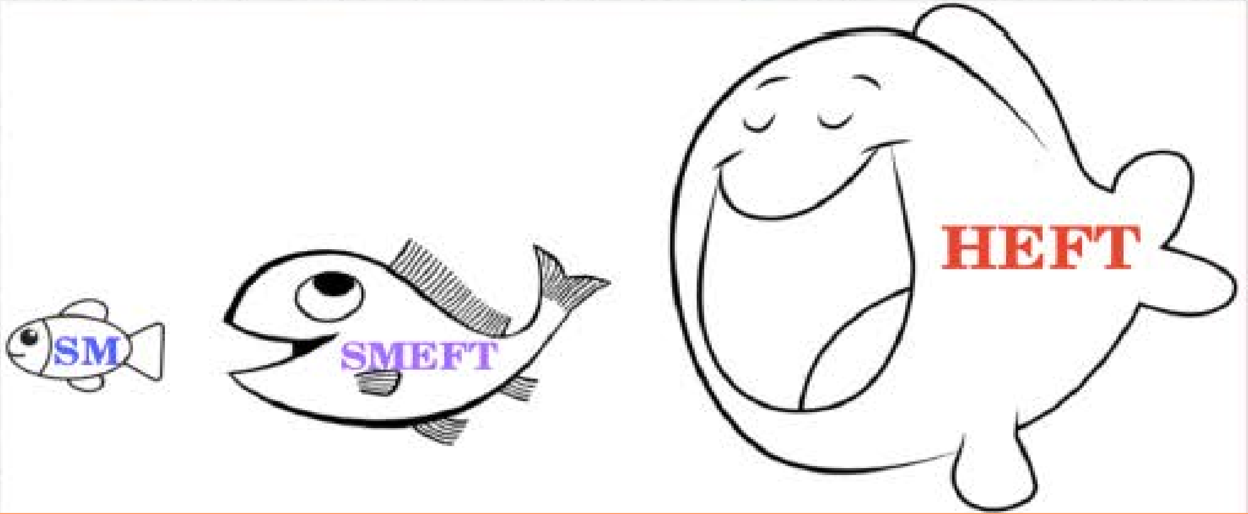
$$\mathcal{L}_{\rm HEFT} = \frac{1}{2}\partial_\mu h \partial^\mu h - V(h) + \frac{1}{2}\mathcal{F}(h)\partial_\mu \omega^i \partial^\mu \omega^j \left(\delta_{ij} + \frac{\omega^i \omega^j}{v^2 - \omega^2}\right)$$

$+ \ldots$

What is their relation?

SMEFT ⇒ HEFT connection

- In HEFT: $\mathcal{F}(h)_{HEFT} = 1 + a_1 \frac{h}{v} + a_2 \left(\frac{h}{v}\right)^2 + a_3 \left(\frac{h}{v}\right)^3 + \ldots$
- In the SM: $\mathcal{F}(h)_{SM} = \left(1 + \frac{h}{v}\right)^2$
- In SMEFT?



$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ (v + h_{\rm SMEFT}) + i\phi_3 \end{pmatrix}$$

Relevant SMEFT at the TeV scale:

$$\mathcal{L}_{\rm SMEFT} = |\partial H|^2 + \frac{c_{H\Box}}{\Lambda^2}(H^\dagger H)\Box(H^\dagger H)$$

To polar-like coordinates:

$$\mathcal{L}_{\rm SMEFT} = \frac{1}{2}\left(1 - 2(v+h)^2 \frac{c_{H\Box}}{\Lambda^2}\right)(\partial_\mu h)^2 + \frac{1}{2}(v+h)^2(\partial_\mu \boldsymbol{n} \cdot \partial^\mu \boldsymbol{n})$$

- SMEFT in the *HEFT-form*[(x)] looks like...

$$\mathcal{L}_{\rm SMEFT} = \frac{v^2}{4}\mathcal{F}(h_1)\langle D_\mu U^\dagger D^\mu U\rangle + \frac{1}{2}(\partial_\mu h_1)^2 - V(h) - \frac{c_{H\Box}\left[(v+h_1)^3 - v^3\right]}{3\Lambda^2}V'(h_1)$$

HEFT (x)

$$\mathcal{F}(h_1) = \left(1+\frac{h_1}{v}\right)^2 + \frac{2v^3 c_{H\Box}}{\Lambda^2}\left(1+\frac{h_1}{v}\right)\left(\frac{h_1^3}{3v^3}+\frac{h_1^2}{v^2}+\frac{h_1}{v}\right) + \mathcal{O}\left(\frac{c_{H\Box}^2}{\Lambda^4}\right) =$$

$$= 1 + \left(\frac{h_1}{v}\right)\left(2+2\frac{c_{H\Box}v^2}{\Lambda^2}\right) + \left(\frac{h_1}{v}\right)^2\left(1+4\frac{c_{H\Box}v^2}{\Lambda^2}\right) +$$

$$+ \left(\frac{h_1}{v}\right)^3\left(8\frac{c_{H\Box}v^2}{3\Lambda^2}\right) + \left(\frac{h_1}{v}\right)^4\left(2\frac{c_{H\Box}v^2}{3\Lambda^2}\right),$$

SMEFT correlated coeff.

$$a_1 = 2a = 2\left(1+v^2\frac{c_{H\Box}}{\Lambda^2}\right), \quad a_2 = b = 1+4v^2\frac{c_{H\Box}}{\Lambda^2}, \quad a_3 = \frac{8v^2}{3}\frac{c_{H\Box}}{\Lambda^2}, \quad a_4 = \frac{2v^2}{3}\frac{c_{H\Box}}{\Lambda^2}$$

Naturally extend to dim8 and further, and to quadratic terms

$$\begin{aligned}\mathcal{F}(h_1) = 1 &+ \left(\frac{h_1}{v}\right)\left(2 + 2\frac{c_{H\Box}^{(6)}v^2}{\Lambda^2} + 3\frac{(c_{H\Box}^{(6)})^2v^4}{\Lambda^4} + 2\frac{c_{H\Box}^{(8)}v^4}{\Lambda^4}\right) + \\ &+ \left(\frac{h_1}{v}\right)^2\left(1 + 4\frac{c_{H\Box}^{(6)}v^2}{\Lambda^2} + 12\frac{(c_{H\Box}^{(6)})^2v^4}{\Lambda^4} + 6\frac{c_{H\Box}^{(8)}v^4}{\Lambda^4}\right) + \\ &+ \left(\frac{h_1}{v}\right)^3\left(8\frac{c_{H\Box}^{(6)}v^2}{3\Lambda^2} + 56\frac{(c_{H\Box}^{(6)})^2v^4}{3\Lambda^4} + 8\frac{c_{H\Box}^{(8)}v^4}{\Lambda^4}\right) + \\ &+ \left(\frac{h_1}{v}\right)^4\left(2\frac{c_{H\Box}^{(6)}v^2}{3\Lambda^2} + 44\frac{(c_{H\Box}^{(6)})^2v^4}{3\Lambda^4} + 6\frac{c_{H\Box}^{(8)}v^4}{\Lambda^4}\right) + \\ &+ \left(\frac{h_1}{v}\right)^5\left(88\frac{(c_{H\Box}^{(6)})^2v^4}{15\Lambda^4} + 12\frac{c_{H\Box}^{(8)}v^4}{5\Lambda^4}\right) + \\ &+ \left(\frac{h_1}{v}\right)^6\left(44\frac{(c_{H\Box}^{(6)})^2v^4}{45\Lambda^4} + 2\frac{c_{H\Box}^{(8)}v^4}{5\Lambda^4}\right) + \mathcal{O}(\Lambda^{-6})\,.\end{aligned}$$

(*) Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, PRD 106 (2022) 5, 5; arXiv: 2207.09848 [hep-ph]

SMEFT $\Leftarrow$ HEFT connection

From HEFT to SMEFT one has to solve

$$h_{\rm HEFT} = \mathcal{F}^{-1}\left((1 + h_{\rm SMEFT}/v)^2\right)$$

and in order to have an analytic Lagrangian:

$$\mathcal{L}_{\rm SMEFT} = \underbrace{|\partial H|^2}_{=\mathcal{L}_{\rm SM}} + \underbrace{\frac{1}{2}\left[\frac{8|H|^2}{v^2}\left((\mathcal{F}^{-1})'\,(2|H|^2/v^2)\right)^2 - 1\right]\frac{(\partial|H|^2)^2}{2|H|^2}}_{=\Delta\mathcal{L}_{\rm BSM}}$$

Possible non-analyticity

$\Rightarrow$ Provides conditions on the derivatives of the flare function $\mathcal{F}(h)$.

$\Rightarrow$ Correlation of HEFT parameters by assuming an analytic SMEFT.

SMEFT vs HEFT: potential issues

- **Theory:**

HEFT Lagrangian becomes singular in *SMEFT-form*

(coordinates)

- **Phenomenology:**

SMEFT predicts correlations absent in experiment?

(in principle, also absent in HEFT)

• Theory:

HEFT Lagrangian becomes singular in *SMEFT-form*

(coordinates)

$$\mathcal{L}_{\rm SMEFT} = \underbrace{|\partial H|^2}_{=\mathcal{L}_{\rm SM}} \quad + \quad \underbrace{\frac{1}{2}\left[\left(\frac{1}{v}(F^{-1})'\left(\sqrt{2|H|^2/v^2}\right)\right)^2 \; - \; 1\right]\frac{(\partial|H|^2)^2}{2|H|^2}}_{=\Delta\mathcal{L}_{\rm BSM}}$$

Potentially singular term

- If we want the Lagrangian non-singular around H=0 then:

$$\boxed{\mathcal{F}(h_1^*) = F(h_1^*)^2 = 0}$$ must have a double zero.

$$\boxed{\mathcal{F}'(h_1^*) = 0\,,\ \mathcal{F}''(h_1^*) = \frac{2}{v^2}}$$

$$\boxed{\mathcal{F}'''(h_1^*) = 0}$$

$$\boxed{\mathcal{F}^{(2\ell+1)}(h_1^*) = 0}$$

(*) Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, PRD 106 (2022) 5, 5; arXiv: 2207.09848 [hep-ph]
(*) Alonso,Jenkins,Manohar, JHEP 08 (2016) 101
(*) Cohen,Craig,Lu,Sutherland, JHEP 03 (2021) 237; JHEP 12 (2021) 003

•Phenomenology:

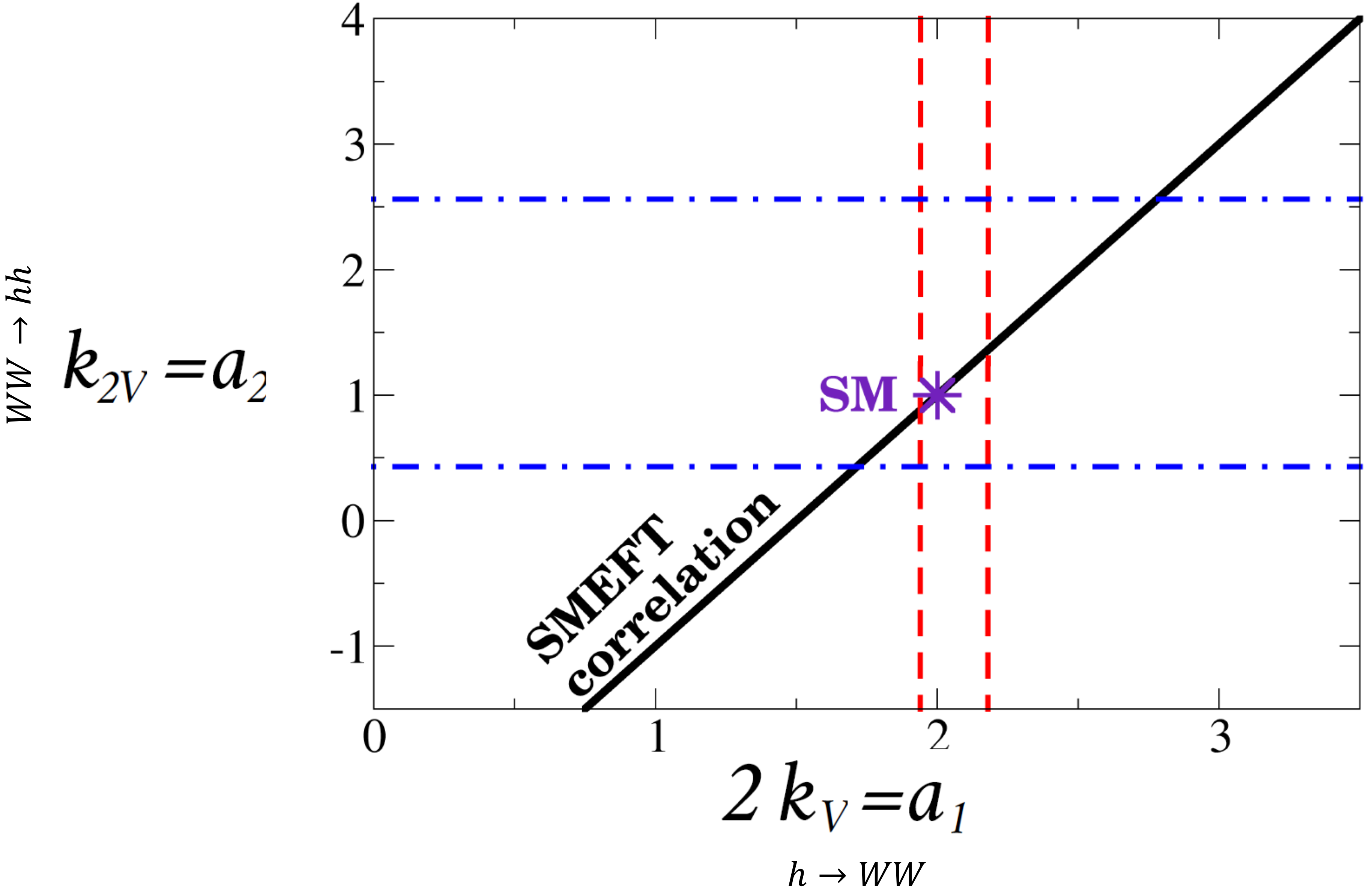
SMEFT predicts correlations absent in experiment?

(in principle, also absent in HEFT)

(*) Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, PRD 106 (2022) 5, 5; arXiv: 2207.09848 [hep-ph]
(*) For other studies on SMEFT correlations see, e.g., Brivio,Corbett,Éboli,Gavela,González-Fraile,González-García,Merlo,Rigolin, JHEP 03 (2014) 024 & Agrawal,Saha,Xu,Yu,Yuan, PRD 101 (2020) 7, 075023

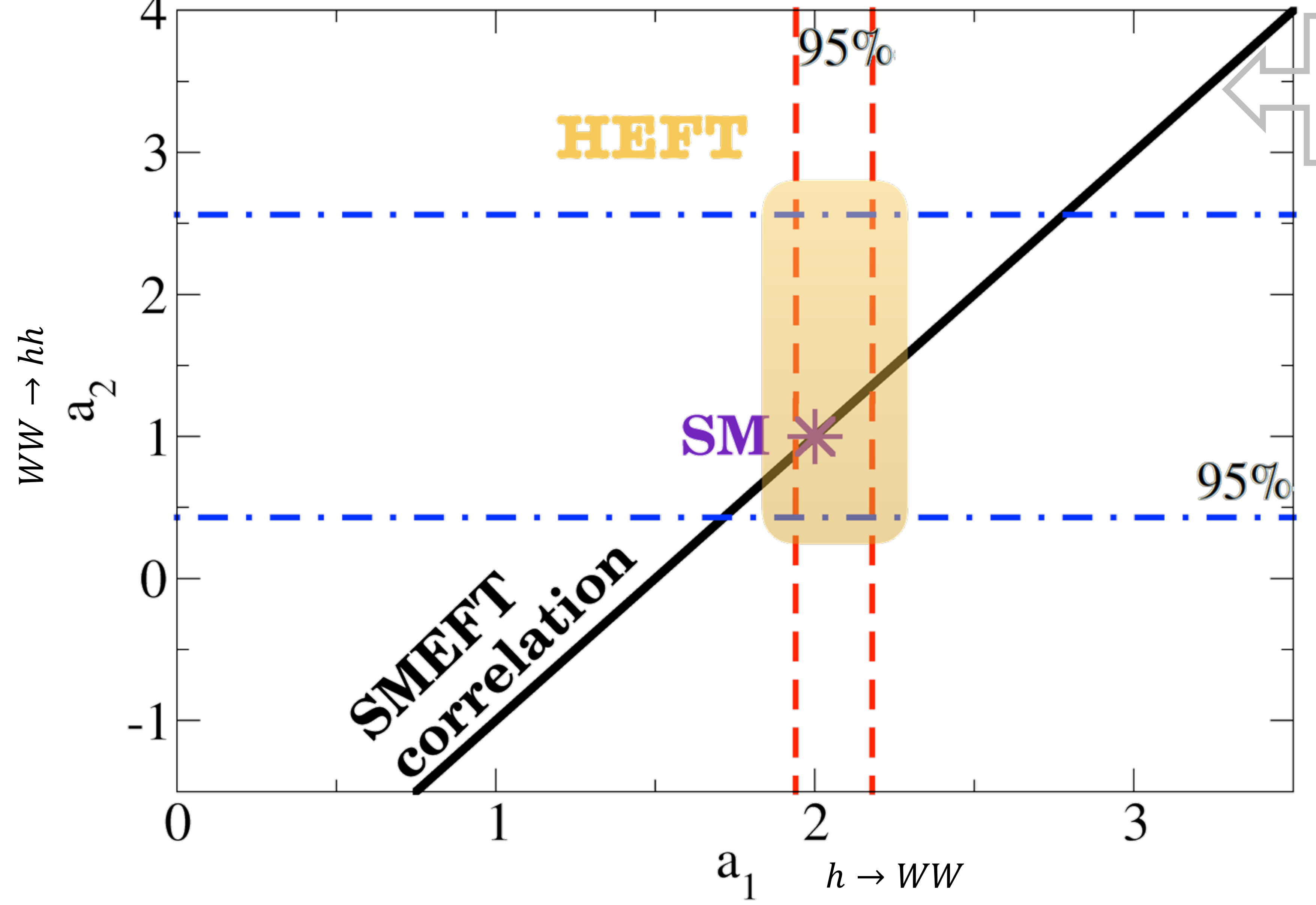
- W/o relying on a specific SMEFT Lagrangian, we obtain:

Valid SMEFT	$\Longrightarrow$	Double zero of $\mathcal{F}(h)$ at some h_*	$\Longrightarrow$	Specific correlations between a_i coefficients from expanding $\mathcal{F}(h)$ around $h = 0$



WW → hh
$k_{2V} = a_2$
4
3
2
1
0
-1
SM
SMEFT correlation
0
1
2
3
$2\,k_V = a_1$
h → WW

ATLAS-CONF-2020-027 $a_1/2 = a \in [0.97, 1.09]$



SMEFT at order $1/\Lambda^2$ predicts correlation $a_2 = 2a_1 - 3$

$a_2 = b = \kappa_{2V} \in [-0.1, 2.2]$

CMS-HIG-20-005

Blue and red: best available exp. bounds

(*) Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, PRD 106 (2022) 5, 5; arXiv: 2207.09848 [hep-ph]

Correlations accurate at order Λ^{-2}	**Correlations accurate at order Λ^{-4}**	**Λ^{-4} Assuming SMEFT perturbativity**
$\Delta a_2 = 2\Delta a_1$		$\lvert\Delta a_2\rvert \leq 5\lvert\Delta a_1\rvert$
$a_3 = \frac{4}{3}\Delta a_1$	$\left(a_3 - \frac{4}{3}\Delta a_1\right) = \frac{8}{3}(\Delta a_2 - 2\Delta a_1) - \frac{1}{3}\left(\Delta a_1\right)^2$	
$a_4 = \frac{1}{3}\Delta a_1$	$\left(a_4 - \frac{1}{3}\Delta a_1\right) = \frac{5}{3}\Delta a_1 - 2\Delta a_2 + \frac{7}{4}a_3 =$ $= \frac{8}{3}(\Delta a_2 - 2\Delta a_1) - \frac{7}{12}\left(\Delta a_1\right)^2$	those for a_3, a_4, a_5, a_6
$a_5 = 0$	$a_5 = \frac{8}{5}\Delta a_1 - \frac{22}{15}\Delta a_2 + a_3 =$ $= \frac{6}{5}(\Delta a_2 - 2\Delta a_1) - \frac{1}{3}\left(\Delta a_1\right)^2$	all the same
$a_6 = 0$ SMEFT	$a_6 = \frac{1}{6}a_5$ SMEFT	SMEFT

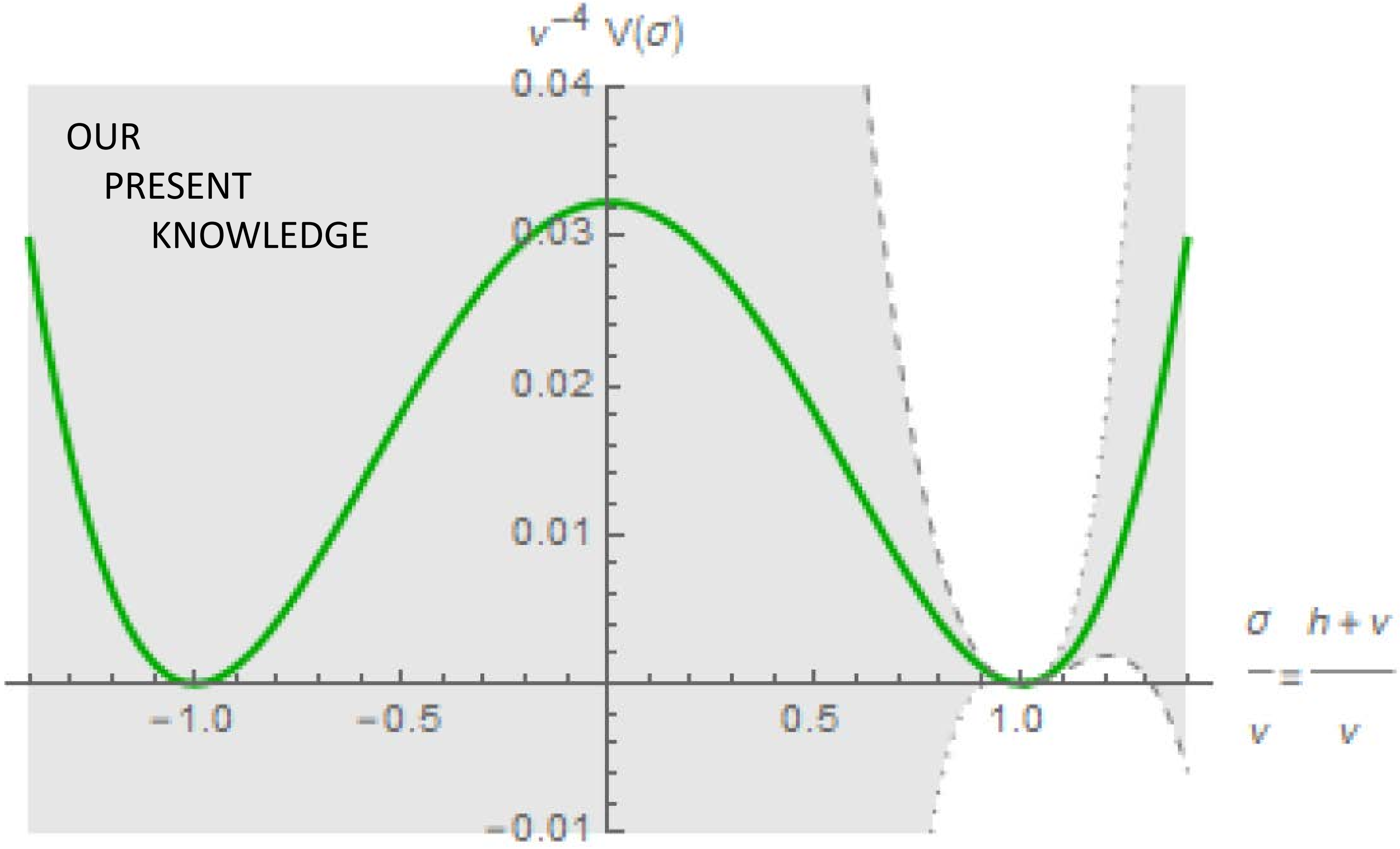
$$\Delta a_1 := a_1 - 2 = 2a - 2$$

$$\Delta a_2 := a_2 - 1 = b - 1$$

$$a_1 = \left(2 + 2\frac{c^{(6)}_{H\Box}v^2}{\Lambda^2} + 3\frac{(c^{(6)}_{H\Box})^2 v^4}{\Lambda^4} + 2\frac{c^{(8)}_{H\Box}v^4}{\Lambda^4}\right) \qquad a_2 = \left(1 + 4\frac{c^{(6)}_{H\Box}v^2}{\Lambda^2} + 12\frac{(c^{(6)}_{H\Box})^2 v^4}{\Lambda^4} + 6\frac{c^{(8)}_{H\Box}v^4}{\Lambda^4}\right)$$

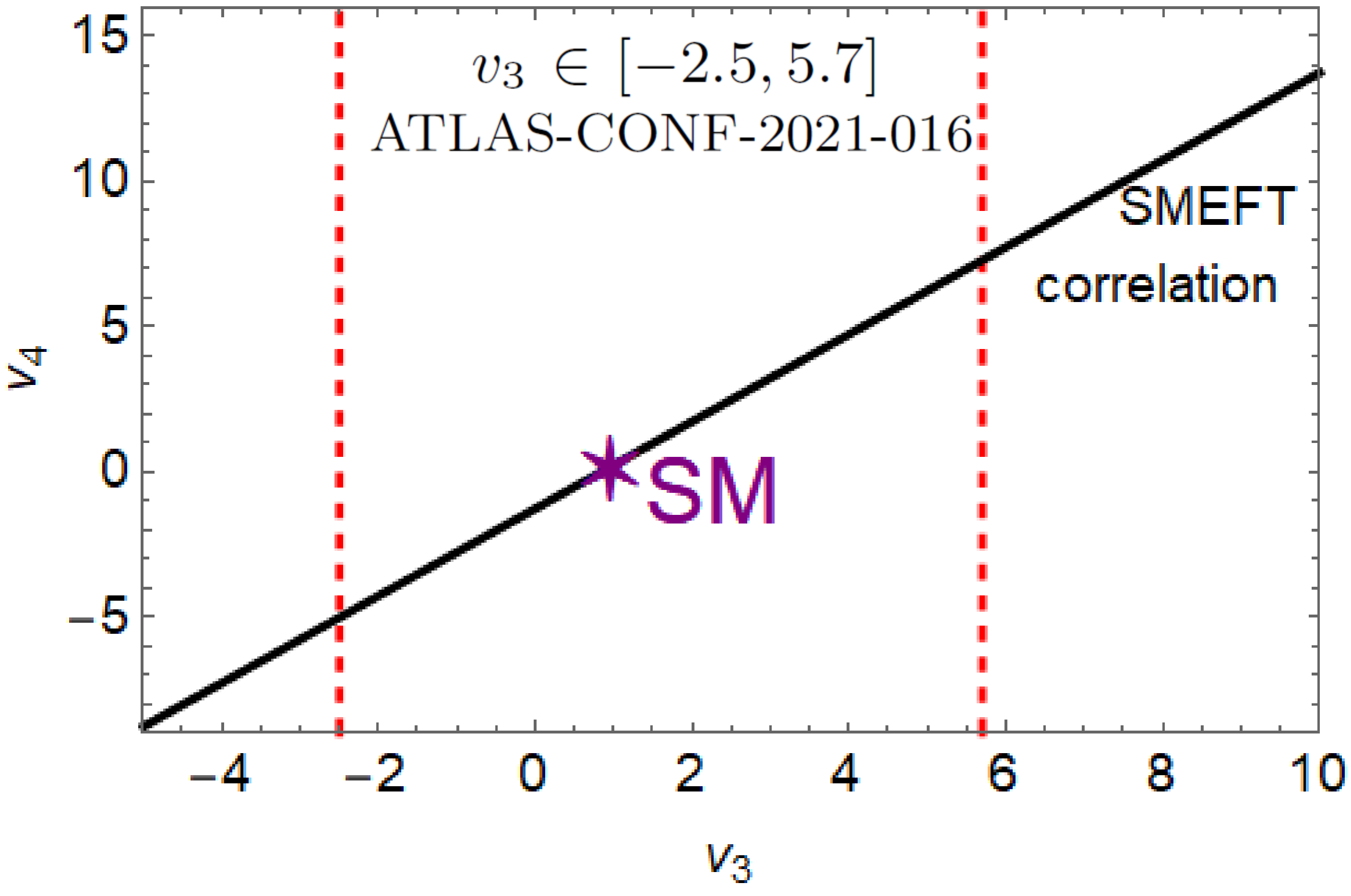
(*) Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, 2204.01763 [hep-ph]

Other correlations: Higgs potential



$$V_{\rm HEFT} = \frac{m_h^2 v^2}{2}\left[\left(\frac{h_{\rm HEFT}}{v}\right)^2 + v_3\left(\frac{h_{\rm HEFT}}{v}\right)^3 + + v_4\left(\frac{h_{\rm HEFT}}{v}\right)^4 + \dots\right],$$

with $v_3 = 1$, $v_4 = 1/4$ and $v_{n\geq 5} = 0$ in the SM



$\Delta v_4 = \frac{3}{2}\Delta v_3 - \frac{1}{6}\Delta a_1$	$\Delta v_4 \in [-3.8, 8.6]$
SMEFT	
$v_5 = 6 v_6 = \frac{3}{4}\Delta v_3 - \frac{1}{8}\Delta a_1$	$v_5 = 6 v_6 \in [-1.9, 4.3]$

$a_1/2 \in [0.97, 1.09]$

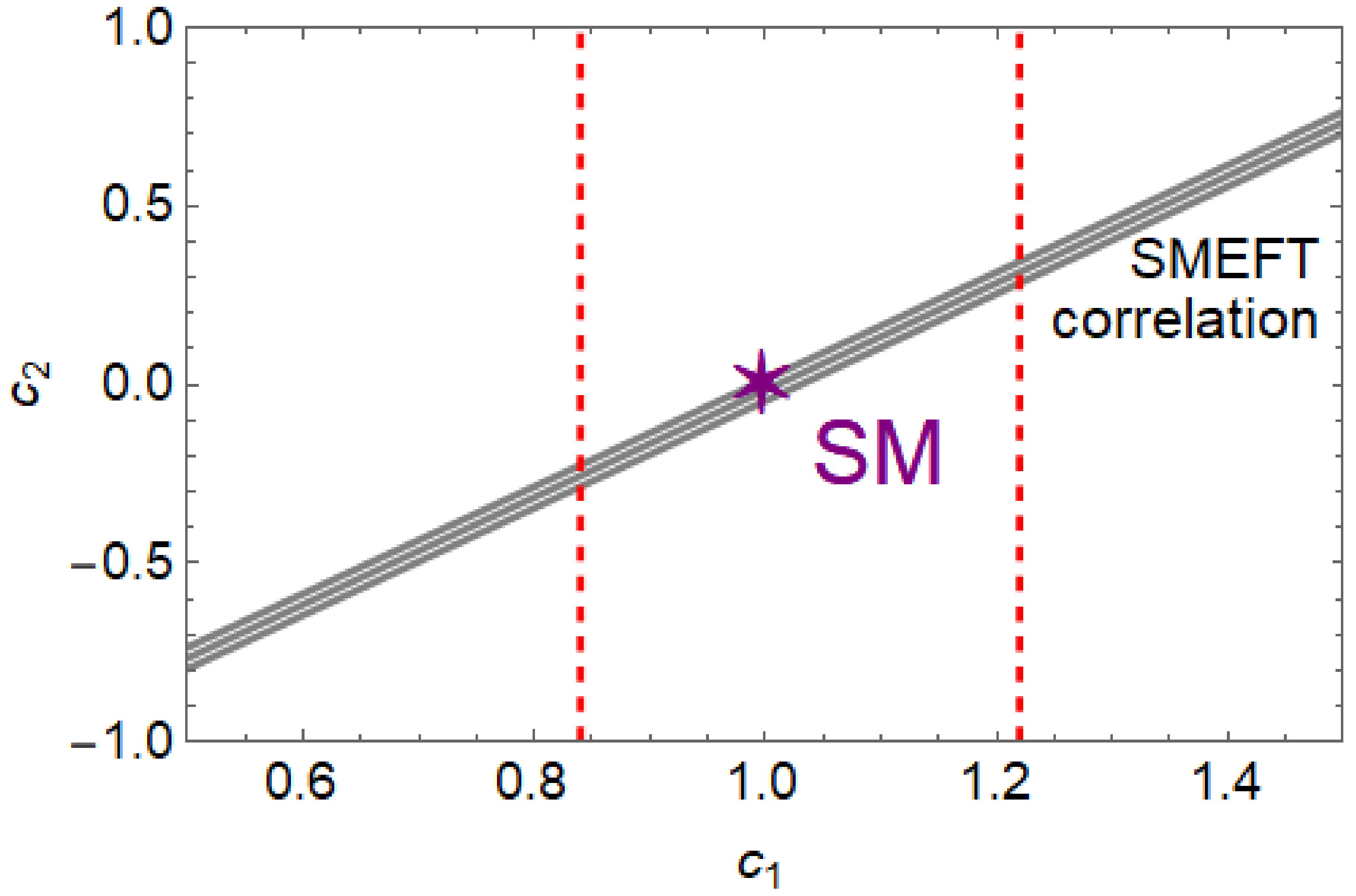
(*) Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, PRD 106 (2022) 5, 5; arXiv: 2207.09848 [hep-ph]

Other correlations: Yukawa's

$$\mathcal{L}_Y = -\mathcal{G}(h) M_t \bar{t}t \sqrt{1 - \frac{\omega^2}{v^2}} + \ldots$$

$$\mathcal{G}(h_{\rm HEFT}) = 1 + c_1 \frac{h_{\rm HEFT}}{v} + c_2 \left(\frac{h_{\rm HEFT}}{v}\right)^2 + \ldots$$

(with $c_1 = 1$, $c_{i\geq 2} = 0$ in the Standard Model)



$c_2 = 3c_3 = \frac{3}{2}(c_1 - 1) - \frac{1}{4}\Delta a_1$ **SMEFT**	$c_2 = 3c_3 \in [-0.27, 0.35]$

(*) Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, PRD 106 (2022) 5, 5; arXiv: 2207.09848 [hep-ph]

Conclusions

- We identified **potential issues** when HEFT described as SMEFT

- The problem is not the realization / choice-of-coordinates

  - **Theory** potential problem:

    HEFT written in SMEFT-form turns singular (?)

  - **Phenomenology** potential problem:

    SMEFT incompatible with data (?)

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BACKUP

Falsifying SMEFT: correlations

Correlations accurate at order Λ^{-2}	Correlations accurate at order Λ^{-4}	Λ^{-4} Assuming SMEFT perturbativity
$\Delta a_2 = 2\Delta a_1$		$\lvert\Delta a_2\rvert \leq 5\lvert\Delta a_1\rvert$
$a_3 = \frac{4}{3}\Delta a_1$	$\left(a_3 - \frac{4}{3}\Delta a_1\right) = \frac{8}{3}(\Delta a_2 - 2\Delta a_1) - \frac{1}{3}(\Delta a_1)^2$	
$a_4 = \frac{1}{3}\Delta a_1$	$\left(a_4 - \frac{1}{3}\Delta a_1\right) = \frac{5}{3}\Delta a_1 - 2\Delta a_2 + \frac{7}{4}a_3 = \frac{8}{3}(\Delta a_2 - 2\Delta a_1) - \frac{7}{12}(\Delta a_1)^2$	those for a_3, a_4, a_5, a_6
$a_5 = 0$	$a_5 = \frac{8}{5}\Delta a_1 - \frac{22}{15}\Delta a_2 + a_3 = \frac{6}{5}(\Delta a_2 - 2\Delta a_1) - \frac{1}{3}(\Delta a_1)^2$	all the same
$a_6 = 0$	$a_6 = \frac{1}{6}a_5$	

$$a_1 = \left(2 + 2\frac{c^{(6)}_{H\Box}v^2}{\Lambda^2} + 3\frac{(c^{(6)}_{H\Box})^2 v^4}{\Lambda^4} + 2\frac{c^{(8)}_{H\Box}v^4}{\Lambda^4}\right) \qquad a_2 = \left(1 + 4\frac{c^{(6)}_{H\Box}v^2}{\Lambda^2} + 12\frac{(c^{(6)}_{H\Box})^2 v^4}{\Lambda^4} + 6\frac{c^{(8)}_{H\Box}v^4}{\Lambda^4}\right).$$

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(*) Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, 2204.01763 [hep-ph]

Consistent SMEFT range at order Λ^{-2}	Consistent SMEFT range at order Λ^{-4}	Perturbativity of Λ^{-4} SMEFT $\lvert\Delta a_2\rvert \leq 5\lvert\Delta a_1\rvert$
$\Delta a_2 \in [-0.12, 0.36]$ $a_3 \in [-0.08, 0.24]$ $a_4 \in [-0.02, 0.06]$ $a_5 = 0$ $a_6 = 0$	ATLAS $a_3 \in [-4.1, 4.0]$ $a_4 \in [-4.2, 3.9]$ $a_5 \in [-1.9, 1.8]$ $a_6 = a_5$	ATLAS $a_3 \in [-3.1, 1.7]$ $a_4 \in [-3.3, 1.5]$ $a_5 \in [-1.5, 0.6]$ $a_6 = a_5$
	CMS $a_3 \in [-3.2, 3.0]$ $a_4 \in [-3.3, 3.0]$ $a_5 \in [-1.5, 1.3]$ $a_6 = a_5$	CMS $a_3 \in [-3.1, 1.7]$ $a_4 \in [-3.3, 1.5]$ $a_5 \in [-1.5, 0.6]$ $a_6 = a_5$

$$a_1/2 = a \in [0.97$$

- ATLAS

$$a_2 = b = \kappa_{2V} \in [$$

- CMS

$$a_2 = b = \kappa_{2V} \in [$$

(*) Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, 2204.01763 [hep-ph]

• A history recollection on the $\mathcal{L}_{p^4}$ renormalization (1):

Higgs-less 1-loop
RENORMALIZATION

(*) Herrero,Ruiz Morales, NPB 418 (1994) 431-455

Higgs-full:
1-LOOP CALCULATIONS of PARTICULAR OBSERVABLES

A small sample of 1-loop HEFT observable computations:
(x) Delgado,Dobado,Llanes-Estrada, PRL114 (2015) 22, 221803
(x) Espriu,Mescia,Yencho, PRD88 (2013) 055002
(x) Delgado,Garcia-Garcia,Herrero, JHEP 11 (2019) 065
(x) Fabbrichesi,Pinamonti(,Tonero,Urbano, PRD93 (2016) 1, 015004
(x) Corbett,Éboli,Gonzalez-Garcia, PRD 93 (2016) 1, 015005
(x) de Blas,Eberhardt,Krause, JHEP 07 (2018) 048
(x) Quezada,Dobado,SC, PoS ICHEP2020 (2021) 076; in preparation

• A history recollection on the $\mathcal{L}_{p^4}$ renormalization (2):

$\mathcal{O}(p^4)$ HEFT renormalization: scalar loops & true $\mathcal{O}(D^4)$ divergences

(*) Guo,Ruiz-Femenia,SC, PRD92 (2015) 074005

$$\mathcal{L} = \dots + \frac{v^2}{4}\mathcal{F}_C(h)\langle D_\mu U^\dagger D^\mu U\rangle$$

c_k	Operator $\mathcal{O}_k$	Γ_k	$\Gamma_{k,0}$
c_1	$\frac{1}{4}\langle f_+^{\mu\nu}f_{+\mu\nu} - f_-^{\mu\nu}f_{-\mu\nu}\rangle$	$\frac{1}{24}(\mathcal{K}^2-4)$	$-\frac{1}{6}(1-a^2)$
(c_2-c_3)	$\frac{i}{2}\langle f_+^{\mu\nu}[u_\mu,u_\nu]\rangle$	$\frac{1}{24}(\mathcal{K}^2-4)$	$-\frac{1}{6}(1-a^2)$
c_4	$\langle u_\mu u_\nu\rangle\langle u^\mu u^\nu\rangle$	$\frac{1}{96}(\mathcal{K}^2-4)^2$	$\frac{1}{6}(1-a^2)^2$
c_5	$\langle u_\mu u^\mu\rangle^2$	$\frac{1}{192}(\mathcal{K}^2-4)^2+\frac{1}{128}\mathcal{F}_C^2\Omega^2$	$\frac{1}{8}(a^2-b)^2+\frac{1}{12}(1-a^2)^2$
c_6	$\frac{1}{v^2}(\partial_\mu h)(\partial^\mu h)\langle u_\nu u^\nu\rangle$	$\frac{1}{16}\Omega(\mathcal{K}^2-4)-\frac{1}{96}\mathcal{F}_C\Omega^2$	$-\frac{1}{6}(a^2-b)(7a^2-b-6)$
c_7	$\frac{1}{v^2}(\partial_\mu h)(\partial_\nu h)\langle u^\mu u^\nu\rangle$	$\frac{1}{24}\mathcal{F}_C\Omega^2$	$\frac{2}{3}(a^2-b)^2$
c_8	$\frac{1}{v^4}(\partial_\mu h)(\partial^\mu h)(\partial_\nu h)(\partial^\nu h)$	$\frac{3}{32}\Omega^2$	$\frac{3}{2}(a^2-b)^2$
c_9	$\frac{(\partial_\mu h)}{v}\langle f_-^{\mu\nu}u_\nu\rangle$	$\frac{1}{24}\mathcal{F}'_C\Omega$	$-\frac{1}{3}a(a^2-b)$
c_{10}	$\frac{1}{2}\langle f_+^{\mu\nu}f_{+\mu\nu} + f_-^{\mu\nu}f_{-\mu\nu}\rangle$	$-\frac{1}{48}(\mathcal{K}^2+4)$	$-\frac{1}{12}(1+a^2)$

$\mathcal{O}(p^4)$ HEFT renormalization: scalar loops & GEOMETRIC APPROACH

A deeper understanding through geometry:
(x) Alonso,Jenkins,Manohar, PLB 754 (2016) 335-342; PLB 756 (2016) 358-364; JHEP 08 (2016) 101

- Beautiful geometric connection to this result *
  provided by the curvature (x) of the scalar manifold metric $g_{ij}(\phi) = \begin{bmatrix} F(h)^2 g_{ab}(\varphi) & 0 \\ 0 & 1 \end{bmatrix}$, with $\mathcal{L} = \frac{1}{2} g_{ij} D_\mu \phi^i D^\mu \phi^j$

$$\mathcal{R}_4 = \left(1 - v^2 (F')^2\right) F^2 \quad = \quad \left(1 - \mathcal{K}^2/4\right) \mathcal{F}_C \,,$$

$$\mathcal{R}_2 = \left(1 - v^2 (F')^2\right) - \frac{v^2 F'' F}{(N_\varphi - 1)} \quad = \quad \left(1 - \mathcal{K}^2/4\right) - \frac{\mathcal{F}_C \Omega}{8} \,,$$

$$\mathcal{R}_0 = 2\mathcal{F}_C^{-1} \mathcal{R}_2 - \mathcal{F}_C^{-2} \mathcal{R}_4 \,,$$

$$F = \mathcal{F}_C^{1/2} \qquad N_\varphi = 3$$

with Λ^{-2} = the Riemann $\mathbb{R}_{ijmn} \propto \mathscr{R}_{0,2,4} / v^2$ *(loosely speaking, the curvature $\mathbb{R}$)*

- NDA gives you the suppression of individual diagrams ~ $1/(4\pi v)^2$
  but the full loop suppression is ~ $g^2 \mathbb{R} / (4\pi)^2$ & ~ $\mathbb{R}^2 / (4\pi)^2$

  EFT as an expansion $\mathcal{M} \sim R\,\mathbf{p}^2 + \frac{R^2\,\mathbf{p}^4}{(4\pi)^2} + \frac{R^3\,\mathbf{p}^6}{(4\pi)^4} + \ldots$ in the curvature?

- **SM:** $\mathbb{R}_{ijmn} = 0$ ➔ No $O(p^4)$ renormalization

* Guo,Ruiz-Femenia,SC, PRD92 (2015) 074005

(x) Alonso,Jenkins,Manohar, PLB754 (2016) 335; PLB756 (2016) 358; JHEP 1608 (2016) 101

• A history recollection on the $\mathcal{L}_{p^4}$ renormalization (3):

$\mathcal{O}(p^4)$ HEFT renormalization:
scalar + gauge + fermion loops
(FULL)

(*) Buchalla,Cata,Celis,Knecht,Krause, NPB 928 (2018) 93-106

(*) Alonso,Kanshin,Saa, PRD 97 (2018) 3, 035010

(*) Buchalla,Catà,Celis,Knecht,Krause, PRD 104 (2021) 7, 076005

Introduction and motivation.
SMEFT $\subset$ HEFT: an overview.
Correlations of HEFT parameters when assuming SMEFT's validity. Explicit computation.
Measurements: $W_L W_L \to n \times h$ to discern between SMEFT and pure-HEFT.
Based on "The flair of Higgsflare" https://arxiv.org/abs/2204.01763.

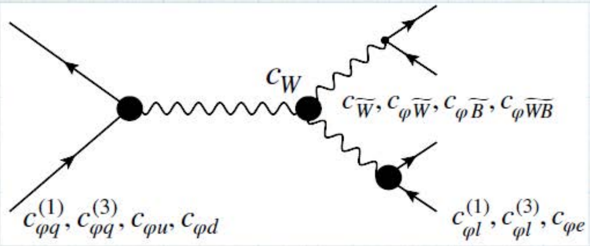
Outline

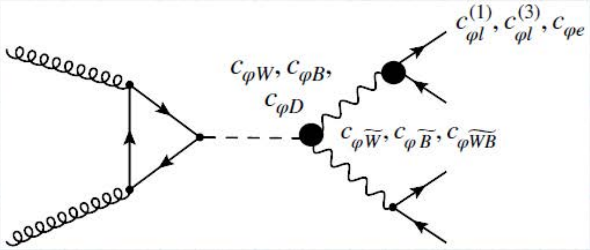
* The SMEFT: LHC's favourite
* HEFT: the old classic
* Geometrical interpretations
* HEFT in terms of SMEFT

3

SMEFT operators

* Warsaw basis -> 59/2499 operators
* dim 8 basis (Murphy et al) -> 993/44807





$$V_{EFT} = V_{SM}\left(1 + \frac{g_6}{\Lambda^2}\right)$$

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Comparing with LHC data

* Amplitude analogous to SM one:

    * $$\sigma_{EFT} = \sigma_{SM} + \underbrace{\sigma_{int,6}}_{linear} + \underbrace{\sigma_{pure,6} + \sigma_{int,8}}_{quadratic} + \ldots$$

* Not uniquely defined (results are truncation-dependent)

* Other than that, technically similar to SM-LHC computations

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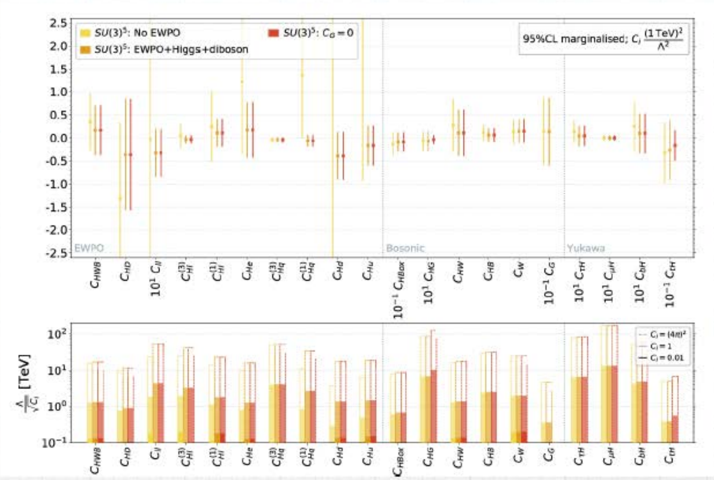
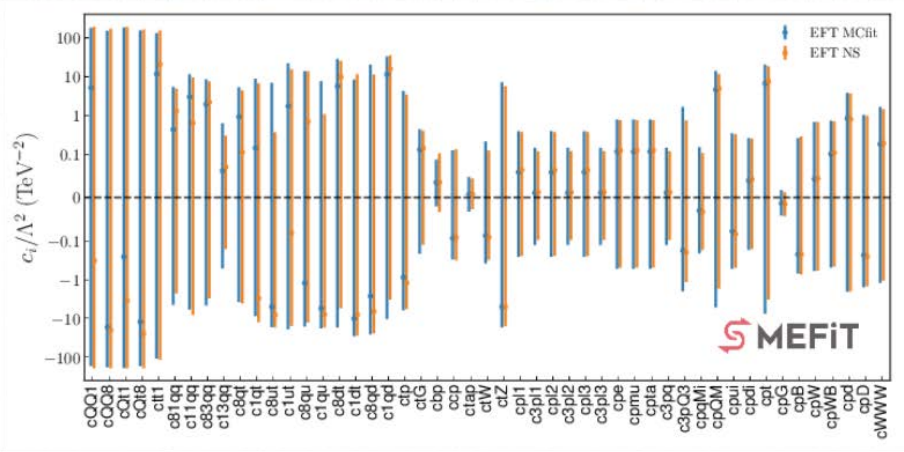
LHC Global fits

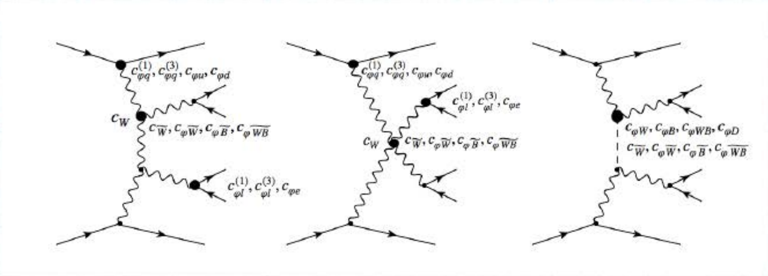
* In the absence of new particles, our main effort goes into constraining SMEFT coefficients

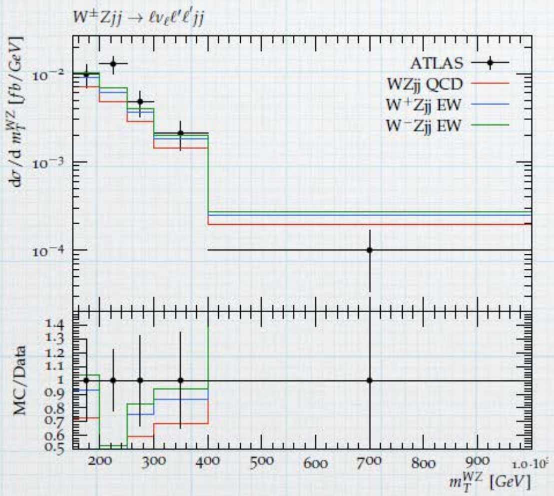


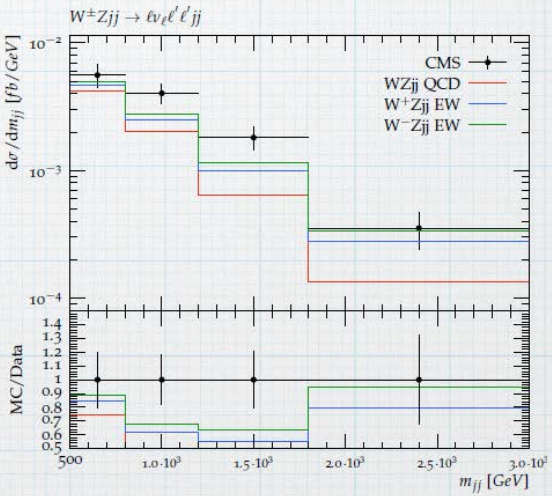


SM-to-SMEFT relatively easy to implement on the technical tools

fitmaker, smefit, et al.







HEPData

See for example 2101.03180

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SMEFT mimics the SM structures

- In particular:
- $V^{SM}_{HHH} = v V^{SM}_{HHHH}$ and $V^{SM}_{WWH} = v V^{SM}_{WWHH}$
- (consequence of the EWSB mechanism)

This is the main feature that we can use to falsify smeft

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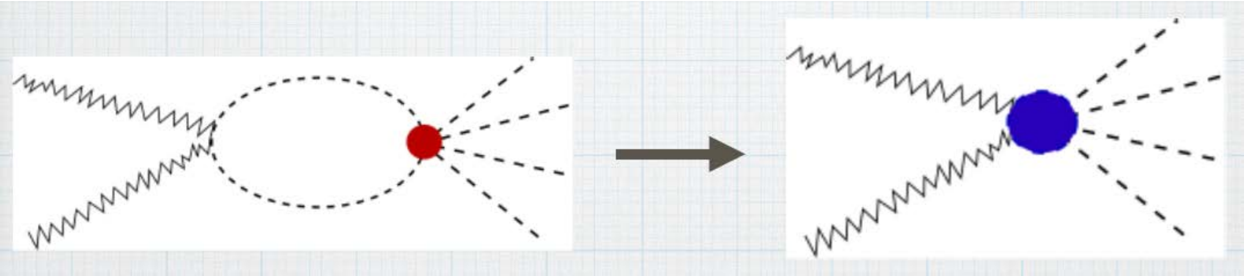
SMEFT@NLO

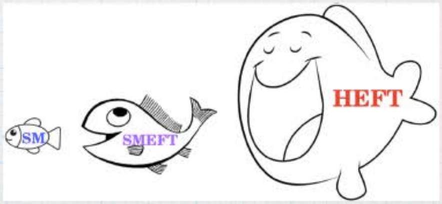
• Manohar, Jenkins, Trott, Alonso



See e.g. 1505.03706. Ghezzi, Gómez-Ambrosio, Passarino, Uccirati

HEFT: *an old classic*

- Originally the non-linear sigma model (for Pions)
- In principle a QCD Lagrangian -> inspired the EWChL
- Very natural for the study of the Higgs-Goldstone interactions
- I.e: scattering of longitudinal gauge bosons -> Vector boson fusion/scattering
- Natural for strongly coupled new physics



EWChL HEFT natural to study VBF/VBS

- Madrid UCM and UAM
  - Strongly coupled theories beyond the Standard Model. Antonio Dobado, Domènec Espriu. Prog.Part.Nucl.Phys. 115 (2020) 103813
  - Unitarity, analyticity, dispersion relations, and resonances in strongly interacting WL WL, ZL ZL, , and hh scattering. R.Delgado , A Dobado, F Llanes-Estrada. Phys.Rev.D 91 (2015) 7, 075017
  - Production of vector resonances at the LHC via WZ-scattering: a unitarized EChL analysis. R.L. Delgado, A. Dobado, D. Espriu, C. Garcia-Garcia, M.J. Herrero et al. JHEP 11 (2017) 098
  - One-loop $\gamma\gamma\to$ WL WL and $\gamma\gamma\to$ ZL ZL from the Electroweak Chiral Lagrangian with a light Higgs-like scalar. R.L. Delgado, A. Dobado, M.J. Herrero, J.J. Sanz-Cillero. JHEP 07 (2014) 149

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And refs therein…

SMEFT

- ω_a and h fit in a left-$SU(2)$ doublet
- Higgs always in the combination: $(h+v)$
- Higher symmetry
- Natural when h is a fundamental field
- ET usually based in a cutoff Λ expansion: $O(d)/\Lambda^{d-4}$ ($d =$ operator dimension: 4,6,8 ...)

$$\mathcal{O}_H = (H^\dagger H)^3 \,, \qquad \mathcal{O}_{HD} = (H^\dagger D_\mu H)^* (H^\dagger D^\mu H) \,,$$
$$\mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H) \,.$$

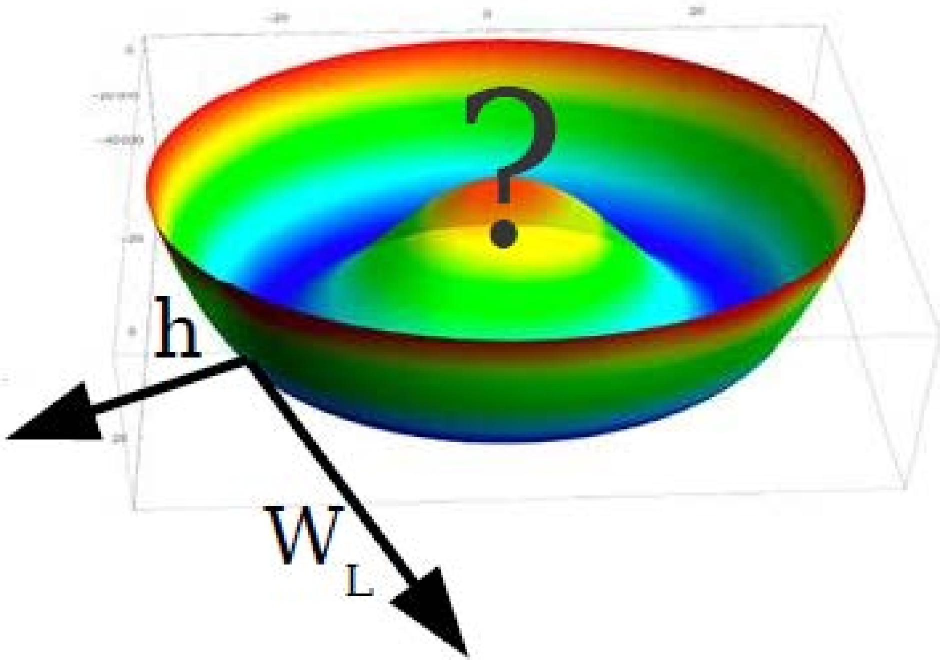
HEFT

- h is a $SU(2)$ singlet and ω_a are coordinates on a coset: $SU(2)_L \times SU(2)_R / SU(2)_V \simeq SU(2) \simeq S3$
- Lesser symmetry; more independent higher-dimension effective operators but less model dependent
- Derivative expansion
- ECLh with $\mathcal{F}(h)$ insertions
- Typical for composite models of the SBS (h as a GB) (Strongly interacting and consistent with the presence of the GAP)

Dobado and Espriu, Prog.Part.Nucl.Phys. 115 (2020) 103813

Geometric distinction HEFT/SMEFT

- Several works have provided field-redefinition invariant criteria to distinguish SMEFT from HEFT:
  - R. Alonso, E. E. Jenkins, and A. V. Manohar,
    "A Geometric Formulation of Higgs Effective Field Theory: Measuring the Curvature of Scalar Field Space," Phys. Lett. B754 (2016) 335–342, arXiv:1511.00724 [hep-ph].
    "Sigma Models with Negative Curvature," Phys.Lett.B756,358(2016),arXiv:1602.00706 [hep-ph].
    "Geometry of the Scalar Sector," JHEP 08 (2016) 101, arXiv:1605.03602 [hep-ph]."
    (Cohen et al., 2021, p. 95)
  - T. Cohen, N. Craig, X. Lu, and D. Sutherland:
    "Is SMEFT Enough?", JHEP 03, 237, arXiv:2008.08597 [hep-ph].
    "Unitarity Violation and the Geometry of Higgs EFTs",
    (2021), arXiv:2108.03240 [hep- ph].



SMEFT

$$\vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_4 + i\phi_3 \end{pmatrix}$$

HEFT

$$h \quad \text{and} \quad \vec{n} = \begin{pmatrix} n_1 = \pi_1/v \\ n_2 = \pi_2/v \\ n_3 = \pi_3/v \\ n_4 = \sqrt{1 - n_1^2 - n_2^2 - n_3^2} \end{pmatrix}$$

* SMEFT scalar sector —> Linear sigma model
* HEFT —> Non-linear sigma model

Strongly interacting Higgs bosons

Thomas Appelquist and Claude Bernard
Phys. Rev. D **22**, 200 – Published 1 July 1980

Recent works highlighting the EFT geometry

- R. Alonso, E. E. Jenkins, and A. V. Manohar,
  - "A Geometric Formulation of Higgs Effective Field Theory: Measuring the Curvature of Scalar Field Space," Phys. Lett. B754 (2016) 335–342, arXiv:1511.00724 [hep-ph].
  - "Sigma Models with Negative Curvature," Phys.Lett.B756,358(2016),arXiv:1602.00706 [hep-ph].
  - "Geometry of the Scalar Sector," JHEP 08 (2016) 101, arXiv:1605.03602 [hep-ph]." (Cohen et al., 2021, p. 95)
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  - "Is SMEFT Enough?", JHEP 03, 237, arXiv:2008.08597 [hep-ph].
  - "Unitarity Violation and the Geometry of Higgs EFTs", (2021), arXiv:2108.03240 [hep- ph].

we now know that HEFT and SMEFT can be understood geometrically

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And refs therein…

* These works show us that SMEFT vs HEFT is more than linear vs nonlinear realisations...
  * SMEFT exists if: $\exists h^* \to \mathcal{F}(h) = 0$
  * And $\mathcal{F}(h)$ is analytic in a certain region
* Consequences:
  * $\exists F(h) \implies \mathcal{F}(h) = F(h)^2$
  * Double 0 of $\mathcal{F}(h)$
  * Odd derivatives vanish (even derivatives of $F(h)$)

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The flair of the Higgsflare: motivation

flair

noun

UK /fleəʳ/ US /fler/

C1 [S]

natural ability to do something well:

- *He has a flair* ***for*** *languages.*

$$\mathcal{F}(h) = \left(1 + a_1\frac{h}{v} + a_2\frac{h^2}{v^2} + a_3\frac{h^3}{v^3} + \ldots + a_n\frac{h^n}{v^n}\right)$$

See 2204.01763

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See 2204.01763

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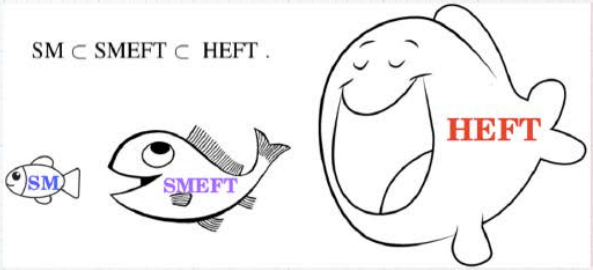
Here is where HEFT kicks in

Write SMEFT
in HEFT form:



$$|\partial H|^2 + \frac{1}{2}B(|H|)^2(\partial(|H|^2))^2 \;\rightarrow\; \frac{v^2}{4}\mathcal{F}(h)\langle D_\mu U^\dagger D^\mu U\rangle + \frac{1}{2}(\partial h_{HEFT})^2$$

$$dh_{HEFT} = \sqrt{1 + (v + h_{SMEFT})^2 B(h_{SMEFT})}\;\; dh_{SMEFT}$$

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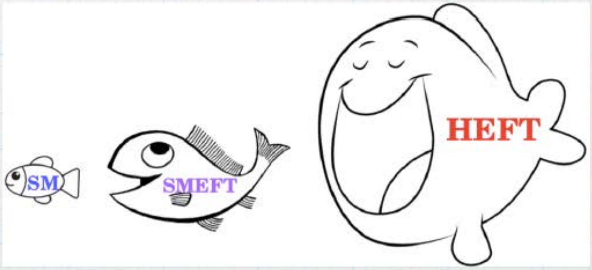
The Flare Function

* In HEFT: $\mathscr{F}(h)_{HEFT} = 1 + a_1 \frac{h}{v} + a_2 \left(\frac{h}{v}\right)^2 + a_3 \left(\frac{h}{v}\right)^3 + \ldots$
* In the SM: $\mathscr{F}(h)_{SM} = \left(1 + \frac{h}{v}\right)^2$
* In SMEFT?



Falsifying SMEFT

- Relevant SMEFT operators for the Higgs sector (dim 6):
- $$\mathcal{O}_H = (H^\dagger H)^3\,, \qquad \mathcal{O}_{HD} = (H^\dagger D_\mu H)^*(H^\dagger D^\mu H)\,,$$
  $$\mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H)\,.$$
  - At high energies they decouple and only one survives: $\mathcal{O}_{H\Box}$

Alonso & West 2109.13290

The Flare function in SMEFT

$$\mathcal{L}_{\rm SMEFT} = \frac{v^2}{4}\left(1+\frac{h_1}{v}\right)^2 \langle D_\mu U^\dagger D^\mu U\rangle + \frac{1}{2}\left(1-\frac{2c_{H\Box}(h_1+v)^2}{\Lambda^2}\right)(\partial_\mu h_1)^2 \quad - \quad V(h_1)$$

$$= \frac{v^2}{4}\mathcal{F}(h_1)\langle D_\mu U^\dagger D^\mu U\rangle + \frac{1}{2}(\partial_\mu h_1)^2 - V(h) - \frac{c_{H\Box}\left[(v+h_1)^3 - v^3\right]}{3\Lambda^2}V'(h_1)\,.$$

$$\mathcal{F}(h_1) = \left(1+\frac{h_1}{v}\right)^2 + \frac{2v^3 c_{H\Box}}{\Lambda^2}\left(1+\frac{h_1}{v}\right)\left(\frac{h_1^3}{3v^3}+\frac{h_1^2}{v^2}+\frac{h_1}{v}\right) + \mathcal{O}\left(\frac{c_{H\Box}^2}{\Lambda^4}\right) =$$

$$= 1 + \left(\frac{h_1}{v}\right)\left(2+2\frac{c_{H\Box}v^2}{\Lambda^2}\right) + \left(\frac{h_1}{v}\right)^2\left(1+4\frac{c_{H\Box}v^2}{\Lambda^2}\right) +$$

$$+ \left(\frac{h_1}{v}\right)^3\left(8\frac{c_{H\Box}v^2}{3\Lambda^2}\right) + \left(\frac{h_1}{v}\right)^4\left(2\frac{c_{H\Box}v^2}{3\Lambda^2}\right),$$

$$a_1 = 2a = 2\left(1+v^2\frac{c_{H\Box}}{\Lambda^2}\right)\,,\quad a_2 = b = 1+4v^2\frac{c_{H\Box}}{\Lambda^2}\,,\quad a_3 = \frac{8v^2}{3}\frac{c_{H\Box}}{\Lambda^2}\,,\quad a_4 = \frac{2v^2}{3}\frac{c_{H\Box}}{\Lambda^2}\,.$$

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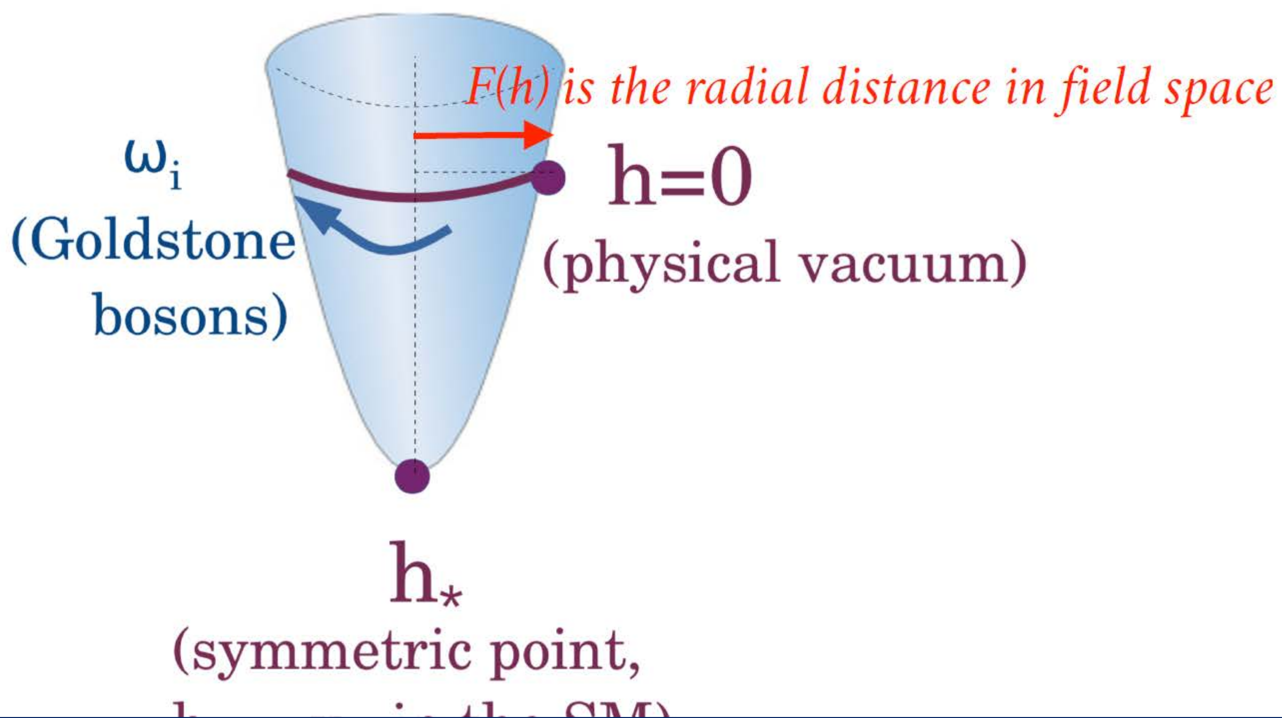
Geometric distinction HEFT/SMEFT

In a nutshell, SMEFT is valid provided:

- $\exists h_* \in \mathbb{R}$ where $\mathcal{F}(h_*) = 0$, and
- Because of the need for $\mathcal{L}_{\rm SMEFT}$ analyticity, $\mathcal{F}$ is analytic between our vacuum $h = 0$ and h_*, particularly around h_*. Moreover its odd derivatives vanish at symmetric point.
- Similar criteria for the potential $V(h)$.



At high energies (TeV region) only (D=6) derivative operators are relevant:

Subleading

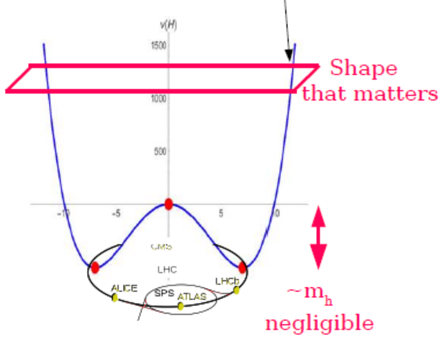
$$\mathcal{O}_H = (H^\dagger H)^3 , \qquad \mathcal{O}_{HD} = (H^\dagger D_\mu H)^* (H^\dagger D^\mu H) ,$$

Custodial-violating

$$\mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H) .$$

$A(H)$ can be set to 1

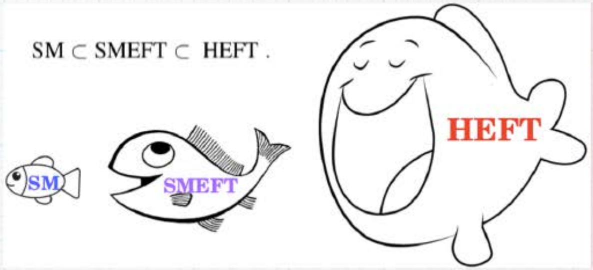
Relevant Operator



$\Rightarrow$ Cleaner measurement of the Flare function $\mathcal{F}$ at high energies.

Here is where HEFT kicks in

Write SMEFT
in HEFT form:



$$|\partial H|^2 + \frac{1}{2}B(|H|)^2(\partial(|H|^2))^2 \;\rightarrow\; \frac{v^2}{4}\mathcal{F}(h)\langle D_\mu U^\dagger D^\mu U\rangle + \frac{1}{2}(\partial h_{HEFT})^2$$

$$dh_{HEFT} = \sqrt{1+(v+h_{SMEFT})^2 B(h_{SMEFT})}\;\; dh_{SMEFT}$$

13

Falsifying SMEFT

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$$\mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H)\,.$$

  * At high energies they decouple and only one survives: $\mathcal{O}_{H\Box}$

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Alonso & West 2109.13290

HEFT: *an old classic*

* First differences: Power counting

LO

$$\mathcal{L}_{\text{NLO HEFT}} = \frac{1}{2}\left[1 + 2a\frac{h}{v} + b\left(\frac{h}{v}\right)^2\right]\partial_\mu\omega^i\partial^\mu\omega^j\left(\delta_{ij} + \frac{\omega^i\omega^j}{v^2 - \boldsymbol{\omega}^2}\right) + \frac{1}{2}\partial_\mu h\partial^\mu h$$

$$+ \frac{4\alpha_4}{v^4}\partial_\mu\omega^i\partial_\nu\omega^i\partial^\mu\omega^j\partial^\nu\omega^j + \frac{4\alpha_5}{v^4}\partial_\mu\omega^i\partial^\mu\omega^i\partial_\nu\omega^j\partial^\nu\omega^j + \frac{g}{v^4}(\partial_\mu h\partial^\mu h)^2$$

$$+ \frac{2d}{v^4}\partial_\mu h\partial^\mu h\partial_\nu\omega^i\partial^\nu\omega^i + \frac{2e}{v^4}\partial_\mu h\partial^\nu h\partial^\mu\omega^i\partial_\nu\omega^i \,,$$

NLO

15

[67] A combination of measurements of Higgs boson production and decay using up to 139 fb^{-1} of proton–proton collision data at $\sqrt{s} =$ 13 TeV collected with the ATLAS experiment, (2020).

[68] A. Tumasyan et al. (CMS), Search for Higgs boson pair production in the four b quark final state in proton-proton collisions at $\sqrt{s}$ = 13 TeV, (2022), arXiv:2202.09617 [hep-ex].

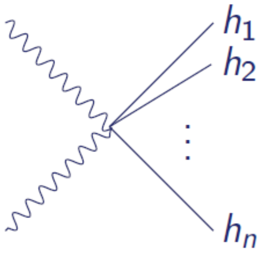
[69] G. Aad et al. (ATLAS), Search for the $HH \to b\bar{b}b\bar{b}$ process via vector-boson fusion production using proton-proton collisions at $\sqrt{s} =$ 13 TeV with the ATLAS detector, JHEP **07**, 108, [Erratum: JHEP 01, 145 (2021), Erratum: JHEP 05, 207 (2021)], arXiv:2001.05178 [hep-ex].

High energy measurements

In this region the potential is subleading. The flare function $\mathcal{F}$ encodes relevant physics (it accompanies the GB kinetic term)

$$\mathcal{F}(h_{\mathrm{HEFT}}) = 1 + \sum_{n=1}^{\infty} a_n \left(\frac{h_{\mathrm{HEFT}}}{v}\right)^n .$$

At high energies (Equivalence Theorem) $\omega \simeq W_L$
$\Rightarrow \omega\omega \to n \times h$ can falsify SMEFT.



$$= -\frac{n!\, a_n}{2v^n}\, s$$

Measure $\mathcal{F}$ expansion in multiHiggs final states

$$T_{\omega\omega\to \underline{h}} = -\frac{a_1 s}{2v}$$

$$T_{\omega\omega\to \underline{hh}} = \frac{s}{v^2}\left(\frac{a_1^2}{4} - a_2\right),$$

Linear in the highest parameter

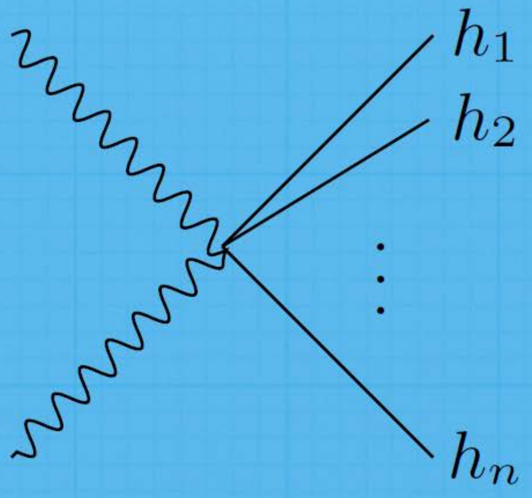
$$\begin{aligned}
T_{\omega\omega\to \underline{hhh}} = & -\frac{s}{8v^3}\left(a_1^3\left[4f_1f_3^2\left(\frac{z_{23}(f_1z_{23}-1)}{f_3(z_3-2f_1z_{23})+f_2z_2} + \frac{z_{13}(f_1z_{13}-1)}{f_1(z_1-2f_3z_{13})+f_3z_3}\right) +\right.\right.\\
& +2f_3\left(f_1\left(\frac{z_{23}-2f_2z_{23}}{-2f_1f_3z_{23}+f_2z_2+f_3z_3} + \frac{z_{13}-2f_1z_{13}}{-2f_1f_3z_{13}+f_1z_1+f_3z_3} + z_{13} + z_{23}\right) + 3(z_3-2)\right) +\\
& \left.+\frac{2f_1f_2z_{12}(2f_1(f_2z_{12}-1)-2f_2+1)}{f_1(z_1-2f_2z_{12})+f_2z_2} + 2f_1(f_2z_{12}+3z_1-6) + 6f_2z_2 - 12f_2 + 9\right] +\\
& +4a_1a_2\left[\frac{f_1^2\left(2z_1(-2f_2z_{12}+f_3(z_{13}+z_{23})-3)-4f_2z_{12}(f_3(z_{13}+z_{23})-2)+3z_1^2\right)}{2f_1f_2z_{12}-f_1z_1-f_2z_2} +\right.\\
& +\frac{2f_1f_2(-2f_2z_{12}(z_2+1)+z_2(f_3(z_{13}+z_{23})+3z_1-3)+z_{12})+3f_2^2z_2^2}{2f_1f_2z_{12}-f_1z_1-f_2z_2} + 6(f_2+f_3-1) -\\
& \left.\left.-\frac{2f_1f_3z_{23}(2f_3(f_1z_{23}-1)-2f_2+1)}{f_3(z_3-2f_1z_{23})+f_2z_2} - \frac{2f_1f_3z_{13}(2f_1(f_3z_{13}-1)-2f_3+1)}{f_1(z_1-2f_3z_{13})+f_3z_3} - 3f_3z_3\right] + 24a_3\right).
\end{aligned}$$

($f_i \equiv ||\vec{p}_i||/\sqrt{s}$; $z_i(\omega_1, h_i) \equiv 2\sin^2(\theta_i/2)$; $z_{ij}(h_i, h_j) \equiv 2\sin^2(\theta_{ij}/2)$) We provide all tree level amplitudes.

* At high energies ($\approx$ 1TeV)

Equivalence Theorem

$W_L W_L \to hhh \ldots \quad \approx \pi\pi \to hhh \ldots$



$$= -\frac{n! a_n}{2v^n}\, s$$

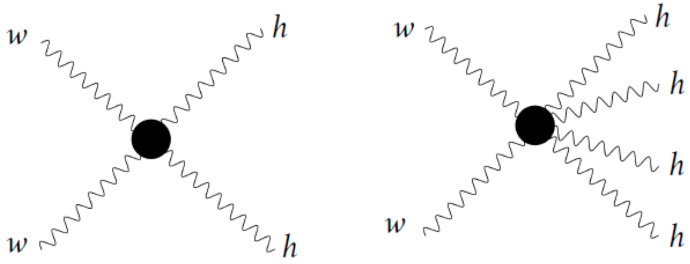
$$T_{\omega\omega\to n\times h} = \frac{s}{v^n}\sum_{i=1}^{p(n)}\left(\psi_i(q_1, q_2, \{p_k\})\prod_{j=1}^{|\mathrm{IP}[n]_i|} a_{\mathrm{IP}[n]_i^j}\right)$$

SMEFT Cross Sections

At the TeV-scale, linear-dimension-6 SMEFT predicts:

$$\boxed{\frac{\sigma(\omega\omega \to nh)}{\sigma(\omega\omega \to mh)} = \text{independent of } c_{H\Box}} \ .$$

$\Rightarrow$ Violation of this would shed doubts on SMEFT validity.



Falsifying SMEFT: Ratios of XSECS

In HEFT:

$$T_{\omega\omega\to nh} = f(a_1,\ldots,a_n)$$

$$T_{\omega\omega\to hh} = \frac{s}{v^2}(a^2-b) = \frac{s}{v^2}\left(\frac{a_1^2}{4}-a_2\right)$$

$$T_{\omega\omega\to nh} \propto \left(\frac{s}{v^{n-2}\Lambda^2}\right) c_{H\Box} \quad \text{in SMEFT up to } \mathcal{O}\left(\Lambda^{-2}\right)$$

$$\frac{\sigma(\omega\omega\to nh)}{\sigma(\omega\omega\to mh)} = \text{independent of } c_{H\Box}$$

SMEFT is a special case of HEFT.
SMEFT is falsifiable studying correlations induced in HEFT parameters.
TeV-scale measurements of $W_L W_L \to n \times H$ are needed to assess if SMEFT is applicable.

Experimental application

* Ideally future colliders will be able to measure multihiggs production at a good enough accuracy to test these correlations.

* Already a measurement of double H production at HL-LHC would provide greater insight on the a1/a2 values.

Measurements of a1/a2

A combination of measurements of Higgs boson production and decay using up to 139 fb^{-1} of proton–proton collision data at 13 TeV collected with the ATLAS experiment, (2020).

A. Tumasyan et al. (CMS), Search for Higgs boson pair production in the four b quark final state in proton-proton collisions at 13 TeV, (2022), arXiv:2202.09617 [hep-ex].

G. Aad et al. (ATLAS), Search for the HH → bbbb process via vector-boson fusion production using proton-proton collisions at s = √ 13 TeV with the ATLAS detector, JHEP **07**, 108, [Erratum: JHEP 01, 145 (2021), Erratum: JHEP 05, 207 (2021)], arXiv:2001.05178

28

EW Chiral Lagrangian (or HEFT)

- Electroweak Chiral Lagrangian : EW GB **transform non-linearly** and a **Higgs-like** field which **transforms linearly** under $SU(2)_L x SU(2)_R$ which breaks to the **Custodial Symmetry** $SU(2)_{L+R}$.

$$SU(2)_L \times SU(2)_R \xrightarrow{SSB} SU(2)_{L+R}$$

- Systematic expansion in **chiral power counting** (different to the SMEFT canonical expansion). **Renormalizable order by order.**

$$\mathscr{L}_{EChL} = \mathscr{L}_2 + \mathscr{L}_4 + \ldots$$

- It is often used the Equivalence Theorem , where we relate the gauge bosons with the would-be-Goldstones at high energies.

$$A(W_L^a W_L^b \to W_L^c W_L^d) = A(\omega^a \omega^b \to \omega^c \omega^d) + O\left(\frac{M_W}{\sqrt{s}}\right)$$

- **HOWEVER:** small BSM deviations ~ corrections to naïve-EqTh (if close to SM)

➔ We needed to go beyond naïve Equivalence Theorem: **physical $W_L W_L$ scattering**

O(p^4) Lagrangian:
(x) Buchalla, Cata, JHEP 1207 (2012) 101;
Buchalla,Catà,Krause, NPB 880 (2014) 552-573
(x) Alonso,Gavela,Merlo,Rigolin,Yepes, PLB 722 (2013) 330-335;
Brivio et al, JHEP 1403 (2014) 024
(x) Pich,Rosell,Santos,SC, PRD93 (2016) no.5, 055041;
JHEP 1704 (2017) 012;
(x) Krause,Pich,Rosell,Santos,SC, JHEP 1905 (2019) 092

Basic Works:
(*) Apelquist,Bernard '80; Longhitano '80, '81
(*) Feruglio, Int. J. Mod. Phys. A 8 (1993) 4937
(*) Grinstein,Trott, PRD 76 (2007) 073002

Counting:
* Weinberg '79
* Manohar,Georgi, NPB234 (1984) 189
* Georgi,Manohar NPB234 (1984) 189
* Hirn,Stern '05
* Pich,Rosell,Santos,SC JHEP 1704 (2017) 012
* Buchalla,Catá,Krause PLB 731 (2014) 80-86

For a recent SMEFT vs HEFT comparison:
(*) Gomez-Ambrosio,Llanes-Estrada, Salas-Bernardez,SC, 2204.01763 [hep-ph]

The lagrangian at lowest order (chiral dimension 2)

$$\mathscr{L}_2 = \frac{v^2}{4}\mathscr{F}(h)\mathrm{Tr}\left[(D_\mu U)^\dagger D^\mu U\right] + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h) + i\bar{Q}\partial Q - v\mathscr{G}(h)\left[\bar{Q}'_L U H_Q Q'_R + \mathrm{h.c.}\right]$$

GB + h
+ YM + matter

Just the top for this case

Spherical parametrization

$$U = \sqrt{1 - \frac{\omega^2}{v^2}} + i\frac{\bar{\omega}}{v}$$

GB $\bar{\omega} = \tau^a \omega^a$

$$Q^{(\prime)} = \begin{pmatrix} \mathscr{U}^{(\prime)} \\ \mathscr{D}^{(\prime)} \end{pmatrix} \quad \begin{cases} \mathscr{U}' = (u,c,t)' \\ \mathscr{D}' = (d,s,b)' \end{cases}$$

Quarks

Analytic functions of powers of the Higgs field. Inspired by most of low energy HEFT models.

$$\mathscr{G}(h) = 1 + c_1\frac{h}{v} + \ldots, \quad \mathscr{F}(h) = 1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \ldots$$

→ **Recover the SM**

$$V(h) = \frac{M_h^2}{2}h^2 + d_3\frac{M_h^2}{2v}h^3 + \ldots$$

↑

$$a = b = 1$$

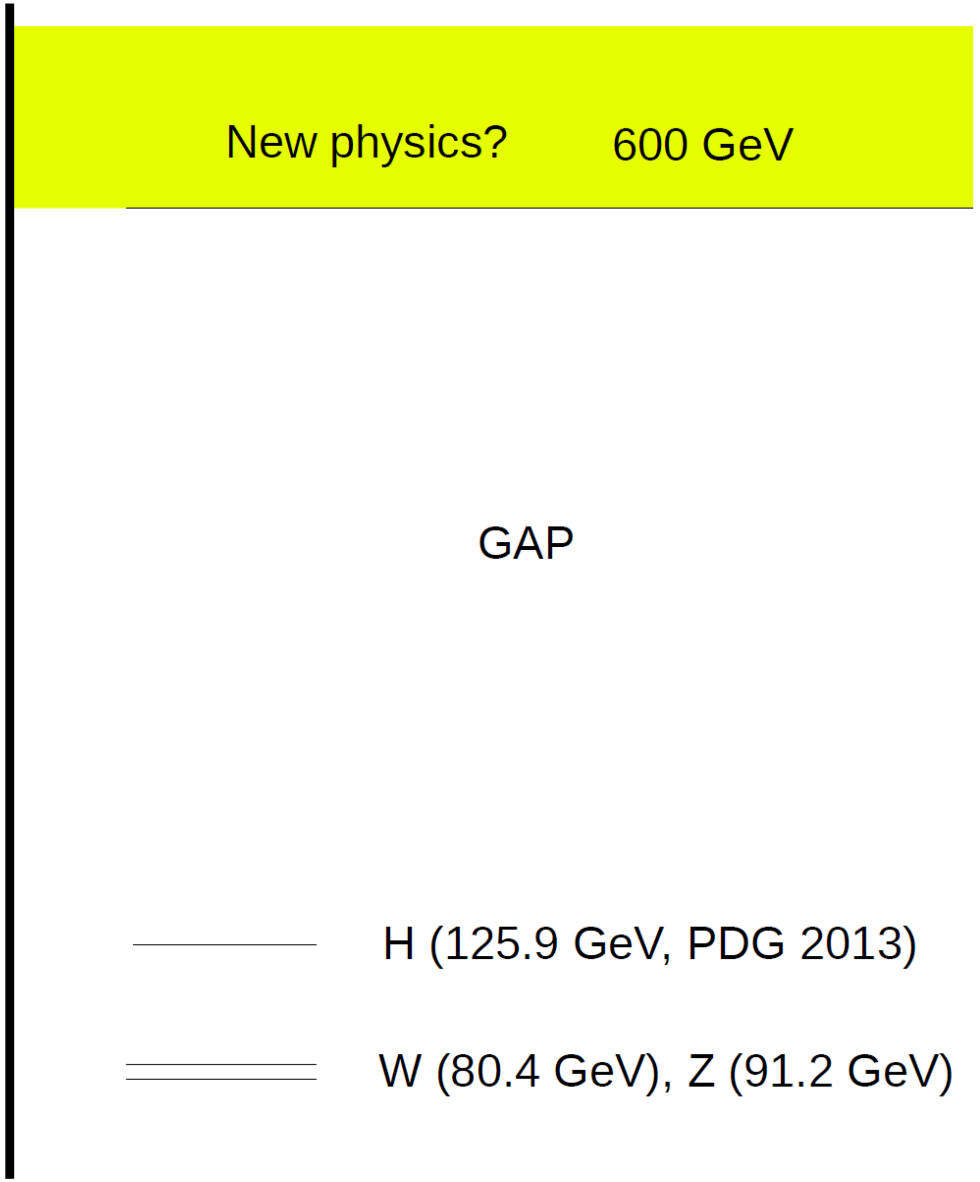
$$c_1 = 1$$

$$c_2 = c_3 = \ldots c_n = 0$$

Modifications on the Higgs SM couplings and beyond!

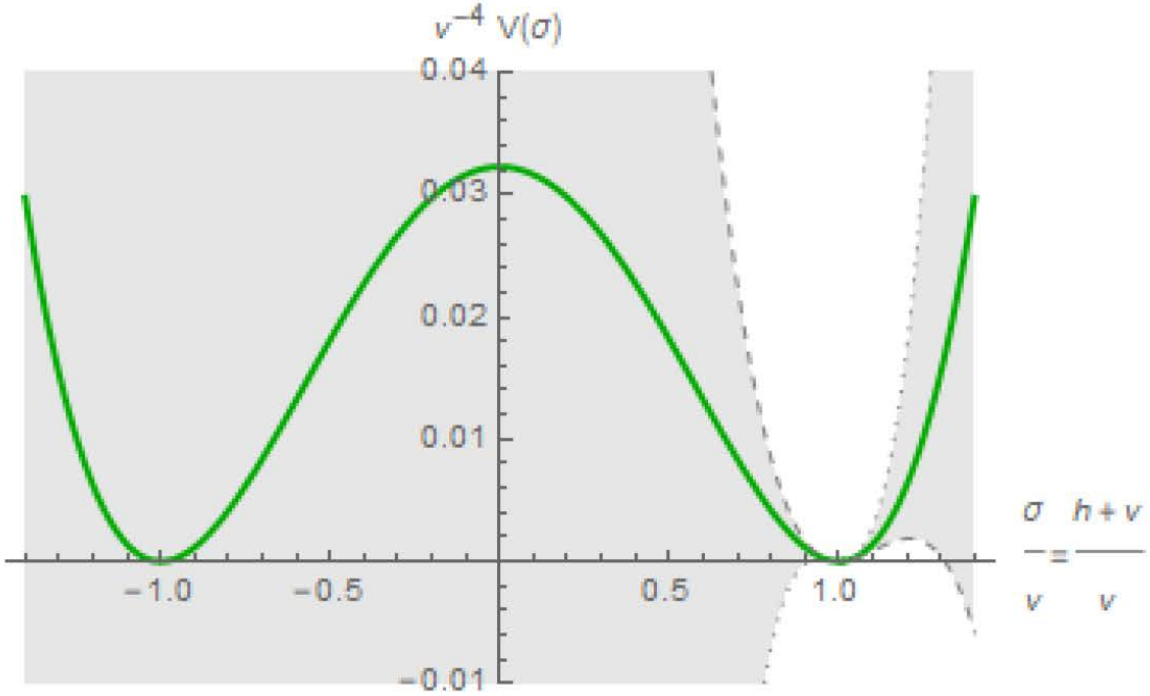
Falsifying the SM:

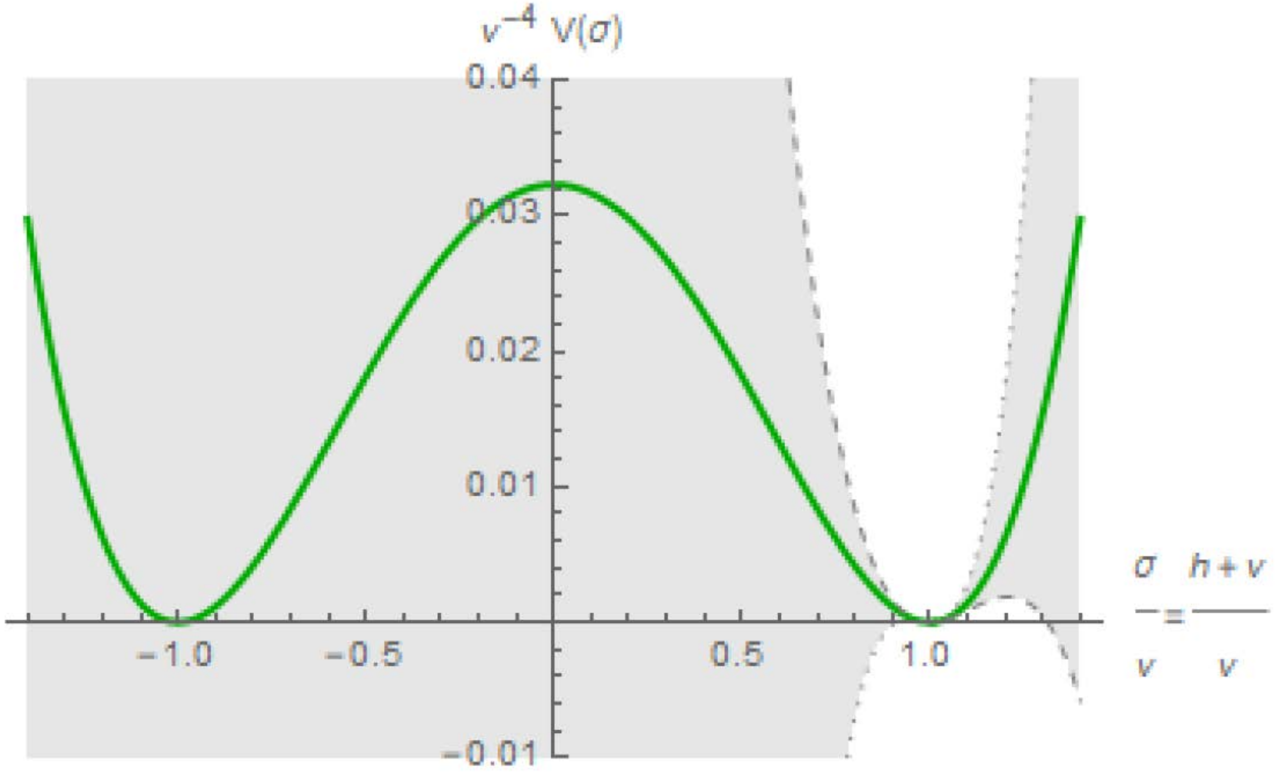
- Discover new particles, or
- Discover new forces



One of the most uncharted and promising sectors in SM

- Nature of Higgs boson and EW gauge bosons? Composite or not?
- Measurable: Higgs self interaction and its coupling to electroweak gauge bosons.





$v^{-4}\,V(\sigma)$
0.04
0.03
0.02
0.01
-0.01
-1.0
-0.5
0.5
1.0
$\frac{\sigma}{v} = \frac{h+v}{v}$

At high energies (TeV region) only (D=6) derivative operators are relevant:

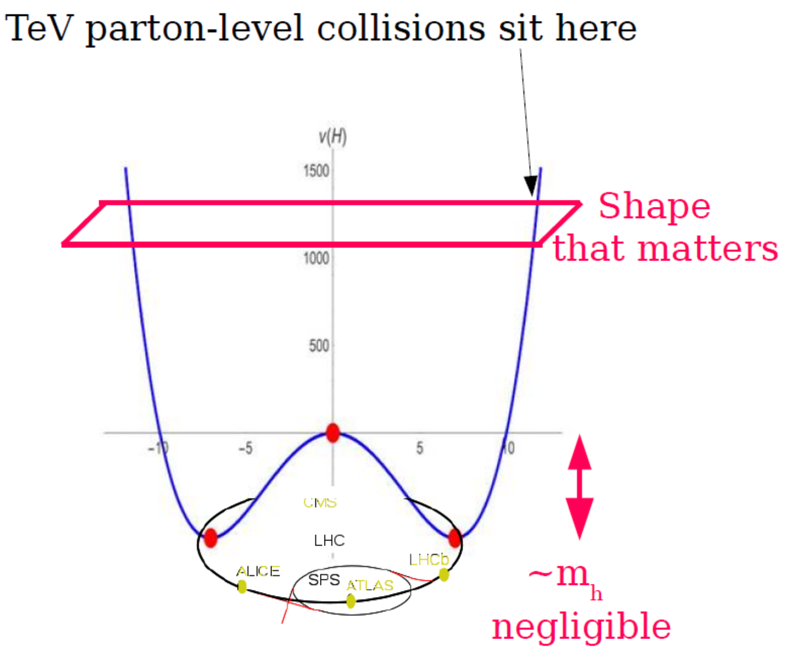
subdominant (no derivatives) *custodial violating*

$$\mathcal{O}_H = (H^\dagger H)^3 \,, \qquad \mathcal{O}_{HD} = (H^\dagger D_\mu H)^* (H^\dagger D^\mu H) \,,$$

$$\mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H) \,.$$

Only relevant operator

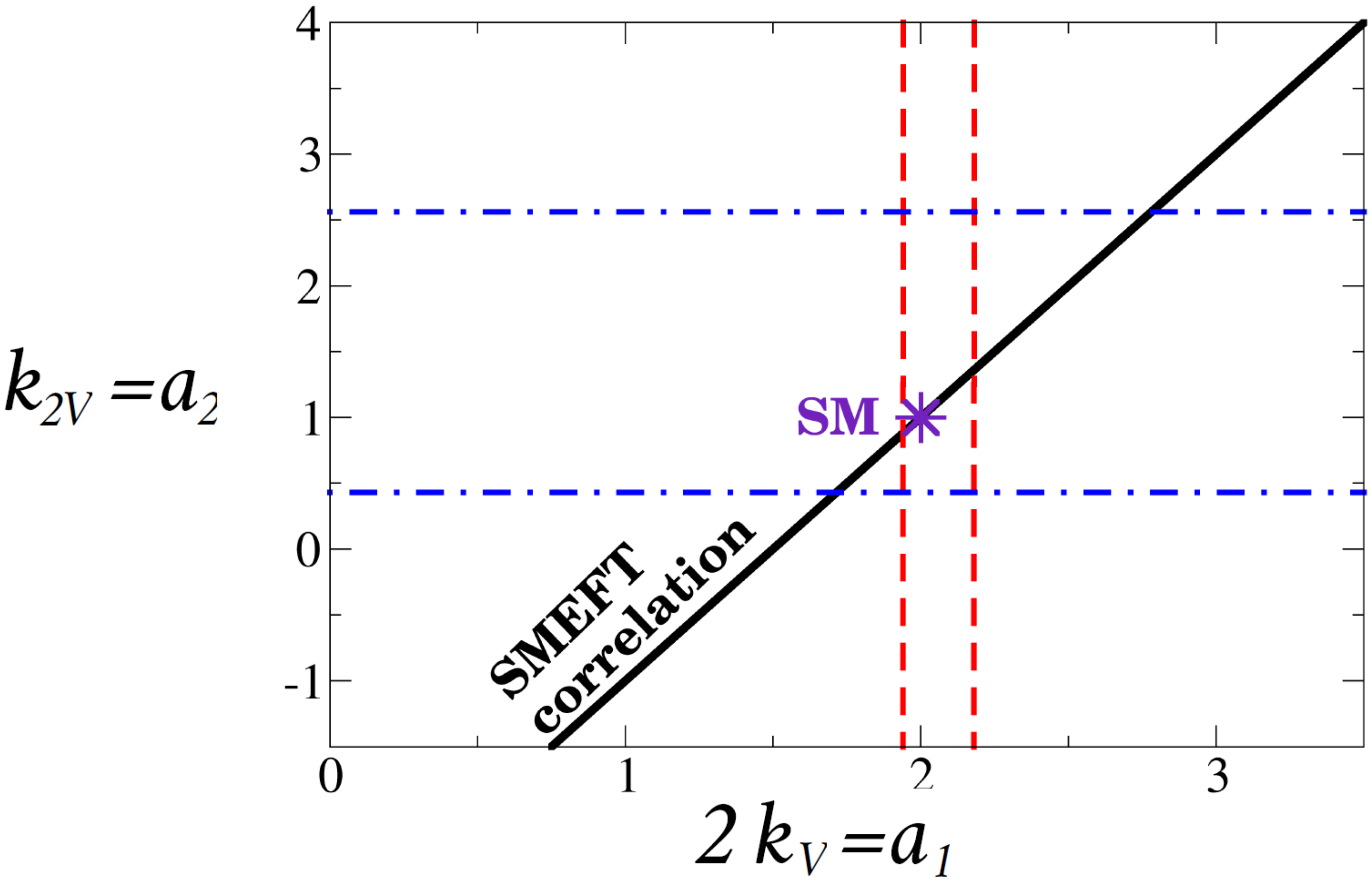
$A(H)$ can be set to 1

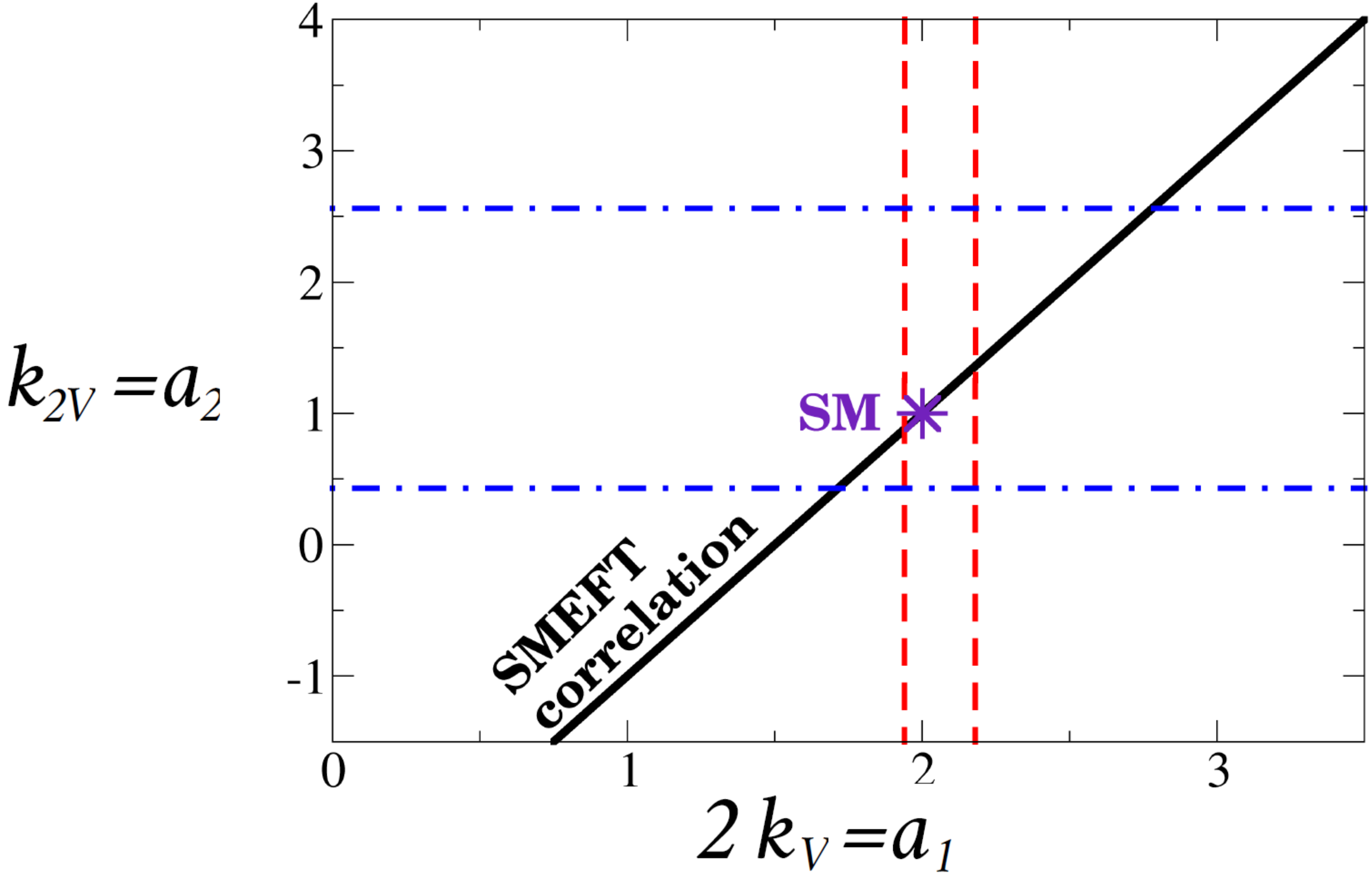


$\Rightarrow$ Cleaner measurement of the Flare function $\mathcal{F}$ at high energies.

Correlations among HEFT parameters due to SMEFT structure:

(Bands from single Higgs production at ATLAS (ATLAS-CONF-2020-027) and Higgs Pair production at CMS `https://arxiv.org/abs/2202.09617`)





$k_{2V} = a_2$
$2\,k_V = a_1$
SM
SMEFT correlation
4
3
2
1
0
-1
0
1
2
3

Correlations in the potential

HEFT correlations from the Custodial preserving SMEFT operators

$$\mathcal{O}_H := (H^\dagger H)^3 \,, \quad \mathcal{O}_{H\Box} := (H^\dagger H)\Box(H^\dagger H) \,.$$

$$v_3 = 1 + \frac{3v^2 c_{H\Box}}{\Lambda^2} + \frac{\mu^2 c_H}{\lambda^2\Lambda^2} \,, \quad v_4 = \frac{1}{4} + \frac{25 v^2 c_{H\Box}}{6\Lambda^2} + \frac{3}{2}\frac{\mu^2 c_H}{\lambda^2\Lambda^2} \,,$$

$$v_5 = \frac{2v^2 c_{H\Box}}{\Lambda^2} + \frac{3}{4}\frac{\mu^2 c_H}{\lambda^2\Lambda^2} \,, \quad v_6 = \frac{v^2 c_{H\Box}}{3\Lambda^2} + \frac{1}{8}\frac{\mu^2 c_H}{\lambda^2\Lambda^2} \,,$$

$$v_{n\geq 7} = 0 \,,$$

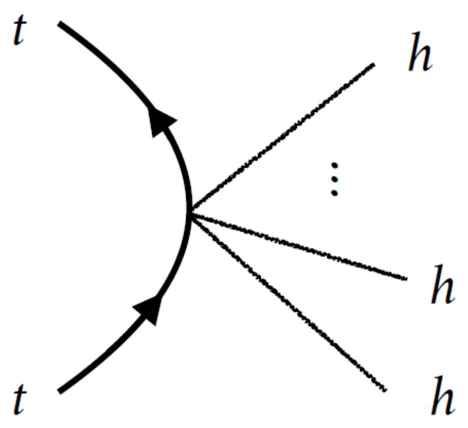
with $m_h^2 = -2\mu^2 \left(1 + \frac{2c_{H\Box} v^2}{\Lambda^2} + \frac{3}{4}\frac{\mu^2 c_H}{\lambda^2\Lambda^2}\right)$,

$2\langle |H|^2\rangle = v^2 = -\frac{\mu^2}{\lambda}\left(1 - \frac{3}{4}\frac{\mu^2 c_H}{\lambda^2\Lambda^2}\right)$.

Correlations in the top-Yukawa

The Yukawa Lagrangian in HEFT:

$$\mathcal{L}_Y = -\mathcal{G}(h) M_t \bar{t} t \sqrt{1 - \frac{\omega^2}{v^2}} \, ,$$



with the function

$$\mathcal{G}(h_{\rm HEFT}) = 1 + c_1 \frac{h_{\rm HEFT}}{v} + c_2 \left(\frac{h_{\rm HEFT}}{v} \right)^2 + \dots$$

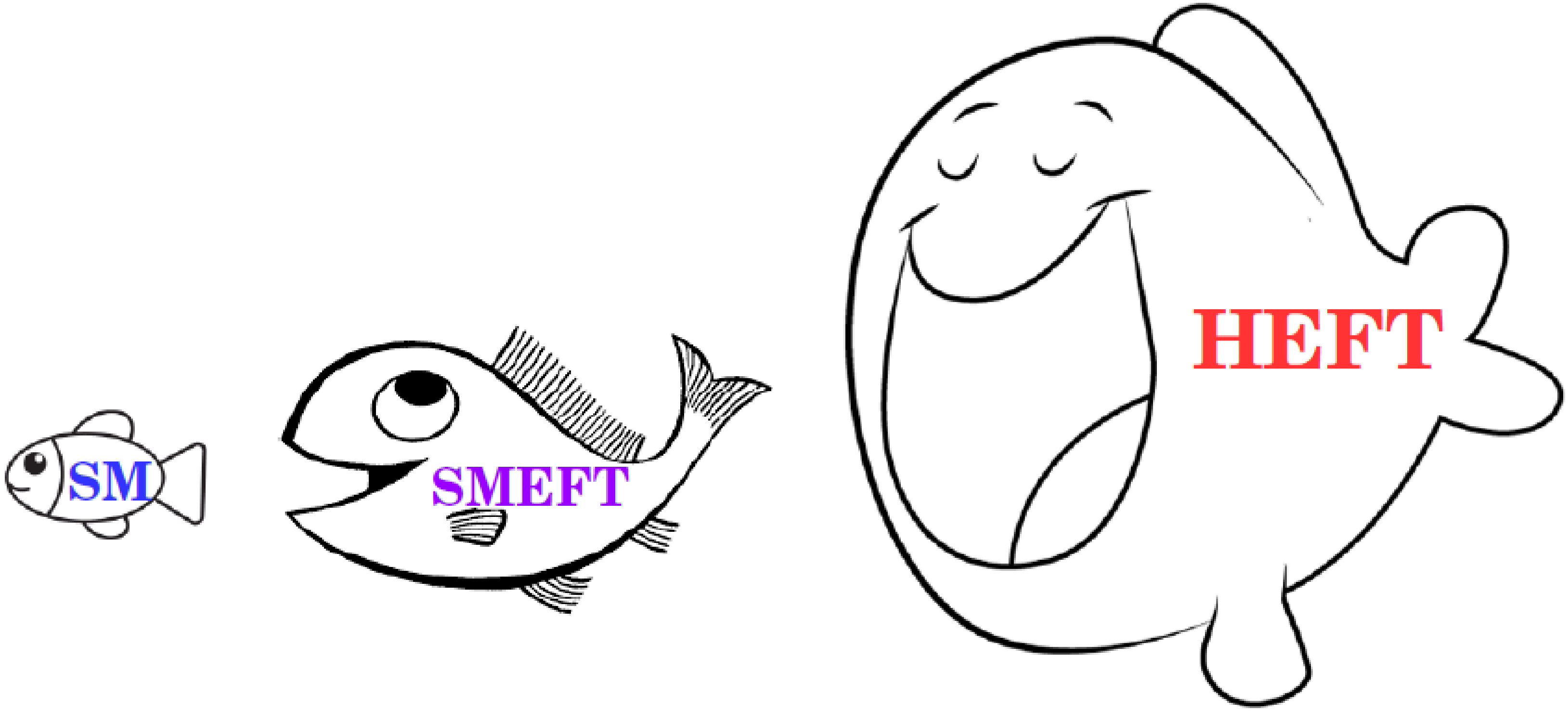
(with $c_1 = 1$, $c_{i \geq 2} = 0$ in the Standard Model).

If SMEFT applies, $\mathcal{G}(h)$ must have only odd powers of $(h - h^*)$ around the symmetric point h^*), we obtain the correlations

$$c_2 = 3c_3 = \frac{3}{2}(c_1 - 1) - \frac{1}{4}\Delta a_1 \qquad c_2 = 3c_3 \in [-0.27, 0.35]$$

$c_1 \in [0.84, 1.22]$ J. de Blas et al., JHEP 07 (2018), 048

SM vs SMEFT vs HEFT



similarities
& differences

- **Expansion** in (non-linear) HEFT: *

Finite pieces from loops (amplitude dependent) (+)

$$\mathcal{M}(2\to 2) \approx \frac{p^2}{v^2} + \left(\frac{\mathcal{F}_k(\mu)\, p^4}{v^2} - \frac{\Gamma_k\, p^4}{16\pi^2 v^2} \ln\frac{p^2}{\mu^2} + \ldots \right) + \mathcal{O}(p^6)$$

LO (tree)

NLO (tree)

suppression
$\sim 1/M^2$ + …

(heavier states)

** Catà, EPJC74 (2014) 8, 2991
** Pich,Rosell,Santos,SC, [1501.07249]; ‘forthcoming FTUAM-15-20
** Pich,Rosell and SC, JHEP 1208 (2012) 106;
PRL 110 (2013) 181801

NLO (1-loop)

Typical loop suppression
$\sim \Gamma_k/(16\pi^2 v^2)$

(non-linearity)

100% determined by $\mathcal{L}_2$

*** Guo,Ruiz-Femenia,SC, PRD92 (2015) 074005
*** Alonso,Jenkins,Manohar, PLB 754 (2016) 335-342
*** Alonso,Kanshin,Saa, PRD 97 (2018) no.3, 035010
*** Buchalla,Cata,Celis,Knecht,Krause, NPB 928 (2018) 93-106
*** Buchalla,Catà,Celis,Knecht,Krause, PRD 104 (2021) 7, 076005

- Indeed, the SM has this arrangement but with $\frac{p^2}{16\pi^2 v^2} \sim \frac{g^{(\prime)2}}{(4\pi)^2}, \frac{\lambda}{(4\pi)^2}, \frac{\lambda_f^2}{(4\pi)^2} \ll 1$; **hence**

• A history recollection on the $\mathcal{L}_{p^4}$ renormalization (1):

Higgs-less 1-loop RENORMALIZATION

(*) Herrero,Ruiz Morales, NPB 418 (1994) 431-455

Higgs-full: 1-LOOP CALCULATIONS of PARTICULAR OBSERVABLES

A small sample of 1-loop HEFT observable computations:

(x) Delgado,Dobado,Llanes-Estrada, PRL114 (2015) 22, 221803
(x) Espriu,Mescia,Yencho, PRD88 (2013) 055002
(x) Delgado,Garcia-Garcia,Herrero, JHEP 11 (2019) 065
(x) Fabbrichesi,Pinamonti(,Tonero,Urbano, PRD93 (2016) 1, 015004
(x) Corbett,Éboli,Gonzalez-Garcia, PRD 93 (2016) 1, 015005
(x) de Blas,Eberhardt,Krause, JHEP 07 (2018) 048
(x) Quezada,Dobado,SC, 2207.01458 [hep-ph]; in preparation
(x) Herrero,Morales, PRD 106 (2022) 7, 073008

• A history recollection on the $\mathcal{L}_{p^4}$ renormalization (2):

$\mathcal{O}(p^4)$ HEFT renormalization:
scalar loops
& true $\mathcal{O}(D^4)$ divergences

(*) Guo,Ruiz-Femenia,SC, PRD92 (2015) 074005

$$\mathcal{L} = \dots + \frac{v^2}{4}\mathcal{F}_C(h)\langle D_\mu U^\dagger D^\mu U\rangle$$

c_k	Operator $\mathcal{O}_k$	Γ_k	$\Gamma_{k,0}$
c_1	$\frac{1}{4}\langle f_+^{\mu\nu}f_{+\mu\nu} - f_-^{\mu\nu}f_{-\mu\nu}\rangle$	$\frac{1}{24}(\mathcal{K}^2-4)$	$-\frac{1}{6}(1-a^2)$
(c_2-c_3)	$\frac{i}{2}\langle f_+^{\mu\nu}[u_\mu,u_\nu]\rangle$	$\frac{1}{24}(\mathcal{K}^2-4)$	$-\frac{1}{6}(1-a^2)$
c_4	$\langle u_\mu u_\nu\rangle\langle u^\mu u^\nu\rangle$	$\frac{1}{96}(\mathcal{K}^2-4)^2$	$\frac{1}{6}(1-a^2)^2$
c_5	$\langle u_\mu u^\mu\rangle^2$	$\frac{1}{192}(\mathcal{K}^2-4)^2+\frac{1}{128}\mathcal{F}_C^2\Omega^2$	$\frac{1}{8}(a^2-b)^2+\frac{1}{12}(1-a^2)^2$
c_6	$\frac{1}{v^2}(\partial_\mu h)(\partial^\mu h)\langle u_\nu u^\nu\rangle$	$\frac{1}{16}\Omega(\mathcal{K}^2-4)-\frac{1}{96}\mathcal{F}_C\Omega^2$	$-\frac{1}{6}(a^2-b)(7a^2-b-6)$
c_7	$\frac{1}{v^2}(\partial_\mu h)(\partial_\nu h)\langle u^\mu u^\nu\rangle$	$\frac{1}{24}\mathcal{F}_C\Omega^2$	$\frac{2}{3}(a^2-b)^2$
c_8	$\frac{1}{v^4}(\partial_\mu h)(\partial^\mu h)(\partial_\nu h)(\partial^\nu h)$	$\frac{3}{32}\Omega^2$	$\frac{3}{2}(a^2-b)^2$
c_9	$\frac{(\partial_\mu h)}{v}\langle f_-^{\mu\nu}u_\nu\rangle$	$\frac{1}{24}\mathcal{F}'_C\Omega$	$-\frac{1}{3}a(a^2-b)$
c_{10}	$\frac{1}{2}\langle f_+^{\mu\nu}f_{+\mu\nu}+f_-^{\mu\nu}f_{-\mu\nu}\rangle$	$-\frac{1}{48}(\mathcal{K}^2+4)$	$-\frac{1}{12}(1+a^2)$

$\mathcal{O}(p^4)$ HEFT renormalization:
scalar loops
& GEOMETRIC APPROACH

A deeper understanding through geometry:
(x) Alonso,Jenkins,Manohar, PLB 754 (2016) 335-342;
PLB 756 (2016) 358-364; JHEP 08 (2016) 101

Low-energy EFT (SM + …): representations

• Higgs field representation: SMEFT vs HEFT, a matter of taste? (+)

1) Linear* (SMEFT): in terms of a doublet $\phi = (1+h/v)\, U(\omega^a) \langle\phi\rangle$

$$\mathcal{L}^{L}_{EFT} = (D_\mu\phi)^\dagger D_\mu\phi - \frac{1}{\Lambda^2}(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + \ldots$$
$$= \frac{(v+h)^2}{4}\langle (D_\mu U)^\dagger D_\mu U\rangle + \frac{1}{2}(1+P(h))\,(\partial_\mu h)^2 + \ldots$$

$$\frac{dh^{NL}}{dh^{L}} = \sqrt{1+P(h^L)}$$

$$h^{NL} = \int_0^{h^L}\sqrt{1+P(h)}\,dh$$

? ? ?

$$\frac{v^2}{2}\mathcal{F}_C(h^{NL}) = \frac{(v+h^L)^2}{2} = \phi^\dagger\phi$$

if there exists an $SU(2)_L \times SU(2)_R$ fixed point $\mathcal{F}_C(h^*)=0$ (x)

$$\mathcal{L}^{NL}_{EFT} = \frac{v^2}{4}\mathcal{F}_c(h)\langle (D_\mu U)^\dagger D_\mu U\rangle + \frac{1}{2}(\partial_\mu h)^2 + \ldots$$

$$\mathcal{F}_C(h) = 1 + \frac{2ah}{v} + \frac{bh^2}{v^2} + \mathcal{O}(h^3)$$

2) Non-linear* (HEFT or EWχL): in terms of 1 singlet h + 3 NGB in $U(\omega^a)$

(+) SC, arXiv:1710.07611 [hep-ph]; PoS EPS-HEP2017 (2017) 460
* Jenkins,Manohar,Trott, JHEP 1310 (2013) 087
* LHCHXSWG Yellow Report [1610.07922]

(x) Transformations:
Giudice,Grojean,Pomarol,Rattazzi, JHEP 0706 (2007) 045
Alonso,Jenkins,Manohar, JHEP 1608 (2016) 101

Always possible to write a SMEFT as a HEFT

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ (v + h_{\rm SMEFT}) + i\phi_3 \end{pmatrix}$$

$$\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3, h + v)$$

Change to polar-like coordinates:

$$\boldsymbol{\phi} = (1 + h/v)\boldsymbol{n} \quad \text{with } \boldsymbol{n} = (\omega_1, \omega_2, \omega_3, \sqrt{v^2 - \omega_1^2 - \omega_2^2 + \omega_3^2})$$

Generic SMEFT operators

$$\mathcal{L}_{\rm SMEFT} = \overbrace{A(|H|^2)}|\partial H|^2 + \frac{1}{2}\overbrace{B(|H|^2)}(\partial(|H|^2))^2 - V(|H|^2) + \mathcal{O}(\partial^4)$$

In polar coordinates

$$\mathcal{L}_{\text{polar}-\text{SMEFT}} = \frac{1}{2}(v+h)^2 A(h)(\partial_\mu \boldsymbol{n}\cdot\partial^\mu \boldsymbol{n}) + \frac{1}{2}\Big(A(h)+(v+h)^2 B(h)\Big)(\partial h)^2$$

$$\mathcal{L}_{\rm polar-SMEFT} = \frac{1}{2}(v+h)^2 A(h)(\partial_\mu \boldsymbol{n}\cdot\partial^\mu \boldsymbol{n}) + \frac{1}{2}\Big(A(h)+(v+h)^2 B(h)\Big)(\partial h)^2$$

Field redefinition

$$L_{\rm LO\ HEFT} = \frac{1}{2}\mathcal{F}(h)\partial_\mu\omega^i\partial^\mu\omega^j\left(\delta_{ij}+\frac{\omega^i\omega^j}{v^2-\omega^2}\right)+\frac{1}{2}\partial_\mu h\partial^\mu h$$

Identify the Flare function and canonicalize higgs kinetic term and :

$$\mathcal{F}(h_{\rm HEFT}) = \left(1+\frac{h_{\rm SMEFT}(h_{\rm HEFT})}{v}\right)^2 A(h_{\rm SMEFT})$$

$$dh_{\rm HEFT} = \sqrt{A(h_{\rm SMEFT})+(v+h_{\rm SMEFT})^2 B(h_{\rm SMEFT})}\; dh_{\rm SMEFT}$$

Always possible to find a HEFT from a given SMEFT

Relevant SMEFT at the TeV scale:

$$\mathcal{L}_{\rm SMEFT} = |\partial H|^2 + \frac{c_{H\Box}}{\Lambda^2}(H^\dagger H)\Box(H^\dagger H)$$

To polar-like coordinates:

$$\mathcal{L}_{\rm SMEFT} = \frac{1}{2}\left(1 - 2(v+h)^2\frac{c_{H\Box}}{\Lambda^2}\right)(\partial_\mu h)^2 + \frac{1}{2}(v+h)^2(\partial_\mu \boldsymbol{n}\cdot\partial^\mu \boldsymbol{n})\,.$$

Canonical Higgs kinetic term by solving:

$$h_{\rm HEFT} = \int_0^h \sqrt{1-(v+t)^2\frac{2c_{H\Box}}{\Lambda^2}}\,dt$$

Yields:

$$h = h_{\rm HEFT} + \frac{1}{3}\left(\frac{c_{H\Box}}{\Lambda^2}\right)(h^3_{\rm HEFT} + 3h^2_{\rm HEFT}v + 3h_{\rm HEFT}v^2) + \mathcal{O}\left(\frac{c^2_{H\Box}}{\Lambda^4}\right)\,.$$

The HEFT function coupling Higgses to the GB kinetic term becomes correlated:

Correlated coefficients

$$\mathcal{F}(h_{\rm HEFT}) =$$
$$1 + \left(\frac{h_{\rm HEFT}}{v}\right)\left(2 + 2\frac{c_{H\Box} v^2}{\Lambda^2}\right) + \left(\frac{h_{\rm HEFT}}{v}\right)^2\left(1 + 4\frac{c_{H\Box} v^2}{\Lambda^2}\right) +$$
$$+ \left(\frac{h_{\rm HEFT}}{v}\right)^3\left(8\frac{c_{H\Box} v^2}{3\Lambda^2}\right) + \left(\frac{h_{\rm HEFT}}{v}\right)^4\left(2\frac{c_{H\Box} v^2}{3\Lambda^2}\right).$$

Whereas in a general HEFT:

Uncorrelated coefficients

$$\mathcal{F}(h_{\rm HEFT}) = 1 + \sum_{n=1}^{\infty} a_n \left(\frac{h_{\rm HEFT}}{v}\right)^n .$$

• **In summary:** SMEFT in the *HEFT-form* looks like...

$$\mathcal{L}_{\rm SMEFT} = \frac{v^2}{4}\left(1+\frac{h_1}{v}\right)^2 \langle D_\mu U^\dagger D^\mu U\rangle + \frac{1}{2}\left(1-\frac{2c_{H\Box}(h_1+v)^2}{\Lambda^2}\right)(\partial_\mu h_1)^2 \;-\; V(h_1)$$

$$= \frac{v^2}{4}\mathcal{F}(h_1)\langle D_\mu U^\dagger D^\mu U\rangle + \frac{1}{2}(\partial_\mu h_1)^2 - V(h) - \frac{c_{H\Box}\left[(v+h_1)^3 - v^3\right]}{3\Lambda^2}V'(h_1)\,.$$

$$\mathcal{F}(h_1) = \left(1+\frac{h_1}{v}\right)^2 + \frac{2v^3 c_{H\Box}}{\Lambda^2}\left(1+\frac{h_1}{v}\right)\left(\frac{h_1^3}{3v^3}+\frac{h_1^2}{v^2}+\frac{h_1}{v}\right) + \mathcal{O}\left(\frac{c_{H\Box}^2}{\Lambda^4}\right) =$$

$$= 1 + \left(\frac{h_1}{v}\right)\left(2+2\frac{c_{H\Box}v^2}{\Lambda^2}\right) + \left(\frac{h_1}{v}\right)^2\left(1+4\frac{c_{H\Box}v^2}{\Lambda^2}\right) +$$

$$+ \left(\frac{h_1}{v}\right)^3\left(8\frac{c_{H\Box}v^2}{3\Lambda^2}\right) + \left(\frac{h_1}{v}\right)^4\left(2\frac{c_{H\Box}v^2}{3\Lambda^2}\right),$$

$$a_1 = 2a = 2\left(1+v^2\frac{c_{H\Box}}{\Lambda^2}\right), \quad a_2 = b = 1+4v^2\frac{c_{H\Box}}{\Lambda^2}, \quad a_3 = \frac{8v^2}{3}\frac{c_{H\Box}}{\Lambda^2}, \quad a_4 = \frac{2v^2}{3}\frac{c_{H\Box}}{\Lambda^2}$$

$$\mathcal{F}(h_1) = 1 + \left(\frac{h_1}{v}\right)\left(2 + 2\frac{c_{H\Box}^{(6)} v^2}{\Lambda^2} + 3\frac{(c_{H\Box}^{(6)})^2 v^4}{\Lambda^4} + 2\frac{c_{H\Box}^{(8)} v^4}{\Lambda^4}\right.$$

$$+ \left(\frac{h_1}{v}\right)^2 \left(1 + 4\frac{c_{H\Box}^{(6)} v^2}{\Lambda^2} + 12\frac{(c_{H\Box}^{(6)})^2 v^4}{\Lambda^4} + 6\frac{c_{H\Box}^{(8)}}{\Lambda^4}\right.$$

$$+ \left(\frac{h_1}{v}\right)^3 \left(8\frac{c_{H\Box}^{(6)} v^2}{3\Lambda^2} + 56\frac{(c_{H\Box}^{(6)})^2 v^4}{3\Lambda^4} + 8\frac{c_{H\Box}^{(8)} v^4}{\Lambda^4}\right)$$

Naturally extend to dim8 and further, and to quadratic terms

$$+ \left(\frac{h_1}{v}\right)^4 \left(2\frac{c_{H\Box}^{(6)} v^2}{3\Lambda^2} + 44\frac{(c_{H\Box}^{(6)})^2 v^4}{3\Lambda^4} + 6\frac{c_{H\Box}^{(8)} v^4}{\Lambda^4}\right)$$

$$+ \left(\frac{h_1}{v}\right)^5 \left(88\frac{(c_{H\Box}^{(6)})^2 v^4}{15\Lambda^4} + 12\frac{c_{H\Box}^{(8)} v^4}{5\Lambda^4}\right) +$$

$$+ \left(\frac{h_1}{v}\right)^6 \left(44\frac{(c_{H\Box}^{(6)})^2 v^4}{45\Lambda^4} + 2\frac{c_{H\Box}^{(8)} v^4}{5\Lambda^4}\right) + \mathcal{O}(\Lambda^{-6})$$

(*) Gómez-Ambrosio,Llanes-Estrada,Salas-Bernárdez,SC, PRD 106 (2022) 5, 5; arXiv: 2207.09848 [hep-ph]